Athabasca University

MATH 265

Assignment 1

Stanley Zheng

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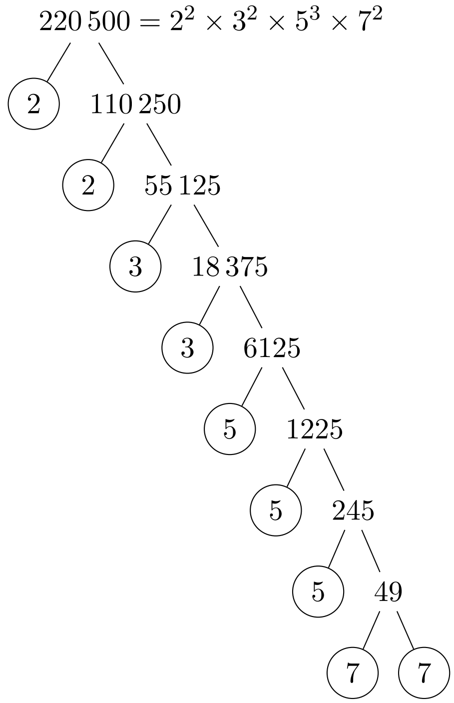
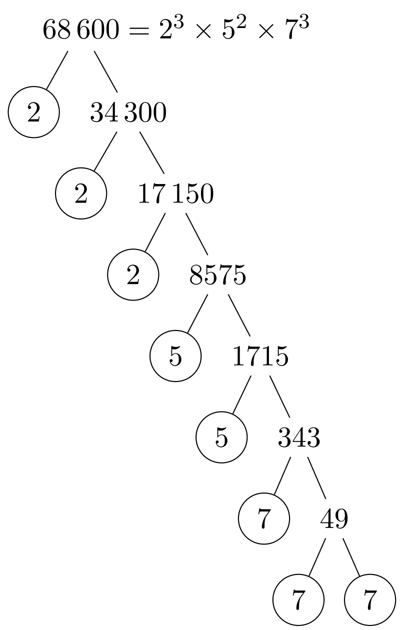
1. (a) We can start by prime factoring the integer 220500.



We can then square root the prime factors and simplify.

$$\sqrt{2^2 \times 3^2 \times 5^3 \times 7^2} = 2 \times 3 \times 5 \times 7\sqrt{5} = \boxed{210\sqrt{5}}$$

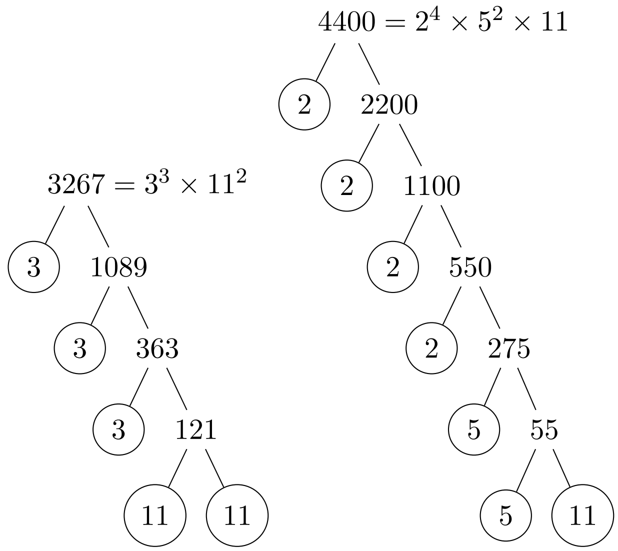
(b) Again, we can start by prime factorizing.



Next, we can cube root the factors of 68600 and simplify.

$$\sqrt[3]{2^3 \times 5^2 \times 7^3} = \boxed{14\sqrt[3]{25}}$$

(c) Again, we can start by prime factoring our two integers, 3267 and 4400.



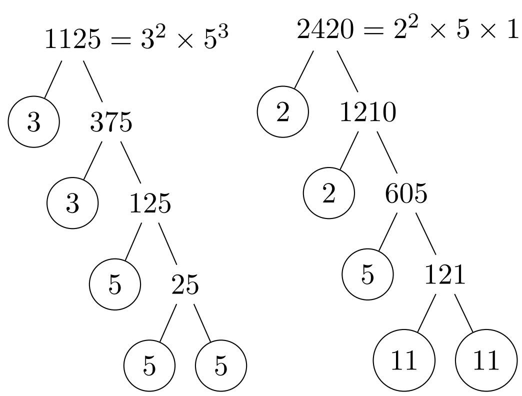
Next, we can simplify each radical on its own.

$$3\sqrt[3]{11^2} \times 2^2 \times 5 \times \sqrt{11} = 3 \times 11^{2/3} \times 20 \times 11^{1/2}$$

Since we can add the exponents of like terms when multiplying, our equation becomes

$$60 \times 11 \times 11^{1/6} = \boxed{660\sqrt[6]{11}}$$

(d) We can start by prime factorizing 1125 and 2420



Next, we can swap the prime factors into the radicals and simplify.

$$\sqrt{1125} + \sqrt{2420} = 3 \times 5\sqrt{5} + 2 \times 11\sqrt{5} = \boxed{37\sqrt{5}}$$

2.

Interval	Inequality	Representation on the Real Line
$(-\infty, -1]$	$x \leq -1$	
$(\sqrt{3}, \infty)$	$x > \sqrt{3}$	
$(-3, \pi)$	$-3 < x < \pi$	
$[\frac{3}{5}, \infty)$	$\frac{3}{5} \leq x$	

3. (a) $[-2, \infty)$ since the question specifies more than including negative two.
   (b) $(0, 2]$ since the question specifies "and", we must choose the more restrictive interval.
   (c) $(3, \infty)$ since the question specifies "or", we must choose the less restrictive interval.

4. (a) We can begin by simplifying the fraction inside of the parenthesis.

$$\left(\frac{5x^3yz^2}{xy^2z}\right)^{-\frac{3}{2}} = (\frac{5x^2z}{y})^{-\frac{3}{2}}$$
$$= \frac{y^{\frac{3}{2}}}{5^{\frac{3}{2}}x^3z^{\frac{3}{2}}}$$

   (b) We can start by expanding $(uv^2w^3 - u^2w)^2$ and then multiplying by v^{-2}.

$$(uv^2w^3 - 3u^2w)^2(v^{-2}) = (uv^2w^3 - 3u^2w) \times (uv^2w^3 - 3u^2w) \times (v^{-2})$$
$$= (u^2v^4w^6 - 3u^3v^2w^4 - 3u^3v^2w^4 + 9u^4w^2)(v^{-2})$$
$$= u^2v^2w^6 - 6u^3w^4 + \frac{9u^4w^2}{v^2}$$

   (c) We can start by converting all of the radicals into fractional exponents in order to make it easier to visualize and add.

$$(\sqrt{x^3} + x^2y^{-2}z)(xy^2z^3) = (x^{\frac{3}{2}} + x^2y^{-2}z)(xy^2z^3)$$
$$= x^{\frac{5}{2}}y^2z^3 + x^3z^4$$

5. (a) We can start by multiplying out the left side of the subtraction and the right side of the subtraction, then grouping like terms.

$$\sqrt{2}(3x - \sqrt{2}x^2 + 1) - \sqrt{18}(1 - 4x)^3 = (3\sqrt{2}x - \sqrt{2}\sqrt{2}x^2 + 1\sqrt{2}) - \sqrt{18}(1 - 4x)(16x^2 - 8x + 1)$$
$$= (3\sqrt{2}x - 2x^2 + \sqrt{2}) - \sqrt{18}(-64x^3 + 48x^2 - 12x + 1)$$
$$= (3\sqrt{2}x - 2x^2 + \sqrt{2}) - (-64\sqrt{18}x^3 + 48\sqrt{18}x^2 - 12\sqrt{18} + \sqrt{18})$$

$\sqrt{18}$ can be simplified into $3\sqrt{2}$

$$= (3\sqrt{2}x - 2x^2 + \sqrt{2}) - (-64 \cdot 3\sqrt{2}x^3 + 48 \cdot 3\sqrt{2}x^2 - 12 \cdot 3\sqrt{2} + 3\sqrt{2})$$
$$= (3\sqrt{2}x - 2x^2 + \sqrt{2}) - (-192\sqrt{2}x^3 + 144\sqrt{2}x^2 - 36\sqrt{2} + 3\sqrt{2})$$
$$= 192\sqrt{2}x^3 - 144\sqrt{2}x^2 - 2x^2 + 39\sqrt{2}x - 2\sqrt{2}$$

(b) We can begin by expanding the left side and right side of the addition sign, then grouping like terms.

$$\begin{aligned}(t-u)^2+5(3t-u+4u^2)(1+u) &= (t^2-2tu+u^2)+(4u^3+3u^2+3tu+3t-u)\\ &= (t^2-2tu+u^2)+(20u^3+15u^2+15tu+15t-5u)\\ &= 20u^3+16u^2+t^2+13tu+15t-5u\end{aligned}$$

(c) We can start by expanding the cubed trinomial, then multiplying by the binomial and grouping like terms.

$$\begin{aligned}(1-3x+x^2)^3(2-2x^2) &= (x^4-6x^3+11x^2-6x+1)(1-3x+x^2)(2-2x^2)\\ &= (x^6-9x^5+30x^4-45x^3+30x^2-9x+1)(2-2x^2)\\ &= -2x^8+18x^7-58x^6+72x^5-72x^3+58x^2-18x+2\end{aligned}$$

6. (a) We can factor by grouping like terms together

$$\begin{aligned}2y^3+6y^2+y+3 &= 2y^2(y+3)+1(y+3)\\ &= (2y^2+1)(y+3)\end{aligned}$$

(b) We can start by taking out a common factor of 3

$$\begin{aligned}3x^2-18xy+24y^2 &= 3(x^2-6xy+8y^2)\\ &= 3(x-4y)(x-2y)\end{aligned}$$

(c) Again, we can start by taking out a common factor of $2x$. The resulting trinonmial is a perfect square trinomial, so we factor it as such

$$\begin{aligned}50x^3+20x^2+2x &= 2x(25x^2+10x+1)\\ &= 2x(5x+1)^2\end{aligned}$$

7. (a) We can start the simplification by factoring all polynomials and eliminating common factors from the fractions. Then, we can multiply both fractions to get a lowest common denominator

$$\begin{aligned}\frac{1}{9x^2-y^2}-\frac{12x^2-10xy+2y^2}{9x^2-6xy+y^2} &= \frac{1}{(3x+y)(3x-y)}-\frac{2(6x^2-5xy+y^2)}{(3x-y)^2}\\ &= \frac{1}{(3x+y)(3x-y)}-\frac{2(2x-y)(3x-y)}{(3x-y)^2}\\ &= \frac{1}{(3x+y)(3x-y)}-\left(\frac{2(2x-y)}{(3x-y)}\right)\left(\frac{3x+y}{3x+y}\right)\\ &= \frac{1}{(3x+y)(3x-y)}-\frac{2(2x-y)(3x+y)}{(3x-y)(3x+y)}\\ &= \frac{-12x^2+2xy+2y^2+1}{9x^2-y^2}\end{aligned}$$

(b) Similarly to the previous question, we can factor all polynomials and simplify the fractions.

$$\begin{aligned}
\frac{\sqrt{x^2+5x+4}}{x^2+8x+16} - \frac{x^2-3x-4}{x^2-16} &= \frac{\sqrt{(x+4)(x+1)}}{(x+4)^2} - \frac{(x-4)(x+1)}{(x+4)(x-4)} \\
&= \frac{\sqrt{(x+4)(x+1)}}{(x+4)^2} - \frac{x+1}{x+4} \\
&= \frac{\sqrt{(x+4)(x+1)}}{(x+4)^2} - \left(\frac{x+1}{x+4}\right)\left(\frac{x+4}{x+4}\right) \\
&= \frac{-x^2-5x-4+\sqrt{x^2+5x+4}}{x^2+8x+16}
\end{aligned}$$

(c) Again, we factor all polynomials and simplify the fractions. However, since the operation between the fractions is now multiplication, we can eliminate factors across both fractions.

$$\begin{aligned}
\left(\frac{9x^3+6x^2+x}{27x^3+1}\right)\left(\frac{6x-1}{3x^2+x}\right) &= \left(\frac{x(3x+1)^2}{(3x+1)(9x^2-3x+1)}\right)\left(\frac{6x-1}{x(3x+1)}\right) \\
&= \frac{6x-1}{9x^2-3x+1}
\end{aligned}$$

8. (a) We can calculate the discriminant with the equation b^2-4ac. If the discriminant is 0, then there is 1 real solution. If the discriminant is more than 0, then the quadratic equation has 2 real solutions, and if the discriminant is less than 0, the quadratic equation has 2 complex solutions. Therefore, if the discriminant is more than 0, we can try to factor the quadratic equation or use the quadratic formula.

$$b^2-4ac = 3^2-4(2)(-6) = 57$$

This equation is not factorable, so we can use the quadratic formula.

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
x &= \frac{-3 \pm \sqrt{3^2-4(2)(-6)}}{2 \cdot 2} \\
x &= \frac{-3 \pm \sqrt{57}}{4}
\end{aligned}$$

(b) Let $y = x^2$. Our equation becomes $y^2+6y-3=0$. Next, we can find the discriminant to see whether this quadratic equation has real solutions.

$$b^2-4ac = 6^2-4(1)(-3) = 48$$

Since this quadratic equation has 2 real solutions and is not factorable, we can use the quadratic formula.

$$\begin{aligned}
y &= \frac{-b \pm \sqrt{b^2-4ac}}{2a} \\
y &= \frac{-6 \pm \sqrt{6^2-4(1)(-3)}}{2 \cdot 1} \\
y &= -3 \pm 2\sqrt{3}
\end{aligned}$$

We substituted $y = x^2$ earlier, so we must substitute back $x^2 = y$.

$$x^2 = -3 \pm 2\sqrt{3}$$
$$x = \pm\sqrt{\pm 2\sqrt{3} - 3}$$

Since the question is asking only for real solutions, our two solutions are $\boxed{x = \pm\sqrt{2\sqrt{3} - 3}}$

(c) We can begin by moving all terms to one side.

$$12x^2 - 8x = 0$$

It is immediately apparent that this equation can be factored into zero pairs, from which we can attain the roots.

$$4x(3x - 2) = 0$$

Looking at the zero pairs, our solutions are $\boxed{x = \frac{2}{3}, 0}$

(d) We can start by multiplying by a lowest common multiple of 6 in order to eliminate the fractions, then finding the discriminant

$$2x^2 + 14x + 15 = 0$$

$$b^2 - 4ac = 14^2 - 4(2)(15) = 76$$

Since the discriminant is positive, we know we have two real solutions. We can then plug the quadratic equation into the quadratic formula since it is not factorable.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-14 \pm \sqrt{14^2 - 4(2)(15)}}{2 \cdot 2}$$
$$x = \frac{-7 \pm \sqrt{19}}{2}$$

(e) We can begin by multiplying by 2 to eliminate the fractions, then move all terms to one side and find the discriminant.

$$3x^2 - 2x + 3 = 0$$

$$b^2 - 4ac = 2^2 - 4(3)(3) = -32$$

Since the discriminant is negative, this quadratic equation has no real roots.

9. (a) In order to rationalize this fraction, we can multiply both the numerator and denominator by the conjugate of the denominator, which in this case, is $5-\sqrt{2}$

$$\begin{aligned}\frac{\sqrt{2}-6}{5+\sqrt{2}} &= \left(\frac{2\sqrt{6}}{5+\sqrt{2}}\right)\left(\frac{5-\sqrt{2}}{5-\sqrt{2}}\right)\\ &= \frac{5\sqrt{2}-\sqrt{2}\times\sqrt{2}-30+6\sqrt{2}}{25-2}\\ &= \frac{11\sqrt{2}-32}{23}\end{aligned}$$

(b) We can start by multiplying by the conjugate of the denominator.

$$\begin{aligned}\frac{5x-2}{\sqrt{2+x}-\sqrt{6x}} &= \left(\frac{5x-2}{\sqrt{2+x}-\sqrt{6x}}\right)\left(\frac{\sqrt{2+x}+\sqrt{6x}}{\sqrt{2+x}+\sqrt{6x}}\right)\\ &= \frac{5x\sqrt{2+x}+5x\sqrt{6x}-2\sqrt{2+x}-2\sqrt{6x}}{2+x-6x}\end{aligned}$$

To continue simplifying, we can factor the numerator by grouping like terms

$$\begin{aligned}\frac{5x\sqrt{2+x}+5x\sqrt{6x}-2\sqrt{2+x}-2\sqrt{6x}}{2+x-6x} &= \frac{5x)(\sqrt{2+x}+\sqrt{6x})-2(\sqrt{2+x}+\sqrt{6x})}{2-5x}\\ &= \frac{-(2-5x)(\sqrt{2+x}+\sqrt{6x})}{2-5x}\\ &= -\sqrt{2+x}-\sqrt{6x}\end{aligned}$$

(c) We can start by multiplying by the conjugate of the denominator. Then, we can factor by grouping like terms and simplify the fraction.

$$\begin{aligned}\frac{\sqrt{8xy^3}+5\sqrt{y}}{2y-\sqrt{y}} &= \left(\frac{\sqrt{8xy^3}+5\sqrt{y}}{2y-\sqrt{y}}\right)\left(\frac{2y+\sqrt{y}}{2y+\sqrt{y}}\right)\\ &= \frac{2y\sqrt{8xy^3}+\sqrt{y}\sqrt{8xy^3}+5\times 2y\sqrt{y}+5\times\sqrt{y}\sqrt{y}}{(2y+\sqrt{y})(2y-\sqrt{y})}\\ &= \frac{\sqrt{8xy^3}(2+\sqrt{y})+5\sqrt{y}(2y+\sqrt{y})}{(2y+\sqrt{y})(2y-\sqrt{y})}\\ &= \frac{(2\sqrt{y}+1)(2y\sqrt{2x})+5}{4y-1}\end{aligned}$$

10. (a) We can start by adding the two fractions.

$$\frac{\pi}{5}+\frac{3\pi}{8}=\frac{8\pi}{40}+\frac{15\pi}{40}=\frac{23\pi}{40}$$

In order to convert from radians into degrees, we multiply by $\frac{180}{\pi}$.

$$\begin{aligned}
\frac{23\pi}{40}\text{rad} &= \left(\frac{23\pi}{40}\right)\left(\frac{180^\circ}{\pi}\right) \\
&= \frac{4140^\circ}{40} \\
&= \boxed{\frac{207^\circ}{2}}
\end{aligned}$$

(b) We can start by multiplying by $\frac{180^\circ}{\pi}$

$$\begin{aligned}
\frac{7\pi}{4} - \sqrt{2}\pi\text{rad} &= \left(\frac{7\pi}{4} - \sqrt{2}\pi\right)\left(\frac{180^\circ}{\pi}\right) \\
&= 315^\circ - 180\sqrt{2}^\circ
\end{aligned}$$

11. (a) In order to convert from degrees to radians, we can multiply by $\frac{\pi}{180}$.

$$\begin{aligned}
\left(\frac{38}{3}\right)^\circ &= \left(\frac{38}{3}\right)^\circ \left(\frac{\pi}{180}\right) \\
&= \frac{38\pi}{540} \\
&= \boxed{\frac{19\pi}{270}\text{rad}}
\end{aligned}$$

(b) Again, we can multiply by $\frac{\pi}{180}$. However, this time, the fraction $\frac{90}{5}$ can be reduced to 18.

$$\begin{aligned}
\left(\frac{90}{5}\right) &= 18^\circ \left(\frac{\pi}{180}\right) \\
&= \frac{18\pi}{180} \\
&= \boxed{\frac{1}{10}\text{rad}}
\end{aligned}$$

12. (a) We can separate the fraction $\frac{7\pi}{6}$ into $\frac{\pi}{6} + \pi$. Since $\tan(a\pi + x) = \tan x$, we have

$$\tan\left(\frac{\pi}{6}\right) = \tan(30^\circ) = \boxed{\frac{1}{\sqrt{3}}}$$

(b) We can use the trigonometric identity of $\cos A \sin B = \frac{1}{2}(\sin(A+B) - \sin(A-B))$. Since we have $A = B$, our equation becomes $\frac{1}{2}\sin(\frac{5\pi}{4})$

The trigonometry unit circle shows that $\sin(\frac{5\pi}{4}) = -\sin(\frac{\pi}{4})$. We know that $\sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.

As a result, we have $\cos\left(\frac{5\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right) = \boxed{-\frac{\sqrt{2}}{4}}$

(c) We can use the trigonometric identity $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$.

$$\frac{1+\cos(\frac{\pi}{4})}{2}$$

We know from the trigonometry unit circle that $\cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ Therefore, we have

$$\cos^2\left(\frac{\pi}{8}\right) = \frac{1+\frac{\sqrt{2}}{2}}{2} = \boxed{\frac{\sqrt{2}+2}{4}}$$

13. We can start by looking at the smaller triangle. Since the circle has a radius of 1, the hypotenuse of the smaller triangle is 1 unit long. We can call the x-axis side of the smaller triangle a and the y-axis side of the smaller triangle b. Then, a is $\cos\theta$, and b is $\sin\theta$. Then, side y on the big triangle is $z\sin\theta$ and side x is $z\cos\theta$. Since the smaller triangle and the bigger triangle are similar triangles, they share the property that the ratio of corresponding sides are equal, as such

$$\frac{z}{1} = \frac{x}{a} = \frac{y}{b}$$

From here, we can substitute a and b for the functions defined earlier, and isolate for $\sin\theta$ and $\cos\theta$.

$$z = \frac{x}{\cos\theta} = \frac{y}{\sin\theta}$$
$$\cos\theta = \frac{x}{z}$$
$$\sin\theta = \frac{y}{z}$$

We know that $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. However, when we divide the numerator and denominator by the hypotenuse, we get

$$\tan\theta = \frac{(\text{opposite})/(\text{hypotenuse})}{(\text{adjacent})/(\text{hypotenuse})} = \frac{\sin\theta}{\cos\theta}$$

Therefore, we have

$$\tan\theta = \frac{\frac{y}{z}}{\frac{x}{z}} = \frac{y}{x}$$