PARTIAL DIFFERENTIAL EQUATIONS
Quiz 2, time 15 minutes

Instructor: James Vickers

Date: Wednesday 13th November

1. The function $f(x)$ is **even** and has **period** 2π. The function is defined for $0 \leqslant x \leqslant \pi$ by

$$f(x) = \pi - x, \qquad 0 \leqslant x \leqslant \pi. \tag{1}$$

(a) [3 marks] Sketch the graph of $f(x)$ for the region $-3\pi \leqslant x \leqslant 3\pi$.

(b) [3 marks] Using the defintion of the derivative $f'(x)$ as the slope of the of the graph of f at the point x, sketch the graph of $f'(x)$ for the region $-3\pi \leqslant x \leqslant 3\pi$.

2. You are given that the function $f(x)$ defined in Question 1. has Fourier Cosine series

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(1-(-1)^n)}{n^2}\cos(nx) \tag{2}$$

(No need to calculate this!)

(a) [3 marks] By evaluating $f(x)$ at $x = 0$ show that

$$\frac{\pi^2}{8} = \sum_{m=1}^{\infty} \frac{1}{(2m-1)^2} \tag{3}$$

(b) [3 marks] Say why it is permitted to differentiate the Fourier series for $f(x)$ given in equation (2) term by term to obtain the Fourier series for $f'(x)$.

(c) [3 marks] Differentiate the Fourier series given by (2) to obtain the Fourier Sine series for $f'(x)$.

(d) [3 marks] Is it permitted to differentiate the Fourier series given by (2) a second time to obtain a Fourier series for $f''(x)$?

Total 18 marks