Estimating the square root of a number using the Monte Carlo method

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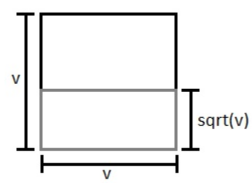
Our goal estimate the square root of the number v

Consider the following rectangle inscribed in a square:



The ratio of the area Ar of the rectangle to the area As of the square is:

$$\frac{Ar}{As} = \frac{v\sqrt{v}}{v^2}$$
$$= \sqrt{v}/v$$
$$= 1/\sqrt{v} \text{ (I)}$$

Let X and Y be uniform random variables continuous in $[0,1]$, and let $g(X_i, Y_i)$ be the indicator function which tells us if the point (X_i, Y_i) is inside the rectangle:

$g(X_i, Y_i) = 1$, if $(\sqrt{v} * Y_i)^2 \leq 1^2 \Rightarrow v * Y_i^2 \leq 1$

$g(X_i, Y_i) = 0$, otherwise

The function $g(X_i, Y_i)$ depends only on Y_i, we may rewrite it as $g(Y_i)$. From (I) we see that:

$E[g(Y_i)] = \dfrac{Ar}{Aq} = \dfrac{1}{\sqrt{v}}$, therefore $\sqrt{v} = \dfrac{1}{E[g(Y_i)]}$

We need to estimate $E[g(Y_i)]$, from the law of the large numbers we know that:

$M_n \to E[g(Y_i)] = \dfrac{1}{\sqrt{v}}$, and

$M_n = \dfrac{1}{n} \sum_{i=1}^{n} g(Y_i)$, by definition

Therefore:

$$\frac{1}{\sqrt{v}} \to \frac{1}{n} \sum_{i=1}^{n} g(Y_i)$$
$$\sqrt{v} \to \frac{n}{\sum_{i=1}^{n} g(Y_i)}$$