Estimating the square root of a number using the Monte Carlo method

Stanley Cortes de Sousa

June 18, 2020

Our goal estimate the square root of the number v

Consider a rectangle of base v and height $h = \sqrt{v}$ inscribed in a square of side v:

$v = h^2$ (I)

The ratio of the area Ar of the rectangle to the area As of the square is:

$$\frac{Ar}{As} = \frac{vh}{v^2}$$
$$= h/v, \text{ using (I)}$$
$$= h/h^2$$
$$= 1/h \text{ (II)}$$

Now consider a continuous uniform distribution $U = unif(0, 1)$

And let the random variables X and Y be samples of U

We define the indicator function $g(x, y)$, $x \in X$ e $y \in Y$ as follows:

$g(x, y) = 1$, if $v * y^2 \leq 1\ \forall y \in Y$. The point (x, y) is inside the rectangle

$g(x, y) = 0$, otherwise

From this definition of $g(x, y)$ its easy to see that we can rewrite it as $g(y)$

How can we calculate $E[g(y)]$?

From (II) we know that:

$$E[g(y)] = \frac{Ar}{Aq} = \frac{1}{h}$$

On the other hand:

$$M_n = \frac{1}{n} \sum_{i=1}^{n} g(Y_i)$$

From law of the large numbers we know that:

$$M_n \to \frac{1}{h}$$

Therefore, h can be estimated by $\frac{1}{M_n}$

$$\sqrt(v) \to \frac{1}{M_n} = \frac{n}{\sum_{i=1}^{n} g(Y_i)}$$