

$$6. \quad r = \sqrt{x^2 + y^2}$$

$$R = \sqrt{r^2 + z^2} \\ = \sqrt{x^2 + y^2 + z^2}$$

$$\sin \theta = \frac{\sqrt{x^2 + y^2}}{\sqrt{R^2}}$$

$$F = \int_{-L/2}^{L/2} \frac{GMm}{R^2} \sin \theta \, dz$$

$$= \frac{GMm \sqrt{x^2 + y^2}}{L} \int_{-L/2}^{L/2} \frac{dz}{(x^2 + y^2 + z^2)^{3/2}}$$

$$F = F_x \hat{i} + F_y \hat{j}$$

$$F_x = -F \cos \phi \quad F_y = -F \sin \phi$$

$$\cos \phi = \frac{x}{L} \quad \sin \phi = \frac{y}{L}$$

$$\ddot{x} = \frac{F_x}{m} = -\frac{GMx}{r^2 \sqrt{r^2 + L^2/4}}$$

$$\ddot{y} = \frac{F_y}{m} = -\frac{GM y}{r^2 \sqrt{r^2 + L^2/4}}$$

$$\frac{dx}{dt} = \dot{x}$$

$$\frac{dy}{dt} = \dot{y}$$

$$\frac{d\dot{x}}{dt} = -\frac{GMx}{r^2 \sqrt{r^2 + L^2/4}}$$

$$\frac{d\dot{y}}{dt} = -\frac{GM y}{r^2 \sqrt{r^2 + L^2/4}}$$

$$\text{where } r = \sqrt{x^2 + y^2}$$

