



ATHABASCA UNIVERSITY

MATH 270

Assignment 4

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1. a) $\vec{v} + \vec{w} = (0 + 7, 1 + 4, -4 + (-1), 1 + (-2), 3) = (7, 5, -5, -1, 5)$
 b) $3(2\vec{u} - \vec{v}) = 3(2(1, 2, -3, 5, 0) - (0, 4, -1, 1, 2))$
 c) $(3\vec{u} - \vec{v}) - (2\vec{u} + 4\vec{w})$
 d) $\frac{1}{2}((7, 1, -4, -2, 3) - 5(0, 4, -1, 1, 2) + 2(1, 2, -3, 5, 0)) + (1, 2, -3, 5, 0)$
2. a i) The Euclidean distance can be found with the equation $\|u-v\| = \sqrt{(u-v) \cdot (u-v)}$.
 Substituting our vectors, we have $u-v = (1, 2, -3, 0) - (5, 1, 2, -2) = (-4, 1, -1, 2)$.
 The Euclidean distance is

$$\sqrt{(-4, 1, -1, 2) \cdot (-4, 1, -1, 2)} = \sqrt{16 + 1 - 1 + 4} = \sqrt{20}$$

To find the angle, we must find $\|u\|$ and $\|v\|$ by square rooting and squaring.

$$\|u\| = \sqrt{u^2} = \sqrt{1 \cdot 1 + 2 \cdot 2 + (-3) \cdot (-3) + 0 \cdot 0} = \sqrt{14}$$

$$\|v\| = \sqrt{v^2} = \sqrt{5 \cdot 5 + 1 \cdot 1 + 2 \cdot 2 + (-2) \cdot (-2)} = \sqrt{34}$$

We know that $\cos \theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$.

$$\frac{(1, 2, -3, 0) \cdot (5, 1, 2, -2)}{\sqrt{14} \cdot \sqrt{34}} = \frac{5 + 2 - 3 + 0}{\sqrt{476}} = \frac{1}{\sqrt{476}} = \frac{1}{2\sqrt{119}}$$

Since $\cos \theta > 0$, we know that the angle between the vectors is acute.

- ii) We can use the same methodology as above.

$$u - v = (0, 1, 1, 1, 2) - (2, 1, 0, -1, 3) = (-2, 0, 1, 2, -1)$$

Our Euclidean distance is

$$\sqrt{(-2, 0, 1, 2, -1) \cdot (-2, 0, 1, 2, -1)} = \sqrt{4 + 0 + 1 + 4 + 1} = 10$$

We know that $\cos \theta = \frac{u \cdot v}{\|u\| \cdot \|v\|}$. Substituting in, we have

$$\begin{aligned} \cos \theta &= \frac{(0, 1, 1, 1, 2) \cdot (2, 1, 0, -1, 3)}{\sqrt{(0, 1, 1, 1, 2) \cdot (0, 1, 1, 1, 2)} \cdot \sqrt{(2, 1, 0, -1, 3) \cdot (2, 1, 0, -1, 3)}} \\ &= \frac{0 + 1 + 0 + -1 + 6}{\sqrt{0 + 1 + 1 + 1 + 4} \cdot \sqrt{4 + 1 + 0 + 1 + 9}} \\ &= \frac{6}{\sqrt{105}} \end{aligned}$$

Again, since $\cos \theta > 0$, the angle θ is acute.

- b i) We can find whether $|u \cdot v| < \|u\| \|v\|$.

$$\|u\| = \sqrt{4 \cdot 4 + 1 \cdot 1 + 1 \cdot 1} = \sqrt{18}$$

$$\|v\| = \sqrt{1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3} = \sqrt{14}$$

$$u \cdot v = 4 \cdot 1 + 1 \cdot 2 + 1 \cdot 3 = 9$$

Then, $|u \cdot v| = 9 < \sqrt{252} = \|u\| \|v\|$, so therefore, the Cauchy-Schwarz inequality holds.

ii) Again, we can calculate $\|u\|$, $\|v\|$, and $|u \cdot v|$.

$$\|u\| = \sqrt{1 \cdot 1 + 2 \cdot 2 + 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3} = \sqrt{19}$$

$$\|v\| = \sqrt{0 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 + 5 \cdot 5 + (-2) \cdot (-2)} = \sqrt{31}$$

$$u \cdot v = \sqrt{1 \cdot 0 + 2 \cdot 1 + 1 \cdot 1 + 2 \cdot 5 + (-2) \cdot 3} = \sqrt{7}$$

Then, $|u \cdot v| = 7 < \sqrt{539} = \|u\|\|v\|$, so therefore, the Cauchy-Schwarz inequality holds.

3. a) We can begin by finding a point on either plane. For the plane $2x - y + z = 1$, we can let $x, z = 0$ so we find point $(0, -1, 0)$.

Next, we need to find the distance between this point and the second plane, $2x - y + z = -1$. We can an equation to find the distance between this point and the plane.

$$\frac{|(2 \cdot 0 + (-1) \cdot (-1) + 1 \cdot 0 + 1)|}{\sqrt{2^2 + (-1)^2 + 1^2}} = \boxed{\frac{2}{\sqrt{6}}}$$

- b) Two vectors are orthogonal if $|a \cdot b| = 0$.

$$|a \cdot b| = -ab + ab = 0$$

Therefore, these two vectors are orthogonal.

We can set the dot product of \vec{v} and a second vector, $\vec{u} = (a, b)$ to 0.

$$|(a, b) \cdot (2, -3)| = 0$$

$$2a - 3b = 0$$

A few possible values include $(a, b) = (3, 2)$ or $(a, b) = (6, 4)$

- ii) We begin by finding orthogonal vectors to $(-3, 4)$ like the previous question.

$$-3a + 4b = 0$$

One possible solution is $(a, b) = (4, 3)$ or $(a, b) = (8, 6)$. To find the unit vector, we multiply a vector by the reciprocal of its magnitude.

$$\frac{1}{\|(4, 3)\|}(4, 3) = \frac{1}{\sqrt{4 \cdot 4 + 3 \cdot 3}}(4, 3) = \frac{1}{5}(4, 3) = \left(\frac{4}{5}, \frac{3}{5}\right)$$

Since the original vector, $(-3, 4)$ is of dimension 2, we can make our unit vector negative and it will still be orthogonal. As such, our two unit vectors are

$$\boxed{\left(\frac{4}{5}, \frac{3}{5}\right), \left(-\frac{4}{5}, -\frac{3}{5}\right)}$$

4. a) We can begin by manipulating this system into an augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & 0 \\ 1 & 2 & 3 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -4 & 0 \\ 0 & 1 & -7 & 0 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 17 & 0 \\ 0 & 1 & -7 & 0 \end{array} \right]$$

Therefore, our solutions are $x_1 = -17x_3$ and $x_2 = 7x_3$. We can form a vector: $(x_1, x_2, x_3) = (-17x_3, 7x_3, a)$. Finally, we can check whether this vector is orthogonal to the coefficient vectors. Note that $a = x_3$

$$1(-17x_3) + 3(7x_3) - 4(a) = -17x_3 - 4a + 21x_3 = 0$$

$$1(-17x_3) + 2(7x_3) + 3(a) = -17x_3 + 14x_3 + 3a = 0$$

b) i)

$$-3x_1 + 2x_2 - 1x_3 = 0$$

$$-2x_2 - 2x_3 = 0$$