$$\min \quad z = -4x_1 - x_2 - 2x_3$$

$$
\begin{array}{ll}
① & -x_1 + 2x_2 + 3x_3 + x_4 = 2 \\
② & 2x_1 + 4x_2 - x_3 + x_5 = 3 \\
③ & -3x_1 + x_2 + 2x_3 + x_6 = 3
\end{array}
$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

Variabili di Slack

$$\underline{c}^T = [-4 \; -1 \; -2 \; 0 \; 0 \; 0]$$

$$
A = \begin{array}{c}
\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\
\left[\begin{array}{ccc|ccc} -1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & -1 & 0 & 1 & 0 \\ -3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]
\end{array}
\qquad
\underline{b} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \begin{array}{c} x_4 \\ x_5 \\ x_6 \end{array}
$$

B

$\boxed{1}$ OTTIMALITÁ

$B = \{4, 5, 6\}$

$N = \{1, 2, 3\}$

$$A_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_B^{-1} = A_B$$

$$\underline{c}_B^T = [0 \; 0 \; 0]$$

NON SONO TUTTI ≤ 0

$$z_1 - c_1 = [0\,0\,0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} - (-4) = 0 - (-4) = 4 \quad x_1$$

$$z_2 - c_2 = [0\,0\,0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} - (-1) = 0 - (-1) = 1$$

$$z_3 - c_3 = [0\,0\,0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} - (-2) = 0 - (-2) = 2$$

ILLIMITATEZZA

$$\underline{y}_1 = A_B^{-1} \underline{a}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}$$

Esiste almeno una componente > 0 in $\underline{y}_1$

$$\underline{\bar{b}} = A_B^{-1} \underline{b} = \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \begin{array}{c} x_4 \\ x_5 \\ x_6 \end{array} \qquad B = \{4, 5, 6\}$$

Minimi Rapporti

$$\min \left\{ \frac{3}{2} \right\} = \frac{3}{2} \rightarrow x_5 \text{ entra in } x_1$$

$\boxed{2}$ OTTIMALITÀ

$$B = \{4, 1, 6\} \qquad N = \{5, 2, 3\} \qquad \underline{c}_B^T = [0\ -4\ 0]$$

$$A_B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$$A_B^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] =$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 3/2 & 1 \end{array}\right] = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix}$$

$$z_5 - c_5 = [0\ -4\ 0] \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 0 = [0\ -2\ 0] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 0 = -2 - 0 = -2$$

$$z_2 - c_2 = [0\ -4\ 0] \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} - (-1) = [0\ -2\ 0] \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} + 1 = -8 + 1 = -7$$

$$z_3 - c_3 = [0\ -4\ 0] \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} - (-2) = [0\ -2\ 0] \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} + 2 = 2 + 2 = 4$$

NON SONO TUTTI ≤ 0

x_3

ILLIMITATEZZA

$$\underline{y}_3 = A_B^{-1} \underline{a}_3 = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ -1/2 \\ 2 \end{bmatrix}$$

Esiste almeno una componente > 0 in $\underline{y}_1$

$$\underline{\bar{b}} = A_B^{-1} \underline{b} = \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & 3/2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} = \begin{array}{c} \begin{bmatrix} 7/2 \\ 3/2 \\ 15/2 \end{bmatrix} \end{array} \begin{array}{l} x_4 \\ x_1 \\ x_6 \end{array}$$

$B = \{4, 1, 6\}$

Minimi Rapporti

$$\min \left\{ \frac{7/2}{5/2}, \frac{15/2}{2} \right\} = \min \{1{,}4, 3{,}75\} = \frac{7}{5}$$

x_4 esce in x_3

$\boxed{3}$ OTTIMALITÁ

$$B = \{3, 1, 6\} \qquad N = \{5, 2, 4\} \qquad \underline{C}^T_B = [-2\ -4\ 0]$$

$$A_B = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & -3 & 1 \end{bmatrix}$$

$$A_B^{-1} = \begin{bmatrix} 2/5 & 1/5 & 0 \\ 1/5 & 3/5 & 0 \\ -1/5 & 7/5 & 1 \end{bmatrix}$$

$$z_5 - c_5 = [-2\ -4\ \ 0] \begin{bmatrix} 2/5 & 1/5 & 0 \\ 1/5 & 3/5 & 0 \\ -1/5 & 7/5 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 0 = \left[-\frac{8}{5}\ -\frac{14}{5}\ \ 0\right] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 0 = -\frac{14}{5}$$

$$z_2 - c_2 = [-2\ -4\ \ 0] \begin{bmatrix} 2/5 & 1/5 & 0 \\ 1/5 & 3/5 & 0 \\ -1/5 & 7/5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} - (-1) = \left[-\frac{8}{5}\ -\frac{14}{5}\ \ 0\right] \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} + 1 = -\frac{72}{5} + 1 = -\frac{67}{5}$$

$$z_4 - c_4 = [-2\ -4\ \ 0] \begin{bmatrix} 2/5 & 1/5 & 0 \\ 1/5 & 3/5 & 0 \\ -1/5 & 7/5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 0 = \left[-\frac{8}{5}\ -\frac{14}{5}\ \ 0\right] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - 0 = -\frac{8}{5}$$

Tutti sono ≤ 0, quindi questa base é OTTIMA.

BASE AMMISSIBILE ?

$$B = \{3, 1, 6\}$$

$$\underline{x}_B = A_B^{-1} \underline{b}$$

$$\underline{x}_B = \begin{bmatrix} 2/5 & 1/5 & 0 \\ 1/5 & 3/5 & 0 \\ -1 & 7/5 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 7/5 \\ 11/5 \\ 34/5 \end{bmatrix}$$

Tutti ≥ 0

É ammissibile.

IL SIMPLESSO HA OTTIMO FINITO

$$B = \{3, 1, 6\}$$