AN ETF TRACKING ERROR STRATEGY

Stanley Sayianka

2022-07-19

INTRODUCTION

An Exchange Traded Funds (ETF)is a type of investment instrument whose price/value is based on the real time value of the underlying asset they track. They are mostly created to give investors exposure to certain indexes, commodities, or stocks in a particular sector[1]. Since their early days, ETFs have garnered a lot of interest in the investment communities as compared to traditional mutual fund indexes, due to their lower fee structures, lower maintenance fees and tax efficiency. They are commonly used as a means of passive investments(*buy and hold*) or as a means of long-term investments.

[1] Certain ETFs are designed to track indexes such as: SPX tracks the Standard and Poors' 500 index. Certain ETFs designed to give investors exposure to commodities, precious metals include: the gold fund GLD, the gold miners fund GDX. Certain ETFs are also designed to track large-cap stocks, technology sector stocks e.g the NASDAQ Composite, oil-mining stocks

Among the reasons ETFs have become a popular investment vehicles are: ETFs are tradable on an exchange implying that investors can trade(long and short) shares of the ETF all throughout the day through brokerage services. This is especially an advantage over mutual funds, which can only be sold and bought at the end of the day. Agapova (2010) documents that ETFs are attractive to investors who are more sensitive to taxation in capital gains and with higher liquidity demands[2]. Many ETFs nowadays have related options contracts which make them a suitable investment opportunity for sophisticated investors.

[2] Agapova (2010), Guedj and Huang (2008) show that there exists a significant negative relationship between the fund flows into traditional index mutual funds and fund flows in ETFs.

A new investment opportunity emerged to members of the wider East African Community, as well as foreign investors with access to the Nairobi Securities Exchange after ABSA Bank[3] launched the ABSA NewGold ETF[4]. The purpose of the NewGold ETF is to allow investors to invest in and have exposure to the Gold Bullion, since it tracks the price of the Gold bullion. The NewGold ETF(*which shall be referred from now by its Ticker: GLD*) is denominated in Kenyan shillings equivalent of the real time international market price of gold (in $).

[3] Formerly known as Barclays Bank

[4] Ticker: GLD

The GLD ETF is the 7th such product under the Barclays Bank Group to be launched in Africa and was was launched at a value of USD 1.4 Billion, making it the largest Gold ETF in Africa and the 7th largest world wide[5]. The GLD ETF price movements will be determined by the price of gold movement, with each security referencing $\frac{1}{100}$ troy ounces of the gold bullion.

[5] Speaking at its launch, the BBK Managing director Jeremy Awori stated: *"NewGold ETF will introduce depth and range to the NSE further entrenching the Exchange as the undisputed regional financial hub and based on our interactions with brokers and fund managers the market is hungry for this product"*

This research study aims to investigate the tracking efficiency of the GLD with respect to the gold bullion[6]. In doing so we attempt to form a strategy that takes advantage of any large tracking deviations as measured by the tracking difference and error[7]. The questions we seek to answer are shown below:

[6] We use the XAUUSD, gold spot prices

[7] The tracking difference is the difference between the returns of a portfolio, and a tracking basket, while the tracking error is the standard deviation of the tracking difference

- How efficiently does the GLD track the gold bullion ?
- Can we construct strategies aimed at taking advantage of any large tracking differences between the gold bullion and the GLD ?

To answer the above questions, we use a ratio-based method as well as regression-based techniques. We seek to quantify the deviations between the ABSA NewGold ETF and the gold bullion[8]. The goal of this study is to design a strategy that takes advantage of large deviations in tracking error(*if any*).

[8] Which shall be represented by XAUUSD. XAU/USD pair listed in NYSE tells the investor how many US Dollars(the quote currency) are needed to purchase one Gold Ounce (the base currency)

DATA & METHODOLOGY

Data

The data used in this study includes: The GLD Closing prices, which we fetch from the **African 'Xchanges**[9]. Data on gold spot prices XAU/USD is fetched from the YahooFinance! website[10]. Data on Kenyan conversion rates USD/KES is fetched from the Central Bank of Kenya website.

[9] The African 'Xchanges website

[10] The yahoo finance website

Methodology

In this study, we aim to construct a strategy which takes advantage of the large deviations in tracking errors in GLD-XAUUSD. To do so we use a ratio-based model and regression based methods to quantify such deviations. In the regression model that we use: the XAUUSD prices is the explanatory variable and the GLD prices is the response variable. This is because the GLD tracks the price of the gold bullion(*XAU/USD in this case*), hence the movements in GLD are "explained" by XAUUSD.

The regression based methods we implement include: a full-dataset regression, rolling-horizon regression, and an expanding horizon regression with an aim of comparing the performance of the regression based approaches. For the rolling-regression, we use a look-back period equivalent to the half-life of mean reversion.[11]

[11] This approach is advocated for in Ernest P. Chan's book, where he states that setting the lookback of regression equal to the half-life or a small multipleof the half-life is a good starting point.

The deviation between the GLD-XAUUSD is quantified using a spread time series. For the regression modelling case, the spread time series could also be thought of as the residuals from the fitted models[12]. For the ratio based method, the spread could be thought of the daily ratio of the closing prices of GLD and XAUUSD i.e. $\frac{GLD}{XAUUSD}$.

[12] Because the residuals quantify the unexplained variation or variation in the response variable which is not attributable to the explanatory variable

Due to the nature of tracking portfolios, we expect the spreads to exhibit stationarity[13], and have a tendancy of mean reversion. To analyze this, we turn to the Ornstein-Uhlenbeck process which is a stochastic process commonly used to model mean-reverting processes. We model the spread to obtain useful statistics such as:

[13] Stationarity in this sense is *weak stationarity*, i.e. having a constant mean throughout the process, and a covariance which only depends on the time lag

- Long term mean
- Long term volatility
- Half life of mean reversion
- Speed of mean reversion

These statistics will be useful in designing trading rules such as in the rolling-regression based methods and the adaptive bands, we will use the half-life of mean reversion as an estimate of our look-back period. For the regression based approach, we use the long-term mean and standard deviation as the unbiased estimate of our mean value and volatility of the spread series.

ANALYSIS

In this section, we begin by exploring the relationship that GLD has with XAU/USD, through visualization. We expect there to be a strong positive relationship, since GLD tracks XAU/USD. The data we use for back-testing, estimating parameters and exploring the relationship between GLD and XAU/USD is data from 2017-March to 2019-March, while the data we used for validation and walk-forward analysis is data from 2019-March to date.

For this analysis, since the XAUUSD is quoted in terms of 1 USD per troy ounce, we rescale by dividing the spot prices by 100, so we have the price of the gold bullion in terms of 1 USD for $\frac{1}{100}$ troy ounce. We also investigate whether re-scaling the spot prices of gold(in USD) to Kenyan shillings has a significant effect on the relationship between the GLD and the XAU. To do so we create a feature *XAU/KES* by multiplying the spot prices of XAUUSD with the currency conversion rates USD/KES.

The relationship between the XAU/USD(KES) and GLD is illustrated below:

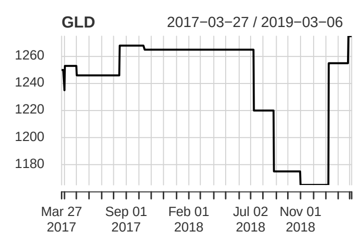


Figure 1: Close prices for GLD

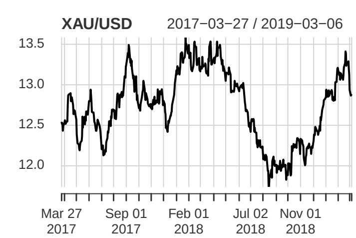


Figure 2: Gold Spot Prices(XAU/USD)

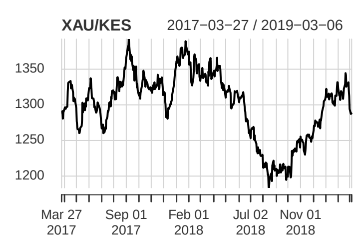
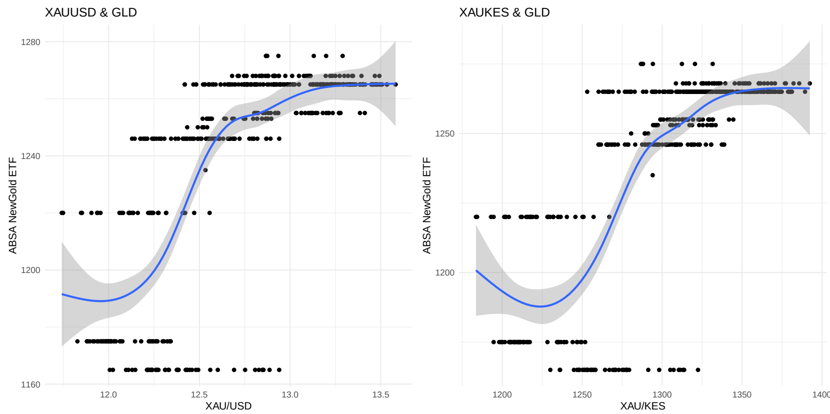


Figure 3: Constructed Gold Spot Prices(XAU/KES)



The relationship between the GLD and XAU/USD(KES) is not strictly linear, since it has some quadratic nature to it, which we investigate later using a linear model. There is no visible changes between the two where we use XAU/USD in the first chart and XAU/KES in the second chart.

Tracking Portfolios

A tracking portfolio is simply a portfolio/basket of stocks which tracks (say) an index. They are designed to perfectly mimic the price movements of the basket and are usually tradable. It is common however to find out that the tracking is not essentially perfect, and thus the tracking basket lags as compared to the index it is tracking. ETFs are created using their underlying composition's "in-kind redemption" which ensures that the prices of the ETF do not diverge significantly from the underlying assets in its composition. Therefore divergence does actually occur, but with some degree of minimality.

This divergence is commonly quantified using the tracking difference and tracking error. The tracking difference is the difference between the returns of the index and the tracking portfolo, and the volatility of this difference series is the tracking error, thus: Given an index i with returns R_i, and a portfolio p with returns R_p, at any time t, we define the the tracking difference and tracking difference error mathematically as:

$$\epsilon_t = R_{i,t} - R_{p,t}$$
$$\sigma_\epsilon = \sqrt{Var(\epsilon_t)}$$

where:

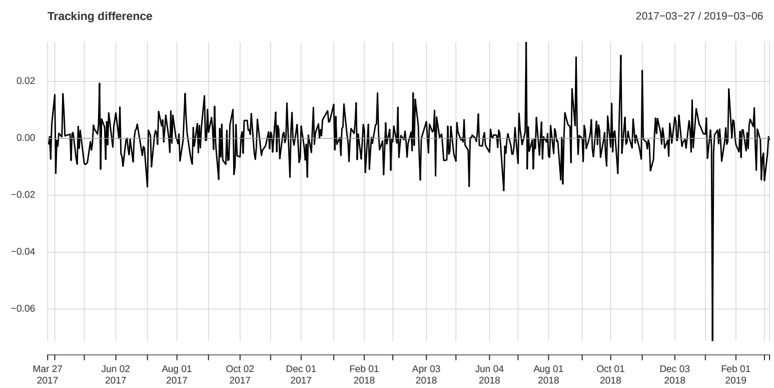
ϵ_t : The tracking difference at time t.

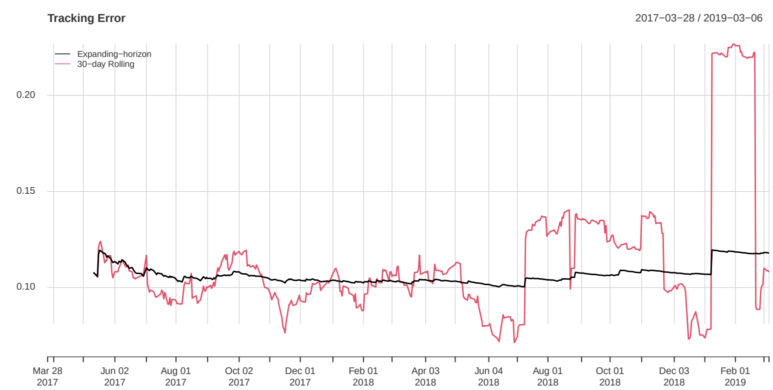
σ_ϵ : The tracking error at time t.

Portfolio managers, and fund managers develop techniques for designing portfolios with an objective of minimizing the tracking error relative to a certain benchmark. This can be translas

$$Min\ E(R_i - R_p)^2 = Var(R_i) + Var(R_p) - 2 * Cov(R_i, R_p)$$

A plot of the tracking difference and a 30-day tracking error is shown for the GLD~XAU:





As can be seen the tracking difference is mean reverting around value of 0, with one downward spike in early 2019. The tracking error which quantifies the volatility of the tracking difference is fluctuating mildly for the 30-day rolling-period, while for the expanding-horizon it is steady for the period under investigation. The average volatility for the tracking difference is 10% for the expanding horizon, and 11% for the rolling window type. This suggests that the deviation is quite signiicant between the GLD and XAUUSD.

A ratio-based model

For the ratio based model, we take a simple ratio of the GLD and XAUUSD closing prices. The ratio obtained is then used as the spread for the trade. A ratio based model essentially assumes a regression model, where the residual series are 0 i.e

$$\frac{GLD}{XAUUSD} = \beta$$

Taking the ratio in this manner(where GLD is the numerator) makes sense since, by transforming it to a linear equation, is equivalent to a regression model where XAUUSD is the explanatory variable, while GL is the response variable. The linear model will be of the form

$$GLD = \beta * XAUUSD$$

The chart below shows the normalized ratio between the two

The normalized ratio between the two form a stationary spread series[14], and it is evident that the normalized ratio is mean reverting around 0.

[14] The ADF test is 0.02298, which is significant at the 5% level

Regression modelling

In this section, we analyze regression models constructed using the back-test duration between the price series of XAU(*Acting as the explanatory variable*) and GLD(*Acting as the response variable*). We fit both intercept and no-intercept regression models for the following:

- A simple linear regression $GLD \sim XAU$[15].
- A simple linear regression $GLD \sim XAU^2$
- A simple linear regression $GLD \sim log_e(XAU)$

[15] XAU simple means for both XAU-USD, and XAUKES

The results are shown in the tables:

Table 1: R-squared: 0.49371750897013

term	estimate	std.error	statistic	p.value
(Intercept)	502.18973	34.126972	14.71533	0
XAUUSD	57.95613	2.678702	21.63590	0

Table 2: R-squared: 0.999374402366886

term	estimate	std.error	statistic	p.value
XAUUSD	97.3507	0.1111736	875.6639	0

Table 3: R-squared: 0.488284001119858

term	estimate	std.error	statistic	p.value
(Intercept)	871.853752	17.2472385	50.55034	0
I(XAUUSD^2)	2.268892	0.1060107	21.40248	0

Table 4: R-squared: 0.997238443532893

term	estimate	std.error	statistic	p.value
I(XAUUSD^2)	7.615158	0.0182909	416.3364	0

Table 5: R-squared: 0.498812785165452

term	estimate	std.error	statistic	p.value
(Intercept)	-639.7847	86.01698	-7.437889	0
log(XAUUSD)	739.0825	33.81437	21.857051	0

Table 6: R-squared: 0.999524452463359

term	estimate	std.error	statistic	p.value
log(XAUUSD)	487.5982	0.4854467	1004.432	0

Table 7: R-squared: 0.52957720971604

term	estimate	std.error	statistic	p.value
(Intercept)	508.758773	31.4863559	16.15807	0
XAUKES	0.562788	0.0242133	23.24291	0

Table 8: R-squared: 0.9993814340355577

term	estimate	std.error	statistic	p.value
XAUKES	0.9537747	0.0010831	880.6301	0

Table 9: R-squared: 0.526570709691611

term	estimate	std.error	statistic	p.value
(Intercept)	871.528569	15.9944372	54.48948	0
I(XAUKES^2)	0.000218	0.0000094	23.10339	0

Table 10: R-squared: 0.997096554072067

term	estimate	std.error	statistic	p.value
I(XAUKES^2)	0.0007308	1.8e-06	406.0071	0

Table 11: R-squared: 0.531996903377814

term	estimate	std.error	statistic	p.value
(Intercept)	-3958.0815	222.56775	-17.78372	0
log(XAUKES)	725.0841	31.04503	23.35589	0

Table 12: R-squared: 0.999338673363561

term	estimate	std.error	statistic	p.value
log(XAUKES)	172.9954	0.2031259	851.6659	0

It is evident that there is no significant change when using XAUKES(denomination: KES), as when compared to XAUUSD(denomination: USD), thus for the rest of the analysis, we proceed with using the XAUUSD. From the regression models fitted above, it is also interesting to note that the no-intercept regression models fitted have a superior explanatory power[16]. We proceed therefore with the simple GLD~XAUUSD regression model to quantify the spread.

[16] As seen by their significantly higher R^2 values of ~0.99

The normalized residual series from the first **GLD~XAUUSD** is plotted below, where the normalized spread is given by:

$$Z_t = \frac{X_t - \mu}{\sigma}$$

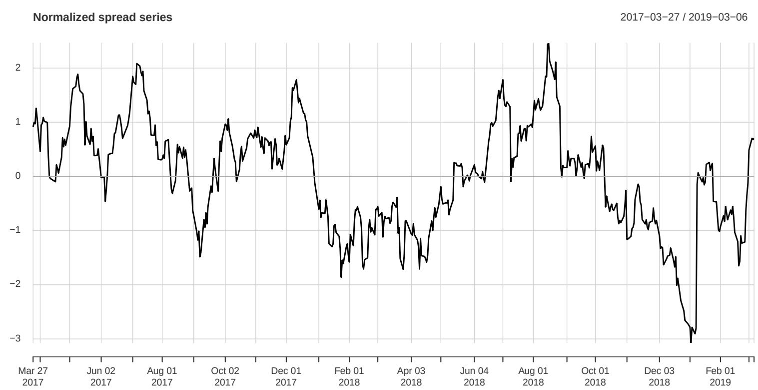
where:

X_t : The spread series

Z_t : The normalized spread series

μ : The mean of the spread series, during the whole back-test period.

σ : The volatility of the spread during the back-test period.



To test the spread for stationarity, we use the Augmented Dickey Fuller test for stationarity. We obtain a p-value equal to 0.02391, thus we conclude that the spread is stationary at the 5% level. The stationarity of the residuals is important in conducting a mean-reversion strategy, since the stationarity ensures that the spread does indeed have a constant mean, so that any deviations of the spread from the mean value is corrected with a reversion towards the constant mean value. In our case here, the long term mean value is: 0.

The Ornstein-Uhlenbeck Process

The OU process is a continuous-time stochastic process used to model mean-reverting processes. The stochastic differential equation for a OU-Process is:

$$dX_t = \kappa(\theta - X_t)dt + \sigma dB_t, for\ dX_t^2 = \sigma^2 dt$$

dX_t : The process increment between time t and dt

θ : The expected value of the process in the long run, and is assumed constant. It is commonly referred to as the drift component.

κ : The speed of reversion of the process towards its long term expected value and is assumed constant.

dt : An infintesimal increase in time t

$\sigma, (\sigma > 0)$: Instantaneous diffusion term of the process, which is used to measure volatility and is assumed constant.

dB_t : Increment in interval $(t, t+dt)$ of a standard Brownian motion, under a probability measure P and is distributed as a $N(0,t)$ random variable.

Solving the SDE[17], we obtain that the mean reverting process X_t :

$$X_t \sim \mathrm{Normal}(X_0 e^{-kt} + \text{`}(1-e^{-kt}), \frac{\text{œ}^2}{2\text{ˇ}}(1-e^{-2\text{ˇ}t}))$$

The long-term mean and variance for the process is:

$$\lim_{t\to\infty} E(X_t) = \lim_{t\to\infty}[X_0 e^{-kt} + \theta(1-e^{-kt})] = \theta$$

$$\lim_{t\to\infty} Var(X_t) = \lim_{t\to\infty}[\frac{\sigma^2}{2\kappa}(1-e^{-2\kappa t})] = \frac{\sigma^2}{2\kappa}$$

We model the normalized spread series using the discretized version of the OU-Process by noting that the OU-Process is the continuous time analog of the AR(1) process as shown below:

$$X_t = X_{t-1}e^{-\kappa\Delta t} + \theta(1-e^{-\kappa\Delta t}) + \sigma\sqrt{\frac{1}{2\kappa}(1-e^{-\kappa\Delta t})}\epsilon_t,\ \epsilon_t \sim N(0,1)$$

Using the AR(1) process of the form: $X_t = \beta_0 + \beta_1 X_{t-1} + \epsilon_t$, where:
where:
$\beta_0 = \theta(1-e^{-\kappa\Delta t})$
$\beta_1 = e^{-\kappa\Delta t}$
$SE(Standard\ error) = \sigma\sqrt{\frac{1}{2\kappa}(1-e^{-\kappa\Delta t})}$

This is an interesting way of modelling the series, since it simply breaks down to fitting a regression model on the spread series and its one-day lag. The coefficients of the regression model are then translated back to their equivalent OU-Process values as follows:

$$\hat{\theta} = \frac{\beta_0}{1-\beta_1}$$

$$\hat{\kappa} = \frac{1}{\Delta t} log_e(\frac{1}{\beta_1})$$

$$\hat{\sigma} = SE\sqrt{\frac{1-\beta_1^2}{2\hat{\kappa}}}$$

The half-life of mean reversion, which depends only on the speed of reversion to mean is given as:

$$h = \frac{ln(2)}{\kappa}$$

The summary table below gives the estimates of the parameters of the Ornstein-Uhlenbeck process for our normalized price series.

[17] Stochastic Differential Equation

	Mean(theta)	Reversion speed(Kappa)	Half-life	Variance
Normalized ratio	-0.0127223	0.0439143	15.78410	0.0871269
Normalized regression spread	-0.0117290	0.0442218	15.67434	0.0877856

Both normalized spread series have near-similar estimates for the Ornstein-Uhlenbeck statistics. The long-term mean of the both processes is around 0. The reversion speed of the processes is quite low[18]. The half life is 15 days which is short and desirable for mean reversion trading.

The volatility is about 29%. Due to their similarity, we only proceed with back-testing the regression based spread series.

[18] The higher the value of the reversion speed, the more attractive the mean-reversion trade becomes

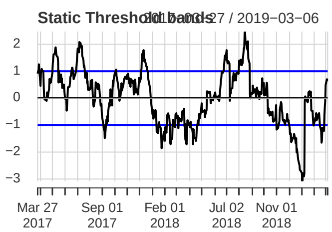


Figure 4: Equidistant static threshold bands for the spread

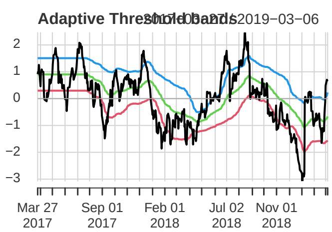


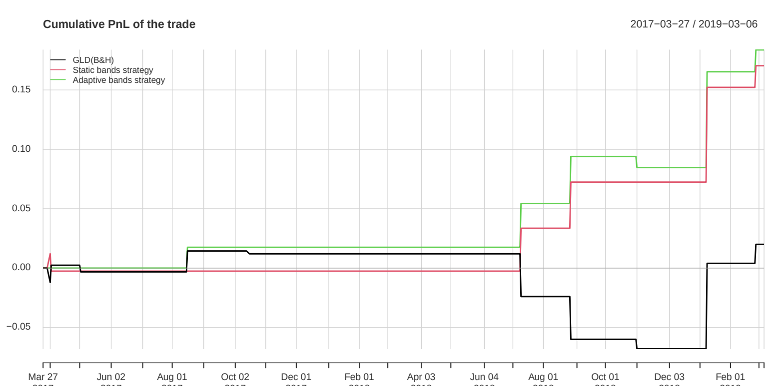
Figure 5: Equidistant adaptive threshold bands for the spread

TRADE EXECUTION

In this section, we back-test the trading strategy in two ways:

- We LONG the spread, when it is below the lower static threshold value($-1 * sd$), and we SHORT the spread when it is above the upper static threshold($1 * sd$). We close any open positions on reversion to the mean value.
- We LONG the spread, when it is below the lower adaptive threshold value($-1 * sd$), and we SHORT the spread when it is above the upper adaptive threshold($1 * sd$). We close any open positions on reversion to the mean value. For the adaptive spread bands, we use a look-back period of some multiple of 15 days(half-life of mean reversion). For this analysis, a back-test optimization reveals that a 60/75 day rolling Exponential moving band(EMA) delivers better returns

The cumulative profit & Loss of the spread trades is shown below:



The trading statistics are summarized below:

	GLD(B&H)	Static bands strategy	Adaptive bands strategy
Annualized Return(%)	1.0500	8.6100	9.2600
Annualized Volatility(%)	7.1200	6.8100	6.8200
Annualized Sharpe (Rf=0%)	0.1469	1.2648	1.3589

The mean reversion strategy using static bands delivers a return of 8.6 % p.a while the adaptive-band strategy delivers a 9.3 % returns p.a, clearly outperforming a buy-and-hold strategy of the GLD ETF. The sharpe ratio which is simply a measure of units of returns per risk is sufficiently higher for the mean reversion strategy.

CONCLUSION

It is evident that using a mean-reversion trade to take advantage of the tracking inefficiency in the GLD~XAUUSD is profitable consistently. It is important to note however that for our case the GLD was not volatile enough during the period under investigation due to low demand in the kenyan market, which makes the strategy have very few trades (11 trades for static bands, and 15 trades for adaptive bands during the period 2017-March to 2019 March)[19]. The strategy however is able to outperform a Buy and hold version of the GLD ETF significantly delivering 8% annual returns for static bands, and 9.3% for adaptive rolling bands. In the backtesting of the strategy, we do not include transaction costs. Including transaction costs might erode some profits of the GLD mean-reversion trade, however the strategy will still be able to perform.

[19] The trading strategy using static bands closed a total of 1 position with negative turnover, 5 positions with 0 net profit, and 4 trades with high positive turnover. Using the adaptive bands, it closed a total of 5 trades with positive turnover, 1 with negative turnover and 7 with 0 net profit.

To conclude the study, we find that there indeed exists significant deviation between the GLD and the XAUUSD(*Proxy for the gold bullion*). This tracking deviation could significantly be exploited for profitable trading under a mean-reversion framework. We recommend using adaptive bands, with the look-back set to some multiple of the half-life of reversion, for trading such an ETF with some tracking error.

REFERENCES

`https://www.absabank.co.ke/content/dam/kenya/absa/pdf/`

news-press-release/new-gold-eft-brochure.pdf

Chan, E. P. (2021). Quantitative trading: how to build your own algorithmic trading business. John Wiley & Sons.

Chan, E. (2013). Algorithmic trading: winning strategies and their rationale (Vol. 625). John Wiley & Sons.