(1) $\begin{cases} y_{n+1} = y_n + h(ak_1 + bk_2), & \text{where } k_1 = f(t_n, y_n) \\ & k_2 = f(t_n + \alpha h, y_n + \beta h k_1) \end{cases}$

$a, b, \alpha, \beta \in \mathbb{R}$ parameters

$a + b = 1, \quad b\alpha = 1/2, \quad b\beta = 1/2, \quad \alpha = \beta \neq 0$

Conclusion

If $a = 1 - \frac{1}{2\alpha}$ and $b = \frac{1}{2\alpha}$ where $\alpha = \beta \neq 0$

$$T_n = \frac{1}{6}h^2\left[f_{tt} + 2ff_{ty} + f^2 f_{yy} + f_t f_y + f_y^2 f\right] - b\left[\frac{1}{2}\alpha^2 h^2 f_{tt} + \alpha\beta h^2 f f_{ty} + \frac{1}{2}\beta^2 h^2 f^2 f_{yy}\right] + \mathcal{O}(h^3)$$

$= \mathcal{O}(h^2)$ that is the method is of order 2

However, the method has no chances to be of order 3

Reason:

$$T_n = h^2\left[\left(\frac{1}{6} - \frac{1}{2}\alpha b\right)f_{tt} + \left(\frac{1}{3} - b\alpha\beta\right)ff_{ty} + \left(\frac{1}{6} - \frac{1}{2}b\beta^2\right)f^2 f_{yy} + \frac{1}{6}\underbrace{\left(f_t f_y + f_y^2 f\right)}_{\text{non-zero}}\right] + \mathcal{O}(h^3)$$

Example $\begin{cases} y'(t) = y(t) \\ y(0) = 1 \end{cases}$ Here, $f(t,y) = y$, $f_t = 0$, $f_{tt} = 0$, $f_{ty} = 0$

$f_y = 1$, $f_{yy} = 0$ and

$$T_n = \frac{1}{6}h^2 \underset{\uparrow\ f(t_n, y(t_n))}{f} + \mathcal{O}(h^3) = \frac{1}{6}h^2 \underbrace{y(t_n)}_{= e^{t_n}} + \mathcal{O}(h^3) = \underbrace{\frac{1}{6}h^2 e^{t_n}}_{\neq 0} + \mathcal{O}(h^3)$$

This example shows that the method has no chances to be of order 3.

Example for $\alpha = \beta = 1$, $a = b = 1/2$, from (1), we get another method:

$y_{n+1} = y_n + \frac{1}{2}hf(t_n, y_n) + \frac{1}{2}hf(t_{n+1}, y_n + hf(t_n, y_n)) \rightarrow$ of order 2 the truncation error in this case is given by

$$T_n = h^2\left[\overbrace{\left(\frac{1}{6}-\frac{1}{4}\right)}^{\frac{2-3}{12}} f_{tt} + \underbrace{\left(\frac{1}{3}-\frac{1}{2}\right)}_{\frac{2-3}{6}} f f_{ty} + \underbrace{\left(\frac{1}{6}-\frac{1}{4}\right)}_{\frac{1}{12}} f^2 f_{yy} + \frac{1}{6}\left(f_t f_y + f f_y^2\right)\right]$$
$$+ O(h^3)$$

$$= h^2\left[-\frac{1}{12} f_{tt} - \frac{1}{6} f f_{ty} - \frac{1}{12} f^2 f_{yy} + \frac{1}{6} f_t f_y + \frac{1}{6} f f_y^2\right] + O(h^3)$$