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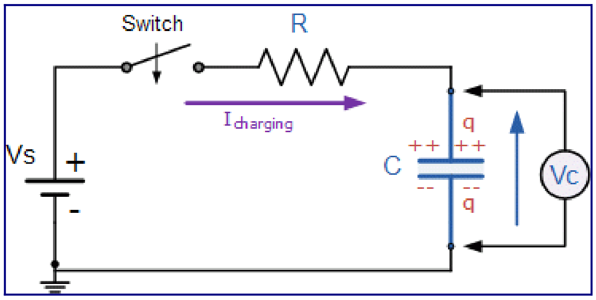

RC Charging Circuit

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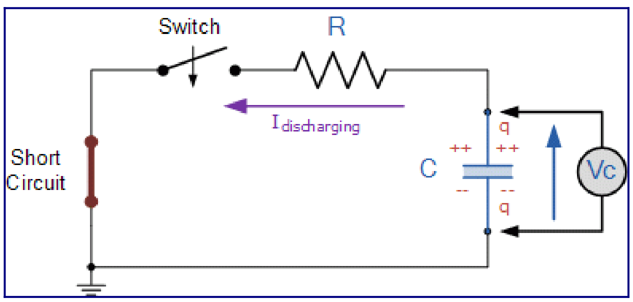

RC Discharging Circuit

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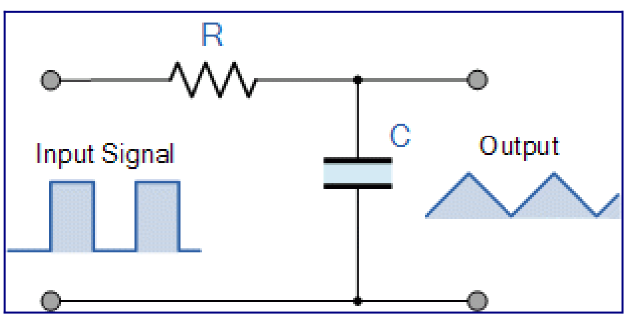
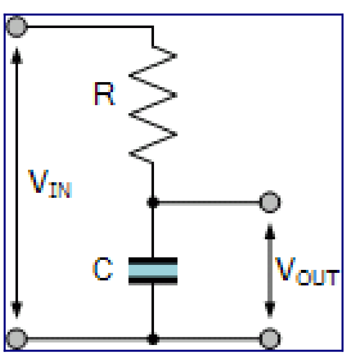


RC Waveforms

In the previous RC Charging and Discharging tutorials, we saw how a capacitor has the ability to both charge and discharges itself through a series connected resistor. The time taken for this capacitor to either fully charge or fully discharge is equal to five RC time constants o...



RC Integrator

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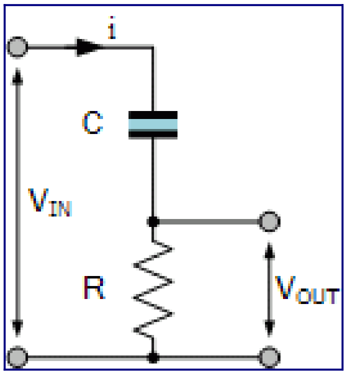
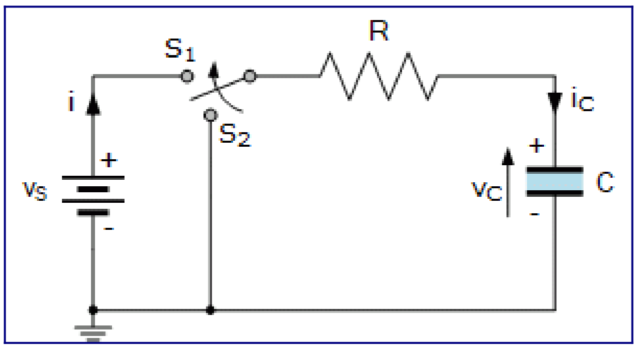
	**RC Differentiator** For a passive RC differentiator circuit, the input is connected to a capacitor while the output voltage is taken from across a resistance being the exact opposite to the RC Integrator Circuit. A passive RC differentiator is nothing more than a capacitance in series with a resist...
	**Tau – The Time Constant** Tau, symbol τ, is the greek letter used in electrical and electronic calculations to represent the time constant of a circuit as a function of time. But what do we mean by a circuits time constant and transient response. Electrical and electronic circuits are not always in a…

https://www.electronics-tutorials.ws/rc/rc_1.html

RC Charging Circuit Tutorial & RC Time Constant

All Electrical or Electronic circuits or systems suffer from some form of "time-delay" between its input and output terminals when either a signal or voltage, continuous, (DC) or alternating (AC), is applied to it.

This delay is generally known as the circuits **time delay** or **Time Constant** which represents the time response of the circuit when an input step voltage or signal is applied. The resultant time constant of any electronic circuit or system will mainly depend upon the reactive components either capacitive or inductive connected to it. Time constant has units of, **Tau –** τ

When an increasing DC voltage is applied to a discharged Capacitor, the capacitor draws what is called a "charging current" and "charges up". When this voltage is reduced, the capacitor begins to discharge in the opposite direction. Because capacitors can store electrical energy they act in many ways like small batteries, storing or releasing the energy on their plates as required.

The electrical charge stored on the plates of the capacitor is given as: $Q = CV$. This charging (storage) and discharging (release) of a capacitors energy is never instant but takes a certain amount of time to occur with the time taken for the capacitor to charge or discharge to within a certain percentage of its maximum supply value being known as its **Time Constant** (τ).

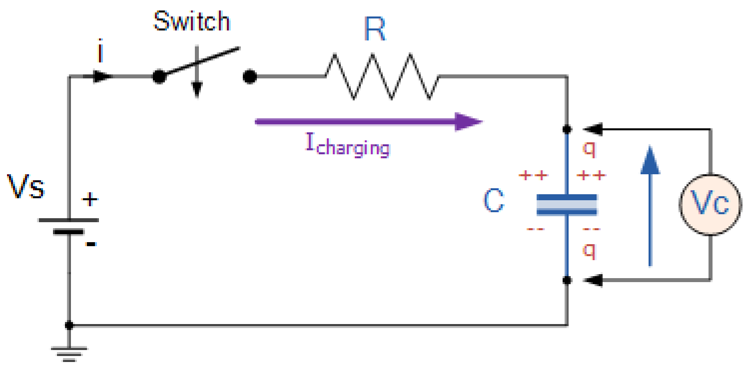
If a resistor is connected in series with the capacitor forming an RC circuit, the capacitor will charge up gradually through the resistor until the voltage across it reaches that of the supply voltage. The time required for the capacitor to be fully charge is equivalent to about **5 time constants** or 5T. Thus, the transient response or a series RC circuit is equivalent to 5 time constants.

This transient response time T, is measured in terms of $\tau = R \times C$, in seconds, where R is the value of the resistor in ohms and C is the value of the capacitor in Farads. This then forms the basis of an RC charging circuit were 5T can also be thought of as "5 x RC".

RC Charging Circuit

The figure below shows a capacitor, (C) in series with a resistor, (R) forming a **RC Charging Circuit** connected across a DC battery supply (Vs) via a mechanical switch. at time zero, when the switch is first closed, the capacitor gradually charges up through the resistor until the voltage across it reaches the supply voltage of the battery. The manner in which the capacitor charges up is shown below.

RC Charging Circuit



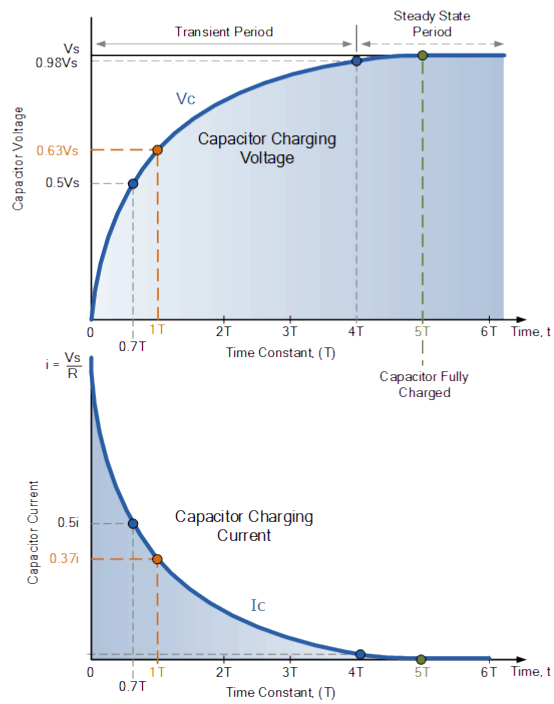
Let us assume above, that the capacitor, C is fully “discharged” and the switch (S) is fully open. These are the initial conditions of the circuit, then t = 0, i = 0 and q = 0. When the switch is closed the time begins at t = 0 and current begins to flow into the capacitor via the resistor.

Since the initial voltage across the capacitor is zero, (Vc = 0) at t = 0 the capacitor appears to be a short circuit to the external circuit and the maximum current flows through the circuit restricted only by the resistor R. Then by using Kirchhoff’s voltage law (KVL), the voltage drops around the circuit are given as:

$$V_S - R \times i(t) - V_C(t) = 0$$

The current now flowing around the circuit is called the **Charging Current** and is found by using Ohms law as: i = Vs/R.

RC Charging Circuit Curves



The capacitor (C), charges up at a rate shown by the graph. The rise in the RC charging curve is much steeper at the beginning because the charging rate is fastest at the start of charge but soon tapers off exponentially as the capacitor takes on additional charge at a slower rate.

As the capacitor charges up, the potential difference across its plates begins to increase with the actual time taken for the charge on the capacitor to reach 63% of its maximum possible fully charged voltage, in our curve 0.63Vs, being known as one full Time Constant, (T).

This 0.63Vs voltage point is given the abbreviation of 1T, (one time constant).

The capacitor continues charging up and the voltage difference between Vs and Vc reduces, so too does the circuit current, i. Then at its final condition greater than five time constants (5T) when the capacitor is said to be fully charged, $t = \infty$, $i = 0$, $q = Q = CV$. At infinity the charging current finally diminishes to zero and the capacitor acts like an open circuit with the supply voltage value entirely across the capacitor as $V_c = V_s$.

So mathematically we can say that the time required for a capacitor to charge up to one time constant, (1T) is given as:

RC Time Constant, Tau

$$\tau \equiv R \times C$$

This RC time constant only specifies a rate of charge where, R is in Ω and C in Farads.

Since voltage V is related to charge on a capacitor given by the equation, $V_c = Q/C$, the voltage across the capacitor (Vc) at any instant in time during the charging period is given as:

$$V_C = V_S\left(1 - e^{(-t/RC)}\right)$$

Where:

- Vc is the voltage across the capacitor
- Vs is the supply voltage
- e is an irrational number presented by Euler as: 2.7182
- t is the elapsed time since the application of the supply voltage
- RC is the *time constant* of the RC charging circuit

After a period equivalent to 4 time constants, (4T) the capacitor in this RC charging circuit is said to be virtually fully charged as the voltage developed across the capacitors plates has now reached 98% of its maximum value, 0.98Vs. The time period taken for the capacitor to reach this 4T point is known as the **Transient Period**.

After a time of 5T the capacitor is now said to be fully charged with the voltage across the capacitor, (Vc) being aproximately equal to the supply voltage, (Vs). As the capacitor is therefore fully charged, no more charging current flows in the circuit so IC = 0. The time period after this 5T time period is commonly known as the **Steady State Period**.

Then we can show in the following table the percentage voltage and current values for the capacitor in a RC charging circuit for a given time constant.

RC Charging Table

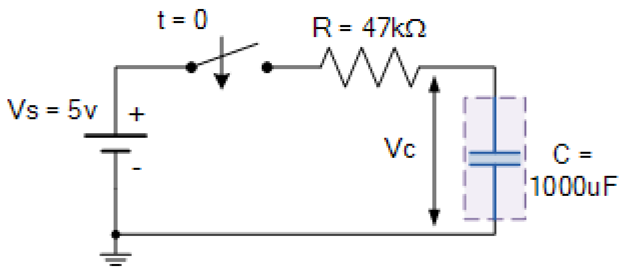
Time Constant	RC Value	Percentage of Maximum	
		Voltage	**Current**
0.5 time constant	0.5T = 0.5RC	39.3%	60.7%
0.7 time constant	0.7T = 0.7RC	50.3%	49.7%
1.0 time constant	1T = 1RC	63.2%	36.8%
2.0 time constants	2T = 2RC	86.5%	13.5%
3.0 time constants	3T = 3RC	95.0%	5.0%
4.0 time constants	4T = 4RC	98.2%	1.8%
5.0 time constants	5T = 5RC	99.3%	0.7%

Notice that the charging curve for a RC charging circuit is exponential and not linear. This means that in reality the capacitor never reaches 100% fully charged. So for all practical purposes, after five time constants (5T) it reaches 99.3% charge, so at this point the capacitor is considered to be fully charged.

As the voltage across the capacitor Vc changes with time, and is therefore a different value at each time constant up to 5T, we can calculate the value of capacitor voltage, Vc at any given point, for example.

RC Charging Circuit Example No1

Calculate the RC time constant, τ of the following circuit.



The time constant, τ is found using the formula T = R x C in seconds.

Therefore the time constant τ is given as: T = R x C = 47k x 1000uF = 47 Secs

a) **What will be the value of the voltage across the capacitors plates at exactly 0.7 time constants?**

At 0.7 time constants (0.7T) Vc = 0.5Vs. Therefore, Vc = 0.5 x 5V = 2.5V

b) **What value will be the voltage across the capacitor at 1 time constant?**

At 1 time constant (1T) Vc = 0.63Vs. Therefore, Vc = 0.63 x 5V = 3.15V

c) **How long will it take to “fully charge” the capacitor from the supply?**

We have learnt that the capacitor will be fully charged after 5 time constants, (5T).

1 time constant (1T) = 47 seconds, (from above). Therefore, 5T = 5 x 47 = 235 secs

d) **The voltage across the Capacitor after 100 seconds?**

The voltage formula is given as $Vc = V(1 - e^{(-t/RC)})$ so this becomes: $Vc = 5(1 - e^{(-100/47)})$

Where: V = 5 volts, t = 100 seconds, and RC = 47 seconds from above.

Therefore, $Vc = 5(1 - e^{(-100/47)}) = 5(1 - e^{-2.1277}) = 5(1 - 0.1191) =$ 4.4 volts

We have seen here that the charge on a capacitor is given by the expression: Q = CV, where C is its fixed capacitance value, and V is the applied voltage. We have also learnt that when a voltage is firstly applied to the plates of the capacitor it charges up at a rate determined by its RC time constant, τ and will be considered fully charged after five time constsants, or 5T.

In the next tutorial we will examine the current-voltage relationship of a discharging capacitor and look at the discharging curves associated with it when the capacitors plates are effectively shorted together.

https://www.electronics-tutorials.ws/rc/rc_2.html

RC Discharging Circuit

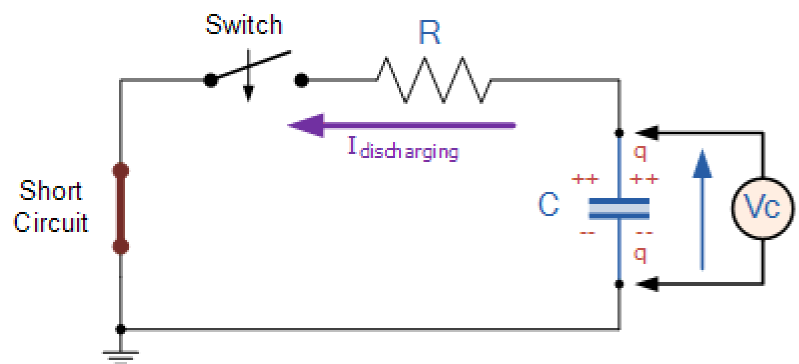
When a voltage source is removed from a fully charged RC circuit, the capacitor, C will discharge back through the resistance, R.

In the previous *RC Charging Circuit* tutorial, we saw how a Capacitor, C charges up through the resistor until it reaches an amount of time equal to 5 time constants known as 5T, and then remains fully charged as long as a constant supply is applied to it.

If this fully charged capacitor is now disconnected from its DC battery supply voltage, the stored energy built up during the charging process would stay indefinitely on its plates, (assuming an ideal capacitor and ignoring any internal losses), keeping the voltage stored across its connecting terminals at a constant value.

If the battery was replaced by a short circuit, when the switch is closed the capacitor would discharge itself back through the resistor, R as we now have a **RC discharging circuit**. As the capacitor discharges its current through the series resistor the stored energy inside the capacitor is extracted with the voltage Vc across the capacitor decaying to zero as shown below.

RC Discharging Circuit

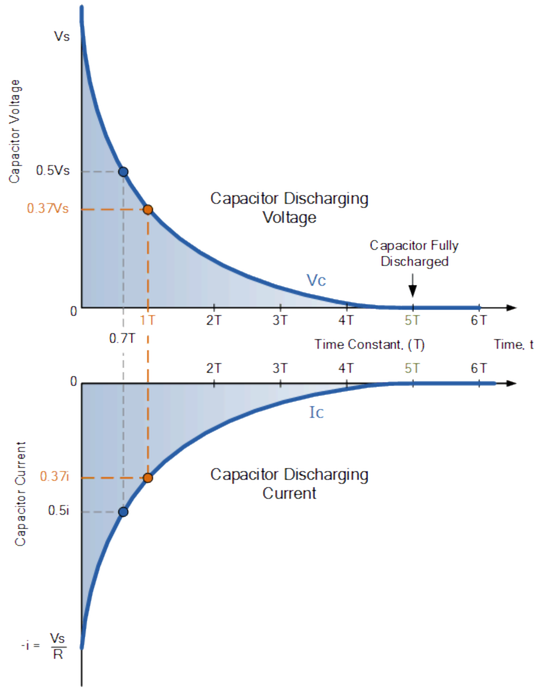


As we saw in the previous tutorial, in a **RC Discharging Circuit** the time constant (τ) is still equal to the value of 63%. Then for a RC discharging circuit that is initially fully charged, the voltage across the capacitor after one time constant, 1T, has dropped by 63% of its initial value which is 1 – 0.63 = 0.37 or 37% of its final value.

Thus the time constant of the circuit is given as the time taken for the capacitor to discharge down to within 63% of its fully charged value. So one time constant for an RC discharge circuit is given as the voltage across the plates representing 37% of its final value, with its final value being zero volts (fully discharged), and in our curve this is given as 0.37Vs.

As the capacitor discharges, it does not lose its charge at a constant rate. At the start of the discharging process, the initial conditions of the circuit are: t = 0, i = 0 and q = Q. The voltage across the capacitors plates is equal to the supply voltage and $V_C = V_S$. As the voltage at t = 0 across the capacitors plates is at its highest value, maximum discharge current therefore flows around the RC circuit.

RC Discharging Circuit Curves



When the switch is first closed, the capacitor starts to discharge as shown. The rate of decay of the RC discharging curve is steeper at the beginning because the discharging rate is fastest at the start, but then tapers off exponentially as the capacitor looses charge at a slower rate. As the discharge continues, V_C reduces resulting in less discharging current.

We saw in the previous RC charging circuit that the voltage across the capacitor, C is equal to 0.5Vc at 0.7T with the steady state fully discharged value being finally reached at 5T.

For a RC discharging circuit, the voltage across the capacitor (V_C) as a function of time during the discharge period is defined as:

$$V_C = V_S \times e^{-t/RC}$$

Where:

- V_C is the voltage across the capacitor
- V_S is the supply voltage
- t is the elapsed time since the removal of the supply voltage
- RC is the *time constant* of the RC discharging circuit

Just like the previous RC Charging circuit, we can say that in a **RC Discharging Circuit** the time required for a capacitor to discharge itself down to one time constant is given as:

$$\tau \equiv R \times C$$

Where, R is in Ω and C in Farads.

Thus we can show in the following table the percentage voltage and current values for the capacitor in a RC discharging circuit for a given time constant.

RC Discharging Table

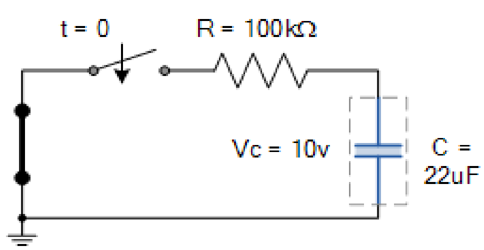
Time Constant	RC Value	Percentage of Maximum	
		Voltage	Current
0.5 time constant	0.5T = 0.5RC	60.7%	39.3%
0.7 time constant	0.7T = 0.7RC	49.7%	50.3%
1.0 time constant	1T = 1RC	36.8%	63.2%
2.0 time constants	2T = 2RC	13.5%	86.5%
3.0 time constants	3T = 3RC	5.0%	95.0%
4.0 time constants	4T = 4RC	1.8%	98.2%
5.0 time constants	5T = 5RC	0.7%	99.3%

Note that as the decaying curve for a RC discharging circuit is exponential, for all practical purposes, after five time constants the voltage across the capacitor's plates is much less than 1% of its inital starting value, so the capacitor is considered to be fully discharged.

So an RC circuit's time constant is a measure of how quickly it either charges or discharges.

RC Discharging Circuit Example No1

A capacitor is fully charged to 10 volts. Calculate the RC time constant, τ of the following RC discharging circuit when the switch is first closed.



The time constant, τ is found using the formula T = R*C in seconds.

Therefore the time constant τ is given as: T = R*C = 100k x 22uF = <u>2.2 Seconds</u>

a) What value will be the voltage across the capacitor at 0.7 time constants? At 0.7 time constants (0.7T) Vc = 0.5Vc. Therefore, Vc = 0.5 x 10V = 5V
b) What value will be the voltage across the capacitor after 1 time constant? At 1 time constant (1T) Vc = 0.37Vc. Therefore, Vc = 0.37 x 10V = 3.7V
c) How long will it take for the capacitor to “fully discharge” itself, (equal to 5 time constants) 1 time constant (1T) = 2.2 seconds. Therefore, 5T = 5 x 2.2 = 11 Seconds

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RC Waveforms

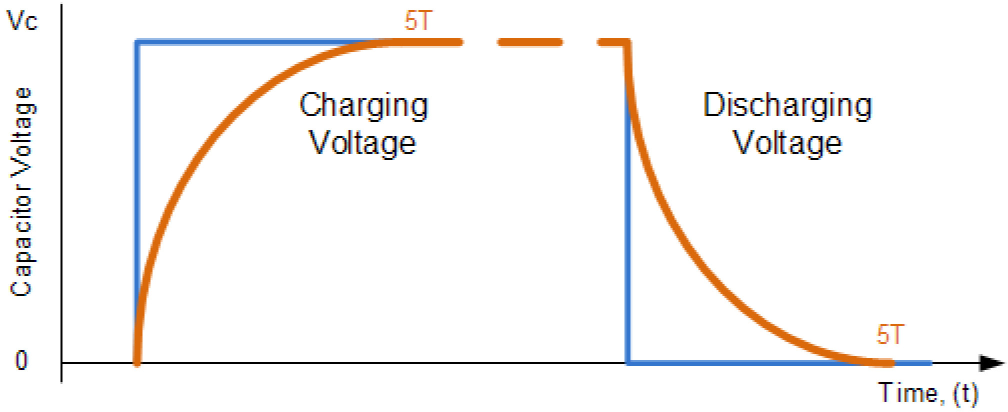
RC circuits can produce useful output waveforms such as square, triangular and sawtooth, when a periodic waveform are applied to its input

In the previous RC Charging and Discharging tutorials, we saw how a capacitor has the ability to both charge and discharges itself through a series connected resistor. The time taken for this capacitor to either fully charge or fully discharge is equal to five RC time constants or 5T when a constant DC voltage is either applied or removed.

But what would happen if we changed this constant DC supply to a pulsed or square-wave waveform that constantly changes from a maximum value to a minimum value at a rate determined by its time period or frequency. How would this affect the output **RC waveform** for a given RC time constant value?

We saw previously that the capacitor charges up to 5T when a voltage is applied and discharges down to 5T when it is removed. In RC charging and discharging circuits this 5T time constant value always remains true as it is fixed by the resistor-capacitor (RC) combination. Then the actual time required to fully charge or discharge the capacitor can only be changed by changing the value of either the capacitor itself or the resistor in the circuit and this is shown below.

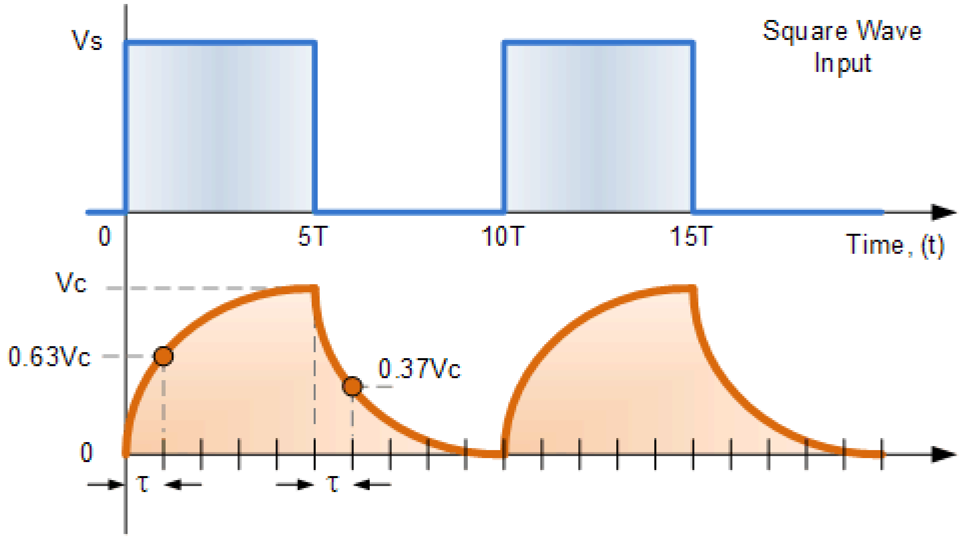
Typical RC Waveform



Square Wave Signal

Useful wave shapes can be obtained by using RC circuits with the required time constant. If we apply a continuous *square wave* voltage waveform to the RC circuit whose pulse width matches that exactly of the 5RC time constant (5T) of the circuit, then the voltage waveform across the capacitor would look something like this:

A 5RC Input Waveform

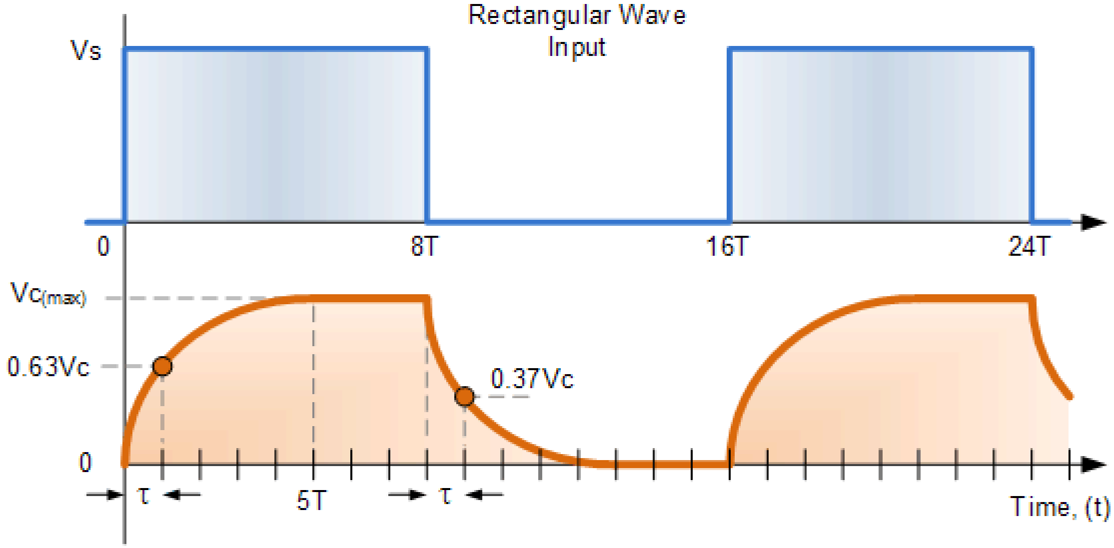


The voltage drop across the capacitor alternates between charging up to Vc and discharging down to zero according to the input voltage. Here in this example, the frequency (and therefore the resulting time period, $f = 1/T$) of the input square wave voltage waveform exactly matches twice that of the 5RC time constant.

This (10RC) time constant allows the capacitor to fully charge during the “ON” period (0-to-5RC) of the input waveform and then fully discharge during the “OFF” period (5-to-10RC) resulting in a perfectly matched RC waveform.

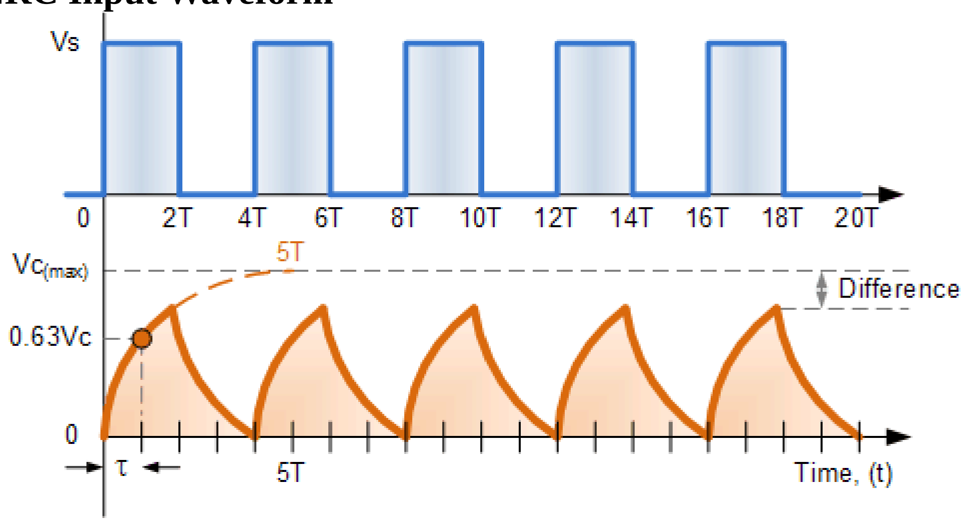
If the time period of the input waveform is made longer (lower frequency, $f < 1/10RC$) for example an “ON” half-period pulse width equivalent to say “8RC”, the capacitor would then stay fully charged longer and also stay fully discharged longer producing an RC waveform as shown.

A Longer 8RC Input Waveform



If however we now reduced the total time period of the input waveform (higher frequency, $f >$ 1/10RC), to say "4RC", the capacitor would not have sufficient time to either fully charge during the "ON" period or fully discharge during the "OFF" period. Therefore the resultant voltage drop across the capacitor, Vc would be less than its maximum input voltage producing an RC waveform as shown below.

A Shorter 4RC Input Waveform

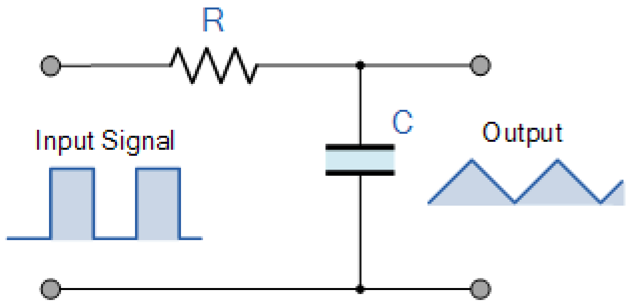


Then by varying the RC time constant or the frequency of the input waveform, we can vary the voltage across the capacitor producing a relationship between Vc and time, t. This relationship can be used to change the shape of various waveforms so that the output waveform across the capacitor barely resembles that of the input.

Frequency Response

The RC Integrator

The **Integrator** is a type of **Low Pass Filter** circuit that converts a square wave input signal into a triangular waveform output. As seen above, if the 5RC time constant is long compared to the time period of the input RC waveform the resultant output will be triangular in shape and the higher the input frequency the lower will be the output amplitude compared to that of the input.

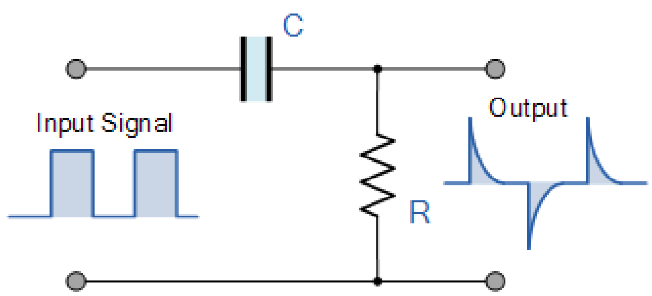


From which we derive an ideal voltage output for the integrator as:

$$V_{out} = \frac{1}{RC} \int_0^t V_{in}\, dt$$

The RC Differentiator

The **Differentiator** is a **High Pass Filter** type of circuit that can convert a square wave input signal into high frequency spikes at its output. If the 5RC time constant is short compared to the time period of the input waveform, then the capacitor will become fully charged more quickly before the next change in the input cycle.



When the capacitor is fully charged the output voltage across the resistor is zero. The arrival of the falling edge of the input waveform causes the capacitor to reverse charge giving a negative output spike, then as the square wave input changes during each cycle the output spike changes from a positive value to a negative value.

From which we have an ideal voltage output for the Differentiator as:

$$V_{out} = RC \frac{dV_{in}}{dt}$$

Alternating Sine Wave Input Signal

If we now change the input RC waveform of these RC circuits to that of a sinusoidal **Sine Wave** voltage signal the resultant output RC waveform will remain unchanged and only its amplitude will be affected. By changing the positions of the Resistor, R or the Capacitor, C a simple first order *Low Pass* or a *High Pass* filters can be made with the frequency response of these two circuits dependant upon the input frequency value.

Low-frequency signals are passed from the input to the output with little or no attenuation, while high-frequency signals are attenuated significantly to almost zero. The opposite is also true for a High Pass filter circuit. Normally, the point at which the response has fallen 3dB (cut-off frequency, f_C) is used to define the filters bandwidth and a loss of 3dB corresponds to a reduction in output voltage to 70.7 percent of the original value.

RC Filter Cut-off Frequency

$$fc = \frac{1}{2\pi RC} \text{ in Hertz}$$

where RC is the time constant of the circuit previously defined and can be replaced by tau, T. This is another example of how the *Time Domain* and the *Frequency Domain* concepts are related.

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RC Integrator

The RC integrator is a series connected RC network that produces an output signal which corresponds to the mathematical process of integration.

For a passive RC integrator circuit, the input is connected to a resistance while the output voltage is taken from across a capacitor being the exact opposite to the *RC Differentiator Circuit*. The capacitor charges up when the input is high and discharges when the input is low.

In Electronics, the basic series connected resistor-capacitor (RC) circuit has many uses and applications from basic charging/discharging circuits to high-order filter circuits. This two component passive RC circuit may look simple enough, but depending on the type and frequency of the applied input signal, the behaviour and response of this basic RC circuit can be very different.

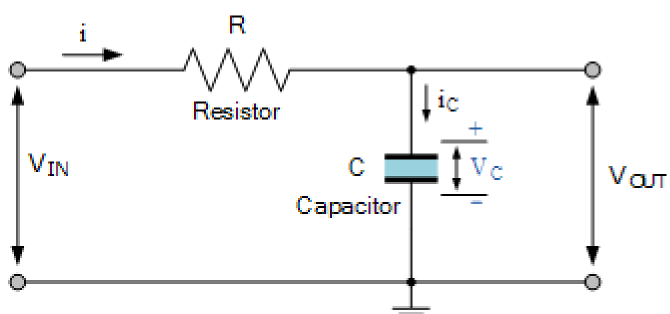
A passive RC network is nothing more than a resistor in series with a capacitor, that is a fixed resistance in series with a capacitor that has a frequency dependant reactance which decreases as the frequency across its plates increases. Thus at low frequencies the reactance, Xc of the capacitor is high while at high frequencies its reactance is low due to the standard capacitive reactance formula of $Xc = 1/(2\pi f C)$, and we saw this effect in our tutorial about *Passive Low Pass Filters*.

If the input signal is a sine wave, an **rc integrator** will simply act as a simple low pass filter (LPF) above its cut-off point with the cut-off or corner frequency corresponding to the RC time constant (tau, τ) of the series network. Thus when fed with a pure sine wave, an RC integrator acts as a passive low pass filter reducing its output above the cut-off frequency point.

As we have seen previously, the RC time constant reflects the relationship between the resistance and the capacitance with respect to time with the amount of time, given in seconds, being directly proportional to resistance, R and capacitance, C.

Thus the rate of charging or discharging depends on the RC time constant, $\tau = RC$. Consider the circuit below.

RC Integrator



For an RC integrator circuit, the input signal is applied to the resistance with the output taken across the capacitor, then V_{OUT} equals V_C. As the capacitor is a frequency dependant element, the amount of charge that is established across the plates is equal to the time domain integral of the current. That is it takes a certain amount of time for the capacitor to fully charge as the capacitor can not charge instantaneously only charge exponentially.

Therefore the capacitor current can be written as:

$$i_{C(t)} = C\frac{dV_{C(t)}}{dt}$$

This basic equation above of i_C = C(dVc/dt) can also be expressed as the instantaneous rate of change of charge, Q with respect to time giving us the following standard equation of: i_C = dQ/dt where the charge Q = C x Vc, that is capacitance times voltage.

The rate at which the capacitor charges (or discharges) is directly proportional to the amount of the resistance and capacitance giving the time constant of the circuit. Thus the time constant of a RC integrator circuit is the time interval that equals the product of R and C.

Since capacitance is equal to Q/Vc where electrical charge, Q is the flow of a current (i) over time (t), that is the product of i x t in coulombs, and from Ohms law we know that voltage (V) is equal to i x R, substituting these into the equation for the RC time constant gives:

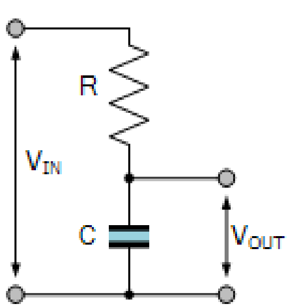
RC Time Constant

$$RC = R\frac{Q}{V} = R\frac{i\times T}{i\times R} = \cancel{R}\frac{\cancel{i}\times T}{\cancel{i}\times \cancel{R}} = T$$

$$\therefore T = RC$$

Then we can see that as both i and R cancel out, only T remains indicating that the time constant of an RC integrator circuit has the dimension of time in seconds, being given the Greek letter tau, τ. Note that this time constant reflects the time (in seconds) required for the capacitor to charge up to 63.2% of the maximum voltage or discharge down to 36.8% of maximum voltage.

Capacitor Voltage



We said previously that for the RC integrator, the output is equal to the voltage across the capacitor, that is: V_{OUT} equals V_C. This voltage is proportional to the charge, Q being stored on the capacitor given by: Q = VxC.

The result is that the output voltage is the integral of the input voltage with the amount of integration dependent upon the values of R and C and therefore the time constant of the network.

We saw above that the capacitors current can be expressed as the rate of change of charge, Q with respect to time. Therefore, from a basic rule of differential calculus, the derivative of Q with respect to time is dQ/dt and as i = dQ/dt we get the following relationship of:

$Q = \int i dt$ (the charge Q on the capacitor at any instant in time)

Since the input is connected to the resistor, the same current, i must pass through both the resistor and the capacitor ($i_R = i_C$) producing a V_R voltage drop across the resistor so the current, (i) flowing through this series RC network is given as:

$$i(t) = \frac{V_{IN}}{R} = \frac{V_R}{R} = C\frac{dV}{dt}$$

therefore:

$$V_{OUT} = V_C = \frac{Q}{C} = \frac{\int i dt}{C} = \frac{1}{C}\int i(t) dt$$

As $i = V_{IN}/R$, substituting and rearranging to solve for V_{OUT} as a function of time gives:

$$V_{OUT} = \frac{1}{C}\int\left(\frac{V_{IN}}{R}\right)dt = \frac{1}{RC}\int V_{IN} dt$$

So in other words, the output from an RC integrator circuit, which is the voltage across the capacitor is equal to the time Integral of the input voltage, V_{IN} weighted by a constant of 1/RC. Where RC represents the time constant, τ.

Then assuming the initial charge on the capacitor is zero, that is $V_{OUT} = 0$, and the input voltage V_{IN} is constant, the output voltage, V_{OUT} is expressed in the time domain as:

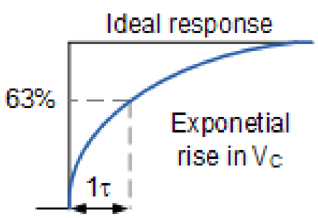
RC Integrator Formula

$$V_{OUT} = \frac{1}{RC}\int_0^t V_{IN(t)} dt$$

So an RC integrator circuit is one in which the output voltage, V_{OUT} is proportional to the integral of the input voltage, and with this in mind, lets see what happens when we apply a single positive pulse in the form of a step voltage to the RC integrator circuit.

Single Pulse RC Integrator

When a single step voltage pulse is applied to the input of an RC integrator, the capacitor charges up via the resistor in response to the pulse. However, the output is not instant as the voltage across the capacitor cannot change instantaneously but increases exponentially as the capacitor charges at a rate determined by the RC time constant, τ = RC.



We now know that the rate at which the capacitor either charges or discharges is determined by the RC time constant of the circuit. If an ideal step voltage pulse is applied, that is with the leading edge and trailing edge considered as being instantaneous, the voltage across the capacitor will increase for charging and decrease for discharging, exponentially over time at a rate determined by:

Capacitor Charging

$$V_{C(t)} = V\left(1 - e^{-\left(\frac{t}{RC}\right)}\right)$$

Capacitor Discharging

$$V_{C(t)} = V\left(e^{-\left(\frac{t}{RC}\right)}\right)$$

So if we assume a capacitor voltage of one volt (1V), we can plot the percentage of charge or discharge of the capacitor for each individual R time constant as shown in the following table.

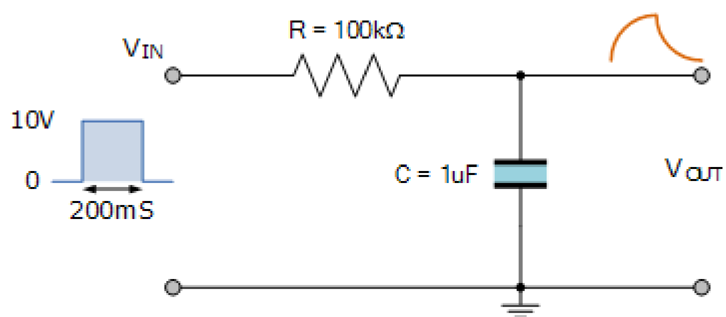
Time Constant	Capacitor Charging	Capacitor Discharging
τ	% Charged	% Discharged
0.5	39.4%	60.6%
0.7	50%	50%
1	63.2%	36.7%
2	86.4%	13.5%
3	95.0%	4.9%
4	98.1%	1.8%
5	99.3%	0.67%

Note that at 5 time constants or above, the capacitor is considered to be 100 percent fully charged or fully discharged.

So now lets assume we have an RC integrator circuit consisting of a 100kΩ resistor and a 1uF capacitor as shown.

RC Integrator Circuit Example

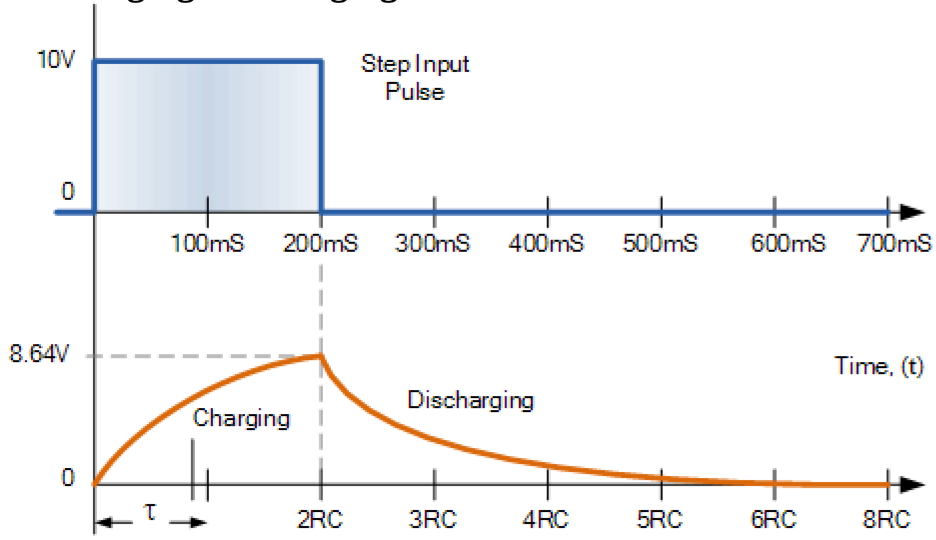
The time constant, τ of the RC integrator circuit is therefore given as: RC = 100kΩ x 1uF = 100ms.



So if we apply a step voltage pulse to the input with a duration of say, two time constants (200mS), then from the table above we can see that the capacitor will charge to 86.4% of its fully charged value. If this pulse has an amplitude of 10 volts, then this equates to 8.64 volts before the capacitor discharges again back through the resistor to the source as the input pulse returns to zero.

If we assume that the capacitor is allowed to fully discharge in a time of 5 time constants, or 500mS before the arrival of the next input pulse, then the graph of the charging and discharging curves would look something like this:

RC Integrator Charging/Discharging Curves



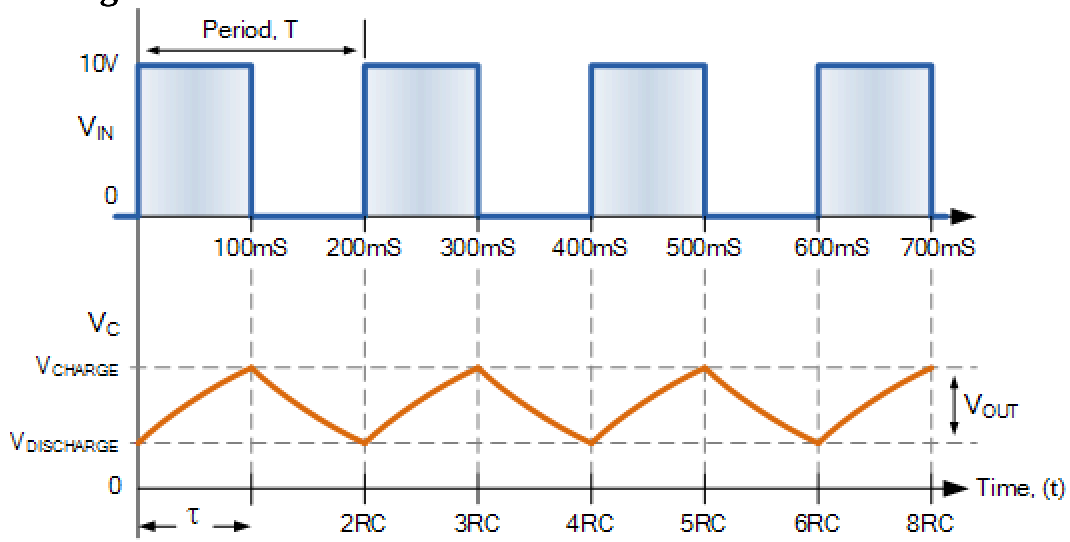
Note that the capacitor is discharging from an initial value of 8.64 volts (2 time constants) and not from the 10 volts input.

Then we can see that as the RC time constant is fixed, any variation to the input pulse width will affect the output of the RC integrator circuit. If the pulse width is increased and is equal too or greater than 5RC, then the shape of the output pulse will be similar to that of the input as the output voltage reaches the same value as the input.

If however the pulse width is decreased below 5RC, the capacitor will only partially charge and not reach the maximum input voltage resulting in a smaller output voltage because the capacitor cannot charge as much resulting in an output voltage that is proportional to the integral of the input voltage.

So if we assume an input pulse equal to one time constant, that is 1RC, the capacitor will charge and discharge not between 0 volts and 10 volts but between 63.2% and 38.7% of the voltage across the capacitor at the time of change. Note that these values are determined by the RC time constant.

Fixed RC Integrator Time Constant



So for a continuous pulse input, the correct relationship between the periodic time of the input and the RC time constant of the circuit, integration of the input will take place producing a sort of ramp up, and then a ramp down output. But for the circuit to function correctly as an integrator, the value of the RC time constant has to be large compared to the inputs periodic time. That is RC ⋙ T, usually 10 times greater.

This means that the magnitude of the output voltage (which was proportional to 1/RC) will be very small between its high and low voltages severely attenuating the output voltage. This is because the capacitor has much less time to charge and discharge between pulses but the average output DC voltage will increase towards one half magnitude of the input and in our pulse example above, this will be 5 volts (10/2).

RC Integrator as a Sine Wave Generator

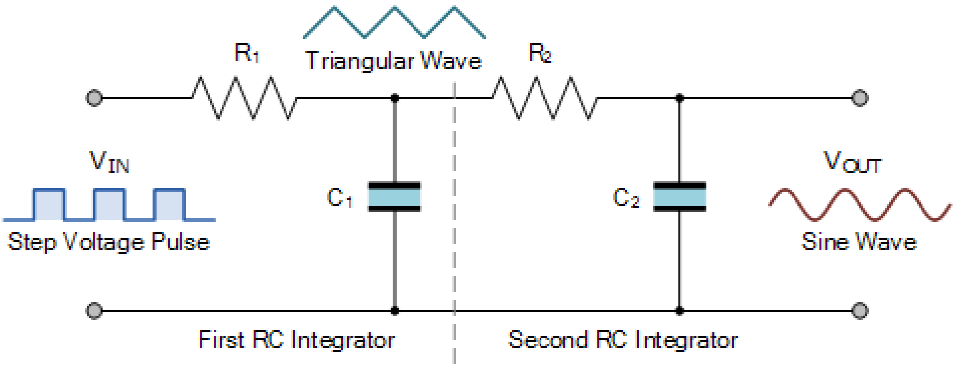
We have seen above that an *RC integrator* circuit can perform the operation of integration by applying a pulse input resulting in a ramp-up and ramp-down triangular wave output due to the charging and discharging characteristics of the capacitor. But what would happen if we reversed the process and applied a triangular waveform to the input, would we get a pulse or square wave output?

When the input signal to an RC integrator circuit is a pulse shaped input, the output is a triangular wave. But when we apply a triangular wave, the output becomes a sine wave due to the integration over time of the ramp signal.

There are many ways to produce a sinusoidal waveform, but one simple and cheap way to electronically produce a sine waves type waveform is to use a pair of passive RC integrator circuits connected together in series as shown.

Sine Wave RC Integrator

Here the first RC integrator converts the original pulse shaped input into a ramp-up and ramp-down triangular waveform which becomes the input of the second RC integrator. This second RC integrator circuit rounds off the points of the triangular waveform converting it into a sine wave as it is effectively performing a double integration on the original input signal with the RC time constant affecting the degree of integration.



As the integration of a ramp produces a sine function, (basically a round-off triangular waveform) its periodic frequency in Hertz will be equal to the period T of the original pulse. Note also that if we reverse this signal and the input signal is a sine wave, the circuit does not act as an integrator, but as a simple low pass filter (LPF) with the sine wave, being a pure waveform does not change shape, only its amplitude is affected.

RC Integrator Summary

We have seen here that the RC integrator is basically a series RC low-pass filter circuit which when a step voltage pulse is applied to its input produces an output that is proportional to the integral of its input. This produces a standard equation of: $Vo = \int Vi\,dt$ where Vi is the signal fed to the integrator and Vo is the integrated output signal.

The integration of the input step function produces an output that resembles a triangular ramp function with an amplitude smaller than that of the original pulse input with the amount of attenuation being determined by the time constant. Thus the shape of the output waveform depends on the relationship between the time constant of the circuit and the frequency (period) of the input pulse.

An RC integrators time constant is always compared to the period, T of the input, so a long RC time constant will produce a triangular wave shape with a low amplitude compared to the input signal as the capacitor has less time to fully charge or discharge. A short time constant allows the capacitor more time to charge and discharge producing a more typical rounded shape.

By connecting two RC integrator circuits together in parallel has the effect of a double integration on the input pulse. The result of this double integration is that the first integrator circuit converts the step voltage pulse into a triangular waveform and the second integrator circuit converts the triangular waveform shape by rounding off the points of the triangular waveform producing a sine wave output waveform with a greatly reduced amplitude.

https://www.electronics-tutorials.ws/rc/rc-differentiator.html
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RC Differentiator

The passive RC differentiator is a series connected RC network that produces an output signal which corresponds to the mathematical process of differentiation.

For a passive RC differentiator circuit, the input is connected to a capacitor while the output voltage is taken from across a resistance being the exact opposite to the *RC Integrator Circuit*.

A passive RC differentiator is nothing more than a capacitance in series with a resistance, that is a frequency dependant device which has reactance in series with a fixed resistance (the opposite to an integrator). Just like the integrator circuit, the output voltage depends on the circuits RC time constant and input frequency.

Thus at low input frequencies the reactance, X_C of the capacitor is high blocking any d.c. voltage or slowly varying input signals. While at high input frequencies the capacitors reactance is low allowing rapidly varying pulses to pass directly from the input to the output.

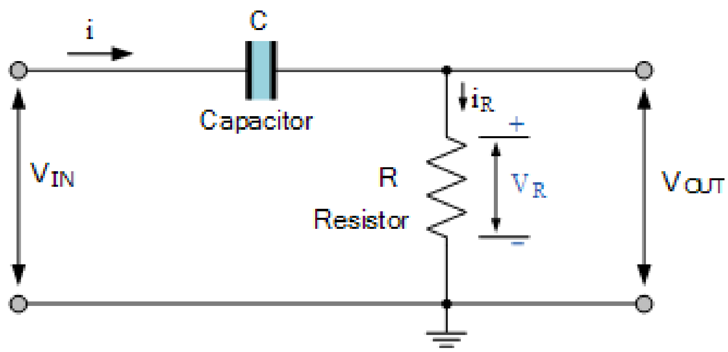
This is because the ratio of the capacitive reactance (X_C) to resistance (R) is different for different frequencies and the lower the frequency the less output. So for a given time constant, as the frequency of the input pulses increases, the output pulses more and more resemble the input pulses in shape.

We saw this effect in our tutorial about *Passive High Pass Filters* and if the input signal is a sine wave, an **rc differentiator** will simply act as a simple high pass filter (HPF) with a cut-off or corner frequency that corresponds to the RC time constant (tau, τ) of the series network.

Thus when fed with a pure sine wave an RC differentiator circuit acts as a simple passive high pass filter due to the standard capacitive reactance formula of $X_C = 1/(2\pi f C)$.

But a simple RC network can also be configured to perform differentiation of the input signal. We know from previous tutorials that the current through a capacitor is a complex exponential given by: $i_C = C(dVc/dt)$. The rate at which the capacitor charges (or discharges) is directly proportional to the amount of resistance and capacitance giving the time constant of the circuit. Thus the time constant of a RC differentiator circuit is the time interval that equals the product of R and C. Consider the basic RC series circuit below.

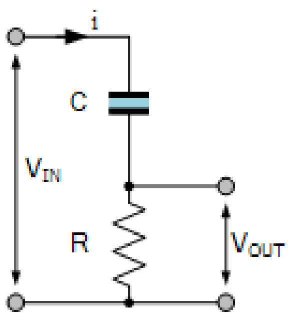
RC Differentiator Circuit



For an RC differentiator circuit, the input signal is applied to one side of the capacitor with the output taken across the resistor, then V_{OUT} equals V_R. As the capacitor is a frequency dependant element, the amount of charge that is established across the plates is equal to the time domain integral of the current. That is it takes a certain amount of time for the capacitor to fully charge as the capacitor can not charge instantaneously only charge exponentially.

We saw in our tutorial about *RC Integrators* that when a single step voltage pulse is applied to the input of an RC integrator, the output becomes a sawtooth waveform if the RC time constant is long enough. The RC differentiator will also change the input waveform but in a different way to the integrator.

Resistor Voltage



We said previously that for the RC differentiator, the output is equal to the voltage across the resistor, that is: V_{OUT} equals V_R and being a resistance, the output voltage can change instantaneously.

However, the voltage across the capacitor can not change instantly but depends on the value of the capacitance, C as it tries to store an electrical charge, Q across its plates. Then the current flowing into the capacitor, that is i_t depends on the rate of change of the charge across its plates. Thus the capacitor current is not proportional to the voltage but to its time variation giving: i = dQ/dt.

As the amount of charge across the capacitors plates is equal to Q = C x Vc, that is capacitance times voltage, we can derive the equation for the capacitors current as:

Capacitor Current

$$i_{(t)} = \frac{dQ}{dt} = \frac{d(C \times dV_C)}{dt} = C\frac{dV_C}{dt} = C\frac{dV_{IN}}{dt}$$

Therefore the capacitor current can be written as:

$$i_{C(t)} = C\frac{dV_{IN(t)}}{dt}$$

As V_{OUT} equals V_R where V_R according to ohms law is equal too: i_R x R. The current that flows through the capacitor must also flow through the resistance as they are both connected together in series. Thus:

$$V_{OUT} = V_R = R \times i_R$$

$$i_C = C\frac{dV_{IN}}{dt}$$

$$\text{As } i_R = i_C, \text{ therefore:}$$

$$V_{OUT} = RC\frac{dV_{IN}}{dt}$$

Thus the standard equation given for an RC differentiator circuit is:

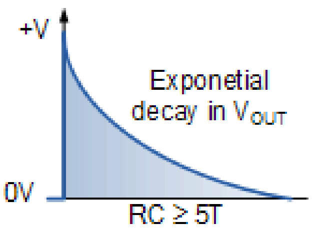
RC Differentiator Formula

$$V_{OUT} = RC\frac{dV_{IN}}{dt}$$

Then we can see that the output voltage, V_{OUT} is the derivative of the input voltage, V_{IN} which is weighted by the constant of RC. Where RC represents the time constant, τ of the series circuit.

Single Pulse RC Differentiator

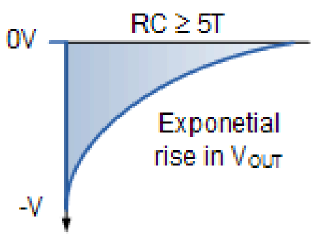
When a single step voltage pulse is firstly applied to the input of an RC differentiator, the capacitor "appears" initially as a short circuit to the fast changing signal. This is because the slope dv/dt of the positive-going edge of a square wave is very large (ideally infinite), thus at the instant the signal appears, all the input voltage passes through to the output appearing across the resistor.



After the initial positive-going edge of the input signal has passed and the peak value of the input is constant, the capacitor starts to charge up in its normal way via the resistor in response to the input pulse at a rate determined by the RC time constant, τ = RC.

As the capacitor charges up, the voltage across the resistor, and thus the output decreases in an exponentially way until the capacitor becomes fully charged after a time constant of 5RC (5T), resulting in zero output across the resistor. Thus the voltage across the fully charged capacitor equals the value of the input pulse as: $V_C = V_{IN}$ and this condition holds true so long as the magnitude of the input pulse does not change.

If now the input pulse changes and returns to zero, the rate of change of the negative-going edge of the pulse pass through the capacitor to the output as the capacitor can not respond to this high dv/dt change. The result is a negative going spike at the output.



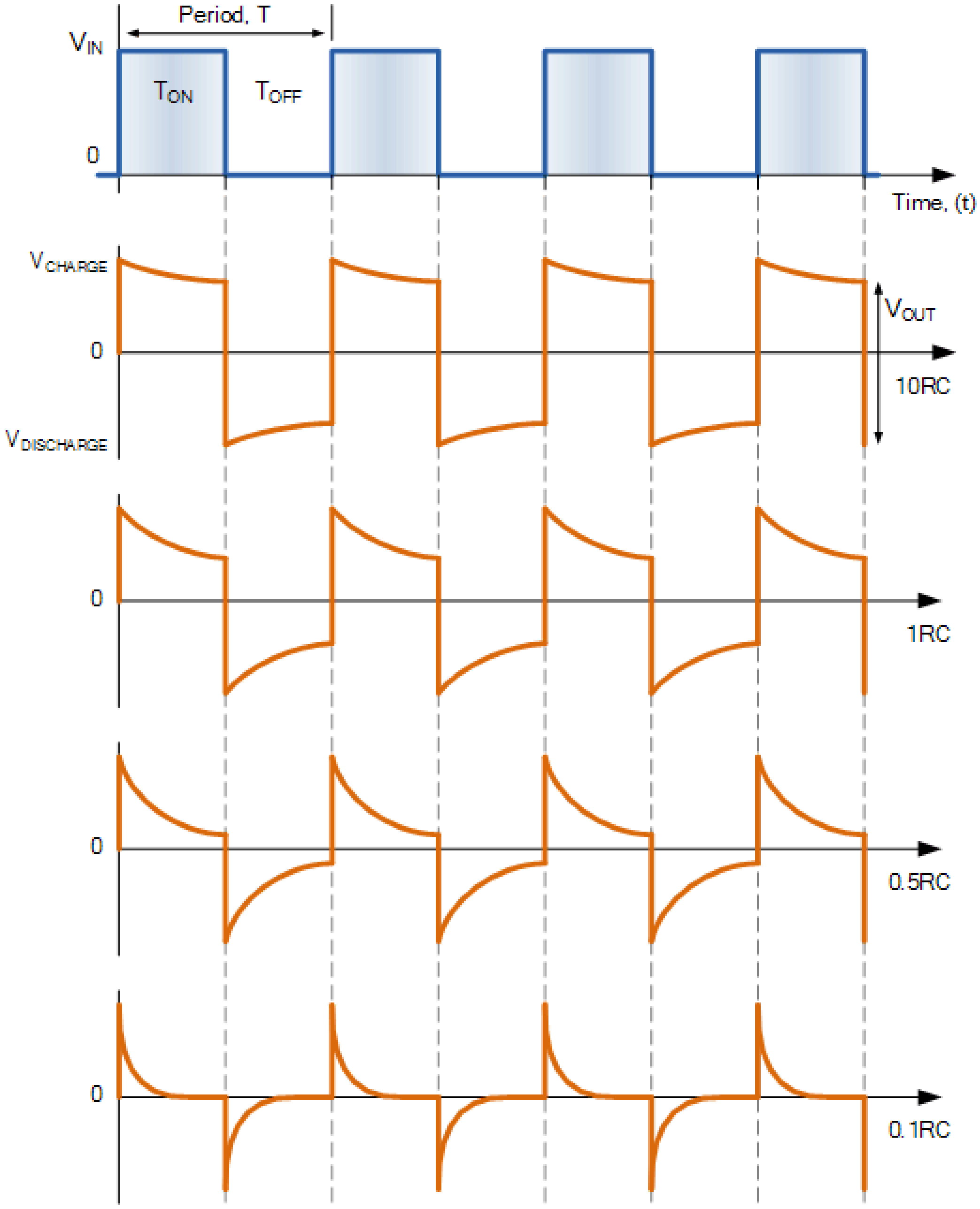
After the initial negative-going edge of the input signal, the capacitor recovers and starts to discharge normally and the output voltage across the resistor, and therefore the output, starts to increases exponentially as the capacitor discharges.

Thus whenever the input signal is changing rapidly, a voltage spike is produced at the output with the polarity of this voltage spike depending on whether the input is changing in a positive or a negative direction, as a positive spike is produced with the positive-going edge of the input signal, and a negative spike produced as a result of the negative-going input signal.

Thus the RC differentiator output is effectively a graph of the rate of change of the input signal which has no resemblance to the square wave input wave, but consists of narrow positive and negative spikes as the input pulse changes value.

By varying the time period, T of the square wave input pulses with respect to the fixed RC time constant of the series combination, the shape of the output pulses will change as shown.

RC Differentiator Output Waveforms



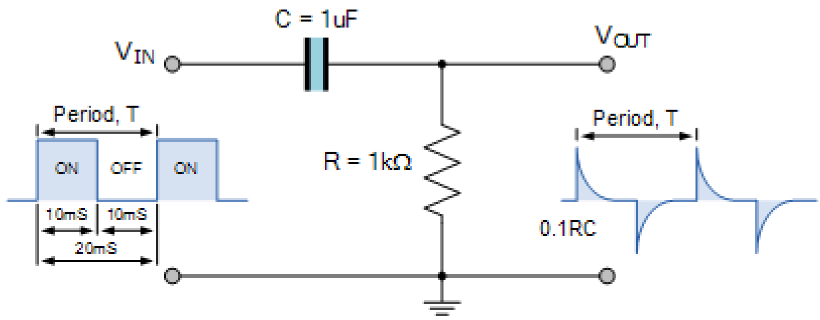
Then we can see that the shape of the output waveform depends on the ratio of the pulse width to the RC time constant. When RC is much larger (greater than 10RC) than the pulse width the output waveform resembles the square wave of the input signal. When RC is much smaller (less than 0.1RC) than the pulse width, the output waveform takes the form of very sharp and narrow spikes as shown above.

So by varying the time constant of the circuit from 10RC to 0.1RC we can produce a range of different wave shapes. Generally a smaller time constant is always used in RC differentiator circuits to provide good sharp pulses at the output across R. Thus the differential of a square wave pulse (high dv/dt step input) is an infinitesimally short spike resulting in an RC differentiator circuit.

Lets assume a square wave waveform has a period, T of 20mS giving a pulse width of 10mS (20mS divided by 2). For the spike to discharge down to 37% of its initial value, the pulse width must equal the RC time constant, that is RC = 10mS. If we choose a value for the capacitor, C of 1uF, then R equals 10kΩ.

For the output to resemble the input, we need RC to be ten times (10RC) the value of the pulse width, so for a capacitor value of say, 1uF, this would give a resistor value of: 100kΩ. Likewise, for the output to resemble a sharp pulse, we need RC to be one tenth (0.1RC) of the pulse width, so for the same capacitor value of 1uF, this would give a resistor value of: 1kΩ, and so on.

RC Differentiator Example



So by having an RC value of one tenth the pulse width (and in our example above this is 0.1 x 10mS = 1mS) or lower we can produce the required spikes at the output, and the lower the RC time constant for a given pulse width, the sharper the spikes. Thus the exact shape of the output waveform depends on the value of the RC time constant.

RC Differentiator Summary

We have seen here in this **RC Differentiator** tutorial that the input signal is applied to one side of a capacitor and the the output is taken across the resistor. A differentiator circuit is used to produce trigger or spiked typed pulses for timing circuit applications.

When a square wave step input is applied to this RC circuit, it produces a completely different wave shape at the output. The shape of the output waveform depending on the periodic time, T (an therefore the frequency, f) of the input square wave and on the circuit's RC time constant value.

When the periodic time of the input waveform is similar too, or shorter than, (higher frequency) the circuits RC time constant, the output waveform resembles the input waveform, that is a square wave profile. When the periodic time of the input waveform is much longer than, (lower frequency) the circuits RC time constant, the output waveform resembles narrow positive and negative spikes.

<u>The positive spike at the output is produced by the leading-edge of the input square wave, while the negative spike at the output is produced by the falling-edge of the input square wave. Then the output of an RC differentiator circuit depends on the rate of change of the input voltage as the effect is very similar to the mathematical function of differentiation.</u>

https://www.electronics-tutorials.ws/rc/time-constant.html
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Tau – The Time Constant

Tau is the time constant of an RC circuit that takes to change from one steady state condition to another steady state condition when subjected to a step change input condition

Tau, symbol **τ**, is the greek letter used in electrical and electronic calculations to represent the *time constant* of a circuit as a function of time. But what do we mean by a circuits time constant and transient response.

Electrical and electronic circuits are not always in a stable or steady state condition, but can be subjected to sudden step changes in the form of changing voltage levels or input conditions. For example the opening or closing of an input switch or sensor.

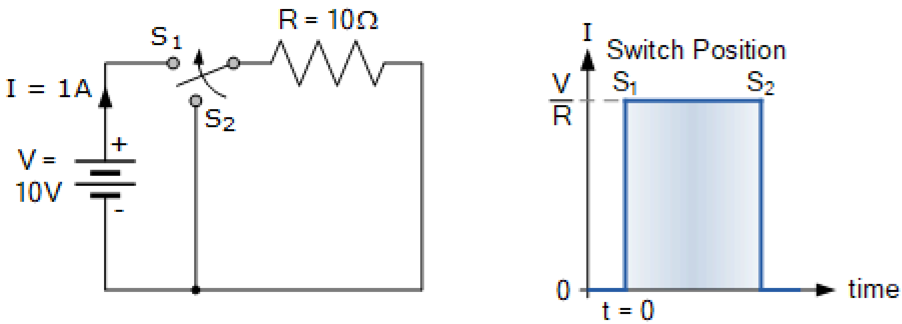
However, whenever a voltage or state change occurs, the circuit may not respond instantaneously to the change, but may take an amount of time no matter how small if reactive components such as capacitors and inductors are present within the circuit.

The change of state from one stable condition to another generally occurs at a rate determined by the time constant of the circuit which itself will be an exponentially value. Then the time constant of a circuit defines how the transient response of the circuits currents and voltages are changing over a set period of time.

We have seen in these tutorials that when subjected to a steady state DC voltage, a capacitor will act as an open circuit, an inductor will act as a short circuit and a resistor will act as a current limiting device. If the voltage across a capacitor as well as the current through an inductor cannot change instantly, then what will be their transient response when subjected to a step change condition.

But before we start applying some form of transient analysis to a capacitive circuit, let’s first remind ourselves of the V-I characteristics of an ordinary resistive circuit as shown below.

Resistive Circuit



 [2022-03-28]

With the switch in position S_2, the 10Ω resistor is shorted and therefore not connected to the 10 volt supply voltage (V). As a result, zero current flows through the resistor, so $I_R = 0$. However, when the switch is moved to position S_1 at time t = 0, a step voltage of 10 volts is applied directly across the 10Ω resistor resulting in a current of 1 ampere (I = V/R) flowing around the closed circuit.

As the resistor is of a fixed non-inductive value, the current changes instantly from 0 to 1 ampere within a fraction of a second as soon as the switch is moved to position S_1. Likewise, if the switch is returned back to position S_2, the supply voltage (V) is removed so the circuit current will immediately drop to zero again as shown in the above graph.

Then for a resistive cicruit the change of electrical state from one to another is almost instant, as there is nothing to resist this change. Therefore, resistors only limit the flow of electrical current around a circuit to a value determined by Ohms Law, that is V/R and as such there is no time constant or transient response associated with them.

Now let’s consider the transient response of a resistor connected in series with a capacitor forming a simple RC circuit. What would be the V-I characteristics of this combination when subjected to an input step voltage change as before.

RC Time Constant

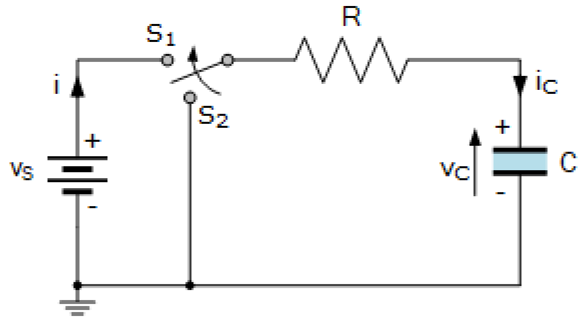
We have seen above that a resistance response instantly to any change in voltage applied to it. But a resistor is a passive linear device that does not store energy but instead dissipates energy in the form of heat. However, a capacitor (C) consists of two conducting plates (electrodes) separated by a dielectric insulating material that has the ability to store electrical energy in the form of an electrostatic charge (Q coulombs) within itself.

The result is that unlike the resistor, the capacitor cannot react instantly to quick or step changes in applied voltage so there will always be a short period of time immediately after the voltage is firstly applied for the circuit current and voltage across the capacitor to change state. In other words, there will be a certain amount of time required for the capacitor to change the amount of energy stored within its electric field, either increasing or decreasing in value.

The amount of time for the circuit to respond is expressed in multiples of R x C, that is the product of “Ohms x Farads” given in seconds (s). The current through the capacitor is given by: $i_C = C(dv/dt)$. Where: *dv* represents the change in voltage and *dt* represents the change in time.

Consider the simple resistor-capacitor RC circuit below.

Resistor-Capacitor (RC) Circuit



With the switch in position S_2 for a while, the resistor-capacitor combination is shorted and therefore not connected to the supply voltage, V_S. As a result, zero current flows around the circuit, so I = 0 and $V_C = 0$.

When the switch is moved to position S_1 at time t = 0, a step voltage (V) is applied to the RC circuit. At this instant in time, the fully discharged capacitor behaves like a short circuit due to the sudden *dv/dt* change of condition the exact moment the switch is closed to position S_1.

This change causes the circuit current to increase to a value limited only by the resistance of the circuit, the same as before. Thus when the switch S_1 is initially closed at t = 0, the current flowing around the closed circuit is approximately equal to V_R/R amperes, as V_R = I*R and $V_C = 0$.

At the same instant the switch is moved to position S_1, as well as current flow, the discharged capacitor starts to charge-up as it attempts to store electric charge onto its plates. The result is that the voltage, V_C across the capacitors starts to gradually increase while the circuit current begins decreasing at a rate determined by time constant, *tau*, of the RC combination.

Thus we can define the voltage growth across the capacitors plates, (V_C) starting from t = 0 as being:

$$i = C\frac{dv}{dt} = \frac{V - V_C}{R}$$

$$V = V_C + iR$$

$$V = V_C + \left(C\frac{dv}{dt} \times R \right)$$

$$\therefore -\frac{dv}{V - V_C} = -\frac{dt}{RC}$$

Integrating both sides gives:

$$\log_e\left(\frac{V - V_C}{V} \right) = -\frac{t}{RC}$$

$$= \frac{V - V_C}{V} = e^{(-t/RC)}$$

$$\therefore V_C = V\left[1 - e^{(-t/RC)} \right]$$

Thus the exponential natural growth of the voltage across the capacitor as it attempts to store charge onto its plates is given as:

$$V_C = V\left(1 - e^{\left(\frac{-t}{RC} \right)} \right)$$

Where:

- V_C is the voltage across the capacitor
- V is the supply voltage
- e is the base of Natural Logs
- t is the time duration since the switch closed
- RC is the *time constant* tau of the RC circuit

We can show the exponential rate of growth of the voltage across the capacitor over time in the following table assuming normalised values for the supply voltage of 1 volt, and an RC time constant of one (1).

Capacitor Voltage Growth Over Time

Time (s)	0.5	0.7	1	2	3	4	5	6
Voltage (V_C)	0.393V	0.503V	0.632V	0.864V	0.950V	0.981V	0.993V	0.997V

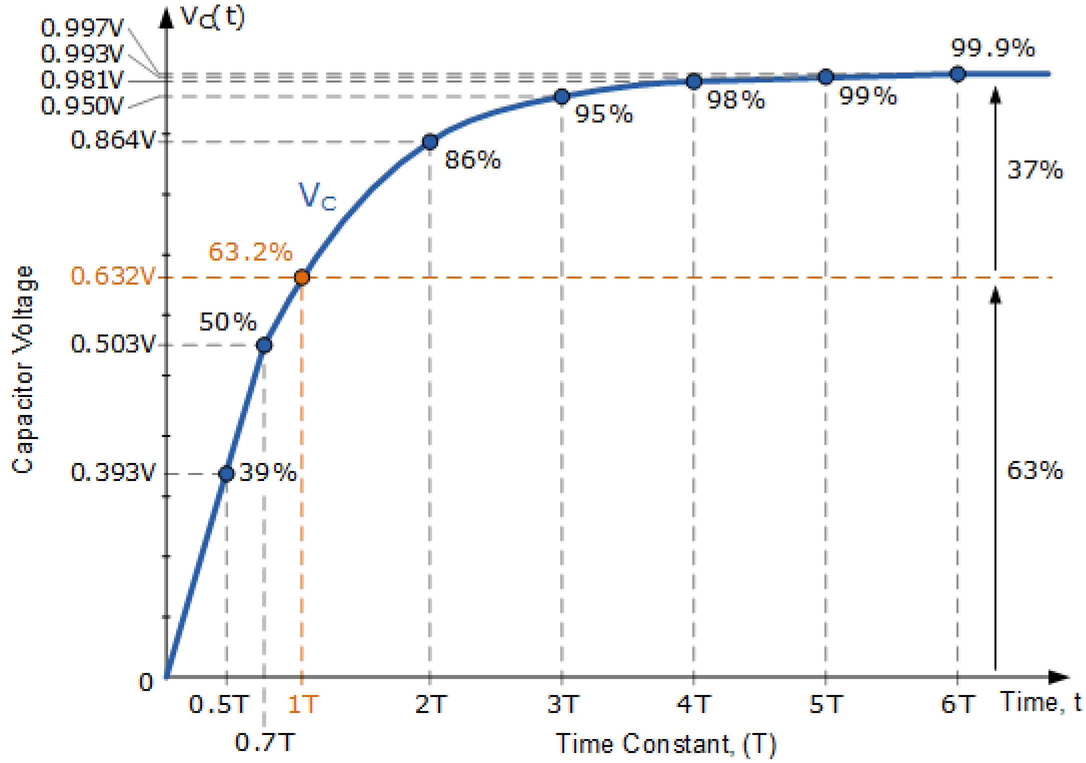
Clearly, we can see from the above table that the values of: $V_C = V(1 - e^{-t/\tau})$ increase over time from t = 0 to t = 6 seconds (6T) in our example, the voltage across the capacitor is an exponentially increasing function because as time (t) increases, the term $e^{-t/\tau}$ gets smaller and smaller, so the voltage across the capacitor, V_C gets larger towards that of the supply voltage driving the change.

Thus at time t = 0, the value of the function is zero, but as time (t) continues to grow towards ∞, the point at which t = RC when $1 - e^{-1}$ produces a value of *0.632* or 63.2% (0.632*100%) of its final steady state value.

Therefore, for an exponentially increasing function, the time constant, **Tau** (τ) is defined as the time taken for the function to reach 63.2% of its final steady state value at a rate starting from time, t = 0. Thus every time interval of tau, (**τ**) the voltage across the capacitor increases by e^{-1} of its previous value and the smaller the time constant tau, the faster is the rate of change.

We can show the variation of the voltage across the capacitor with respect to time graphically as follows:

Exponential Voltage Growth Over Time



Then we can see that the transient response of a capacitor to a step-input is not instant or linear, but increases exponentially at a rate determined by the time constant of the RC circuit and that one time constant is equal to a factor of $1 - e^{-1} = 0.6321$.

Tau on its own does not describe how long it takes the capacitor to become fully charged and theoretically due to its exponentially increasing transient curve, a capacitor never becomes 100% fully charged.

However, after a time period equal too or greater than the equivalent of 5 time constants, that is ≥ 5τ or 5RC, from when the inital change in condition occured, the exponential growth has slowed to less than 1% of its maximum value so for most practical applications we can say that it has reached its final state or steady state condition with no more change taking place with time. That is, at 5T the capacitor is "fully charged".

Tau Example No1

An RC series circuit has resistance of 50Ω and capacitance of 160μF. What is its time constant, tau of the circuit and how long does the capacitor take to become fully charged.

1. Time Constant, τ = RC. Therefore:

$$\tau = RC = 50 \times 160 \times 10^{-6} = 8 \text{ ms}$$

2. Time duration to fully charged:

$$5T = 5\tau = 5RC = 5 \times 50 \times 160 \times 10^{-6} = 40 \text{ ms, or } 0.04\text{s}$$

Tau Example No2

A circuit consists of a resistance of 40Ω and a capacitance of 350uF connected together in series. If the capacitor is fully discharged, what will be the time taken for the voltage across the capacitors plates to reach 45% of its final steady state value once charging begins.

Data given: R = 40Ω, C = 350uF, t is the time at which the capacitor voltage becomes 45% of its final value, that is 0.45V

$$V_C = V\left(1 - e^{\left(-\frac{1}{RC}t\right)}\right)$$

$$\therefore\ 0.45V = V\left(1 - e^{\left(-\frac{1}{40 \times 350 \times 10^{-6}}t\right)}\right)$$

$$0.45V = V\left(1 - e^{-71.43t}\right)$$

$$(1 - 0.45) = 0.55 = e^{-71.43t}$$

$$\log_e(0.55) = -0.5978 = -71.43t$$

$$\therefore\ t = \frac{-0.5978}{-71.43} = 8.37\text{ms}$$

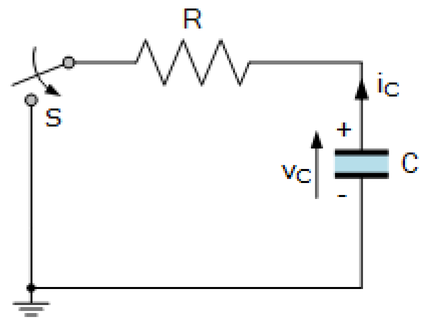
Then it takes 8.37 milli-seconds for the voltage across the capacitor to reach 45% of its 5T steady state condition when the time constant, tau is 14 ms and 5T is 70 ms.

Hopefully now we understand that the time constant of a series RC circuit is the time interval that equals 0.632V (usually taken as 63.2%) of its maximum value (V) at the end of one time constant, (1T) resulting from the product of R and C. Also, the symbol for time constant is a τ (Greek letter tau), and that $\tau = RC$, where R is in ohms, C is in farads, and τ is in seconds.

But what about a capacitor that is already fully charged (V_C >5T), what will be the V-I characteristics of the capacitor as it discharges back down to zero volts, and will the decay of the capacitor's voltage follow the same exponential curve shape.

RC Transient Discharge Curve

The discharging of a fully-charged capacitor is similar to the charging process. The DC power supply used to charge the capacitor originally is disconnected and replaced by a short circuit as shown.



Assuming initial conditions in that switch (S) is "open" and the capacitor has been fully charged (V_C >5T). When switch (S) is closed at time t = 0, the capacitor begins to discharged through the resistor with the amount of time required to discharge depending on the value of the resistor. Since initially $V_C = V_R = V$, the decay of voltage is given as:

Exponential Decay of Voltage Equation

$$V_{(t)} = V_C\left(e^{\left(-\frac{1}{RC}t\right)}\right)$$

Where; $V_{(t)}$ is the voltage across the capacitors plates, and V_C is the initial capacitor voltage value before decay begins.

Previously the exponential function was for voltage growth. For an exponentially decaying function, the time required for the voltage to reach zero volts at a constant rate is still dependant on the RC time constant. Thus time constant is a measure of the "rate of decay".

Therefore for an exponentially decaying function, the time constant, tau (τ) is also defined as the time required for the decaying voltage to reach approximately 36.8% of its final steady state value when the decay started at time t = 0. Thus if τ is one time constant, that is: τ = RC, and the RC circuit was at its fully charged steady state condition at t = 0, then:

$$V_{(t)} = V_C\left(e^{\left(-\frac{1}{RC}t\right)}\right)$$

$$V_{(t)} = V_C\left(e^{(-1)}\right) = V_C\left(\frac{1}{e}\right)$$

$$V_{(t)} = V_C(0.3678)$$

$$\therefore\ V_{(t)} = 0.368V_C$$

Thus at time t = 0, the value of the function is at its maxium, but as time (t) moves towards ∞, the point at which t = RC when e^{-1} produces a value of *0.368* or 36.8% (0.368*100%) of its final steady state value, which is zero volts (fully discharged).

Again we can show the exponential rate of decay of the voltage across the capacitor over time in the following table with normalised values for the supply voltage of 1 volt, and an RC time constant of one (1).

Capacitor Voltage Decay Over Time

Time (s)	0.5	0.7	1	2	3	4	5	6
Voltage (V_t)	$0.607V_C$	$0.497V_C$	$0.368V_C$	$0.135V_C$	$0.049V_C$	$0.018V_C$	$0.007V_C$	$0.002V_C$

One point to notice here. The time constant, **tau** of a series RC circuit from its inital value at t = 0 to **τ** will always be 63.2% whether the capacitor is charging or discharging. For an exponential growth the inital condition is 0V, (zero volts) since the capacitor is fully discharged.

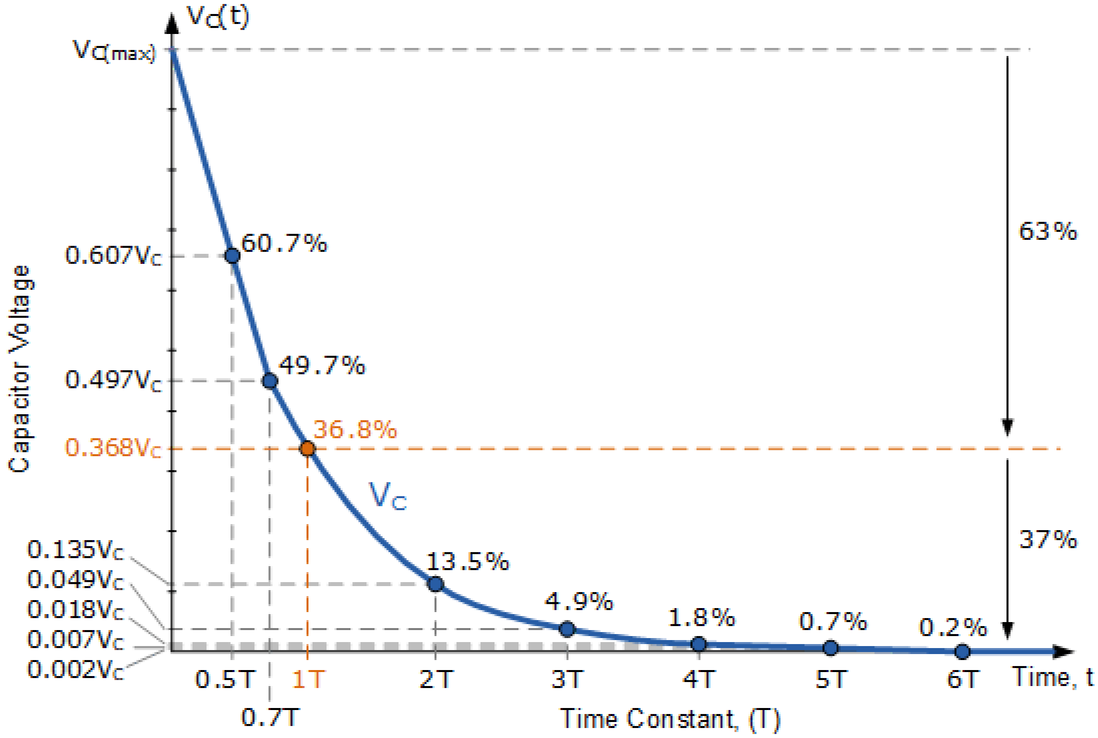
Thus the voltage grows exponentially upwards to 63.2% of V_{MAX} at one time constant, 1T. But we could also think of the capacitor voltage at 1T as being 36.8% away from its final steady state value at 5T. That is fully charged.

The same idea is also true for an exponential decay. For a fully charged capacitor the inital steady state condition is $V_{C(max)}$, so the capacitor will discharge down to 36.8% of its final steady state condition of zero volts (0V) after 5T. But again, we can also think of the voltage across the capacitor at time 1T, as being 63.2% down from its inital starting when the capacitor was fully charged at t = 0.

Then the value of one time constant 1T, from the inital starting condition to 1T will always be 0.632V, or 63.2% of its final steady state condition. Likewise at 1T, the capacitor voltage will always be 0.368V, or 36.8% away from its final steady state condition after 5T as either fully charged at $V_{C(max)}$ or fully discharged at 0V.

We can show the decay of voltage with respect to time graphically as follows:

Exponential Voltage Decay Over Time



Again the rate of voltage decay over time relies greatly on the value of the RC time constant, tau.

Tau Example No3

An RC series circuit has a time constant, tau of 5ms. If the capacitor is fully charged to 100V, calculate: 1) the voltage across the capacitor at time: 2ms, 8ms and 20ms from when discharging started, 2) the elapsed time at which the capacitor voltage decays to 56V, 32V and 10V.

The voltage across a discharging capacitor is given as:

$$\mathbf{V_C(t) = V_C \times e^{-t/RC} \ \ Volts}$$

RC time constant is given as 5ms, therefore 1/RC = 200. V_C = 100V.

1a). Capacitor voltage after 2ms $V_C(0.002) = 100\ e^{-200t} = 100\ e^{-0.4} = 100 \times 0.67 = 67.0$ volts
1b). Capacitor voltage after 8ms $V_C(0.008) = 100\ e^{-200t} = 100\ e^{-1.6} = 100 \times 0.202 = 20.2$ volts
1c). Capacitor voltage after 20ms $V_C(0.02) = 100\ e^{-200t} = 100\ e^{-4} = 100 \times 0.018 = 1.8$ volts

Capacitor voltage (V_C) at time duration from t = 0.

2a). Elapsed time (t_1) when $V_C(t)$ = 56 volts $56 = 100\ e^{-200t}$, therefore: $-200t_1 = \ln(56/100) = -0.5798$ Thus: $t_1 = -0.5798 \div -200 = 2.9$ ms
2b). Elapsed time (t_2) when $V_C(t)$ = 32 volts $32 = 100\ e^{-200t}$, therefore: $-200t_2 = \ln(32/100) = -1.1394$ Thus: $t_2 = -1.1394 \div -200 = 5.7$ ms
2c). Elapsed time (t_3) when $V_C(t)$ = 10 volts $10 = 100\ e^{-200t}$, therefore: $-200t_3 = \ln(10/100) = -2.3026$ Thus: $t_3 = -2.3026 \div -200 = 11.5$ ms

Tau the Time Constant Summary

We have seen here in this tutorial about **Time Constant**, **Tau**, symbol **τ** that the *transient response* of an RC circuit is the time it takes to change from one steady state condition to another steady state condition when subjected to a step change input condition.

When the capacitor is charges up from its initial zero voltage state to its final steady state voltage (V), the time duration is defined as: $\tau = RC$. That is the product of R and C. This produces an exponentially growing function for V_C with this RC time constant measured in seconds, and the smaller the time constant, the faster the rate of voltage change.

We have also seen for an exponentially growing function that the value after one time constant, 1T is 63.2% of its final steady state value. That is for an exponentially increasing function, it is the time required for the voltage to reach its final steady state value after 5T.

The value of V(t)for an exponentially growing function at time $t = \tau$ is given as:

$$\mathbf{V(t) = V(\ 1 - e^{-1}\ ) = 0.632V}$$

Likewise, for an exponentially decaying function, the value after one time constant, 1T is 36.8% of its final steady state value. That is for an exponentially decaying function it is time required for the voltage to reach zero value.

The value of V(t) for an exponentially decaying function at time $t = \tau$ is given as:

$$\mathbf{V(t) = V(\ e^{-1}\ ) = 0.368V}$$

Either way, from t = 0 to **τ** will always be 63.2% of the time duartion, and from 1T to 5T will always be 36.8% of the time duration, exponentially increasing or decreasing.