P6

@ weight function $w(x) = \frac{1}{\sqrt{x}}$

sampling interval $\to (0,1)$

normalising $w(x)$ we get

$$\int_0^1 A \cdot \frac{1}{\sqrt{x}} = 1$$

$$\Rightarrow A \cdot \int_0^1 \frac{1}{\sqrt{x}} = 1$$

$$\Rightarrow A \cdot \left[2\sqrt{x}\right]_0^1 = 1$$

$$\Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$$

so $p(x) = A \, w(x) = \frac{1}{2\sqrt{x}}$

$$\boxed{p(x) = \frac{1}{2\sqrt{x}}}$$

suppose z is the uniformly distributed random variable on $(0,1)$.

$$\Rightarrow \int_0^{x(z)} p(t) \, dt = z$$

$$\Rightarrow \sqrt{x(z)} = z$$

$$\Rightarrow \boxed{x = z^2} \quad \text{transformation formula.}$$