Project 1 **Last name:** _____

First name: _____

Consider a population of foxes (predators) and rabbits (prey). The foxes kill their prey, that is, rabbits, for food. Let F_n and R_n be the respective population of foxes and rabbits at the end of year n. We try to formulate a mathematical model to examine the long-term behavior of the two species under a few assumptions. We assume that

- Rabbits are the only source of food for the foxes and the foxes are the only predators for rabbits.
- The rabbit population will grow if there are no foxes and without the rabbit population, the foxes will die out.
- The rate at which the population of foxes grows increases with the presence of the population of rabbits and the rate at which the population of rabbits grows decreases with the presence of the population of foxes.

Under these assumptions, the dynamic model for this scenario is given by

$$\begin{aligned} \Delta F_n &= F_{n+1} - F_n = -\alpha F_n + \beta R_n \\ \Delta R_n &= R_{n+1} - R_n = \gamma R_n - \delta F_n, \end{aligned} \tag{1}$$

where ΔF_n and ΔR_n are the rates of change in the populations of foxes and rabbits, respectively and α, β, γ, and δ are positive constants, $0 < \alpha, \gamma < 1$. Here, α is the rate at which the foxes die if no rabbits are available for food and β is the rate at which the population of foxes grows when the food (rabbits) is available. Similarly, the population of rabbits grows at a rate γ when no foxes are around and decreases at a rate δ in the presence of the population of foxes.

Build your own Matlab file to compute and graphically illustrate the solutions F_n and R_n, where $n = 1, 2, \ldots, N$ and N, F_1, and R_1 are given. For the graphical illustration, use the horizontal axis for n over the interval from 1 to 30 and the vertical axis for F_n and R_n. Write supporting documentation describing each part of your file (insert comments, beginning with the % sign, to describe each part of your file).

Submit the following items

- all of your Matlab files needed to compute and graphically illustrate F_n and R_n defined by (1),
- a figure presenting four subplots:
  1. subplot(2,2,1) illustrating F_n and R_n computed with the initial values $F_1 = 500$, $R_1 = 200$ and the model parameters $\alpha = 0.5$, $\beta = 0.4$, $\gamma = 0.1$, $\delta = 0.17$
  2. subplot(2,2,2) illustrating F_n and R_n computed with the initial values $F_1 = 500$, $R_1 = 200$ and the model parameters $\alpha = 0.5$, $\beta = 0.4$, $\gamma = 0.1$, $\delta = 0.05$
  3. subplot(2,2,3) illustrating F_n and R_n computed with the initial values $F_1 = 10$, $R_1 = 200$ and the model parameters $\alpha = 0.5$, $\beta = 0.4$, $\gamma = 0.1$, $\delta = 0.05$
  4. subplot(2,2,4) illustrating F_n and R_n computed with the initial values $F_1 = 2300$, $R_1 = 200$ and the model parameters $\alpha = 0.5$, $\beta = 0.4$, $\gamma = 0.1$, $\delta = 0.05$
- supporting documentation, described above.