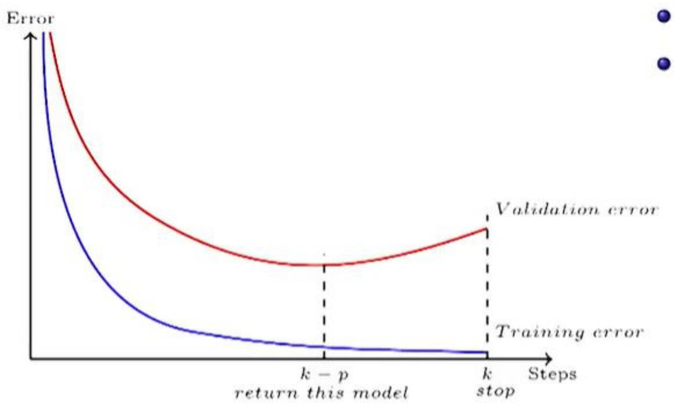
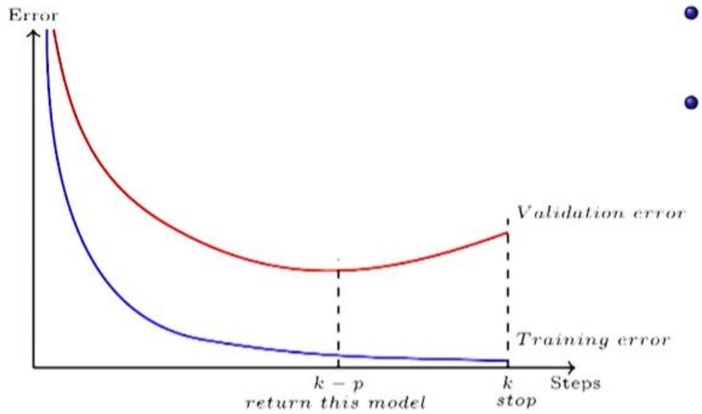
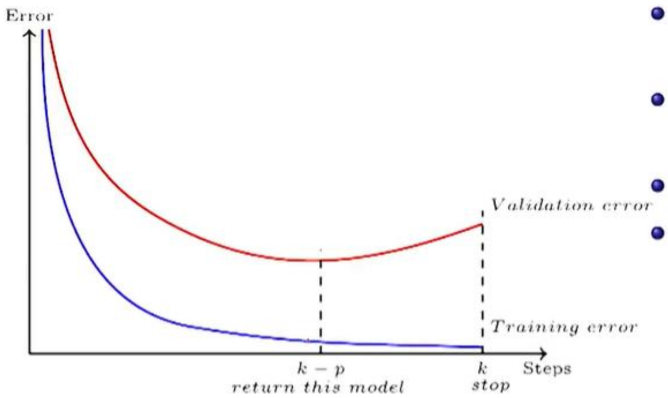
Module 8.9 : Early stopping



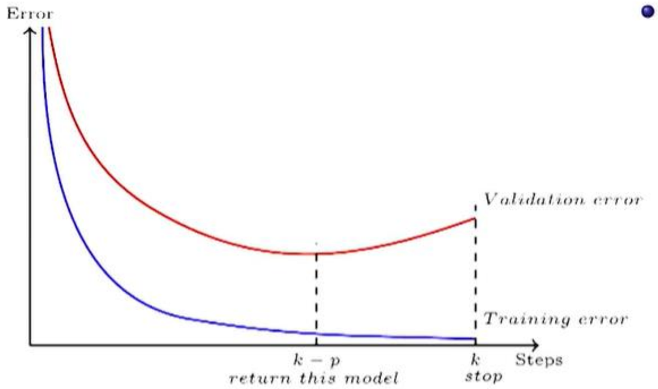
- Track the validation error
- Have a patience parameter p



- Very effective and the mostly widely used form of regularization
- Can be used even with other regularizers (such as l_2)



- Very effective and the mostly widely used form of regularization
- Can be used even with other regularizers (such as l_2)
- How does it act as a regularizer ?
- We will first see an intuitive explanation and then a mathematical analysis



- Recall that the update rule in SGD is

$$\omega_{t+1} = \omega_t + \eta \nabla \omega_t$$
$$= \omega_0 + \eta \sum_{i=1}^{t} \nabla \omega_i$$

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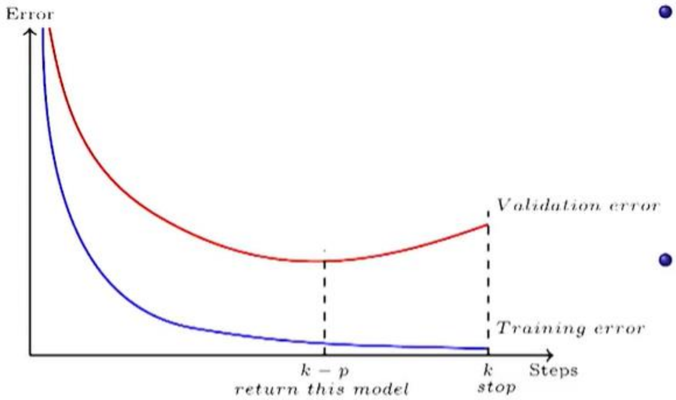

- Recall that the update rule in SGD is

$$\omega_{t+1} = \omega_t + \eta \nabla \omega_t$$
$$= \omega_0 + \eta \sum_{i=1}^{t} \nabla \omega_i$$

- Let τ be the maximum value of $\nabla \omega_i$ then

$$\omega_{t+1} \leq \omega_0 + \eta t \tau$$

- Recall that the Taylor series approximation for $L(\omega)$ is

$$L(\omega) = L(\omega^*) + (\omega - \omega^*)^T \nabla L(\omega^*) + \frac{1}{2}(\omega - \omega^*)^T H (\omega - \omega^*)$$
$$= L(\omega^*) + \frac{1}{2}(\omega - \omega^*)^T H (\omega - \omega^*) \qquad [\ \omega^* \ is\ optimal\ so\ \nabla L(\omega^*)\ is\ 0\ ]$$
$$\nabla(L(\omega)) = H(\omega - \omega^*)$$

Now the SGD update rule is:

$$\omega_t = \omega_{t-1} + \eta \nabla L(\omega_{t-1})$$
$$= \omega_{t-1} + \eta H(\omega_{t-1} - \omega^*)$$
$$= (I + \eta H)\omega_{t-1} - \eta H \omega^*$$

$$\omega_t = (I + \eta H)\omega_{t-1} - \eta H \omega^*$$

- Using EVD of H as $H = Q\Lambda Q^T$, we get:

$$\omega_t = (I + \eta Q\Lambda Q^T)\omega_{t-1} - \eta Q\Lambda Q^T \omega^*$$

- If we start with $\omega_0 = 0$ then we can show that (See Appendix)

$$\omega_t = Q[I - (I - \varepsilon\Lambda)^t]Q^T \omega^*$$

- Compare this with the expression we had for optimum $\tilde{\omega}$ with L_2 regularization

$$\tilde{\omega} = Q[I - (\Lambda + \alpha I)^{-1}\alpha]Q^T \omega^*$$

- We observe that $\omega_t = \tilde{\omega}$, if we choose ε,t and α such that

$$(I - \varepsilon\Lambda)^t = (\Lambda + \alpha I)^{-1}\alpha$$

Things to be remember

- Early stopping only allows t updates to the parameters.
- If a parameter ω corresponds to a dimension which is important for the loss $\mathscr{L}(\theta)$ then $\frac{\partial \mathscr{L}(\theta)}{\partial \omega}$ will be large
- However if a parameter is not important ($\frac{\partial \mathscr{L}(\theta)}{\partial \omega}$ is small) then its updates will be small and the parameter will not be able to grow large in 't' steps