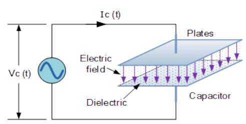
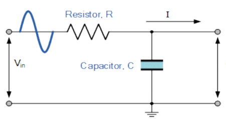
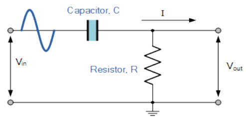
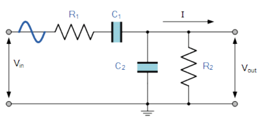
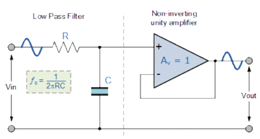
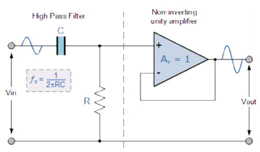
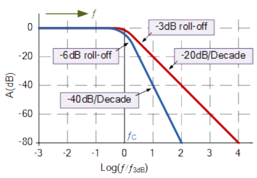
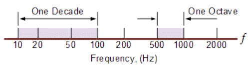
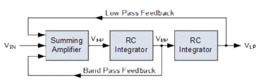
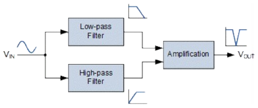
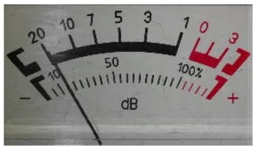
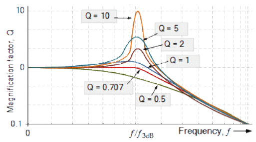
https://www.electronics-tutorials.ws/category/filter

Filters (13)

	**Capacitive Reactance** In the RC Network tutorial we saw that when a DC voltage is applied to a capacitor, the capacitor itself draws a charging current from the supply and charges up to a value equal to the applied voltage. Likewise, when the supply voltage is reduced the charge stored in the capacito...
	**Passive Low Pass Filter** In other words they "filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency. In low frequency applications (up to 100kHz), passive filters are generally constructed using simple RC (Resistor-Capacitor) networks,...
	**Passive High Pass Filter** Where as the low pass filter only allowed signals to pass below its cut-off frequency point, ƒc, the passive high pass filter circuit as its name implies, only passes signals above the selected cut-off point, ƒc eliminating any low frequency signals from the waveform. C...
	**Passive Band Pass Filter** Band Pass Filters can be used to isolate or filter out certain frequencies that lie within a particular band or range of frequencies. The cut-off frequency or ƒc point in a simple RC passive filter can be accurately controlled using just a single resistor in series with a no...
	**Active Low Pass Filter** In the RC Passive Filter tutorials, we saw how a basic first-order filter circuits, such as the low pass and the high pass filters can be made using just a single resistor in series with a non-polarized capacitor connected across a sinusoidal input signal. We also noticed that th…
	**Active High Pass Filter** The basic operation of an Active High Pass Filter (HPF) is the same as for its equivalent RC passive high pass filter circuit, except this time the circuit has an operational amplifier or included within its design providing amplification and gain control. Like the previous acti...
	**Active Band Pass Filter** For a low pass filter this pass band starts from 0Hz or DC and continues up to the specified cut-off frequency point at -3dB down from the maximum pass band gain. Equally, for a high pass filter the pass band starts from this -3dB cut-off frequency and continues up to infinity or...

	**Second Order Filters** Second Order Filters which are also referred to as VCVS filters, because the op-amp is used as a Voltage Controlled Voltage Source amplifier, are another important type of active filter design because along with the active first order RC filters we looked at previously, higher or...
	**Butterworth Filter Design** In applications that use filters to shape the frequency spectrum of a signal such as in communications or control systems, the shape or width of the roll-off also called the "transition band", for a simple first-order filter may be too long or wide and so active filters designed ...
	**State Variable Filter** State variable filters use three (or more) operational amplifier circuits (the active element) cascaded together to produce the individual filter outputs but if required an additional summing amplifier can also be added to produce a fourth Notch filter output response as well. S...
	**Band Stop Filter** By combining a basic RC low-pass filter with a RC high-pass filter we can form a simple band-pass filter that will pass a range or band of frequencies either side of two cut-off frequency points. But we can also combine these low and high pass filter sections to produce another k...
	**Decibels** When designing or working with amplifier and filter circuits, some of the numbers used in the calculations can be very large or very small. For example, if we cascade two amplifier stages together with power or voltage gains of say 20 and 36, respectively, then the total gain wou...
	**Sallen and Key Filter** The Sallen and Key Filter design is a second-order active filter topology which we can use as the basic building blocks for implementing higher order filter circuits, such as low-pass (LPF), high-pass (HPF) and band-pass (BPF) filter circuits. As we have seen in this filters sect...

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Capacitive Reactance

Capacitive Reactance is the complex impedance of a capacitor who's value changes with respect to the applied frequency

In the RC Network tutorial we saw that when a DC voltage is applied to a capacitor, the capacitor itself draws a charging current from the supply and charges up to a value equal to the applied voltage. Likewise, when the supply voltage is reduced the charge stored in the capacitor also reduces and the capacitor discharges.

But in an AC circuit in which the applied voltage signal is continually changing from a positive to a negative polarity at a rate determined by the frequency of the supply, as in the case of a sine wave voltage. For example, the capacitor is either being charged or discharged on a continuous basis at a rate determined by the supply frequency.

As the capacitor charges or discharges, a current flows through it which is restricted by the internal impedance of the capacitor. This internal impedance is commonly known as **Capacitive Reactance** and is given the symbol X_C in Ohms.

Unlike resistance which has a fixed value, for example, 100Ω, 1kΩ, 10kΩ etc, (this is because resistance obeys Ohms Law), **Capacitive Reactance** varies with the applied frequency so any variation in supply frequency will have a big effect on the capacitor's "capacitive reactance" value.

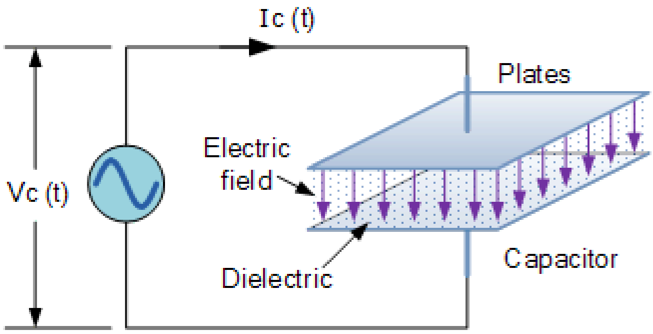
As the frequency applied to the capacitor increases, its effect is to decrease its reactance (measured in ohms). Likewise as the frequency across the capacitor decreases its reactance value increases. This variation is called the capacitor's *complex impedance*.

Complex impedance exists because the electrons in the form of an electrical charge on the capacitor plates, appear to pass from one plate to the other more rapidly with respect to the varying frequency.

As the frequency increases, the capacitor passes more charge across the plates in a given time resulting in a greater current flow through the capacitor appearing as if the internal impedance of the capacitor has decreased. Therefore, a capacitor connected to a circuit that changes over a given range of frequencies can be said to be "Frequency Dependant".

Capacitive Reactance has the electrical symbol “X_C” and has units measured in Ohms the same as resistance (R). It is calculated using the following formula:

Capacitive Reactance



$$Xc = \frac{1}{2\pi fC}$$

Capacitive Reactance Formula

Where:

- Xc = Capacitive Reactance in Ohms, (Ω)
- π (pi) = 3.142 (decimal) or as 22÷7 (fraction)
- *ƒ* = Frequency in Hertz, (Hz)
- C = Capacitance in Farads, (F)

Capacitive Reactance Example No1

Calculate the capacitive reactance value of a 220nF capacitor at a frequency of 1kHz and again at a frequency of 20kHz.

At a frequency of 1kHz:

$$Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi x 1000 x 220x10^{-9}} = 723.4\Omega$$

Again at a frequency of 20kHz:

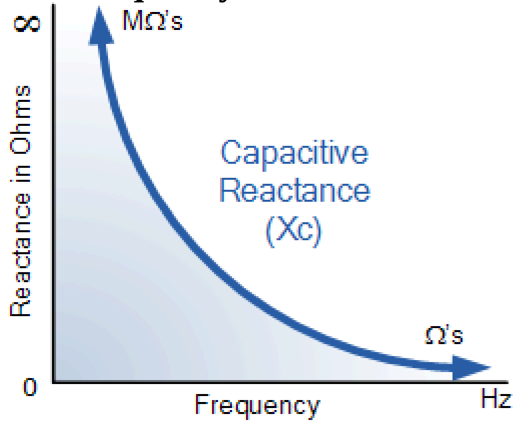
$$Xc = \frac{1}{2\pi fC} = \frac{1}{2\pi x 20000 x 220x10^{-9}} = 36.2\Omega$$

where: *ƒ* = frequency in Hertz and C = capacitance in Farads

Therefore, it can be seen from above that as the frequency applied across the 220nF capacitor increases, from 1kHz to 20kHz, its reactance value, X_C decreases, from approx 723Ω to just 36Ω and this is always true as capacitive reactance, X_C is inversely proportional to frequency with the current passed by the capacitor for a given voltage being proportional to the frequency.

For any given value of capacitance, the reactance of a capacitor, X_C expressed in ohms can be plotted against the frequency as shown below.

Capacitive Reactance against Frequency



By re-arranging the reactance formula above, we can also find at what frequency a capacitor will have a particular capacitive reactance (X_C) value.

Capacitive Reactance Example No2

At which frequency would a 2.2uF Capacitor have a reactance value of 200Ωs?

$$f = \frac{1}{2\pi C X c} = \frac{1}{2\pi x 2.2x10^{-6} x200} = 361.7Hz$$

Or we can find the value of the capacitor in Farads by knowing the applied frequency and its reactance value at that frequency.

Capacitive Reactance Example No3

What will be the value of a capacitor in farads when it has a capacitive reactance of 200Ω and is connected to a 50Hz supply.

$$C = \frac{1}{2\pi f X c} = \frac{1}{2\pi x 50x200} = 15.92uF$$

We can see from the above examples that a capacitor when connected to a variable frequency supply, acts a bit like a "frequency controlled variable resistor" as its reactance (X) is inversely proportional to frequency. At very low frequencies, such as 1Hz our 220nF capacitor has a high capacitive reactance value of approx 723.3KΩ (giving the effect of an open circuit).

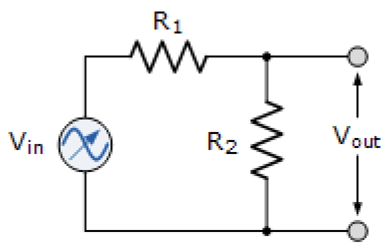
At very high frequencies such as 1Mhz the capacitor has a low capacitive reactance value of just 0.72Ω (giving the effect of a short circuit). So at zero frequency or steady state DC our 220nF capacitor has infinite reactance looking more like an "open-circuit" between the plates and blocking any flow of current through it.

Voltage Divider Revision

We remember from our tutorial about Resistors in Series that different voltages can appear across each resistor depending upon the value of the resistance and that a voltage divider circuit has the ability to divide its supply voltage by the ratio of R2/(R1+R2).

Therefore, when R1 = R2 the output voltage will be half the value of the input voltage. Likewise, any value of R2 greater or less than R1 will result in a proportional change to the output voltage. Consider the circuit below.

Voltage Divider Network



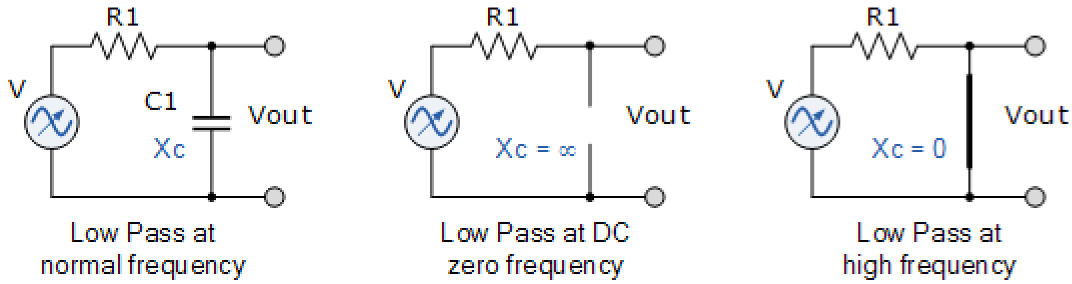
$$V_{out} = V_{in} \frac{R_2}{(R_1 + R_2)}$$

We now know that a capacitor's reactance, X_c (its complex impedance) value changes with respect to the applied frequency. If we now changed resistor R2 above for a capacitor, the voltage drop across the two components would change as the frequency changed because the reactance of the capacitor affects its impedance.

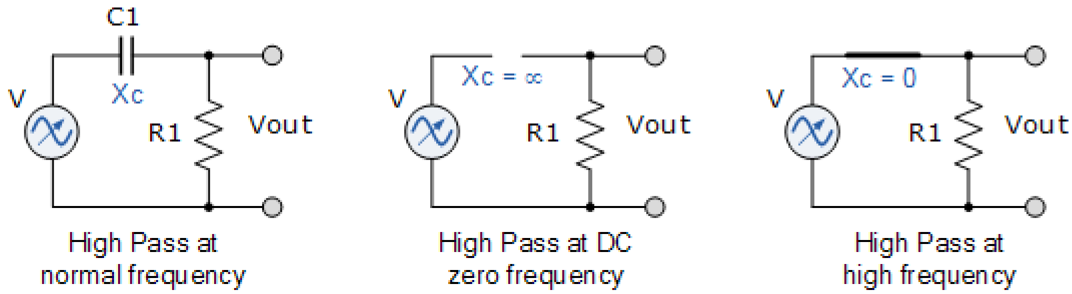
The impedance of resistor R1 does not change with frequency. Resistors are of fixed values and are unaffected by frequency change. Then the voltage across resistor R1 and therefore the output voltage is determined by the capacitive reactance of the capacitor at a given frequency.

This then results in a frequency-dependent RC voltage divider circuit. With this idea in mind, passive **Low Pass Filters** and **High Pass Filters** can be constructed by replacing one of the voltage divider resistors with a suitable capacitor as shown.

Low Pass Filter



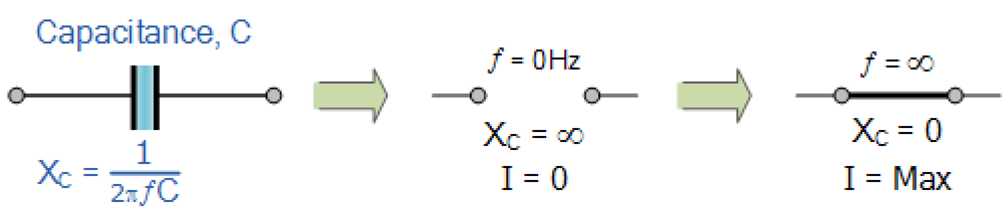
High Pass Filter



The property of **Capacitive Reactance**, makes the capacitor ideal for use in AC filter circuits or in DC power supply smoothing circuits to reduce the effects of any unwanted Ripple Voltage as the capacitor applies an short circuit signal path to any unwanted frequency signals on the output terminals.

Capacitive Reactance Summary

So, we can summarize the behaviour of a capacitor in a variable frequency circuit as being a sort of frequency controlled resistor that has a high capacitive reactance value (open circuit condition) at very low frequencies and low capacitive reactance value (short circuit condition) at very high frequencies as shown in the graph above.



It is important to remember these two conditions and in our next tutorial about the Passive Low Pass Filter, we will look at the use of **Capacitive Reactance** to block any unwanted high frequency signals while allowing only low frequency signals to pass.

https://www.electronics-tutorials.ws/filter/filter_2.html

Passive Low Pass Filter

A Low Pass Filter is a circuit that can be designed to modify, reshape or reject all unwanted high frequencies of an electrical signal and accept or pass only those signals wanted by the circuits designer.

In other words they “filter-out” unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency. In low frequency applications (up to 100kHz), passive filters are generally constructed using simple RC (Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or “attenuating” the rest. The most commonly used filter designs are the:

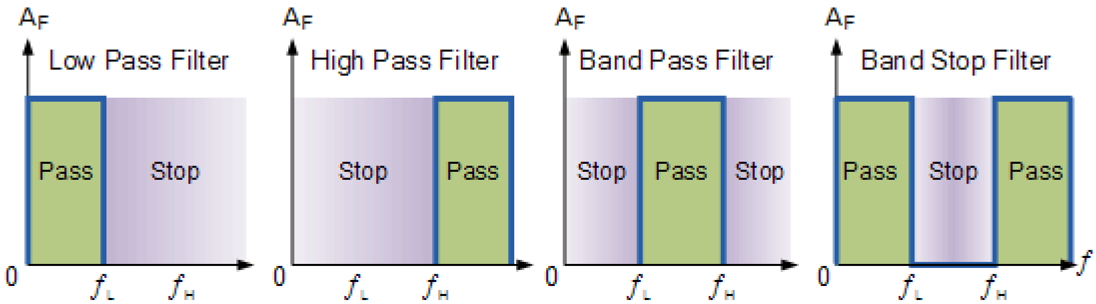
- The Low Pass Filter–the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, ƒc point to pass while blocking those any higher.
- The High Pass Filter–the high pass filter only allows high frequency signals from its cut-off frequency, ƒc point and higher to infinity to pass through while blocking those any lower.
- The Band Pass Filter–the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

Simple First-order passive filters (1st order) can be made by connecting together a single resistor and a single capacitor in series across an input signal, (V_{IN}) with the output of the filter, (V_{OUT}) taken from the junction of these two components.

Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a **Low Pass Filter** or a **High Pass Filter**.

As the function of any filter is to allow signals of a given band of frequencies to pass unaltered while attenuating or weakening all others that are not wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

Ideal Filter Response Curves



Filters can be divided into two distinct types: active filters and passive filters. Active filters contain amplifying devices to increase signal strength while passive do not contain amplifying devices to strengthen the signal. As there are two passive components within a passive filter design the output signal has a smaller amplitude than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one (unity).

A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the “cut-off” or “corner” frequency.

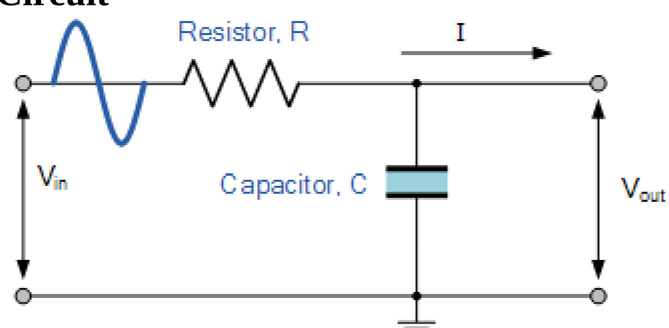
The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

The Low Pass Filter

A simple passive **RC Low Pass Filter** or **LPF**, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below. In this type of filter arrangement the input signal (V_{IN}) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (V_{OUT}) is taken across the capacitor only.

This type of filter is known generally as a “first-order filter” or “one-pole filter”, why first-order or single-pole?, because it has only “one” reactive component, the capacitor, in the circuit.

RC Low Pass Filter Circuit



As mentioned previously in the Capacitive Reactance tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At low frequencies the capacitive reactance, (X_C) of the capacitor will be very large compared to the resistive value of the resistor R.

This means that the voltage potential, V_C across the capacitor will be much larger than the voltage drop, V_R developed across the resistor. At high frequencies the reverse is true with V_C being small and V_R being large due to the change in the capacitive reactance value.

While the circuit above is that of an RC Low Pass Filter circuit, it can also be thought of as a frequency dependant variable potential divider circuit similar to the one we looked at in the Resistors tutorial. In that tutorial we used the following equation to calculate the output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

We also know that the capacitive reactance of a capacitor in an AC circuit is given as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

Opposition to current flow in an AC circuit is called **impedance**, symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

RC Potential Divider Equation

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Low Pass Filter Example No1

A **Low Pass Filter** circuit consisting of a resistor of 4k7Ω in series with a capacitor of 47nF is connected across a 10v sinusoidal supply. Calculate the output voltage (V_{OUT}) at a frequency of 100Hz and again at frequency of 10,000Hz or 10kHz.

Voltage Output at a Frequency of 100Hz.

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{Xc}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

Voltage Output at a Frequency of 10,000Hz (10kHz).

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\Omega$$

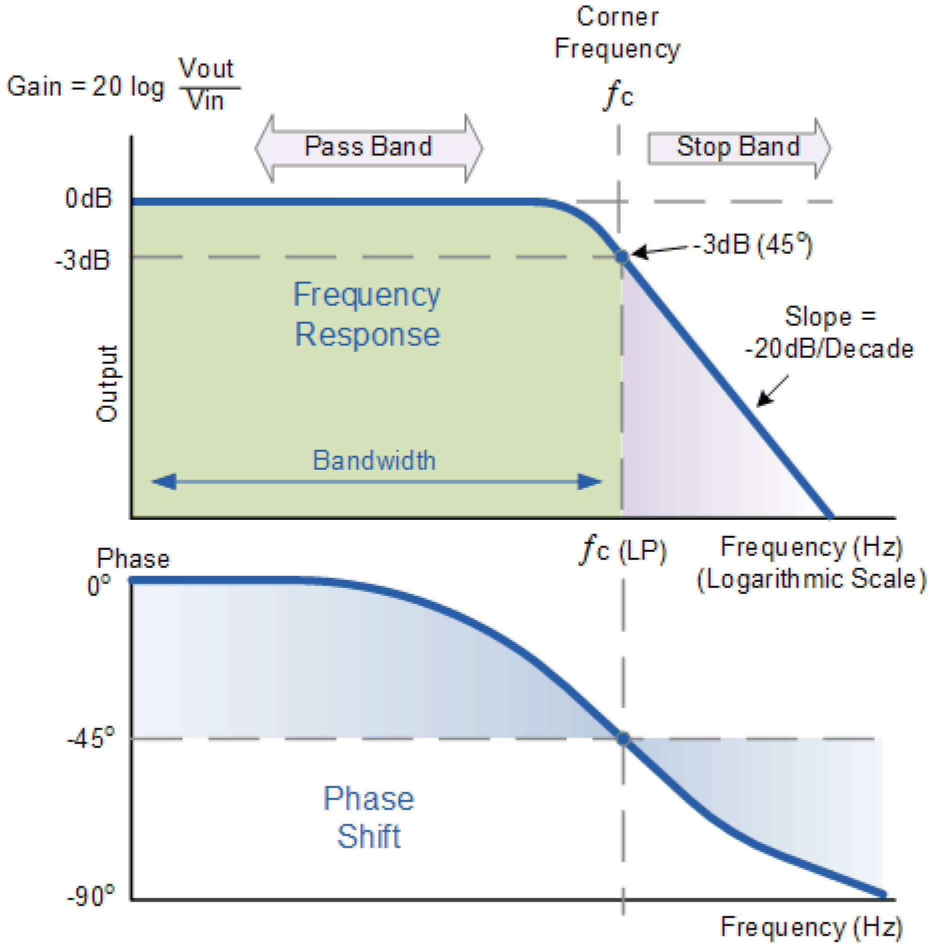
$$V_{OUT} = V_{IN} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

Frequency Response

We can see from the results above, that as the frequency applied to the RC network increases from 100Hz to 10kHz, the voltage dropped across the capacitor and therefore the output voltage (V_{OUT}) from the circuit decreases from 9.9v to 0.718v.

By plotting the networks output voltage against different values of input frequency, the **Frequency Response Curve** or **Bode Plot** function of the low pass filter circuit can be found, as shown below.

Frequency Response of a 1st-order Low Pass Filter



The Bode Plot shows the **Frequency Response** of the filter to be nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its **Cut-off Frequency** point (fc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases to zero at a slope of -20dB/ Decade or (-6dB/Octave) “roll-off”. Note that the angle of the slope, this -20dB/ Decade roll-off will always be the same for any RC combination.

Any high frequency signals applied to the low pass filter circuit above this cut-off frequency point will become greatly attenuated, that is they rapidly decrease. This happens because at very high frequencies the reactance of the capacitor becomes so low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

Then by carefully selecting the correct resistor-capacitor combination, we can create a RC circuit that allows a range of frequencies below a certain value to pass through the circuit unaffected while any frequencies applied to the circuit above this cut-off point to be attenuated, creating what is commonly called a **Low Pass Filter**.

For this type of “Low Pass Filter” circuit, all the frequencies below this cut-off, fc point that are unaltered with little or no attenuation and are said to be in the filters **Pass band** zone. This pass band zone also represents the **Bandwidth** of the filter. Any signal frequencies above this point cut-off point are generally said to be in the filters **Stop band** zone and they will be greatly attenuated.

This “Cut-off”, “Corner” or “Breakpoint” frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or **-3dB** (20 log (Vout/Vin)) of the input. Although R = Xc, the output is **not** half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle (Φ) of the output signal **LAGS** behind that of the input and at the -3dB cut-off frequency (fc) is -45^{o} out of phase. This is due to the time taken to charge the plates of the capacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) “lagging” behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more “out of phase”.

The cut-off frequency point and phase shift angle can be found by using the following equation:

Cut-off Frequency and Phase Shift

$$f\text{c} = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

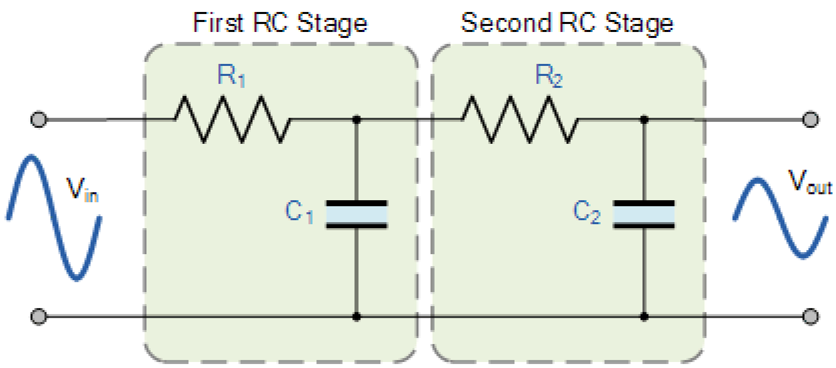
$$\text{Phase Shift } \varphi = -\arctan\left(2\pi f RC\right)$$

Then for our simple example of a “**Low Pass Filter**” circuit above, the cut-off frequency (fc) is given as 720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45^{o}.

Second-order Low Pass Filter

Thus far we have seen that simple first-order RC low pass filters can be made by connecting a single resistor in series with a single capacitor. This single-pole arrangement gives us a roll-off slope of -20dB/decade attenuation of frequencies above the cut-off point at f_{-3dB}. However, sometimes in filter circuits this -20dB/decade (-6dB/octave) angle of the slope may not be enough to remove an unwanted signal then two stages of filtering can be used as shown.

Second-order Low Pass Filter



The above circuit uses two passive first-order low pass filters connected or “cascaded” together to form a second-order or two-pole filter network. Therefore we can see that a first-order low pass filter can be converted into a second-order type by simply adding an additional RC network to it and the more RC stages we add the higher becomes the order of the filter.

If a number (n) of such RC stages are cascaded together, the resulting RC filter circuit would be known as an “n^{th}-order” filter with a roll-off slope of “n x -20dB/decade”.

So for example, a second-order filter would have a slope of -40dB/decade (-12dB/octave), a fourth-order filter would have a slope of -80dB/decade (-24dB/octave) and so on. This means that, as the order of the filter is increased, the roll-off slope becomes steeper and the actual stop band response of the filter approaches its ideal stop band characteristics.

Second-order filters are important and widely used in filter designs because when combined with first-order filters any higher-order n^{th}-value filters can be designed using them. For example, a third order low-pass filter is formed by connecting in series or cascading together a first and a second-order low pass filter.

But there is a downside to cascading together RC filter stages. Although there is no limit to the order of the filter that can be formed, as the order increases, the gain and accuracy of the final filter declines.

When identical RC filter stages are cascaded together, the output gain at the required cut-off frequency (fc) is reduced (attenuated) by an amount in relation to the number of filter stages used as the roll-off slope increases. We can define the amount of attenuation at the selected cut-off frequency using the following formula.

Passive Low Pass Filter Gain at fc

$$\left(\frac{1}{\sqrt{2}}\right)^{n}$$

where “n” is the number of filter stages.

So for a second-order passive low pass filter the gain at the corner frequency fc will be equal to 0.7071 x 0.7071 = 0.5Vin (-6dB), a third-order passive low pass filter will be equal to 0.353Vin (-9dB), fourth-order will be 0.25Vin (-12dB) and so on. The corner frequency, fc for a second-order passive low pass filter is determined by the resistor/capacitor (RC) combination and is given as.

2nd-Order Filter Corner Frequency

$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \ \text{Hz}$$

In reality as the filter stage and therefore its roll-off slope increases, the low pass filters -3dB corner frequency point and therefore its pass band frequency changes from its original calculated value above by an amount determined by the following equation.

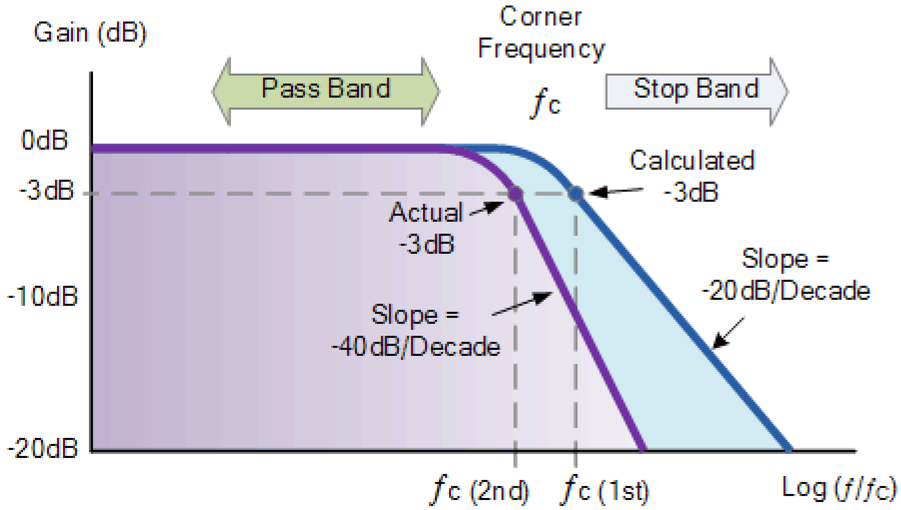
2nd-Order Low Pass Filter -3dB Frequency

$$f_{(-3dB)} = f_C\sqrt{2^{\left(\frac{1}{n}\right)} - 1}$$

where fc is the calculated cut-off frequency, n is the filter order and f_{-3dB} is the new -3dB pass band frequency as a result in the increase of the filters order.

Then the frequency response (bode plot) for a second-order low pass filter assuming the same -3dB cut-off point would look like:

Frequency Response of a 2nd-order Low Pass Filter



In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so R2 = 10 x R1 and C2 = 1/10th C1. Second-order and above filter networks are generally used in the feedback circuits of op-amps, making what are commonly known as Active Filters or as a phase-shift network in RC Oscillator circuits.

Low Pass Filter Summary

So to summarize, the **Low Pass Filter** has a constant output voltage from D.C. (0Hz), up to a specified Cut-off frequency, (f_C) point. This cut-off frequency point is 0.707 or **-3dB** (dB = –20log*$V_{OUT/IN}$) of the voltage gain allowed to pass.

The frequency range "below" this cut-off point f_C is generally known as the **Pass Band** as the input signal is allowed to pass through the filter. The frequency range "above" this cut-off point is generally known as the **Stop Band** as the input signal is blocked or stopped from passing through.

A simple 1st order low pass filter can be made using a single resistor in series with a single non-polarized capacitor (or any single reactive component) across an input signal Vin, whilst the output signal Vout is taken from across the capacitor.

The cut-off frequency or -3dB point, can be found using the standard formula, $fc = 1/(2\pi RC)$. The phase angle of the output signal at fc and is -45^o for a Low Pass Filter.

The gain of the filter or any filter for that matter, is generally expressed in **Decibels** and is a function of the output value divided by its corresponding input value and is given as:

$$Gain\ in\ dB = 20 \log \frac{Vout}{Vin}$$

Applications of passive Low Pass Filters are in audio amplifiers and speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or “hiss” type distortion. When used like this in audio applications the low pass filter is sometimes called a “high-cut”, or “treble cut” filter.

If we were to reverse the positions of the resistor and capacitor in the circuit so that the output voltage is now taken from across the resistor, we would have a circuit that produces an output frequency response curve similar to that of a High Pass Filter, and this is discussed in the next tutorial.

Time Constant

Until now we have been interested in the frequency response of a low pass filter when subjected to sinusoidal waveform. We have also seen that the filters cut-off frequency (fc) is the product of the resistance (R) and the capacitance (C) in the circuit with respect to some specified frequency point and that by altering any one of the two components alters this cut-off frequency point by either increasing it or decreasing it.

We also know that the phase shift of the circuit lags behind that of the input signal due to the time required to charge and then discharge the capacitor as the sine wave changes. This combination of R and C produces a charging and discharging effect on the capacitor known as its **Time Constant** (τ) of the circuit as seen in the RC Circuit tutorials giving the filter a response in the time domain.

The time constant, **tau** (τ), is related to the cut-off frequency fc as:

$$\tau = RC = \frac{1}{2\pi fc}$$

or expressed in terms of the cut-off frequency, fc as:

$$fc = \frac{1}{2\pi RC} \quad or \quad \frac{1}{2\pi\tau}$$

The output voltage, V_{OUT} depends upon the time constant and the frequency of the input signal. With a sinusoidal signal that changes smoothly over time, the circuit behaves as a simple 1st order low pass filter as we have seen above.

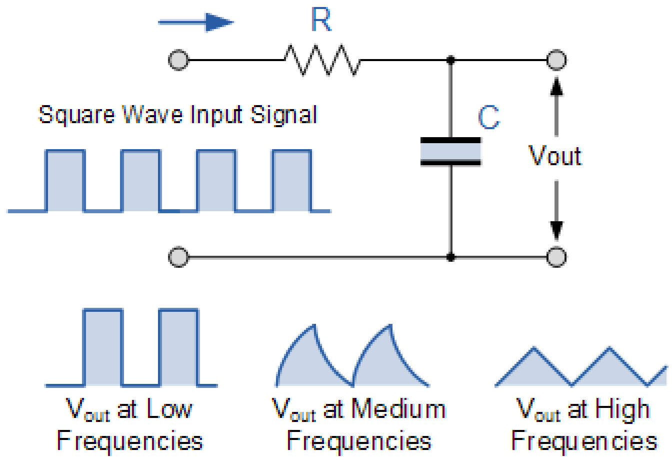
But what if we were to change the input signal to that of a “square wave” shaped “ON/OFF” type signal that has an almost vertical step input, what would happen to our filter circuit now. The output response of the circuit would change dramatically and produce another type of circuit known commonly as an **Integrator**.

The RC Integrator

The **Integrator** is basically a low pass filter circuit operating in the time domain that converts a square wave "step" response input signal into a triangular shaped waveform output as the capacitor charges and discharges. A **Triangular** waveform consists of alternate but equal, positive and negative ramps.

As seen below, if the RC time constant is long compared to the time period of the input waveform the resultant output waveform will be triangular in shape and the higher the input frequency the lower will be the output amplitude compared to that of the input.

The RC Integrator Circuit



This then makes this type of circuit ideal for converting one type of electronic signal to another for use in wave-generating or wave-shaping circuits.

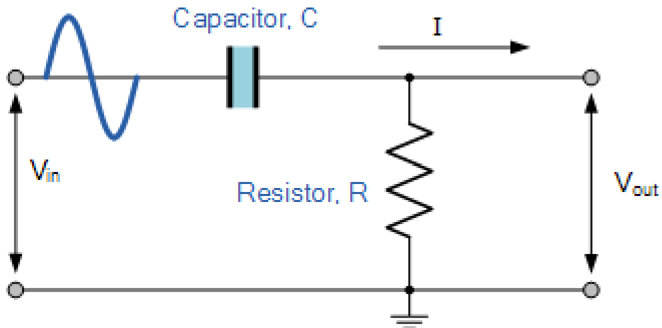
https://www.electronics-tutorials.ws/filter/filter_3.html

Passive High Pass Filter

A High Pass Filter is the exact opposite to the low pass filter circuit as the two components have been interchanged with the filters output signal now being taken from across the resistor.

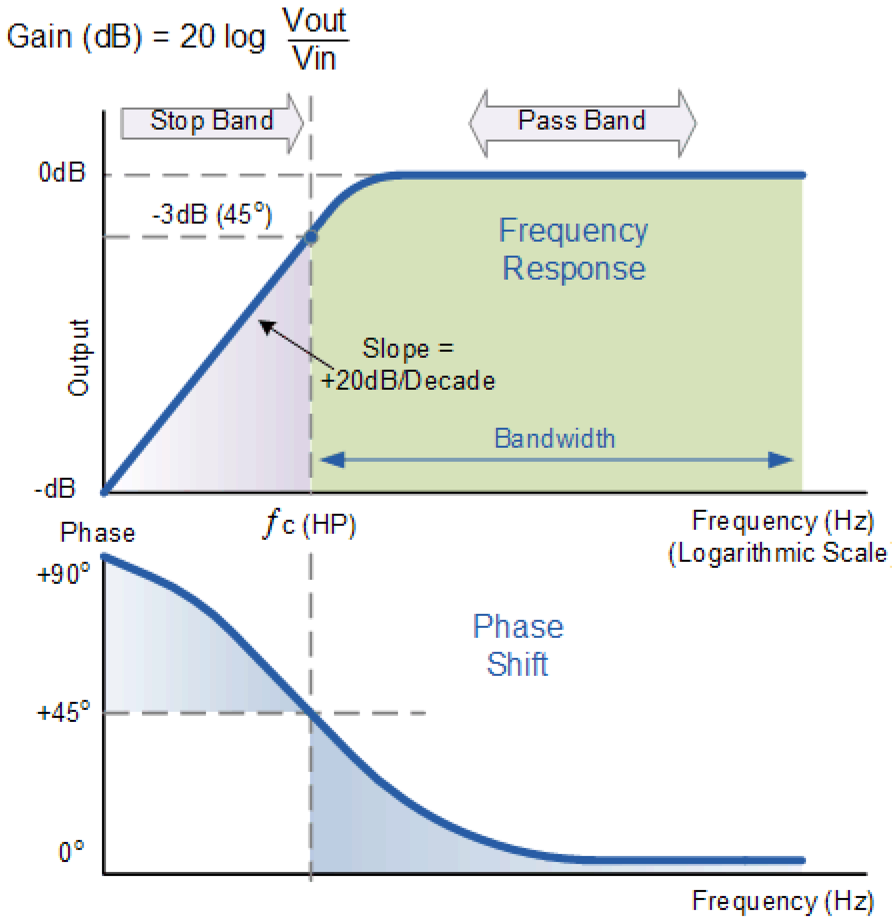
Where as the low pass filter only allowed signals to pass below its cut-off frequency point, fc, the passive high pass filter circuit as its name implies, only passes signals above the selected cut-off point, fc eliminating any low frequency signals from the waveform. Consider the circuit below.

The High Pass Filter Circuit



In this circuit arrangement, the reactance of the capacitor is very high at low frequencies so the capacitor acts like an open circuit and blocks any input signals at V_{IN} until the cut-off frequency point (f_C) is reached. Above this cut-off frequency point the reactance of the capacitor has reduced sufficiently as to now act more like a short circuit allowing all of the input signal to pass directly to the output as shown below in the filters response curve.

Frequency Response of a 1st Order High Pass Filter



The **Bode Plot** or Frequency Response Curve above for a passive high pass filter is the exact opposite to that of a low pass filter. Here the signal is attenuated or damped at low frequencies with the output increasing at +20dB/Decade (6dB/Octave) until the frequency reaches the cut-off point (ƒc) where again R = Xc. It has a response curve that extends down from infinity to the cut-off frequency, where the output voltage amplitude is 1/√2 =70.7% of the input signal value or -3dB (20 log (Vout/Vin)) of the input value.

Also we can see that the phase angle (Φ) of the output signal **LEADS** that of the input and is equal to **+45^{o}** at frequency ƒc. The frequency response curve for this filter implies that the filter can pass all signals out to infinity. However in practice, the filter response does not extend to infinity but is limited by the electrical characteristics of the components used.

The cut-off frequency point for a first order high pass filter can be found using the same equation as that of the low pass filter, but the equation for the phase shift is modified slightly to account for the positive phase angle as shown below.

Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC}$$

$$Phase\ Shift\ \phi = arctan\ \frac{1}{2\pi fRC}$$

The circuit gain, Av which is given as Vout/Vin (magnitude) and is calculated as:

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + Xc^2}} = \frac{R}{Z}$$

$$\text{at low } f: \ Xc \rightarrow \infty, \ \ Vout = 0$$
$$\text{at high } f: \ Xc \rightarrow 0, \ \ Vout = Vin$$

High Pass Filter Example No1

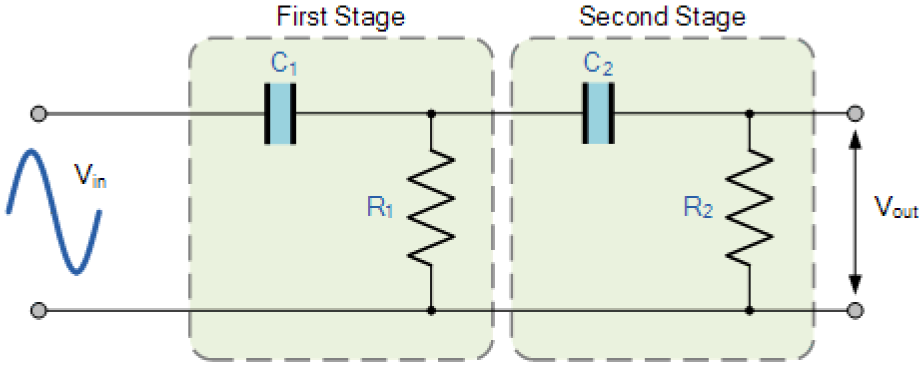
Calculate the cut-off or “breakpoint” frequency (fc) for a simple passive high pass filter consisting of an 82pF capacitor connected in series with a 240kΩ resistor.

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi\ x\ 240{,}000 x 82 x 10^{-12}} = 8{,}087\ Hz\ or\ 8kHz$$

Second-order High Pass Filter

Again as with low pass filters, high pass filter stages can be cascaded together to form a second order (two-pole) filter as shown.

Second-order High Pass Filter



The above circuit uses two first-order filters connected or cascaded together to form a second-order or two-pole high pass network. Then a first-order filter stage can be converted into a second-order type by simply using an additional RC network, the same as for the 2^{nd}-order low pass filter. The resulting second-order high pass filter circuit will have a slope of 40dB/decade (12dB/octave).

As with the low pass filter, the cut-off frequency, fc is determined by both the resistors and capacitors as follows.

$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \text{ Hz}$$

In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so $R_2 = 10*R_1$ and C_2 = 1/10th of C_1.

High Pass Filter Summary

We have seen that the **Passive High Pass Filter** is the exact opposite to the low pass filter. This filter has no output voltage from DC (0Hz), up to a specified cut-off frequency (fc) point. This lower cut-off frequency point is 70.7% or **-3dB** (dB = -20log V_{OUT}/V_{IN}) of the voltage gain allowed to pass.

The frequency range “below” this cut-off point fc is generally known as the **Stop Band** while the frequency range “above” this cut-off point is generally known as the **Pass Band**.

The cut-off frequency, corner frequency or -3dB point of a high pass filter can be found using the standard formula of: fc = 1/(2πRC). The phase angle of the resulting output signal at fc is **+45^o**. Generally, the high pass filter is less distorting than its equivalent low pass filter due to the higher operating frequencies.

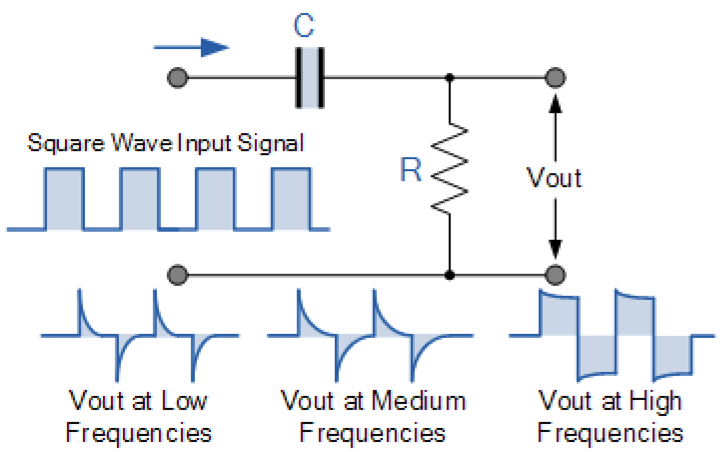
A very common application of this type of passive filter, is in audio amplifiers as a coupling capacitor between two audio amplifier stages and in speaker systems to direct the higher frequency signals to the smaller “tweeter” type speakers while blocking the lower bass signals or are also used as filters to reduce any low frequency noise or “rumble” type distortion. When used like this in audio applications the high pass filter is sometimes called a “low-cut”, or “bass cut” filter.

The output voltage Vout depends upon the time constant and the frequency of the input signal as seen previously. With an AC sinusoidal signal applied to the circuit it behaves as a simple 1st Order high pass filter. But if we change the input signal to that of a “square wave” shaped signal that has an almost vertical step input, the response of the circuit changes dramatically and produces a circuit known commonly as an **Differentiator**.

The RC Differentiator

Up until now the input waveform to the filter has been assumed to be sinusoidal or that of a sine wave consisting of a fundamental signal and some harmonics operating in the frequency domain giving us a frequency domain response for the filter. However, if we feed the **High Pass Filter** with a **Square Wave** signal operating in the time domain giving an impulse or step response input, the output waveform will consist of short duration pulse or spikes as shown.

The RC Differentiator Circuit



Each cycle of the square wave input waveform produces two spikes at the output, one positive and one negative and whose amplitude is equal to that of the input. The rate of decay of the spikes depends upon the time constant, (RC) value of both components, (t = R x C) and the value of the input frequency. The output pulses resemble more and more the shape of the input signal as the frequency increases.

https://www.electronics-tutorials.ws/filter/filter_4.html

Passive Band Pass Filter

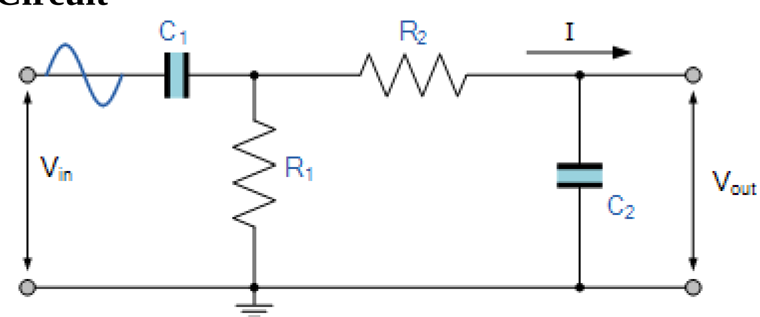
Passive Band Pass Filters can be made by connecting together a low pass filter with a high pass filter

Band Pass Filters can be used to isolate or filter out certain frequencies that lie within a particular band or range of frequencies. The cut-off frequency or fc point in a simple RC passive filter can be accurately controlled using just a single resistor in series with a non-polarized capacitor, and depending upon which way around they are connected, we have seen that either a Low Pass or a High Pass filter is obtained.

One simple use for these types of passive filters is in audio amplifier applications or circuits such as in loudspeaker crossover filters or pre-amplifier tone controls. Sometimes it is necessary to only pass a certain range of frequencies that do not begin at 0Hz, (DC) or end at some upper high frequency point but are within a certain range or band of frequencies, either narrow or wide.

By connecting or "cascading" together a single **Low Pass Filter** circuit with a **High Pass Filter** circuit, we can produce another type of passive RC filter that passes a selected range or "band" of frequencies that can be either narrow or wide while attenuating all those outside of this range. This new type of passive filter arrangement produces a frequency selective filter known commonly as a **Band Pass Filter** or **BPF** for short.

Band Pass Filter Circuit



Unlike the low pass filter which only pass signals of a low frequency range or the high pass filter which pass signals of a higher frequency range, a **Band Pass Filters** passes signals within a certain "band" or "spread" of frequencies without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is commonly known as the filters **Bandwidth**.

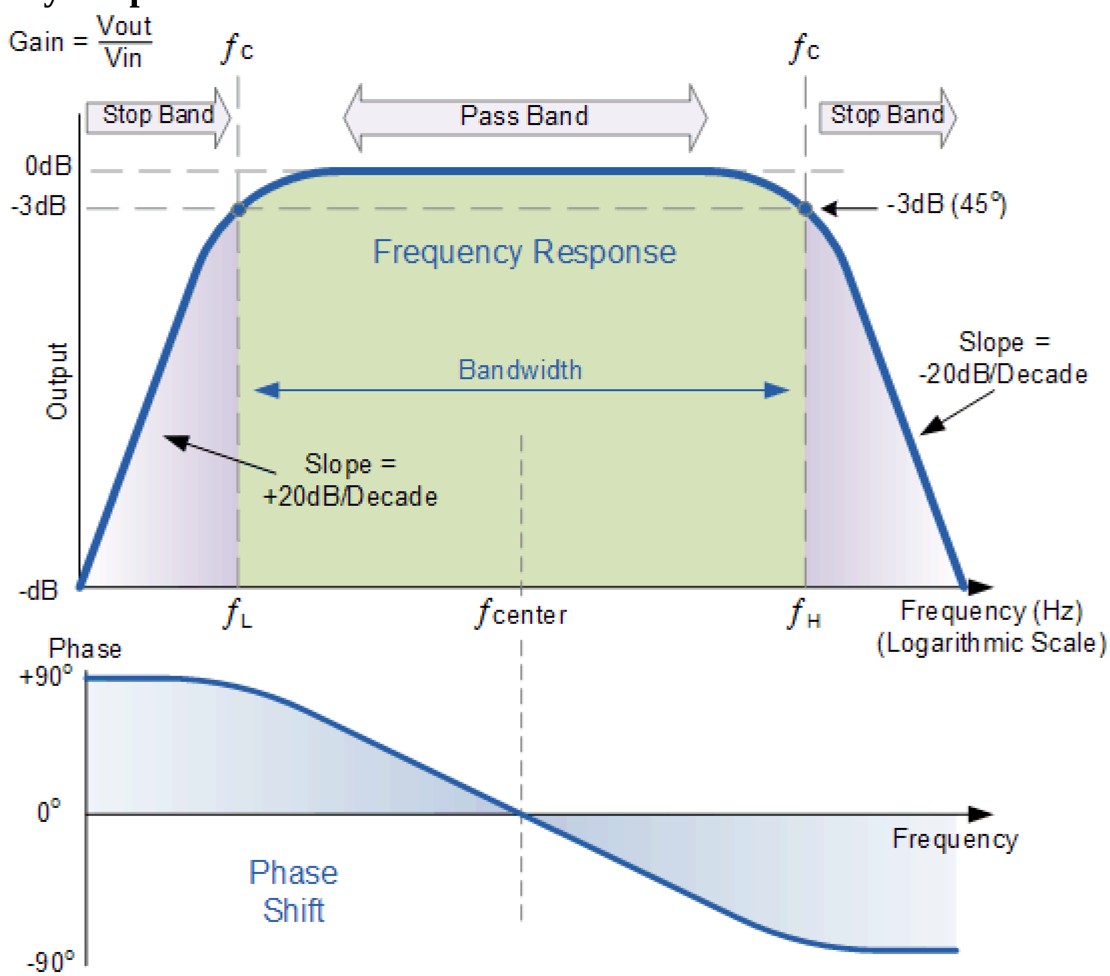
Bandwidth is commonly defined as the frequency range that exists between two specified frequency cut-off points (fc), that are 3dB below the maximum centre or resonant peak while attenuating or weakening the others outside of these two points.

Then for widely spread frequencies, we can simply define the term "bandwidth", BW as being the difference between the lower cut-off frequency (fc_{LOWER}) and the higher cut-off frequency

(fc_{HIGHER}) points. In other words, BW = $f_H - f_L$. Clearly for a pass band filter to function correctly, the cut-off frequency of the low pass filter must be higher than the cut-off frequency for the high pass filter.

The "ideal" **Band Pass Filter** can also be used to isolate or filter out certain frequencies that lie within a particular band of frequencies, for example, noise cancellation. Band pass filters are known generally as second-order filters, (two-pole) because they have "two" reactive component, the capacitors, within their circuit design. One capacitor in the low pass circuit and another capacitor in the high pass circuit.

Frequency Response of a 2nd Order Band Pass Filter



The **Bode Plot** or frequency response curve above shows the characteristics of the band pass filter. Here the signal is attenuated at low frequencies with the output increasing at a slope of +20dB/Decade (6dB/Octave) until the frequency reaches the "lower cut-off" point f_L. At this frequency the output voltage is again $1/\sqrt{2}$ = 70.7% of the input signal value or **-3dB** (20*log(V_{OUT}/V_{IN})) of the input.

The output continues at maximum gain until it reaches the "upper cut-off" point f_H where the output decreases at a rate of -20dB/Decade (6dB/Octave) attenuating any high frequency signals. The point of maximum output gain is generally the geometric mean of the two -3dB value between the lower and upper cut-off points and is called the “Centre Frequency” or “Resonant Peak” value fr. This geometric mean value is calculated as being $f r^2 = f_{(UPPER)} x f_{(LOWER)}$.

A band pass filter is regarded as a second-order (two-pole) type filter because it has “two” reactive components within its circuit structure, then the phase angle will be twice that of the previously seen first-order filters, ie, **180^o**. The phase angle of the output signal **LEADS** that of the input by **+90^o** up to the centre or resonant frequency, fr point were it becomes “zero” degrees (0^o) or “in-phase” and then changes to **LAG** the input by **-90^o** as the output frequency increases.

The upper and lower cut-off frequency points for a band pass filter can be found using the same formula as that for both the low and high pass filters, For example.

$$fc = \frac{1}{2\pi RC}\ Hz$$

Then clearly, the width of the pass band of the filter can be controlled by the positioning of the two cut-off frequency points of the two filters.

Band Pass Filter Example No1.

A second-order **band pass filter** is to be constructed using RC components that will only allow a range of frequencies to pass above 1kHz (1,000Hz) and below 30kHz (30,000Hz). Assuming that both the resistors have values of 10kΩ, calculate the values of the two capacitors required.

The High Pass Filter Stage

The value of the capacitor C1 required to give a cut-off frequency f_L of 1kHz with a resistor value of 10kΩ is calculated as:

$$C_1 = \frac{1}{2\pi f_L R} = \frac{1}{2\pi x 1{,}000 x 10{,}000} = 15.9\,nF$$

Then, the values of R1 and C1 required for the high pass stage to give a cut-off frequency of 1.0kHz are: R1 = 10kΩ and to the nearest preferred value, C1 = 15nF.

The Low Pass Filter Stage

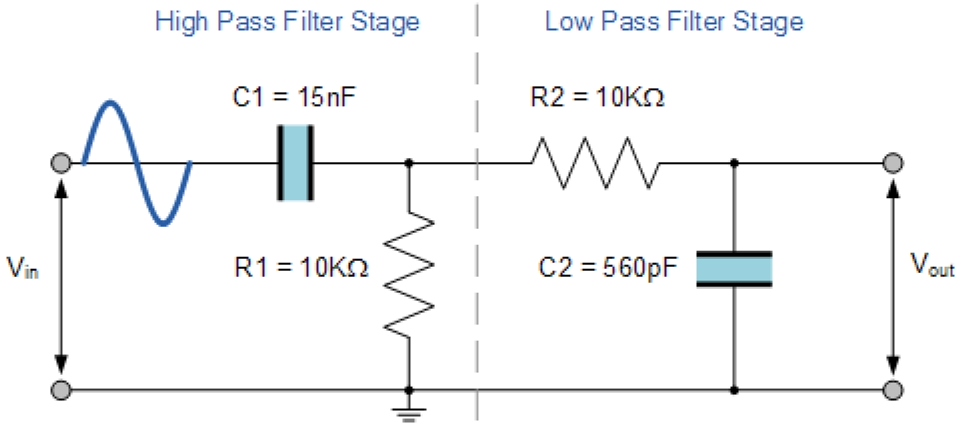
The value of the capacitor C2 required to give a cut-off frequency f_H of 30kHz with a resistor value of 10kΩ is calculated as:

$$C_2 = \frac{1}{2\pi f_H R} = \frac{1}{2\pi x 30{,}000 x 10{,}000} = 530\,pF$$

Then, the values of R2 and C2 required for the low pass stage to give a cut-off frequency of 30kHz are, R = 10kΩ and C = 530pF. However, the nearest preferred value of the calculated capacitor value of 530pF is 560pF, so this is used instead.

With the values of both the resistances R1 and R2 given as 10kΩ, and the two values of the capacitors C1 and C2 found for both the high pass and low pass filters as 15nF and 560pF respectively, then the circuit for our simple passive **Band Pass Filter** is given as.

Completed Band Pass Filter Circuit



Band Pass Filter Resonant Frequency

We can also calculate the "Resonant" or "Centre Frequency" (fr) point of the band pass filter were the output gain is at its maximum or peak value. This peak value is not the arithmetic average of the upper and lower -3dB cut-off points as you might expect but is in fact the "geometric" or mean value. This geometric mean value is calculated as being $fr^2 = fc_{(UPPER)} x fc_{(LOWER)}$ for example:

Centre Frequency Equation

$$fr = \sqrt{f_L \; x \; f_H}$$

- Where, f_r is the resonant or centre frequency
- f_L is the lower -3dB cut-off frequency point
- f_H is the upper -3db cut-off frequency point

and in our simple example above, the calculated cut-off frequencies were found to be f_L = 1,060 Hz and f_H = 28,420 Hz using the filter values.

Then by substituting these values into the above equation gives a central resonant frequency of:

$$fr = \sqrt{f_L \; x \; f_H} = \sqrt{1{,}060 \; x \; 28{,}420} = 5{,}48 \, kHz$$

Band Pass Filter Summary

A simple passive **Band Pass Filter** can be made by cascading together a single **Low Pass Filter** with a **High Pass Filter**. The frequency range, in Hertz, between the lower and upper -3dB cut-off points of the RC combination is know as the filters "Bandwidth".

The width or frequency range of the filters bandwidth can be very small and selective, or very wide and non-selective depending upon the values of R and C used.

The centre or resonant frequency point is the geometric mean of the lower and upper cut-off points. At this centre frequency the output signal is at its maximum and the phase shift of the output signal is the same as the input signal.

The amplitude of the output signal from a band pass filter or any passive RC filter for that matter, will always be less than that of the input signal. In other words a passive filter is also an attenuator giving a voltage gain of less than 1 (Unity). To provide an output signal with a voltage gain greater than unity, some form of amplification is required within the design of the circuit.

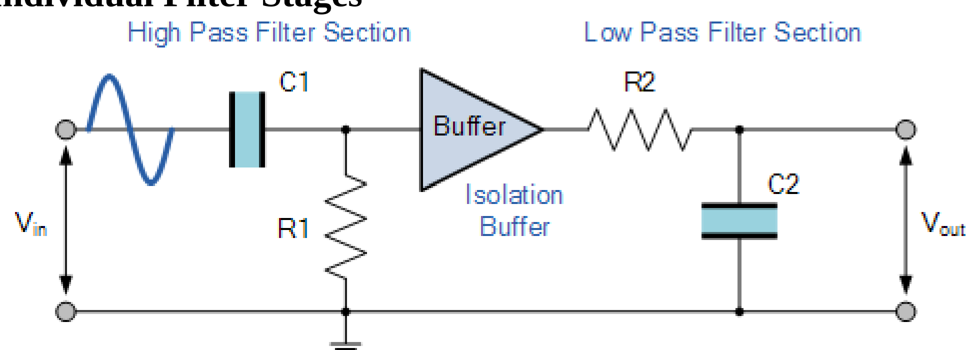
A **Passive Band Pass Filter** is classed as a second-order type filter because it has two reactive components within its design, the capacitors. It is made up from two single RC filter circuits that are each first-order filters themselves.

If more filters are cascaded together the resulting circuit will be known as an "n^{th}-order" filter where the "n" stands for the number of individual reactive components and therefore poles within the filter circuit. For example, filters can be a 2^{nd}-order, 4^{th}-order, 10^{th}-order, etc.

The higher the filters order the steeper will be the slope at n times -20dB/decade. However, a single capacitor value made by combining together two or more individual capacitors is still one capacitor.

Our example above shows the output frequency response curve for an "ideal" band pass filter with constant gain in the pass band and zero gain in the stop bands. In practice the frequency response of this Band Pass Filter circuit would not be the same as the input reactance of the high pass circuit would affect the frequency response of the low pass circuit (components connected in series or parallel) and vice versa. One way of overcoming this would be to provide some form of electrical isolation between the two filter circuits as shown below.

Buffering Individual Filter Stages



One way of combining amplification and filtering into the same circuit would be to use an Operational Amplifier or Op-amp, and examples of these are given in the Operational Amplifier section. In the next tutorial we will look at filter circuits which use an operational amplifier within their design to not only to introduce gain but provide isolation between stages. These types of filter arrangements are generally known as **Active Filters**.

https://www.electronics-tutorials.ws/filter/filter_5.html

Active Low Pass Filter

By combining a basic RC Low Pass Filter circuit with an operational amplifier we can create an Active Low Pass Filter circuit complete with amplification

In the RC Passive Filter tutorials, we saw how a basic first-order filter circuits, such as the low pass and the high pass filters can be made using just a single resistor in series with a non-polarized capacitor connected across a sinusoidal input signal.

We also noticed that the main disadvantage of passive filters is that the amplitude of the output signal is less than that of the input signal, ie, the gain is never greater than unity and that the load impedance affects the filters characteristics.

With passive filter circuits containing multiple stages, this loss in signal amplitude called “Attenuation” can become quiet severe. One way of restoring or controlling this loss of signal is by using amplification through the use of **Active Filters**.

As their name implies, **Active Filters** contain active components such as operational amplifiers, transistors or FET’s within their circuit design. They draw their power from an external power source and use it to boost or amplify the output signal.

Filter amplification can also be used to either shape or alter the frequency response of the filter circuit by producing a more selective output response, making the output bandwidth of the filter more narrower or even wider. Then the main difference between a “passive filter” and an “active filter” is amplification.

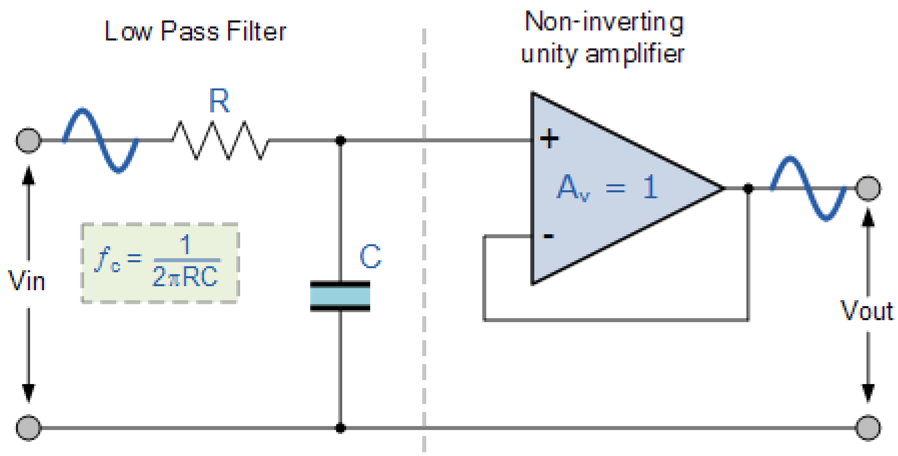
An active filter generally uses an operational amplifier (op-amp) within its design and in the Operational Amplifier tutorial we saw that an Op-amp has a high input impedance, a low output impedance and a voltage gain determined by the resistor network within its feedback loop.

Unlike a passive high pass filter which has in theory an infinite high frequency response, the maximum frequency response of an active filter is limited to the Gain/Bandwidth product (or open loop gain) of the operational amplifier being used. Still, active filters are generally much easier to design than passive filters, they produce good performance characteristics, very good accuracy with a steep roll-off and low noise when used with a good circuit design.

Active Low Pass Filter

The most common and easily understood active filter is the **Active Low Pass Filter**. Its principle of operation and frequency response is exactly the same as those for the previously seen passive filter, the only difference this time is that it uses an op-amp for amplification and gain control. The simplest form of a low pass active filter is to connect an inverting or non-inverting amplifier, the same as those discussed in the Op-amp tutorial, to the basic RC low pass filter circuit as shown.

First Order Low Pass Filter

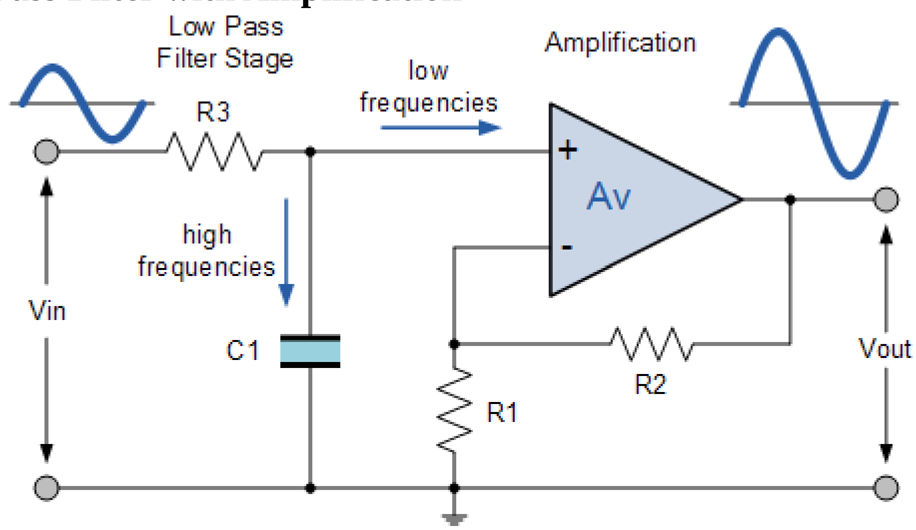


This first-order low pass active filter, consists simply of a passive RC filter stage providing a low frequency path to the input of a non-inverting operational amplifier. The amplifier is configured as a voltage-follower (Buffer) giving it a DC gain of one, Av = +1 or unity gain as opposed to the previous passive RC filter which has a DC gain of less than unity.

The advantage of this configuration is that the op-amps high input impedance prevents excessive loading on the filters output while its low output impedance prevents the filters cut-off frequency point from being affected by changes in the impedance of the load.

While this configuration provides good stability to the filter, its main disadvantage is that it has no voltage gain above one. However, although the voltage gain is unity the power gain is very high as its output impedance is much lower than its input impedance. If a voltage gain greater than one is required we can use the following filter circuit.

Active Low Pass Filter with Amplification



The frequency response of the circuit will be the same as that for the passive RC filter, except that the amplitude of the output is increased by the pass band gain, A_F of the amplifier. For a non-inverting amplifier circuit, the magnitude of the voltage gain for the filter is given as a function of the feedback resistor (R_2) divided by its corresponding input resistor (R_1) value and is given as:

$$\text{DC gain} = \left(1 + \frac{R_2}{R_1}\right)$$

Therefore, the gain of an active low pass filter as a function of frequency will be:

Gain of a first-order low pass filter

$$\textit{Voltage Gain}, (Av) = \frac{Vout}{Vin} = \frac{A_F}{\sqrt{1 + \left(\frac{f}{fc}\right)^2}}$$

- Where:
- A_F = the pass band gain of the filter, (1 + R2/R1)
- f = the frequency of the input signal in Hertz, (Hz)
- fc = the cut-off frequency in Hertz, (Hz)

Thus, the operation of a low pass active filter can be verified from the frequency gain equation above as:

1. At very low frequencies, $f < f$c $\quad \frac{Vout}{Vin} \cong A_F$

2. At the cut-off frequency, $f = f$c $\quad \frac{Vout}{Vin} = \frac{A_F}{\sqrt{2}} = 0.707\,A_F$

3. At very high frequencies, $f > f$c $\quad \frac{Vout}{Vin} < A_F$

Thus, the **Active Low Pass Filter** has a constant gain A_F from 0Hz to the high frequency cut-off point, f_C. At f_C the gain is $0.707A_{F,}$ and after f_C it decreases at a constant rate as the frequency increases. That is, when the frequency is increased tenfold (one decade), the voltage gain is divided by 10.

In other words, the gain decreases 20dB (= 20*log(10)) each time the frequency is increased by 10. When dealing with filter circuits the magnitude of the pass band gain of the circuit is generally expressed in *decibels* or *dB* as a function of the voltage gain, and this is defined as:

Magnitude of Voltage Gain in (dB)

$$Av(dB) = 20\log_{10}\left(\frac{Vout}{Vin}\right)$$

$$\therefore\ -3dB = 20\log_{10}\left(0.707\frac{Vout}{Vin}\right)$$

Active Low Pass Filter Example No1

Design a non-inverting active low pass filter circuit that has a gain of ten at low frequencies, a high frequency cut-off or corner frequency of 159Hz and an input impedance of 10KΩ.

The voltage gain of a non-inverting operational amplifier is given as:

$$A_F = 1 + \frac{R_2}{R_1} = 10$$

Assume a value for resistor R1 of 1kΩ rearranging the formula above gives a value for R2 of:

$$R_2 = (10 - 1)\ x\ R_1 = 9\ x\ 1k\Omega = 9k\Omega$$

So for a voltage gain of 10, R1 = 1kΩ and R2 = 9kΩ. However, a 9kΩ resistor does not exist so the next preferred value of 9k1Ω is used instead. Converting this voltage gain to an equivalent decibel dB value gives:

$$Gain\ in\ dB = 20\log A = 20\log 10 = 20dB$$

The cut-off or corner frequency (*ƒ*c) is given as being 159Hz with an input impedance of 10kΩ. This cut-off frequency can be found by using the formula:

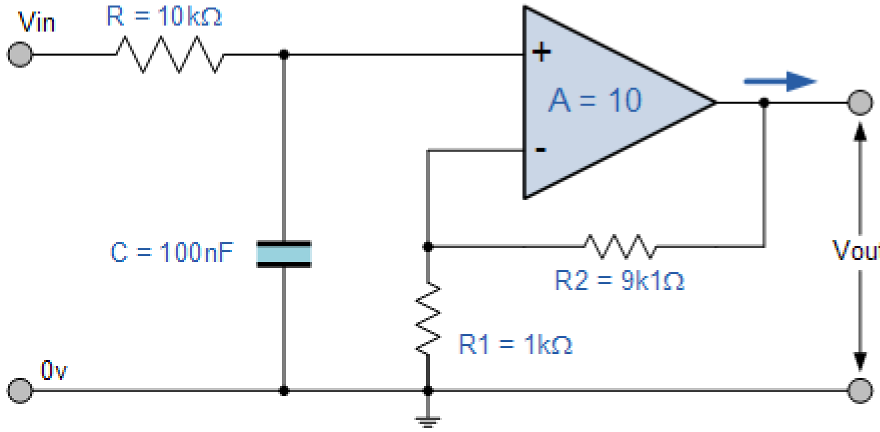
$$fc = \frac{1}{2\pi RC}\ Hz$$ where *ƒ*c = 159Hz and R = 10kΩ.

By rearranging the above standard formula we can find the value of the filter capacitor C as:

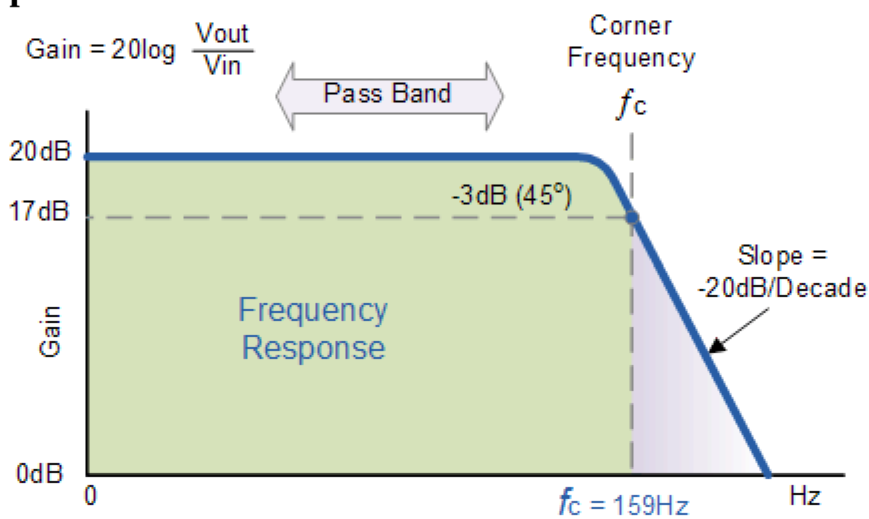
$$C = \frac{1}{2\pi\, fc\, R} = \frac{1}{2\pi\ x\ 159\ x\ 10k\Omega} = 100nF$$

Thus the final low pass filter circuit along with its frequency response is given below as:

Low Pass Filter Circuit



Frequency Response Curve



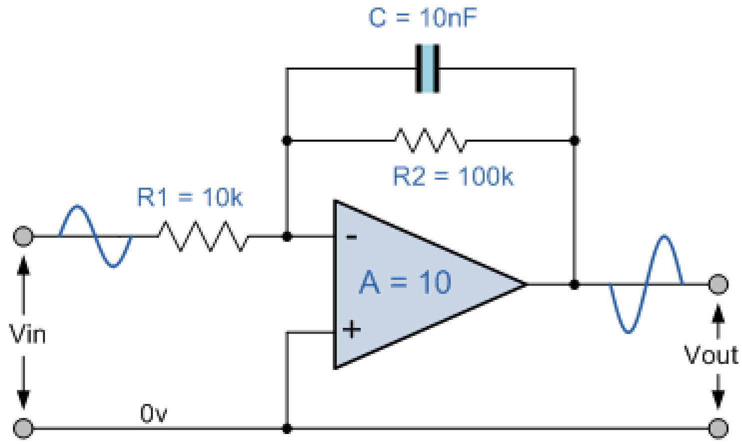
If the external impedance connected to the input of the filter circuit changes, this impedance change would also affect the corner frequency of the filter (components connected together in series or parallel). One way of avoiding any external influence is to place the capacitor in parallel with the feedback resistor R2 effectively removing it from the input but still maintaining the filters characteristics.

However, the value of the capacitor will change slightly from being 100nF to 110nF to take account of the 9k1Ω resistor, but the formula used to calculate the cut-off corner frequency is the same as that used for the RC passive low pass filter.

$$fc = \frac{1}{2\pi C R_2} \quad Hertz$$

Examples of different first-order *active low pass filter circuit* configurations are given as:

Simplified Inverting Amplifier Filter Circuit

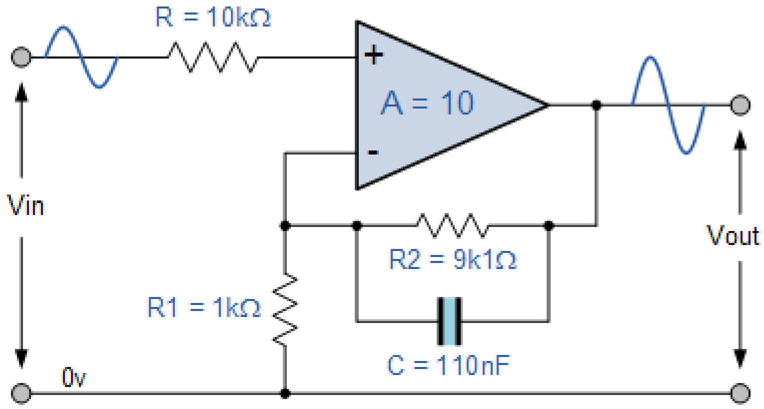


Here the capacitor has been moved from the op-amps input to its feedback circuit in parallel with R2. This parallel combination of C and R2 sets the -3dB point as before, but allows the amplifiers gain to roll-off indefinitely beyond the corner frequency.

At low frequencies the capacitors reactance is much higher than R2, so the dc gain is set by the standard inverting formula of: -R2/R1 = 10, for this example. As the frequency increases the capacitors reactance decreases reducing the impedance of the parallel combination of Xc||R2, until eventually at a high enough frequency, Xc reduces to zero.

The advantage here is that the circuits input impedance is now just R1 and the output signal is inverted. With the corner frequency determining components in the feedback circuit, the RC set-point is unaffected by variations in source impedance and the dc gain can be adjusted independently of the corner frequency.

Unity Gain Non-inverting Amplifier Filter Circuit



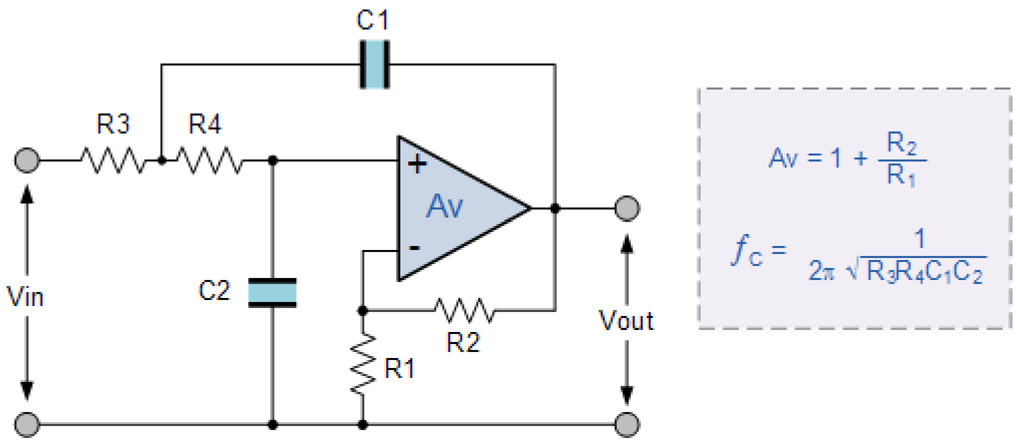
Here due to the position of the capacitor in parallel with the feedback resistor R2, the low pass corner frequency is set as before but at high frequencies the reactance of the capacitor dominates shorting out R2 reducing the amplifiers gain. At a high enough frequency the gain bottoms out at unity (0dB) as the amplifier effectively becomes a voltage follower so the gain equation becomes 1 + 0/R1 which equals 1 (unity).

Applications of **Active Low Pass Filters** are in audio amplifiers, equalizers or speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or “hiss” type distortion. When used like this in audio applications the active low pass filter is sometimes called a “Bass Boost” filter.

Second-order Low Pass Active Filter

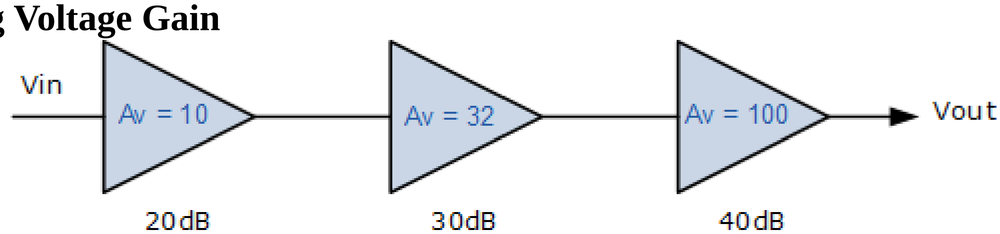
As with the passive filter, a first-order low-pass active filter can be converted into a second-order low pass filter simply by using an additional RC network in the input path. The frequency response of the second-order low pass filter is identical to that of the first-order type except that the stop band roll-off will be twice the first-order filters at 40dB/decade (12dB/octave). Therefore, the design steps required of the second-order active low pass filter are the same.

Second-order Active Low Pass Filter Circuit



When cascading together filter circuits to form higher-order filters, the overall gain of the filter is equal to the product of each stage. For example, the gain of one stage may be 10 and the gain of the second stage may be 32 and the gain of a third stage may be 100. Then the overall gain will be 32,000, (10 x 32 x 100) as shown below.

Cascading Voltage Gain



$$Av = Av_1 \times Av_2 \times Av_3$$

$$Av = 10 \times 32 \times 100 = 32{,}000$$

$$Av(dB) = 20\log_{10}(32{,}000)$$

$$Av(dB) = 90dB$$

$$90dB = 20dB + 30dB + 40dB$$

Second-order (two-pole) active filters are important because higher-order filters can be designed using them. By cascading together first and second-order filters, filters with an order value, either odd or even up to any value can be constructed. In the next tutorial about *filters*, we will see that Active High Pass Filters, can be constructed by reversing the positions of the resistor and capacitor in the circuit.

https://www.electronics-tutorials.ws/filter/filter_6.html

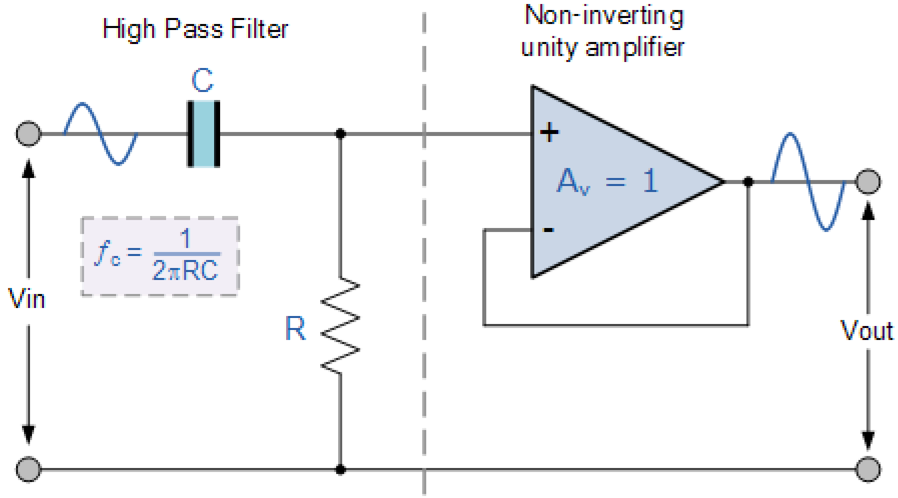
Active High Pass Filter

An Active High Pass Filter can be created by combining a passive RC filter network with an operational amplifier to produce a high pass filter with amplification

The basic operation of an **Active High Pass Filter** (HPF) is the same as for its equivalent RC passive high pass filter circuit, except this time the circuit has an operational amplifier or included within its design providing amplification and gain control.

Like the previous active low pass filter circuit, the simplest form of an *active high pass filter* is to connect a standard inverting or non-inverting operational amplifier to the basic RC high pass passive filter circuit as shown.

First Order High Pass Filter

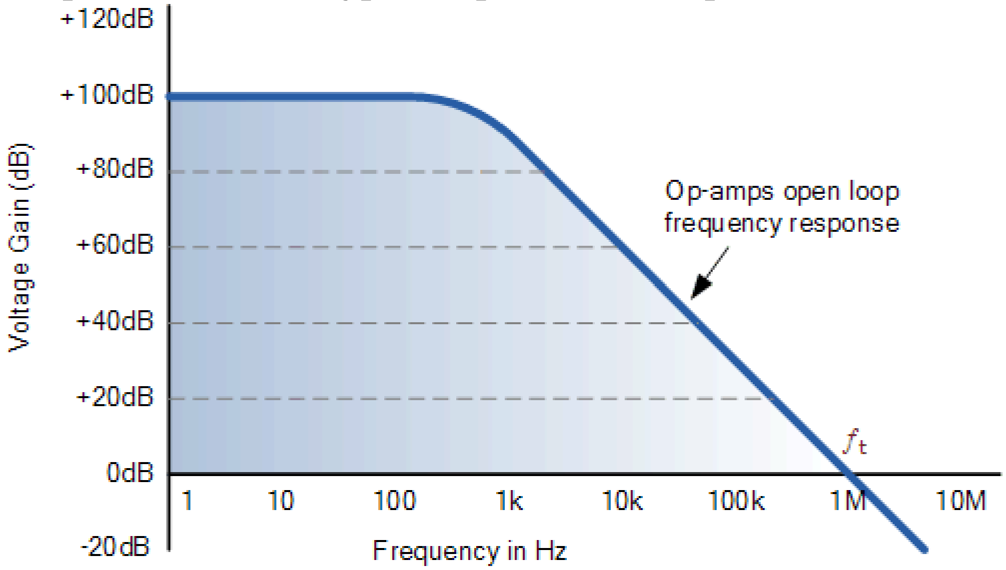


Technically, there is no such thing as an **active high pass filter**. Unlike Passive High Pass Filters which have an "infinite" frequency response, the maximum pass band frequency response of an active high pass filter is limited by the open-loop characteristics or bandwidth of the operational amplifier being used, making them appear as if they are band pass filters with a high frequency cut-off determined by the selection of op-amp and gain.

In the Operational Amplifier tutorial we saw that the maximum frequency response of an op-amp is limited to the Gain/Bandwidth product or open loop voltage gain (A_V) of the operational amplifier being used giving it a bandwidth limitation, where the closed loop response of the op amp intersects the open loop response.

A commonly available operational amplifier such as the uA741 has a typical "open-loop" (without any feedback) DC voltage gain of about 100dB maximum reducing at a roll off rate of -20dB/Decade (-6db/Octave) as the input frequency increases. The gain of the uA741 reduces until it reaches unity gain, (0dB) or its “transition frequency” (ft) which is about 1MHz. This causes the op-amp to have a frequency response curve very similar to that of a first-order low pass filter and this is shown below.

Frequency response curve of a typical Operational Amplifier



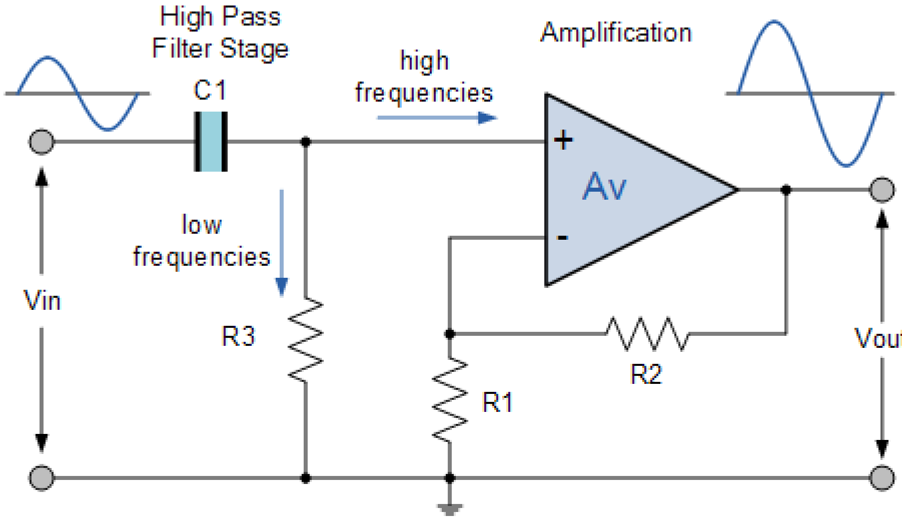
Then the performance of a “high pass filter” at high frequencies is limited by this unity gain crossover frequency which determines the overall bandwidth of the open-loop amplifier. The gain-bandwidth product of the op-amp starts from around 100kHz for small signal amplifiers up to about 1GHz for high-speed digital video amplifiers and op-amp based active filters can achieve very good accuracy and performance provided that low tolerance resistors and capacitors are used.

Under normal circumstances the maximum pass band required for a closed loop active high pass or band pass filter is well below that of the maximum open-loop transition frequency. However, when designing active filter circuits it is important to choose the correct op-amp for the circuit as the loss of high frequency signals may result in signal distortion.

Active High Pass Filter

A first-order (single-pole) **Active High Pass Filter** as its name implies, attenuates low frequencies and passes high frequency signals. It consists simply of a passive filter section followed by a non-inverting operational amplifier. The frequency response of the circuit is the same as that of the passive filter, except that the amplitude of the signal is increased by the gain of the amplifier and for a non-inverting amplifier the value of the pass band voltage gain is given as 1 + R2/R1, the same as for the low pass filter circuit.

Active High Pass Filter with Amplification



This *first-order high pass filter*, consists simply of a passive filter followed by a non-inverting amplifier. The frequency response of the circuit is the same as that of the passive filter, except that the amplitude of the signal is increased by the gain of the amplifier.

For a non-inverting amplifier circuit, the magnitude of the voltage gain for the filter is given as a function of the feedback resistor (R2) divided by its corresponding input resistor (R1) value and is given as:

Gain for an Active High Pass Filter

$$\text{Voltage Gain, (Av)} = \frac{\text{Vout}}{\text{Vin}} = \frac{A_F\left(\frac{f}{fc}\right)}{\sqrt{1+\left(\frac{f}{fc}\right)^2}}$$

- Where:
- A_F = the Pass band Gain of the filter, (1 + R2/R1)
- f = the Frequency of the Input Signal in Hertz, (Hz)
- fc = the Cut-off Frequency in Hertz, (Hz)

Just like the low pass filter, the operation of a high pass active filter can be verified from the frequency gain equation above as:

1. At very low frequencies, $f < fc$ $\qquad \frac{\text{Vout}}{\text{Vin}} < A_F$

2. At the cut-off frequency, f=fc $$\frac{Vout}{Vin} = \frac{A_F}{\sqrt{2}} = 0.707\,A_F$$

3. At very high frequencies, f>fc $$\frac{Vout}{Vin} \cong A_F$$

Then, the **Active High Pass Filter** has a gain A_F that increases from 0Hz to the low frequency cut-off point, f_C at 20dB/decade as the frequency increases. At f_C the gain is 0.707*$A_{F,}$ and after f_C all frequencies are pass band frequencies so the filter has a constant gain A_F with the highest frequency being determined by the closed loop bandwidth of the op-amp.

When dealing with filter circuits the magnitude of the pass band gain of the circuit is generally expressed in *decibels* or *dB* as a function of the voltage gain, and this is defined as:

Magnitude of Voltage Gain in (dB)

$$Av(dB) = 20\log_{10}\left(\frac{Vout}{Vin}\right)$$

$$\therefore\ -3dB = 20\log_{10}\left(0.707\,\frac{Vout}{Vin}\right)$$

For a first-order filter the frequency response curve of the filter increases by 20dB/decade or 6dB/octave up to the determined cut-off frequency point which is always at -3dB below the maximum gain value. As with the previous filter circuits, the lower cut-off or corner frequency (fc) can be found by using the same formula:

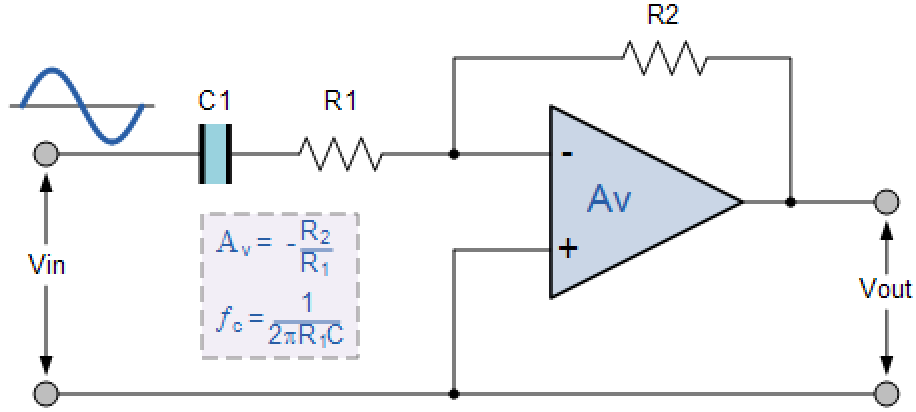
$$fc = \frac{1}{2\pi RC}\ Hz$$

The corresponding phase angle or phase shift of the output signal is the same as that given for the passive RC filter and **leads** that of the input signal. It is equal to **+45^{o}** at the cut-off frequency fc value and is given as:

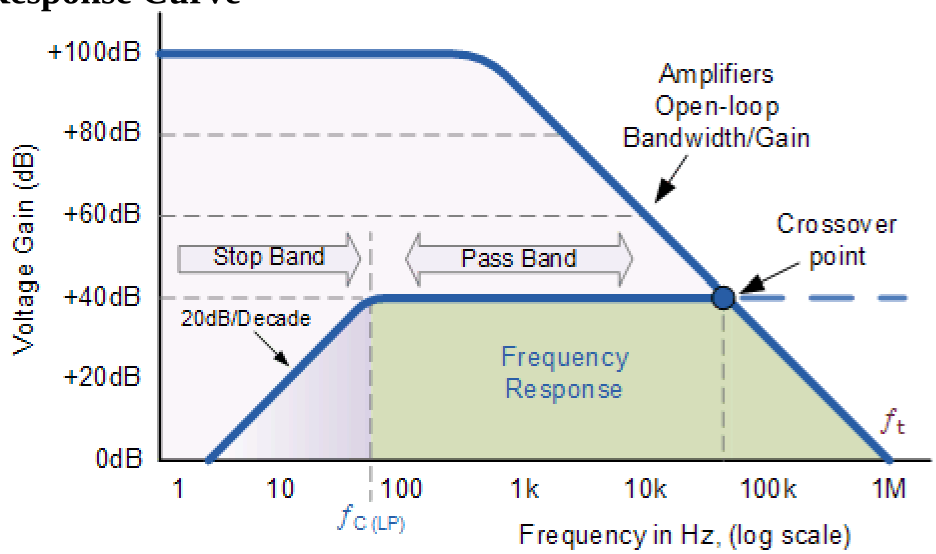
$$Phase\ Shift\ \ \phi = tan^{-1}\left(\frac{1}{2\pi fRC}\right)$$

A simple first-order active high pass filter can also be made using an inverting operational amplifier configuration as well, and an example of this circuit design is given along with its corresponding frequency response curve. A gain of 40dB has been assumed for the circuit.

Inverting Operational Amplifier Circuit



Frequency Response Curve



Active High Pass Filter Example No1

A first order active high pass filter has a pass band gain of two and a cut-off corner frequency of 1kHz. If the input capacitor has a value of 10nF, calculate the value of the cut-off frequency determining resistor and the gain resistors in the feedback network. Also, plot the expected frequency response of the filter.

With a cut-off corner frequency given as 1kHz and a capacitor of 10nF, the value of R will therefore be:

$$R = \frac{1}{2\pi f_C C} = \frac{1}{2\pi.1000.10\times 10^{-9}} = 15.92k\Omega$$

or 16kΩ to the nearest preferred value.

Thus the pass band gain of the filter, A_F is therefore given as being: 2.

$$A_F = 1 + \frac{R_2}{R_1}, \quad \therefore 2 = 1 + \frac{R_2}{R_1} \quad \text{and} \quad \frac{R_2}{R_1} = 1$$

As the value of resistor, R_2 divided by resistor, R_1 gives a value of one. Then, resistor R_1 must be equal to resistor R_2, since the pass band gain, A_F=2. We can therefore select a suitable value for the two resistors of say, 10kΩ each for both feedback resistors.

So for a high pass filter with a cut-off corner frequency of 1kHz, the values of R and C will be, 10kΩ and 10nF respectively. The values of the two feedback resistors to produce a pass band gain of two are given as: R_1=R_2=10kΩ

The data for the frequency response bode plot can be obtained by substituting the values obtained above over a frequency range from 100Hz to 100kHz into the equation for voltage gain:

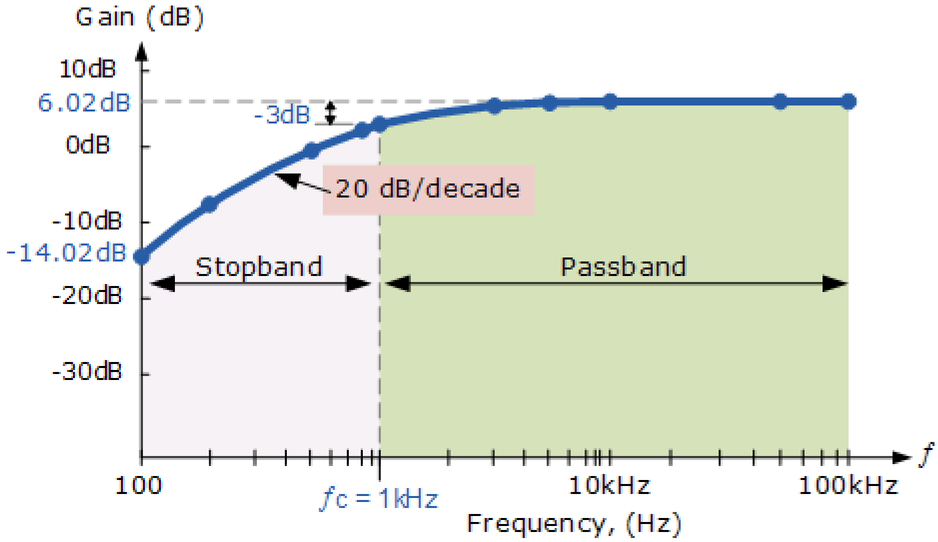
$$\text{Voltage Gain, } (Av) = \frac{Vout}{Vin} = \frac{A_F\left(\frac{f}{fc}\right)}{\sqrt{1 + \left(\frac{f}{fc}\right)^2}}$$

This then will give us the following table of data.

Frequency, ƒ (Hz)	Voltage Gain (*Vo/Vin*)	Gain, (dB) 20log(*Vo/Vin*)
100	0.20	-14.02
200	0.39	-8.13
500	0.89	-0.97
800	1.25	1.93
1,000	1.41	3.01
3,000	1.90	5.56
5,000	1.96	5.85
10,000	1.99	5.98
50,000	2.00	6.02
100,000	2.00	6.02

The frequency response data from the table above can now be plotted as shown below. In the stop band (from 100Hz to 1kHz), the gain increases at a rate of 20dB/decade. However, in the pass band after the cut-off frequency, f_C=1kHz, the gain remains constant at 6.02dB. The upper-frequency limit of the pass band is determined by the open loop bandwidth of the operational amplifier used as we discussed earlier. Then the bode plot of the filter circuit will look like this.

The Frequency Response Bode-plot for our example

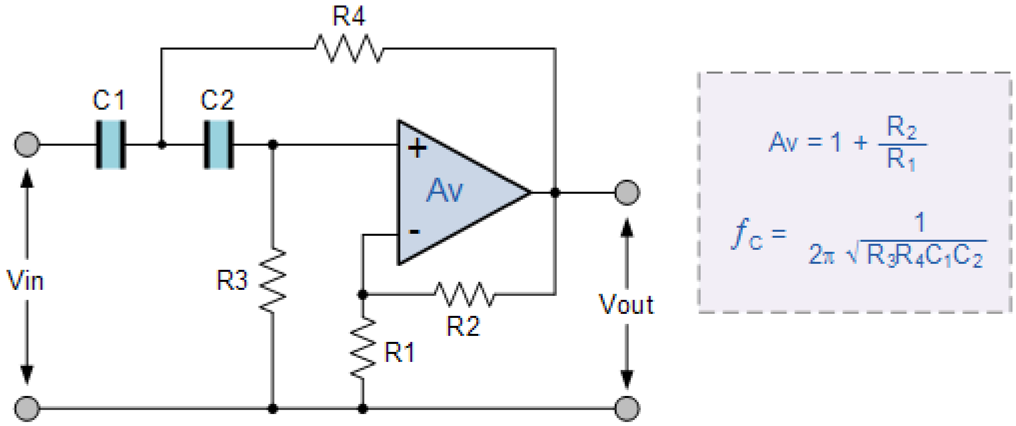


Applications of **Active High Pass Filters** are in audio amplifiers, equalizers or speaker systems to direct the high frequency signals to the smaller tweeter speakers or to reduce any low frequency noise or "rumble" type distortion. When used like this in audio applications the active high pass filter is sometimes called a "Treble Boost" filter.

Second-order High Pass Active Filter

As with the passive filter, a first-order high pass active filter can be converted into a second-order high pass filter simply by using an additional RC network in the input path. The frequency response of the second-order high pass filter is identical to that of the first-order type except that the stop band roll-off will be twice the first-order filters at 40dB/decade (12dB/octave). Therefore, the design steps required of the second-order active high pass filter are the same.

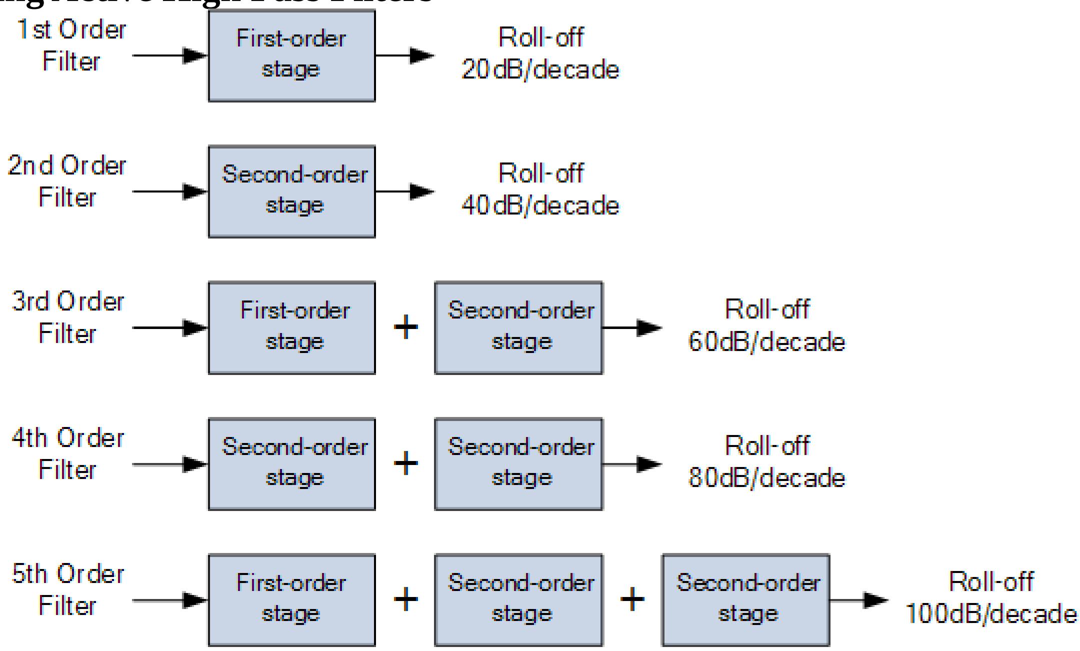
Second-order Active High Pass Filter Circuit



Higher-order high pass active filters, such as third, fourth, fifth, etc are formed simply by cascading together first and second-order filters. For example, a third order high pass filter is formed by cascading in series first and second order filters, a fourth-order high pass filter by cascading two second-order filters together and so on.

Then an **Active High Pass Filter** with an even order number will consist of only second-order filters, while an odd order number will start with a first-order filter at the beginning as shown.

Cascading Active High Pass Filters



Although there is no limit to the order of a filter that can be formed, as the order of the filter increases so to does its size. Also, its accuracy declines, that is the difference between the actual stop band response and the theoretical stop band response also increases.

If the frequency determining resistors are all equal, R1 = R2 = R3 etc, and the frequency determining capacitors are all equal, C1 = C2 = C3 etc, then the cut-off frequency for any order of filter will be exactly the same. However, the overall gain of the higher-order filter is fixed because all the frequency determining components are equal.

In the next tutorial about filters, we will see that Active Band Pass Filters, can be constructed by cascading together a high pass and a low pass filter.

https://www.electronics-tutorials.ws/filter/filter_7.html

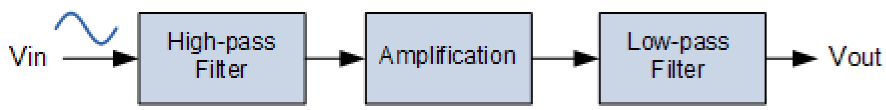
Active Band Pass Filter

The principal characteristic of a **Band Pass Filter** or any filter for that matter, is its ability to pass frequencies relatively unattenuated over a specified band or spread of frequencies called the "Pass Band".

For a low pass filter this pass band starts from 0Hz or DC and continues up to the specified cut-off frequency point at -3dB down from the maximum pass band gain. Equally, for a high pass filter the pass band starts from this -3dB cut-off frequency and continues up to infinity or the maximum open loop gain for an active filter.

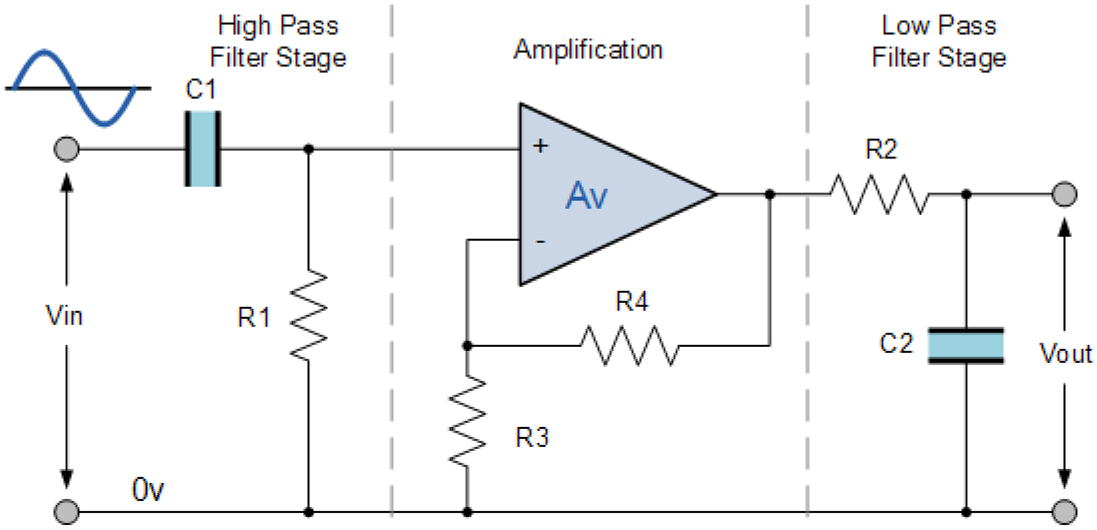
However, the **Active Band Pass Filter** is slightly different in that it is a frequency selective filter circuit used in electronic systems to separate a signal at one particular frequency, or a range of signals that lie within a certain "band" of frequencies from signals at all other frequencies. This band or range of frequencies is set between two cut-off or corner frequency points labelled the "lower frequency" (f_L) and the "higher frequency" (f_H) while attenuating any signals outside of these two points.

Simple **Active Band Pass Filter** can be easily made by cascading together a single Low Pass Filter with a single High Pass Filter as shown.

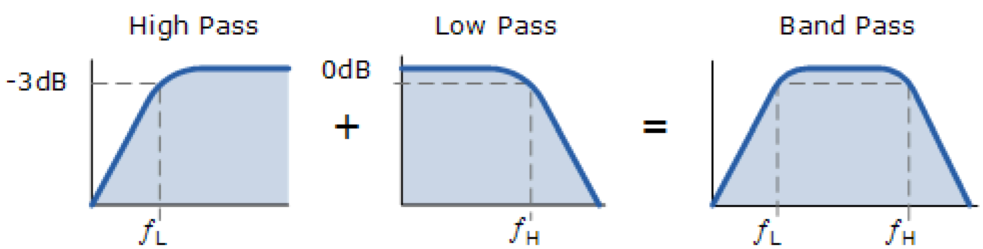


The cut-off or corner frequency of the low pass filter (LPF) is higher than the cut-off frequency of the high pass filter (HPF) and the difference between the frequencies at the -3dB point will determine the "bandwidth" of the band pass filter while attenuating any signals outside of these points. One way of making a very simple **Active Band Pass Filter** is to connect the basic passive high and low pass filters we look at previously to an amplifying op-amp circuit as shown.

Active Band Pass Filter Circuit



This cascading together of the individual low and high pass passive filters produces a low "Q-factor" type filter circuit which has a wide pass band. The first stage of the filter will be the high pass stage that uses the capacitor to block any DC biasing from the source. This design has the advantage of producing a relatively flat asymmetrical pass band frequency response with one half representing the low pass response and the other half representing high pass response as shown.

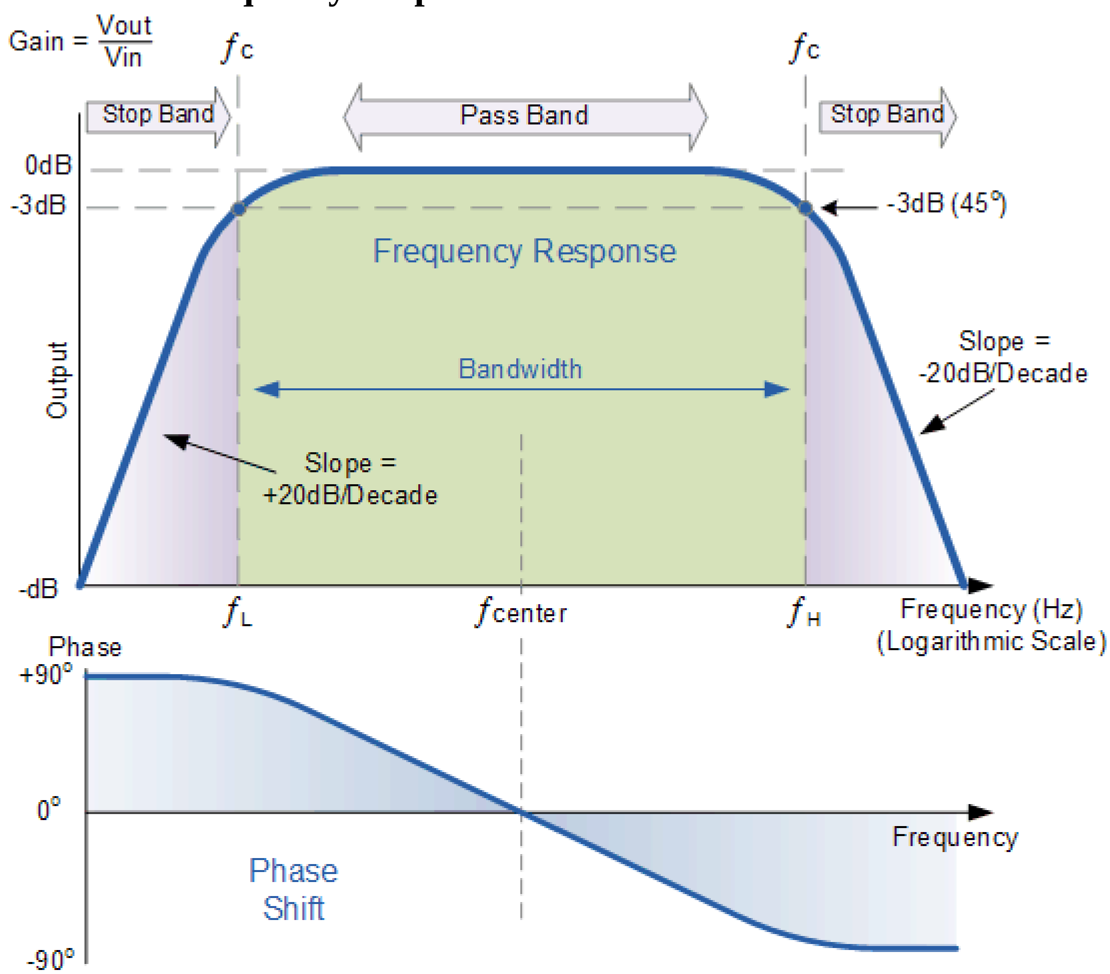


The higher corner point (f_H) as well as the lower corner frequency cut-off point (f_L) are calculated the same as before in the standard first-order low and high pass filter circuits. Obviously, a reasonable separation is required between the two cut-off points to prevent any interaction between the low pass and high pass stages. The amplifier also provides isolation between the two stages and defines the overall voltage gain of the circuit.

The bandwidth of the filter is therefore the difference between these upper and lower -3dB points. For example, suppose we have a band pass filter whose -3dB cut-off points are set at 200Hz and 600Hz. Then the bandwidth of the filter would be given as: Bandwidth (BW) = 600 – 200 = 400Hz.

The normalised frequency response and phase shift for an active band pass filter will be as follows.

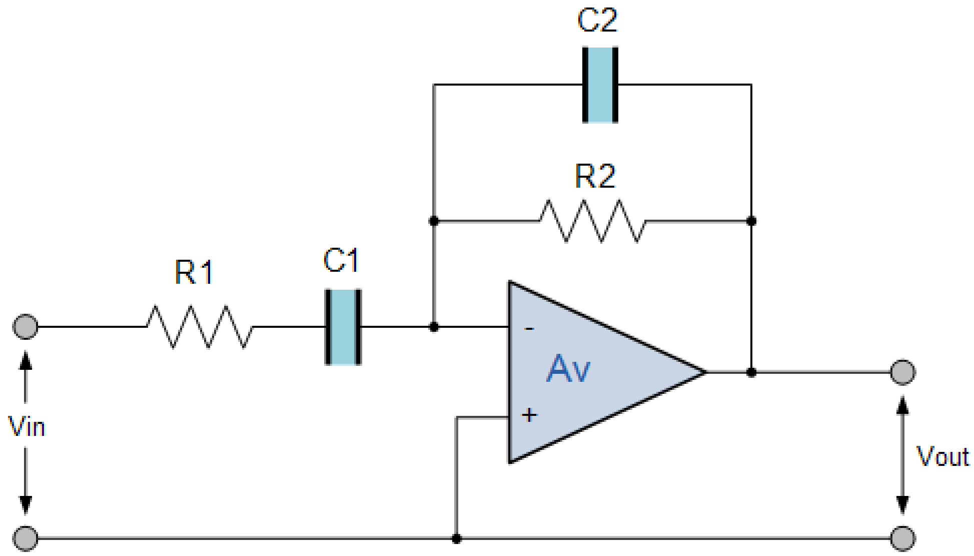
Active Band Pass Frequency Response



While the above passive tuned filter circuit will work as a band pass filter, the pass band (bandwidth) can be quite wide and this may be a problem if we want to isolate a small band of frequencies. Active band pass filter can also be made using inverting operational amplifier.

So by rearranging the positions of the resistors and capacitors within the filter we can produce a much better filter circuit as shown below. For an active band pass filter, the lower cut-off -3dB point is given by f_{C1} while the upper cut-off -3dB point is given by f_{C2}.

Inverting Band Pass Filter Circuit



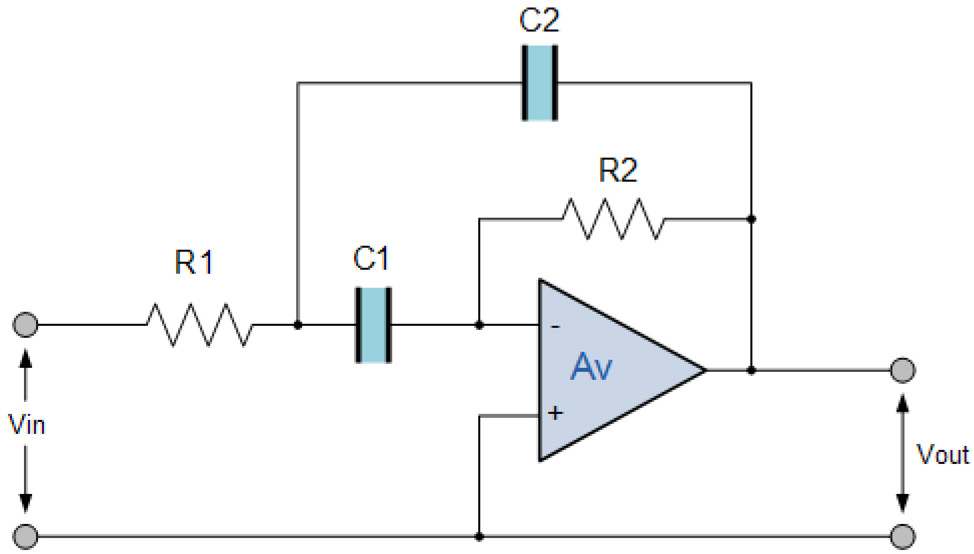
$$Voltage\ Gain = -\frac{R_2}{R_1}, \quad fc_1 = \frac{1}{2\pi R_1 C_1}, \quad fc_2 = \frac{1}{2\pi R_2 C_2}$$

This type of band pass filter is designed to have a much narrower pass band. The centre frequency and bandwidth of the filter is related to the values of R1, R2, C1 and C2. The output of the filter is again taken from the output of the op-amp.

Multiple Feedback Band Pass Active Filter

We can improve the band pass response of the above circuit by rearranging the components again to produce an infinite-gain multiple-feedback (IGMF) band pass filter. This type of active band pass design produces a “tuned” circuit based around a negative feedback active filter giving it a high “Q-factor” (up to 25) amplitude response and steep roll-off on either side of its centre frequency. Because the frequency response of the circuit is similar to a resonance circuit, this center frequency is referred to as the resonant frequency, (fr). Consider the circuit below.

Infinite Gain Multiple Feedback Active Filter



This active band pass filter circuit uses the full gain of the operational amplifier, with multiple negative feedback applied via resistor, R_2 and capacitor C_2. Then we can define the characteristics of the IGMF filter as follows:

$$f_r = \frac{1}{2\pi\sqrt{R_1R_2C_1C_2}} \qquad Q_{BP} = \frac{f_r}{BW_{(3dB)}} = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

$$\text{Maximum Gain, }(Av) = -\frac{R_2}{2R_1} = -2Q^2$$

We can see then that the relationship between resistors, R_1 and R_2 determines the band pass "Q-factor" and the frequency at which the maximum amplitude occurs, the gain of the circuit will be equal to -$2Q^2$. Then as the gain increases so to does the selectivity. In other words, high gain – high selectivity.

Active Band Pass Filter Example No1

An active band pass filter that has a voltage gain Av of one (1) and a resonant frequency, fr of 1kHz is constructed using an infinite gain multiple feedback filter circuit. Calculate the values of the components required to implement the circuit.

Firstly, we can determine the values of the two resistors, R_1 and R_2 required for the active filter using the gain of the circuit to find Q as follows.

$$Av = 1 = -2Q^2 \quad \therefore \; Q_{BP} = \sqrt{\frac{1}{2}} = 0.7071$$

$$Q = 0.7071 = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \quad \therefore \; \frac{R_2}{R_1} = \left(\frac{0.7071}{\frac{1}{2}}\right)^2 = 2$$

Then we can see that a value of Q = 0.7071 gives a relationship of resistor, R_2 being twice the value of resistor R_1. Then we can choose any suitable value of resistances to give the required ratio of two. Then resistor R_1 = 10kΩ and R_2 = 20kΩ.

The center or resonant frequency is given as 1kHz. Using the new resistor values obtained, we can determine the value of the capacitors required assuming that $C = C_1 = C_2$.

$$f_r = 1{,}000\text{Hz} = \frac{1}{2\pi C\sqrt{R_1 R_2}}$$

$$\therefore \; C = \frac{1}{2\pi f_r \sqrt{R_1 R_2}} = \frac{1}{2\pi 1000\sqrt{10{,}000 \times 20{,}000}} = 11.2\text{nF}$$

The closest standard value is 10nF.

Resonant Frequency Point

The actual shape of the frequency response curve for any passive or active band pass filter will depend upon the characteristics of the filter circuit with the curve above being defined as an "ideal" band pass response. An active band pass filter is a **2nd Order** type filter because it has "two" reactive components (two capacitors) within its circuit design.

As a result of these two reactive components, the filter will have a peak response or **Resonant Frequency** (ƒr) at its "center frequency", ƒc. The center frequency is generally calculated as being the geometric mean of the two -3dB frequencies between the upper and the lower cut-off points with the resonant frequency (point of oscillation) being given as:

$$fr = \sqrt{f_L \; x \; f_H}$$

- Where:

- f_r is the resonant or Center Frequency
- f_L is the lower -3dB cut-off frequency point
- f_H is the upper -3db cut-off frequency point

and in our simple example in the text above of a filters lower and upper -3dB cut-off points being at 200Hz and 600Hz respectively, then the resonant center frequency of the active band pass filter would be:

$$fr = \sqrt{200 \times 600} = \sqrt{120{,}000} = 346\,Hz$$

The "Q" or Quality Factor

In a **Band Pass Filter** circuit, the overall width of the actual pass band between the upper and lower -3dB corner points of the filter determines the **Quality Factor** or **Q-point** of the circuit. This **Q Factor** is a measure of how "Selective" or "Un-selective" the band pass filter is towards a given spread of frequencies. The lower the value of the Q factor the wider is the bandwidth of the filter and consequently the higher the Q factor the narrower and more "selective" is the filter.

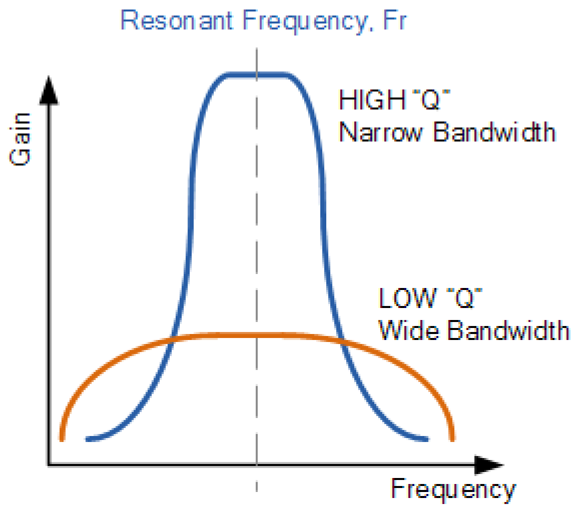
The **Quality Factor, Q** of the filter is sometimes given the Greek symbol of **Alpha**, (α) and is known as the **alpha-peak frequency** where:

$$\alpha = \frac{1}{Q}$$

As the quality factor of an active band pass filter (Second-order System) relates to the "sharpness" of the filters response around its centre resonant frequency (fr) it can also be thought of as the "Damping Factor" or "Damping Coefficient" because the more damping the filter has the flatter is its response and likewise, the less damping the filter has the sharper is its response. The damping ratio is given the Greek symbol of **Xi**, (ξ) where:

$$\xi = \frac{\alpha}{2}$$

The "Q" of a band pass filter is the ratio of the **Resonant Frequency**, (fr) to the **Bandwidth**, (BW) between the upper and lower -3dB frequencies and is given as:



$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}}$$

So for our simple example above, if the bandwidth (BW) is 400Hz, that is $f_H - f_L$, and the center resonant frequency, f_r is 346Hz. Then the quality factor "**Q**" of the band pass filter will be given as:

346Hz / 400Hz = **0.865**. Note that **Q** is a ratio, it has no units.

When analysing active filters, generally a normalised circuit is considered which produces an "ideal" frequency response having a rectangular shape, and a transition between the pass band and the stop band that has an abrupt or very steep roll-off slope. However, these ideal responses are not possible in the real world so we use approximations to give us the best frequency response possible for the type of filter we are trying to design.

Probably the best known filter approximation for doing this is the Butterworth or maximally-flat response filter. In the next tutorial we will look at higher order filters and use Butterworth approximations to produce filters that have a frequency response which is as flat as mathematically possible in the pass band and a smooth transition or roll-off rate.

https://www.electronics-tutorials.ws/filter/second-order-filters.html

Second Order Filters

Second Order (or two-pole) Filters consist of two RC filter sections connected together to provide a -40dB/decade roll-off rate.

Second Order Filters which are also referred to as VCVS filters, because the op-amp is used as a Voltage Controlled Voltage Source amplifier, are another important type of active filter design because along with the active first order RC filters we looked at previously, higher order filter circuits can be designed using them.

In this analogue filters section tutorials we have looked at both passive and active filter designs and have seen that first order filters can be easily converted into second order filters simply by using an additional RC network within the input or feedback path. Then we can define second order filters as simply being: “two 1st-order filters cascaded together with amplification”.

Most designs of second order filters are generally named after their inventor with the most common filter types being: *Butterworth*, *Chebyshev*, *Bessel* and *Sallen-Key*. All these types of filter designs are available as either: low pass filter, high pass filter, band pass filter and band stop (notch) filter configurations, and being second order filters, all have a 40-dB-per-decade roll-off.

The Sallen-Key filter design is one of the most widely known and popular 2nd order filter designs, requiring only a single operational amplifier for the gain control and four passive RC components to accomplish the tuning.

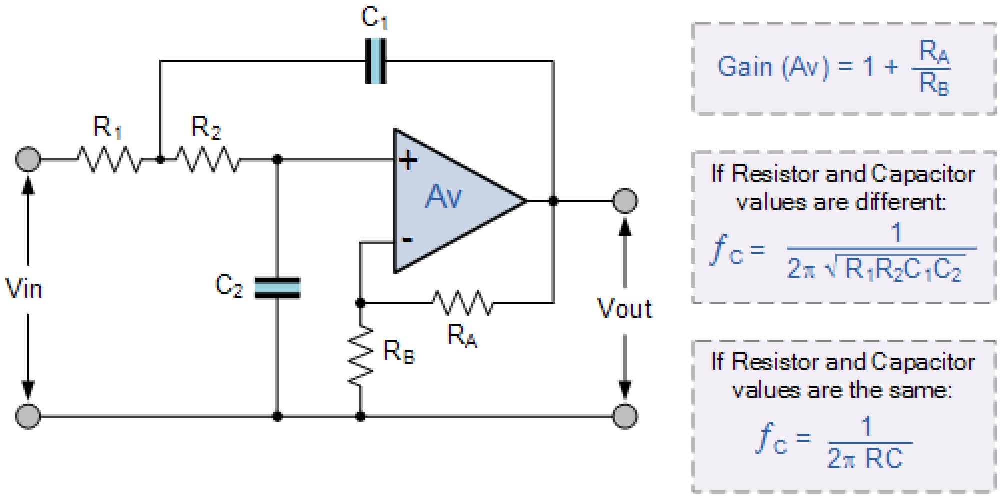
Most active filters consist of only op-amps, resistors, and capacitors with the cut-off point being achieved by the use of feedback eliminating the need for inductors as used in passive 1st-order filter circuits.

Second order (two-pole) active filters whether low pass or high pass, are important in Electronics because we can use them to design much higher order filters with very steep roll-off’s and by cascading together first and second order filters, analogue filters with an n^{th} order value, either odd or even can be constructed up to any value, within reason.

Second Order Low Pass Filter

Second order low pass filters are easy to design and are used extensively in many applications. The basic configuration for a Sallen-Key second order (two-pole) low pass filter is given as:

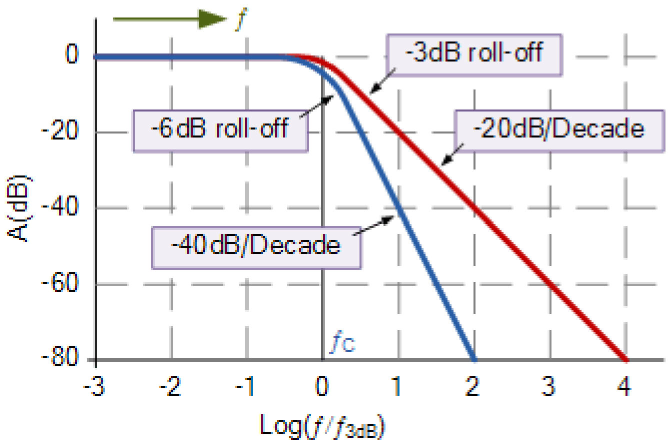
Second Order Low Pass Filter



This second order low pass filter circuit has two RC networks, R1 – C1 and R2 – C2 which give the filter its frequency response properties. The filter design is based around a non-inverting op-amp configuration so the filters gain, A will always be greater than 1. Also the op-amp has a high input impedance which means that it can be easily cascaded with other active filter circuits to give more complex filter designs.

The normalised frequency response of the second order low pass filter is fixed by the RC network and is generally identical to that of the first order type. The main difference between a 1st and 2nd order low pass filter is that the stop band roll-off will be twice the 1st order filters at 40dB/decade (12dB/octave) as the operating frequency increases above the cut-off frequency *ƒ*c, point as shown.

Normalised Low Pass Frequency Response



The frequency response bode plot above, is basically the same as that for a 1st-order filter. The difference this time is the steepness of the roll-off which is -40dB/decade in the stop band. However, second order filters can exhibit a variety of responses depending upon the circuits voltage magnification factor, Q at the the cut-off frequency point.

In active second order filters, the damping factor, ζ (zeta), which is the inverse of Q is normally used. Both Q and ζ are independently determined by the gain of the amplifier, A so as Q decreases the damping factor increases. In simple terms, a low pass filter will always be low pass in its nature but can exhibit a resonant peak in the vicinity of the cut-off frequency, that is the gain can increases rapidly due to resonance effects of the amplifiers gain.

Then Q, the quality factor, represents the "peakiness" of this resonance peak, that is its height and narrowness around the cut-off frequency point, f_C. But a filters gain also determines the amount of its feedback and therefore has a significant effect on the frequency response of the filter.

Generally to maintain stability, an active filters gain must not be more than 3 and is best expressed as:

The Quality Factor, "Q"

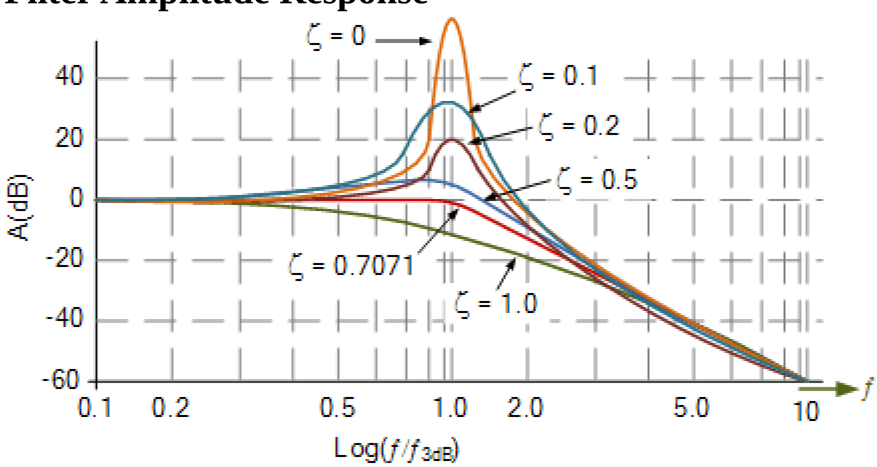
$$A = 3 - (2 \times \zeta)$$

$$\text{Where:} \quad \zeta = \frac{3 - A}{2} = \frac{1}{2Q}$$

$$\therefore \ A = 3 - \frac{1}{Q}$$

Then we can see that the filters gain, A for a non-inverting amplifier configuration must lie somewhere between 1 and 3 (the damping factor, ζ between zero an 2). Therefore, higher values of Q, or lower values of ζ gives a greater peak to the response and a faster initial roll-off rate as shown.

Second Order Filter Amplitude Response



The amplitude response of the second order low pass filter varies for different values of damping factor, ζ. When ζ=1.0 or more (2 is the maximum) the filter becomes what is called "overdamped" with the frequency response showing a long flat curve. When ζ=0, the filters output peaks sharply at the cut-off point resembling a sharp point at which the filter is said to be “underdamped”.

Then somewhere in between, ζ=0 and ζ=2.0, there must be a point where the frequency response is of the correct value, and there is. This is when the filter is “critically damped” and occurs when ζ=0.7071.

One final note, when the amount of feedback reaches 4 or more, the filter begins to oscillate by itself at the cut-off frequency point due to the resonance effects, changing the filter into an oscillator. This effect is called self oscillation. Then for a low pass second order filter, both Q and ζ play a critical role.

We can see from the normalised frequency response curves above for a 1st order filter (red line) that the pass band gain stays flat and level (called maximally flat) until the frequency response reaches the cut-off frequency point when: $f=f_r$ and the gain of the filter reduces past its corner frequency at 1/√2, or 0.7071 of its maximum value. This point is generally referred to as the filters -3dB point and for a first order low pass filter the damping factor will be equal to one, (ζ=1).

However, this -3dB cut-off point will be at a different frequency position for second order filters because of the steeper -40dB/decade roll-off rate (blue line). In other words, the corner frequency, f_r changes position as the order of the filter increases. Then to bring the second order filters -3dB point back to the same position as the 1st order filter’s, we need to add a small amount of gain to the filter.

So for a Butterworth second order low pass filter design the amount of gain would be: **1.586**, for a Bessel second order filter design: **1.268**, and for a Chebyshev low pass design: **1.234**.

Second Order Filter Example No1

A **Second Order Low Pass Filter** is to be design around a non-inverting op-amp with equal resistor and capacitor values in its cut-off frequency determining circuit. If the filters characteristics are given as: **Q=5**, and **ƒc=159Hz**, design a suitable low pass filter and draw its frequency response.

Characteristics given: R1=R2, C1=C2, Q=5 and f_c=159Hz

From the circuit above we know that for equal resistances and capacitances, the cut-off frequency point, f_c is given as:

$$f_C = \frac{1}{2\pi RC}$$

Choosing a suitable value of say, 10kΩ’s for the resistors, the resulting capacitor value is calculated as:

$$f_C = \frac{1}{2\pi RC} \qquad \therefore\ C = \frac{1}{2\pi R f_C}$$

$$C = \frac{1}{2\pi R f_C} = \frac{1}{2\pi \times 10k\Omega \times 159Hz}$$

$$\therefore\ C = 100nF \text{ or } 0.1\mu F$$

Then for a cut-off corner frequency of **159Hz**, R=10kΩ and C=0.1uF.

with a value of **Q=5**, the filters gain, A is calculated as:

$$Q = 5, \quad \text{and} \quad A = 3 - \frac{1}{Q}$$

$$\therefore\ A = 3 - \frac{1}{5} = 3 - 0.2 = 2.8$$

We know from above that the gain of a non-inverting op-amp is given as:

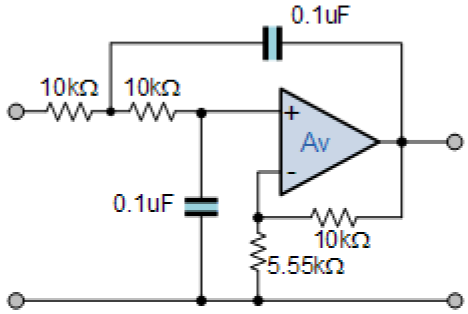
$$A = \frac{Vout}{Vin} = \left(1 + \frac{R_A}{R_B}\right) = 2.8$$

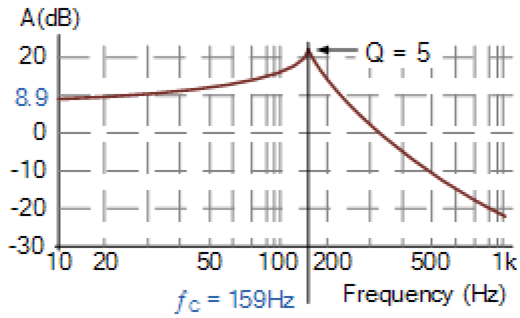
$$\text{Hence:} \quad \frac{R_A}{R_B} = 1.8$$

$$\text{If } R_A = 10k\Omega, \quad \text{then } R_B = 5.55k\Omega$$

Therefore the final circuit for the second order low pass filter is given as:

Second Order Low Pass Filter Circuit





We can see that the peakiness of the frequency response curve is quite sharp at the cut-off frequency due to the high quality factor value, Q=5. At this point the gain of the filter is given as: Q×A=14, or about **+23dB**, a big difference from the calculated value of 2.8, (+8.9dB).

But many books, like the one on the right, tell us that the gain of the filter at the normalised cut-off frequency point, etc, etc, should be at the -3dB point. By lowering the value of Q significantly down to a value of **0.7071**, results in a gain of, A=1.586 and a frequency response which is maximally flat in the passband having an attenuation of -3dB at the cut-off point the same as for a second order butterworth filter response.

So far we have seen that **second order filters** can have their cut-off frequency point set at any desired value but can be varied away from this desired value by the damping factor, ζ. Active filter designs enable the order of the filter to range up to any value, within reason, by cascading together filter sections.

In practice when designing n^{th}-order low pass filters it is desirable to set the cut-off frequency for the first-order section (if the order of the filter is odd), and set the damping factor and corresponding gain for each of the second order sections so that the correct overall response is obtained. To make the design of low pass filters easier to achieve, values of ζ, Q and A are available in tabulated form for active filters as we will see in the Butterworth Filter tutorial. Let’s look at another example.

Second Order Filter Example No2

Design a non-inverting second order Low Pass filter which will have the following filter characteristics: Q = 1, and ƒc = 79.5Hz.

From above, the corner frequency, ƒc of the filter is given as:

$$f_C = \frac{1}{2\pi RC}$$

Choosing a suitable value of 1kΩ for the filters resistors, then the resulting capacitor values are calculated as:

$$f_C = \frac{1}{2\pi RC} \qquad \therefore\ C = \frac{1}{2\pi R f_C}$$

$$C = \frac{1}{2\pi R f_C} = \frac{1}{2\pi \times 1k\Omega \times 79.5Hz}$$

$$\therefore\ C = 2.0\mu F$$

Therefore, for a corner frequency of **79.5Hz**, or 500 rads/s, R=1kΩ and C=2.0uF.

With a value of **Q=1** given, the filters gain A is calculated as follows:

$$Q = \frac{1}{2\xi}, \qquad \therefore\ \xi = \frac{1}{2Q} = \frac{1}{2\times 1} = 0.5$$

$$\xi = 0.5 = \frac{3-A}{2}, \qquad \therefore\ A = 3-2\xi = 2$$

The voltage gain for a non-inverting op-amp circuit was given previously as:

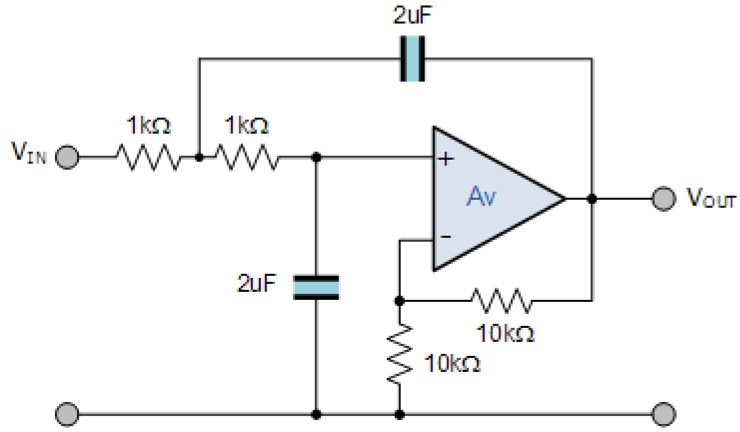
$$A = \frac{Vout}{Vin} = \left(1+\frac{R_A}{R_B}\right) = 2$$

$$\text{Hence:} \quad \frac{R_A}{R_B} = 1$$

$$\text{If } R_A = 10k\Omega, \quad \text{then } R_B = 10k\Omega$$

Therefore the second order low pass filter circuit which has a Q of 1, and a corner frequency of 79.5Hz is given as:

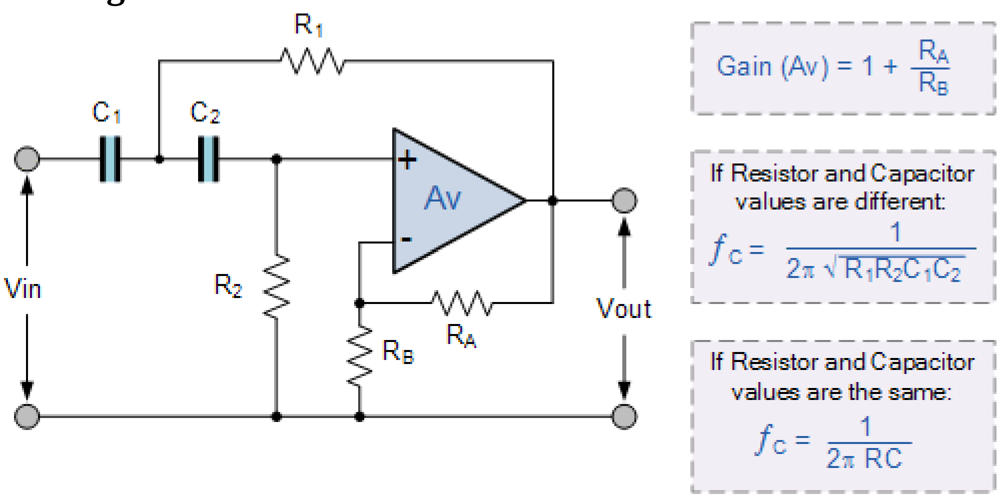
Low Pass Filter Circuit



Second Order High Pass Filter

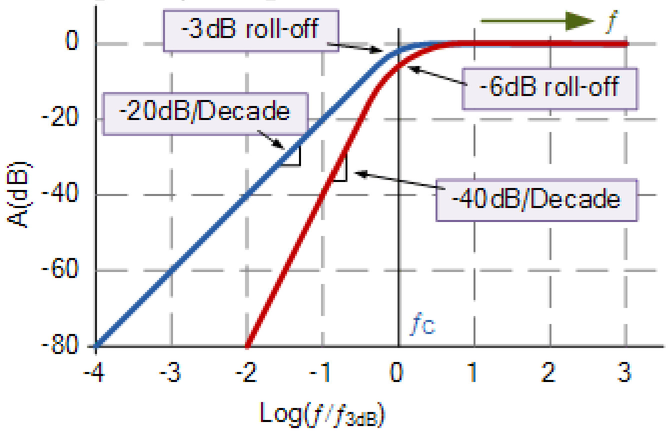
There is very little difference between the second order low pass filter configuration and the second order high pass filter configuration, the only thing that has changed is the position of the resistors and capacitors as shown.

Second Order High Pass Filter



Since second order high pass and low pass filters are the same circuits except that the positions of the resistors and capacitors are interchanged, the design and frequency scaling procedures for the high pass filter are exactly the same as those for the previous low pass filter. Then the bode plot for a 2nd order high pass filter is therefore given as:

Normalised High Pass Frequency Response



As with the previous low pass filter, the steepness of the roll-off in the stop band is -40dB/decade.

In the above two circuits, the value of the op-amp voltage gain, (Av) is set by the amplifiers feedback network. This only sets the gain for frequencies well within the pass band of the filter. We can choose to amplify the output and set this gain value by whatever amount is suitable for our purpose and define this gain as a constant, K.

2nd order Sallen-Key filters are also referred to as positive feedback filters since the output feeds back into the positive terminal of the op-amp. This type of active filter design is popular because it requires only a single op-amp, thus making it relatively inexpensive.

https://www.electronics-tutorials.ws/filter/filter_8.html

Butterworth Filter Design

In the previous filter tutorials we looked at simple first-order type low and high pass filters that contain only one single resistor and a single reactive component (a capacitor) within their RC filter circuit design.

In applications that use filters to shape the frequency spectrum of a signal such as in communications or control systems, the shape or width of the roll-off also called the "transition band", for a simple first-order filter may be too long or wide and so active filters designed with more than one "order" are required. These types of filters are commonly known as "High-order" or "n^{th}-order" filters.

The complexity or filter type is defined by the filters "order", and which is dependant upon the number of reactive components such as capacitors or inductors within its design. We also know that the rate of roll-off and therefore the width of the transition band, depends upon the order number of the filter and that for a simple first-order filter it has a standard roll-off rate of 20dB/decade or 6dB/octave.

Then, for a filter that has an n^{th} number order, it will have a subsequent roll-off rate of 20n dB/decade or 6n dB/octave. So a first-order filter has a roll-off rate of 20dB/decade (6dB/octave), a second-order filter has a roll-off rate of 40dB/decade (12dB/octave), and a fourth-order filter has a roll-off rate of 80dB/decade (24dB/octave), etc, etc.

High-order filters, such as third, fourth, and fifth-order are usually formed by cascading together single first-order and second-order filters.

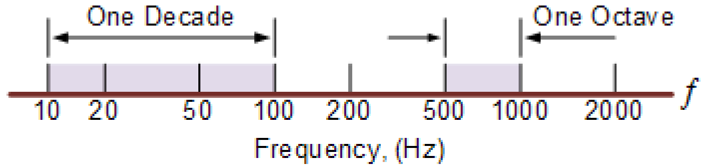
For example, two second-order low pass filters can be cascaded together to produce a fourth-order low pass filter, and so on. Although there is no limit to the order of the filter that can be formed, as the order increases so does its size and cost, also its accuracy declines.

Decades and Octaves

One final comment about *Decades* and *Octaves*. On the frequency scale, a **Decade** is a tenfold increase (multiply by 10) or tenfold decrease (divide by 10). For example, 2 to 20Hz represents one decade, whereas 50 to 5000Hz represents two decades (50 to 500Hz and then 500 to 5000Hz).

An **Octave** is a doubling (multiply by 2) or halving (divide by 2) of the frequency scale. For example, 10 to 20Hz represents one octave, while 2 to 16Hz is three octaves (2 to 4, 4 to 8 and finally 8 to 16Hz) doubling the frequency each time. Either way, *Logarithmic* scales are used extensively in the frequency domain to denote a frequency value when working with amplifiers and filters so it is important to understand them.

Logarithmic Frequency Scale



Since the frequency determining resistors are all equal, and as are the frequency determining capacitors, the cut-off or corner frequency (f_C) for either a first, second, third or even a fourth-order filter must also be equal and is found by using our now old familiar equation:

$$fc = \frac{1}{2\pi RC}\ Hz$$

As with the first and second-order filters, the third and fourth-order high pass filters are formed by simply interchanging the positions of the frequency determining components (resistors and capacitors) in the equivalent low pass filter. High-order filters can be designed by following the procedures we saw previously in the Low Pass filter and High Pass filter tutorials. However, the overall gain of high-order filters is **fixed** because all the frequency determining components are equal.

Filter Approximations

So far we have looked at a low and high pass first-order filter circuits, their resultant frequency and phase responses. An ideal filter would give us specifications of maximum pass band gain and flatness, minimum stop band attenuation and also a very steep pass band to stop band roll-off (the transition band) and it is therefore apparent that a large number of network responses would satisfy these requirements.

Not surprisingly then that there are a number of “approximation functions” in linear analogue filter design that use a mathematical approach to best approximate the transfer function we require for the filters design.

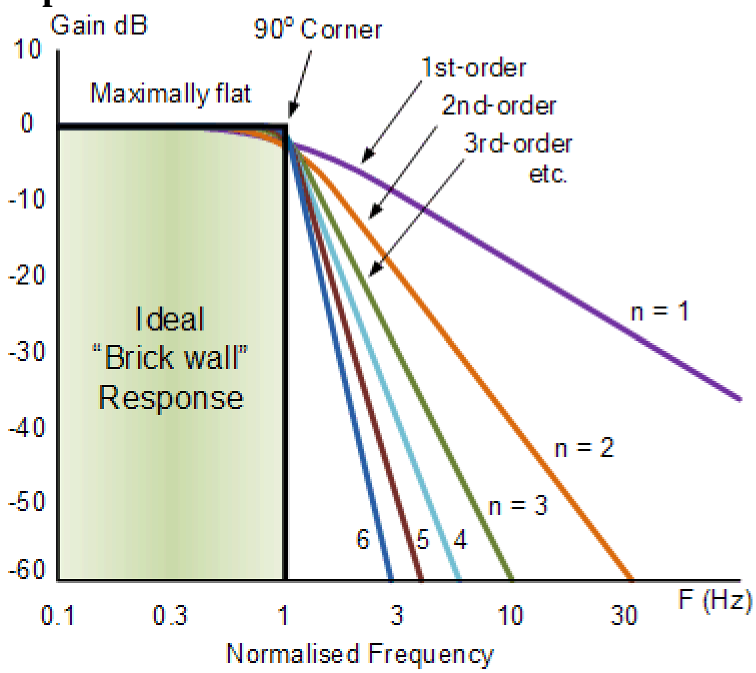
Such designs are known as **Elliptical**, **Butterworth**, **Chebyshev**, **Bessel**, **Cauer** as well as many others. Of these five “classic” linear analogue filter approximation functions only the **Butterworth Filter** and especially the *low pass Butterworth filter* design will be considered here as its the most commonly used function.

Low Pass Butterworth Filter Design

The frequency response of the **Butterworth Filter** approximation function is also often referred to as “maximally flat” (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 0.707.

However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter, and the standard Butterworth approximations, for different filter orders are given below.

Ideal Frequency Response for a Butterworth Filter



Note that the higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal "brick wall" response.

In practice however, Butterworth's ideal frequency response is unattainable as it produces excessive passband ripple.

Where the generalised equation representing a "nth" Order Butterworth filter, the frequency response is given as:

$$H_{(j\omega)} = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2n}}}$$

Where: n represents the filter order, Omega ω is equal to $2\pi f$ and Epsilon ε is the maximum pass band gain, (A_{max}). If A_{max} is defined at a frequency equal to the cut-off -3dB corner point (fc), ε will then be equal to one and therefore ε^2 will also be one. However, if you now wish to define A_{max} at a different voltage gain value, for example 1dB, or 1.1220 (1dB = $20*\log A_{max}$) then the new value of epsilon, ε is found by:

$$H_1 = \frac{H_0}{\sqrt{1+\varepsilon^2}}$$

- Where:
- H_0 = the Maximum Pass band Gain, A_{max}.
- H_1 = the Minimum Pass band Gain.

Transpose the equation to give:

$$\frac{H_0}{H_1} = 1.1220 = \sqrt{1+\varepsilon^2} \quad gives \quad \varepsilon = 0.5088$$

The **Frequency Response** of a filter can be defined mathematically by its **Transfer Function** with the standard Voltage Transfer Function H(jω) written as:

$$H(j\omega) = \left[\frac{Vout(j\omega)}{Vin(j\omega)}\right]$$

- Where:
- Vout = the output signal voltage.
- Vin= the input signal voltage.
- j= to the square root of -1 (√-1)
- ω= the radian frequency ($2\pi f$)

Note: (jω) can also be written as (s) to denote the **S-domain.** and the resultant transfer function for a second-order low pass filter is given as:

$$H(s) = \frac{Vout}{Vin} = \frac{1}{S^2 + S + 1}$$

Normalised Low Pass Butterworth Filter Polynomials

To help in the design of his low pass filters, Butterworth produced standard tables of normalised second-order low pass polynomials given the values of coefficient that correspond to a cut-off corner frequency of 1 radian/sec.

n	Normalised Denominator Polynomials in Factored Form
1	$(1+s)$
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$
6	$(1+0.518s+s^2)(1+1.414s+s^2)(1+1.932s+s^2)$
7	$(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)$
8	$(1+0.390s+s^2)(1+1.111s+s^2)(1+1.663s+s^2)(1+1.962s+s^2)$

9	$(1+s)(1+0.347s+s^2)(1+s+s^2)(1+1.532s+s^2)(1+1.879s+s^2)$
10	$(1+0.313s+s^2)(1+0.908s+s^2)(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$

Filter Design – Butterworth Low Pass

Find the order of an active low pass Butterworth filter whose specifications are given as: A_{max} = 0.5dB at a pass band frequency (ωp) of 200 radian/sec (31.8Hz), and A_{min} = -20dB at a stop band frequency (ωs) of 800 radian/sec. Also design a suitable Butterworth filter circuit to match these requirements.

Firstly, the maximum pass band gain A_{max} = 0.5dB which is equal to a gain of **1.0593**, remember that: 0.5dB = 20*log(A) at a frequency (ωp) of 200 rads/s, so the value of epsilon ε is found by:

$$1.0593 = \sqrt{1 + \varepsilon^2}$$

$$\therefore \ \varepsilon = 0.3495 \ \ and \ \ \varepsilon^2 = 0.1221$$

Secondly, the minimum stop band gain A_{min} = -20dB which is equal to a gain of **10** (-20dB = 20*log(A)) at a stop band frequency (ωs) of 800 rads/s or 127.3Hz.

Substituting the values into the general equation for a Butterworth filters frequency response gives us the following:

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_S}{\omega_p}\right)^{2n}}}$$

$$\frac{1}{10} = \frac{1}{\sqrt{1 + 0.1221 \left(\frac{800}{200}\right)^{2n}}}$$

$$(10)^2 = 1 + 0.1221 \; x \; 4^{2n}$$

$$\therefore \; 100 - 1 = 0.1221 \; x \; 4^{2n}$$

$$4^{2n} = \frac{99}{0.1221} = 810.811$$

$$4^n = \sqrt{810.811} = 28.475$$

$$\therefore \; n = \frac{\log 28.475}{\log 4} = 2.42$$

Since n must always be an integer (whole number) then the next highest value to 2.42 is n = 3, therefore a "**a third-order filter is required"** and to produce a third-order **Butterworth filter**, a second-order filter stage cascaded together with a first-order filter stage is required.

From the normalised low pass Butterworth Polynomials table above, the coefficient for a third-order filter is given as $(1+s)(1+s+s^2)$ and this gives us a gain of 3-A = 1, or A = 2. As A = 1 + (Rf/R1), choosing a value for both the feedback resistor Rf and resistor R1 gives us values of 1kΩ and 1kΩ respectively as: (1kΩ/1kΩ) + 1 = 2.

We know that the cut-off corner frequency, the -3dB point (ω_o) can be found using the formula 1/CR, but we need to find ω_o from the pass band frequency ω_p then,

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$3dB \;=\; 1.414 \; at \;\; \omega = \omega_O$$

$$\frac{1}{1.414} \;=\; \frac{1}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$2 \;=\; 1 \;+\; \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}$$

$$\therefore\; 1 \;=\; \varepsilon \left(\frac{\omega_O}{\omega_P}\right)^{n}$$

$$\omega_O{}^{n} \;=\; \frac{\omega_P{}^{n}}{\varepsilon}$$

$$\omega_O{}^{3} \;=\; \frac{200^3}{0.3495}$$

$$\omega_O{}^{3} \;=\; 22.889 \; x \; 10^6$$

$$\therefore\; \omega_O \;=\; 283.93 \;\simeq\; 284 \, rads/s$$

So, the cut-off corner frequency is given as 284 rads/s or 45.2Hz, (284/2π) and using the familiar formula 1/CR we can find the values of the resistors and capacitors for our third-order circuit.

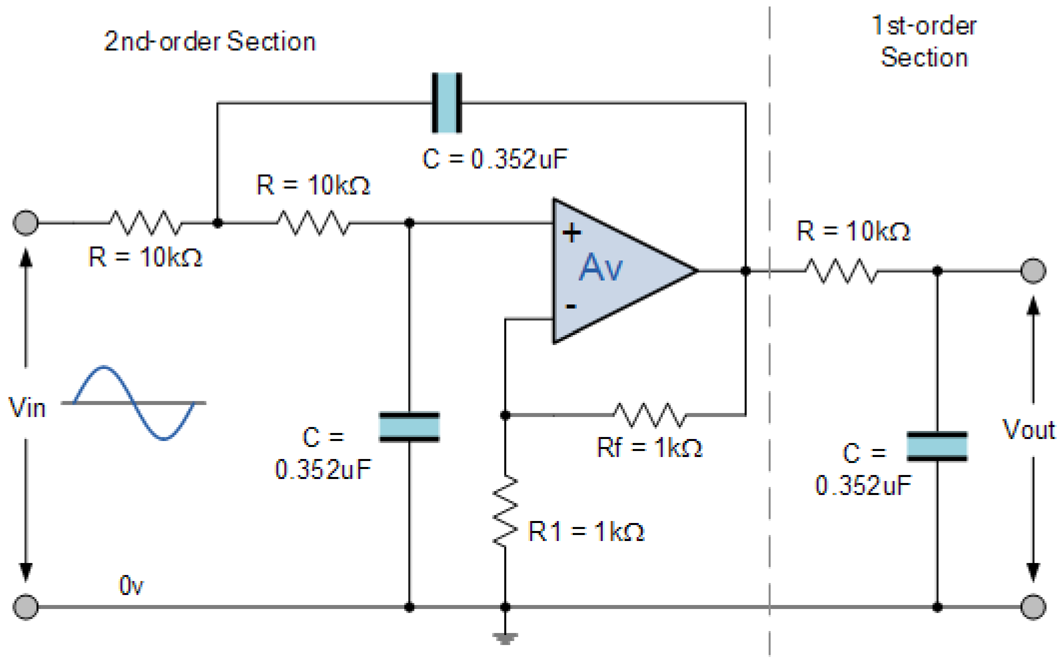
$$284 rads/s \;=\; \frac{1}{CR} \quad use\; a\; value\; of\; R = 10k\Omega$$

$$\therefore\; Capacitor\;\; C \;=\; \frac{1}{284 \, x \, 10{,}000} \;=\; 0.352 uF$$

Note that the nearest preferred value to 0.352uF would be 0.36uF, or 360nF.

Third-order Butterworth Low Pass Filter

and finally our circuit of the third-order low pass **Butterworth Filter** with a cut-off corner frequency of 284 rads/s or 45.2Hz, a maximum pass band gain of 0.5dB and a minimum stop band gain of 20dB is constructed as follows.



So for our 3rd-order Butterworth Low Pass Filter with a corner frequency of 45.2Hz, C = 360nF and R = 10kΩ.

https://www.electronics-tutorials.ws/filter/state-variable-filter.html

State Variable Filter

The **state variable filter** is a type of multiple-feedback filter circuit that can produce all three filter responses, *Low Pass*, *High Pass* and *Band Pass* simultaneously from the same single active filter design.

State variable filters use three (or more) operational amplifier circuits (the active element) cascaded together to produce the individual filter outputs but if required an additional summing amplifier can also be added to produce a fourth *Notch filter* output response as well.

State variable filters are second-order RC active filters consisting of two identical op-amp integrators with each one acting as a first-order, single-pole low pass filter, a summing amplifier around which we can set the filters gain and its damping feedback network. The output signals from all three op-amp stages are fed back to the input allowing us to define the state of the circuit.

One of the main advantages of a state variable filter design is that all three of the filters main parameters, Gain (A), corner frequency, f_C and the filters Q can be adjusted or set independently without affecting the filters performance.

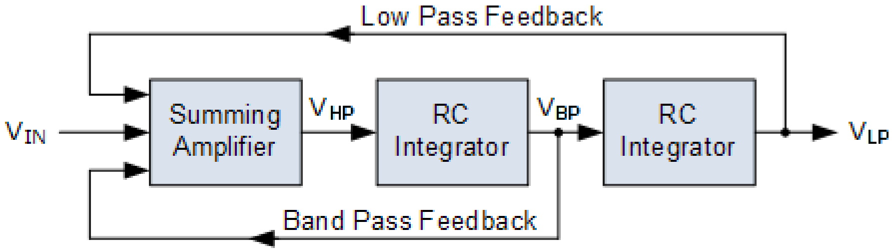
In fact if designed correctly, the -3dB corner frequency, (fc) point for both the low pass amplitude response and the high pass amplitude response should be identical to the center frequency point of the band pass stage. That is $f_{LP(-3dB)}$ equals $f_{HP(-3dB)}$ which equals $f_{BP(center)}$. Also the damping factor, (ζ) for the band pass filter response should be equal to 1/Q as Q will be set at -3dB, (0.7071).

Although the filter provides low pass (LP), high pass (HP) and band pass (BP) outputs the main application of this type of filter circuit is as a state variable band pass filter design with the center frequency set by the two RC integers.

While we have seen before that a band pass filters characteristics can be obtained by simply cascading together a low pass filter with a high pass filter, state variable band pass filters have the advantage that they can be tuned to be highly selective (high Q) offering high gains at the center frequency point.

There are several state variable filter designs available all based on the standard filter design with both inverting and non-inverting variations available. However, the basic filter design will be the same for both variations as shown in the following block diagram representation.

State Variable Filter Block Diagram

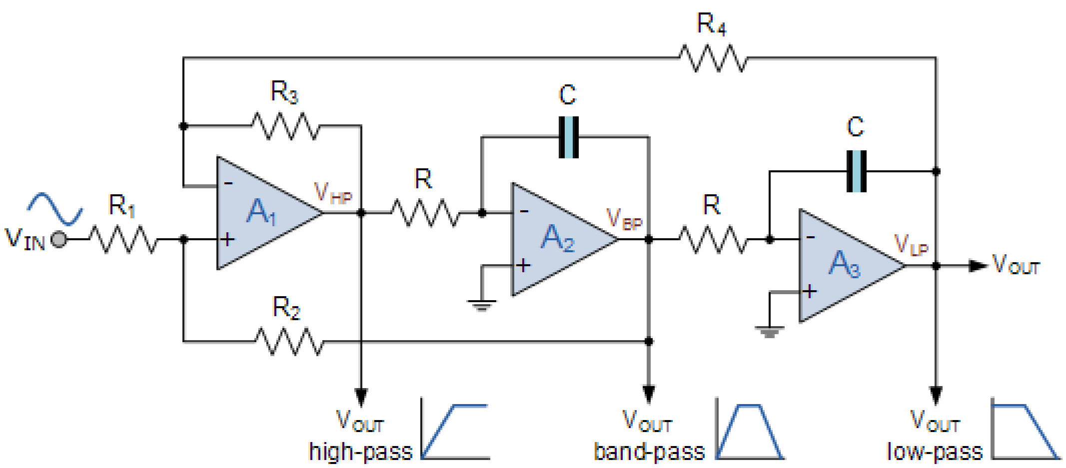


Then we can see from the basic block diagram above that the state variable filter has three possible outputs, V_{HP}, V_{BP} and V_{LP} with one each from the three op-amps. A notch filter response can also be realized by the addition of a fourth op-amp.

With a constant input voltage, V_{IN} the output from the summing amplifier produces a high pass response which also becomes the input of the first RC Integrator. The output from this integrator produces a band pass response which becomes the input of the second RC Integrator producing a low pass response at its output. As a result, separate transfer functions for each individual output with respect to the input voltage can be found.

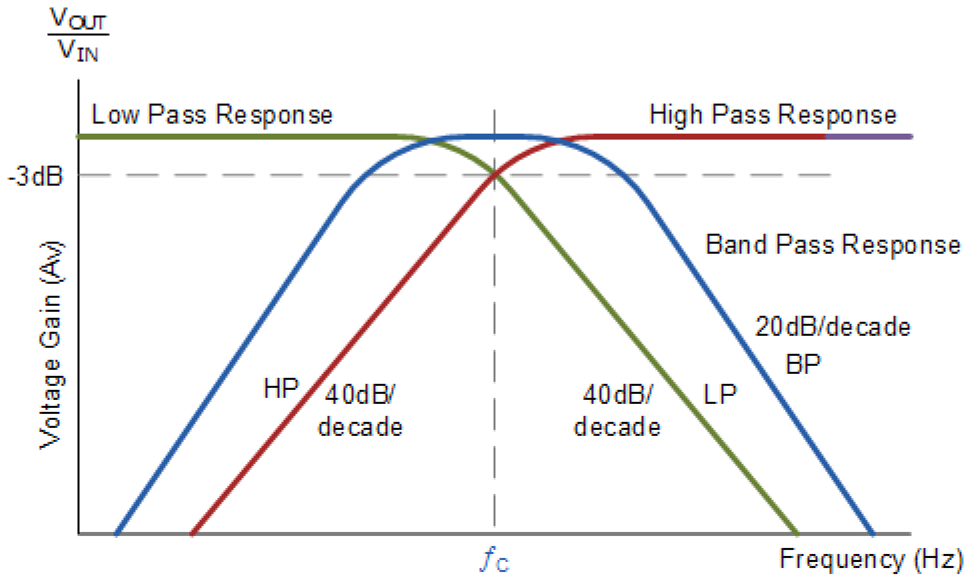
The basic non-inverting state variable filter design is therefore given as:

State Variable Filter Circuit



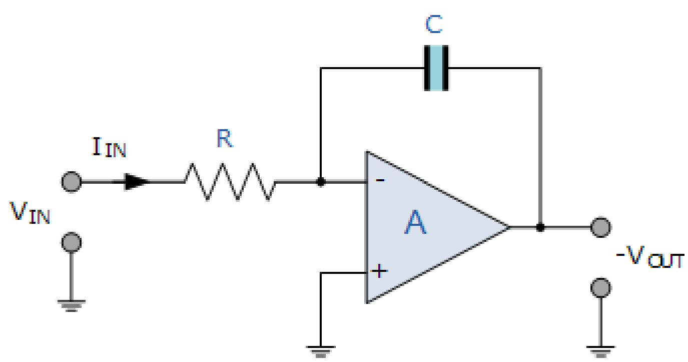
and the amplitude response of the three outputs from the state variable filter will look like:

Normalised Response of a State Variable Filter



One of the main design elements of a state variable filter is its use of two op-amp integrators. As we saw in the Integrator tutorial, op-amp integrators use a frequency dependant impedance in the form of a capacitor within their feedback loop. As a capacitor is used the output voltage is proportional to the integral of the input voltage as shown.

Op-amp Integrator Circuit



$$V_{OUT} = -\frac{1}{RC}\int_0^t V_{IN}\,dt$$

To simplify the math's a little, this can also be re-written in the frequency domain as:

$$V_{OUT} = -\frac{1}{2\pi f_c\, RC} V_{IN}$$

The output voltage Vout is a constant 1/RC times the integral of the input voltage Vin with respect to time. Integrators produce a phase lag with the minus sign (–) indicating a 180° phase shift because the input signal is connected directly to the inverting input terminal of the op-amp.

In the case of op-amp A2 above, its input signal is connected to the output of the proceeding op-amp, A1 so its input is given as V_{HP} and its output as V_{BP}. Then from above, the expression for op-amp, A2 can be written as:

$$V_{BP} = -\frac{1}{2\pi f_C RC} V_{HP}$$

Then by rearranging this formula we can find the transfer function of the inverting integrator, A2

Op-amp A2 Transfer Function

$$\frac{V_{OUT}}{V_{IN}} = \frac{V_{BP}}{V_{HP}} = -\frac{1}{2\pi f_C RC}$$

Exactly the same assumption can be made as above to find the transfer function for the other op-amp integrator, A3

Op-amp A3 Transfer Function

$$\frac{V_{OUT}}{V_{IN}} = \frac{V_{LP}}{V_{BP}} = -\frac{1}{2\pi f_C RC}$$

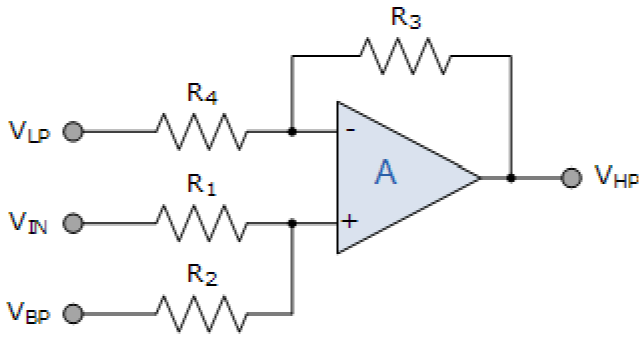
So the two op-amp integrators, A2 and A3 are connected together in cascade, so the output from the first (V_{BP}) becomes the input of the second. So we can see that the band pass response is created by integrating the high pass response and the low pass response is created by integrating the band pass response. Therefore the transfer function between V_{HP} and V_{LP} is given as:

$$\frac{V_{LP}}{V_{HP}} = -\frac{1}{2\pi f_C RC} \times -\frac{1}{2\pi f_C RC} = \frac{1}{(2\pi f_C RC)^2}$$

Note that each integrator stage provides an inverted output but the summed output will be positive since they are inverting integrators. If exactly the same values for R and C are used so that the two circuits have the same integrator time constant, the two amplifier circuits can be regarded with one single integrator circuit having a corner frequency, f_C.

As well as the two integrator circuits, the filter also has a differential summing amplifier providing a weighted summation of its inputs. The advantage here is that the inputs to the summing amplifier, A1 combines oscillatory feedback, damping and input signals to the filter as all three outputs are fed back to the summing inputs.

Amplifier Summing Circuit



Operational amplifier, A1 is connected as an adder–subtracter circuit. That is it sums the input signal, V_{IN} with the V_{BP} output of op-amp A2 and the subtracts from it the V_{LP} output of op-amp A3, thus:

$$i_1 = \frac{V_{IN} - (+V)}{R_1} + \frac{(+V) - V_{BP}}{R_2} = 0$$

$$\text{Then: } V_{IN} R_2 = +V(R_1 + R_2) - V_{BP} R_1$$

$$\therefore \quad +V = \frac{V_{IN} R_2 + V_{BP} R_1}{(R_1 + R_2)}$$

and

$$i_2 = \frac{V_{HP} - (-V)}{R_3} + \frac{(-V) - V_{LP}}{R_4} = 0$$

$$\text{Then: } V_{HP} R_4 = -V(R_3 + R_4) - V_{LP} R_3$$

$$\therefore \quad -V = \frac{V_{LP} R_3 + V_{HP} R_4}{(R_3 + R_4)}$$

As the differential inputs, +V and -V of an operational amplifier are the same, that is: +V–-V, we can rearrange the two expressions above to find the transfer function for the output of A1, the high pass output.

$$\frac{V_{IN}R_2 + V_{BP}R_1}{(R_1 + R_2)} = \frac{V_{LP}R_3 + V_{HP}R_4}{(R_3 + R_4)}$$

Therefore:

$$(V_{IN}R_2 + V_{BP}R_1)(R_3 + R_4) = (V_{LP}R_3 + V_{HP}R_4)(R_1 + R_2)$$

$$V_{IN}R_2(R_3 + R_4) + V_{BP}R_1(R_3 + R_4) = V_{LP}R_3(R_1 + R_2) + V_{HP}R_4(R_1 + R_2)$$

Rearrange for output:

$$V_{HP} = V_{IN}\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + V_{BP}\frac{R_1(R_3 + R_4)}{R_4(R_1 + R_2)} - V_{LP}\frac{R_3(R_1 + R_2)}{R_4(R_1 + R_2)}$$

$$\therefore\ V_{HP} = V_{IN}\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + V_{BP}\frac{R_1(R_3 + R_4)}{R_4(R_1 + R_2)} - V_{LP}\frac{R_3}{R_4}$$

We know from above, that V_{BP} and V_{LP} are the outputs from the two integrators, A2 and A3 respectively. By substituting the integrator equations for A2 and A3 into the above equation, we get the transfer function of the state variable filter to be:

State Variable Filter Transfer Function

$$\frac{V_{OUT}}{V_{IN}} = \frac{V_{LP}}{V_{IN}} = \frac{\dfrac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} \times \left(\dfrac{1}{RC}\right)}{\dfrac{R_3}{R_4\,RC} + \left(\dfrac{R_1(R_3 + R_4)}{R_4(R_1 + R_2)} \times \dfrac{1}{2\pi f\,RC}\right) + \left(\dfrac{1}{2\pi f\,RC}\right)^2}$$

We said previously that a **State Variable Filter** produces three filter responses, *Low Pass*, *High Pass* and *Band Pass* and that the band pass response is that of a very narrow high Q filter and this is evident in the SVF's transfer function above as it resembles that of a standard second-order response.

Normalised 2nd-order Transfer Function

$$\frac{V_{OUT}}{V_{IN}} = \frac{A_o\left(\dfrac{f}{f_o}\right)}{\left[1 + 2\zeta\dfrac{f}{f_o} + \left(\dfrac{f}{f_o}\right)^2\right]}$$

The Filters Corner Frequency, f_C

If we make both the integrators input resistors and feedback capacitors the same, then the state variable filters corner frequency can be easily tuned without affective its overall Q. Likewise, the value of Q can be varied without altering the corner frequency. Then the corner frequency is given as:

State Variable Filter Corner Frequency

$$2\pi f_C = \sqrt{\frac{R_3}{R_4 (RC)^2}} \quad \therefore f_C = \sqrt{\frac{R_3}{R_4 (2\pi RC)^2}}$$

If we make feedback resistors R3 and R4 the same values, then the corner frequency of each filter output from the state variable filter simply becomes:

$$f_{C(HP)} = f_{C(BP)} = f_{C(LP)} = \frac{1}{2\pi RC}$$

Then tuning of the state variable corner frequency is accomplished simply by varying either the tuning resistor, R or the capacitor, C.

State variable filters are characterised not only by their individual output responses, but also by the filters "Q", Quality factor. Q relates to the "sharpness" of the band pass filters amplitude response curve and the higher the Q, the higher or sharper the output response resulting in a filter that is highly selective.

For a band pass filter, Q is defined as the center frequency divided by the filters -3dB bandwidth, that is $Q = f_C/BW$. But Q can also be found from the denominator of the above transfer function as it is the reciprocal of the damping factor (ζ). Then Q is given as:

The Q Factor of a State Variable Filter

$$Q = \frac{f_C}{BW} = \frac{1}{2\zeta} = \frac{R_1 (R_3 + R_4)}{R_4 (R_1 + R_2)} \sqrt{\frac{R_3}{R_4} \times \frac{RC}{RC}}$$

Again, if resistors R3 and R4 are equal and both integrator components R and C are equal, then the final square root expression would reduce to: √1 or simply 1 as the numerator and denominator cancel each other out.

State Variable Filter Example No1

Design a *State Variable Filter* which has a corner (natural undamped) frequency, f_C of 1kHz and a quality factor, Q of 10. Assume both the frequency determining resistors and capacitors are equal. Determine the filters DC gain and draw the resulting circuit and Bode plot.

We said above that if both the resistor, R and the feedback capacitor, C of the two integrator circuits are the same values, that is R=R and C=C, the cut-off or corner frequency point for the filter is given simply as:

Filter's Corner Frequency

$$fc = \frac{1}{2\pi RC}\ Hz$$

We can choose a value for either the resistor, or the capacitor to find the value of the other. If we assume a suitable value of 10nF for the capacitor then the value of the resistor will be:

$$R = \frac{1}{2\pi f_C C} = \frac{1}{2\pi\times 1000\times 10nF} = 15.9k\Omega$$

Giving C=10nF and R=15.9kΩ, or 16kΩ to the nearest preferred value.

The value of Q is given as **10**. This relates to the filters damping coefficient as:

$$Q = 10 = \frac{1}{2\zeta} \qquad \therefore\ 2\zeta = \frac{1}{10} = 0.1$$

In the state variable transfer function above, the 2ζ part is replaced by the resistor combination giving:

$$2\zeta = \frac{1}{Q} = \frac{1}{10} = \frac{R_1(R_3+R_4)}{R_4(R_1+R_2)}\sqrt{\frac{R_3}{R_4}\times\frac{RC}{RC}} = 0.1$$

We know from above that R=16kΩ and C=10nF, but if we assume that the two feedback resistors, R3 and R4 are the same and equal to 10kΩ, then the above equation reduces down to:

$$0.1 = \frac{R_1(R_3+R_4)}{R_4(R_1+R_2)} = \frac{R_1(10k\Omega+10k\Omega)}{10k\Omega(R_1+R_2)}$$

Assuming a suitable value for the input resistor, R1 of say 1kΩ, then we can find the value of R2 as follows:

$$\therefore\ R_2 = \frac{R_1(R_3+R_4)}{0.1\times R_4} - R_1$$

$$= \frac{1k\Omega(10k\Omega+10k\Omega)}{0.1\times 10k\Omega} - 1k\Omega = 19k\Omega$$

From the normalised transfer function above, the DC passband gain is defined as A_o and from the equivalent state variable filter transfer function this equates to:

The SVF Filters DC Passband Gain

$$A_o = \frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} = \frac{19k\Omega(10k\Omega+10k\Omega)}{10k\Omega(1k\Omega+19k\Omega)}$$

$$\therefore\ A_o = 1.9 = 5.57\,dB$$

Therefore the DC voltage gain of the filter is calculated at 1.9, which basically equates to R2/R3. Also the maximum gain of the filter at f_C can be calculated as: A_oxQ as follows.

The SVF Filters Maximum Gain

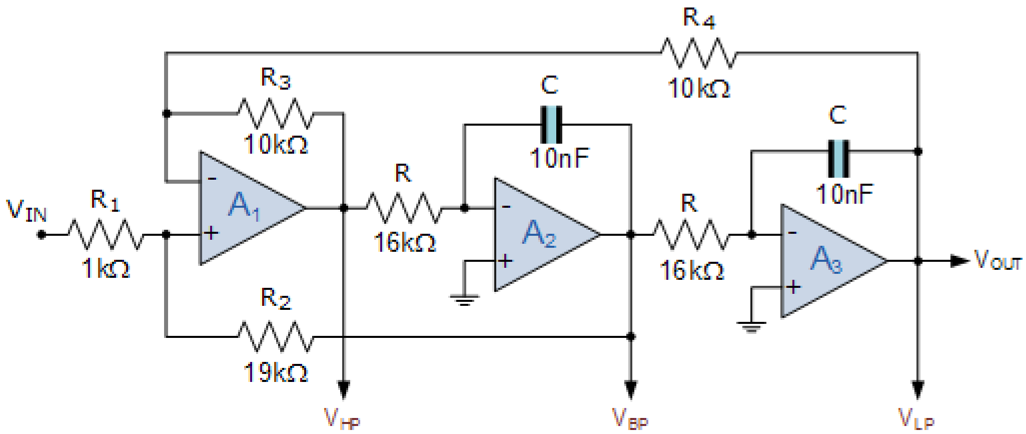
$$A_{(f_c)} = \frac{V_{OUT}}{V_{IN}} = \frac{A_o}{2\zeta} = A_o\times Q$$

$$\therefore\ A_o\times Q = 1.9\times 10 = 19 = 25.6\,dB$$

State Variable Filter Circuit

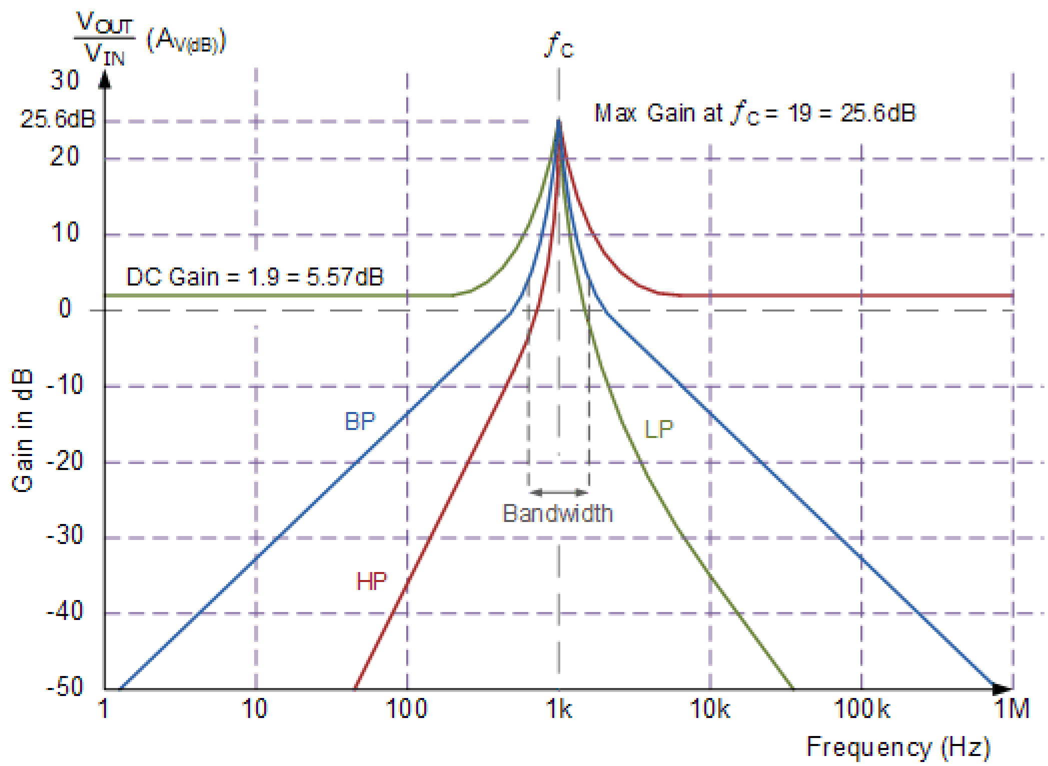
Then the design of the state variable filter circuit will be: R=16kΩ, C=10nF, R1=1kΩ, R2=19kΩ and R3=R4=10kΩ as shown.

State Variable Filter Design



We can now plot the individual output response curves for the state variable filter circuit over a range of frequencies from 1Hz to 1MHz onto a Bode Plot as shown.

State Variable Filter Bode Plot



Then we can see from the filters response curves above, that the DC gain of the filter circuit is at 5.57dB which equates to an open loop voltage gain, A_o or 1.9 as calculated above. The response also shows that the output curves peaks at a maximum voltage gain of 25.6dB at the corner frequency due to the value of **Q**. As Q also relates the band pass filters center frequency to its bandwidth, the bandwidth of the filter will therefore be: $f_o/10$=100Hz.

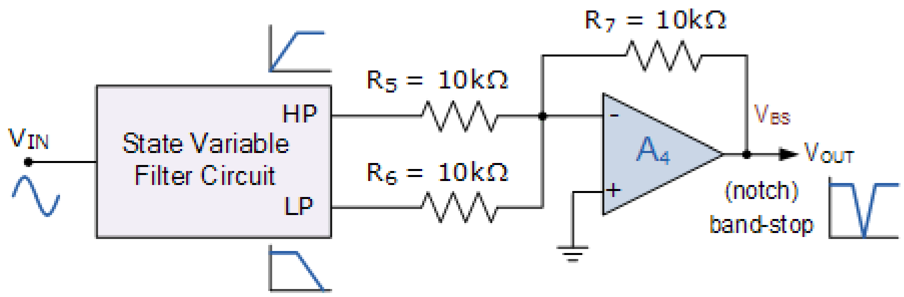
We have seen in this *state variable filter* tutorial that instead of an active filter producing one type of frequency response, we can use multiple-feedback techniques to produce all three filter responses, *Low Pass*, *High Pass* and *Band Pass* simultaneously from the same single active filter design.

But as well as the three basic filter responses, we can add an additional op-amp circuit onto the basic state variable filter design above to produce a fourth output response resembling that of a standard *Notch Filter*.

Notch Filter Design

A **notch filter** filter is basically the opposite of a band pass filter, in that it rejects or stops a specific band of frequencies. Then a notch filter is also known as a “band stop filter”. To obtain the response of a notch filter from the basic state variable filter design, we have to sum together the high pass and low pass output responses using another op-amp summing amplifier, A4 as shown.

Notch Filter Circuit



Here to keep things simple we have assumed that the two input resistors, R5 and R6 as well as the feedback resistor, R7 all have the same value of 10kΩ the same as for R3 and R4. This therefore gives the notch filter a gain of 1, unity.

The output response of the notch filter and bandpass filter are related with the center frequency of the bandpass response being equal to the point of zero response of the notch filter, and in this example will be 1kHz.

Also the bandwidth of the notch is determined by the circuits Q, exactly the same as for the pass band response. The downward peak is therefore equal to the center frequency divided by the -3dB bandwidth, that is the frequency difference between the -3dB points either side of the notch. Note that the quality factor Q has nothing to do with the actual depth of the notch.

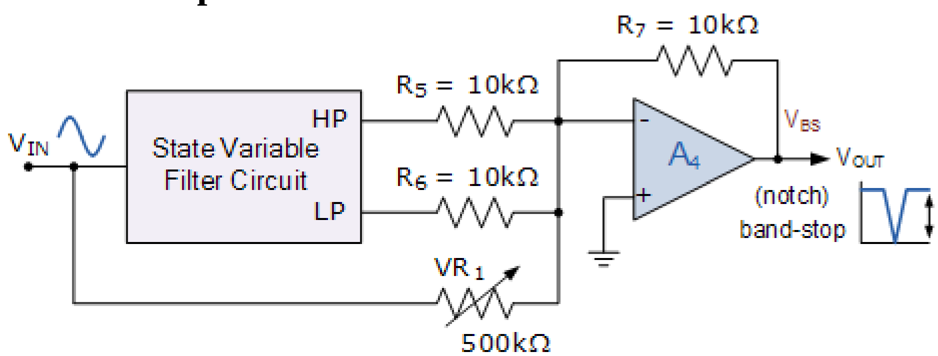
This basic notch filter (band-stop) design has only two inputs applied to its summing amplifier, the low pass output, V_{LP} and the high pass output, V_{HP}. However, there are two more signals available for us to use from the basic state variable filter circuit, the band pass output, V_{BP} and the input signal itself, V_{IN}.

If one of these two signals is also used as an input to the notch filter summing amplifier along with the low pass and high pass signals, then the depth of the notch can be controlled.

Depending upon how you wanted to control the output from the notch filter section would depend upon which one of the two available signals you would use. If it was required that the output notch changes from a negative response to a positive response at the undamped natural frequency f_o then the band pass output signal V_{BP} would be used.

Likewise, if it was required that the output notch only varies in its downward negative depth, then the input signal, V_{IN} would be used. If either one of these two additional signals was connected to the op-amp summing amplifier through a variable resistor then the depth and direction of the notch could be fully controlled. Consider the modified notch filter circuit below.

Variable Notch Filter Depth



State Variable Filter Summary

The **State Variable Filter**, (**SVF**) circuit is a second-order active RC filter design that use multiple feedback techniques to produce three different frequency response outputs, namely: *Low Pass*, *High Pass* and *Band Pass* from the same single filter. The advantage of the state variable filter over other basic filter designs is that the three main filter parameters, Gain, Q and fc can be adjusted independently.

We have also seen here that the filter is also easy to tune as the corner frequency, fc can be set and adjusted by varying either R or C without affecting the filters damping factor. However, at higher corner frequencies and larger damping factors the filter can become unstable so is best used with low Q, less than 10, and at low corner frequencies.

The basic state variable filter design uses three op-amp sections to produce its outputs, but we have also seen that with the addition of a fourth op-amp section summing the low pass and high pass sections together, a notch (band-stop) filter output response can also be achieved at the desired center frequency.

https://www.electronics-tutorials.ws/filter/band-stop-filter.html

Band Stop Filter

A band Stop Filter known also as a Notch Filter, blocks and rejects frequencies that lie between its two cut-off frequency points passes all those frequencies either side of this range

By combining a basic RC low-pass filter with a RC high-pass filter we can form a simple band-pass filter that will pass a range or band of frequencies either side of two cut-off frequency points. But we can also combine these low and high pass filter sections to produce another kind of RC filter network called a band stop filter that can block or at least severely attenuate a band of frequencies within these two cut-off frequency points.

The **Band Stop Filter**, (BSF) is another type of frequency selective circuit that functions in exactly the opposite way to the Band Pass Filter we looked at before. The band stop filter, also known as a *band reject filter*, passes all frequencies with the exception of those within a specified stop band which are greatly attenuated.

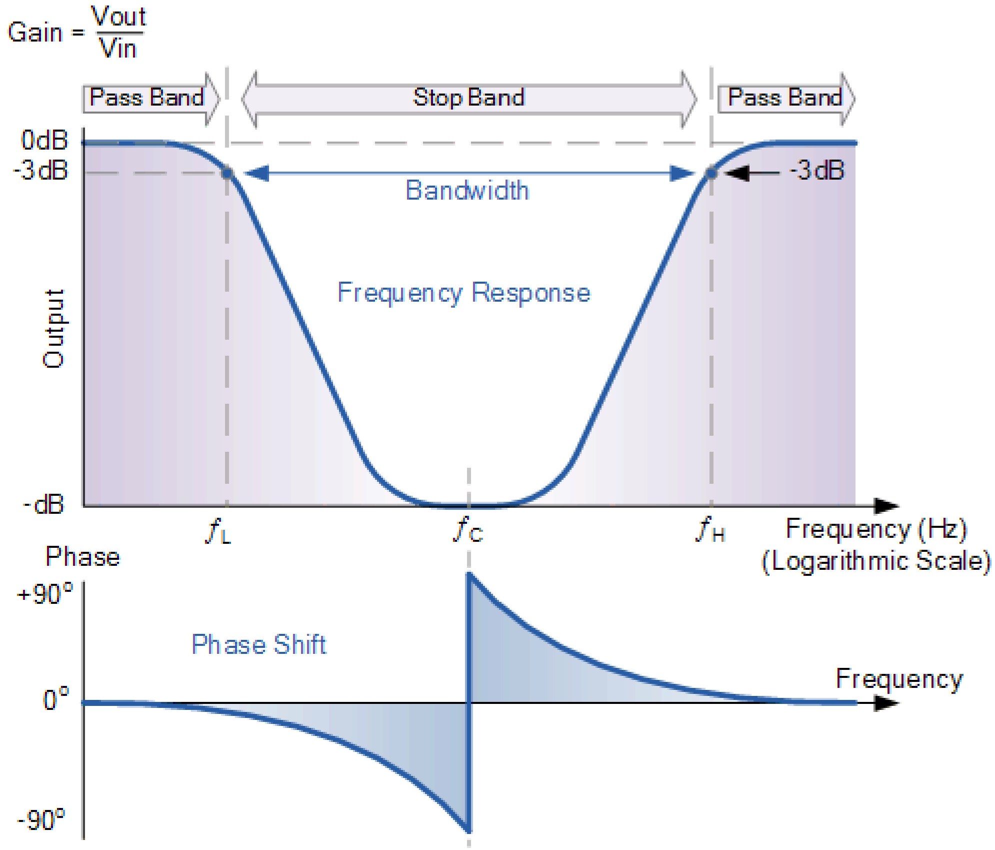
If this stop band is very narrow and highly attenuated over a few hertz, then the band stop filter is more commonly referred to as a *notch filter*, as its frequency response shows that of a deep notch with high selectivity (a steep-side curve) rather than a flattened wider band.

Also, just like the band pass filter, the band stop (band reject or notch) filter is a second-order (two-pole) filter having two cut-off frequencies, commonly known as the -3dB or half-power points producing a wide stop band bandwidth between these two -3dB points.

Then the function of a band stop filter is too pass all those frequencies from zero (DC) up to its first (lower) cut-off frequency point f_L, and pass all those frequencies above its second (upper) cut-off frequency f_H, but block or reject all those frequencies in-between. Then the filters bandwidth, BW is defined as: ($f_H - f_L$).

So for a wide-band band stop filter, the filters actual stop band lies between its lower and upper -3dB points as it attenuates, or rejects any frequency between these two cut-off frequencies. The frequency response curve of an ideal band stop filter is therefore given as:

Band Stop Filter Response

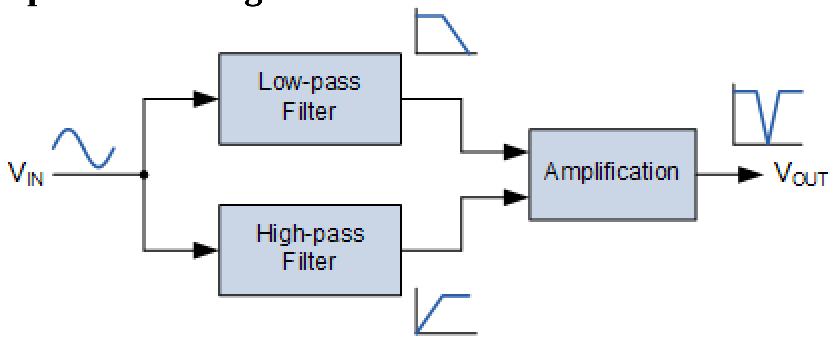


We can see from the amplitude and phase curves above for the band pass circuit, that the quantities f_L, f_H and f_C are the same as those used to describe the behaviour of the band-pass filter. This is because the band stop filter is simply an inverted or complimented form of the standard band-pass filter. In fact the definitions used for bandwidth, pass band, stop band and center frequency are the same as before, and we can use the same formulas to calculate bandwidth, BW, center frequency, f_C, and quality factor, Q.

The ideal band stop filter would have infinite attenuation in its stop band and zero attenuation in either pass band. The transition between the two pass bands and the stop band would be vertical (brick wall). There are several ways we can design a “Band Stop Filter”, and they all accomplish the same purpose.

Generally band-pass filters are constructed by combining a low pass filter (LPF) in series with a high pass filter (HPF). Band stop filters are created by combining together the low pass and high pass filter sections in a “parallel” type configuration as shown.

Typical Band Stop Filter Configuration

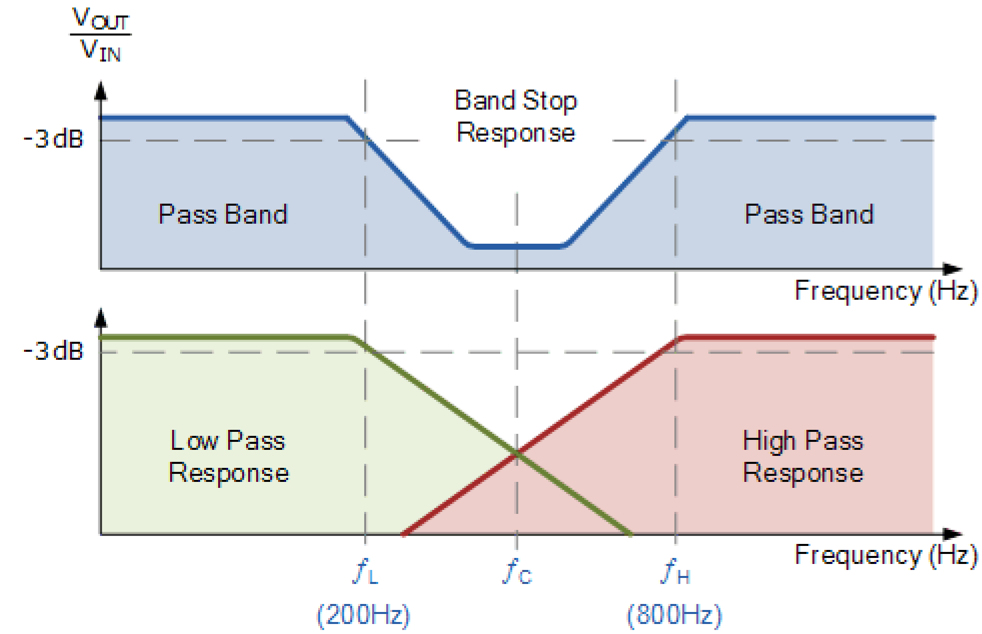


The summing of the high pass and low pass filters means that their frequency responses do not overlap, unlike the band-pass filter. This is due to the fact that their start and ending frequencies are at different frequency points. For example, suppose we have a first-order low-pass filter with a cut-off frequency, f_L of 200Hz connected in parallel with a first-order high-pass filter with a cut-off frequency, f_H of 800Hz. As the two filters are effectively connected in parallel, the input signal is applied to both filters simultaneously as shown above.

All of the input frequencies below 200Hz would be passed unattenuated to the output by the low-pass filter. Likewise, all input frequencies above 800Hz would be passed unattenuated to the output by the high-pass filter. However, and input signal frequencies in-between these two frequency cut-off points of 200Hz and 800Hz, that is f_L to f_H would be rejected by either filter forming a notch in the filters output response.

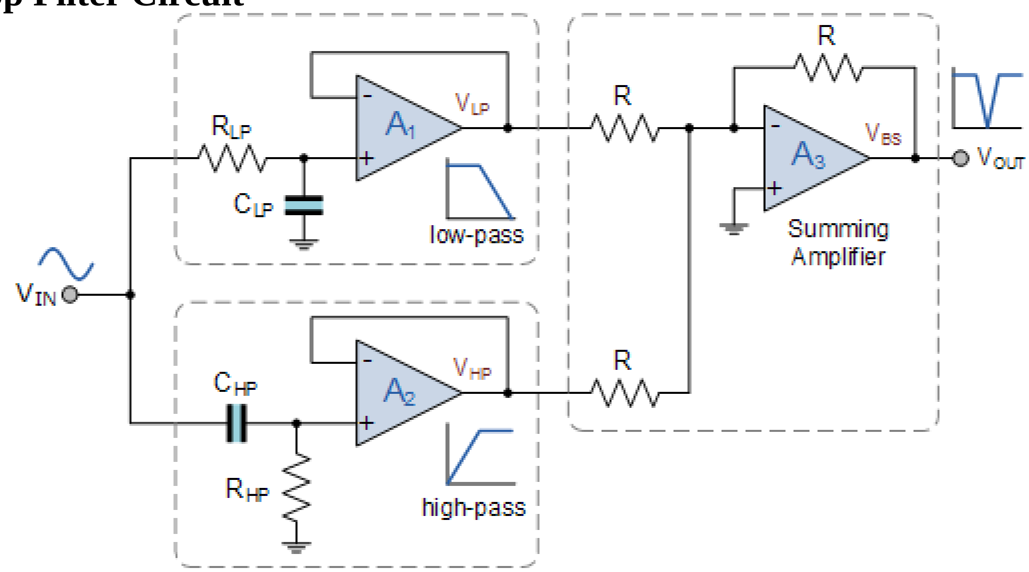
In other words a signal with a frequency of 200Hz or less and 800Hz and above would pass unaffected but a signal frequency of say 500Hz would be rejected as it is too high to be passed by the low-pass filter and too low to be passed by the high-pass filter. We can show the effect of this frequency characteristic below.

Band Stop Filter Characteristics



The transformation of this filter characteristic can be easily implemented using a single low pass and high pass filter circuits isolated from each other by non-inverting voltage follower, ($Av=1$). The output from these two filter circuits is then summed using a third operational amplifier connected as a voltage summer (adder) as shown.

Band Stop Filter Circuit



The use of operational amplifiers within the band stop filter design also allows us to introduce voltage gain into the basic filter circuit. The two non-inverting voltage followers can easily be converted into a basic non-inverting amplifier with a gain of $Av=1+Rf/Rin$ by the addition of input and feedback resistors, as seen in our non-inverting op-amp tutorial.

Also if we require a band stop filter to have its -3dB cut-off points at say, 1kHz and 10kHz and a stop band gain of -10dB in between, we can easily design a low-pass filter and a high-pass filter with these requirements and simply cascade them together to form our wide-band band-pass filter design.

Now we understand the principle behind a **Band Stop Filter**, let us design one using the previous cut-off frequency values.

Band Stop Filter Example No1

Design a basic wide-band, RC band stop filter with a lower cut-off frequency of 200Hz and a higher cut-off frequency of 800Hz. Find the geometric center frequency, -3dB bandwidth and Q of the circuit.

$$f = \frac{1}{2\pi RC}\ Hz$$

The upper and lower cut-off frequency points for a band stop filter can be found using the same formula as that for both the low and high pass filters as shown.

Assuming a capacitor, C value for both filter sections of 0.1uF, the values of the two frequency determining resistors, R_L and R_H are calculated as follows.

Low Pass Filter Section

$$f_L = \frac{1}{2\pi R_L C} = 200\text{Hz} \quad \text{and} \quad C = 0.1\text{uF}$$

$$\therefore R_L = \frac{1}{2\pi \times 200 \times 0.1 \times 10^{-6}} = 7958\Omega \text{ or } 8k\Omega$$

High Pass Filter Section

$$f_H = \frac{1}{2\pi R_H C} = 800\text{Hz} \quad \text{and} \quad C = 0.1\text{uF}$$

$$\therefore R_H = \frac{1}{2\pi \times 800 \times 0.1 \times 10^{-6}} = 1990\Omega \text{ or } 2k\Omega$$

From this we can calculate the geometric center frequency, f_C as:

$$f_C = \sqrt{f_L \times f_H} = \sqrt{200 \times 800} = 400\text{Hz}$$

$$f_{BW} = f_H - f_L = 800 - 200 = 600\text{Hz}$$

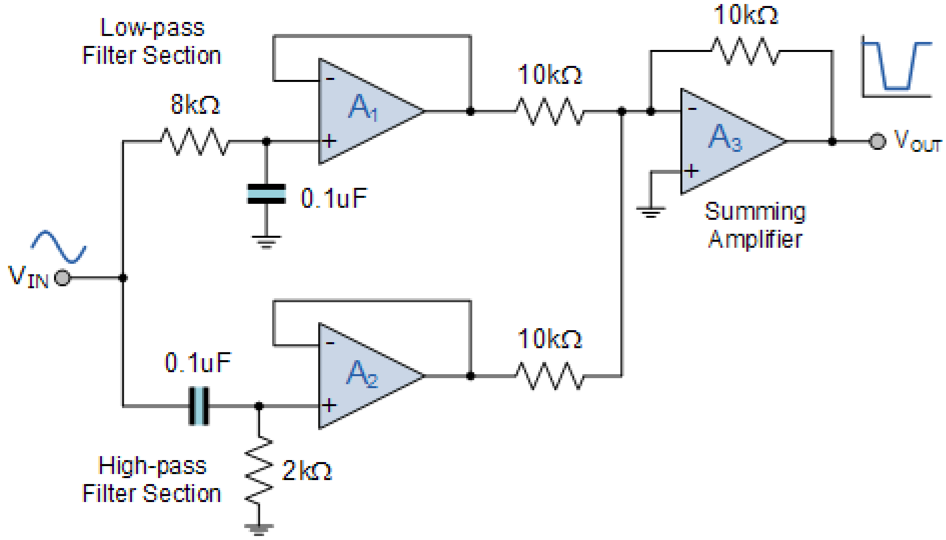
$$Q = \frac{f_C}{f_{BW}} = \frac{400}{600} = 0.67 \text{ or } -3.5\text{dB}$$

Now that we know the component values for the two filter stages, we can combine them into a single voltage adder circuit to complete our filter design. The magnitude and polarity of the adders output will be at any given time, the algebraic sum of its two inputs.

If we make the op-amps feedback resistor and its two input resistors the same values, say 10kΩ, then the inverting summing circuit will provide a mathematically correct sum of the two input signals with zero voltage gain.

Then the final circuit for our band stop (band-reject) filter example will be:

Band Stop Filter Design



We have seen above that simple band stop filters can be made using first or second order low and high pass filters along with a non-inverting summing op-amp circuit to reject a wide band of frequencies. But we can also design and construct band stop filters to produce a much narrower frequency response to eliminate specific frequencies by increasing the selectivity of the filter. This type of filter design is called a “Notch Filter”.

Notch Filters

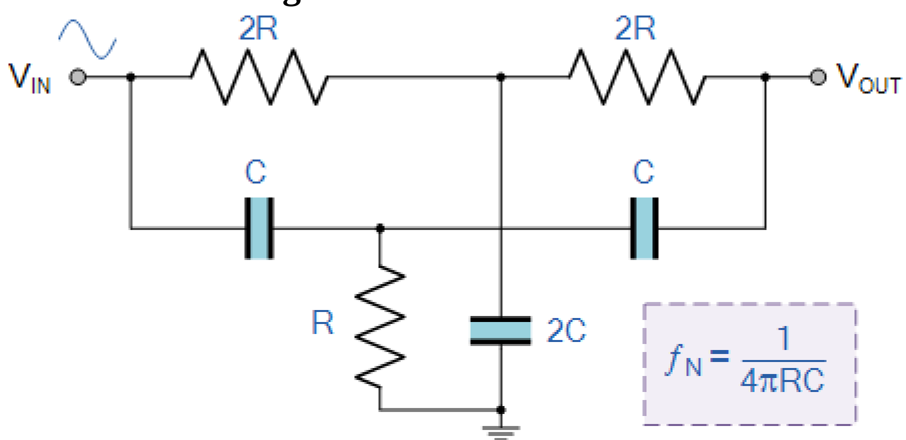
Notch filters are a highly selective, high-Q, form of the band stop filter which can be used to reject a single or very small band of frequencies rather than a whole bandwidth of different frequencies. For example, it may be necessary to reject or attenuate a specific frequency generating electrical noise (such as mains hum) which has been induced into a circuit from inductive loads such as motors or ballast lighting, or the removal of harmonics, etc.

But as well as filtering, variable notch filters are also used by musicians in sound equipment such as graphic equalizers, synthesizers and electronic crossovers to deal with narrow peaks in the acoustic response of the music. Then we can see that notch filters are widely used in much the same way as low-pass and high-pass filters.

Notch filters by design have a very narrow and very deep stop band around their center frequency with the width of the notch being described by its selectivity Q in exactly the same way as resonance frequency peaks in RLC circuits.

The most common notch filter design is the twin-T notch filter network. In its basic form, the twin-T, also called a parallel-tee, configuration consists of two RC branches in the form of two tee sections, that use three resistors and three capacitors with opposite and opposing R and C elements in the tee part of its design as shown, creating a deeper notch.

Basic Twin-T Notch Filter Design



The upper T-pad configuration of resistors 2R and capacitor 2C form the low-pass filter section of the design, while the lower T-pad configuration of capacitors C and resistor R form the high-pass filter section. The frequency at which this basic twin-T notch filter design offers maximum attenuation is called the “notch frequency”, f_N and is given as:

Twin-T Notch Filter Equation

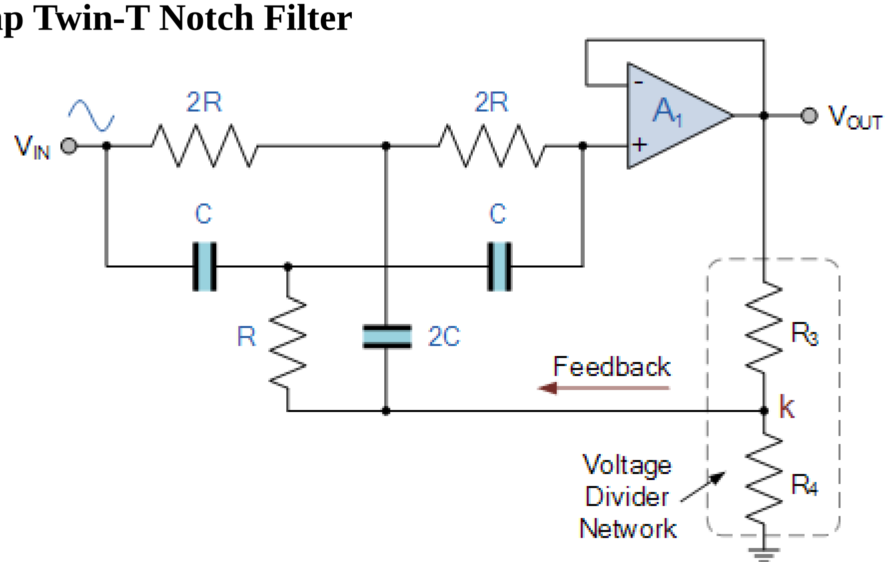
$$f_N = \frac{1}{4\pi RC}$$

Being a passive RC network, one of the disadvantages of this basic twin-T notch filter design is that the maximum value of the output (Vout) below the notch frequency is generally less than the maximum value of output above the notch frequency due in part to the two series resistances (2R) in the low-pass filter section having greater losses than the reactances of the two series capacitors (C) in the high-pass section.

As well as uneven gains either side of the notch frequency, another disadvantage of this basic design is that it has a fixed Q value of **0.25**, in the order of -12dB. This is because at the notch frequency, the reactances of the two series capacitors equals the resistances of the two series resistors, resulting in the currents flowing in each branch being out-of-phase by 180^{o}.

We can improve on this by making the notch filter more selective with the application of positive feedback connected to the center of the two reference legs. Instead of connecting the junction of R and 2C to ground, (0v) but instead connect it to the central pin of a voltage divider network powered by the output signal, the amount of the signal feedback, set by the voltage divider ratio, determines the value of Q, which in turn, determines to some extent, the depth of the notch.

Single Op-amp Twin-T Notch Filter



Here the output from the twin-T notch filter section is isolated from the voltage divider by a single non-inverting op-amp buffer. The output from the voltage divider is fed back to “ground” point of R and 2C. The amount of signal feedback, known as the feedback fraction k, is set by the resistor ratio and is given as:

$$k = \frac{R_4}{R_3 + R_4} = 1 - \frac{1}{4Q}$$

The value of Q is determined by the R3 and R4 resistor ratio, but if we wanted to make Q fully adjustable, we could replace these two feedback resistors with a single potentiometer and feed it into another op-amp buffer for increased negative gain. Also, to obtain the maximum notch depth at the given frequency, resistors R3 and R4 could be eliminated and the junction of R and 2C connected directly to the output.

Band Stop Filter Example No2

Design a two op-amp narrow-band, RC notch filter with a center notch frequency, f_N of 1kHz and a -3dB bandwidth of 100 Hz. Use 0.1uF capacitors in your design and calculate the expected notch depth in decibels.

Data given: f_N=1000Hz, BW=100Hz and C=0.1uF.

1. Calculate value of R for the given capacitance of 0.1uF

$$R = \frac{1}{4\pi f_N C} = \frac{1}{4\pi \times 1000 \times 0.1 \times 10^{-6}}$$

$$\therefore R = 795\Omega \text{ or } 800\Omega$$

2. Calculate value of Q

$$Q = \frac{f_N}{BW} = \frac{1000}{100} = 10$$

3. Calculate value of feedback fraction k

$$k = 1 - \frac{1}{4Q} = 1 - \frac{1}{4 \times 10} = 0.975$$

4. Calculate the values of resistors R3 and R4

$$k = 0.975 = \frac{R_4}{R_3 + R_4}$$

Assume $R_4 = 10k\Omega$, then R_3 equals:

$$R_3 = R_4 - 0.975R_4 = 10000 - 0.975 \times 10000$$

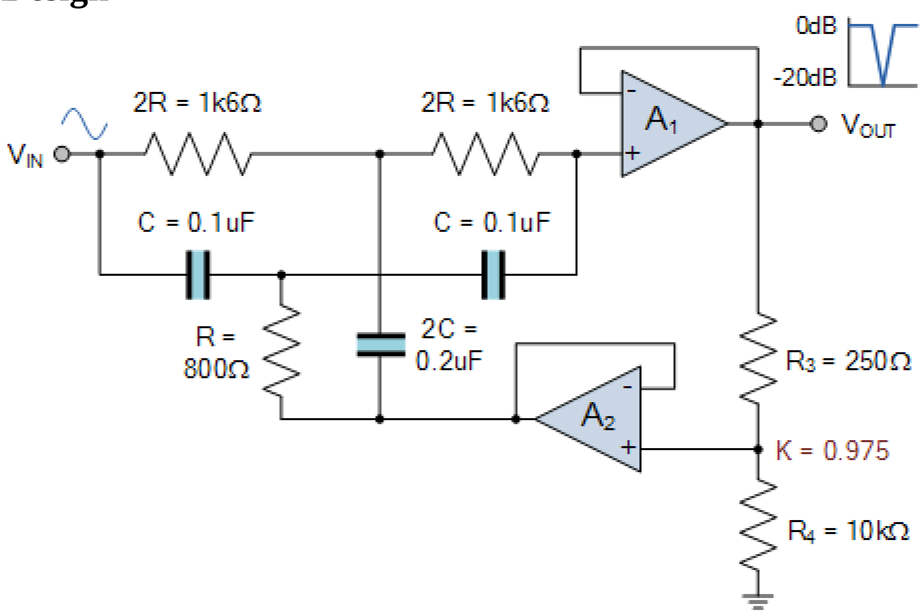
$$\therefore R_3 = 250\Omega$$

5. Calculate expected notch depth in decibels, dB

$$\frac{1}{Q} = \frac{1}{10} = 0.1$$

$$f_{N(dB)} = 20\log(0.1) = -20dB$$

Notch Filter Design



Band Stop Filter Summary

We have seen here that an ideal **band stop filter** has a frequency response which is the inverse of the band-pass filter. Band stop filters block or “reject” frequencies that lie between its two cut-off frequency points (f_L and f_H) but passes all those frequencies either side of this range. The range of frequencies above f_L and below f_H is called the stop band.

Band stop filters accomplish this by summing the outputs of a high pass with that of a low pass filter (especially for the wide band design) with the filters output being the difference. A band stop filter design with a wide stop band is also referred to as a *band reject filter* and a band stop filter design with a narrow stop band is referred to as a *notch filter*. Either way, band stop filters are second-order filters.

Notch filters are designed to provide high attenuation at and near a single frequency with little or no attenuation at all other frequencies. Notch filters use a twin-T parallel resistance-capacitance (RC) network to obtain a deep notch. Higher values of Q can be obtained by feeding back some of the output to the junction of the two tees.

To make the notch filter more selective and with adjustable values of Q, we can connect the junction of the resistance and the capacitance in the two tees to the central point of a voltage divider network connected to the filters output signal. A properly designed notch filter can produce attenuation of more than -60dB at the notch frequency.

Band Stop Filters have many uses in electronics and communication circuits and as we have seen here, they can be used to remove a band of unwanted frequencies from a system, allowing other frequencies to pass with minimum loss. Notch filters can be highly selective and can be designed to reject or attenuate a specific frequency or harmonic content generating electrical noise, such as mains hum within a circuit.

https://www.electronics-tutorials.ws/filter/decibels.html

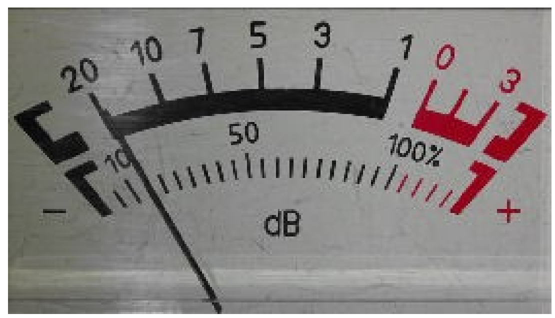
Decibels

The decibel is the base-10 logarithm ratio used to express an increase or decrease in power, voltage, or current in a circuit.

When designing or working with amplifier and filter circuits, some of the numbers used in the calculations can be very large or very small. For example, if we cascade two amplifier stages together with power or voltage gains of say 20 and 36, respectively, then the total gain would be 720 (20*36).

Likewise if we cascaded together to first-order RC filter circuits with attenuations of 0.7071 each, the the total attenuation would be 0.5 (0.7071*0.7071). Remembering of course that if a circuits output is positive, then it produces amplification or gain, and if its output is negative, then it produces attenuation or loss.

When analysing circuits in the *frequency domain*, it is more convenient to compare the amplitude ratio of the output to input values on a logarithmic scale rather than on a linear scale. So if we use the logarithmic ratio of two quantities, P_1 and P_2 we end up with a new quantity or level which can be presented using *Decibels*.



Unlike voltage or current which is measured in volts and amperes respectively, the **decibel**, or simple **dB** for short, is just a ratio of two values, well actually the ratio of one value against another known or fixed value, so therefore the decible is a dimensionless quantity, but does have the “Bel” as its units after the telephone inventor, Alexander Graham Bell.

The ratio of any two values, where one is fixed or known and of the same qunatity or units, whether power, voltage or current, can be represented using decibels (dB) where “deci” means one tenth (1/10th) of a Bel. Clearly then there are 10 decibels (10dB) per Bel or 1 Bel = 10 decibels.

The decibel is commonly used to show the ratio of power change (increasing or decreasing) and is commonly defined as the value which is ten times the Base-10 logarithm of two power levels. So for example, 1 watt to 10 watts is the same power ratio as 10 watts to 100 watts, that is 10:1, so while there is a large difference in the number of watts, 9 compared to 90, the decibel ratio would be exactly the same.

Hopefully then we can see that the decibel (dB) value is a ratio used for comparing and calculating levels of change in power and is not the power itself. So if we have two quantities of power, for example: P_1 and P_2, the ratio of these two values is represented by the equation:

$$dB=10\log_{10}[P_2/P_1]$$

Where, P_1 represents the input power and P_2 represents the output power, (P_{OUT}/P_{IN}).

As the decibel represents the Base-10 logarithmic change of two power levels, we can expand this equation further by using antilogarithms to show by how much change one decibel (1dB) really is.

$$dB=10\log_{10}[P_2/P_1]$$

If P_2/P_1 is equal to 1, that is $P_1 = P_2$ then:

$$dB=10\log_{10}[1]=\log_{10}[1/10]=\log_{10}[0.1]=\text{antilog}[0.1]$$

Thus a dB change in value equals: $10^{0.1}=1.259$

Clearly then the logarithmic change of two powers has a ratio of 1.259, meaning that a 1dB change represents an increase (or decrease) in power of 25.9% (or 26% rounded-off).

So if a circuit or system has a gain of say 5 (7dB), and it is increased by 26%, then the new power ratio of the circuit will be: 5*1.26 = 6.3, so $10\log_{10}(6.3) = 8$dB. An increase in gain of +1dB, proving again that a +1dB change represents a logarithmic increase in power of 26% and not a linear change.

Decibel Example No1

An audio amplifier delivers 100 watts into an 8 ohm speaker load when fed by a 100mW input signal. Calculate the power gain of the amplifier in decibels.

$$\text{Power Gain} = A_P = 10\times\log_{10}\left[\frac{P_2}{P_1}\right]\text{dB}$$

$$A_P = 10\times\log_{10}\left[\frac{100}{0.1}\right] = 10\times\log_{10}(1000) = 30\,\text{dB}$$

We can express the power gain of the amplifier in units of decibels regardless of its input or output values, as an amplifier delivering 40 watts output for 40mW input will also have a power gain of 30 dB, and so on.

We could also, if we so wished, convert this amplifiers decibels value back into a linear value by first converting from decibels (dB) to a Bel remembering that a decibel is 1/10th of a Bel. For example:

A 100 watt audio amplifier has a power gain ratio of 30dB. What will be its maximum input value.

$$A_P = 10 \times \log_{10}\left[\frac{P_2}{P_1}\right] \text{dB}$$

$$\therefore \frac{100}{P_1} = \text{Anti-log}^{\frac{A_P}{10}} = 10^{\frac{30}{10}} = 10^3 = 1000$$

$$\frac{100}{P_1} = 1000, \quad \therefore P_1 = \frac{100}{1000} = 0.1\,\text{W or } 100\,\text{mW}$$

So the result is 100mW as declared in example No1.

One of the advantages of using the base 10 logarithm ratio of two powers is that when dealing with multiple amplifier, filter or attenuator stages cascaded together, we can simply add or subtract their decibel values instead of multiplying or dividing their linear values. In other words, a circuits overall gain (+dB), or attenuation (-dB) is the sum of the individual gains and attenuations for all stages connected between the input and output.

For example, if a single stage amplfier has a power gain of 20dB and it supplies a passive resistive network that has an attenuation of 2, before the signal is amplified again using a second amplifier stage with a gain of 200. Then the total power gain of the circuit between the input and output in decibels would be:

For the passive circuit, an attenuation of 2 is the same as saying the circuit has a positive gain of 1/2 = 0.5, thus the power gain of the passive section is:

$$\text{dB Gain} = 10\log_{10}[0.5] = -3\text{dB} (\text{note a negative value})$$

The second stage amplifier has a gain of 200, thus the power gain of this section is:

$$\text{dB Gain} = 10\log_{10}[200] = +23\text{dB}$$

Then the overall gain of the circuit will be:

$$20 - 3 + 23 = +40\text{dB}$$

We can double check our answer of 40dB by multiplying the individual gains of each stage in the usual way as follows:

A power gain of 20dB in decibels is equal to a gain of 100, as $10^{(20/10)} = 100$. So:

$$100 \text{x} 0.5 \text{x} 200 = 10{,}000 (\text{or } 10{,}000 \text{ times greater})$$

Converting this back to a decibel value gives:

dB Gain=10log$_{10}$[10,000]=40dB

Then clearly we can see that a gain of 10,000 is equal to a power gain ratio of +40dB as shown above and that we can use the decibel value to express large ratios of powers with much smaller numbers as 40dB is a power ratio of 10,000, whereas -40dB is a power ratio of 0.0001. So using decibels makes the maths a little easier.

Decibels of Voltage and Current

Any power level can be expressed as a voltage or current if we know the resistance. According to Ohms Law, $P = V^2/R$ and $P = I^2R$. As V and I relate to the current through and the voltage across the same resistance, if (and only if) we make R = R = 1, then the dB values for the ratios of voltage (V_1, and V_2) as well as for current (I_1, and I_2) will be given as:

$$dB_v = 10\log_{10}\left[\frac{V_2^2/R}{V_1^2/R}\right] = 10\log_{10}\left[\frac{V_2^2}{V_1^2}\right]$$

$$10\log_{10}\left[\frac{V_2^2}{V_1^2}\right] = 10\log_{10}\left[\frac{V_2}{V_1}\right]^2$$

$$\therefore\ dB_v = 10\log_{10}\left[\frac{V^2}{V^1}\right]^2 = 20\log_{10}\left[\frac{V_2}{V_1}\right]$$

that is 20log(voltage gain), and for the current gain would be:

$$dB_i = 10\log_{10}\left[\frac{I_2^2 \times \cancel{R}}{I_1^2 \times \cancel{R}}\right] = 10\log_{10}\left[\frac{I_2^2}{I_1^2}\right] = 10\log_{10}\left[\frac{I_2}{I_1}\right]^2$$

$$\therefore\ dB_i = 10\log_{10}\left[\frac{I^2}{I^1}\right]^2 = 20\log_{10}\left[\frac{I_2}{I_1}\right]$$

Thus the only difference between defining the power, voltage, and current decibel (dB) calculations is the constant of 10 and 20, and that for the dB ratio to be correct in all instancies the two quantities must both have the same units, either watts, milli-watts, volts, milli-volts, amperes or milli-amperes, or any other unit.

Decibel Example No2

A passive resistive network is used to provide an attenuation (loss) of 10dB, with an input voltage is 12V. What will be the networks output voltage value.

$$dB = 20\log_{10}\left[\frac{V_{OUT}}{V_{IN}}\right]$$

$$-10 = 20\log_{10}\left[\frac{V_{OUT}}{12}\right]$$

$$-0.5 = \log_{10}\left[\frac{V_{OUT}}{12}\right]$$

$$0.3162 = \frac{V_{OUT}}{12}$$

$$\therefore\ V_{OUT} = 0.3162 \times 12 = 3.79\ \text{Volts}$$

As *decibels* represents a logarithmic change in terms of power, voltage or current, we can construct a table to show the specific gains and their equivalent decibel values below.

Decibel Table of Gains

dB Value	Power Ratio 10log(A)	Voltage/Current Ratio 20log(A)
-20dB	0.01	0.1
-10dB	0.1	0.3162
-6dB	0.25	1/2 = 0.5
-3dB	1/2 = 0.5	1/√2 = 0.707
-1dB	0.79	0.89
0dB	1	1
1dB	1.26	1.1
3dB	2	√2 = 1.414

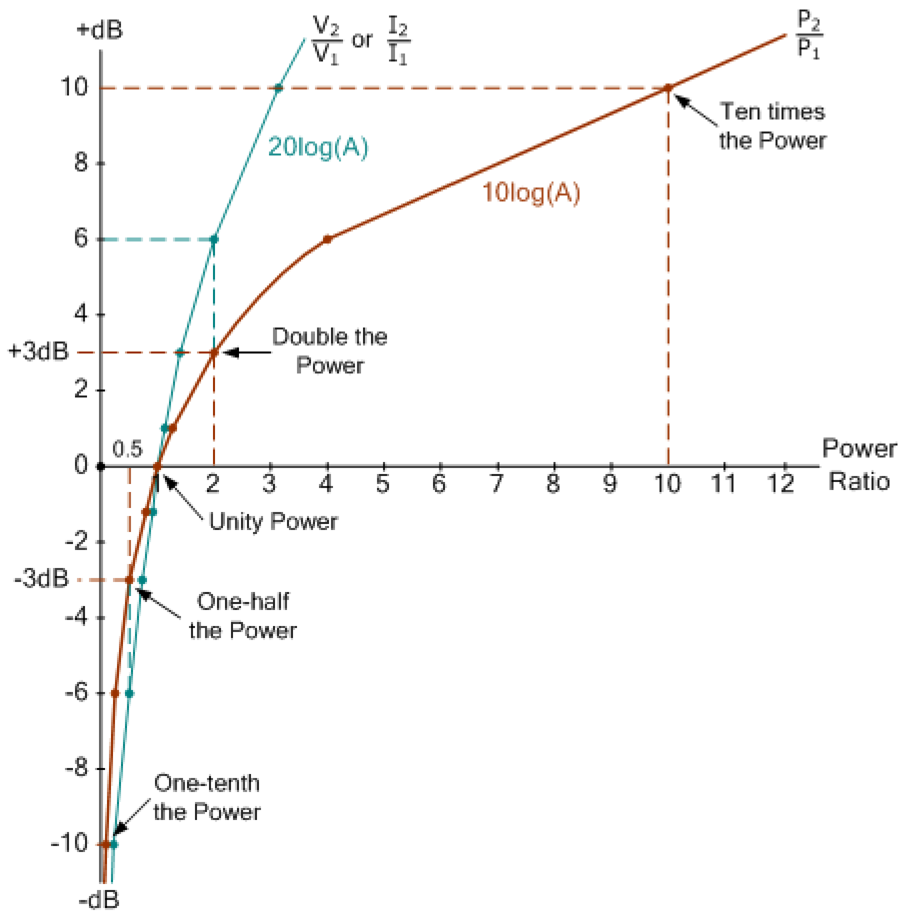
6dB	4	2
10dB	10	√10 = 3.162
20dB	100	10
30dB	1000	31.62

We can see from the above decibel table that at 0dB the ratio gain for power, voltage and current is equal to "1" (unity). This means that the circuit (or system) produces no gain or loss between the input and output signals. So zero dB corresponds to a unity gain i.e. A = 1 and not zero gain.

We can also see that at +3dB the output of the circuit (or system) has doubled its input value, meaning a positive dB gain (amplification) so A > 1. Likewise, at -3dB the output the circuit is at half its input value, meaning a negative dB gain (attenuation) so A < 1. This -3dB value is commonly called the "half-power" point and defines the corner frequency in filter networks.

It is all well and good tabulating the power gains against decibels in a reference table, but when dealing with amplifier and filters, Electrical Engineers prefer to use Bode Plots, charts or graphs as a visual display of the circuits (or systems) frequency response characteristics. Then using the data values in the table above we can create the following "decibel" Bode plot showing the various positions of the power points.

Decibel Power Bode Plot



Then we can clearly see that the power curve is not linear but follows the logarithmic ratio of 1.259.

Decibel Tutorial Summary

We have seen in this tutorial about the **Decibel** (dB) that it is a Base-10 logarithmic unit of power change and that the decibel unit is a 1/10th dimensionless value of a Bel (1 Bel = 10 decibels or 1dB = 0.1B). The decibel allows us to present large ratios of powers using small numbers and we have seen above that 30dB is equivalent to a power ratio of 1000 with the most commonly used decibel values being: 3dB, 6dB, 10dB and 20dB (and their negative equivalents). However, 20dB is not double the power of 10dB.

The decibel also shows us that any change in power by the same ratio will have the same decibel ratio. For example, doubling the power from 1 watt to 2 watts is the same ratio as 10 watts to 20 watts, that is a +3dB change, while a -3dB change means that the power ratio will be halved.

If the dB ratio is positive in value, then it means amplification or gain is present as the output power is greater than the input power ($P_{OUT} > P_{IN}$). If however the dB power ratio is of a negative value, then this means an attenuation or loss is affecting the circuit as the output power will be less than the circuits input power ($P_{OUT} < P_{IN}$). Clearly then 0dB means the power ratio is one with no reduction or gain of the signal.

https://www.electronics-tutorials.ws/filter/sallen-key-filter.html

Sallen and Key Filter

Sallen-Key Filter topology is used as the building block to implement higher order active filters.

The **Sallen and Key Filter** design is a second-order active filter topology which we can use as the basic building blocks for implementing higher order filter circuits, such as low-pass (LPF), high-pass (HPF) and band-pass (BPF) filter circuits.

As we have seen in this filters section, electronic filters, either passive or active, are used in circuits where a signals amplitude is only required over a limited range of frequencies. The advantage of using *Sallen-Key Filter* designs is that they are simple to implement and understand.

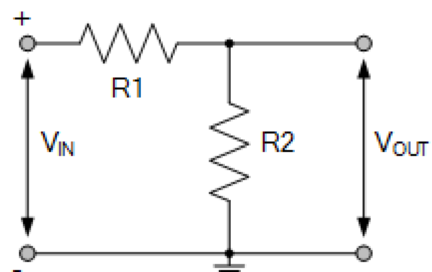
The Sallen and Key topology is an active filter design based around a single non-inverting operational amplifier and two resistors, thus creating a voltage-controlled voltage-source (VCVS) design with filter characteristics of, high input impedance, low output impedance and good stability, and as such allows individual Sallen-key filter sections to be cascaded together to produce much higher order filters.

But before we look at the design and operation of the *Sallen-key filter*, let’s first remind ourselves of the characteristics of a single resistor-capacitor, or RC network when subjected to a range of input frequencies.

The Voltage Divider

When two (or more) resistors are connected together across a DC supply voltage, different voltage values will be developed across each resistor creating what is basically called a voltage divider or potential divider network.

Resistive Voltage Divider



The basic circuit shown consists of two resistors in series connected across an input voltage, V_{IN}.

Ohm’s Law tells us that the voltage dropped across a resistor is the sum of the current flowing through it multiplied by its resistive value, V = I*R, so if the two resistors are equal, then the voltage dropped across both resistors, R1 and R2 will also be equal and is split equally between them.

The voltage developed or dropped across resistor R2 represents the output voltage, V_{OUT} and is given by the ratio of the two resistors and the input voltage. Thus the transfer function for this simple voltage divider network is given as:

Resistive Voltage Divider Transfer Function

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R_2}{(R_1 + R_2)}$$

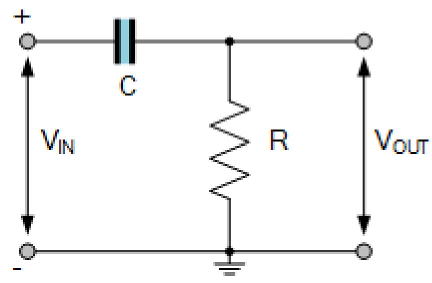
$$\text{Thus:} \quad V_{OUT} = V_{IN} \frac{R_2}{(R_1 + R_2)}$$

But what would happen to the output voltage, V_{OUT} if we changed the input voltage to an AC supply or signal, and varied its frequency range. Well actually nothing, as resistors are generally not affected by changes in frequency (wirewounds excluded) so their frequency response is zero, allowing AC, $Irms^2$*R voltages to be developed or dropped across the resistors just the same as it would be for steady state DC voltages.

The RC Voltage Divider

If we change resistor R1 above to a capacitor, C as shown, how would that affect our previous transfer function. We know from our tutorials about Capacitors that a capacitor behaves like an open circuit once charged when connected to a DC voltage supply.

RC Voltage Divider



Thus when a steady state DC supply is connected to V_{IN}, the capacitor will be fully charged after 5 time constants (5T = 5RC) and in which time it draws no current from the supply. Therefore there is no current flowing through the resistor, R and no voltage drop developed across it, so no output voltage. In other words, capacitors block steady state DC voltages once charged.

If we now change the input supply to an AC sinusoidal voltage, the characteristics of this simple RC circuit completely changes as the DC or constant part of the signal is blocked. So now we are analysing the RC circuit in the frequency domain, that is the part of the signal that depends on time.

In an AC circuit, a capacitor has the property of **capacitive reactance**, $\mathbf{X_C}$ but we can still analyse the RC circuit in the same way as we did with the resistor only circuits, the difference is that the impedance of the capacitor now depends on frequency.

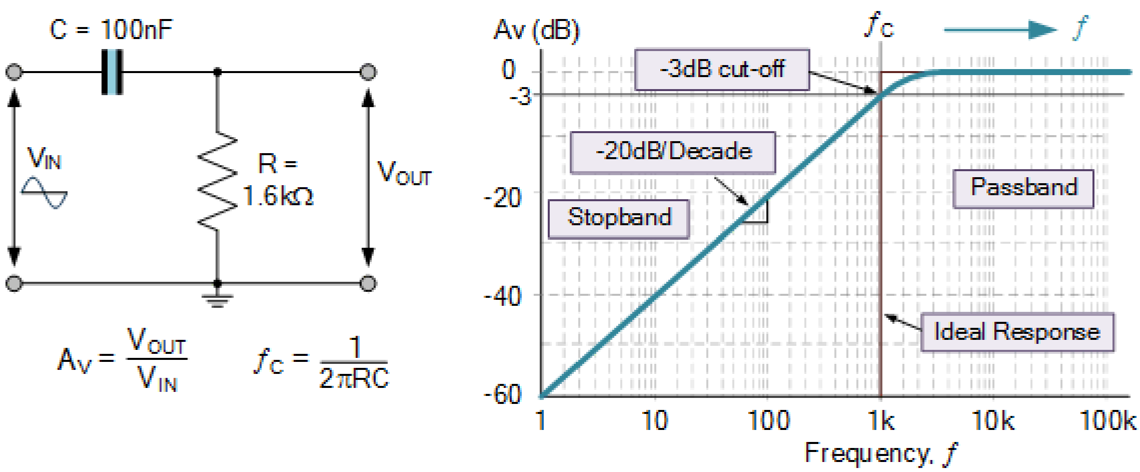
For AC circuits and signals, capacitive reactance (X_C), is the opposition to alternating current flow through a capacitor measured in Ohm's. Capacitive reactance is frequency dependant, that is at low frequencies ($f \cong 0$) the capacitor behaves like an open circuit and blocks them.

At very high frequencies ($f \cong \infty$) the capacitor behaves like a short circuit and pass the signals directly to the output as $V_{OUT} = V_{IN}$. However, somewhere in between these two frequency extremes the capacitor has an impedance given by X_C. So our voltage divider transfer function from above becomes:

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R}{(R + X_C)}$$

Thus changes in frequency, causes changes in X_C, which causes changes in the magnitude of the output voltage. Consider the circuit below.

RC Filter Circuit



The graph shows the frequency response of this simple 1st-order RC circuit. At low frequencies the voltage gain is extremely low, as the input signal is being block by the reactance of the capacitor. At high frequencies the voltage gain is high (unity) as the reactance is causes the capacitor to effectively become a short-circuit to these high frequencies, so $V_{OUT} = V_{IN}$

However, there becomes a frequency point where the reactance of the capacitor is equal to the resistance of the resistor, that is: $X_C = R$ and this is called the "critical frequency" point, or more commonly called the **cut-off frequency**, or **corner frequency** $\boldsymbol{f_C}$.

As the cut-off frequency occurs when X_C = R the standard equation used to calculate this critical frequency point is given as:

Cut-off Frequency Equation

$$f_C = \frac{1}{2\pi RC}$$

The cut-off frequency, f_C defines where the circuit, in this example, changes from attenuating or blocking all frequencies below, f_C and starts to pass all the frequencies above this f_C point. Thus the circuit is called a “high pass filter”.

The cut-off frequency is where the ratio of the input-to-output signal has a magnitude of 0.707 and when converted to decibels is equal to –3dB. This is often referred to as a filters 3dB down point.

As the reactance of the capacitor is related to frequency, that is capacitive reactance (X_C) varies inversely with applied frequency, we can modify the above voltage divider equation to obtain the transfer function of this simple RC high pass filter circuit as shown.

RC Filter Circuit

$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{\left(R + X_C\right)}$$

$$\text{As:}\quad X_C = \frac{1}{2\pi f C}$$

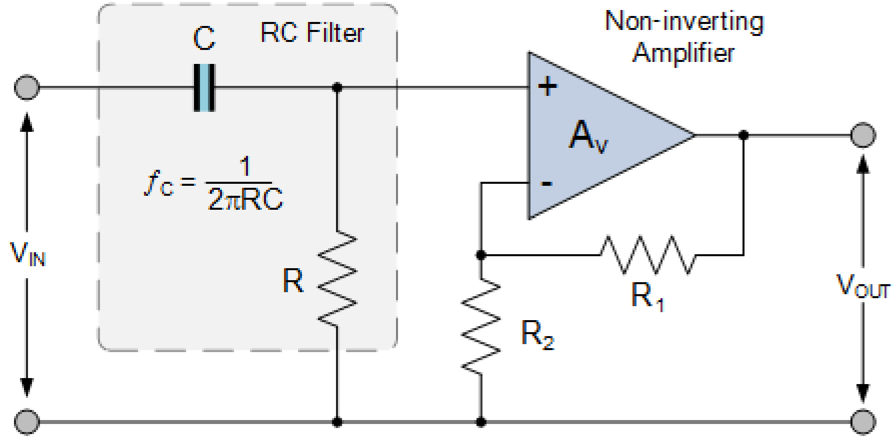
$$\therefore \frac{V_{OUT}}{V_{IN}} = \frac{R}{\left(R + \frac{1}{2\pi f C}\right)} = \frac{2\pi f RC}{1 + 2\pi f RC}$$

One of the main disadvantages of an RC filter is that the output amplitude will always be less than the input so it can never be greater than unity. Also the external loading of the output by more RC stages or circuits will have an affect on the filters characteristics. One way to overcome this problem is to convert the passive RC filter into an “Active RC Filter” by adding an operational amplifier to the basic RC configuration.

By adding an operational amplifier, the basic RC filter can be designed to provide a required amount of voltage gain at its output, thus changing the filter from an attenuator to an amplifier. Also due to the high input impedance and low output impedance of an operational amplifier prevents external loading of the filter allowing it to be easily adjusted over a wide frequency range without altering the designed frequency response.

Consider the simple active RC high-pass filter below.

Active High Pass Filter



The RC filter part of the circuit responds the same as above, that is passing high frequencies but blocking low frequencies, with the cut-off frequency set by the values of R and C. The operational amplifier, or op-amp for short, is configured as a non-inverting amplifier whose voltage gain is set by the ratio of the two resistors, R_1 and R_2.

Then the closed loop voltage gain, A_V in the passband of a *non-inverting operational amplifier* is given as:

Cut-off Frequency Equation

$$A_V = 1 + \frac{R_1}{R_2}$$

RC Filter Example No1

A simple 1st-order active high-pass filter is require to have a cut-off frequency of 500Hz and a passband gain of 9dB. Calculate the required components assuming a standard 741 operational amplifier is used.

From above we have seen that the cut-off frequency, f_C is determined by the values of R and C in the frequency-selective RC circuit. If we assume a value for R of 5kΩ (any reasonable value would do), then the value of C is calculated as:

$$f_C = \frac{1}{2\pi RC} = 500\text{Hz}$$

We choose $R = 5\text{k}\Omega$

$$\therefore\ C = \frac{1}{2\pi f_C R} = \frac{1}{2\pi \times 500 \times 5000} = 63.65\text{nF}$$

The calculated value of C is 63.65nF, so the nearest preferred value used is 62nF.

The gain of the high pass filter in the passband region is to be +9dB which equates to a voltage gain, A_V of 2.83. Assume an arbitrary value for feedback resistor, R_1 of 15kΩ, this gives a value for resistor R_1 of:

$$9\text{dB} = 20\log(A)$$

$$\therefore\ A = 2.83$$

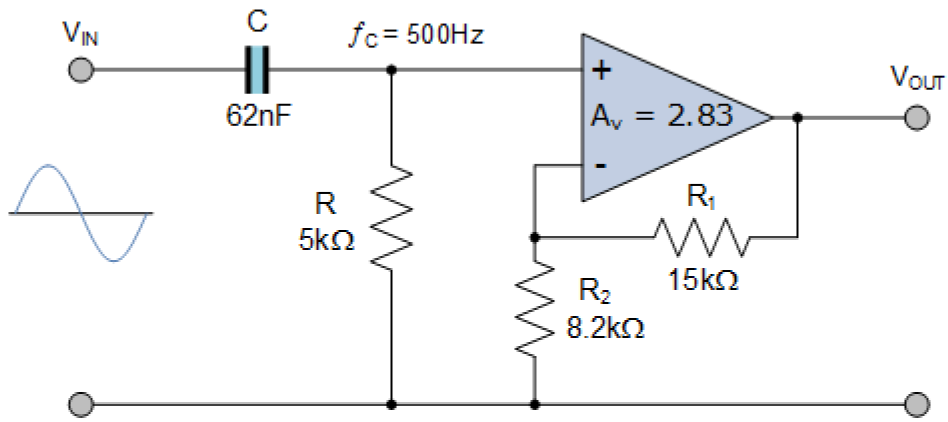
$$A = \frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_1}{R_2} = 2.83$$

Assume a value of $R_1 = 15\text{k}\Omega$

$$\therefore\ R_2 = \frac{R_1}{A - 1} = \frac{15000}{2.83 - 1} = 8197\Omega$$

Again the calculated value of R_2 is 8197Ω. The nearest preferred value would be 8200Ω or 8.2kΩ. This then gives us the final circuit for our active high-pass filter example of:

High Pass Filter Circuit

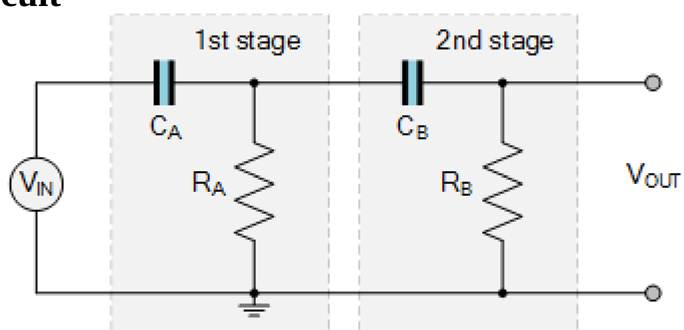


We have seen that a simple first-order high-pass filters can be made using a single resistor and capacitor producing a cut-off frequency, f_C point where the output amplitude is –3dB down from the input amplitude. By adding a second RC filter stage to the first, we can convert the circuit into a second-order high-pass filter.

Second-order RC Filter

The simplest second order RC filter consists of two RC sections cascade together as shown. However, for this basic configuration to operate correctly, input and output impedances of the the two RC stages should not affect each others operation, that is they should be non-interacting.

High Pass Filter Circuit



Cascading one RC filter stage with another (identical or different RC values), does not work very well because each successive stage loads the previous one and when more RC stages are added, the cut-off frequency point moves further away from the designed or required frequency.

One way to overcome this problem for a passive filter design is to have the input impedance of the second RC stage at least 10 times greater than the output impedance of the first RC stage. That is $R_B = 10*R_1$ and $C_B = C_A/10$ at the cut-off frequency.

The advantage of increasing the component values by a factor of 10 is that the resulting second-order filter produces a steeper roll-off of 40dB/decade than cascaded RC stages. But what if you wanted to design a 4th or a 6th-order filter, then the calculation of ten times the value of the previous components can be time consuming and complicated.

One simple way to cascade together RC filter stages which do not interact or load each other to create higher-order filters (individual filter sections need not be identical) which can be easily tuned and designed to provide required voltage gain is to use *Sallen-key Filter* stages.

Sallen and Key Filters

Sallen-Key is one of the most common filter configurations for designing first-order (1st-order) and second-order (2nd-order) filters and as such is used as the basic building blocks for creating much higher order filters.

The main advantages of the Sallen-key filter design are:

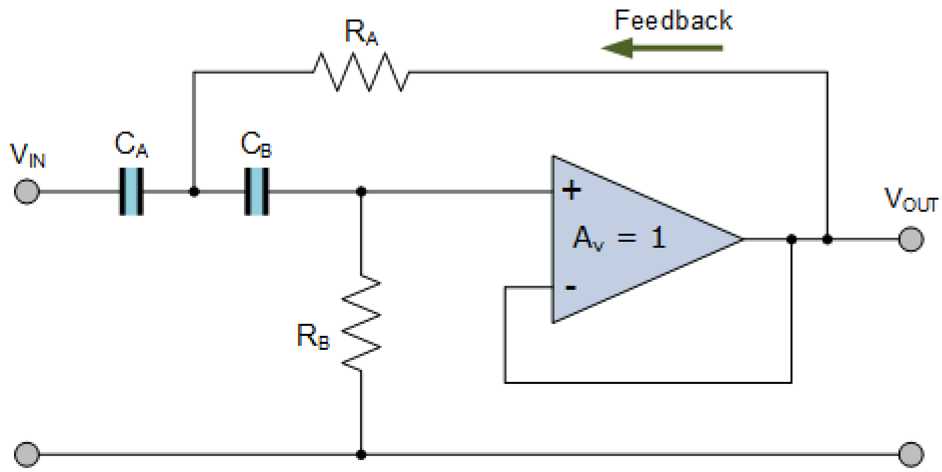
- ✔ Simplicity and Understanding of their Basic Design
- ✔ The use of a Non-inverting Amplifier to Increase Voltage Gain
- ✔ First and Second-order Filter Designs can be Easily Cascaded Together
- ✔ Low-pass and High-pass stages can be Cascaded Together
- ✔ Each RC stage can have a different Voltage Gain
- ✔ Replication of RC Components and Amplifiers
- ✔ Second-order Sallen-key Stages have Steep 40dB/decade roll-off than cascaded RC

However, there are some limitations to the basic Sallen-key filter design in that the voltage gain, A_V and magnification factor, Q are closely related due to the use of an operational amplifier within the Sallen-key design.

Almost any Q value greater than 0.5 can be realised since using a non-inverting configuration, the voltage gain, A_V will always be greater than 1, (unity) but must be less than 3 otherwise it will become unstable.

The simplest form of Sallen-key filter design is to use equal capacitor and resistor values (but the C’s and R’s don’t have to be equal), with the operational amplifier configured as a unity-gain buffer as shown. Note that resistor R_A is no longer connected to ground but instead provides positive feedback for the amplifier.

Sallen-key High Pass Filter Circuit



The passive components C_A, R_A, C_B and R_B form the second-order frequency-selective circuit.

Thus at low frequencies, capacitors C_A and C_B appear as open circuits, so the input signal is blocked resulting in no output. At higher frequencies, C_A and C_B appear to the sinusoidal input signal as short circuits, so the signal is buffered directly to the output.

However, around the cut-off frequency point, the impedance of C_A and C_B will be the same value as R_A and R_B, as noted above, so the positive feedback produced through C_B provides voltage gain and and increase in output signal magnification, Q.

Since we now have two sets of RC networks, the above equation for the cut-off frequency for a Sallen-Key filter is modified too:

Sallen-key Cut-off Frequency Equation

$$f_C = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

If the two series capacitors C_A and C_B are made equal (C_A=C_B=C) and the two resistors R_A and R_B are also made equal (R_A=R_B=R), then the above equation simplifies to the original cut-off frequency equation of:

$$f_C = \frac{1}{2\pi RC}$$

As the operational amplifier is configured as a unity gain buffer, that is A=1, the cut-off frequency, f_C and Q are completely independent of each other making for a simpler filter design. Then the magnification factor, Q is calculated as:

$$Q = \frac{1}{3 - A} = \frac{1}{3 - 1} = \frac{1}{2} = 0.5$$

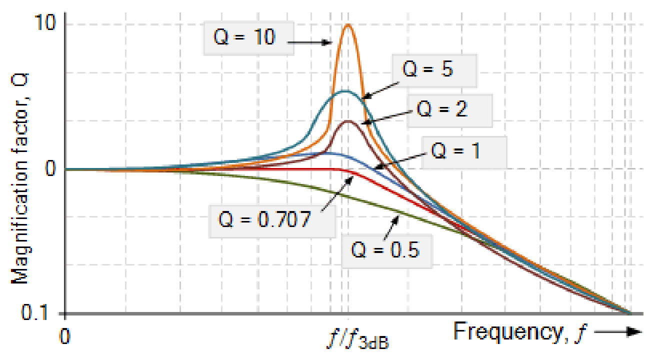
Therefore for the unity-gain buffer configuration, the voltage gain (A_V) of the filter circuit is equal to 0.5, or -6dB (over damped) at the cut-off frequency point, and we would expect to see this because its a second-order filter response, as 0.7071*0.7071 = 0.5. That is -3dB*-3dB = -6dB.

However, as the value of Q determines the response characteristics of the filter, the proper selection of the operational amplifiers two feedback resistors, R_1 and R_2, allows us to select the required passband gain A for the chosen magnification factor, Q.

Note that for a Sallen-key filter topology, selecting the value of A to be very close to the maximum value of 3, will result in high Q values. A high Q will make the filter design sensitive to tolerance variations in the values of feedback resistors R_1 and R_2.

For example, setting the voltage gain to 2.9 (A = 2.9) will result in the value of Q being 10 (1/(3-2.9)), thus the filter becomes extremely sensitive around f_C.

Sallen-key Filter Response



Then we can see that the lower the value of Q the more stable will be the Sallen and Key filter design. While high values of Q can make the design unstable, with very high gains producing a negative Q would lead to oscillations.

Sallen and Key Filter Example No2

Design a second-order high-pass *Sallen and Key Filter* circuit with the following characteristics: f_C=200Hz, and Q=3

To simplify the math's a little, we will assume that the two series capacitors C_A and C_B are equal ($C_A=C_B=C$) and also the two resistors R_A and R_B are equal ($R_A=R_B=R$).

$$f_C = \frac{1}{2\pi RC} = 200\text{Hz}$$

We will choose C = 100nF

$$\therefore R = \frac{1}{2\pi f_C C} = \frac{1}{2\pi \times 200 \times 100 \times 10^{-9}} = 7957\,\Omega$$

The calculated value of R is 7957Ω, so the nearest preferred value used is 8kΩ.

For Q=3, the gain is calculated as:

$$Q = \frac{1}{3 - A} \qquad \therefore A = \frac{3Q - 1}{Q}$$

If Q = 3, then :

$$A = \frac{(3 \times 3) - 1}{3} = 2.667$$

If A=2.667, then the ratio of R_1/R_2=1.667 as shown.

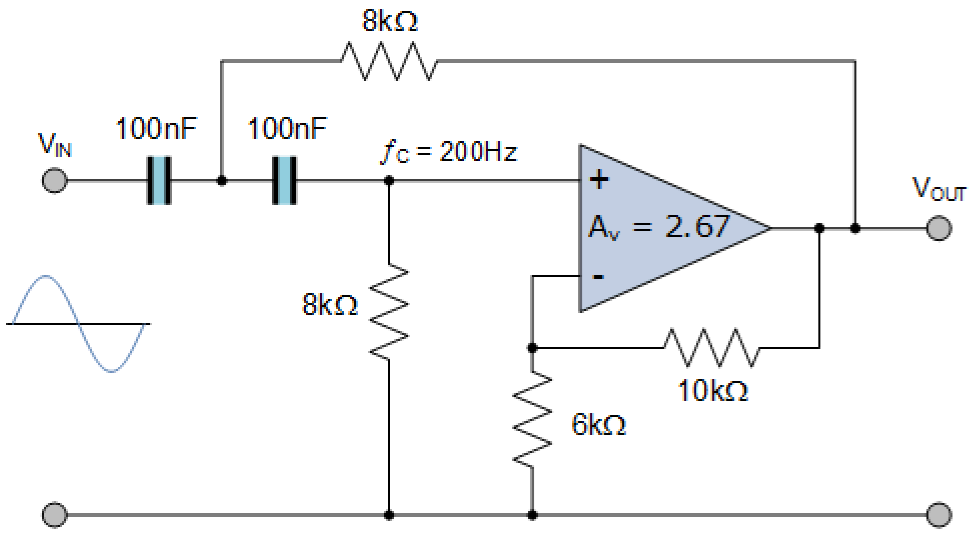
$$A = \frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_1}{R_2} = 2.667$$

Assume a value of R_1 = 10kΩ

$$\therefore R_2 = \frac{R_1}{A - 1} = \frac{10000}{2.667 - 1} = 5998\,\Omega$$

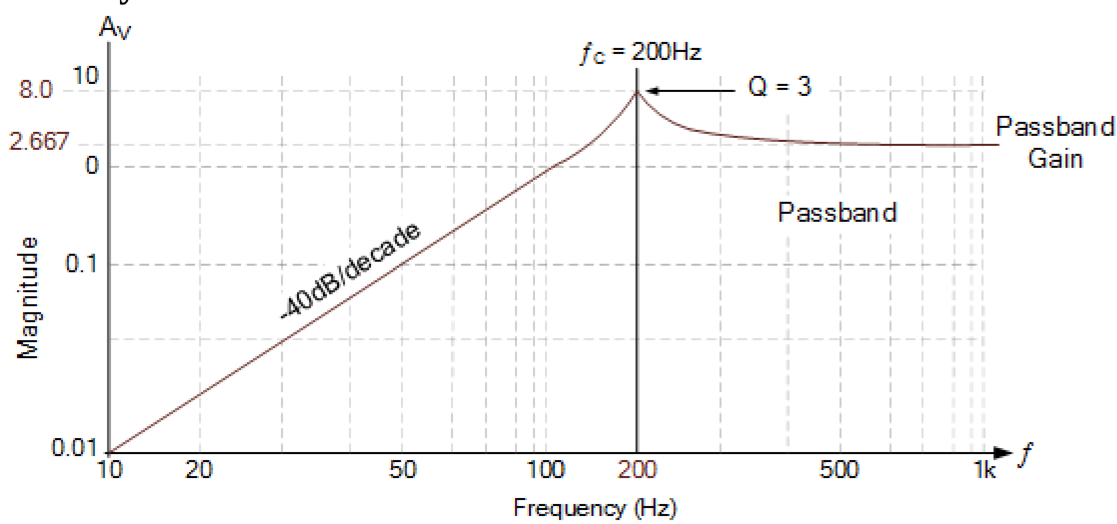
The calculated value of R_2 is 5998Ω, so the nearest preferred value used 6000Ω or 6kΩ. This then gives us the final circuit for our Sallen and Key high-pass filter example of:

Sallen and Key High Pass Filter



Then with a cut-off or corner frequency of 200Hz, a passband gain of 2.667, and a maximum voltage gain at the cut-off frequency of 8 (2.667*3) due to Q = 3, we can show the characteristics of this second-order high-pass Sallen and Key filter in the following Bode plot.

Sallen and Key Filter Bode Plot



Sallen and Key Filter Summary

We have seen here in this tutorial that the Sallen-Key configuration, also known as a *voltage-controlled, voltage-source (VCVS)* circuit is the most widely used filter topologies due mainly to the fact that the operational amplifier used within its design can be configured as a unity gain buffer or as a non-inverting amplifier.

The basic Sallen-key filter configuration can be used to implement different filter responses such as, Butterworth, Chebyshev, or Bessel with the correct selection of RC filter network. Most practical values of R and C can be used remembering that for a specific cut-off frequency point, the values of R and C are inversely proportional. That is as the value of R is made smaller, C becomes larger, and vice versa.

The Sallen-key is a 2^{nd}-order filter design which can be cascaded together with other RC stages to create higher-order filters. Multiple filter stages need not be the same but can each have different cut-off frequency or gain characteristics. For instance, putting together a low-pass stage and a high-pass stage to create a Sallen and Key band-pass filter.

Here we have looked at designing a Sallen-key high-pass filter, but the same rules apply equally for a Sallen-key low-pass design. The voltage gain, A_V of the op-amp determines its response. The voltage gain is preset by the two voltage divider resistors, R_1 and R_2 remembering that the voltage gain must always be less than **3** otherwise, the filter circuit will become unstable and oscillate.