7. (b) $f(x) = x^{a-1} e^{-x}$

$$\frac{df}{dx} = -x^{a-1} e^{-x} + (a-1) x^{a-2} e^{-x}$$

for maxima, $\frac{df}{dx} = 0$

$$\Rightarrow \cancel{} \; x^{a-2} e^{-x} \left(-x + (a-1)\right) = 0$$

$$\Rightarrow \underline{x = a-1} \quad \text{for maxima}$$

(c) $z = \frac{x}{c+x}$

~~maxima occurs at x = a-1~~

maxima should occur when $z = \frac{1}{2}$

$$\Rightarrow \frac{x}{c+x} = \frac{1}{2} \Rightarrow 2x = c + x$$

$$\Rightarrow c = x = \underline{a-1}$$

(d) $f(x) = x^{a-1} e^{-x} = e^{(a-1)\ln x} e^{-x}$

$$= e^{[(a-1)\ln x - x]}$$

this is better than $x^{a-1} e^{-x}$

because,

(i) x^{a-1}, which would blow up at large value of x ~~is replaced by~~ and e^{-x} which would take near 0 values are replaced by an exponential that takes moderate values.