Assignment 3

Complete this assignment after you have finished Unit 4, and submit your work to your tutor for grading.

Total points: **60**
Weight: **10%**

(8 points)

1. Sketch the graph of the function f and its derivative function f' where

$$f(x) = \begin{cases} 4 & \text{if } x \leq -2 \\ x^2 & \text{if } -2 \leq x \leq 3 \\ \dfrac{2}{x-3} & \text{if } x > 3 \end{cases}$$

(16 points)

2. In each of the cases below, give the indicated derivative. You may not need to simplify your answers.

   a. $\dfrac{d}{dx} x \cos\sqrt{x-3}$

   b. $\dfrac{d^2}{x^2} x^2 \tan x \Big|_{x=\pi}$

   c. $\dfrac{d}{dx} \dfrac{\sin x - \cos x}{x^3}$

   d. If $f(0) = 1$, $f'(0) = 2$, $g(0) = 0$ and $g'(0) = -1$, find

   $$\frac{d}{dx} \frac{f(x) - x^2 g(x)}{f(x) + g(x)}\Bigg|_{x=0}$$

(8 points)

3. A softball diamond is a square whose sides are 18 m long. Suppose that a player running from first to second base has a speed of 7.5 m/s at the instant she is 3 m from second base. At what rate is the player's distance from home plate changing at that instant?

   Hint. Draw a diagram and locate the variables that change with respect to time.

(5 points)

4. Let $f(x) = \dfrac{x-1}{\sqrt{x}}$.

   a. Find the equation of the tangent line of the function f at the point $(4, f(4))$.

   b. Use differentials to estimate the value of $f(4.02)$.

(6 points)

5. Use implicit differentiation to prove that the curve $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ (see exercise 27 of page 187 of the textbook) has a vertical tangent line at the point (1,0).

(8 points)

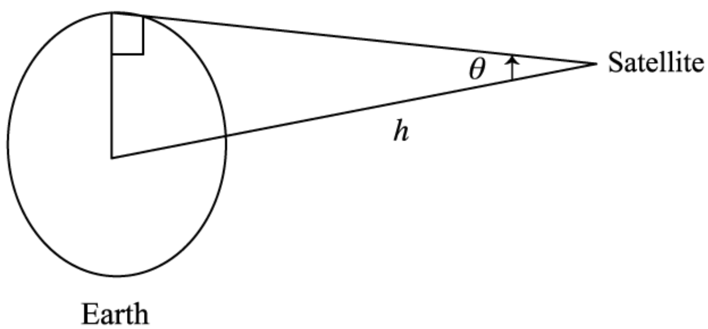
6. One side of a right triangle is known to be 25 cm exactly. The angle opposite to this side is measured to be 60°, with a possible error of ±0.5°.

   a. Use differentials to estimate the errors in the adjacent side and the hypothenuse.

   b. Estimate the percentage errors in the adjacent side and hypothenuse.

(9 points)

7. An Earth-observing satellite can see only a portion of the Earth's surface. The satellite has horizon sensors that can direct the angle θ shown in the accompanying figure. Let r be the radius of the Earth (assumed spherical) and h the distance of the satellite from the Earth's surface.



   a. Show that $h = r(\csc\theta - 1)$.

   b. Using $r = 6378$ km, find the average rate of change of the distance from the satellite to the surface of the Earth, when θ changes from $\pi/4$ to $\pi/3$. What are the units?

   c. At what rate is the distance h changing when $\theta = \pi/6$?