DLP Hash Theory

A collision resistant hash function using DLP can be formulated as follows:
$f(x, y) = g^x \times h^y \ mod\ p$ where g is a primitive root of p, h is another generator from the same cyclic group and p is a prime.

Proof that the above formulation is collision resistant:
Assume that (x_1, y_1) and (x_2, y_2) lead to a collision. Then, $f(x_1, y_1) = f(x_2, y_2)$
That implies that,
$g^{x_1} \times h^{y_1} \ mod\ p = g^{x_2} \times h^{y_2} \ mod\ p$
Implies,
$g^{x_1 - x_2} \ mod\ p = h^{y_2 - y_1} \ mod\ p$
Assume $h = g^u \ mod\ p$
This implies that $g^{x1 - x2 - z \times (y2 - y1)} \ mod\ p = 1$, i.e. $g^a \ mod\ p = 1$. This is highly improbable, as g is given to be the primitive root of p.