



ATHABASCA UNIVERSITY

MATH 270

Final Evaluation Report

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1. We can let $A = (9, 7, 5), B = (3, 1, 2), C = (4, 6, 8)$. We can then find the individual vectors $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{AC}$ from these points.

$$\overrightarrow{AB} = A - B = (9, 7, 5) - (3, 1, 2) = (6, 6, 3)$$

$$\overrightarrow{BC} = B - C = (3, 1, 2) - (4, 6, 8) = (-1, -5, -6)$$

$$\overrightarrow{AC} = A - C = (9, 7, 5) - (4, 6, 8) = (5, 1, -3)$$

Then, we can find each vector's magnitude

$$\|\overrightarrow{AB}\| = \sqrt{6^2 + 6^2 + 3^2} = 9$$

$$\|\overrightarrow{BC}\| = \sqrt{(-1)^2 + (-5)^2 + (-6)^2} = \sqrt{62}$$

$$\|\overrightarrow{AC}\| = \sqrt{5^2 + 1^2 + (-3)^2} = \sqrt{35}$$

Then, substituting in our vectors and simplifying, our inequality becomes

$$(\sqrt{35})^2 \leq 9^2 + (\sqrt{62})^2$$

$$35 \leq 143$$

2.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

We can find the eigenvectors and eigenvalues of the matrix.

We know that for matrix A , there is a λ for which $\det(A - \lambda I) = 0$. We can find the determinant with cofactor expansion and solve for λ

$$\begin{aligned} 0 &= \det \begin{bmatrix} 1 - \lambda & 2 & 3 \\ 4 & 5 - \lambda & 6 \\ 7 & 8 & 9 - \lambda \end{bmatrix} \\ &= (1 - \lambda)((5 - \lambda)(9 - \lambda) - (8 \cdot 6)) - 2(4(9 - \lambda) - 7 \cdot 6) + 3(4 \cdot 8 - 7(5 - \lambda)) \\ &= -\lambda^3 + 15\lambda^2 - 11\lambda - 3 - 8\lambda - 12 + 21\lambda - 9 \\ &= -\lambda^3 + 15\lambda^2 + 18\lambda \\ &= -\lambda(\lambda^2 - 15\lambda - 18) \end{aligned}$$

We have one root, $\lambda = 0$. From here, we can use the quadratic formula to find the other roots.

$$\lambda = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(-18)}}{2} = \frac{15 \pm 3\sqrt{33}}{2}$$

Therefore, our eigenvalues are $\lambda = 0, \frac{15 \pm 3\sqrt{33}}{2}$

Next, to find our eigenvectors, we can use Gaussian Elimination. For each eigenvalue λ , we have $(A - \lambda I)\vec{x} = \vec{0}$. To find \vec{x} , we can create an augmented matrix consisting of $A - \lambda I|0$.

First, we let $\lambda = 0$.

$$\left[\begin{array}{ccc|c} 1-0 & 2 & 3 & 0 \\ 4 & 5-0 & 6 & 0 \\ 7 & 8 & 9-0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array} \right]$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

This gives us the following linear system.

$$x_1 - x_3 = 0$$

$$x_1 + 2x_2 = 0$$

Then, our eigenvector \vec{x} is given by

$$\vec{x} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Next, we can let $\lambda = \frac{15+3\sqrt{33}}{2}$

$$\left[\begin{array}{ccc|c} 1 - \frac{15+3\sqrt{33}}{2} & 2 & 3 & 0 \\ 4 & 5 - \frac{15+3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15+3\sqrt{33}}{2} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 4 & 5 - \frac{15+3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15+3\sqrt{33}}{2} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 0 & \frac{-9\sqrt{33}-33}{8} & \frac{9\sqrt{33}+57}{16} & 0 \\ 0 & \frac{21\sqrt{33}+165}{32} & -\frac{33\sqrt{33}-177}{64} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 0 & 1 & \frac{-3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{11-3\sqrt{33}}{22} & 0 \\ 0 & 1 & \frac{-3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

This gives us the following systems of equations.

$$x_1 + \frac{11-3\sqrt{33}}{22}x_3 = 0$$

$$x_2 + \frac{-3\sqrt{33}-11}{44}x_3 = 0$$

Thus,

$$\vec{x} = x_3 \begin{bmatrix} \frac{-11+3\sqrt{33}}{22} \\ \frac{3\sqrt{33}+11}{44} \\ 1 \end{bmatrix}$$

Finally, we can let $\lambda = \frac{15-3\sqrt{33}}{2}$

$$\left[\begin{array}{ccc|c} 1 - \frac{15-3\sqrt{33}}{2} & 2 & 3 & 0 \\ 4 & 5 - \frac{15-3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15-3\sqrt{33}}{2} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{3\sqrt{33}+13}{32} & \frac{9\sqrt{33}+39}{64} & 0 \\ 4 & 5 - \frac{15-3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15-3\sqrt{33}}{2} & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13-3\sqrt{33}} & -\frac{6}{13-3\sqrt{33}} & 0 \\ 0 & 1 & \frac{3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{11+3\sqrt{33}}{22} & 0 \\ 0 & 1 & \frac{3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

This gives us the following systems of equations.

$$x_1 + \frac{11+3\sqrt{33}}{22}x_3 = 0$$

$$x_2 + \frac{3\sqrt{33}-11}{44}x_3 = 0$$

Thus,

$$\vec{x} = x_3 \begin{bmatrix} -\frac{11+3\sqrt{33}}{22} \\ -\frac{3\sqrt{33}-11}{44} \\ 1 \end{bmatrix}$$

Therefore, our eigenvalues are $\lambda = 0, \frac{15 \pm 3\sqrt{33}}{2}$,

and our eigenvectors are $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} \frac{-11+3\sqrt{33}}{22} \\ \frac{3\sqrt{33}+11}{44} \\ 1 \end{bmatrix}$, $\begin{bmatrix} -\frac{11+3\sqrt{33}}{22} \\ -\frac{3\sqrt{33}-11}{44} \\ 1 \end{bmatrix}$

Reflection

As a High School student and aspiring Data Scientist, MATH270 is opening my eyes to the rigors of postsecondary education. In previous mathematics courses, it is a long journey of learning before being able to apply concepts to real-world concepts. However, I have been able to apply linear algebra in my everyday programming.

Linear Algebra in particular is used widely in machine learning, where efficient matrix operations are used to backpropagate, or make machines "learn". Knowing matrix operations, especially the difference between dot product and cross product, has allowed me to be more efficient in programming and debugging. Other concepts are also widely used, such as rank, the number of linearly independent rows/columns of a matrix, which is used for Principal Component Analysis (PCA). PCA is a tool that is used in machine learning to reduce data dimensionality. For example, the rank of the observation matrix can tell you if it is possible to find a unique solution for your model. If that were the case and you had a full-rank matrix you could just invert the matrix (for linear problems, and assuming it is square) and use that as you model. While this almost never happens, PCA can often filter which data points are most useful, and reduce noise. PCA can also use eigendecomposition, where matrices are broken down into eigenvalues and eigenvectors to simplify other operations, such as powers, without using as many computer resources.

Euclidean distance is used in machine learning regularization. L2 regression, or Ridge regression, is a technique used to regularize neural network to keep the coefficients of the model small and, in turn, the model less complex. This way, less computing power is needed as smaller numbers are multiplied together. L2 regularization also normalizes data, allowing neural networks to generalize better on data they have never encountered before.

Another topic I found interesting is the determinant - the fact that there is a scalar representation of a matrix, or more specifically, the product of all eigenvalues of a matrix. The determinant represents the multiplicative change when transforming the space using a matrix. However, linear algebra's uses extend far beyond machine learning. Topics such as traffic flow, Leontief economic models, and cryptography all use linear algebra. There are also many topics I would like to explore in the future - Markov chains and graph theory. Overall, I have really enjoyed MATH270 - especially its applicability to my career of interest. Linear algebra has expanded my intuition for the building blocks of machine learning and AI.