

let $H^s(x)$ be the M-D Hash function. To be provably secure, $H^s(x) = H^s(x')$ s.t. $x \neq x'$ should be improbable

let L & L' be the lengths of x & x'

~~if~~ If $L \neq L'$,

the last step transforms for both x & x' will be

$$z_{B+1} = H^s(z_B || L)$$

$$z'_{B+1} = H^s(z'_B || L')$$

since $H^s(x) = H^s(x')$

~~we~~
$$H^s(z_B || L) = H^s(z'_B || L')$$

since $L' \neq L$, we know that $z_B || L$ & $z'_B || L$ are different strings. the probability that H^s collides is negligible (DLP hash theorem)

~~if~~

If $L \geq L'$, $x_{B+1} = x'_{B+1}$

we have $x \neq x'$ & $|x| \geq |x'|$

so we must have an i such that $x_i \neq x'_i$

let $i^* \in B_H$ be the greatest such

$$z_{i^*-1} || x_{i^*} \neq z_{i^*-1} || x_{i^*} \neq z_{i^*-1} || x_{i^*}$$

if $i^* = B_H$

$$H^S(x) = H^S(x') \Rightarrow h^S(z_B || x_{B_H}) = h^S(z_B || x'_{B_H})$$

$\Rightarrow z_B || x_{B_H} \neq z'_B || x'_{B_H}$ are two different strings for which h collides

improbable because of DLP hash

if $i^* \in B_H$, i^* is the maximum

$$\text{such that } z_{i^*-1} || x_{i^*} \neq z_{i^*-1} || x_{i^*}$$

$$\& z_{i^*} = z_{i^*}$$

$$\Rightarrow h^S(z_{i^*-1} || x_{i^*}) = h^S(z_{i^*-1} || x_{i^*})$$

again these strings are different but h collides \Rightarrow improbable