AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

(AIMS RWANDA, KIGALI)

Name: Akor stanley
Course: Physical Problem Solving

Assignment Number: 2
Date: November 13, 2020

Question 1

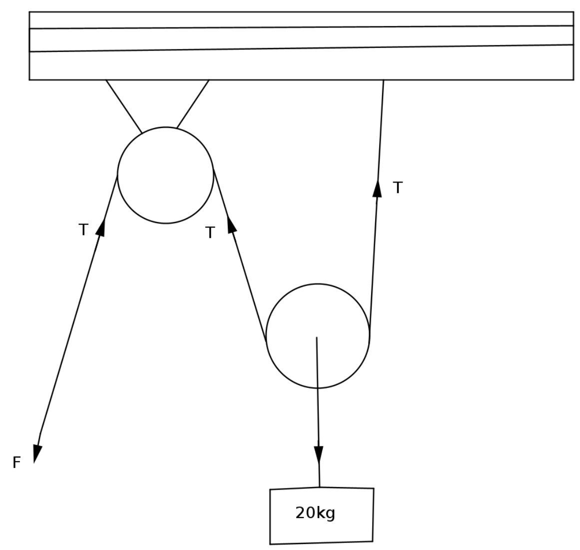

Figure 1: Force Distribution on The Canister

(a) The equation of the forces acting on the mass is given by;

$$mg - 2T = 0 \tag{1}$$
$$T - F = 0 \tag{2}$$

Where T, a, g, are defined as the tension on the string,acceleration of the body and acceleration due to gravity respectively. Since speed of the system is constant, its acceleration will be zero. Solving equation (1) and (2) above and subsitituing the values of mass=20kg and $g = 9.8ms^{-2}$, we obatin:

$$\begin{aligned} F_{net} &= \frac{mg}{2} \\ &= \frac{20 \times 9.8}{2} \\ &= 98N \end{aligned}$$

(b) Distance to pull the cord equals $2 \times 10cm = 20cm$.

(c) The work done by pulling the cord is the product of the net force in the system and the distance moved by it.

$$\begin{aligned} W &= F \times d \\ &= 98 \times 0.2 \\ &= 19.6J \end{aligned}$$

(d) The net work done on the carnister is given by;

$$\begin{aligned} W &= -mgh \\ &= -20 \times 9.8 \times 0.1 \\ &= -19.6J \end{aligned}$$

The negative sign indicates that the direction of gravity is opposite to the displacement of the carnister.

(e) The net force exerted on the ceiling is the sum of all the forces acting on the ceiling.

$$F_{net} = mg + F \tag{3}$$

But $mg = 2T$ and $F = T$;

$$\begin{aligned} F_{net} &= 3T \\ &= 3 \times 98 \\ &= 294N \end{aligned}$$

Question 2

$m_A = 10kg$
$m_B = 3kg$
$\mu_s = 0.56$
$\mu_k = 0.25$
$g = 9.8ms^{-2}$
$\theta = 40°$

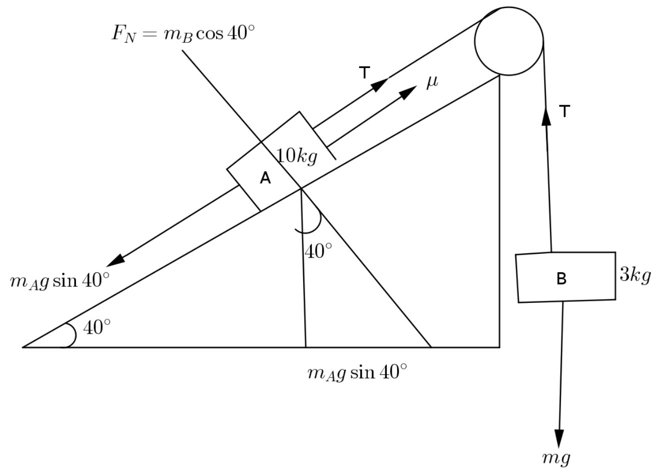

Figure 2: Force Distribution on The Blocks

(a) The component of the weight of A along the incline is given by;

$$\begin{aligned}\sum F_x &= mg\sin\theta \\ &= 10 \times 9.8 \times \sin 40° \\ &= 62.993N\end{aligned}$$

The component of the weight of A perpendicular to the incline is given by;

$$\begin{aligned}\sum F_y &= mg\cos\theta \\ &= 10 \times 9.8 \times \cos 40^\circ \\ &= 75.072N\end{aligned}$$

(b) The maximum static friction that can act between A and the incline is given by;

$$\begin{aligned}F_s^{max} &= \mu_s mg\cos\theta \\ &= 0.56 \times 10 \times 9.8 \times \cos 40^\circ \\ &= 42.040N\end{aligned}$$

(c) In order to determine the acceleration of the system, we shall consider the net force acting on the two bodies.

$$m_A g\sin\theta - T = 10 \times a \tag{4}$$
$$-m_B g + T = 3 \times a \tag{5}$$

Solving equations (4) and (5) simultaneously, we obtain the acceleration formular.

$$a = \frac{m_A \sin 40^\circ - m_B g}{13}$$

Substituting the values of $m_A = 10kg, m_B = 3kg, \theta = 40^\circ$, $g = 9.8ms^{-2}$ into the formular, we shall obtain the acceleration as $2.584ms^{-2}$.

(d) To determine if body A will be at rest or in motion, we shall calculate the acceleration of the system and then draw a conclusion from there.

$$m_A g\sin 40^\circ - \mu_s m_A g\cos 40^\circ - T = 10 \times a \tag{6}$$
$$T - m_B g = 3 \times a \tag{7}$$

Solving equations (6) and (7) simultaneously, we obtain the acceleration equation.

$$\begin{aligned}a &= \frac{m_A g\sin 40^\circ - \mu_s m_A g\cos 40^\circ - m_B g}{13} \\ &= \frac{10 \times 9.8\sin 40^\circ - 0.56 \times 10 \times 9.8\cos 40^\circ - 3 \times 9.8}{13} \\ &= -0.649ms^{-2}\end{aligned}$$

From our calculation, the acceleration of body A is $-0.649ms^{-2}$, this means that body A will be accelerated upwards towards body B.

(e) If body A was moving upwards,we can generate two equations to describe the forces on body A ;

$$T - mg\sin 40^\circ - \mu_k mg\cos 40^\circ = m_A a \tag{8}$$
$$-T + m_B g = m_B a \tag{9}$$

Solving equations (8) and (9) simultaneously, we obtain;

$$a = \frac{m_B g - m_A g \sin 40^\circ - \mu_k m_A g \cos 40^\circ}{m_A + m_B} \tag{10}$$

Substituting the values of m_A, m_B, μ_k and g into equation (10) we obtain an acceleration of -4.027ms^{-2}.

(f) If the system started with A moving down the inclined plane, we can generate two equations to describe the system with respect to the movement of body A.

$$m_A g \sin 40^\circ - \mu_k m_A g \cos 40^\circ - T = m_A a \tag{11}$$

$$T - m_B g = m_B a \tag{12}$$

Solving equations (11) and (12) simultaneously, we obtain;

$$a = \frac{m_A g \sin 40^\circ - m_B g - \mu_k m_A g \cos 40^\circ}{m_A + m_B} \tag{13}$$

Substituting the values of m_A, m_B ,μ_k and g into equation (13) we obtain an acceleration of 1.140ms^{-2}.

Question 3

(a) The sping constant can be obtained by taking the sum of forces acting on the string, two forces act on the string, these are the gravitational force mg acting downwards and the reverse tension force T acting upwards, these two forces are in equilibrium.

$$T = mg \tag{14}$$

From Hooke's law, the applied force is given as $F = k \times x$, equating the gravitational force and the Hooke's force, we obtain;

$$k \times x = mg \tag{15}$$

$$\begin{aligned} k &= \frac{mg}{x} \\ &= \frac{0.08 \times 9.8}{0.04} \\ &= 19.6 Nm^{-1} \end{aligned}$$

(b) When the block is compressed by an additional 0.1m, the total compression is 0.14m,The elastic potential energy of the string is given by;

$$U = \frac{kx^2}{2} \tag{16}$$

$$\begin{aligned} U &= \frac{19.6 \times 0.14 \times 0.14}{2} \\ &= 0.192 J \end{aligned}$$

(c) To determine how high the spring will rise, we equate the elastic potential energy to the gravitational potential enery, that is $U_e = U_g$,we obtained $U_e = 0.192J$.

$$\begin{aligned} mgh &= 0.192 \\ h &= \frac{0.192}{9.8 \times 0.08} \\ &= 0.244m \end{aligned}$$