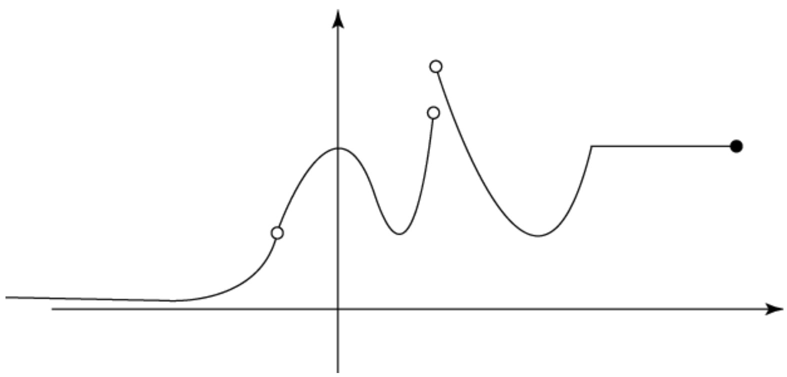
# Assignment 4

Complete this assignment after you have finished Unit 7, and submit your work to your tutor for grading.

Total points: **118**
Weight: 1**0%**

**(6 points)**

1. Consider the graph of the function $g$ shown below. **Hint.** Start by labelling the relevant points on the graph.



   a. What is the domain of the function $g$?

   b. Where is the function continuous?

   c. Identify on the graph the local maximum.

   d. Identify on the graph the local minimum.

   e. Does it have an absolute maximum value? Explain.

   f. Does it have an absolute minimum value? Explain.

**(8 points)**

2. Sketch the graph of one and only one function $f$ which satisfies all the conditions listed below.

   a. $f(-x) = -f(x)$

   b. $\lim_{x \to 4^-} f(x) = \infty$

   c. $\lim_{x \to 4^+} f(x) = -\infty$

   d. $\lim_{x \to \infty} f(x) = 2$

   e. $f''(x) > 0$ on the interval $(0, 4)$

**(8 points)**

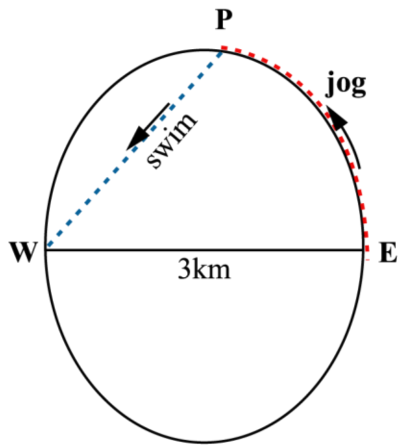
3. Sketch the graph of the function

$$f(x) = \frac{x^2 - 4}{x^2 + 6}$$

   Clearly indicate each of the steps as listed on pages 232-234 of the textbook.

**(8 points)**

4. The shoreline of a lake is a circle with diameter 3 km. Peter stands at point E and wants to reach the diametrically opposite point W. He intends to jog along the north shore to a point P and then swim the straight line distance to W. If he swims at a rate of 3 km/h and jogs at a rate of 24 km/h. How far should he jog in order to arrive at point W in the least amount of time?




**(10 points)**

5. a. Sketch the graphs of the curves $y = \sin x$ and $y = x^2$ showing their points of intersection.

   b. Use the Intermediate Value Theorem to identify an interval where the equation $\sin x - x^2 = 0$ has a non-zero solution.

   c. Use Newton's method to approximate the non-zero solution of the equation $\sin x - x^2 = 0$.

**(4 points)**

6. The velocity of an ant running along the edge of a shelf is modeled by the function

$$v(t) = \begin{cases} 5t, & 0 \le t < 1 \\ 6\sqrt{t}, & 1 \le t \le 2 \end{cases}$$

where $t$ is in seconds and $v$ is in centimeters per second. Estimate the time at which the ant is 4 cm from its starting position.

**(16 points)**

7. Calculate the indefinite integrals listed below

   a. $\displaystyle\int \frac{3x-9}{\sqrt{x^2-6x+1}}\,dx$

   b. $\displaystyle\int \frac{3-\tan\theta}{\cos^2\theta}\,d\theta$

   c. $\displaystyle\int \frac{(2-x+x^2)^2}{\sqrt{x}}\,dx$

   d. $\displaystyle\int \cos^2(3x)\,dx$

**(4 points)**

8. Use the Mean Value Theorem to show that for any real numbers $a$, $b$

$$|\cos a - \cos b| \le |a-b|$$

**(4 points)**

9. Let $f(x) = 3x^3 + \sqrt{x} - 2$.

   a. Find an interval where the function $f$ has one root.

   b. Use Rolle's theorem to show that the function $f$ has exactly one root. **Hint.** See Example 2 on page 283 of the textbook.

**(4 points)**

10. Use the identity $\cos^2 x + \sin^2 x = 1$ to integrate $\int \cos^3 x \sin^2 x \, dx$.

**(12 points)**

11. Evaluate each of the definite integrals listed below

    a. $\int_0^{\pi/6} \cos^2(3x) \, dx$

    b. $\int_0^{\pi} \sin(2x) \sin x \, dx$

    c. $\int_{-2}^{2} x^2 + \cos(2x) \, dx$

**(6 points)**

12. Apply the fundamental theorem of calculus to find the following derivative $\dfrac{d}{dx}\int_{-x}^{x^2} \tan(3t) \, dt$.

**(8 points)**

13. A circular swimming pool has a diameter of 24 ft., the sides are 5 ft. high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 lb/ft$^3$.)

**(8 points)**

14. a. Sketch the region bounded by the curves $y = \dfrac{1}{x^2}$, $y = 8x$, and $y = 64x$.

    b. Find the area of the region sketched in part *a*.

**(8 points)**

15. A motorcycle starting from rest, speeds up with a constant acceleration of 2.6 m/s$^2$. After it has traveled 120 m, it slows down with a constant acceleration of –1.5 m/s until it attains a velocity of 12 m/s. What is the distance traveled by the motorcycle at that point?

**(6 points)**

16. a. The temperature of a 10 m long metal bar is 15°C at one end and 30°C at the other end. Assuming that the temperature increases linearly from the cooler end to the hotter end, what is the average temperature of the bar?

    b. Explain why there must be a point on the bar where the temperature is the same as the average, and find it.