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Course: Probability and Statistics	Date: November 14, 2020

Question 1

(1) The expression for the events A, B happening and a third event C not happening is the intercept of A, B and the intercept of $C's$ complement $i.e$ $A \cap B \cap \bar{C}$.

(2) The expression for exactly two events amongst the 3 events to occur is given as; $\left(A \cap B \cap \bar{C}\right) \cup \left(A \cap \bar{B} \cap C\right) \cup \left(\bar{A} \cap \cap B \cap C\right)$

(3) P(E) $=\left(A \cap B \cap \bar{C}\right)$, This can be interpreted as the Probability of $A \cap B$ without $(A \cap B \cap C)$

$$\begin{aligned} P(E) &= P\left(A \cap B\right) - P\left(A \cap B \cap C\right) \\ &= 0.15 - 0.08 \\ &= 0.07 \end{aligned}$$

$$\begin{aligned} P\left(F\right) &= \left(A \cap B \cap \bar{C}\right) \cup \left(A \cap \bar{B} \cap C\right) \cup \left(\bar{A} \cap B \cap C\right) \\ &= P\left(A \cap B\right) - P\left(A \cap B \cap C\right) + P\left(A \cap C\right) - P\left(A \cap B \cap C\right) + P\left(B \cap C\right) - P\left(A \cap B \cap C\right) \\ &= (0.15 - 0.08) + ((0.2 - 0.08)) + (0.15 - 0.08) \\ &= 0.26 \end{aligned}$$

Question 2

(1)

$$\begin{aligned} A_1 &= \{aaa, abc, acb\} \\ A_2 &= \{aaa, bac, cab\} \\ A_3 &= \{aaa, bca, cba\} \end{aligned}$$

(2) No

(3) Yes

Question 3

(1) The probability of picking a defective item is:
$P(D|M_1) = 0.06, \quad P(D|M_2) = 0.05, \quad P(D|M_3) = 0.08, \quad P(D|M_1) = 0.08, \quad P(M_1) = 0.2, \quad P(M_2) = 0.2, \quad P(M_3) = 0.3, \quad P(M_3) = 0.3.$

Applying Baye's theorem, we can obtain the probability of picking a defective item $i.e \quad P(D)$.

$$\begin{aligned} P(D) &= P(D|M_1)\,P(M_1) + P(D|M_2)\,P(M_2) + P(D|M_3)\,P(M_3) + P(D|M_4)\,P(M_4) \\ &= (0.06)(0.2) + (0.05)(0.2) + (0.08)(0.3) + (0.08)(0.3) \\ &= 0.07 \end{aligned}$$

(2)

$$\begin{aligned} P(M_2|D) &= \frac{P(M_2 \cap D)}{P(D)} \\ &= \frac{0.2 \times 0.05}{0.07} \\ &= \frac{1}{7} \end{aligned}$$

Question 4

Let $T_1 = \frac{1}{3}$ represent the probability with which the first person succeeds and $R_1 = \frac{1}{4}$ represent the probability of the second person's success. The probability of person 1 succeeding before 2 can be obtained as follows;

Let T represent the required probability, This probability can be obtained from the countability theorem as;

$$P(T) = \sum_{i=0}^{\infty} P(T_i)$$

$P\left(T_1 = \frac{1}{3}\right)$, probability $T_2 = \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)$, this occurs when both person 1 and 2 fails and then on the second trial person 1 succeeds, $T_3 = \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$, we can can obtain $T_4, T_5, ..., T_n$ by multipying the subsequent terms by $\frac{1}{2}$. Thus the required probability is;

$$\begin{aligned} P(T) &= \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^n \\ &= \left(\frac{1}{3}\right)\sum_{i=0}^{\infty}\left(\frac{1}{2}\right)^n \qquad\qquad when \quad n = 1, \end{aligned}$$

Recall that, $\sum_{i=0}^{\infty} = \frac{1}{1-x}, \qquad |x| < 0$, where $x = \frac{1}{2}$ in this case. We can then apply this condition to obtain our solution as;

$$\begin{aligned} P\left(T\right) &= \left(\frac{1}{3}\right)\sum_{i=0}^{\infty}\left(\frac{1}{1-\frac{1}{2}}\right)^{1} \\ &= \frac{2}{3} \end{aligned}$$