

## Linear Algebra MATH270 Midterm Cheat Sheet

One parameter linear equation means that there is one variable in the solution. Eg.  $\begin{bmatrix} 1 & 0 & \frac{2}{a} & \frac{2}{a} \\ 0 & 1 & \frac{2}{a} & \frac{2}{a} \end{bmatrix}$

**Leontief** consumption matrices are productive when  $(I - C)^{-1}$ . The greatest dollar value is the sector that requires the largest amount of inputs from the other sectors.

**Symmetrical** matrices are their own transposes.

**Diagonal** matrices consist of only the main diagonal. It is invertible iff all diagonal entries are nonzero.

**Triangular** matrices consist of 0's below or above the main diagonal. They can be used to solve systems through back or forwards substitution.

### Properties of matrix arithmetic

- a)  $A + B = B + A$
- b)  $A + (B + C) = (A + B) + C$
- c)  $A(BC) = (AB)C$
- d)  $A(B + C) = AB + AC$
- e)  $(B + C)A = BA + CA$
- f)  $a(B + C) = aB + aC$
- g)  $(a + b)C = aC + bC$
- h)  $a(bC) = (ab)C$
- i)  $a(BC) = (aB)C = B(aC)$
- j)  $(A + B)^2 = A^2 + AB + BA + B^2$

### Properties of inverse matrices

- a)  $(AB)^{-1} = B^{-1}A^{-1}$
- b)  $(A^T)^{-1} = (A^{-1})^T$

Equivalent statements theorem (all true or all false)

- $A$  is invertible.
- $A\mathbf{x} = \mathbf{0}$  has only the trivial solution
- The reduced row echelon form of  $A$  is  $I_n$
- $A$  can be expressed as a product of elementary matrices.
- $A\mathbf{x} = \mathbf{b}$  is consistent for every  $n \times 1$  matrix  $\mathbf{b}$  and has exactly one solution

If  $A$  is an **invertible**  $m \times n$  matrix, then for each  $n \times 1$  matrix  $\mathbf{b}$ , the systems of equations  $A\mathbf{x} = \mathbf{b}$  has exactly one solution, namely,  $\mathbf{x} = A^{-1}\mathbf{b}$

A matrix is called **linearly independent** if the vector equation  $x_1\mathbf{v}_1 + \dots + x_n\mathbf{v}_n = \mathbf{0}$  has **only** the trivial solution. If it has more solutions, it is not linearly independent.

For a **homogenous** system, the system either has only the trivial solution, or more than one solution.

For a non-homogenous system, either the system has a single unique solution, more than one solution, or no solution at all.

A homogenous system of  $m$  linear equations in  $n$  unknowns always has a non-trivial solution if  $m < n$ .

A system is **consistent** when it has at least one solution. If the system  $x_1v_1 + \dots + x_nv_n = \mathbf{b}$ , we say that  $\mathbf{b}$  is a linear combination of the vectors. No solutions means the system is inconsistent.

A **consistent** system will have a **unique** solution if and only if the columns of the coefficient matrix are linearly independent vectors (if the homogenous linear equation has non-trivial solutions)

Extras:

A system of linear equations with coefficient matrix  $A$  will be inconsistent for certain values on the right hand side if the row echelon form of  $A$  contains a row of zeros. If the row echelon form of the coefficient matrix  $A$  does not contain a row of zeros, then the system is always consistent, regardless of what the right hand side is.

A system has infinite solutions if it has arbitrary variables

Diagonal matrices can be raised to the power like an integer.

Assignment 1 Q1 (Parametric equations?), Q3 (Try this, no fractions), Q4 (Try this, see if it is defined and find rules), Q5 (Try this)

Assignment 2 Q2b & Q2c (Try this), Q3 (Theorem 1.6.2)

## Stanley Zheng - MATH270 Reflection

First of all, my sincere apologies for the delay. I should have read the expectations prior to handing in my solution, and I hope you will accept this reflection.

Throughout MATH270, I've learned not only about linear algebra, but the ins-and-outs of remote learning and post-secondary education. I am a high school student interested in computer science (data science and AI in particular), and linear algebra has been very eye-opening to me. First of all, the way the proofs differ from other math I have done in class; instead of being taught rules to memorize, we are taught proofs for rules that are intuitive. I am still very new to proofs and unsure how to eloquently write my own proofs, but I prefer this hands-on style of learning. The proofs that are taught are elegant, though some of the notation is still a bit alien to me.

I most enjoyed learning about properties of operations and inverse matrices in linear algebra. In the field of artificial intelligence, matrix operations are performed rapidly and a crucial part of building intuition. Recommender neural networks use matrix factorization, and linear algebra is crucial for linear regression. I have had to use the transposes of matrices and matrix multiplication to find optimal image resolutions, among others. The future units, especially eigenvectors and eigenvalues, also have an application in every AI, which I am excited to learn about. Linear algebra has helped me build an intuition for how the code I write works, and how I can improve it. I also enjoyed learning about Leontief consumption matrices, as they are another real-world application of linear algebra. It is inspiring to me to think that the mathematics I am doing has a real-world impact and was used by many great minds before.

Among the topics I have explored while studying for the midterm are many definitions. When I started this course, my friend told me that formulas are to calculus what definitions are to linear algebra. They're both absolutely crucial.

I had some troubles working through the definitions, as many sounded similar, but had vastly different definitions, for example, consistent and dependent systems. I therefore spent a lot of time learning definitions, about what a trivial solution is (a 0 vector), the definition of a linearly dependent and independent matrix, properties of homogenous systems, one or two parameter solutions, and when matrices are consistent and have unique solutions.

There were also many cases where I thought back on my previous mathematic experiences including tests or math contests, where I could have applied linear algebra, particularly Gaussian elimination, where I could have solved a problem much quicker and more efficiently with techniques I now know. Thus far, this course, though vastly different from any math courses I have taken to date, has helped me build my mathematical intuition and logic. I look forwards to completing the second half of the class in a timely manner.