Estimating Muon Lifetime

Stanley Yu (sy2751)* and Adam He (ash2223)†
PHYS 3081 E.K.A. Advanced Physics Laboratory, Columbia University
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To better characterize the behavior of subatomic particles, we estimate the mean lifetime of muons produced by cosmic rays colliding with particles in Earth's upper atmosphere. Using 1000 individually collected muon double events collected over 36 hours and 18 minutes, we find that the mean muon lifetime in a vacuum is $2.22\mu s \pm 0.0764\mu s$. The original muon lifetime measurement was corrected for the accidental events involving multiple unrelated events occurring within the same interval and for the bias resulting from the finite window size. This result is within .45% of the currently accepted mean muon lifetime. We also plot a histogram and fit the plot to an exponential decay model to demonstrate that the distribution of muon lifetimes behaves exponnentially as expected.

I. INTRODUCTION

Ultrahigh-energy cosmic rays bombard the atoms of oxygen, nitrogen, and other molecules comprising Earth's atmospheric gases, emitting gamma radiation and creating an air shower of subatomic particles including muons. Specifically, a cosmic ray proton colliding with atomic nuclei will typically produce a secondary proton or neutron alongside other energetic hadrons, mainly pions. Since pions are unstable, they may decay into other fundamental particles. Neutral pions immediately decay into two photons, generating an electromagnetic shower of electron-positron pairs and gamma rays.[2] On the other hand, the fate of charged pions depends on their energy. High-energy charged pions will interact with more atmospheric particles and create more neutral pions that propagate the electromagnetic portion of the air shower. Low-energy charged pions instead decay into muons and muon neutrinos, which survive to Earth's surface because of time dilation.

$$\pi^+ = \mu^+ + \nu \tag{1}$$

$$\pi^- = \mu^- + \nu \tag{2}$$

The muon is an elementary particle classified as a lepton in the Standard Model of particle physics and possesses a similar charge and spin to the electron but with a mass about 207 times the electron's.[4] Without relativity, muons would only survive without decaying a distance of approximately 456 ($2.197\mu s * \ln(2) * 0.9997c$) meters (as seen from the Earth). This is only enough to cover the upper region of the atmosphere, which, in total, spans about 12 kilometers. However, muons are detectable in significant amounts from the ground and even deep underground, highlighting the penetrative nature of these particles. Comparing the difference between the number of muons expected from a Newtonian perspective and that from a relativistic perspective allows for the determination of the mean muon lifetime T_0.

$$M_{Newton} = e^{-Z/T_0} \tag{3}$$

$$M_{relativity} = e^{-Z/\gamma T_0} \tag{4}$$

M is the number of muons measured at sea level, while N is the number measured in the upper atmosphere. Z is the time it takes for a muon to traverse the distance between the two in the rest frame of the Earth. The relativistic properties of muons are of great interest to scientists and were studied closely in the Rossi-Hall experiment (1941). By setting up detectors at different altitudes on a mountain in Colorado, Bruno Rossi and David B. Hall measured the muons' momentum and computed their mean lifetime, obtaining a proper lifetime of $2.4 \pm 0.3\mu s$.[5] Their result verified time dilation as predicted by special relativity, an example of the significant achievements possible through our understanding and application of muons.

Modern experiments have more precisely determined the mean muon lifetime to be $2.1969803(2.2)\mu s$ using more than $2 * 10^{12}$ recorded decays. Such observations were made possible by precisely setting up the proper equipment to construct a time-structured, low-energy muon beam directed towards a segmented plastic scintillator array.[6] These experiments capture pions to take advantage of their decay into muons, which are detected by the scintillator array. With this setup, we can measure the time from some of the muons' collision with the scintillation material and its subsequent decay into electrons, neutrinos, and antineutrinos. Then, we can quantify the number of radioactive particles N left after a given amount of time t by modeling a familiar exponential distribution:

$$N(t) = N_0 \exp(-\lambda t) \tag{5}$$

where N_0 is the number of unstable particles at $t = 0$. By definition, the exponential time constant is the reciprocal of the decay rate, $\tau = 1/\lambda$. Differentiating (5) then gives the rate of decay in terms of the mean lifetime:

$$\frac{dN}{dt} = (\frac{-1}{\tau})N_0 \exp(\frac{-t}{\tau}) \tag{6}$$

* stanley.yu@columbia.edu
† ash2223r@institution.edu

Taking the expectation of particles decaying according to this exponential distribution then gives the mean lifetime:

$$E[t] = \frac{1}{N_0}\int_0^\infty t\frac{dN}{dt}dt \tag{7}$$

$$= (\frac{-1}{\tau})\int_0^\infty t\exp(\frac{-t}{\tau})dt \tag{8}$$

Thus, to accurately determine the mean lifetime, we design an experimental procedure to record the muons' rate of decay and plot a waveform showing the total time interval from collision to decay, which we can feed into a computer program to graph and analyze.

II. METHODS

The experiment's objective is to use a scintillation detector to record the time interval between the moment the muon enters the detector and the moment it decays to an electron, among other particles. Although both muons and antimuons result from cosmic ray showers, we do not distinguish between the two because their lifetimes in a vacuum are equivalent, and, in air, the muon lifetime is only slightly shorter than the antimuon lifetime due to muon capture.

The E.K.A. Laboratory generously provides a detector constructed from a plastic scintillator and photomultiplier tube. The scintillation detector takes advantage of the small flash of light known as a scintillation that occurs when charged particles strike certain materials to produce an electrical pulse amplified by the photomultiplier tube. It consists of a scintillating material optically coupled to a photomultiplier via a light guide. The resulting electrical signal produced from the setup is then fed into other equipment for analysis. Directly from the detector, the electrical pulse generated by the muon capture and decay feeds into a discriminator, which filters out low-amplitude noise. The oscilloscope then allows us to view the resulting signal.

The following discusses additional details about the equipment and their components: the plastic scintillator, photomultiplier tube, oscilloscope, and program used for waveform analysis.

A. Plastic Scintillator

The plastic scintillator attached to the photomultiplier tube is a solution of organic scintillators in a solid plastic solvent. Organic scintillators are aromatic hydrocarbon compounds containing linked or condensed benzene-ring structures. Scintillation light in these compounds arises from transitions made by the free valence electrons of the molecules. When a muon travels to the scintillator, the ionization energy from the penetrating radiation excites both the electron and vibrational levels. When the electron relaxes, it emits a photon of light, which is guided towards the photomultiplier tube. Plastics offer an extremely fast signal with a decay constant of about 2-3 ns and a high light output.[3]

B. Photomultiplier Tube

Photomultiplier tubes (PMT) are highly sensitive electron tube devices for detecting ultraviolet, visible, and infrared light and converting it into a measurable electric current. The PMT consists of a photocathode made of photosensitive material, an optical input system with a focusing electrode, a dynode string that acts as a multiplier, and an anode. It awaits the plastic scintillator's emission of a photon of light, which causes the photocathode to emit an electorn via the photoelectric effect. This electron is directed towards the dynode string, causing an electron cascade that is collected at the anode to produce a current for measuring and analysis.[3]

C. Discriminator

A discriminator is an electornic circuit that responds only to input signals exceeding a threshold value by producing a standard logic signal. This allows us to block out low amplitude noise from our detector and act as a simple analog-to-digital converter for processing by other electronics.

D. LeCroy WaveSurfer 3034z Oscilloscope

The oscilloscope is an electronic device that visualizes changing signal voltages as a two-dimensional plot of signal versus time. The LeCroy WaveSurfer 3034z Oscilloscope also presents many options for adjusting the rate and type of data received. To record the relevant observations, we chose a threshold voltage of -1.555 V and set the interval trigger to $[200ns, 10\mu s]$, with each waveform comprising about 40,000 points, each representing a 2-tuple of time and amplitude. For the background calculation, we chose an interval trigger of $[20\mu s, 30\mu s]$. A diagram from the LeCroy WaveSurfer 3000z Manual is shown in Figure 1 describing the range of functionality and adjustments available for our data collection.

E. Waveform Analysis Program

The resulting waveform from the oscilloscope can be ported over to a computer for data analysis. To graph the data and calculate the lifetimes from our observations, we wrote a program in Python 3 to extract the lifetime from each double event recorded, plot a histogram of the data, and calculate the mean lifetime.

The software uses the popular plotting library `matplotlib` for visualizing all the data, including the

FIG. 1. View of the front of the oscilloscope from the LeCroy WaveSurfer 3000z Manual. The following components are labeled: (A) Touch screen display, which includes a signal reading, trigger level indicator, and setup dialog (B) Front panel, which includes the knobs to adjust the offset and voltage per division (C) Ground and calibration tunnels (D) USB 2.0 ports (E) Mixed signal interface (F) Channel inputs (G) Rotating / tilting feet (H) Power button

waveform and the histogram. As shown below, the function `plot_lifetime` instantiates a list of all the relevant filenames that the program will parse through to extract the muon lifetime. It calls the function `read_waveform_data`, which simply scans all the points and compares it to the given threshold value to find the interval between the two recorded signals. After compiling a list of all the lifetimes, the program invokes `matplotlib` to bin the results into 10 bins and displays the histogram to the user.

```
def plot_lifetime(data_path):
    fnames = [os.path.join(data_path, f)
        for f in os.listdir(data_path)
        if os.path.isfile(
            os.path.join(data_path, f)
    )]

    lifetimes = []
    for fn in tqdm(fnames):
        res_x, res_y =
            read_waveform_data(fn)
        lifetimes.append(
            1e6 * find_distance(
                res_x, res_y
        ))

    ...

    fig.tight_layout()
    plt.show()

    return avg(lifetimes)
```

Using this histogram, we can then manually compile and fit the graph to an exponential distribution for further analysis. The exponential model is fitted using `scipy`, which is a scientific computing library for Python, and error bars representing one standard deviation of uncertainty are also plotted on the histogram, as shown in Figure 2. The entire software codebase is available on GitHub.[7]

III. RESULTS

Over the course of approximately 36 hours and 18 minutes, we collected a total of 1000 double events for analysis. From our calculations, we determined a mean muon lifetime prior to corrections of $2.4158 \pm 0.0764 \mu s$. See Figure 2 for a histogram of the muon lifetimes with 10 bins.

We perform two corrections. The first accounts for accidental doubles, which are two unrelated events that occur within the same window. The accidental doubles rate D_{acc} can be calculated using the singles rate R multiplied by the probability of two events, $E1$ and $E2$ occuring within the same window:

$$D_{acc} = R * P[E1|E2] = R * R * \nabla t = R^2 \nabla t \tag{9}$$

where ∇t is the width of the window. The singles rate is displayed on the counter and observed to be $9.0185 \mu s^{-1}$. Then the corrected muon doubles rate, also known as the true doubles rate D_μ, is calculated by subtracting the total doubles rate from the accidental one.

$$D_\mu = D - D_{acc} \tag{10}$$

We can find the total doubles rate by dividing the number of events we collected over the total sampling interval. Using this, we can calculate the background-corrected mean muon lifetime T_μ as the following:

$$D_\mu T_\mu = DT - D_{acc} T_{acc} \tag{11}$$

$$T_\mu = \frac{DT - D_{acc} T_{acc}}{D_\mu} \tag{12}$$

where T is the measured muon lifetime and $T_a cc$ can be found by taking the median time of the window. Using our measured mean muon lifetime, we find that the background-corrected mean muon lifetime is $2.1037 \mu s$.

For our second correction, we account for the finite window size bias. Since the interval is finite, we have filtered out muons that decay either very quickly or very slowly. Although rare, these values can disproportionately affect the mean. We can find the correction factor k by taking the ratio of the expectation of the characteristic lifetime, which can be approximated using the expectation of the exponential distribution, to the expectation of the average experimental muon lifetime:

$$k = \frac{\int_0^\infty t e^{-t/T_\mu}}{\int_{t_0}^{t_1} e^{-t/T_\mu}} \tag{13}$$

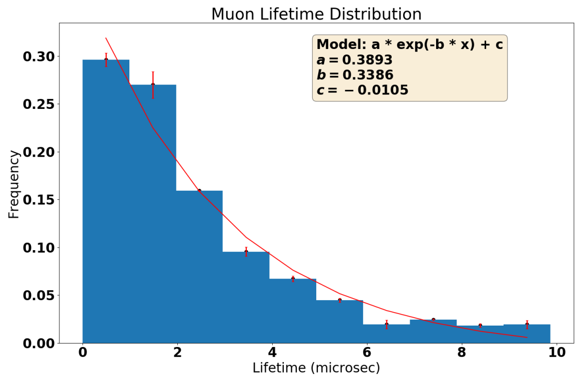

FIG. 2. Histogram of the muon lifetimes observed with a trigger interval of $[200ns, 10\mu s]$. The number of bins used is 10, and an exponential decay model is fitted to the curve, with parameters shown in the yellow box. The mean muon lifetime can be found by taking the reciprocal of the parameter b. See the appendix for the expanded figure including histograms using 20 and 30 bins.

where t_0 and t_1 are the start and end of the window. This correction factor is calculated to be 1.0569, which we multiply by our result from the first correction to get the unbiased, background-corrected mean muon lifetime of $2.2234\mu s \pm 0.0764\mu s$.

IV. DISCUSSION

Our findings determined a mean muon lifetime within about 4.7% of the accepted value. We can explain this error in a number of ways. First, it is possible for pile-ups to occur when a muon is waiting in the scintillator to decay that another stray particle may enter and "pre-empt" the muon. The rate of pile-up events D_{pu} can be calculated as the doubles rate multiplied by the probability that a singles event will occur in the time δt between the two events of the double. We can then take the worst-case scenario for long-lived events with $\delta t = 10\mu s$:

$$D_p u \leq DR\delta t \approx 6.099 * 10^{-11} \tag{14}$$

Since this is an extremely small number, even a larger number of pile-up events would drastically affect the results of the calculation, so we determined that it is not a significant correction needed to be applied. However, since the accepted value was determined using much more accurate experimental setups, it is possible for the difference between mean lifetimes to be attributed in part to pile-up events.

Another method of further reducing this error in future work is taking additional data. By increasing the number of events collected, we could arrive at a better fit for the exponential model. Additional measurements can also be done to find the total cosmic ray rate to see if the value aligns with our expectation to verify our experimental setup. This would involve turning off the interval trigger and recording as many samples as possible. With this method, we could reduce the error in our measured values, including R, to prevent error propagation.

In our second analysis method, we histogram the data by choosing a bin size of 10 and fitting the exponential decay model to the plot. The results produce a mean muon lifetime, which is found by taking $\frac{1}{b}$, to be $2.95\mu s$. This value has a significantly large error compared to the accepted value, which indicates that there are errors within our histogram method. One potential source of error that could be fixed is subtracting the background from each bin to avoid counting accidental events. We could also improve the precision by weighting the frequencies by the errors to have the exponential decay model fit better relative to the uncertainty in the data.

Looking at the histogram produced, it is possible to see that a potential source of error can come from the $[1\mu s, 2\mu s]$ bucket, which deviates further from the best-fit than other points. This could be a result of an error in the experimental apparatus, which, as described earlier, could be verified by finding the total cosmic ray rate and calculating the expectation to see if the results match with our expectations. Another source of error could be the inherent randomness in the data collected, which is likely since the decay of the muons is a probabilistic event that becomes more apparent over time. Although we can never eliminate randomness, it can be reduced with additional data-taking or repeated trials.

V. CONCLUSION

The unbiased, background-corrected mean muon lifetime determined is $2.2234\mu s \pm 0.0764\mu s$. Further corrections could be made to account for error propagation in measured values from the experimental devices as well as the sample size. The histogram produced demonstrates the expected exponential decay behavior of the muon lifetimes collected. Although the data collection experiment is somewhat simple, understanding the muon's behavior has allowed for verification of other physical phenomena. For instance, the Rossi-Hall experiment verified the relativistic effects of time dilation by calculating the then-accepted value for the muon lifetime. [5]

Similarly, the determination of an extremely accurate value for the mean muon lifetime also gives the most precise value for the Fermi coupling constant, which is central to explaining beta decay as proposed by Enrico Fermi in 1933. Recent developments have also explored the possibility of creating a muon collider, which could succeed the Large Hadron Collider and offer a 10-fold increase in energy for the creation of new particles. Understanding the behavior and lifetime of muons would allow scientists to create muon beams with significantly high luminosity. Due to their penetrative nature, muons also have other useful applications that take advantage of our knowledge of muon behavior, including studying the atomic structures of materials, catalyzing fusion, and

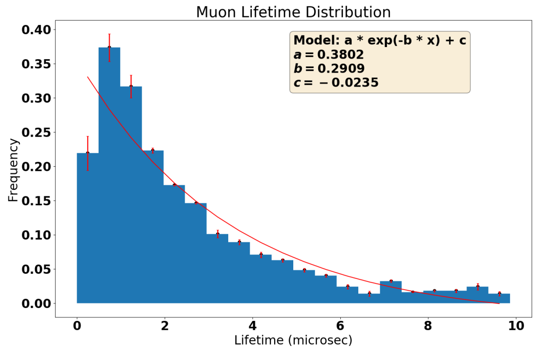

FIG. 3. Histogram of the muon lifetimes observed with a trigger interval of $[200ns, 10\mu s]$. The number of bins used is 20, and an exponential decay model is fitted to the curve, with parameters shown in the yellow box.

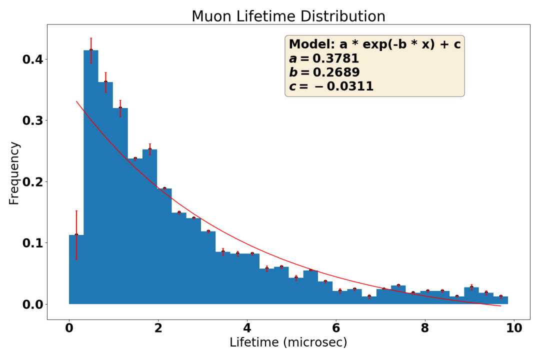

FIG. 4. Histogram of the muon lifetimes observed with a trigger interval of $[200ns, 10\mu s]$. The number of bins used is 30, and an exponential decay model is fitted to the curve, with parameters shown in the yellow box.

elucidate extremely dense materials that X-rays cannot penetrate.[1]

APPENDIX

The following is the lab instruction sheet provided by Professor May for the lab for reference: `http://www.columbia.edu/~mm21/exp_files/muon.pdf`. We also used the linked sheet for determining which corrections to make during the experiment: `http://www.columbia.edu/~mm21/exp_files/muoncorr.pdf`

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