Project 3 **Last name:** _____

First name: _____

An initial influenza A infection attacks the upper respiratory tract and upper bronchial tubes. In certain cases, it may develop uncontrollably, attacking the lower lungs. The system of differential equations governing the spread of influenza A virus within an individual host can be written in the form (Beauchemin and Handel, Public Health 2011)

$$\begin{cases} \dfrac{dT}{dt} &= -\beta TV, \\ \dfrac{dI}{dt} &= \beta TV - \delta I, \\ \dfrac{dV}{dt} &= pI - cV, \end{cases} \tag{1}$$

where β, δ, p, c are nonnegative parameters, t represents time, $T = T(t)$ stands for epithelial cells of the respiratory tract, $I = I(t)$ represents the infected cells, and $V = V(t)$ represents the influenza A virus.

Apply the numerical method

$$\begin{aligned} \xi_1 &= y_i, \\ \xi_2 &= y_i + h a_{2,1} f(t_i, \xi_1), \\ \xi_3 &= y_i + h\Big(a_{3,1} f(t_i, \xi_1) + a_{3,2} f(t_i + c_2 h, \xi_2)\Big), \\ \xi_4 &= y_i + h\Big(a_{4,1} f(t_i, \xi_1) + a_{4,2} f(t_i + c_2 h, \xi_2) + a_{4,3} f(t_i + c_3 h, \xi_3)\Big), \\ y_{i+1} &= y_i + h\Big(b_1 f(t_i, \xi_1) + b_2 f(t_i + c_2 h, \xi_2) + b_3 f(t_i + c_3 h, \xi_3) + b_4 f(t_i + c_4 h, \xi_4)\Big) \end{aligned}$$

where $a_{2,1} = a_{3,2} = \frac{1}{2}$, $a_{3,1} = a_{4,1} = a_{4,2} = 0$, $a_{4,3} = 1$, $c_2 = c_3 = \frac{1}{2}$, $c_4 = 1$, $b_1 = b_4 = \frac{1}{6}$, $b_2 = b_3 = \frac{1}{3}$, to build your own Matlab files to compute numerical solutions of system (1) supplemented by any initial conditions $T(0) = T_0$, $I(0) = I_0$, $V(0) = V_0$ and to graphically illustrate the dynamics of the influenza A virus.

For the computations, use a suitable step size h of your choice. For the graphical illustration, use the horizontal axis for time t over the interval from 0 to 2 and the vertical axis for $T(t)$, $I(t)$, $V(t)$ (all three in one figure, clearly labelled). Write supporting documentation describing each part of your files, what each part does, and how the parts work together.

Submit the following items

- all of your Matlab files needed to compute and graphically illustrate the solutions of (1),
- a discussion on how you choose the step size h in order to compute the solutions,
- a figure illustrating three curves of $T(t)$, $I(t)$, $V(t)$ versus t for $0 \le t \le 2$, generated by your files with the initial values $T_0 = 30000$, $I_0 = 0$, $V_0 = 10$ and the parameters $\beta = 0.02$, $\delta = 0.01$, $p = 0.025$, $c = 0.05$; label each curve by the corresponding function,
- supporting documentation, described above.