$$\begin{cases} -\frac{d}{dt}\left(\sin(t)\frac{du}{dt}\right) + 2\sin(t)u(t) = 2\sin(2t) \\ u(0) = 1, \quad u(\pi) = -1 \end{cases}$$

$$\psi(t) = \frac{B-A}{b-a}(t-a) + A = \frac{-1-1}{\pi}(t-0) + 1 = \frac{-2t}{\pi} + 1$$

$p(t) = \sin(t)$; $r(t) = 2\sin(t)$; $f(t) = 2\sin(2t)$

ⓐ $$A(\varphi_i, \varphi_i) = \int_0^{\pi} p(t)(\varphi_i'(t))^2 dt + \int_0^{\pi} r(t)(\varphi_i(t))^2 dt = a_{ii}$$

$$= \int_{t_{i-1}}^{t_i} \sin(t)(\underbrace{\varphi_i'(t)}_{1/h})^2 dt + \int_{t_i}^{t_{i+1}} \sin(t)(\underbrace{\varphi_i'(t)}_{-1/h})^2 dt + 2\int_{t_{i-1}}^{t_i} \sin(t)(\varphi_i(t))^2 dt$$

$$+ 2\int_{t_i}^{t_{i+1}} \sin(t)(\varphi_i(t))^2 dt$$

$$\approx 1/h \sin(t_{i-1/2}) + 1/h \sin(t_{i+1/2}) + \frac{2h}{2}\left[\sin(t_{i-1})(\overbrace{\varphi_i(t_{i-1})}^{0})^2 + \sin(t_i)(\overbrace{\varphi_i(t_i)}^{1})^2\right]$$

$$+ \frac{2h}{2}\left[\sin(t_i)(\overbrace{\varphi_i(t_i)}^{1})^2 + \sin(t_{i+1})(\overbrace{\varphi_i(t_{i+1})}^{0})^2\right]$$

$$A(\varphi_i, \varphi_i) = 1/h \sin(t_{i-1/2}) + 1/h \sin(t_{i+1/2}) + 2h\sin(t_i) = a_{ii}$$

ⓑ $$A(\varphi_{i-1}, \varphi_i) = A(\varphi_i, \varphi_{i-1}) = \int_0^{\pi} p(t)\varphi_{i-1}'(t)\varphi_i'(t) dt + \int_0^{\pi} r(t)\varphi_i(t)\varphi_{i-1}(t) dt$$

$$= \int_{t_{i-1}}^{t_i} \sin(t)\varphi_{i-1}'(t)\varphi_i'(t) dt + \int_{t_{i-1}}^{t_i} 2\sin(t)\varphi_{i-1}(t)\varphi_i(t) dt$$

$$\approx \frac{-1}{h^2}\sin(t_{i-1/2})\cdot h + \frac{2h}{2}\left[\sin(t)\overbrace{\varphi_{i-1}(t_{i-1})}^{=1}\overbrace{\varphi_i(t_{i-1})}^{=0} + \sin(t)\overbrace{\varphi_{i-1}(t_i)}^{=0}\overbrace{\varphi_i(t_i)}^{=1}\right]$$

$$A(\varphi_{i-1},\varphi_i) \approx \frac{-1}{h^2}\sin(t_{i-1/2})\cdot h + \frac{2h}{2}\Big[\sin(t_{i-1})\underbrace{\varphi_{i-1}(t_{i-1})}_{=1}\underbrace{\varphi_i(t_{i-1})}_{=0}\Big] + \frac{2h}{2}\Big[\sin(t_i)\underbrace{\varphi_{i-1}(t_i)}_{=0}\underbrace{\varphi_i(t_i)}_{=1}\Big]$$

$$A(\varphi_{i-1},\varphi_i) = \frac{-1}{h}\sin(t_{i-1/2}) = a_{i-1,i} = a_{i,i-1}$$

ⓒ $b_i = \langle f,\varphi_i\rangle - A(\varphi,\varphi_i)$

$$\langle f,\varphi_i\rangle = \int_0^{\pi} f(t)\varphi_i(t)\,dt = \int_{t_{i-1}}^{t_i} f(t)\varphi_i(t)\,dt + \int_{t_i}^{t_{i+1}} f(t)\varphi_i(t)\,dt$$

$$\approx \frac{2h}{2}\Big[\sin(2t_{i-1})\underbrace{\varphi_i(t_{i-1})}_{=0}+\sin(2t_i)\underbrace{\varphi_i(t_i)}_{=1}\Big] + \frac{2h}{2}\Big[\sin(2t_i)\underbrace{\varphi_i(t_i)}_{=1}+\sin(2t_{i+1})\underbrace{\varphi_i(t_{i+1})}_{=0}\Big]$$

$$\langle f,\varphi_i\rangle = 2h\sin(2t_i)$$

$$A(\varphi,\varphi_i) = \underbrace{\int_0^{\pi}\sin(t)\varphi'(t)\varphi_i'(t)\,dt}_{*} + \underbrace{\int_0^{\pi} 2\sin(t)\varphi(t)\varphi_i(t)\,dt}_{**} \quad \text{— ①}$$

from *

$$\int_0^{\pi} \sin(t)\varphi'(t)\varphi_i'(t)\,dt = \int_{t_{i-1}}^{t_i} \sin(t)\left(-\frac{2}{\pi}\right)\cdot \frac{1}{h}\,dt + \int_{t_i}^{t_i} \sin(t)\left(-\frac{2}{\pi}\right)\left(-\frac{1}{h}\right)dt$$

$$\approx \frac{h}{2}\cdot\frac{1}{h}\left(\frac{-2}{\pi}\right)\left[\sin(t_i)+\sin(t_{i-1})\right] + \frac{h}{2}\left(\frac{-1}{h}\right)\left(\frac{-2}{\pi}\right)\left[\sin(t_{i+1})+\sin(t_i)\right]$$

$$= \frac{1}{\pi}\left[\sin(t_{i+1})-\sin(t_{i-1})\right]$$

from **

$$\int_0^{\pi} 2\sin(t)\varphi(t)\varphi_i(t)\,dt = \int_{t_{i-1}}^{t_i} 2\sin(t)\varphi(t)\varphi_i(t)\,dt + \int_{t_i}^{t_{i+1}} 2\sin(t)\varphi(t)\varphi_i(t)\,dt$$

$$\approx \frac{h}{2}\left[2\sin(t_i)\varphi_i \overbrace{\varphi_i(t_i)}^{=1} + 2\sin(t_{i+1})\varphi_{i+1}\overbrace{\varphi_i(t_{i+1})}^{=0}\right] + \frac{h}{2}\left[2\sin(t_i)\varphi_i\left(\overbrace{\varphi_i(t_i)}^{=1}\right) + 2\sin(t_{i-1})\varphi_{i-1}\underbrace{\varphi_i(t_{i-1})}_{=0}\right]$$

$$\int_0^{\pi} 2\sin(t)\varphi(t)\varphi_i(t)\,dt = 2h\sin(t_i)\varphi_i$$

$$A(\varphi,\varphi_i) = \frac{1}{\pi}\left[\sin(t_{i+1})-\sin(t_{i-1})\right] + 2h\sin(t_i)\varphi_i$$

substituting * and ** into C

$$b_i = \langle f,\varphi_i\rangle - A(\varphi,\varphi_i)$$

$$\cancel{= \frac{1}{\pi}\left(\sin(t_{i+1}) - \sin(t_{i-1})\right)}$$

$$b_i = 2h\sin(2t_i) + \frac{1}{\pi}\left[\sin(t_{i-1}) - \sin(t_{i+1})\right] - 2h\sin(t_i)\varphi_i$$

$$\varphi_i = -\frac{2}{\pi}t_i + 1$$

$$b_i = 2h\sin(2t_i) + \frac{1}{\pi}\left(\sin(t_{i-1}) - \sin(t_{i+1})\right) + \left(\frac{4ht_i}{\pi} - 2h\right)\sin(t_i)$$