Market Regime Detection In Quantitative Wealth Investment Management

Lingxiao Zhang
lz2086@nyu.edu

Hepu Jin
hj879@nyu.edu

August 11, 2020

Abstract

The purpose of this project is to overcome the challenge that changing market conditions present to traditional portfolio optimization. We formulate a Hidden Markov Model to detect market regimes using standardized absorption ratio, a market risk indicator, calculated by 10 MSCI U.S. sector indices. In this way, we can estimate the expected returns and corresponding covariance matrix of assets based on regimes. By design, these two parameters are calibrated to better describe the properties of different market regimes. Then, these regime-based parameters serve as the inputs of different portfolio weight optimizers, thereby constructing regime-dependent portfolios. In an asset universe consisting of U.S. equities, U.S. bonds and commodities, it is shown that regime-based portfolios have better returns, lower volatility and less tail risks compared with other competing portfolios.

1 Introduction

Portfolio optimization is a process of selecting the best portfolio according to some objectives. Mathematically, it can be viewed as a constrained utility-maximization problem. Although portfolio optimization method such as mean-variance optimization gained wide attention as a quantitative tool in asset management, it has been criticized for its susceptibility to estimation error in its input parameters, namely, expected return and covariance matrix. Chopra and Ziemba [1993] concluded that estimation errors during optimization can lead to overconcentrated portfolios, arguing that errors in expected returns have much larger impact than errors in the covariance matrix [1].

In order to improve the estimation of input parameters for portfolio optimization, we introduce the Hidden Markov Model (HMM) which is a popular choice for modelling the hidden states of financial markets. In our project, we identify hidden states in absorption ratio, a market risk indicator, using HMM. It was firstly introduced by Mark Kritzman in 2011 and defined as the fraction of the total variance of a set of asset returns explained by a fixed number of eigenvectors [2]. It captures the extent to which markets are unified or tightly coupled. After identifying market regimes, we estimate expected returns and covariance matrix of a set of assets separately under different regimes and test performance of regime-based portfolios using regime-based parameters as input.

1.1 Outline of the report

Section 2 introduces intuition and features of absorption ratio. It serves as a systemic risk indicator allowing us to detect market fragility before market crashes. Section 3 introduces the Hidden Markov model using standardized absorption ratio as input. HMM allows us to derive the regime-dependent expected returns and covariance matrix, implicitly capturing the market dynamics. Section 4 shows the empirical results: calculating standardized absorption ratio of 10 MSCI U.S. sector indices and detecting hidden states from it. Section 5 compares regime-based portfolios with basic portfolios using different optimizers. Section 6 summarizes the findings.

2 THE ABSORPTION RATIO

2.1 Principal Component Analysis

Principal Component Analysis (PCA) is an unsupervised learning method that reduces the dimensionality of datasets, increasing interpretability but at the same time minimizing information loss. It does so by creating new uncorrelated variables that successively maximize variance. In our project, PCA is implemented in calculating absorption ratio, finding common risk factors for a set of assets and calculating how much variance these risk factors are able to capture throughout the time.

2.2 Calculate Absorption Ratio

Absorption ratio is defined as the fraction of the total variance of a set of assets explained by a finite set of eigenvectors, as shown in the following equation:

$$AR = \frac{\sum_{i=1}^{n} \sigma_{E_i}^2}{\sum_{j=1}^{N} \sigma_{E_j}^2} \tag{1}$$

where,
AR = absorption ratio
N = number of assets
n = number of eigenvectors
$\sigma_{E_i}^2$ = variance of the i^{th} eigenvector

$\sigma^2_{E_j}$ = variance of the j^{th} asset

Large absorption ratio implies that market risks are more unified, thus indicating a higher level of systemic market risk. On the other hand, low absorption ratio indicates lower systemic risk. Intuitively, it could be understood as when absorption ratio is high, if something bad happened, the market would fall significantly because of the more unified risk. Thus, the absorption ratio is an indicator of market fragility. However, we should not expect that market drawdowns always follow the spikes in absorption ratio since a fragile market does not necessarily guarantee a market crash. Mark Kritzman provided extensive evidence that absorption ratio effectively captures market fragility. It has been shown that most significant U.S. stock market drawdowns were preceded by spikes in the absorption ratio and the absorption ratio often provides an early sign of market turbulence [2].

3 THE HIDDEN MARKOV MODEL

3.1 Introduction of Hidden Markov Model

To detect different market regimes, we will mainly use Hidden Markov Model (HMM), which is based on augmenting the Markov Chain. A Markov Chain makes a very strong assumption that the prediction for the future sequence only relies on the current state [3]. The states before the current state have no influence on the future prediction. More formally, A sequence of discrete random variables $\{Z_t : t \in T\}$ is said to be a first-order Markov chain if, for all $t \in T$, it satisfies the Markov property:

$$P(Z_{t+1}|Z_t, ..., Z_1) = P(Z_{t+1}|Z_t) \quad (2)$$

A Markov Chain is useful when we compute the probability on observable events. However, in many cases, some of the events that we need to predict are not observable: we could not observe them directly. For example, the price, return and volatility of S&P500 index are observable but which regime the market is currently in is not observable. As a result, we introduce the Hidden Markov Model to help detect regimes. In HMM, the probability distribution that generates an observation depends on the state of an unobserved Markov chain. HMM allows us to take care of both observable and hidden events that act as causal factors in the model when making prediction [3]. Generally, we divide the market into two regimes: bullish regime and bearish regime. When the market is in the bullish regime, the average return of the portfolio has a higher possibility to be higher and the volatility is relatively low; when the market is in the bearish regime, the average return of the portfolio is supposed to be lower and the volatility would be higher [4]. Although the market regime is a hidden event, we could rationally make inference from the observable events such as returns data.

In order to explain the HMM more clearly, we will specify some components of the model first [3]:

- $\{X_t : t \in T\}$: A sequence of T observations
- $\{Y_t : t \in T\}$: A sequence of T hidden states with M different states
- $\Gamma = \gamma_{11}, ..., \gamma_{ij}, \gamma_{MM}$: A transition probability matrix with shape $M \times M$, each $\gamma_{ij} = P(Y_{t+1} = j | Y_t = i)$ represents the transition probability from state i to state j, such that $\sum_{j=1}^{M} \gamma_{ij} = 1, \forall i$
- $E = e_i(X_t)$: A sequence of likelihoods called emission probabilities, each representing the probability of an observation X_t being generated from state i
- $\Pi = \pi_1, ..., \pi_M$: An initial probability distribution, each π_i represents the probability that the Markov Chain starts at state i, such that $\sum_{i=1}^{M} \pi_i = 1$

A first-order HMM should follow two assumptions. First, as with a first-order Markov Chain, the probability of a particular hidden state depends only on the previous state. In other words, it should satisfy the Markov property:

$$P(Y_{i+1}|Y_i, ..., Y_1) = P(Y_{i+1}|Y_i) \tag{3}$$

Second, the probability of an output observation X_i depends only on the state that produced the observation Y_i and not on any other states or observations:

$$P(X_i|Y_1, ..., Y_i, ..., Y_T; X_1, ..., X_i, ..., X_T) = P(X_i|Y_i) \tag{4}$$

As an example in our case to detect market regimes, consider the two-state model with Gaussian conditional distributions [4]:

$$X_t \sim N(\mu_y, \sigma_y^2) \tag{5}$$

where

$$\mu_{Y_t} = \begin{cases} \mu_1 & \text{if } Y_t = 0 \\ \mu_2 & \text{if } Y_t = 1 \end{cases}, \sigma_{Y_t}^2 = \begin{cases} \sigma_1^2 & \text{if } Y_t = 0 \\ \sigma_2^2 & \text{if } Y_t = 1 \end{cases}, \Gamma = \begin{bmatrix} 1 - \gamma_{01} & \gamma_{01} \\ \gamma_{10} & 1 - \gamma_{10} \end{bmatrix} \tag{6}$$

In this example, 0 could represent the low volatility regime and 1 could represent the high volatility regime. Mean, variance, emission probabilities and transition probabilities are parameters that need to be estimated. Under different hidden states, X_t is supposed to follow different distributions. If we know which state Y_t is currently in, we could derive the distribution of X_t from the information of Y_t. Furthermore, the sojourn time in one particular state is assumed to follow geometric distribution, which is memory-less, implying that the time until the next transition out of the current state is independent of the time spent in the state. Therefore, HMM can match the tendency of financial markets to change their behavior abruptly and the phenomenon that the new behavior often persists for several periods after a change [4]. Generally, HMM is characterized by three fundamental problems [3]:

- Learning: Given a sequence of observations X and possible hidden states, estimate the parameters Γ and E
- Decoding: Given a sequence of observations X, Γ and E, determine the most possible sequence of hidden states Y
- Likelihood: Given Γ, E and a sequence of observations X, determine the likelihood $P(X|\Gamma; E)$

In the next two sections, we will mostly touch on Learning and Decoding process since only these two processes are useful when detecting market regimes.

3.2 Decoding Process: Viterbi Algorithm

If the transition matrix Γ and emission probabilities E are given, the sequence of hidden states could be inferred. Viterbi algorithm is one of the most commonly used algorithm to conduct the decoding task, which makes use of dynamic programming trellis. The idea is to compute the best path through the hidden states space given a sequence of observation X ,and estimated parameters Γ and E. We will define $v_t(j)$ as the probability that HMM is in state j after seeing the first t observations and passing through the most possible path $Y_1, Y_2, ..., Y_{t-1}$, given Γ and E:

$$v_t(j) = \max_{Y_1,...,Y_{t-1}} P(Y_1, ..., Y_{t-1}, X_1, ..., X_t, Y_t = j|\Gamma; E) \tag{7}$$

The probability is computed by recursively taking the most possible path to lead to the current state. The possibility could also be represented recursively:

$$v_t(j) = \max_{i=1}^{M} v_{t-1}(i)\gamma_{ij}e_j(X_t) \tag{8}$$

With $v_t(j)$, the Viterbi algorithm could be divided into three parts [3]:

A. Initialization:

$$v_1(j) = \pi_j e_j(X_1), 1 \leq j \leq M \tag{9}$$

$$e_{t_1}(j) = 0, 1 \leq j \leq M \tag{10}$$

B. Recursion:

$$v_t(j) = \max_{i=1}^{M} v_{t-1}(i)\gamma_{ij}e_j(X_t), 1 \leq j \leq M, 1 \leq t \leq T \tag{11}$$

$$b_{t_t} = arg \max_{i=1}^{M} v_{t-1}(i)\gamma_{ij}e_j(X_t), 1 \leq j \leq M, 1 \leq t \leq T \tag{12}$$

C. Termination:

$$\text{The best score} : P^* = \max_{i=1}^{M} v_T(i) \tag{13}$$

$$\text{The start of back trace} : Y_T^* = arg \max_{i=1}^{M} v_T(i) \tag{14}$$

3.3 Learning Process: Expectation-Maximization Algorithm

Learning the parameters of HMM is the most fundamental step when using the model. Generally, HMM is trained through Expectation-Maximization(EM) algorithm, which trains transition probabilities Γ and emission probabilities at the same time. EM is an iterative algorithm, which computes an initial guess of the probabilities and then uses the computed probabilities to get a better estimate until convergence. Before introducing the EM algorithm, we will define some probabilities first [3]:

- $\alpha_t(j)[P(X_1, ..., X_t, Y_t = j|\Gamma, E)$: The probability of being in state j after seeing the first t observations
- $\beta_t(i) = P(X_{t+1}, X_{t+2}, ..., X_T)$: The probability of seeing the observations from time $t+1$ to the end, given that the hidden state is in i at time t, Γ and E.
- $\xi_t(ij) = P(Y_t = i, Y_{t+1} = j|X, \Gamma, E)$: The probability of being in state i at time t and state j at time $t+1$
- $\theta_t(j) = P(Y_t = j|X, \Gamma, E)$: The probability of being in state j at time t

Expectation Maximization algorithm could be divided into two steps: Expectation step and Maximization step. In the Expectation step, the expected state occupancy count θ and the expected transition count ξ will be calculated from earlier Γ and E estimation. In the Maximization step, the Γ and E will be re-estimated according to θ and ξ from Expectation step. Algorithm 1 is the pseudo code for the algorithm [3]:

3.4 Training Process

With the learning and decoding process, the most possible path of hidden states could be computed with the estimated parameters. During the regime detection process, we will first use some data to initialize the model and make

Algorithm 1: Expectation Maximization Algorithm

Result: Transition probabilities Γ and emission probabilities E

Initialize Γ and E ;

while Γ *and* E *have not converged* **do**

  Expectation Step: ;

  $\theta_t(j) = \frac{\alpha_t(j)\beta_t(j)}{\alpha_T(Y_F)}, \forall t$ and j ;

  $\xi_t(i,j) = \frac{\alpha_t(i)\gamma_{ij}e_j(X_{t+1})\beta_{t+1}(j)}{\alpha_T(Y_F)}, \forall t, i$ and j ;

  Maximization Step: ;

  $\hat{a_{ij}} = \frac{\sum_{t=1}^{T-1}\xi_t(i,j)}{\sum_{t=1}^{T-1}\sum_{k=1}^{M}\xi_t(i,k)}$;

  $b_j(\hat{v_k}) = \frac{\sum_{t=1, s.t. Y_t=v_k}^{T}\theta_t(j)}{\sum_{t=1}^{T}\theta_t(j)}$;

end

prediction. Then, we feed in one new data point over time and re-estimate the parameters of HMM periodically in order to simulate the real-time situation and prevent from looking into future. In this way, we could dynamically deal with the time-varying property of HMM parameters. Algorithm 2 is the pseudo code of training steps:

Algorithm 2: Periodically re-estimated HMM

Result: A sequence of estimated hidden states

n = days for initialization ;

m = days to re-estimate ;

Initiate HMM model with first n days' data (learning);

for $t = n+1, n+2, ...,$ *number of observations* **do**

  if *m days have passed since last estimation* **then**

    Re-estimate the parameters of HMM model using new data (learning) ;

  end

  Predict the regime on day t (decoding)

end

4 Empirical Results

4.1 The Absorption Ratio

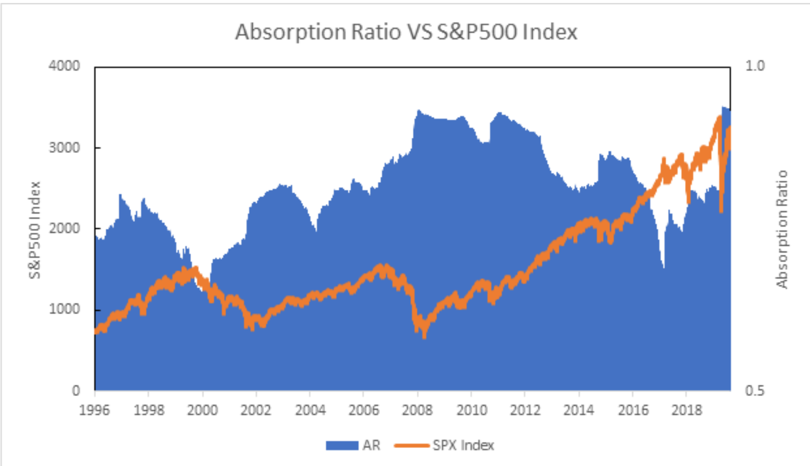
In order to calculate the absorption ratio, we use a window of 500 days to estimate the covariance matrix and eigenvectors, and fix the number of eigenvectors to be 1/5th the number of assets in our sample. The variances $\sigma^2_{E_i}$ and $\sigma^2_{A_j}$ are exponential weighted. This approach weighted recent data more important than the past. The half-life of the exponential weight decay is set to 250 days. Algorithm 3 shows the pseudo code for calculating the absorption ratio:

Algorithm 3: Absorption Ratio

for $t = n, n+1, n+2, \ldots$ **do**
    Collect asset daily returns for the past n days ;
    Run PCA on return data ;
    Calculate absorption ratio at t as $AR_t = \frac{\sum_{i=1}^{num\_pc} \sigma_{E_i}^2}{\sum_{j=1}^{N} \sigma_{E_j}^2}$;
end

Figure 1 shows a time series of the absorption ratio estimated from the returns of 10 MSCI U.S. sector indices (hence, 2 eigenvectors) based on trailing 500-day overlapping windows, along with the level of the S&P500 index from December 12, 1996, through July 21, 2020. From the plot, we see

Figure 1: Relationship between Absorption Ratio and S&P 500 Index



negative correlation between the level of the absorption ratio and the level of U.S. stock prices. In particular, the absorption ratio spiked sharply to its highest level during the global financial crisis of 2008 and the coronavirus outbreak of 2020, coincident with a steep decline in stock prices. However, the absorption ratio remained high after the economy recovered from 2008 financial crisis. Thus, we are more interested in detecting regime shifts in change of the absorption ratio, rather than the level of itself. This leads us to calculate delta absorption ratio, which, as defined below, will be the data input of Hidden Markov model,

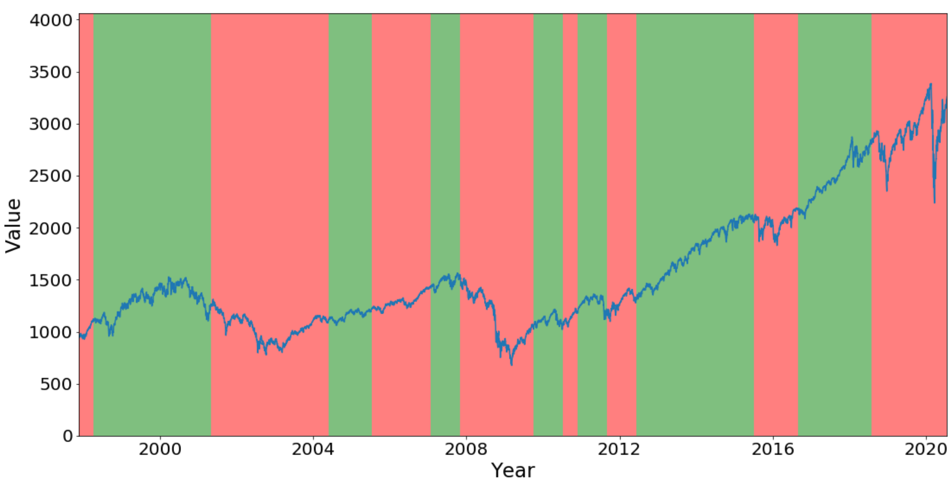
$$\Delta AR = \frac{(AR_{15day} - AR_{1Year})}{\sigma} \tag{15}$$

Where,
ΔAR = standardized shift in absorption ratio
AR_{15day} = 15-day moving average of absorption ratio
AR_{1Year} = one-year moving average of absorption ratio
σ = standard deviation of one-year absorption ratio

4.2 Regime Detection Using HMM

The HMM uses delta absorption ratio as the input to estimate parameters and detect market regimes. We use a look-back window of size 1000 days to train the model and re-estimate the parameters every 100 days. The choice of window size is selected according to the trade-off between accuracy and latency when detecting regimes. In this way, the model could promptly adapt to the time-varying parameters. Figure 2 shows two different market regimes during the same period as calculated delta absorption ratio, from December 12, 1996, to July 21, 2020. The red region represents the high-volatility market situation, while the green region represents the low-volatility market situation. It can be seen that the regime detection is able to largely capture trending periods and highly volatile periods. In particular, the majority of global financial crisis of 2008 and coronavirus outbreak of 2020 occur in the red region. As a result, through the decoding process of HMM, we derive a sequence of hidden states for different market regimes. In the following sections, we will conduct portfolio optimization based on the result of market regime detection.

Figure 2: Two Regimes for the S&P 500 Index



5 PORTFOLIO OPTIMIZATION

5.1 DATA

We consider the following seven asset classes and investment styles: U.S. large cap equities, U.S. small cap equities, U.S. value equities, U.S. growth equities, U.S. Corporate bonds, U.S. Treasuries, and commodities. The sample period is from October 19, 1999 through July 10, 2020, spanning a total of 21 years. We use a daily data frequency. Table 1 shows our data sources.

Table 1: Trading Universe

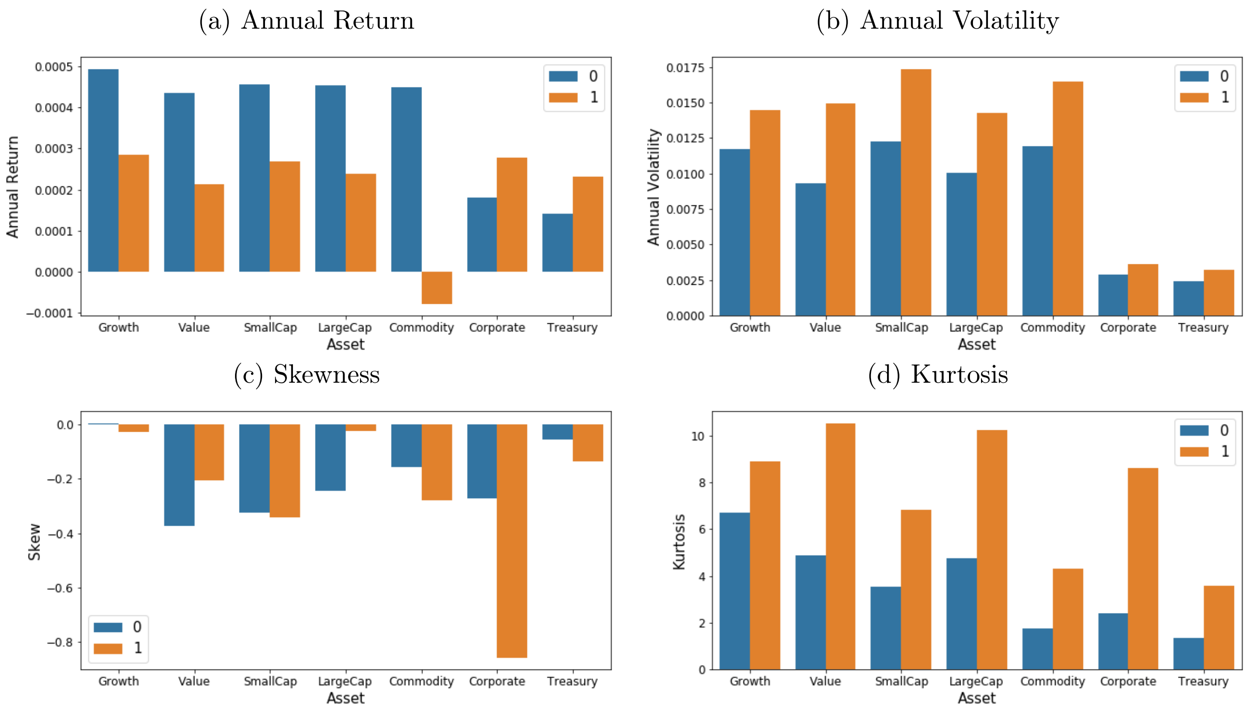
Index	Data
U.S. Growth Equities	iShares Russell 1000 Growth ETF
U.S. Value Euquities	iShares Russell 1000 Value ETF
U.S. Small Cap Equities	Russell 2000 Total Return
U.S. Large Cap Equities	S&P 500
Commodities	Bloomberg Commodity Index Total Return
U.S. Corporate Bonds	Bloomberg Barclays US Corporate Total Return Value Unhedged USD
U.S. Treasuries	Bloomberg Barclays US Treasury Total Return Unhedged USD

5.2 Compare Assets Performance under Different Regimes

Portfolio optimization has been criticized by practitioners saying that it is not useful in practice. For example, Michaud in 1989 argued that the major problem with mean-variance optimization is its tendency to maximize the effects of errors in the input assumptions: expected return and covariance matrix. Unconstrained mean-variance optimization can yield results that are inferior to those of simple equal-weighting schemes [5]. Indeed, expected returns and covariance matrices are often estimated using historical data and assumed to be able to represent the future. However, if they are not stable or calculated incorrectly, the optimal weights determined by optimizers will be far from optimal. By Figure 3, we provide evidence that assets behave differently under different regimes, explaining why simple historical estimates may not good enough.

Figure 3 shows the first four moments of seven assets under regime 0 (low volatility regime) and regime 1 (high volatility regime). It is evident that equities and commodities have significantly higher returns under low volatility regime while Corporate bonds and Treasuries show the opposite. Annual volatilities are high for all assets under high volatility regime. This coincides with the fact that investors are willing to take more risk and invest in equities when market is not volatile. However, when people see market in turbulence, they switch to safer assets like bonds. Skewness of most as-

Figure 3: First Four Monments Of Asset Returns Under Different Regimes



sets become more negative when market risk is high, and kurtosis increases sharply during high volatility period. Thus, asset returns tend to have fatter tails and large negative market shift may be observed more often than expected.

Figure 4 shows two correlation matrices for the seven assets corresponding to two market regimes. Under high volatility regime, equities and commodities become more positively correlated with each other while equities and bonds are more negatively correlated. Given that expected returns and covariance matrices change across different regimes, it is necessary to estimate them differently for different regime.

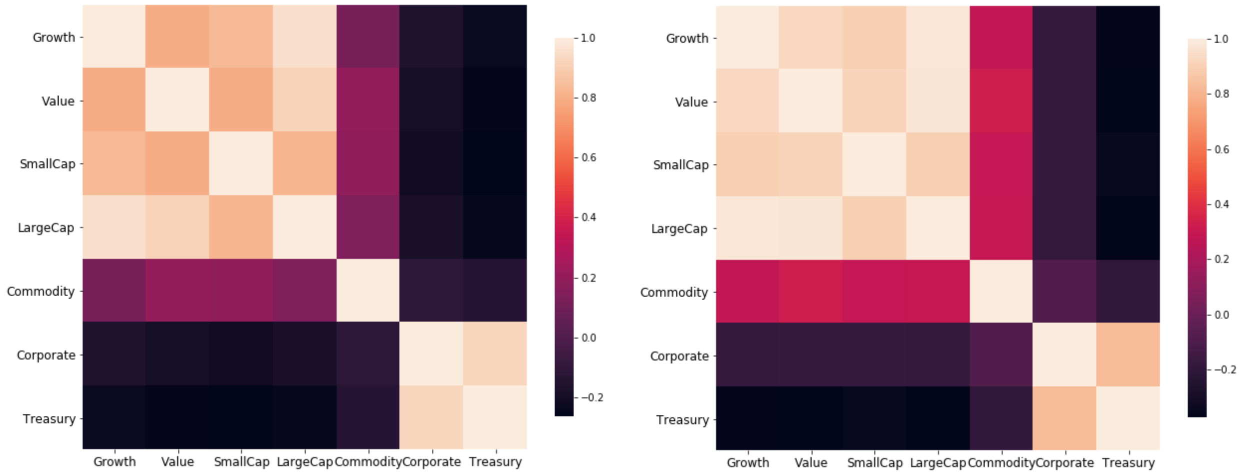
5.3 Investment Models

This section presents four investment models that will be tested under the regime-switching framework: maximum Sharpe Ratio, minimum variance, maximizing return given risk and minimizing risk given return. The portfolio expected return and variance are

$$\mu_p = \mu^T w \tag{16}$$

$$\sigma_p^2 = w^T \sum w \tag{17}$$

Figure 4: Correlation Matrices Of Asset Returns



(a) Correlation Matrix Under Low Volatility Regime (b) Correlation Matrix Under High Volatility Regime

Where μ_p is the portfolio expected return, σ_p^2 is the portfolio variance which is our risk model and w is the vector of asset weights. The problem of choosing optimum weights is equivalent to finding a solution w^* that maximizes the utility subject to some prescribed constraints. The next two subsections will introduce both the basic investment models and the regime-based investment models. Then we perform four separate computational experiments to test performance of the regime-based investment models relative to their basic counterparts.

5.3.1 Basic and Regime-based Models

Maximizing Sharpe Ratio, maximizing return given risk and minimizing risk given return are variants of mean-variance optimization (MVO). The MVO model seeks to attain an optimal balance between risk and return. On the other hand, minimum-variance model seeks to minimize the portfolio risk without imposing any conditions on the portfolio expected return. Thus, the only input is the asset covariance matrix. Without additional constraints, these models may produce unrealistic results such as many zero weights and overweight on some assets. So, we add some constraints to the four optimization models. Firstly, from our experience, many of the assets are set to be zero without any regularization. This may not be ideal for diversification purposes. So, we add a $L2$ regularization term to the utility function. Secondly, we force each asset weight between 0 - 0.5 so we do not allow over weighted or short positions in our portfolios. The four models can be written as below:

Maximum Sharpe ratio:

$$\begin{aligned}
\min_{w} \quad & \frac{\mu^T w - r_f}{\sqrt{w^T \sum w}} + \gamma w^T w \\
\text{s.t.} \quad & 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad\qquad \text{i = 1,2,..,7}
\end{aligned} \tag{18}$$

Maximize return given risk:

$$\begin{aligned}
\max_{w} \quad & \mu^T w + \gamma w^T w \\
\text{s.t.} \quad & w^T \sum w = \sigma^2 \\
& 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad \text{i = 1,2,..,7}
\end{aligned} \tag{19}$$

Minimize risk given return:

$$\begin{aligned}
\min_{w} \quad & w^T \sum w + \gamma w^T w \\
\text{s.t.} \quad & \mu^T w = R \\
& 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad \text{i = 1,2,..,7}
\end{aligned} \tag{20}$$

Minimum variance:

$$\begin{aligned}
\min_{w} \quad & w^T \sum w + \gamma w^T w \\
\text{s.t.} \quad & 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad \text{i = 1,2,..,7}
\end{aligned} \tag{21}$$

Then we introduce the regime-based investment models. The cyclical nature of the market can be captured through the estimated regime-dependent parameters μ_{s_i} and $\sum_{s_i}$. Thus, the regime-dependent variants can easily be attained by replacing μ and $\sum$ with their regime-based counterparts, ie, Maximum Sharpe ratio:

$$
\begin{aligned}
\min_{w} \quad & \frac{\mu_{s_i}^T w - r_f}{\sqrt{w^T \sum_{s_i} w}} + \gamma w^T w \\
\text{s.t.} \quad & 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad\qquad \text{i} = 1,2,..,7
\end{aligned} \tag{22}
$$

Maximize return given risk:

$$
\begin{aligned}
\max_{w} \quad & \mu_{s_i}^T w + \gamma w^T w \\
\text{s.t.} \quad & w^T \sum_{s_i} w = \sigma_{s_i}^2 \\
& 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad \text{i} = 1,2,..,7
\end{aligned} \tag{23}
$$

Minimize risk given return:

$$
\begin{aligned}
\min_{w} \quad & w^T \sum_{s_i} w + \gamma w^T w \\
\text{s.t.} \quad & \mu_{s_i}^T w = R_{s_i} \\
& 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad\quad \text{i} = 1,2,..,7
\end{aligned} \tag{24}
$$

Minimum variance:

$$
\begin{aligned}
\min_{w} \quad & w^T \sum_{s_i} w + \gamma w^T w \\
\text{s.t.} \quad & 1^T w = 1 \\
& 0 \leq w_i \leq 0.5 \qquad\quad \text{i} = 1,2,..,7
\end{aligned} \tag{25}
$$

where R_{s_i} and σ_{s_i} are regime-dependent target return and target volatility respectively.

To further exploit regime information in our models, an additional constraint has been introduced to control the weight of equities and commodities based on regime: under low volatility regime, sum of weights of equities and commodities should be larger than W and under high volatility regime, the sum should be less than $1 - W$.

5.3.2 Computational Experiments

In this section, we present four separate experiments, comparing performance between the portfolios without using regime information and their regime-based counterparts. We refer them to basic portfolio and regime-based portfolio later. For each experiment, we construct the basic and regime-based portfolios at the same time. For each day t, if regime switches, both portfolios need to feed updated parameters into the chosen optimizer and update their asset allocations separately. We also introduce a rebalance policy to restore asset weights regularly. The frequency we choose is 60 days. Algorithm 4 below shows details of how we construct portfolios:

Algorithm 4: Construct Portfolio

Initialize shares of holding for each asset **for** *Every time step* **do**
  if *regime switches* **then**
    Update weights using a chosen optimizer Update new holdings using updated weights
  end
  if *portfolio hasn't been rebalanced for n days* **then**
    Rebalance the portfolio
  end
end

In Table 2, we compare the performance of the regime-dependent portfolios and the basic portfolios using different optimization methods. The empirical results show that regime-based asset allocation is profitable, compared with its basic counterpart. Across the four optimizers, regime-based portfolios have higher annual return and Sharpe ratio, lower annual volatility and maximum drawdown than basic portfolios. In addition, regime-based portfolios have less negative skew and larger kurtosis, implying that they significantly reduce tail risk compared with basic portfolios.

6 CONCLUSION

In this project, we use standardized absorption ratio computed from 10 U.S. sector indices as the input to the changing point detection. Absorption ratio is considered because of its predictive power to the stock market. Our Hidden Markov Model successfully predicts the Financial Crisis in 2008 and the market crash due to coronavirus outbreak in 2020. Even if there are periods misclassified, the results show that they are mostly short-lived and this is outweighted by the benefit of correctly predicting market crashes beforehand.

The experimental results show that regime-based portfolios are able to steadily outperform their basic counterparts over a long investment horizon,

Table 2: Portfolio Performance Table

Optimizer	Maximum Sharpe ratio		Minimum Variance	
Portfolio Type	**Regime-based**	**Basic**	**Regime-based**	**Basic**
Annual Return	7.6%	5.6%	7.7%	6.1%
Annual Volatility	8.7%	15.4%	8%	9.4%
Sharpe Ratio	0.89	0.43	0.97	0.95
MDD	-24%	-48%	-17.3%	-31.4%
Skew	-0.43	-0.64	-0.59	-0.73
Kurtosis	5	7.67	7.29	9.17
Optimizer	**Maximum Return given Risk**		**Minimum Risk given Return**	
Portfolio Type	**Regime-based**	**Basic**	**Regime-based**	**Basic**
Annual Return	7.1%	5.6%	6.2%	5.8%
Annual Volatility	14.1%	16.8%	10.9%	12.2%
Sharpe Ratio	0.56	0.41	0.6	0.53
MDD	-38.4%	-50.9%	-34.1%	-40.2%
Skew	-0.42	-0.64	-0.62	-0.65
Kurtosis	6.56	8.29	8.6	7.45

exhibiting higher returns, lower volatility and less tail risk. These results are consistent over four different optimizers: maximum Sharpe ratio, minimum variance, maximizing return given risk and minimizing risk given return, showing that regime-dependent portfolios are quite resilient.

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