McMaster University Comp Sci 4TB3/6TB3, Winter Term 2018 — Lab 1
For the Labs on January 9 - 11,
Due Sunday, January 14, 11 pm
Max 28 Points

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- Solutions are to be submitted electronically on Avenue under Assessments → Assignments. In this lab, you are asked to write the submission with LaTeX and submit one .tex and one .pdf file. The files should be named yourmacid.tex and yourmacid.pdf, where yourmacid is your McMaster e-mail address, not your student number. Please do not compress these files or submit a compressed directory with the files. The .tex file should be self-contained, i.e. not include any other file, except the style files mentioned below, and should compile with `pdflatex yourmacid.tex`.
- In this lab, you are allowed to work in pairs, provided that you split the work equally and arrive at a common understanding of the solution. However, in that case you must state in your submission the person you worked with, such that similarities in the solution will not be construed as Academic Dishonesty. Working in groups of three or larger is not allowed and will be considered Academic Dishonesty. If you look for someone to work with, we will try to find a match, please contact the TAs.
- You are allowed and encouraged to talk to everyone in the course to get a common understanding of the problem, but you can share a solution only with your collaborator, if you work in a pair.
- If you work on own computer, you will need a TeX distribution, like TeX Live at `tug.org`; for Macs, MacTeX is recommended; on Windows, MiKTeX is popular. You might need to install packages from `ctan.org`. You can also run LaTeX remotely on either `moore.mcmaster.ca` or `mills.mcmaster.ca`.
- This lab asks you to draw parse tree. Fo this, you have to use a LaTeX package: `synttree` is a simple to use package that is included in TeX Live and MiKTeX, but you can use another package at `ctan.org/topic/tree`, like `qtree`. Do not use a package that is not in `ctan.org`
- This week, the solution to the Tutorial Exercise will be presented in the first hour of your lab. You can get help with LaTeX in the lab as well. Attendance at the labs is not checked.

Tutorial Exercise 1 (Generated Language)**.** Let $G = (T, N, P, S)$ be a grammar with $T = \{a\}$, $N = \{S\}$ and productions P given by:

$$S \rightarrow \varepsilon \tag{1}$$
$$S \rightarrow aS \tag{2}$$

Prove that $L(G) = \{a^n \mid n \geq 0\}$ giving formal arguments. *Hint:* prove inclusion in both directions separately. Use induction. See also the Dragon book.

Answer. The elements of $L(G)$ are, by definition, sentences over T that are derived from S, i.e. $L(G) = \{\chi \in T^* \mid S \Rightarrow^+ \chi\}$. The inclusion $L(G) \subseteq \{a^n \mid n \geq 0\}$ means that every element of $L(G)$ is a^n for some $n \geq 0$.

By considering all productions we show that all sentences derived from both S are of the form a^n, i.e. by induction assuming this for all S on the right hand sides we show that this holds for the left hand sides as well. For production (1),we have that $S \Rightarrow \varepsilon = a^0$, hence the inclusion holds. For production 2, we assume that S on the right hand side is of the form a^n, then aS is a^{n+1}, hence S on the left hand side is again of the form a^n, for some n. Thus we have $L(G) \subseteq \{a^n \mid n \geq 0\}$.

We now show that every a^n, for $n \geq 0$, can be generated by G. We do so by induction over n. Obviously $a^0 = \varepsilon$ can be generated by $S \Rightarrow \varepsilon$. Suppose a^n can be generated. Then we must have $S \Rightarrow^* a^n$. We need to show that a^{n+1} can be generated as well, which is done by $S \Rightarrow aS \Rightarrow^* aa^n = a^{n+1}$. Thus we have $L(G) \supseteq \{a^n \mid n \geq 0\}$ and can conclude $L(G) = \{a^n \mid n \geq 0\}$.

Lab Question 1 (Ambiguous Grammar, 8 points)**.** Let $G = (T, N, P, S)$ be a grammar with $T = \{\texttt{a}, \texttt{the}, \texttt{dog}, \texttt{baby}, \texttt{table}, \texttt{scratches}, \texttt{with}\}$, $N = \{S, NP, VP, PP, CN, CV, A, NN, V, PR\}$ and productions P given by:

$$
\begin{array}{rcl}
S & \rightarrow & NP \;\; VP \\
NP & \rightarrow & CN \mid CN \;\; PP \\
VP & \rightarrow & CV \mid CV \;\; PP \\
PP & \rightarrow & PR \;\; CN \\
CN & \rightarrow & A \;\; NN \\
CV & \rightarrow & V \mid V \;\; NP \\
A & \rightarrow & \texttt{a} \mid \texttt{the} \\
NN & \rightarrow & \texttt{cat} \mid \texttt{baby} \mid \texttt{nail} \\
V & \rightarrow & \texttt{scratches} \\
PR & \rightarrow & \texttt{with}
\end{array}
$$

a. Show that G is ambiguous by constructing two parse trees for the following sentence:
`a cat scratches the baby with a nail`.

b. Assuming that the sentence above is in the English language, describe how it could be interpreted in two different ways.

Lab Question 2 (Precedence of Operators, 16 points)**.** Write grammars for expressions made up of *id*, $*$, $/$ (where id is an identifier), e.g.

$$a * b / c * d$$

a. Write a grammar such that $*$ binds tighter than $/$, i.e. the above sentence would be evaluated as $(a * b)/(c * d)$. Draw the parse tree for $a * b/c * d$.

b. Write a grammar such that $/$ binds tighter than $*$, i.e. the above sentence would be evaluated as $a * (b/c) * d$. Draw the parse tree for $a * b/c * d$.

c. Write a grammar such that $+$ and $-$ bind equally strong but associate to the left, i.e. the above sentence would be evaluated as $((a * b)/c) * d$. Draw the parse tree for $a * b/c * d$.

d. Write a grammar such that $+$ and $-$ bind equally strong but associate to the right, i.e. the above sentence would be evaluated as $a * (b/(c * d))$. Draw the parse tree for $a * b/c * d$.

Note that the parentheses are not part of the grammar, they are only used for illustrating precedence.

Lab Question 3 (Generated Language, 4 points)**.** Let $G = (T, N, P, S)$ be a grammar with $T = \{a, b, c\}$, $N = \{A, S\}$ and productions P given by:

$$S \rightarrow A \tag{1}$$
$$A \rightarrow b \tag{2}$$
$$A \rightarrow aAc \tag{3}$$

Prove that $L(G) = \{a^n bc^n \mid n \geq 0\}$ giving formal arguments.