Linear Algebra Equations and theorems

One parameter linear equation means that there is one variable in the solution. Eg. $\begin{bmatrix} 1 & 0 & \frac{2}{a} & \frac{2}{a} \\ 0 & 1 & \frac{2}{a} & \frac{2}{a} \end{bmatrix}$

Leontief consumption matrices are productive when $(I - C)^{-1}$. The greatest dollar value is the sector that requires the largest amount of inputs from the other sectors.
Symmetrical matrices are their own transposes.
Diagonal matrices consist of only the main diagonal. It is invertible iff all diagonal entries are nonzero.
Triangular matrices consist of 0's below or above the main diagonal. They can be used to solve systems through back or forwards substitution.

Properties of matrix arithmetic

a) $A + B = B + A$

b) $A + (B + C) = (A + B) + C$

c) $A(BC) = (AB)C$

d) $A(B + C) = AB + AC$

e) $(B + C)A = BA + CA$

f) $a(B + C) = aB + aC$

g) $(a + b)C = aC + bC$

h) $a(bC) = (ab)C$

i) $a(BC) = (aB)C = B(aC)$

j) $(A + B)^2 = A^2 + AB + BA + B^2$

Properties of inverse matrices

a) $(AB)^{-1} = B^{-1}A^{-1}$

b) $(A^T)^{-1} = (A^{-1})^T$

Equivalent statements theorem (all true or all false)

- A is invertible.
- $A\mathbf{x} = 0$ has only the trivial solution
- The reduced row echelon form of A is I_n
- A can be expressed as a product of elementary matrices.
- $A\mathbf{x} = \mathbf{b}$ is consistent for every $n \times 1$ matrix $\mathbf{b}$ and has exactly one solution

If A is an **invertible** $m \times n$ matrix, then for each $n \times 1$ matrix $\mathbf{b}$, the systems of equations $A\mathbf{x} = \mathbf{b}$ has exactly one solution, namely, $\mathbf{x} = A^{-1}\mathbf{b}$

A matrix is called **linearly independent** if the vector equation $x_1\mathbf{v}_1 + \cdots + x_n\mathbf{v_n} = \mathbf{0}$ has **only** the trivial solution. If it has more solutions, it is not linearly independent.

For a **homogenous** system, the system either has only the trivial solution, or more than one solution.

For a non-homogenous system, either the system has a single unique solution, more than one solution, or no solution at all.

A homogenous system of m linear equations in n unknowns always has a non-trivial solution if $m < n$.

A system is **consistent** when it has at least one solution. If the system $x_1v_1 + \cdots + x_nv_n = \mathbf{b}$, we say that $\mathbf{b}$ is a linear combination of the vectors. No solutions means the system is inconsistent.

A **consistent** system will have a **unique** solution if and only if the columns of the coefficient matrix are linearly independent vectors (if the homogenous linear equation has non-trivial solutions)

The **span** of a system is the sum of the linear combination of each of the column vectors.

A matrix is **orthogonal** if $u \cdot v = 0$.

$||u|| = \sqrt{u \cdot u}$

Cauchy-Schwarz: $|u + v| \leq ||u||||v||$

The **adjoint method** is finding the inverse of a matrix by multiplying by the inverse of the determinant.

Cramer's rule is replacing the variable you are trying to find with the solution vector and then dividing by the determinant of the coefficient matrix

Euclidean distance is the abs value distance/magnitude between two points. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Extras:

A system of linear equations with coefficient matrix A will be inconsistent for certain values on the right hand side if the row echelon form of A contains a row of zeros. If the row echelon form of the coefficient matrix A does not contain a row of zeros, then the system is always consistent, regardless of what the right hand side is.
Vector operations (what does dot/cross product represent and how to find)
Cauchy-Schwarz inequality for vectors
Vector Space Axioms
Contraction and dilation of matrices
Standard matrix for shear, reflection of certain dimensions