Humidity

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Calculating relative humidity at a temperature given the temperature and humitity of another volume.

Relative humidity should be within the range of 40-60%:

$$\phi = \frac{e_w}{e_w^*},$$

e_w - partial pressure of water vapour, e_w^* - saturated vapour pressure of water at a given temperature.

Alternatively

$$\phi = \frac{S_H p}{(0.622 + 0.378 S_H)\, p_{H_20}^*}$$

S_h - specific humidity:
$S_h = \frac{m_v}{m_a}$
p - pressure

Use ideal gas equation - mass, volume, R are all constant. We can find the actual vapour pressure as a function of temperature:

$$e_w = \frac{mR_vT}{V}$$

$$\phi = \frac{mR_vT}{Ve_{w,T}^*}$$

$$e_{w,1}^* = \frac{mR_vT_1}{V\phi_1}$$

$$\frac{mR_vT_2}{V\phi_2} = \frac{mR_vT_1}{V\phi_1}$$
$$\phi_2 = \frac{\phi_1 T_2}{T_1}$$

Saturated vapour pressure, e_w^*, is from:

$$e_w^* = 6.112 \exp\left(\frac{17.62T}{243.12+T}\right)$$

$$\begin{aligned}\phi_2 &= \frac{e_{w,2}}{e^*_{w,2}} \\ &= \frac{\frac{mR_vT_2}{V}}{6.112 \exp\left(\frac{17.62T_2}{243.12+T_2}\right)}\end{aligned}$$

$$\begin{aligned}\frac{m}{V} &= \frac{e_w^*\phi_1}{R_vT_1} \\ &= \frac{\phi_1 6.112 \exp\left(\frac{17.62T}{243.12+T}\right)}{R_vT_1}\end{aligned}$$

$$\phi_2 = \frac{T_2\phi_1 \exp\left(\frac{17.62T_1}{243.12+T_1}\right)}{T_1 \exp\left(\frac{17.62T_2}{243.12+T_2}\right)}$$

$\phi_1 = 85\%$, $T_1 = 15C = 288K$, $T_2 = 22C = 295K$