Assignment 2

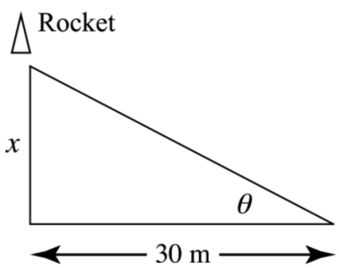
Complete this assignment after you have finished Unit 3, and submit your work to your tutor for grading.

This assignment has one bonus question. There is no penalty if you do not attempt it but you may be rewarded if you do. The maximum grade you can obtain in this assignment is 100%.

Total points: **100**
Weight: **10%**

(6 points)

1. As shown in the figure below, a camera is mounted on a point 30 m from the base of a rocket launching pad. The rocket rises vertically when launched, and the camera's angle is continually adjusted to follow the base of the rocket.



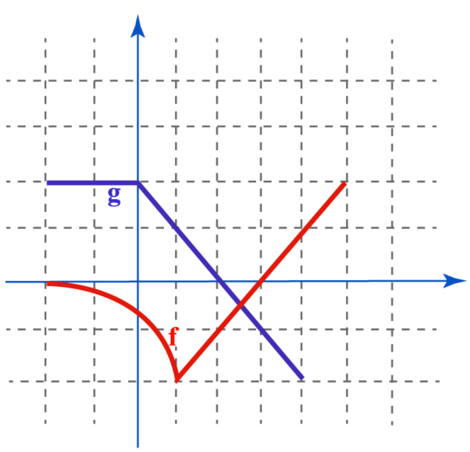
   a. Express the height x as a function of the elevation angle θ.

   b. Give the domain of the function in part *a*.

   c. Give the height of the rocket when the elevation angle is $\pi / 3$.

(10 points)

2. Consider the graphs of the functions f and g shown in the figure below.



Graphs of the function ***f*** and ***g***

   a. Fill in the table below with the corresponding values of f and g.

x	–2	–1	0	1	2	3	4	5
$f(x)$								
$g(x)$								

b. Use the table in part *a* to evaluate each of the expressions listed below.

i. $g \circ f(-2)$

ii. $g \circ f(1)$

iii. $g \circ f(4)$

iv. $f \circ g(0)$

v. $f \circ g(4)$

vi. $f \circ g(-1)$

(12 points)

3. Let $f(x) = \sqrt{1-2x}$, $g(x) = \dfrac{x}{x^2-1}$. Find the formulas for the functions listed below and specify their respective domains.

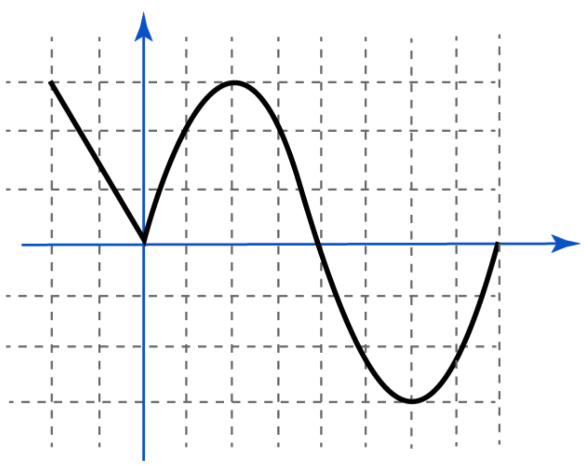
a. $\dfrac{f}{g}$

b. $g \circ f$

c. gf^2

(8 points)

4. Consider the graph of the function f shown below.



Graphs of the function f

Apply the appropriate transformations and sketch the functions listed below. Aim for a neat, labeled graph. Indicate the transformations that you are using in each step.

a. $F(x) = f(2x) - 1$

b. $G(x) = 1 - 2f(x)$

(4 points)

5. Indicate whether each of the limits listed below is well-defined. Explain your answer in each case.

a. $\lim\limits_{x \to 2} \sqrt{x^2 - 4}$

b. $\lim\limits_{x \to \infty} \cot\left(\dfrac{x^2+1}{x+3}\right)$

(6 points)

6. Determine whether the evaluation of the limits given below are correct. If it is correct, explain why. If not, give the correct evaluation.

   a. For $x \to -\infty$ we have $\dfrac{\sqrt{x^2 - x^3}}{x} = \dfrac{\sqrt{-x^3}\sqrt{1-1/x}}{x}$, hence

   $$\lim_{x\to-\infty} \frac{\sqrt{x^2 - x^3}}{x} = \lim_{x\to-\infty} \frac{\sqrt{-x^3}\sqrt{1-1/x}}{x} = \lim_{x\to-\infty} \sqrt{1-1/x}\sqrt{-x} = \infty$$

   b. $$\begin{aligned} \lim_{x\to0} \frac{x^2 \sin(x - \pi/2)}{1-\cos(3x)} &= \lim_{x\to0} \frac{x^2 \sin(x-\pi/2)(1+\cos(3x))}{(1-\cos(3x))(1+\cos(3x))} \\ &= \lim_{x\to0} \frac{x^2 \sin(x-\pi/2)(1+\cos(3x))}{\sin^2(3x)} \\ &= \lim_{x\to0} \left(\frac{x}{\sin(3x)}\right)^2 \sin(x-\pi/2)(1+\cos(3x)) \\ &= \lim_{x\to0} \left(\frac{x}{\sin(3x)}\right)^2 \lim_{x\to0} \sin(x-\pi/2) \lim_{x\to0} 1+\cos(3x) \\ &= \frac{1}{9}(-1)(1) \\ &= -\frac{1}{9} \end{aligned}$$

(28 points)

7. Evaluate each of the limits listed below. Justify your answers by identifying the results of Units 2 and 3 you apply in each case. If a limit does not exist, explain why.

   a. $\displaystyle\lim_{x\to0} \frac{\sin(\pi - x)}{\sqrt{x^2 - x + 1}}$

   b. $\displaystyle\lim_{x\to3} \frac{x^3 - 3x^2 + 4x - 12}{3x^2 - 7x - 6}$

   c. $\displaystyle\lim_{x\to\infty} \cos\left(\frac{(\pi x + 1)(3 - x^2)}{x^3 - \pi}\right)$

   d. $\displaystyle\lim_{x\to\infty} \frac{x\sin(3x)}{x^2 + 1}$

   e. $\displaystyle\lim_{x\to1^+} \sin\left(\frac{\sqrt{x+1}}{x^2 - 1}\right)$

   f. $\displaystyle\lim_{x\to0^+} \frac{\sqrt{x^3 + x^2}}{\sqrt{x+1} - 1}$

   g. $\displaystyle\lim_{x\to0} \frac{\sin^2(3x)}{1 - \cos(2x)}$

(17 points)

8. Let

$$f(x) = \begin{cases} \dfrac{2}{x} & \text{if } x < 0 \\ \sqrt{x} + \cos(\pi x) & \text{if } 0 < x \le 5 \\ \dfrac{x}{x-5} & \text{if } x > 5 \end{cases}$$

Evaluate the limits listed below. If a limit does not exist explain why.

a. $\lim_{x\to 0^-} f(x)$

b. $\lim_{x\to 0^+} f(x)$

c. $\lim_{x\to 5^-} f(x)$

d. $\lim_{x\to 5^+} f(x)$

e. $\lim_{x\to 5} f(x)$

f. $\lim_{x\to \infty} f(x)$

g. $\lim_{x\to -\infty} f(x)$

Where is the function not continuous? Explain.

(9 points)

9. Give the graph of one and only one function which satisfies all the following conditions.

a. Domain of the function $[-5,0) \cup (0,\infty)$

b. The function is continuous on its domain

c. $f(-5) = f(5)$

d. $\lim_{x\to 5} f(x) = 6$

e. $\lim_{x\to 0^-} f(x) = \infty$

f. $\lim_{x\to 0^+} f(x) = 0$

g. $\lim_{x\to \infty} f(x) = -\infty$

Explain why the graph of a function which satisfies all these conditions must intercept the x-axis, meaning that there is at least one number c so that $f(c) = 0$.

Bonus. (10 points)

10. Prove each of the statements below using the formal definition of limits.

a. If $\lim_{x\to\infty} f(x) = -\infty$ and $c > 0$, then $\lim_{x\to\infty} c\, f(x) = -\infty$.

b. If $\lim_{x\to a} g(x) = \infty$ and $g(x) \le f(x)$ for $x \to a$, then $\lim_{x\to a} f(x) = \infty$.