Athabasca University

MATH 270

Final

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1. Let $A = (a,b,c), B = (d,e,f), C = (g,h,i)$. We can now find the individual magnitudes $\overrightarrow{A \cdot B}, \overrightarrow{B \cdot C}, \overrightarrow{A \cdot C}$

$$\overrightarrow{AB} = A - B = (a,b,c) - (d,e,f) = (a-d, b-e, c-f)$$

$$\overrightarrow{BC} = B - C = (d,e,f) - (g,h,i) = (d-g, e-h, f-i)$$

$$\overrightarrow{AC} = A - C = (a,b,c) - (g,h,i) = (a-g, b-h, c-i)$$

Then, we can find the magnitude

$$||\overrightarrow{AB}|| = \sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$$

$$||\overrightarrow{BC}|| = \sqrt{(d-g)^2 + (e-h)^2 + (f-i)^2}$$

$$||\overrightarrow{AC}|| = \sqrt{(a-g)^2 + (b-h)^2 + (c-i)^2}$$

Our inequality is $||\overrightarrow{AC}||^2 \leq ||\overrightarrow{AB}||^2 + ||\overrightarrow{BC}||^2$. Substituting in, we have

$$(a-d)^2 + (b-e)^2 + (c-f)^2 \leq (d-g)^2 + (e-h)^2 + (f-i)^2 + (a-g)^2 + (b-h)^2 + (c-i)^2$$

This vector is similar to the pythagorean theorem/inequality

2.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

We can find the eigenvectors and eigenvalues of the matrix.

We know that for matrix A, there is a λ for which $\det(A - \lambda I) = 0$. We can find the determinant with cofactor expansion and solve for λ

$$0 = \det \begin{bmatrix} 1-\lambda & 2 & 3 \\ 4 & 5-\lambda & 6 \\ 7 & 8 & 9-\lambda \end{bmatrix}$$

$$= (1-\lambda)((5-\lambda)(9-\lambda) - (8 \cdot 6)) - 2(4(9-\lambda) - 7 \cdot 6) + 3(4 \cdot 8 - 7(5-\lambda))$$

$$= -\lambda^3 + 15\lambda^2 - 11\lambda - 3 - 8\lambda - 12 + 21\lambda - 9$$

$$= -\lambda^3 + 15\lambda^2 + 18\lambda \qquad = -\lambda(\lambda^2 - 15\lambda - 18)$$

We have one root, $\lambda = 0$. From here, we can use the quadratic formula to find the other roots.

$$\lambda = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(-18)}}{2} = \frac{15 \pm 3\sqrt{33}}{2}$$

Therefore, our eigenvalues are $\lambda = 0, \frac{15 \pm 3\sqrt{33}}{2}$

Next, to find our eigenvectors, we can use Gaussian Elimination. For each eigenvalue λ, we have $(A - \lambda I)\vec{x} = \vec{0}$. To find $\vec{x}$, we can create an augmented matrix consisting of $A - \lambda I | 0$.

First, we let $\lambda = 0$.

$$\left[\begin{array}{ccc|c} 1-0 & 2 & 3 & 0 \\ 4 & 5-0 & 6 & 0 \\ 7 & 8 & 9-0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -12 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This gives us the following linear system.

$$x_1 - x_3 = 0$$

$$x_1 + 2x_2 = 0$$

Then, our eigenvector $\vec{x}$ is given by

$$\vec{x} = \begin{bmatrix} x_3 \\ -2x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Next, we can let $\lambda = \frac{15+3\sqrt{33}}{2}$

$$\left[\begin{array}{ccc|c} 1 - \frac{15+3\sqrt{33}}{2} & 2 & 3 & 0 \\ 4 & 5 - \frac{15+3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15+3\sqrt{33}}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 4 & 5 - \frac{15+3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9 - \frac{15+3\sqrt{33}}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 0 & \frac{-9\sqrt{33}-33}{8} & \frac{9\sqrt{33}+57}{16} & 0 \\ 0 & \frac{21\sqrt{33}+165}{32} & -\frac{-33\sqrt{33}-177}{64} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -\frac{4}{13+3\sqrt{33}} & -\frac{6}{13+3\sqrt{33}} & 0 \\ 0 & 1 & \frac{-3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{11-3\sqrt{33}}{22} & 0 \\ 0 & 1 & \frac{-3\sqrt{33}-11}{44} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

This gives us the following systems of equations.

$$x_1 + \frac{11 - 3\sqrt{33}}{22} x_3 = 0$$

$$x_2 + \frac{-3\sqrt{33} - 11}{44} x_3 = 0$$

Thus,

$$\vec{x} = x_3 \begin{bmatrix} -\frac{11-3\sqrt{33}}{22} \\ -\frac{-3\sqrt{33}-11}{44} \\ 1 \end{bmatrix}$$

Finally, we can let $\lambda = \frac{15-3\sqrt{33}}{2}$

$$\left[\begin{array}{ccc|c} 1-\frac{15-3\sqrt{33}}{2} & 2 & 3 & 0 \\ 4 & 5-\frac{15-3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9-\frac{15-3\sqrt{33}}{2} & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{3\sqrt{33}+13}{32} & \frac{9\sqrt{33}+39}{64} & 0 \\ 4 & 5-\frac{15-3\sqrt{33}}{2} & 6 & 0 \\ 7 & 8 & 9-\frac{15-3\sqrt{33}}{2} & 0 \end{array}\right]$$