```
clc;
clear all;
format long;
```

**QUESTION 1**

```
tol = 1e-10;
N = 200;
A = 2*eye(N) + 0.5*randn(N)/sqrt(N);
b=ones(N,1);
```

```
[x,count,res] = gmres(A,b,tol);
```

```
Number_of_iterations = count
```

```
Number_of_iterations =
    17
```

```
Relative_residual = res
```

```
Relative_residual =
     4.001613056528999e-11
```

```
x
```

```
x = 200×1
   0.384555256382748
   0.625841897275675
   0.180404306372180
   0.373943620432330
   0.517029276633928
   0.376626553293669
   0.478962197072671
   0.596480920115364
   0.548677881907637
   0.368488253780582
       ⋮
```

**QUESTION 2**

```
%%% 2b %%%%%%
```

```
q = @(z) (1/16)*(231*z.^6 - 315*z.^4 + 105*z.^2 -5);
```

```
p = @(z)(16/231)*q(z)
```

```
p = function_handle with value:
    @(z)(16/231)*q(z)
```

```
syms z
X = companion_matrix(p(z))
```

```
X = 6×6
                   0   1.000000000000000                   0                   0···
                   0                   0   1.000000000000000                   0
                   0                   0                   0   1.000000000000000
                   0                   0                   0                   0
                   0                   0                   0                   0
   0.021645021645022                   0  -0.454545454545455                   0
```

```
eiggs = eig(X)
```

```
eiggs = 6×1
  -0.932469514203151
  -0.661209386466264
   0.932469514203153
   0.661209386466265
  -0.238619186083197
   0.238619186083197
```

```
%% sorted roots
sort(eiggs)
```

```
ans = 6×1
  -0.932469514203151
  -0.661209386466264
  -0.238619186083197
   0.238619186083197
   0.661209386466265
   0.932469514203153
```

```
q(eiggs) % Verify that they are actual roots
```

ans = 6×1

$10^{-13}$ ×

```
  -0.106581410364015
   0.008881784197001
   0.044408920985006
  -0.004440892098501
  -0.006106226635438
  -0.003885780586188
```

**QUESTION 3**

```
%%% 3a %%%%%%%%
```

```
function [v,lam,count] = rqi(A,x0,ep)
N =size(A,2);
v = x0;
I = eye(N);
lam = v'*A*v;
count = 0;
while true
    p = A - lam*I;
    w = p\v;
    v_new = w/norm(w,2);
    lam_new = v_new'*A*v_new;

    nmw = norm(A*v_new - lam_new*v_new);

    count = count + 1;
    if nmw < ep
        break
    end

    v = v_new;
    lam = lam_new;

end
end
```

```
%%% 3b %%%%%%%%
```

```
%rqi(A,x0,ep)
N = 6;
a1 = -2*ones(N,1);
a2 = ones(N-1,1);
A = diag(a2,-1)+diag(a1)+diag(a2,1);
x0 =  (1/sqrt(6)).*ones(N,1);
```

```
ep = 1e-10;
[v,lam,iteration_count] = rqi(A,x0,ep)
```

```
v = 6×1
   0.231939176669697
   0.417913061018633
   0.521107370659655
   0.521107370659655
   0.417913061018633
   0.231939176669697
lam =
```

```
  -0.198062265752671
iteration_count = 
     3
```

```
% verify results
```

```
[vn,d] = eig(A)
```

```
vn = 6×6
   0.231920613924330  -0.417906505941275  -0.521120889169602   0.521120889169603 ···
  -0.417906505941275   0.521120889169603   0.231920613924330   0.231920613924330
   0.521120889169603  -0.231920613924330   0.417906505941275  -0.417906505941275
  -0.521120889169603  -0.231920613924330  -0.417906505941275  -0.417906505941275
   0.417906505941275   0.521120889169602  -0.231920613924330   0.231920613924330
  -0.231920613924330  -0.417906505941275   0.521120889169602   0.521120889169602
d = 6×6
  -3.801937735804839                   0                   0                   0 ···
                   0  -3.246979603717467                   0                   0
                   0                   0  -2.445041867912629                   0
                   0                   0                   0  -1.554958132087371
                   0                   0                   0                   0
                   0                   0                   0                   0
```

```
lam_err = abs(d(end,end) - lam)
```

```
lam_err = 
     1.557509121674627e-09
```

```
v_error = abs(vn(:,end)-(-v))
```

v_error = 6×1

$10^{-4}$ ×

```
   0.185627453672121
   0.065550773586942
   0.135185099473523
   0.135185099472412
   0.065550773586942
   0.185627453672399
```

## QUESTION 4

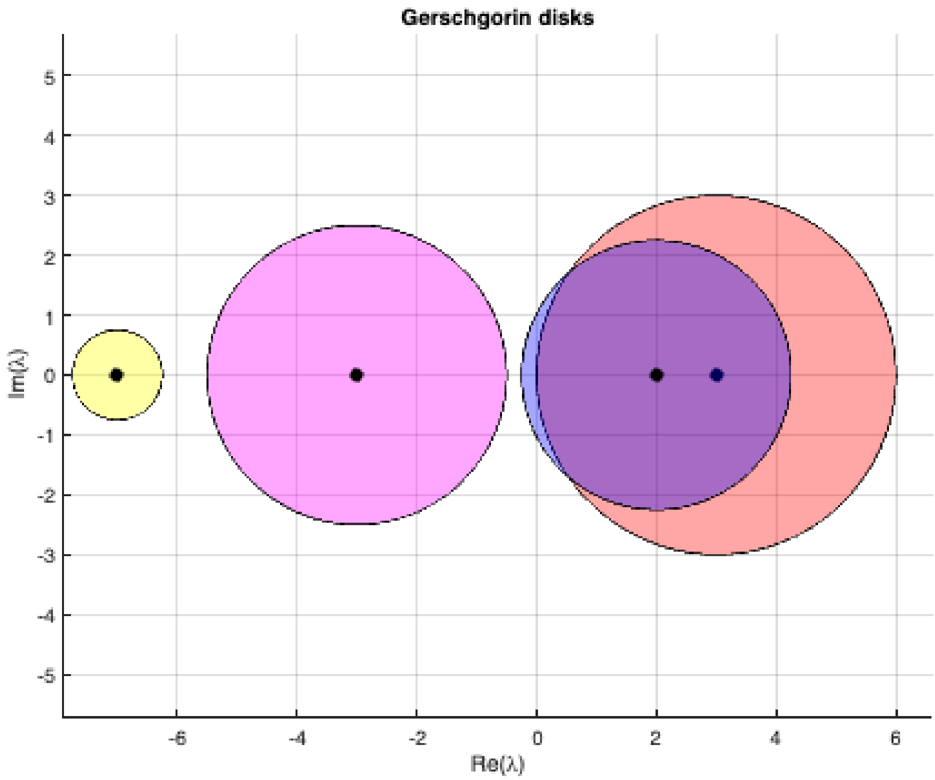
```
% set up matrix

A = [3 -1 -1 1;
    -1 2 -1/4 1 ;
    -1 -1 -3 1/2;
    -1/2 -1/4 0 -7];
```

```
%centre
c1 = A(1,1);
c2 = A(2,2);
c3 = A(3,3);
c4 = A(4,4);

%radius
r1 = 1 + 1+ 1; r2 = 1 + 1/4 +1;
r3 = 1 +1 + 1/2; r4 = 1/2 +1/4;

true_eigen_values = eig(A)
```

```
true_eigen_values = 4×1
    3.645698826149831
    1.533195532698929
   -3.279302662649521
   -6.899591696199242
```

```
close all;


figure(1)
p = nsidedpoly(1000, 'Center', [c1 0], 'Radius', r1);
plot(p, 'FaceColor', 'r')
axis equal
grid on
hold on
plot(c1,0,'k.','MarkerSize',20)
hold on
p2 = nsidedpoly(1000, 'Center', [c2 0], 'Radius', r2);
plot(p2, 'FaceColor', 'b')
axis equal
hold on
plot(c2,0,'k.','MarkerSize',20)
hold on
p3 = nsidedpoly(1000, 'Center', [c3 0], 'Radius', r3);
plot(p3, 'FaceColor', 'm')
axis equal
hold on
plot(c3,0,'k.','MarkerSize',20)
hold on
p4 = nsidedpoly(1000, 'Center', [c4 0], 'Radius', r4);
plot(p4, 'FaceColor', 'y')
axis equal
hold on
plot(c4,0,'k.','MarkerSize',20)
xlabel("Re(\lambda)"); ylabel("Im(\lambda)");
```

```
title("Gerschgorin disks")
```




According to Gerschgorin's theorem, the matrix A has eigenvalues in the union of the Gerschgorin disks $|\lambda - 3| < 3$, $|\lambda - 2| < 2.25$, $|\lambda + 3| < 2.5$ and $|\lambda + 7| < 0.75$. Since $|\lambda - 3| < 3$, $|\lambda - 2| < 2.25$ are disjoint from the other two, the Gerschgorin's theorem tells us that two of the eigenvalues must be in the two disk with the remaining two in the union of the other two disks.

**QUESTION 5**

```
%%% a
```

```
%%% b
```

```
SA = [4 7 -6 10 9;
    4 -6 4 9 5;
    -2 4 6 10 3;
    -4 6 3 -3 7;
    -1 8 0 6 2];

UA = zeros(5,5); UA(1,1) = 4; UA(2,2) = 1; UA(3,3) = 3; UA(4,4) = 9; UA(5,5) = 10;

SB = [6 6 -1 5;
```

```
    8 7 -6 -6;
    -3 3 -5 10;
    -6 -6 -9 -7];

UB = zeros(4,4); UB(1,1) = -7; UB(2,2) = -4; UB(3,3) = -3; UB(4,4) = -5;

C = [-9 10 6 -7;
    -8 -2 -5 3;
    -7 0 -6 5;
    -8 9 0 8;
    -4 -9 -4 5];
```

```
x= silvester(SA,SB,UA,UB,C) % solution
```

```
x = 5×4
   7.164257638512401  10.901274644160024  -0.358273182187601   2.112026215055829
   2.277357282833669   2.038219188954689   0.346773493076260  -0.755989882624110
   3.164640265518682   5.588511648918816  -1.286321203219333   2.534960382610655
  -2.473528353258586  -0.290393814006969  -1.064736389326259   2.264702503984480
  -1.340482562043512  -0.388265945321812  -0.949257360228529   1.772782710005899
```

```
function [x,count,R_norm] = gmres(A,b,tol)
m = size(A,1);
q = zeros(m,m);
h = zeros(m,m);
norm_b = norm(b);
q(:,1) = b/norm_b;

for n = 1:m
    v = A*q(:,n);
    for j = 1:n
        h(j,n) = q(:,j)'*v;
        v = v - h(j,n)*q(:,j);
    end
    h(n+1,n) = norm(v);
    q(:,n+1) = v/h(n+1,n);
    H = h(1:n+1,1:n);

    bb = norm_b*speye(n+1,1);
    y = H\bb;

    xn = q(:,1:n)*y;

    r = A*xn - b;

    R_norm = norm(r)/norm_b;

    count = n;

    if (norm(r)< norm_b*tol)
```

```
        break
    end
end

x = xn;
end


function X = companion_matrix(p)
coeff = coeffs(p,'all');
c = fliplr(coeff);

N =  polynomialDegree(p);
It = eye(N-1,N-1);

X = [zeros(N-1,1),It];
X = [X; -double(c(1:N))];
end


function [v,lam,count] = rqi(A,x0,ep)
N =size(A,2);
v = x0;
I = eye(N);
lam = v'*A*v;
count = 0;
while true
    p = A - lam*I;
    w = p\v;
    v_new = w/norm(w,2);
    lam_new = v_new'*A*v_new;

    nmw = norm(A*v_new - lam_new*v_new);

    count = count + 1;
    if nmw < ep
        break
    end

    v = v_new;
    lam = lam_new;

end
end

function x= silvester(SA,SB,UA,UB,C)

N = size(C,2);
n = size(C,1);
```

```
I = diag(diag(ones(n)));

invers_SA = inv(SA);
invers_SB = inv(SB);

xht = zeros(n,N);
for j = 1:N
    A = UA - UB(j,j)*I;
    cht = SA*C*invers_SB(:,j);
    xht(:,j) = A\cht;
end
x = zeros(n,N);
for i = 1:N
    x(:,i) = invers_SA*xht*SB(:,i);
end
end
```