Project 2 **Last name:** ______________________________

First name: ______________________________

A patient is injected with a dose d of a certain drug regularly, each day for 2 weeks. After each injection, the drug vanishes from the body according to the rule

$$\frac{dM}{dt} = -kM, \tag{1}$$

where $M = M(t)$ is the total mass of the drug in the body at time t measured in days and k is a constant of proportionality.

After the first dose d is injected, the total mass of the drug in the body during the first day is given by $M_1(t)$ such that $M_1(0) = d$, where $M_1(t)$ solves equation (1) for $0 \leq t < 1$. To maintain an appropriate amount of the drug in the body, the second dose d is injected at $t = 1$. The mass of the drug in the body during the second day is given by $M_2(t)$ such that $M_2(1) = M_1(1) + d$, where $M_1(1)$ is the residue of the first dose, and $M_2(t)$ solves equation (1) for $1 \leq t < 2$.

This process repeats itself day after day, meaning that the drug in the body during the n-th day is given by $M_n(t)$ such that $M_n(n-1) = M_{n-1}(n-1) + d$, where $M_{n-1}(n-1)$ is the residue of the dose from the previous day, and $M_n(t)$ solves equation (1) for $n - 1 \leq t < n$, where $n = 2, 3, 4, \ldots, 14$.

Apply the numerical method

$$y_{i+1} = y_i + \frac{1}{2}h\Big(f(t_{i+1}, y_{i+1}) + f(t_i, y_i)\Big)$$

to build your own Matlab files to compute and graphically illustrate the total mass

$$M_T(t) = M_n(t), \quad n - 1 \leq t < n, \tag{2}$$

of the drug in the body for 2 weeks (that is, for $n = 1, 2, 3, \ldots, 14$) with an arbitrary prescribed dose d. For the computations, use a suitable step size h of your choice. For the graphical illustration, use the horizontal axis for time t over the interval from 0 to 14 and the vertical axis for the total mass of the drug. Write supporting documentation describing each part of your files, what each part does, and how the parts work together.

Submit the following items

- all of your Matlab files needed to compute and graphically illustrate $M_T(t)$ defined by (2),
- a discussion on how you chose the step size h in order to compute the total mass $M_T(t)$ for $0 \leq t \leq 14$,
- a figure illustrating five curves of $M_T(t)$ versus t for $0 \leq t \leq 14$, generated by your files with $k = 5$ and five different values of $d = 1, 2, 3, 4, 5$ (each value of d corresponds to a different curve); label each curve by the corresponding value of d,
- supporting documentation, described above.