1 Synchronous generators

1.1 Dynamic model

$$\frac{d\delta_i}{dt} = \omega_i - \omega_0 \tag{1}$$

$$M_i \frac{d\omega_i}{dt} = -D_i(\omega_i - \omega_0) + P_i^m - \frac{E_i V_i \sin(\delta_i - \theta_i)}{X_i} \tag{2}$$

$$\tau_i \frac{dP_i^m}{dt} = -P_i^m - \frac{1}{R_i \omega_0}(\omega_i - \omega_0) + u_i \tag{3}$$

1.2 Real power balance

$$\frac{E_i V_i \sin(\delta_i - \theta_i)}{X_i} = V_i \sum_{k=1}^{n+l} V_k \left[G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k)\right] \tag{4}$$

$$\tag{5}$$

1.3 Reactive power balance

$$-\frac{V_i^2}{X_i} + \frac{E_i V_i}{X_i} \cos(\delta_i - \theta_i) = V_i \sum_{k=1}^{n+l} V_k \left[G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k)\right] \tag{6}$$

1.4 Computing Initial States

Set left hand side to all equations in Section 1.1 to zero. That is,

$$0 = \omega_i - \omega_0, \tag{7}$$

$$0 = -D_i(\omega_i - \omega_0) + P_i^m - \frac{E_i V_i \sin(\delta_i - \theta_i)}{X_i}, \tag{8}$$

$$0 = -P_i^m - \frac{1}{R_i \omega_0}(\omega_i - \omega_0) + u_i. \tag{9}$$

Then, if we assume that V_i and θ_i are known (from solving static power flow) and that u_i is known, we have that

$$\omega_{i,0} = \omega_0, \tag{10}$$

$$P_{i,0}^m = u_i - \frac{1}{R_i \omega_0}(\omega_{i,0} - \omega_0), \tag{11}$$

$$= u_i \tag{12}$$

$$\delta_{i,0} = \theta_{i.0} + \arcsin\left(\frac{X_i(P_{i,0}^m - D_i(\omega_{i,0} - \omega_0))}{E_i V_i}\right), \tag{13}$$

$$= \theta_{i.0} + \arcsin\left(\frac{X_i u_i}{E_i V_{i,0}}\right). \tag{14}$$

2 Inverter-interfaced power supplies

$$\frac{d\delta_i}{dt} = \omega_i - \omega_0 = \frac{1}{H_i}\left[u_i - V_i \sum_{k=1}^{n+l} V_k \left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right]\right] \tag{15}$$

$$0 = \frac{V_i^2}{X_i} - \frac{E_i V_i}{X_i}\cos(\delta_i - \theta_i) + V_i \sum_{k=1}^{n+l} V_k \left[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)\right] \tag{16}$$

For load buses:

$$0 = P_i^d + V_i \sum_{k=1}^{n+l} V_k \left[G_{ik}\cos(\theta_i - \theta_k) + B_{ik}\sin(\theta_i - \theta_k)\right] \tag{17}$$

$$0 = Q_i^d + V_i \sum_{k=1}^{n+l} V_k \left[G_{ik}\sin(\theta_i - \theta_k) - B_{ik}\cos(\theta_i - \theta_k)\right] \tag{18}$$