

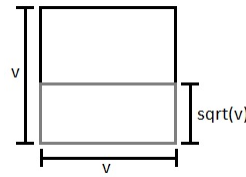
Estimating the square root of a number using the Monte Carlo method

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Our goal estimate the square root of the number v

Consider the following rectangle inscribed in a square:



The ratio of the area Ar of the rectangle to the area As of the square is:

$$\begin{aligned}\frac{Ar}{As} &= \frac{v\sqrt{v}}{v^2} \\ &= \sqrt{v}/v \\ &= 1/\sqrt{v} \text{ (I)}\end{aligned}$$

Let X and Y be iid sequences continuous in $[0, 1]$, and let $g(X_i, Y_i)$ be the indicator function which tells us if the point (X_i, Y_i) is inside the rectangle:

$$\begin{aligned}g(X_i, Y_i) &= 1, \text{ if } (\sqrt{v} * Y_i)^2 \leq 1^2 \Rightarrow v * Y_i^2 \leq 1 \\ g(X_i, Y_i) &= 0, \text{ otherwise}\end{aligned}$$

The function $g(X_i, Y_i)$ depends only on Y_i , we may rewrite it as $g(Y_i)$. From (I) we see that:

$$E[g(Y_i)] = \frac{Ar}{As} = \frac{1}{\sqrt{v}}, \text{ therefore } \sqrt{v} = \frac{1}{E[g(Y_i)]}$$

We need to estimate $E[g(Y_i)]$, from the law of the large numbers we know that:

$$\begin{aligned}M_n &\rightarrow E[g(Y_i)] = \frac{1}{\sqrt{v}}, \text{ and} \\ M_n &= \frac{1}{n} \sum_{i=1}^n g(Y_i), \text{ by definition}\end{aligned}$$

Therefore:

$$\begin{aligned}\frac{1}{\sqrt{v}} &\rightarrow \frac{1}{n} \sum_{i=1}^n g(Y_i) \\ \sqrt{v} &\rightarrow \frac{n}{\sum_{i=1}^n g(Y_i)}\end{aligned}$$