Problem 1. Lizzie and Alex are playing a game on the whiteboard. Initially, n twos are written on the board. On a player's turn, they must either:

(1) change any single positive number to 0, or

(2) subtract one from any positive number of positive numbers on the board.

The game ends once all numbers are 0, and the last player who made a move wins. If Lizzie always plays first, find all n for which Lizzie has a winning strategy.

Solution. Define a position to be an pair of integer (a, b) where a and b are the number of twos and ones on the board respectively. We will say the position is `W`, a winning position, if Lizzie can win in the current turn, or `L`, a losing position, if Alex can win in the current turn. ■

Problem 2. Let $\mathscr{F}$ be the set of functions $f(x, y)$ that are twice continuously differentiable for $x \geq 1$ and $y \geq 1$ that satisfy:

For each $f \in \mathscr{F}$, let:

$$m(f) = \min_{s \geq 1} \Big(f(s+1, s+1) - f(s+1, s) - f(s, s+1) + f(s, s) \Big).$$

Determine $m(f)$ and show it is independent of the choice of f. https://www.google.com