Question 6 a

$P_1 = (A_{11}+A_{22})(B_{11}+B_{22})$

$P_2 = (A_{21}+A_{22})B_{11}$

$P_3 = A_{11}(B_{12}-B_{22})$

$P_4 = A_{22}(B_{21}-B_{11})$

$P_5 = (A_{11}+A_{12})B_{22}$

$P_6 = (A_{21}-A_{11})(B_{11}+B_{12})$

$P_7 = (A_{12}-A_{22})(B_{21}+B_{22})$

$C_{11} = P_1+P_4-P_5+P_7$

$C_{12} = P_3+P_5$

$C_{21} = P_2+P_4$

$C_{22} = P_1-P_2+P_3+P_6$

$$C_{11} = (A_{11}+A_{22})(B_{11}+B_{22}) + A_{22}(B_{21}-B_{11}) - (A_{11}+A_{12})B_{22} + (A_{12}-A_{22})(B_{21}+B_{22})$$
$$= A_{11}B_{11} + \cancel{A_{11}B_{22}} + \cancel{A_{22}B_{11}} + \cancel{A_{22}B_{22}} + \cancel{A_{22}B_{21}} - \cancel{A_{22}B_{11}} - \cancel{A_{11}B_{22}} - \cancel{A_{12}B_{22}}$$
$$+ A_{12}B_{21} + \cancel{A_{12}B_{22}} - \cancel{A_{22}B_{21}} - \cancel{A_{22}B_{22}}$$

$C_{11} = A_{11}B_{11} + A_{12}B_{21}$

$$C_{12} = A_{11}(B_{12}-B_{22}) + (A_{11}+A_{12})B_{22}$$
$$= A_{11}B_{12} - A_{11}B_{22} + A_{11}B_{22} + A_{12}B_{22}$$
$$= A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = (A_{21}+A_{22})B_{11} + A_{22}(B_{21}-B_{11})$$
$$= A_{21}B_{11} + A_{22}B_{11} + A_{22}B_{21} - A_{22}B_{11}$$
$$= A_{21}B_{11} + A_{22}B_{21}$$

$$
\begin{aligned}
C_{22} &= (A_{11}+A_{22})(B_{11}+B_{22}) - (A_{21}+A_{22})B_{11} + A_{11}(B_{12}-B_{22}) + (A_{21}-A_{11})(B_{11}+B_{12}) \\
&= \cancel{A_{11}B_{11}} + \cancel{A_{11}B_{22}} + \cancel{A_{22}B_{11}} + A_{22}B_{22} - \cancel{A_{21}B_{11}} - \cancel{A_{22}B_{11}} + \cancel{A_{11}B_{12}} - \cancel{A_{11}B_{22}} + \cancel{A_{21}B_{11}} \\
&\quad + A_{21}B_{12} - \cancel{A_{11}B_{11}} - \cancel{A_{11}B_{12}} \\
&= A_{21}B_{12} + A_{22}B_{22}
\end{aligned}
$$

6c

$$7 \cdot 7^m - 6 \cdot 4^m < 2n^3 - n^2 \quad , \quad \text{where } n = 2^m$$

$$7 \cdot 7^m - 3 \times 2^{m+1} < 2(2^{3m}) - 2^{2m}$$

$$7 \cdot 7^m - 3 \times 2^{2m+1} < 2^{3m+1} - 2^{2m} \quad — \ (*)$$

solving (*) with wolfram Alpha, we obtain $m \geq 10$. Thus, $n = 2^{10} = 1024$.

Strassen's algorithm is practical for large n. i.e $n \geq 1024$.

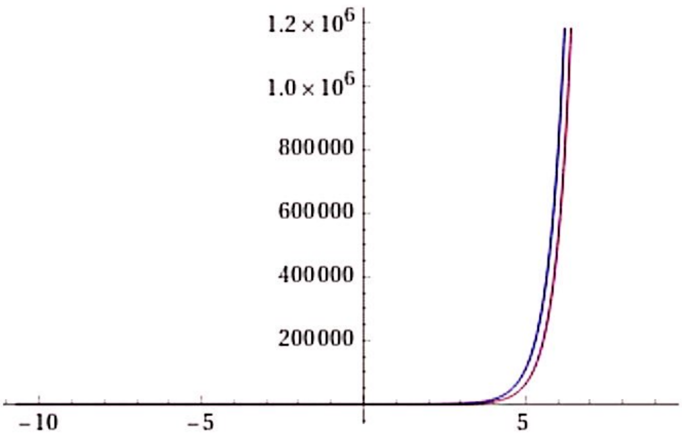
Input

$7 \times 7^m - 6 \times 4^m < 2\left(2^m\right)^3 - \left(2^m\right)^2$

Result

$7^{m+1} - 3 \times 2^{2m+1} < 2^{3m+1} - 2^{2m}$

Inequality plot



Enlarge Data Customize

Alternate forms

$7^{m+1} < 4^m\left(2^{m+1} + 5\right)$

$7^{m+1} < 5 \times 2^{2m} + 2^{3m+1}$

$7^{m+1} - 3 \times 2^{2m+1} < 2^{2m}\left(2^{m+1} - 1\right)$

Number line

Real solutions

Exact forms More digits

$m < 0$

$m > 9.35321$

6d

Base case $m = 1,\ n = 2^1$

$$7\cdot 7^m - 6\cdot 4^m$$
$$7\cdot 7^1 - 6\cdot 4^1 = 25$$

By Analysing the Strassen's algorithm for the case of a 2x2 matrix, we indeed observe that the FLOPs count is 25. Hence we conclude that the base case is true.

Inductive Hypothesis $(m=K)$ Assume the statement is true for $m=K$

$$7\cdot 7^K - 6\times 4^K = A \quad - \circledast$$

Inductive Step $(m=K+1)$

$$7\cdot 7^{K+1} - 6\cdot 4^{K+1}$$
$$= 7(7\cdot 7^K) - 6\cdot 4^{K+1}$$

from $*$ $7\cdot 7^K = A + 6\cdot 4^K$

$$= 7(A + 6\cdot 4^K) - 4\times 6\cdot 4^K$$
$$= 7A + 6\cdot 4^K(7-4)$$
$$= 7A + 3\cdot 6\cdot 4^K$$
$$= 7(7\cdot 7^K - 6\cdot 4^K) + 3\cdot 6\cdot 4^K$$
$$= 7^{K+1} + (3-7)\, 6\cdot 4^K$$
$$= 7^{K+1} - 4\cdot 6\cdot 4^K = 7^{K+1} - 6\cdot 4^{K+1}$$

Therefore, the total arithmetic operation count for Strassen's algorithm is $7\cdot 7^m - 6\cdot 4^m$ for $\forall\, m \geq 1$