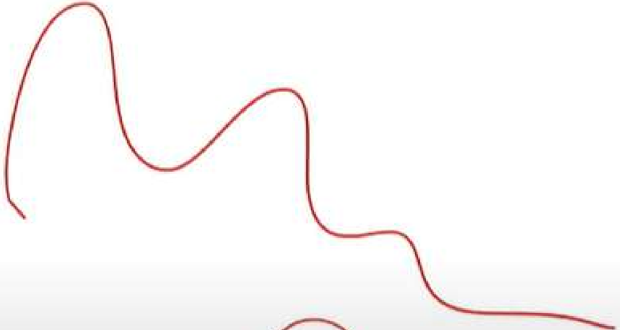
$X \in \mathbb{R}^n \quad X = \{x_1, x_2 \dots x_n\}$

$$\{X = x_i\}_{i=1}^{N}$$

$$\underset{w,b,c}{\text{maximize}} \sum_{i=1}^{N} \log P(X = x_i)$$

$$\downarrow$$

$$E(\underline{w, b, c}, x_i)$$



$\Sigma = I$

μ

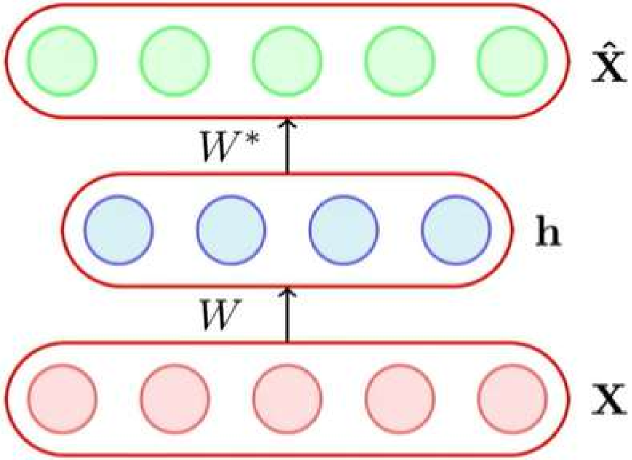
$$\underset{\cancel{\mu}\,\theta}{\text{maximize}} \sum_{i=1}^{N} \log P(X = x_i)$$

μ, Σ

μ, Σ

$$\mu = \theta_1 x_1 + \theta_2 x_2 + \dots$$

$$= \boxed{f_\theta(x)}$$



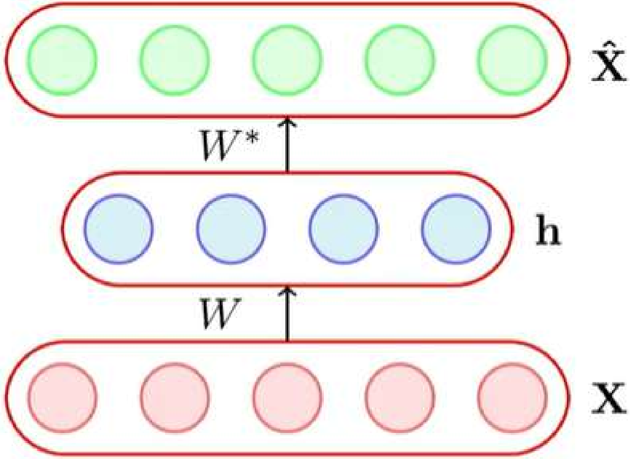
$$\mathbf{h} = g(W\mathbf{X} + \mathbf{b})$$

$$\hat{\mathbf{X}} = f(W^*\mathbf{h} + \mathbf{c})$$

- Before we start talking about VAEs, let us quickly revisit autoencoders
- An autoencoder contains an encoder which takes the input X and maps it to a hidden representation
- The decoder then takes this hidden representation and tries to reconstruct the input from it as $\hat{X}$
- The training happens using the following objective function

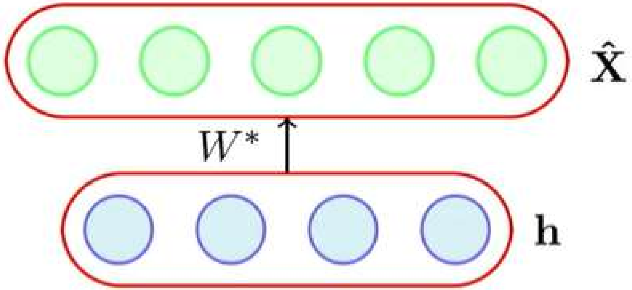
$$\min_{W,W^*,\mathbf{c},\mathbf{b}} \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n}(\hat{x}_{ij} - x_{ij})^2$$

- where m is the number of training instances, $\{x_i\}_{i=1}^{m}$ and each $x_i \in R^n$ (x_{ij} is thus the j-th dimension of the i-th training instance)



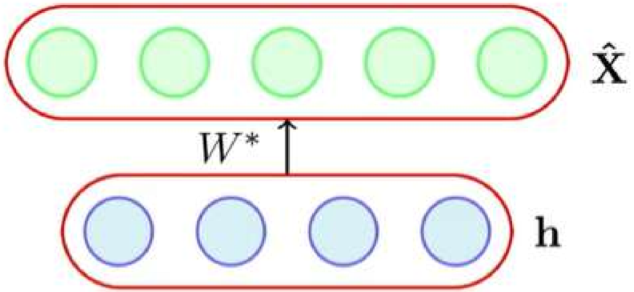
$$\mathbf{h} = g(W\mathbf{X} + \mathbf{b})$$
$$\hat{\mathbf{X}} = f(W^*\mathbf{h} + \mathbf{c})$$

- But where's the fun in this ?
- We are taking an input and simply reconstructing it
- Of course, the fun lies in the fact that we are getting a good *abstraction* of the input
- But RBMs were able to do something more besides abstraction (they were able to do *generation*)
- Let us revisit *generation* in the context of autoencoders



$$\hat{\mathbf{X}} = f(W^*\mathbf{h} + \mathbf{c})$$

- Can we do generation with autoencoders ?
- In other words, once the autoencoder is trained can I remove the encoder, feed a hidden representation h to the decoder and decode a $\hat{X}$ from it ?
- In principle, yes! But in practice there is a problem with this approach
- h is a very high dimensional vector and only a few vectors in this space would actually correspond to meaningful latent representations of our input
- So of all the possible value of h which values should I feed to the decoder (we had asked a similar question before: slide 67, bullet 5 of lecture 19)



$$\hat{\mathbf{X}} = f(W^*\mathbf{h} + \mathbf{c})$$

- Ideally, we should only feed those values of h which are highly *likely*
- In other words, we are interested in sampling from $P(h|X)$ so that we pick only those h's which have a high probability
- But unlike RBMs, autoencoders do not have such a probabilistic interpretation
- They learn a hidden representation h but not a distribution $P(h|X)$
- Similarly the decoder is also deterministic and does not learn a distribution over X (given a h we can get a X but not $P(X|h)$)

RBM → V, H

VAE → X, Z

We will now look at variational autoencoders which have the same structure as autoencoders but they learn a distribution over the hidden variables

$P(h|X)$

$P(x|h)$