

Exploring Semantic Hierarchies to Improve Resolution Theorem Proving on Ontologies

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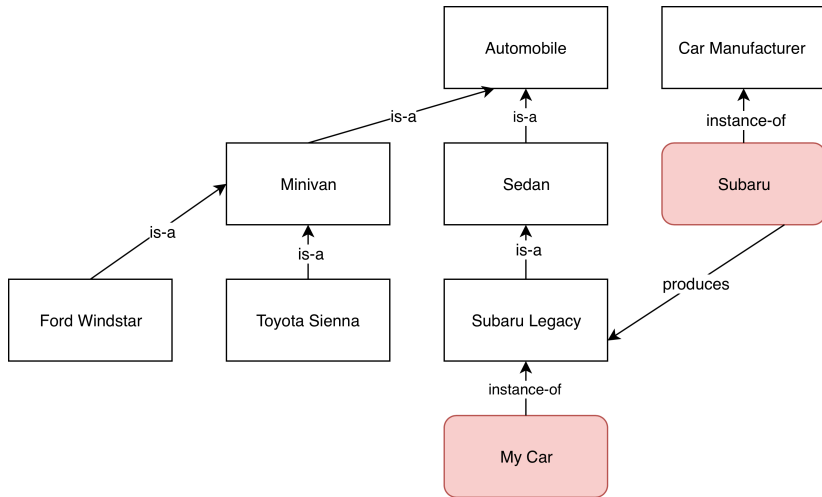
May 2, 2019

Agenda

Defense (2.5 Hours)

- ▶ Honors Thesis (1 Hour)
 - ▶ Presentation (20 min)
 - ▶ Questions (40 min)
- ▶ Honors Reading List (1 Hour)
 - ▶ Reading List Description (5 min)
 - ▶ Reading List Discussion (55 min)
- ▶ Committee Deliberation (30 min)
 - ▶ Level of honors discussion
 - ▶ Suggestions for Revision

Ontologies



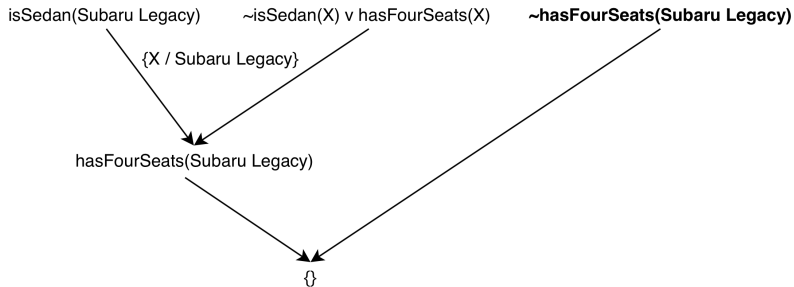
First Order Logic

SubaruLegacy(myCar)

$\forall x \text{ SubaruLegacy}(x) \rightarrow \text{Sedan}(x)$

$\forall x \text{ Sedan}(x) \rightarrow \text{Automobile}(x)$

Theorem Proving



Semantic Hierarchies

Class hierarchy:

Asserted

- owl:Thing
 - biological_process
 - cellular_component
 - 'axon part'
 - 'axoneme part'
 - 'bacterial-type flagellum part'
 - 'biofilm matrix component'
 - cell
 - 'cell cortex part'
 - 'cell division site part'
 - 'cell junction'
 - 'cell part'
 - 'cell projection part'
 - 'cell septum part'
 - 'cell wall part'
 - 'chloroplast part'
 - 'chromosomal part'
 - 'ciliary part'
 - 'collagen and cuticulin-based cuticle ext'
 - 'contractile fiber part'
 - 'cytoplasmic part'

Annotation property hierarchy Datatypes

Data property hierarchy Individuals by type

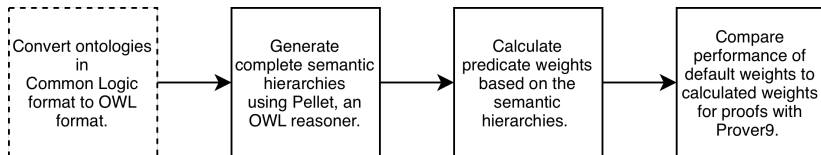
Object property hierarchy

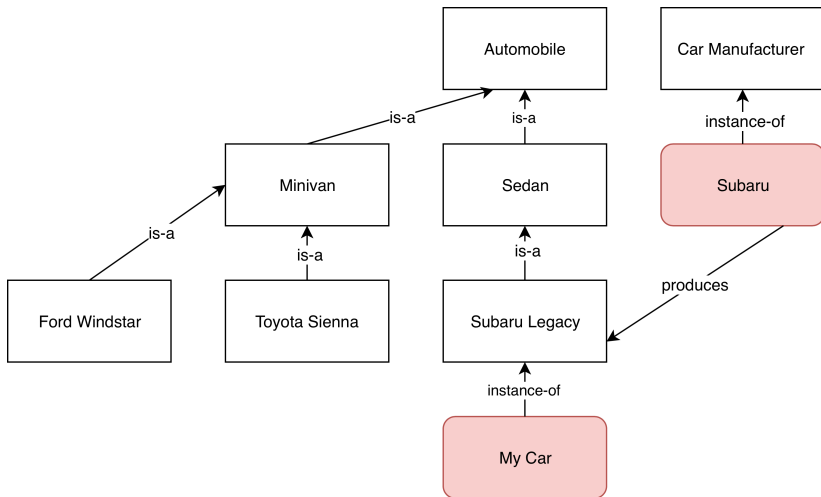
Object property hierarchy:

Asserted

- owl:topObjectProperty
 - aboral_to
 - adjacent_to
 - immediately_anterior_to
 - immediately_deep_to
 - immediately_posterior_to
 - immediately_superficial_to
 - 'anastomoses with'
 - anterior_to
 - attaches_to_part_of
 - 'bearer of'
 - 'boundary of'
 - 'channel for'
 - channels_from
 - channels_into
 - 'child nucleus of'

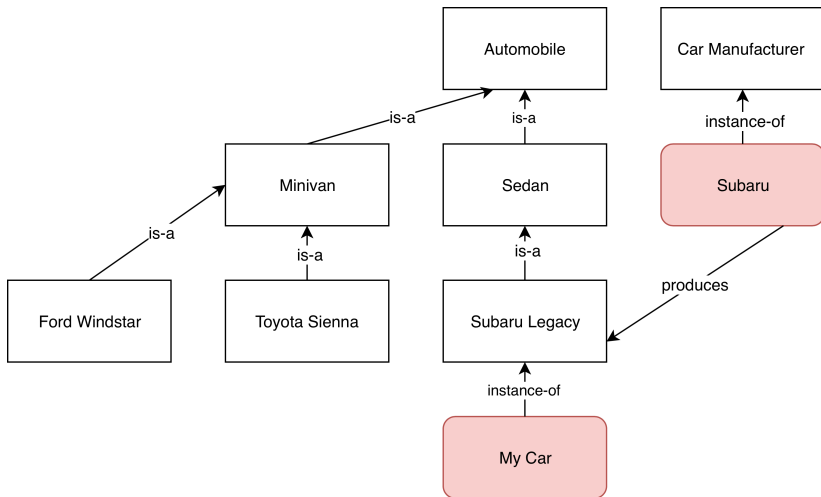
Approach





Function 1 Weights

- ▶ $\text{weight}(\text{SubaruLegacy}(x)) = 1.$
- ▶ $\text{weight}(\text{Sedan}(x)) = 1.$
- ▶ $\text{weight}(\text{Automobile}(x)) = 1.$
- ▶ $\text{weight}(\text{Minivan}(x)) = 2.$
- ▶ $\text{weight}(\text{ToyotaSienna}(x)) = 3.$
- ▶ $\text{weight}(\text{FordWindstar}(x)) = 3.$
- ▶ $\text{weight}(\text{CarManufactuer}(x)) = 10.$
- ▶ $\text{weight}(\text{Produces}(x,y))$ - This is not defined as the conjecture contains no relationships.



Function 2 Weights

- ▶ $\text{weight}(\text{SubaruLegacy}(x)) = 1.$
- ▶ $\text{weight}(\text{FordWindstar}(x)) = 1.$
- ▶ $\text{weight}(\text{Automobile}(x)) = 1.$ (LCA)
- ▶ $\text{weight}(\text{Sedan}(x)) = 2.$
- ▶ $\text{weight}(\text{Minivan}(x)) = 2.$
- ▶ $\text{weight}(\text{ToyotaSienna}(x)) = 3.$
- ▶ $\text{weight}(\text{CarManufactuer}(x)) = 3.$

Conjecture	Default	Function 1	Percent Change 1	Function 2	Percent Change 2
1	85691	9165	-89	9231	-89
2	1803	1803	-	1803	-
3	1803	1803	-	1803	-
4	175	175	-	175	-
5	175	175	-	175	-
6	172	172	-	172	-
7	6357	6337	0	6225	-2
8	6015	5855	-3	2352	-61
9	1802	1802	-	1802	-
Average	11555	3032	-73	2638	-77
Median	1803	1803	0	1803	0
Sum	103993	27287	-74	23738	-77

Table: Results for the multidim_space_voids Ontology

Conjecture	Default	Function 1	Percent Change 1	Function 2	Percent Change 2
1	140734	50476	-64	50476	-64
2	480	754	57	742	55
3	295	295	-	234	-21
4	308	308	-	332	8
5	28188	28188	-	28188	-
6	11793	7830	-34	7830	-34
Average	30300	14642	-52	14634	-52
Median	6137	4292	-30	4286	-30
Sum	181798	87851	-52	87802	-52

Table: Results for the inch Ontology

Conjecture	Default	Function 1	Percent Change 1	Function 2	Percent Change 2
1	426	426	-	410	-4
2	285	285	-	285	-
3	426	426	-	430	-1
4	289	266	-8	324	-12
5	438	438	-	323	-26
6	283	240	-15	283	-
7	495	425	-14	255	-48
Average	377	358	-5	330	-12
Median	426	425	0	323	-24
Sum	2642	2506	-5	2310	-13

Table: Results for the multidim_space_physcont Ontology

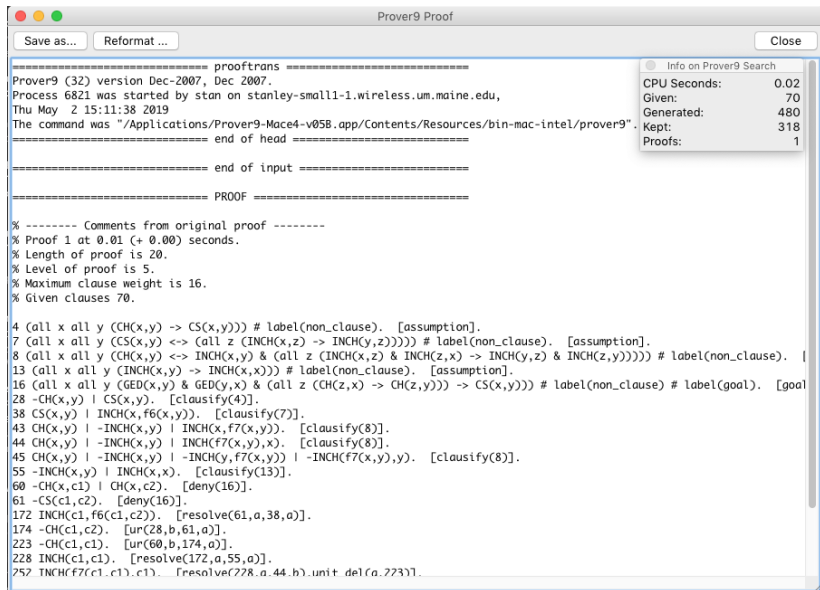
Metric	Default	Function 1	Percent Change 1	Function 2	Percent Change
Average	13111	5347	-59	5175	-61
Median	459	432	-6	420	-8
Sum	288433	117644	-59	113850	-61

Table: Overall Results

Limitations

- ▶ Increase in clauses for inch ontology
- ▶ $(\text{all } x \text{ all } y (\text{GED}(x,y) \ \& \ \text{GED}(y,x) \ \& \ (\text{all } z (\text{CH}(z,x) \rightarrow \text{CH}(z,y))) \rightarrow \text{CS}(x,y)))$.
- ▶ Semantic hierarchy has a depth of 2

Prover9 Output



```
===== prooftrans =====
Prover9 (32) version Dec-2007, Dec 2007.
Process 6821 was started by stan on stanley-small1-1.wireless.um.maine.edu,
Thu May 2 15:11:38 2019
The command was "/Applications/Prover9-Mace4-v05B.app/Contents/Resources/bin-mac-intel/prover9".
===== end of head =====

===== end of input =====

===== PROOF =====

% ----- Comments from original proof -----
% Proof 1 at 0.01 (+ 0.00) seconds.
% Length of proof is 20.
% Level of proof is 5.
% Maximum clause weight is 16.
% Given clauses 70.

4 (all x all y (CH(x,y) -> CS(x,y))) # label(non_clause). [assumption].
7 (all x all y (CS(x,y) <=> (all z (INCH(x,z) -> INCH(y,z))))) # label(non_clause). [assumption].
8 (all x all y (CH(x,y) <=> INCH(x,y) & (all z (INCH(x,z) & INCH(z,x) -> INCH(y,z) & INCH(z,y))))) # label(non_clause). [assumption].
13 (all x all y (INCH(x,y) -> INCH(x,x))) # label(non_clause). [assumption].
16 (all x all y (GED(x,y) & GED(y,x) & (all z (CH(z,x) -> CH(z,y))) -> CS(x,y))) # label(non_clause) # label(goal). [goal]
28 -CH(x,y) | CS(x,y). [clausify(4)].
38 CS(x,y) | INCH(x,f6(x,y)). [clausify(7)].
43 CH(x,y) | -INCH(x,y) | INCH(x,f7(x,y)). [clausify(8)].
44 CH(x,y) | -INCH(x,y) | INCH(f7(x,y),x). [clausify(8)].
45 CH(x,y) | -INCH(x,y) | -INCH(y,f7(x,y)) | -INCH(f7(x,y),y). [clausify(8)].
55 -INCH(x,y) | INCH(x,x). [clausify(13)].
60 -CH(x,c1) | CH(x,c2). [deny(16)].
61 -CS(c1,c2). [deny(16)].
172 INCH(c1,f6(c1,c2)). [resolve(61,a,38,a)].
174 -CH(c1,c2). [ur(28,b,61,a)].
223 -CH(c1,c1). [ur(60,b,174,a)].
228 INCH(c1,c1). [resolve(172,a,55,a)].
252 INCH(f7(c1,c1),c1). [resolve(228,a,44,b).unit.del(a,223)].
```

Info on Prover9 Search

CPU Seconds:	0.02
Given:	70
Generated:	480
Kept:	318
Proofs:	1

Summary

- ▶ When proving specific conjectures with few predicates on large ontologies, semantic hierarchies can focus the search of a resolution theorem prover.
- ▶ Results of the experiments conducted indicate further work might yield lucrative results, especially for exceptionally large ontologies.
- ▶ Tests demonstrated success in a relatively unexplored domain of research.

Thank You

Questions

My Thoughts

- ▶ A Mind for Numbers
- ▶ The Inner Game of Tennis
- ▶ Trying Not to Try
- ▶ I Ching

My Career

- ▶ Algorithms to Live By
- ▶ Black Mirror
- ▶ The Signal and the Noise
- ▶ Disrupted

My Worldview

- ▶ What Every BODY is Saying
- ▶ Mr. Nobody
- ▶ The Lobster
- ▶ BoJack Horseman