Athabasca University

MATH 270

# Midterm

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Solve the following system of 3 linear equations using Gaussian and Gauss-Jordan elimination.

$$
\begin{aligned}
x + 2y + 3z &= 9 \\
4x + 5y + 6z &= 24 \\
3x + y - 2z &= 4
\end{aligned}
$$

We can begin by manipulating this system into an augmented matrix.

$$
\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 4 & 5 & 7 & 24 \\ 3 & 1 & -2 & 4 \end{array}\right]
$$

Next, we can begin manipulating this augmented matrix into row echelon form by adding $-4R_1$ to $R_2$ and adding $-3R_1$ to $R_3$. This creates a leading 1.

$$
\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & -3 & -6 & -12 \\ 0 & -5 & -11 & -23 \end{array}\right]
$$

Finally, we let $R_2 = \frac{R_2}{-3}$ and $R_3 = 5R_2 + R_3$, and our matrix is in row echelon form.

$$
\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]
$$

From here, we can either solve for the leading variables (Gauss-Jordan elimination) or manipulate the matrix into reduced row echelon form (Gaussian elimination). We will begin with Gaussian elimination by adding $-2R_2$ to $R_1$

$$
\left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]
$$

Finally, we let $R_1 = R_3 + R_1$ and $R_2 = -2R_3 + R_2$.

$$
\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]
$$

Our solutions are $\boxed{(x, y, z) = (4, -2, 3)}$. To solve using Gaussian-Jordan elimination, we can solve for the leading variables from our augmented matrix in row echelon form.

$$
\left[\begin{array}{ccc|c} 1 & 2 & 3 & 9 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]
$$

$$
\begin{aligned}
x + 2y + 3z &= 9 \\
y + 2z &= 4 \\
z &= 3
\end{aligned}
$$

Substituting $z = 3$, we get $y + 2(3) = 4$, so $y = -2$. Then, substituting $y = -2$ and $z = 3$ into the first equation, we have $x + 2(-2) + 3(3) = 9$. Therefore, we have $\boxed{(x, y, z) = (4, -2, 3)}$