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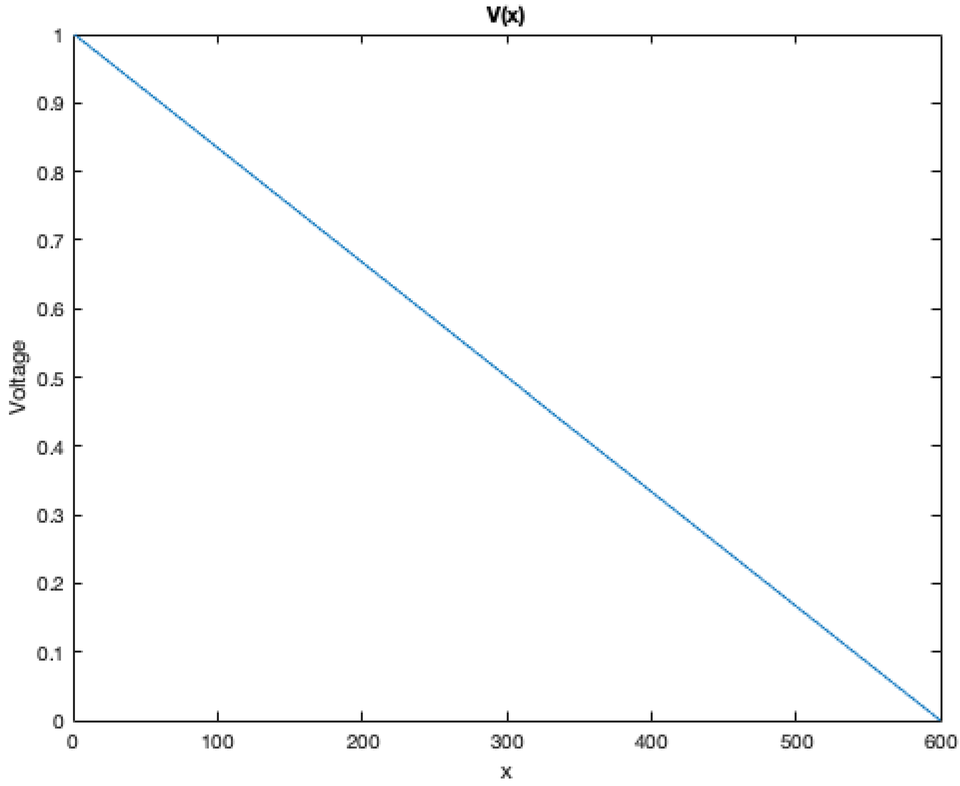
Part 1 A

Part 1 is used to explore the representation of the electrostatic potential in a 2-D rectangular region. Beucase the potential will not change with respect to the y-coordinate, the problem will be treated as a 1-D problem.

```
clear all;
close all;

% Initial conditions
Length = 30;
Width = 20;
v0 = 1;

% Create matrices
N = Length*Width;
F = zeros(N,N);
G = zeros(N,1);

for i=1:N
    if (i == N) % right
        F(i,i) = 1;
        G(i) = 0;
    elseif i == 1 % left
        F(i,i) = 1;
        G(i) = 1;
    else % Middle
        F(i,i)   = -2;
        F(i,i+1) =  1;
        F(i,i-1) =  1;
    end
end

V = F\G;

% Plot graph
figure(1);
plot(V);
xlabel('x');
ylabel('Voltage');
title('V(x)');

% The change is linear. As moving from left to right in the x
 direction
```

```
% shows a decrease in voltage. Voltage is uniform in the y diredtion.
 If
% this was mapped out on a voltage map, there would be a gradient in
 the
% x-direction with the same colour in the y-direction.

clear all
```



Part 1 B

The finite differences method will be used to implement a matrix calculation, $GV = F$. V represents the voltages at different points, F is the matrix used to set the boundary conditions and G represents the relation of voltages throughout. The the equation used:

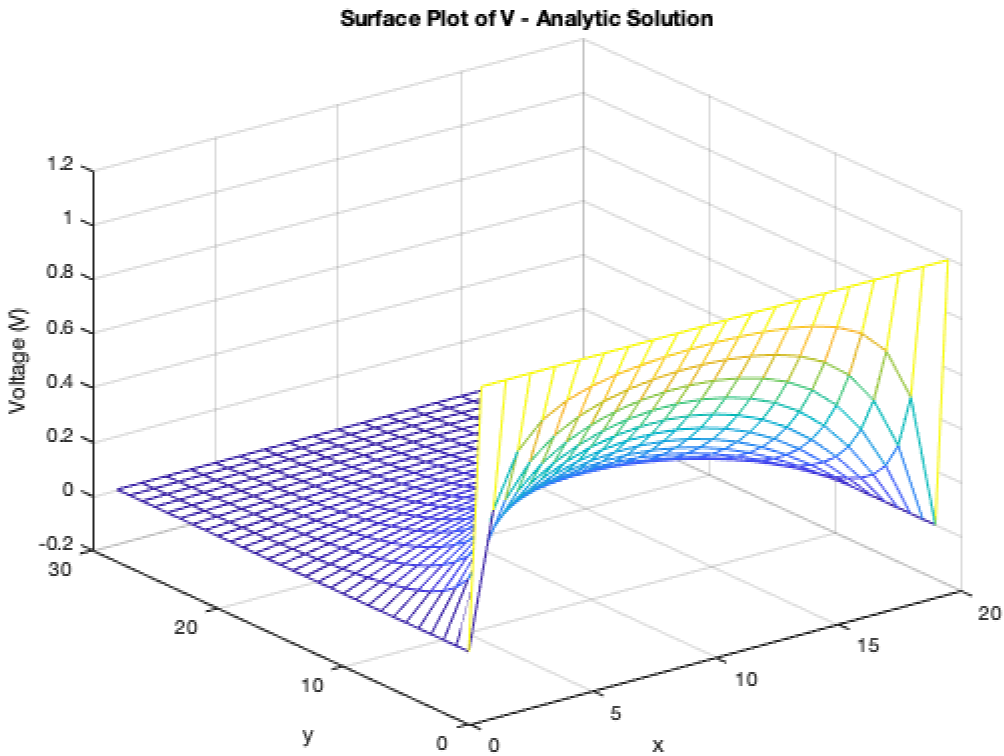
$$\frac{V_{x-1,y} - 2V_{x,y} + V_{x+1,y}}{(\Delta x)^2} + \frac{V_{x,y-1} - 2V_{x,y} + V_{x,y+1}}{(\Delta y)^2} = 0$$

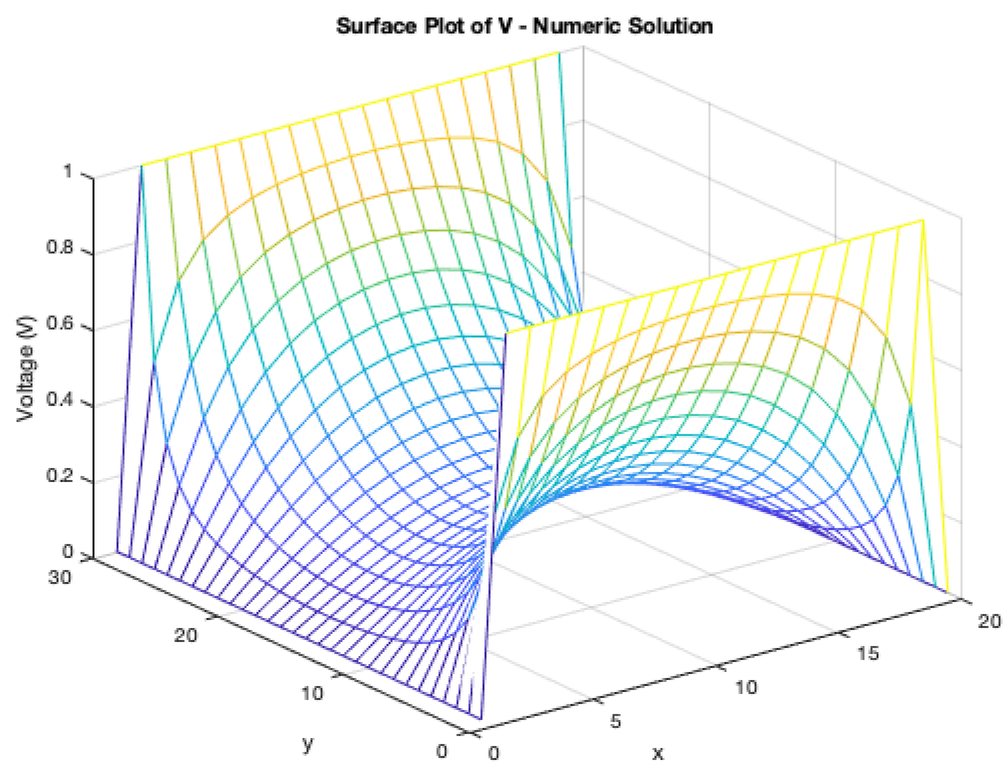
```
dx = 0.25; % spacing along x
dy = 0.25; % spacing along y

Const1 = -2*(1/dx^2 + 1/dy^2);
Const2 = 1/(dx^2);
Const3 = 1/(dy^2);

% Initial conditions
Length = 30;
```

```matlab
Width = 20;
v0 = 1;

% Create Grid
x = linspace(0,Length);
y = linspace(0,Width);

% Create matrices
N = Length*Width;
F = zeros(N,N);
G = zeros(N,1);

% Analytical solution
% Middle
for i = 2:Length-1
    for j = 2:Width-1
        n = i + (j-1)*Length;
        F(n,n) = Const1;
        F(n,n-1) = Const2;
        F(n,n+1) = Const2;
        F(n,n-Length) = Const3;
        F(n,n+Length) = Const3;
        G(n,1) = 0;
    end
end

% Left BC
i = 1;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Right BC
i = Length;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

% Bottom BC
j = 1;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
end

% Top BC
j = Width;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
```

```
end

% To find the solution
v = F\G;

% converting for plot
for i = 1 : Length
    for j = 1 : Width
        n = i + (j-1)*Length;
        Ph(i,j) = v(n);
    end
end

% Plot
figure(2);
mesh(Ph);
xlabel('x');
ylabel('y');
zlabel('Voltage (V)');
title('Surface Plot of V - Analytic Solution');

% It can be seen that the potential is 1 V on one end and 0 V on the
 other.

% Numerical soliution
% Middle
for i = 2:Length-1
    for j = 2:Width-1
        n = i + (j-1)*Length;
        F(n,n) = -4;
        F(n,n-1) = 1;
        F(n,n+1) = 1;
        F(n,n-Length) = 1;
        F(n,n+Length) = 1;
        G(n,1) = 0;
    end
end

% Left BC
i = 1;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Right BC
i = Length;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end
```

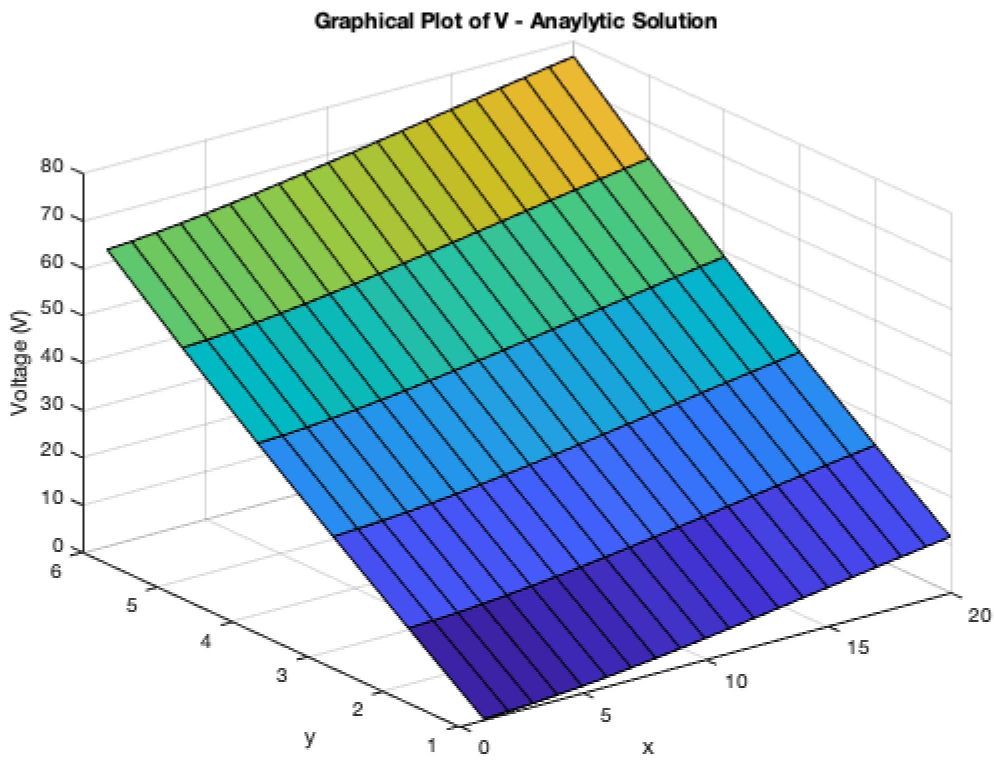
```
% Bottom BC
j = 1;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

% Top BC
j = Width;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

v = F\G;

for i = 1 : 30
    for j = 1 : 20
        n = i + (j-1)*Length;
        Ph(i,j) = v(n);
    end
end

% Plot
figure(3);
mesh(Ph);
xlabel('x');
ylabel('y');
zlabel('Voltage (V)');
title('Surface Plot of V - Numeric Solution');
```


Surface Plot of V - Analytic Solution
Voltage (V)
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
30
20
10
0
y
0
5
10
15
20
x


Surface Plot of V - Numeric Solution
Voltage (V)
1
0.8
0.6
0.4
0.2
0
30
20
10
0
y
0
5
10
15
20
x

The numeric solution has boundry conditions that is elevated on both ends with a potential at 1 V. The middle of the region contains the lowest potential as it is furthest from the influence of both ends.

```
ph2 = Ph;
a = 30;
b = 10;
anew = 0;
for i = 1:Length
    for j = 1:Width
        for n = 1:2:1000
            anew = anew + ((1/n)*(cosh(n*pi*i/a))*(sin(n*pi*j/a))*(1/
(cosh(n*pi*b/a))));
        end
    ph2(i,j) = (4/pi) * anew;
    end
end
figure(4);
surf(ph2);
xlabel('x');
ylabel('y');
zlabel('Voltage (V)');
title('Graphical Plot of V - Anaylytic Solution');
```



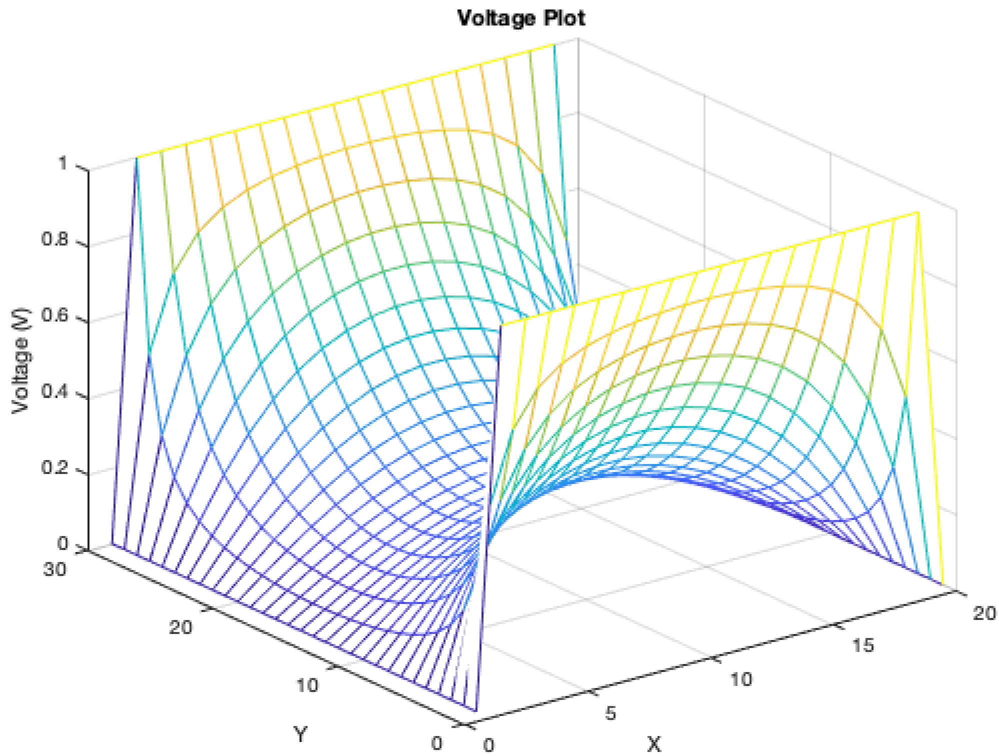
The graph above is a alternate representation of the potential, V using the equation in the problem sheet. The analytic solution is not nearly as accurate as the numeric solution. The graph does not properly show the raised voltages on both ends of the region making the numeric solution more accurate.

Part 2 A

Part 2 will provide an understanding of current flow in a 2D rectangular region by looking at the current density, electric potential, and electric field with a bottleneck region.

```
% Middle
for i = 2:Length-1
    for j = 2:Width-1
        n = i + (j-1)*Length;
        F(n,n) = -4;
        F(n,n-1) = 1;
        F(n,n+1) = 1;
        F(n,n-Length) = 1;
        F(n,n+Length) = 1;
        G(n,1) = 0;
    end
end

% Left BC
i = 1;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Right BC
i = Length;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Bottom BC
j = 1;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

% Top BC
j = Width;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

v = F\G;

for i = 1 : 30
```

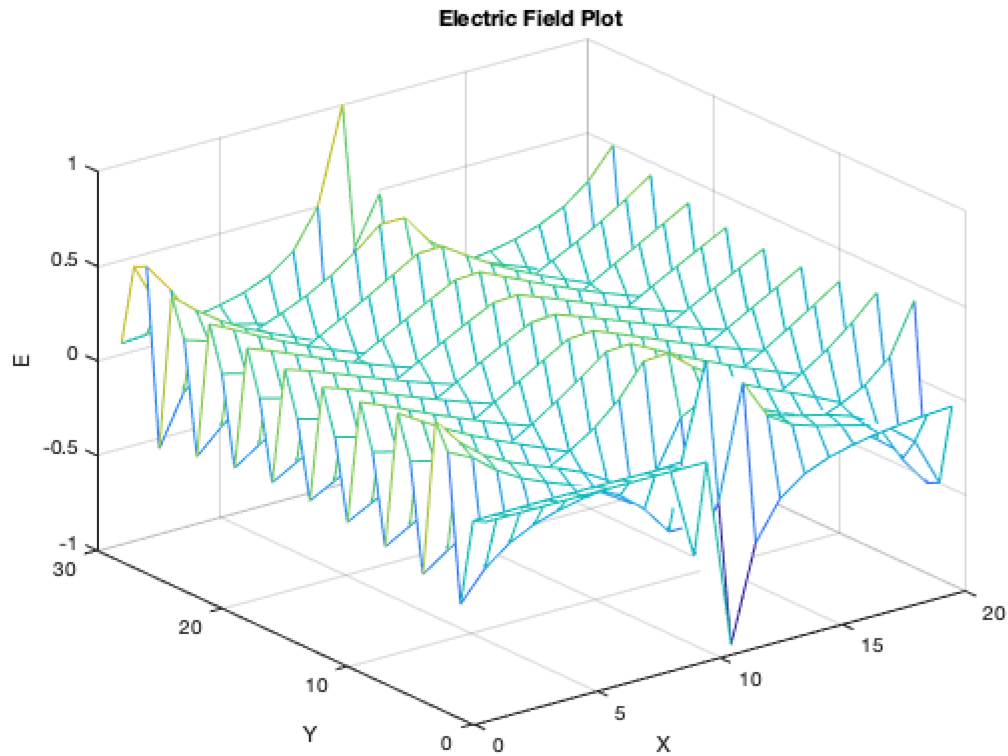
```
    for j = 1 : 20
        n = i + (j-1)*Length;
        Ph(i,j) = v(n);
    end
end

% Plot
figure(5);
mesh(Ph);
xlabel('X');
ylabel('Y');
zlabel('Voltage (V)');
title('Voltage Plot');
```



Electric Field

```
Elecmap = zeros(Length,Width);
for i = 1:Length
    for j = 1:Width
        n = j + (i-1)*Width;
        Elecmap(i,j) = v(n);
    end
end

for i = 1:Length
    for j = 1:Width
        if i == 1
```

```
            Ex(i,j) = (Elecmap(i+1,j) - Elecmap(i,j));
        elseif i == Length
            Ex(i,j) = (Elecmap(i,j) - Elecmap(i-1,j));
        else
            Ex(i,j) = (Elecmap(i+1,j) - Elecmap(i-1,j))*0.5;
        end
        if j == 1
            Ey(i,j) = (Elecmap(i,j+1) - Elecmap(i,j));
        elseif j == Width
            Ey(i,j) = (Elecmap(i,j) - Elecmap(i,j-1));
        else
            Ey(i,j) = (Elecmap(i,j+1) - Elecmap(i,j-1))*0.5;
        end
    end
end

Ex = -Ex;
Ey = -Ey;

Et = Ex + Ey; % electric field plot generated by adding the x and y
 components

figure(6);
mesh(Et);
xlabel('X');
ylabel('Y');
zlabel('E');
title('Electric Field Plot');

% Sigma
sigma = ones(Length,Width);
for i = 1:Length
    for j = 1:Width
        if j <= (Width/3) || j >= (Width*2/3)
            if i >= (Length/3) && i <= (Length*2/3)
                sigma(i,j) = 10^-12;
            end

        end
    end
end

figure(7);
mesh(sigma);
xlabel('X');
ylabel('Y');
zlabel('Conduction');
title('Conductivity Map');
```


Electric Field Plot
1
0.5
0
-0.5
-1
E
30
20
10
0
Y
0
5
10
15
20
X

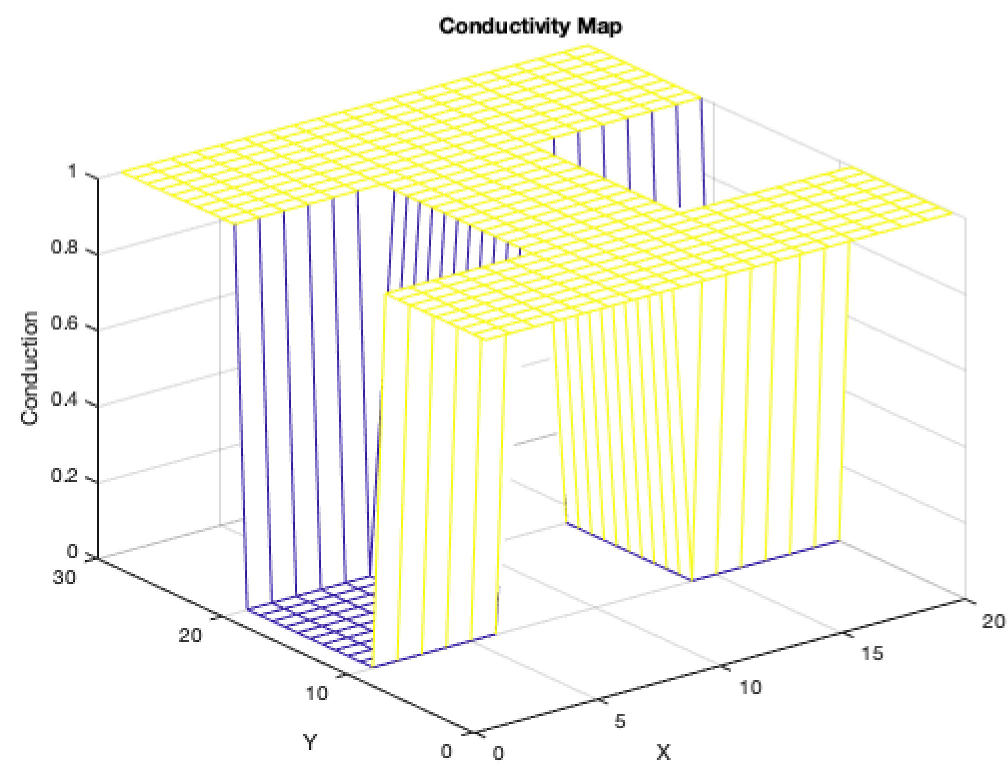
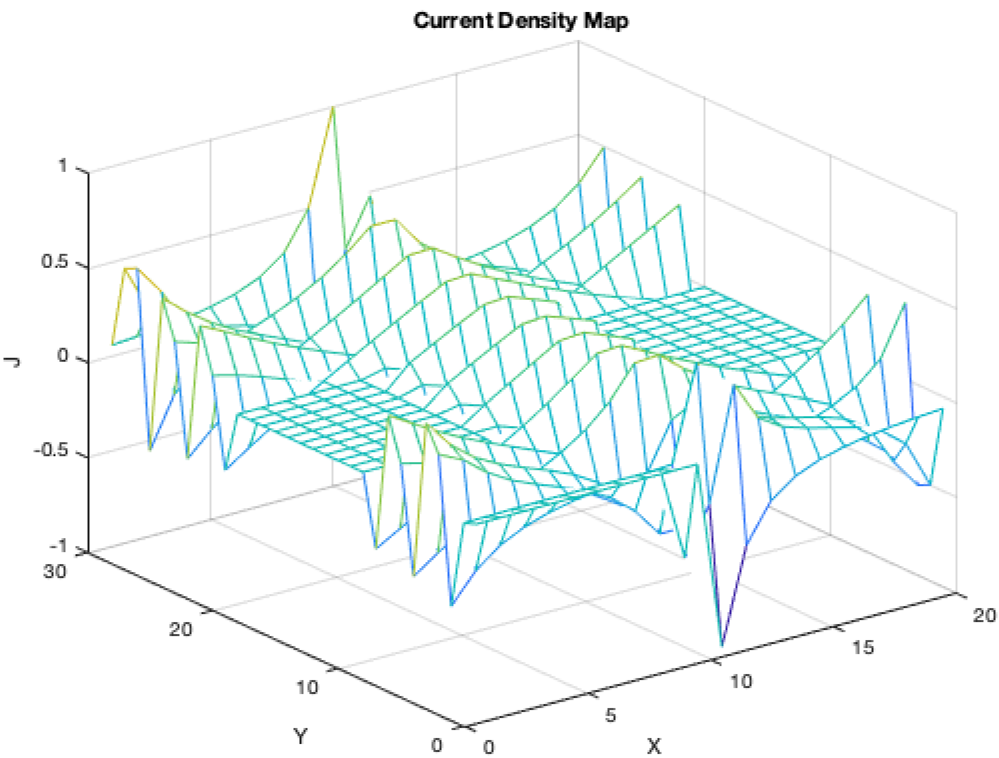

Conductivity Map
1
0.8
0.6
0.4
0.2
0
Conduction
30
20
10
0
Y
0
5
10
15
20
X

Figure 7 shows that the bottleneck area contain a conductivity lower than that of the surrounding area. This means the 2 boxes are areas of high resisitivity which will resist current flow.

```
% Current Density
J = sigma .* Et;
figure(8);
mesh(J);
xlabel('X');
ylabel('Y');
zlabel('J');
title('Current Density Map');
```



Part 2 C

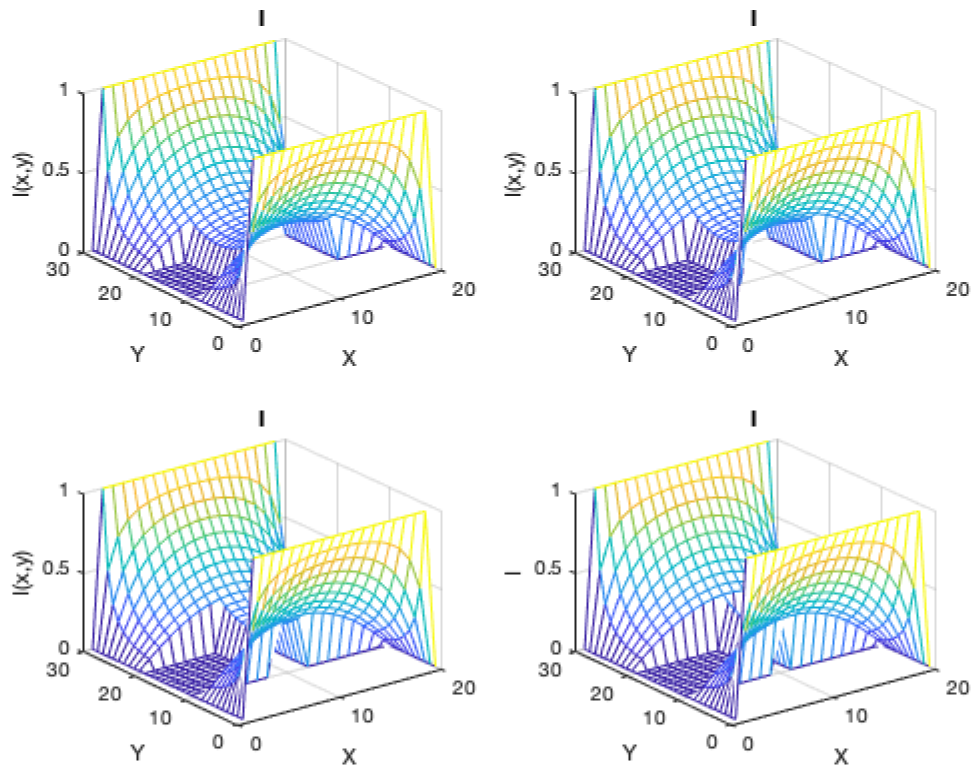
```
% Inputs
Length = 30;
Width = 20;

%Grid
x = linspace(0,Length);
y = linspace(0,Width);
dx = x(2) - x(1);
dy = y(2) - y(1);

% Make matrices
N = Length*Width;
```

```matlab
F = zeros(N,N);
G = zeros(N,1);

% Middle
for i = 2:Length-1
    for j = 2:Width-1
        n = i + (j-1)*Length;
        F(n,n) = -4;
        F(n,n-1) = 1;
        F(n,n+1) = 1;
        F(n,n-Length) = 1;
        F(n,n+Length) = 1;
        G(n,1) = 0;
    end
end

% Left BC
i = 1;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Right BC
i = Length;
for j = 1:Width
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 1;
end

% Bottom BC
j = 1;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

% Top BC
j = Width;
for i = 1:Length
    n = i + (j-1)*Length;
    F(n,n) = 1;
    G(n,1) = 0;
end

v = F\G;

for i = 1 : 30
    for j = 1 : 20
        n = i + (j-1)*Length;
        Ph(i,j) = v(n);
```

```
    end
end

% Sigma
sigma1 = ones(Length,Width);
sigma2 = ones(Length,Width);
sigma3 = ones(Length,Width);
sigma4 = ones(Length,Width);

for i = 1:Length %Changing the lengths and widths
    for j = 1:Width
        if j <= (Width/4) || j >= (Width*3/4)
            if i >= (Length/3) && i <= (Length*2/3)
                sigma1(i,j) = 10^-2;
            end
        end
        if j <= (Width/3.1) || j >= (Width - (Width/3.1))
            if i >= (Length/3) && i <= (Length*2/3)
                sigma2(i,j) = 10^-2;
            end
        end
        if j <= (Width/2.5) || j >= (Width - (Width/2.5))
            if i >= (Length/3) && i <= (Length*2/3)
                sigma3(i,j) = 10^-2;
            end
        end
        if j <= (Width/2.1) || j >= (Width - (Width/2.1))
            if i >= (Length/3) && i <= (Length*2/3)
                sigma4(i,j) = 10^-2;
            end
        end
    end
end

t1 = sigma1;
t2 = sigma2;
t3 = sigma3;
t4 = sigma4;

for i = 1:Length
    for j = 1:Width
        if sigma1(i,j) == (10^-2)
            t1(i,j) = 1 / (10^-2);
        end
    end
end
for i = 1:Length
    for j = 1:Width
        if sigma2(i,j) == (10^-2)
            t2(i,j) = 1 / (10^-2);
        end
    end
end
for i = 1:Length
```

```
    for j = 1:Width
        if sigma3(i,j) == (10^-2)
            t3(i,j) = 1 / (10^-2);
        end
    end
end
for i = 1:Length
    for j = 1:Width
        if sigma4(i,j) == (10^-2)
            t4(i,j) = 1 / (10^-2);
        end
    end
end

Current1 = Ph ./ t1;
C01 = sum(Current1(1,:));
CL1 = sum(Current1(Length,:));
c1 = (C01 + CL1) / 2;
figure(9);
subplot(2,2,1);
mesh(Current1);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('I');

Current2 = Ph ./ t2;
C02 = sum(Current2(1,:));
CL2 = sum(Current2(Length,:));
c2 = (C02 + CL2) / 2;
subplot(2,2,2);
mesh(Current2);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('I');

Current3 = Ph ./ t3;
C03 = sum(Current3(1,:));
CL3 = sum(Current3(Length,:));
c3 = (C03 + CL3) / 2;
subplot(2,2,3);
mesh(Current3);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('I');

Current4 = Ph ./ t4;
C04 = sum(Current4(1,:));
CL4 = sum(Current4(Length,:));
c4 = (C04 + CL4) / 2;
subplot(2,2,4);
mesh(Current4);
```

```
xlabel('X');
ylabel('Y');
zlabel('I');
title('I');
```



Narrowing the boxes should have caused the conductivity to decreased.

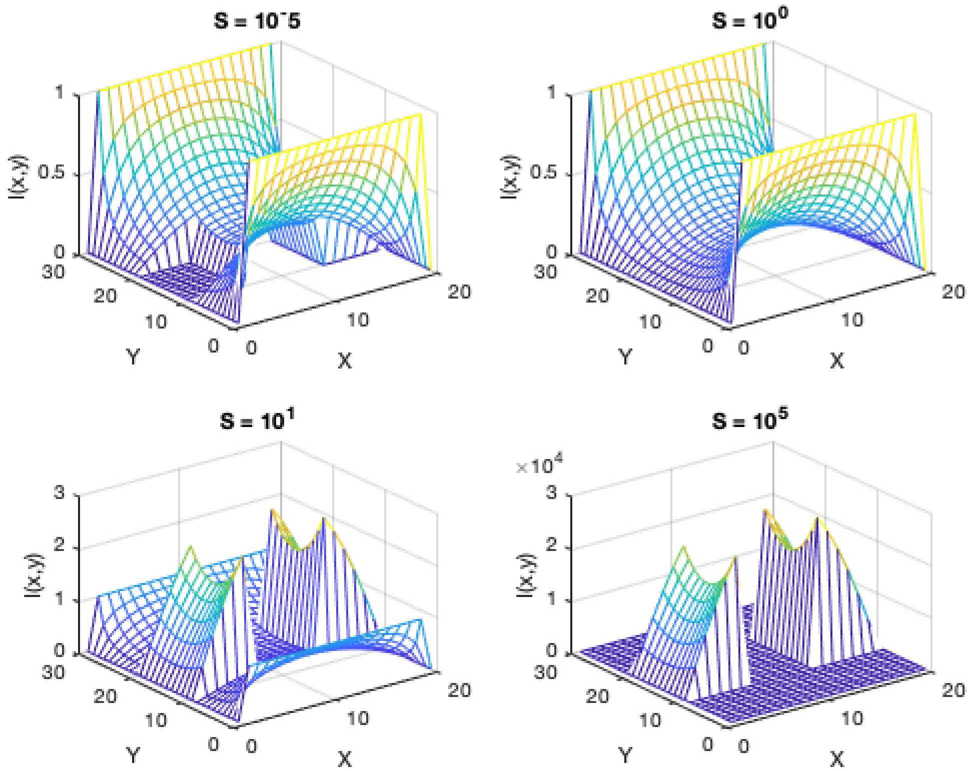
Part 2 D

```
% Middle
for i = 2:Length-1
    for j = 2:Width-1
        n = i + (j-1)*Length;
        F(n,n) = -4;
        F(n,n-1) = 1;
        F(n,n+1) = 1;
        F(n,n-Length) = 1;
        F(n,n+Length) = 1;
        G(n,1) = 0;
    end
end

% Left BC
i = 1;
for j = 1:Width
    n = i + (j-1)*Length;
```

```
        F(n,n) = 1;
        G(n,1) = 1;
    end

    % Right BC
    i = Length;
    for j = 1:Width
        n = i + (j-1)*Length;
        F(n,n) = 1;
        G(n,1) = 1;
    end

    % Bottom BC
    j = 1;
    for i = 1:Length
        n = i + (j-1)*Length;
        F(n,n) = 1;
        G(n,1) = 0;
    end

    % Top BC
    j = Width;
    for i = 1:Length
        n = i + (j-1)*Length;
        F(n,n) = 1;
        G(n,1) = 0;
    end

    v = F\G;

    for i = 1 : 30
        for j = 1 : 20
            n = i + (j-1)*Length;
            Ph(i,j) = v(n);
        end
    end

    % Sigma
    sigma = ones(Length,Width);
    for i = 1:Length
        for j = 1:Width
            if j <= (Width/3) || j >= (Width*2/3)
                if i >= (Length/3) && i <= (Length*2/3)
                    sigma(i,j) = 10^-2;
                end

            end
        end
    end

    % Current Flow I = V/R
    t1 = sigma;
    t2 = sigma;
    t3 = sigma;
```

```
t4 = sigma;

for i = 1:Length
    for j = 1:Width
        if sigma(i,j) == (10^-2)
            t1(i,j) = 1 / (10^-5);
            t2(i,j) = 1 / (10^0);
            t3(i,j) = 1 / (10^1);
            t4(i,j) = 1 / (10^5);
        end
    end
end

Cur = Ph ./ t1;
Ce = sum(Cur(1,:));
CL = sum(Cur(Length,:));
c = (Ce + CL) / 2;
figure(10);
subplot(2,2,1);
mesh(Cur);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('S = 10^-5');

Cur = Ph ./ t2;
Ce = sum(Cur(1,:));
CL = sum(Cur(Length,:));
c = (Ce + CL) / 2;
subplot(2,2,2);
mesh(Cur);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('S = 10^0');

Cur = Ph ./ t3;
Ce = sum(Cur(1,:));
CL = sum(Cur(Length,:));
c = (Ce + CL) / 2;
subplot(2,2,3);
mesh(Cur);
xlabel('X');
ylabel('Y');
zlabel('I(x,y)');
title('S = 10^1');

Cur = Ph ./ t4;
Ce = sum(Cur(1,:));
CL = sum(Cur(Length,:));
c = (Ce + CL) / 2;
subplot(2,2,4);
mesh(Cur);
xlabel('X');
```

```
ylabel('Y');
zlabel('I(x,y)');
title('S = 10^5');
```



As the box conductivity increased, current increased.

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