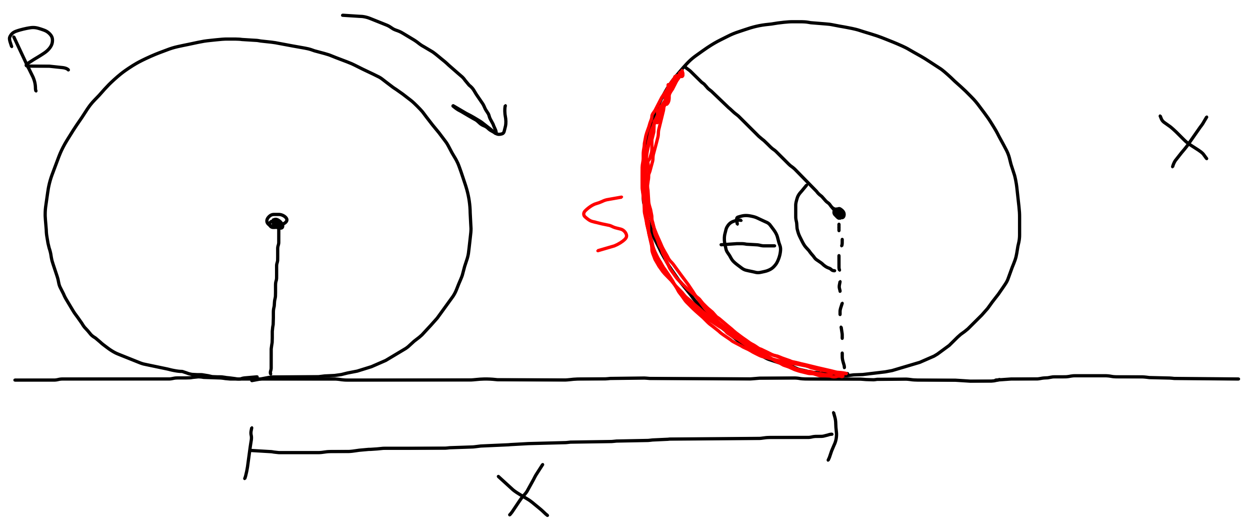
# Final Exam Review

* cumulative, but emphasis on chs. 7-8, 13-14

- ch. 7 - whole thing




$x = S = R\theta$

in radians

$v = R\omega$

angular velocity $\omega_{ave} = \dfrac{\Delta\theta}{\Delta t}$

angular acceleration $\alpha_{ave} = \dfrac{\Delta\omega}{\Delta t}$

$a = R\alpha$

for $\alpha = \text{const}$:

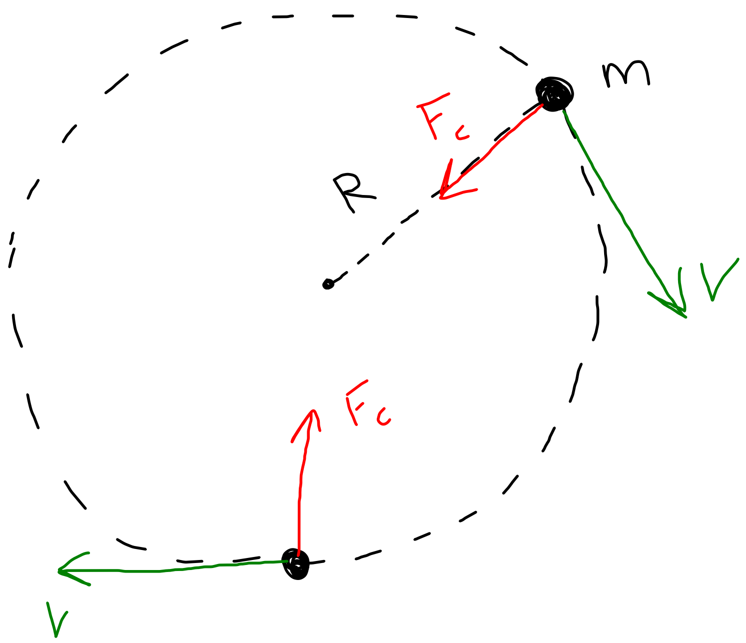
$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$$

$$\theta_f = \theta_i + \tfrac{1}{2}(\omega_i + \omega_f)t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

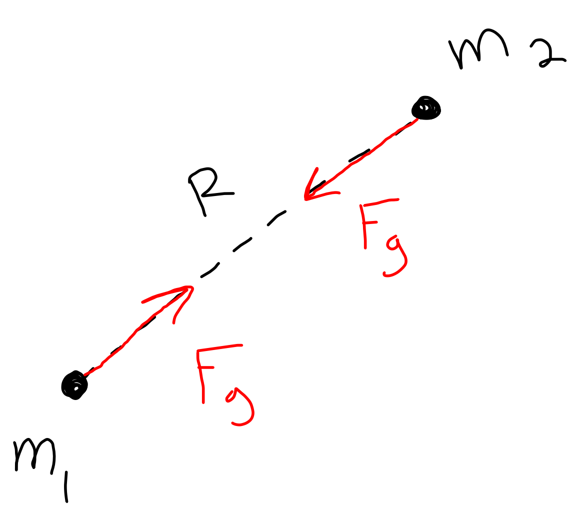
- circular motion (e.g. orbits)




$$F_c = \frac{mv^2}{R}$$ points at center

V is tangent to circle

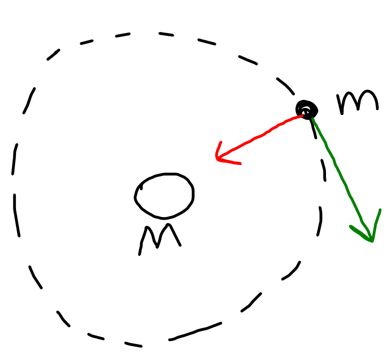
- Newtonian gravity




$$F_g = G\frac{m_1 m_2}{R^2}$$ attractive

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

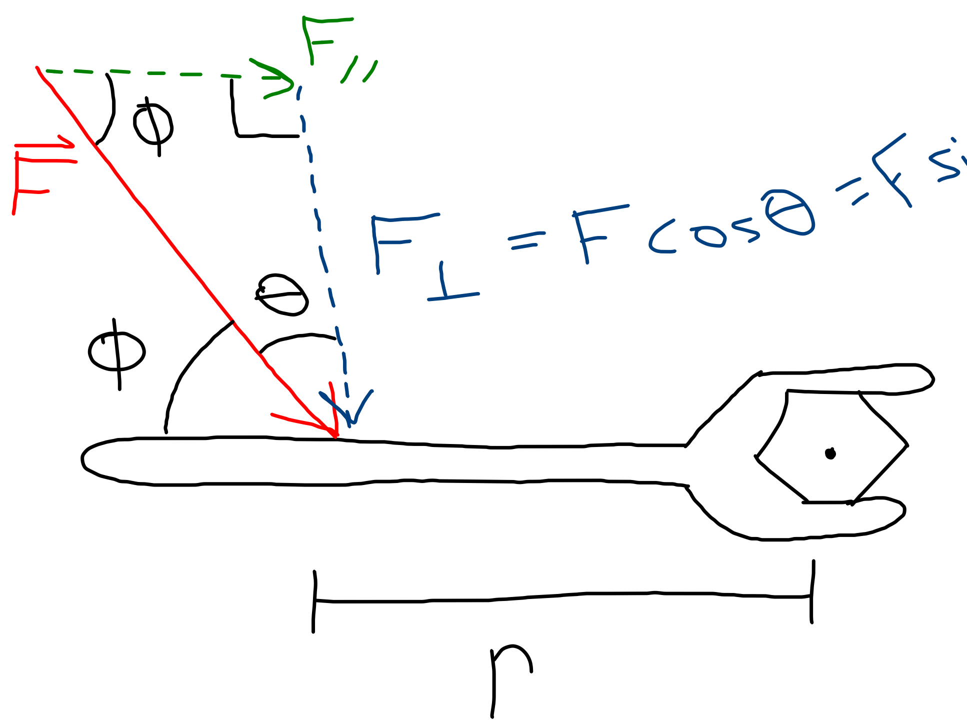
gravitational orbits:
↳ circular




$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{G\cancel{m}M}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$

- ch. 8 – sec. 1, 4




$F_{\perp} = F\cos\theta = F\sin\phi$

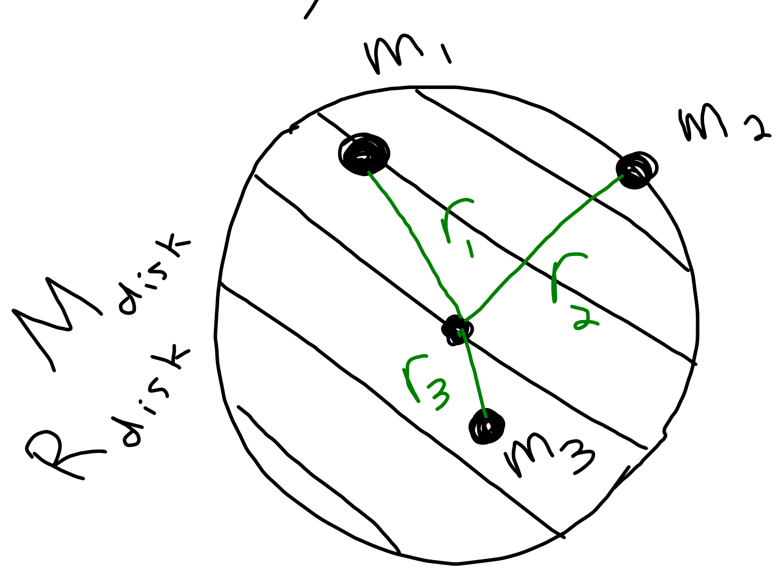
$\tau = r F_{\perp}$ magnitude

direction: cw $\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right)$ or ccw $\left(\begin{smallmatrix}-\\+\end{smallmatrix}\right)$

$$\sum \tau = I\alpha$$ where

$$\left(\sum F = m a\right)$$

$$I = \sum_j m_j r_j^2$$




$$I_{total} = \underbrace{I_{disk}}_{\frac{1}{2} M_{disk} R^2_{disk}} + m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

↖ get from table

- ch. 13

Simple Harmonic Motion

Period, $T$

frequency, $f$

angular frequency, $\omega$

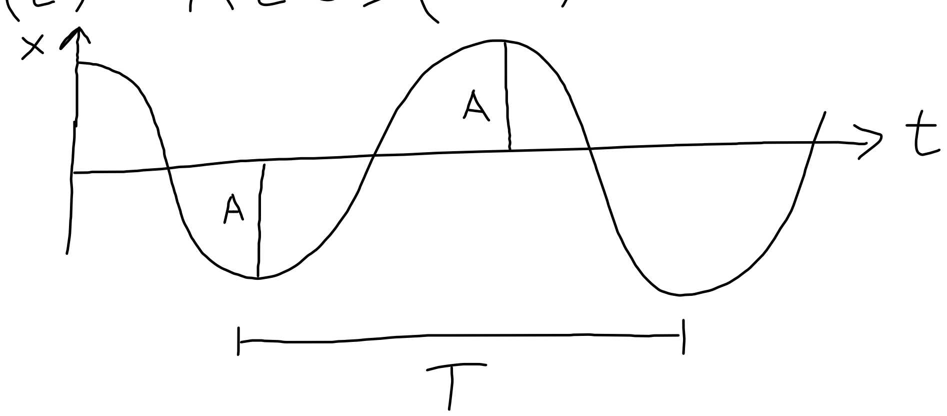
redundant

$T = \frac{1}{f}$, $f = \frac{1}{T}$

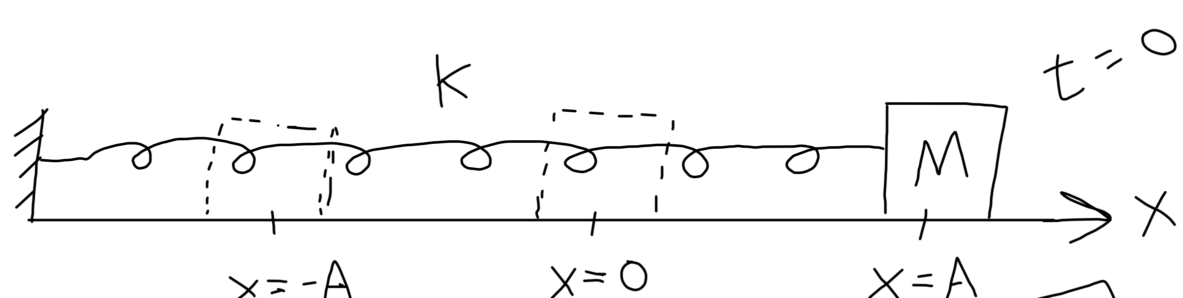
$\omega = 2\pi f$, $f = \frac{\omega}{2\pi}$

$\omega = \frac{2\pi}{T}$, $T = \frac{2\pi}{\omega}$

$x(t) = A\cos(\omega t)$




- spring-mass system

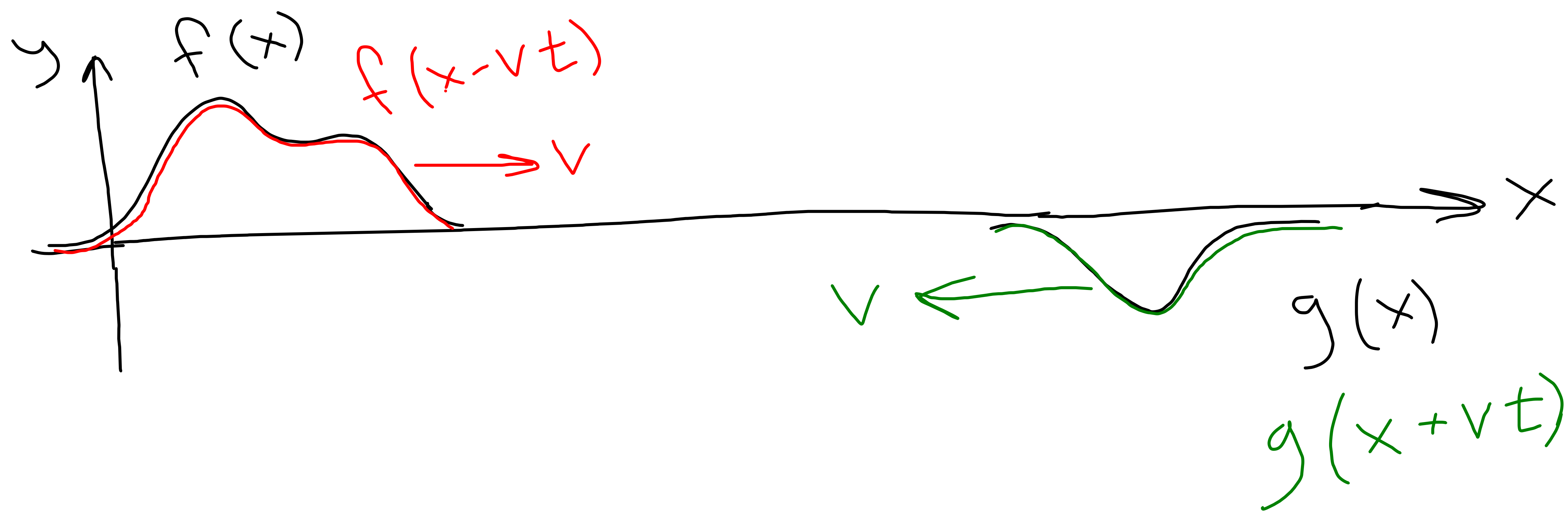



$$\boxed{\omega = \sqrt{\frac{k}{m}}}$$

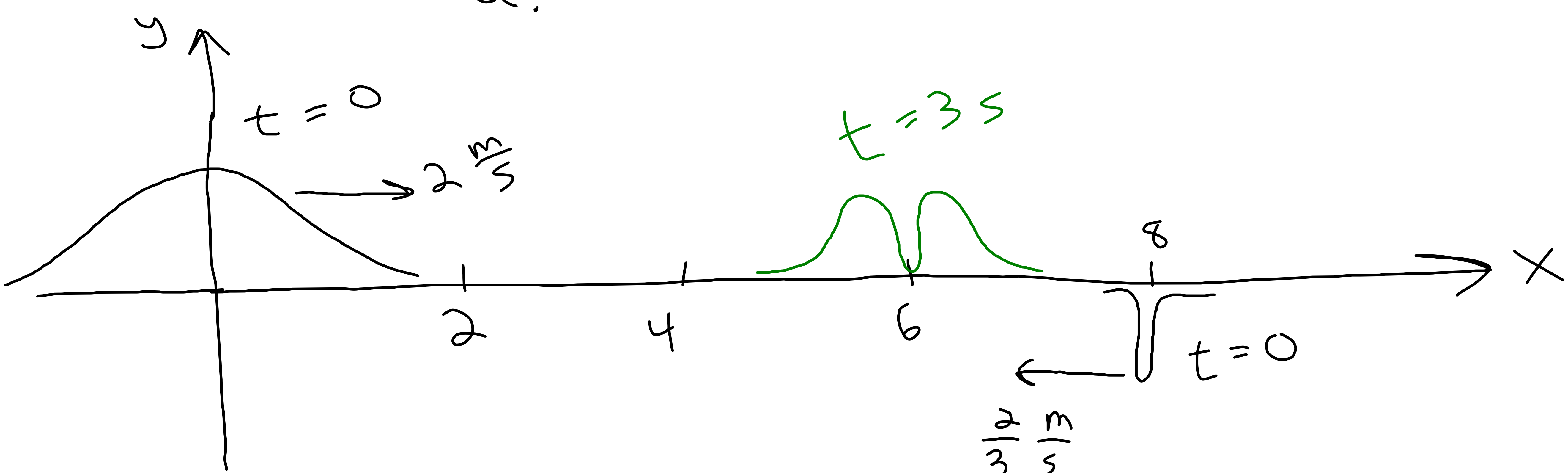
$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

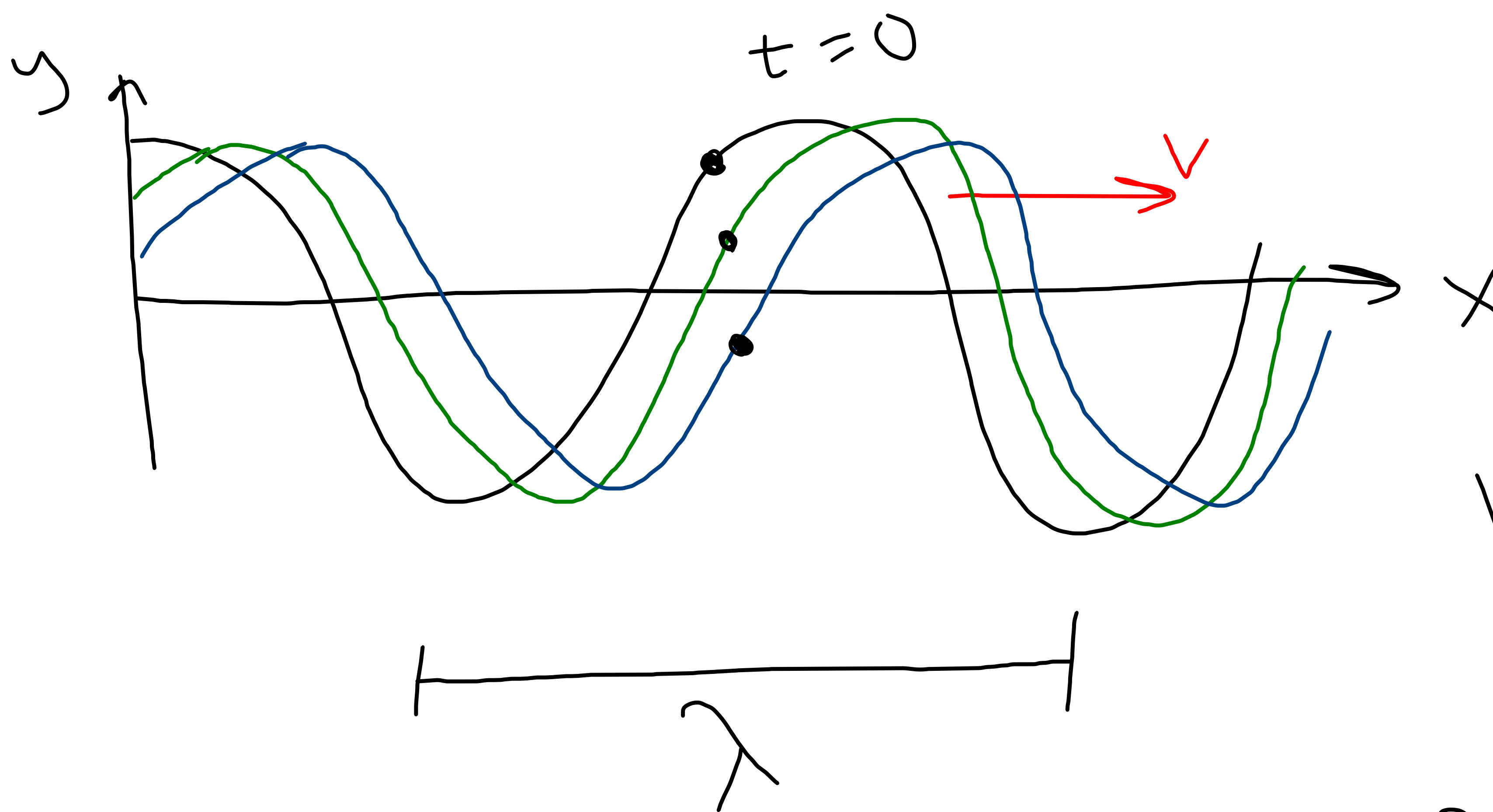
• waves – fixed shape, constant $v$




interference!



sinusoidal waves




$$V = \frac{\lambda}{T}$$

or, $V = \lambda f$

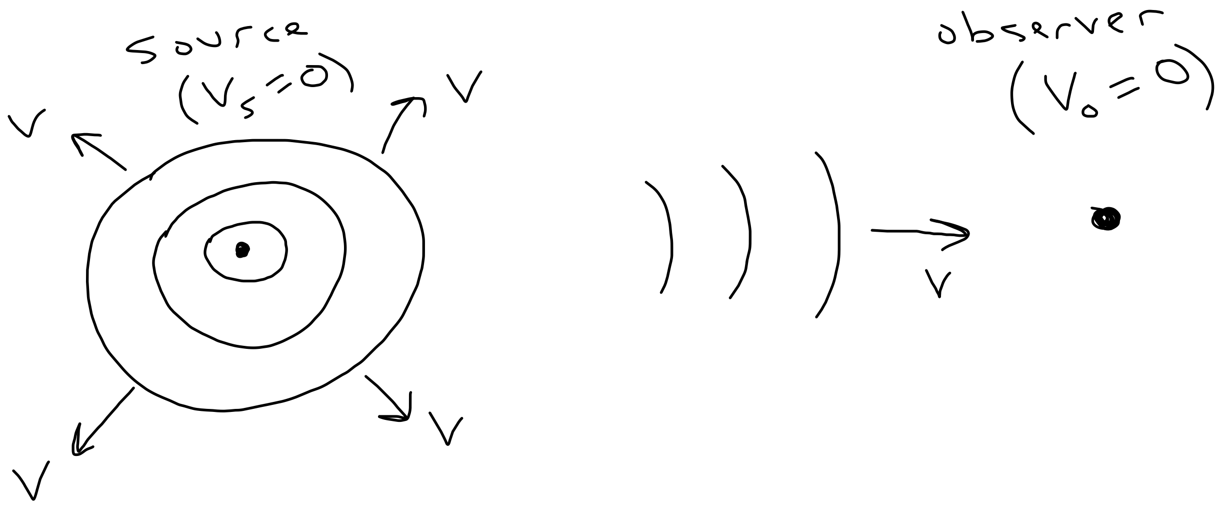
or, $V = \frac{\lambda \omega}{2\pi}$

or, $\frac{2\pi}{\lambda} V = \omega$

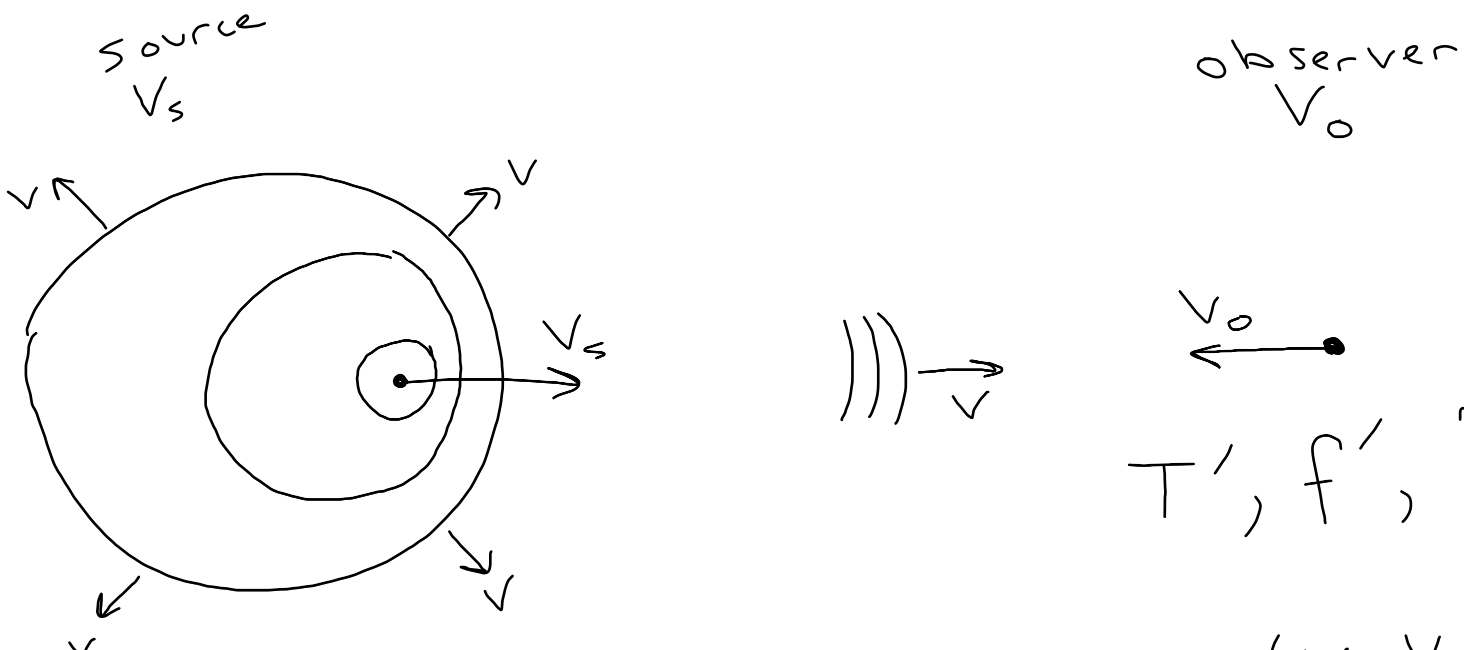
...

$$y = A \cos\left(\frac{2\pi}{\lambda}(x - Vt)\right)$$

• <u>ch. 14</u> - Doppler effect




$T, f, \lambda$




$T', f', \lambda'$

$$T' = \left(\frac{V - V_s}{V + V_o}\right) T$$

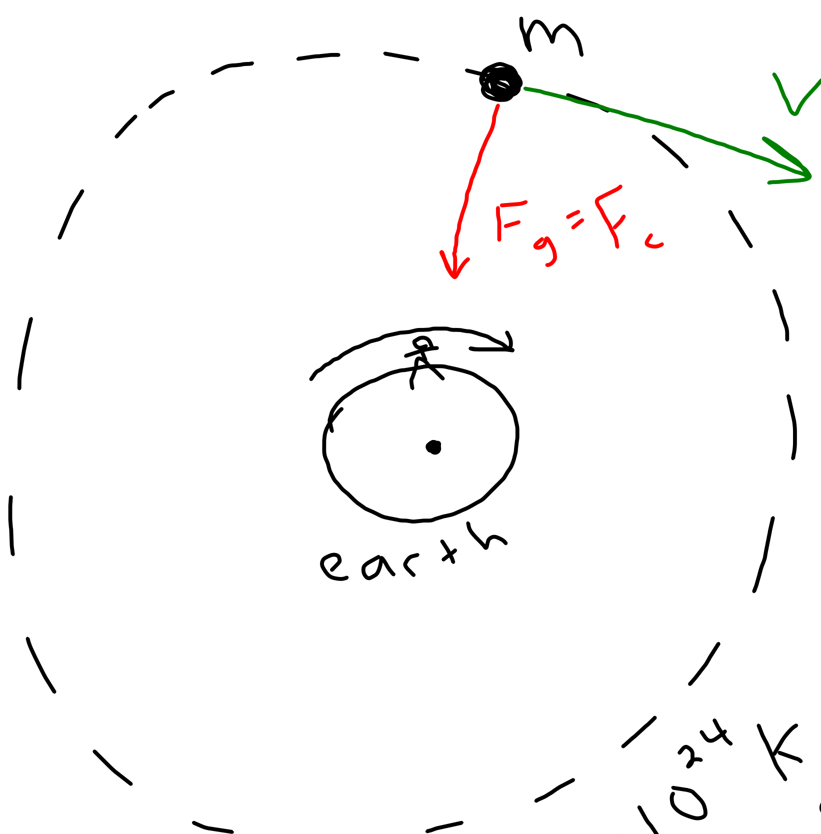
$$f' = \frac{1}{T'} \quad \text{etc.}$$

sign convention:

$V_s > 0$ when moving towards the observer

$V_o > 0$ when moving towards the source

Example: "Geosynchronous orbit" occurs when a satellite orbits the earth about the equator with an orbital period of 24 hours. Therefore, to an observer on earth the satellite appears to never move. For this to occur, what must be the satellite's distance from the earth's center?




$$M_{earth} = 5.97\times10^{24}\ kg$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

$$v = \frac{2\pi r}{24\ hrs}$$

$$v = \frac{2\pi r}{\underbrace{8.64\times10^4\ s}_{T}}$$

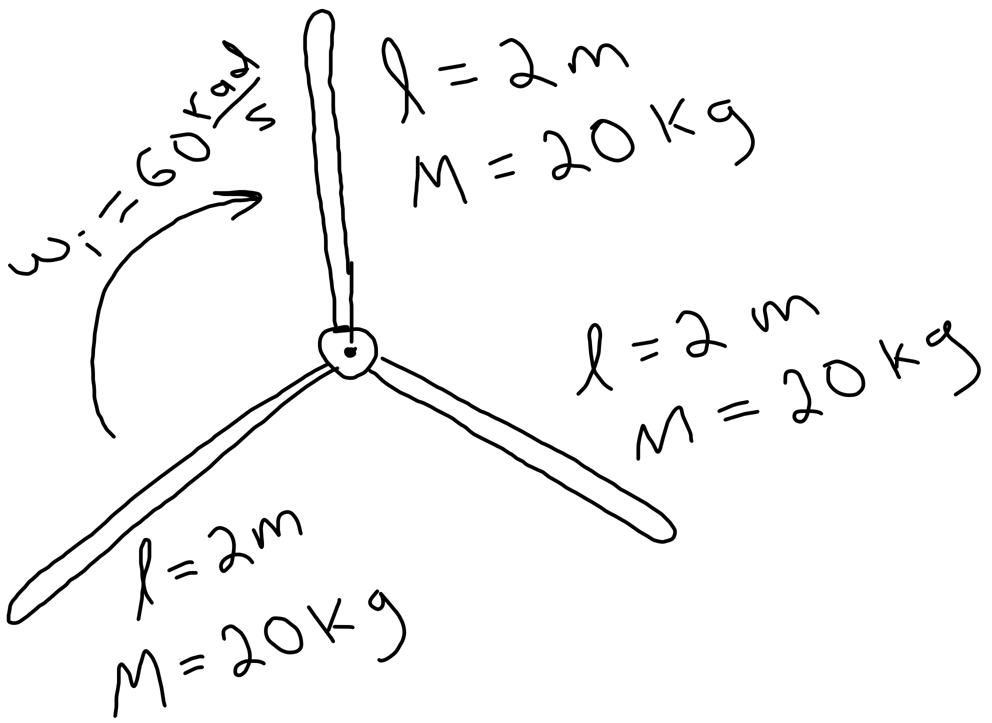
$$\frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \boxed{4.2\times10^7\ m}$$

## Example

Airplane propeller




Engine shuts off

↳ prop. slows to a stop

due to frictional torque $= 50\ N\cdot m$

How much time does this take?

$$I = \frac{1}{3}ML^2 + \frac{1}{3}ML^2 + \frac{1}{3}ML^2 = ML^2 = 80\ kg\cdot m^2$$

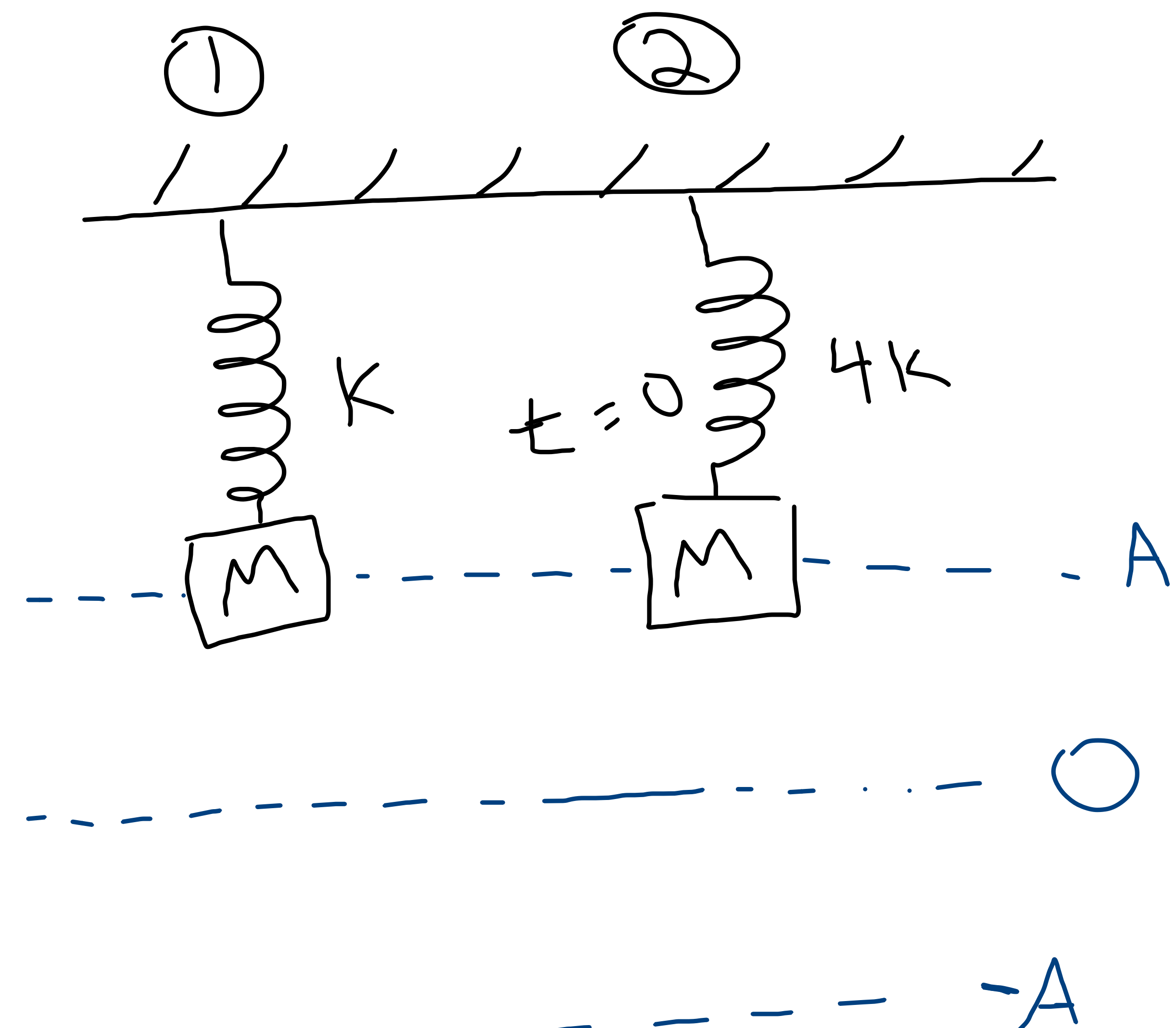
$$\tau = I\alpha \Rightarrow \alpha = \frac{50}{80} = 0.625\ \frac{rad}{s^2}$$

$$\omega_f = \omega_i + \alpha t$$

$$0 = 60 - 0.625\,t$$

$$\boxed{t = 96\,s}$$

## Example




How many cycles each when they are both back at the top together?

1

$$\omega_1 = \sqrt{\frac{K}{M}}$$

$$T_1 = 2\pi\sqrt{\frac{M}{K}}$$

2

$$\omega_2 = \sqrt{\frac{4K}{M}} = 2\sqrt{\frac{K}{M}}$$

$$T_2 = \frac{2\pi}{2}\sqrt{\frac{M}{K}}$$

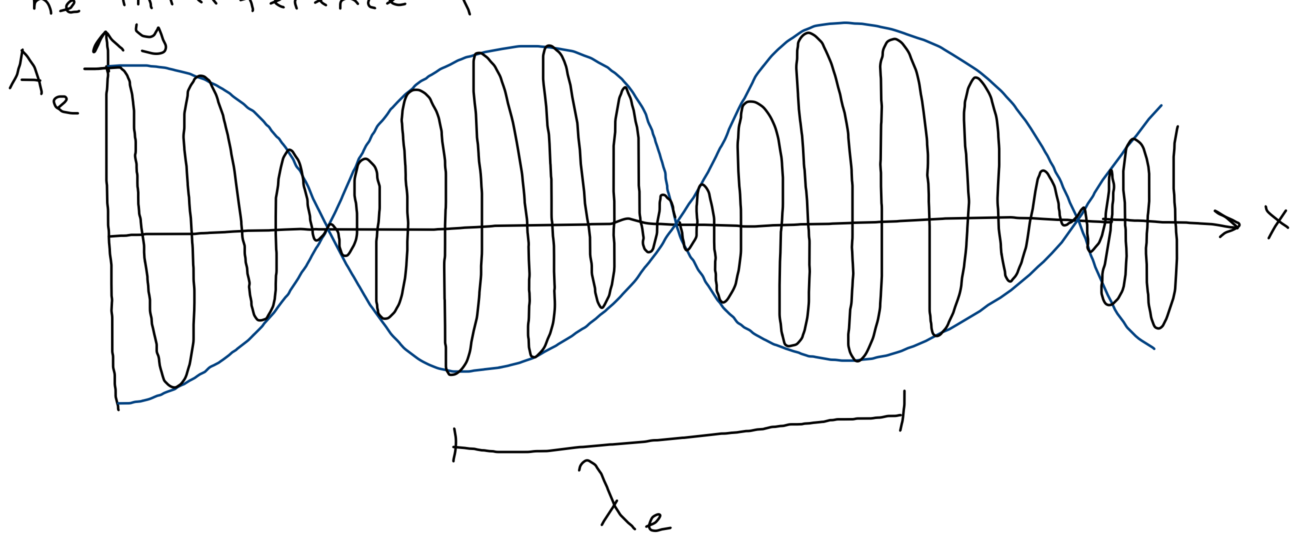
$\boxed{\text{1 cycle} \qquad \text{2 cycles}}$

Consider 2 waves at $t = 0$

$$y_1 = \cos\left(\frac{2\pi}{2} x\right)$$

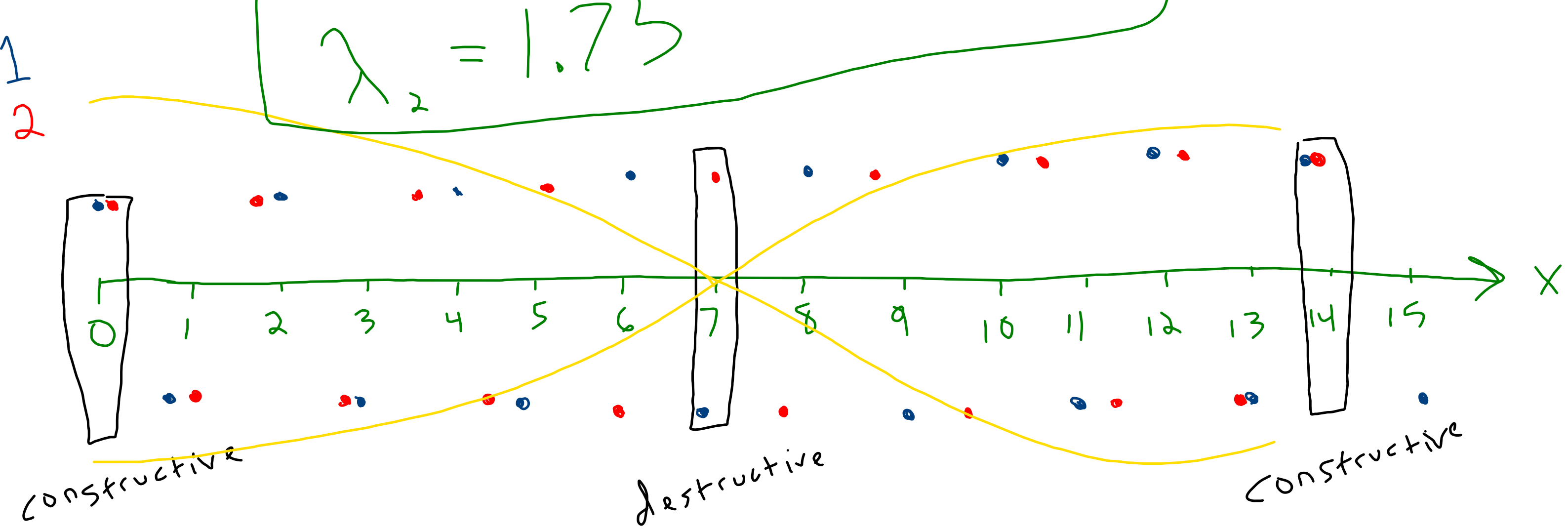
$$y_2 = \cos\left(\frac{2\pi}{1.75} x\right)$$

The interference pattern is this:



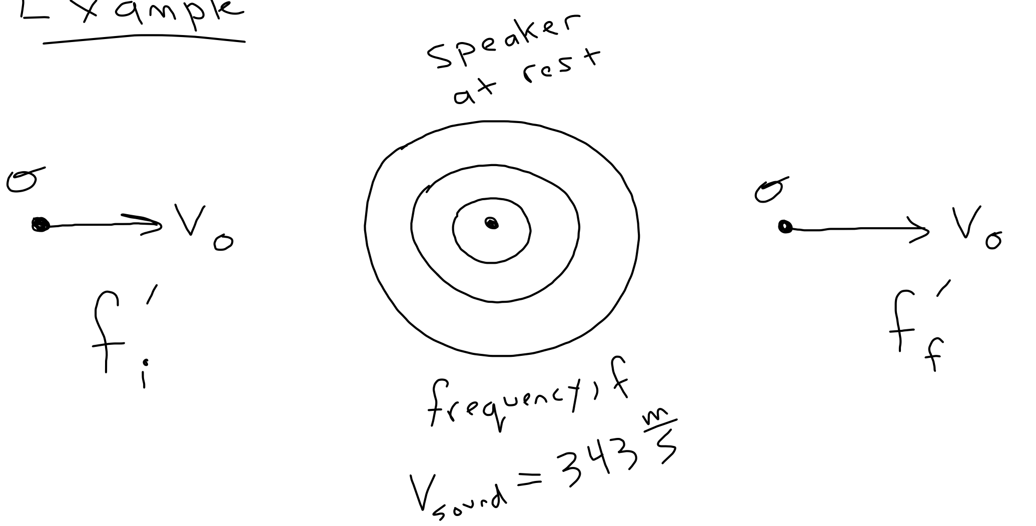

Find $\lambda_1, \lambda_2, \lambda_e, A_e$

$$\boxed{\lambda_1 = 2 \qquad A_e = 2 \qquad \lambda_2 = 1.75}$$




$$\boxed{\lambda_e = 14}$$

<u>Example</u>




given $f_i' - f_f' = \frac{f}{2}$ , find $V_o$ .

sol-

$$T' = \left(\frac{\overset{343}{V} - \overset{0}{V_s}}{\underset{343}{V} + V_o}\right) T$$

$$\frac{1}{f'} = \left(\frac{343}{343 + V_o}\right)\frac{1}{f}$$

$$\frac{1}{f_i'} = \left(\frac{343}{343 + V_o}\right)\frac{1}{f} \qquad \frac{1}{f_f'} = \left(\frac{343}{343 - V_o}\right)\frac{1}{f}$$

$$f_i' = \left(\frac{343 + V_o}{343}\right) f \qquad f_f' = \left(\frac{343 - V_o}{343}\right) f$$

$$\left(\frac{343 + V_o}{343}\right)\cancel{f} - \left(\frac{343 - V_o}{343}\right)\cancel{f} = \frac{\cancel{f}}{2}$$

$$\cancel{1} + \frac{V_o}{343} \cancel{-1} + \frac{V_o}{343} = \frac{1}{2}$$

$$\frac{2V_o}{343} = \frac{1}{2} \longrightarrow V_o = \frac{343}{4}\ \frac{m}{s}$$

$$\boxed{V_o = 85.75\ \frac{m}{s}}$$