Partial Differential Equations:

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Course Overview

Differential equations occupy a central role in mathematics because they allow us to describe a wide variety of real-world systems. The module will aim to stress the importance of both theory and applications of differential equations.

The module begins by revisiting some of the material from your previous course on ordinary differential equations focussing attention on boundary value problems and also on equations with a source term. We then look at how one can express a general periodic function in terms of Fourier series of sine and cosine functions.

The second section of the course introduces some of the basic concepts of partial differential equations (PDEs). It is shown how PDEs may be used to model situations in a wide variety of situations including biology, finance and applied mathematics. The three important classes of second order PDE appropriate for modelling different sorts of phenomena are introduced and the appropriate boundary conditions for each of these are considered. The technique of separation of variables will be used to reduce the problem to that of solving the sort of ordinary differential equations seen at the start of the course and writing the general solution using Fourier series. Throughout the course there will be a strong emphasis on problem solving and examples.

Having successfully completed this module you will be able to:

- Solve simple second order differential equations
- Be able to calculate Fourier series
- Understand the wide applications of partial differential equation
- Prove the orthogonality of eigenfunctions of boundary value problems
- Be able to classify second order partial differential equations and choose the appropriate boundary conditions
- Apply the method of separation of variables to standard PDEs in 2 and 3 dimensions.
- Understand basic numerical methods for solving PDEs

Syllabus:

- *Introduction to PDE's and overview of the course*

- *Revision of material on 2nd Order Ordinary Differential equations*
  2nd Order ODEs with constant coefficients
  Green's Functions
  Boundary value problems
  Eigenvalues and Eigenfunctions,
  Orthogonality of eigenfunctions

- *Fourier Series*
  Periodic functions
  Odd and even functions, Half-range series,
  Convergence

- *2nd order PDE's*
  Type of PDE: Hyperbolic, Parabolic and Elliptic and corresponding boundary conditions
  Examples: Wave equations, Diffusion in physical and biological systems, Black-Scholes equation, applications in finance, Potential theory applications in biology and physics.

- *Hyperbolic PDE's*
  D'Alembert's solution
  Characteristics
  Wave equation
  Cauchy problem
  Separation of variables

- *Parabolic PDE's*
  Heat equation
  Separation of variables
  Applications to diffusion problems in Finance and Biology

- *Elliptic PDE's*
  Laplace's equation
  Dirichlet problem
  2-dim: Harmonic functions
  3-dim: Fundamental solution