Estimating the square root of a number using the Monte Carlo method

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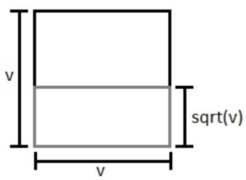
June 19, 2020

Our goal estimate the square root of the number v

Consider the following rectangle inscribed in a square:



The ratio of the area Ar of the rectangle to the area As of the square is:

$$\frac{Ar}{As} = \frac{v\sqrt{v}}{v^2}$$
$$= \sqrt{v}/v$$
$$= 1/\sqrt{v} \text{ (I)}$$

Let X and Y be uniform random variables continuous in $[0,1]$, and let $g(x,y)$ be the indicator function which tells us if the point (x,y) is inside the rectangle:

$$g(x,y) = 1, \text{ if } (\sqrt{v} * y)^2 \leq 1 \Rightarrow v * y^2 \leq 1$$
$$g(x,y) = 0, \text{ otherwise}$$

Since our function $g(x,y)$ depends only on y we can rewrite it as $g(y)$. From (I) We know that:

$$E[g(y)] = \frac{Ar}{As} = \frac{1}{\sqrt{v}}, \text{ therefore } \sqrt{v} = \frac{1}{E[g(y)]}$$

We need to estimate $E[g(y)]$, and we know, by definition, that:

$$M_n = \frac{1}{n}\sum_{i=1}^{n} g(Y_i)$$

From law of the large numbers we know that:

$$M_n \to E[g(y)] = \frac{1}{\sqrt{v}}$$

Therefore, $\sqrt{v}$ can be estimated by $\frac{1}{M_n}$:

$$\sqrt{v} = \frac{n}{\sum_{i=1}^{n} g(Y_i)}$$