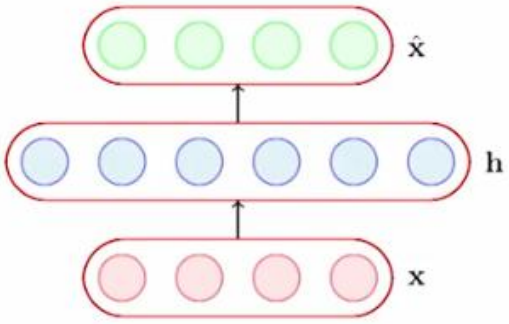
- A contractive autoencoder also tries to prevent an overcomplete autoencoder from learning the identity function.
- It does so by adding the following regularization term to the loss function

$$\Omega(\theta) = \|J_{\mathbf{x}}(\mathbf{h})\|_F^2$$

where $J_{\mathbf{x}}(\mathbf{h})$ is the Jacobian of the encoder.

- Let us see what it looks like.



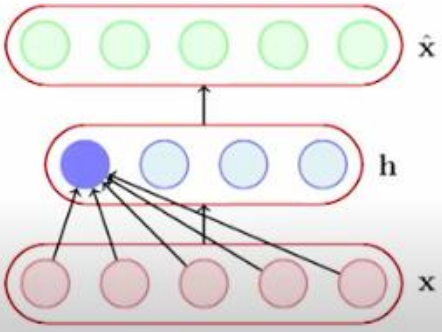

- If the input has $n$ dimensions and the hidden layer has $k$ dimensions then
- In other words, the $(j, l)$ entry of the Jacobian captures the variation in the output of the $l^{th}$ neuron with a small variation in the $j^{th}$ input.

$$J_{\mathbf{x}}(\mathbf{h}) = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \cdots & \cdots & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \cdots & \cdots & \cdots & \frac{\partial h_2}{\partial x_n} \\ \vdots & & \ddots & & \vdots \\ \frac{\partial h_k}{\partial x_1} & \cdots & \cdots & \cdots & \frac{\partial h_k}{\partial x_n} \end{bmatrix}$$

$$\|J_{\mathbf{x}}(\mathbf{h})\|_F^2 = \sum_{j=1}^{n} \sum_{l=1}^{k} \left( \frac{\partial h_l}{\partial x_j} \right)^2$$

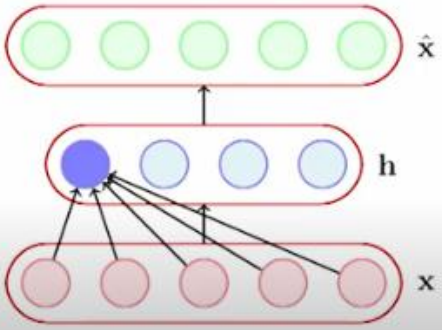
- What is the intuition behind this ?
- Consider $\frac{\partial h_1}{\partial x_1}$, what does it mean if $\frac{\partial h_1}{\partial x_1} = 0$
- It means that this neuron is not very sensitive to variations in the input $x_1$.
- But doesn't this contradict our other goal of minimizing $\mathcal{L}(\theta)$ which requires $\mathbf{h}$ to capture variations in the input.

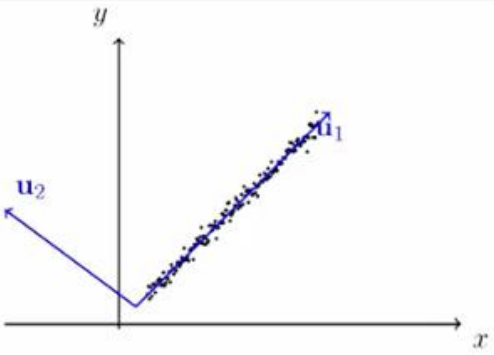
$$\|J_{\mathbf{x}}(\mathbf{h})\|_F^2 = \sum_{j=1}^{n} \sum_{l=1}^{k} \left( \frac{\partial h_l}{\partial x_j} \right)^2$$



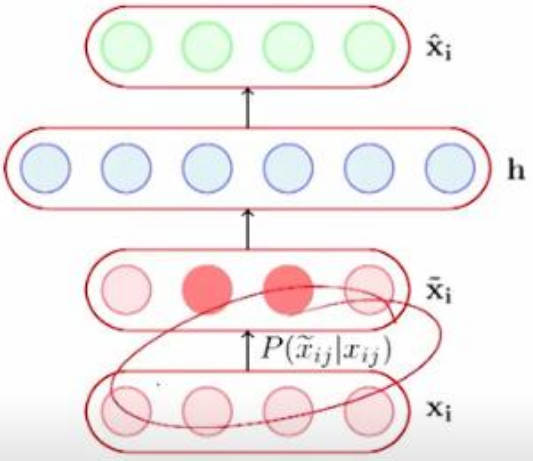
- Indeed it does and that's the idea
- By putting these two contradicting objectives against each other we ensure that $\mathbf{h}$ is sensitive to only very important variations as observed in the training data.
- $\mathcal{L}(\theta)$ - capture important variations in data
- $\Omega(\theta)$ - do not capture variations in data
- Tradeoff - capture only very important variations in the data

$$\|J_{\mathbf{x}}(\mathbf{h})\|_F^2 = \sum_{j=1}^{n}\sum_{l=1}^{k}\left(\frac{\partial h_l}{\partial x_j}\right)^2$$




- Consider the variations in the data along directions $\mathbf{u}_1$ and $\mathbf{u}_2$
- It makes sense to maximize a neuron to be sensitive to variations along $\mathbf{u}_1$
- At the same time it makes sense to inhibit a neuron from being sensitive to variations along $\mathbf{u}_2$ (as there seems to be small noise and unimportant for reconstruction)
- By doing so we can balance between the contradicting goals of good reconstruction and low sensitivity.
- What does this remin




## Regularization

$$\Omega(\theta) = \lambda\|\theta\|^2 \quad \boxed{\text{Weight decaying}}$$

$$\Omega(\theta) = \sum_{l=1}^{k}\rho\log\frac{\rho}{\hat{\rho}_l} + (1-\rho)\log\frac{1-\rho}{1-\hat{\rho}_l} \quad \boxed{\text{Sparse}}$$

$$\Omega(\theta) = \sum_{j=1}^{n}\sum_{l=1}^{k}\left(\frac{\partial h_l}{\partial x_j}\right)^2 \quad \boxed{\text{Contractive}}$$