Project 4 **Last name:** _____

First name: _____

Consider the boundary value problem

$$\begin{cases} -\dfrac{d}{dt}\left(\sin(t)\dfrac{du}{dt}\right) + 2\sin(t)u(t) = 2\sin(2t) \\ u(0) = 1, \quad u(\pi) = -1, \end{cases} \tag{1}$$

on the interval $[0, \pi]$. Use equally spaced points $t_i = ih$, $i = 0, 1, 2, \ldots, n$, where $h = \pi/n$, $n \geq 2$, the piecewise finite element basis hat functions

$$\varphi_i(t) = \begin{cases} \dfrac{t - t_{i-1}}{h}, & \text{for} \quad t_{i-1} \leq t \leq t_i, \\ \dfrac{t_{i+1} - t}{h}, & \text{for} \quad t_i \leq t \leq t_{i+1}, \\ 0, & \text{for} \quad t \leq t_{i-1} \ \text{ or } \ t \geq t_{i+1}, \end{cases}$$

for $i = 1, 2, 3, \ldots, n-1$,

$$\psi(t) = \varphi_0(t) - \varphi_n(t),$$

where

$$\varphi_0(t) = \begin{cases} \dfrac{t_1 - t}{h}, & \text{for} \quad t_0 \leq t \leq t_1, \\ 0, & \text{for} \quad t \geq t_1, \end{cases} \qquad \varphi_n(t) = \begin{cases} 0, & \text{for} \quad t \leq t_{n-1}, \\ \dfrac{t - t_{n-1}}{h}, & \text{for} \quad t_{n-1} \leq t \leq t_n, \end{cases}$$

and $f(t) = 2\sin(2t)$ to derive approximations to $\mathcal{A}(\varphi_{i-1}, \varphi_i)$, $\mathcal{A}(\varphi_i, \varphi_i)$, and $\langle f, \varphi_i \rangle - \mathcal{A}(\psi, \varphi_i)$ for the coefficients c_1, c_2, …, c_{n-1} of the Galerkin approximations

$$u^h(t) = \psi(t) + \sum_{i=1}^{n-1} c_i \varphi_i(t) \tag{2}$$

to the solution u of (1).

Build your own Matlab files to solve the resulting system of linear equations for any $n \geq 2$ and graphically illustrate $u^h(t)$ versus t over the interval $[0, \pi]$. Write supporting documentation describing each part of your files, what each part does, and how the parts work together.

Submit the following items

- derivation of $a_{i-1,i} \approx \mathcal{A}(\varphi_{i-1}, \varphi_i)$, $a_{i,i} \approx \mathcal{A}(\varphi_i, \varphi_i)$, and $b_i \approx \langle f, \varphi_i \rangle - \mathcal{A}(\psi, \varphi_i)$
- all of your Matlab files needed to compute and graphically illustrate the approximate solution $u^h(t)$ defined by (2),
- a figure illustrating three curves of $u^h(t)$ versus t for $0 \leq t \leq \pi$, generated by your files with three different values of $n = 2, 4, 100$ (each value of n corresponds to a different curve); label each curve by the corresponding value of n,
- supporting documentation, described above.