

$$p(\theta) = \frac{\sin \theta}{2}$$

②

range of  $\theta \rightarrow [0, \pi]$ , range of  $\phi \rightarrow [0, 2\pi]$

$$\Rightarrow \int_0^{\pi} p(\theta) d\theta = \int_0^{\pi} \frac{\sin \theta}{2} d\theta$$

$$= \frac{1}{2} [-\cos \theta]_0^{\pi}$$

$$= 1 \quad (p(\theta) \text{ is normalized})$$

$$\int_0^{2\pi} p(\phi) d\phi = 1$$

$$p(\phi) = \frac{1}{2\pi}$$

$$\int_0^{2\pi} p(\phi) d\phi = \int_0^{2\pi} \frac{1}{2\pi} d\phi = 1 \quad (p(\phi) \text{ is normalized})$$

So yes,  $p(\theta)$  and  $p(\phi)$  are normalized.

③

$$\int_0^{\theta(z)} p(\theta) d\theta = z \Rightarrow \int_0^{\theta(z)} \frac{\sin \theta}{2} d\theta = z \Rightarrow \frac{1}{2} [-\cos \theta]_0^{\theta(z)} = z$$

$$\Rightarrow \frac{1 - \cos \theta}{2} = z$$

④

$$\theta = \cos^{-1}(1 - 2z)$$

$$\Rightarrow \theta = \cos^{-1}(1 - 2z)$$