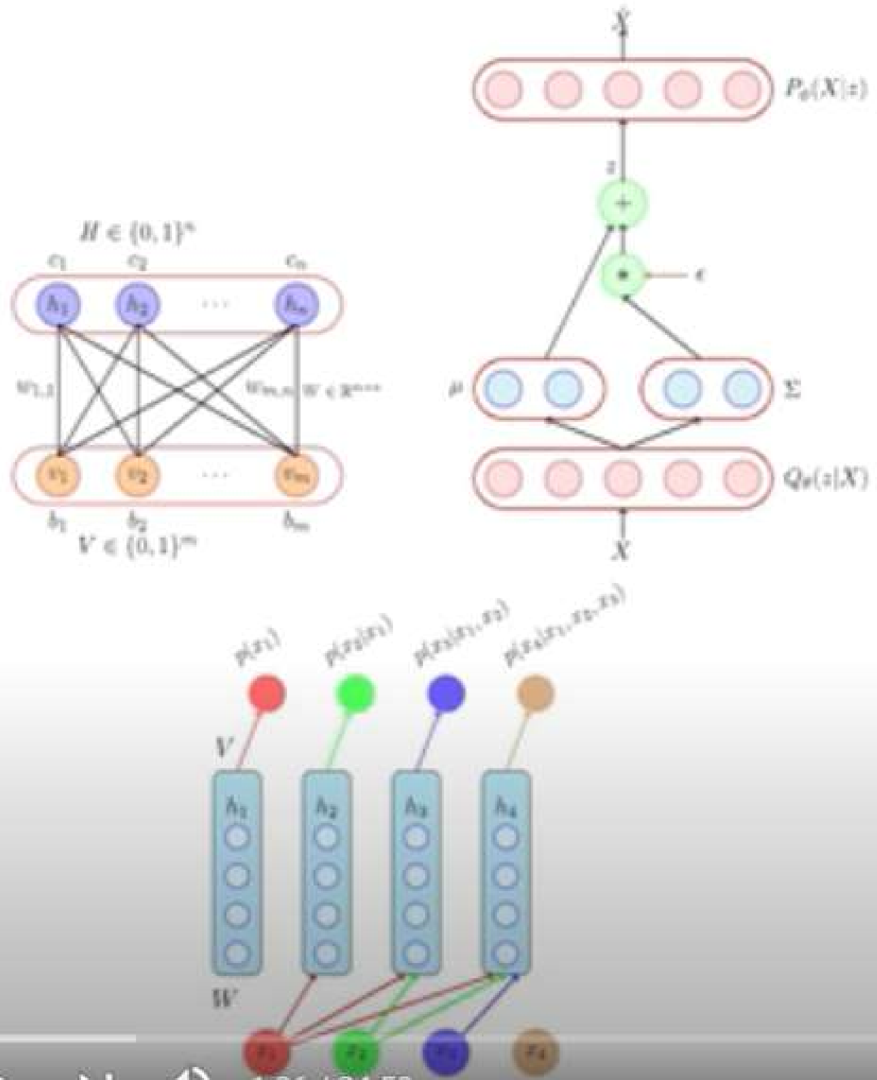
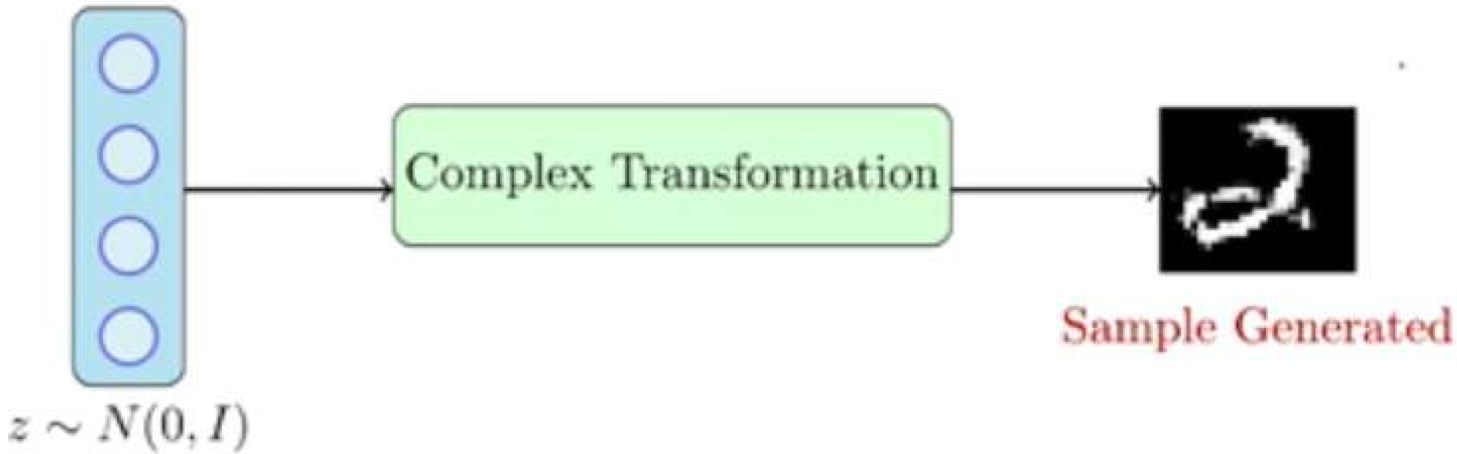
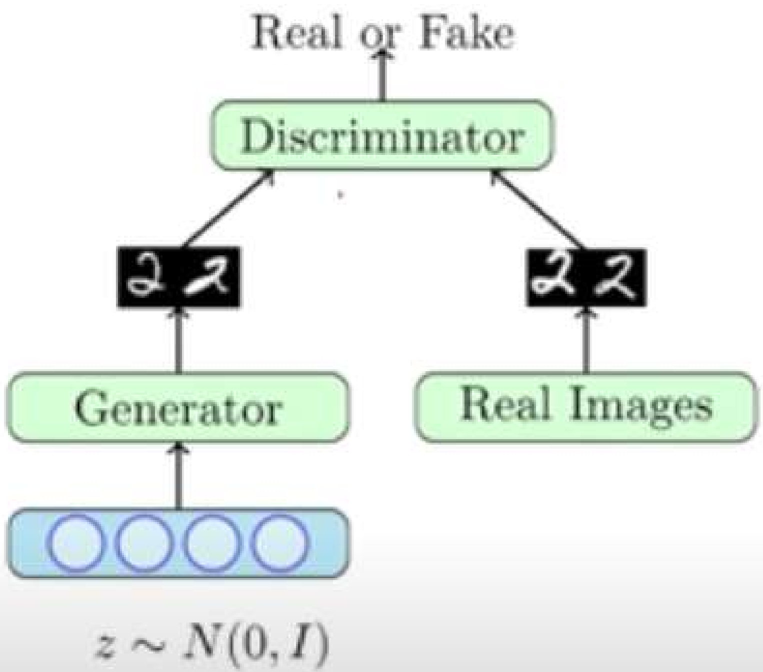


- So far we have looked at generative models which explicitly model the joint probability distribution or conditional probability distribution
- For example, in RBMs we learn $P(X, H)$, in VAEs we learn $P(z|X)$ and $P(X|z)$ whereas in AR models we learn $P(X)$
- What if we are only interested in sampling from the distribution and don't really care about the explicit density function $P(X)$?

- As usual we are given some training data (say, MNIST images) which obviously comes from some underlying distribution
- Our goal is to generate more images from this distribution (*i.e.*, create images which look similar to the images from the training data)
- In other words, we want to sample from a complex high dimensional distribution which is intractable (recall RBMs, VAEs and AR models deal with this intractability in their own way)



- GANs take a different approach to this problem where the idea is to sample from a simple tractable distribution (say, $z \sim N(0, I)$) and then learn a complex transformation from this to the training distribution
- In other words, we will take a $z \sim N(0, I)$, learn to make a series of complex transformations on it so that the output looks as if it came from our training distribution



- What can we use for such a complex transformation? A Neural Network
- How do you train such a neural network? Using a two player game
- There are two players in the game: a generator and a discriminator
- The job of the generator is to produce images which look so natural that the discriminator thinks that the images came from the real data distribution
- The job of the discriminator is to get better and better at distinguishing between true images and generated (fake) images

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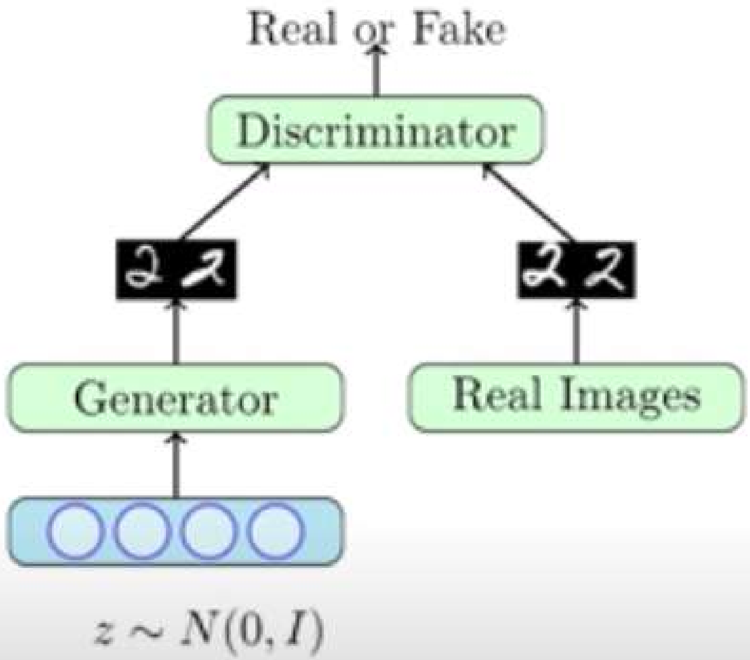
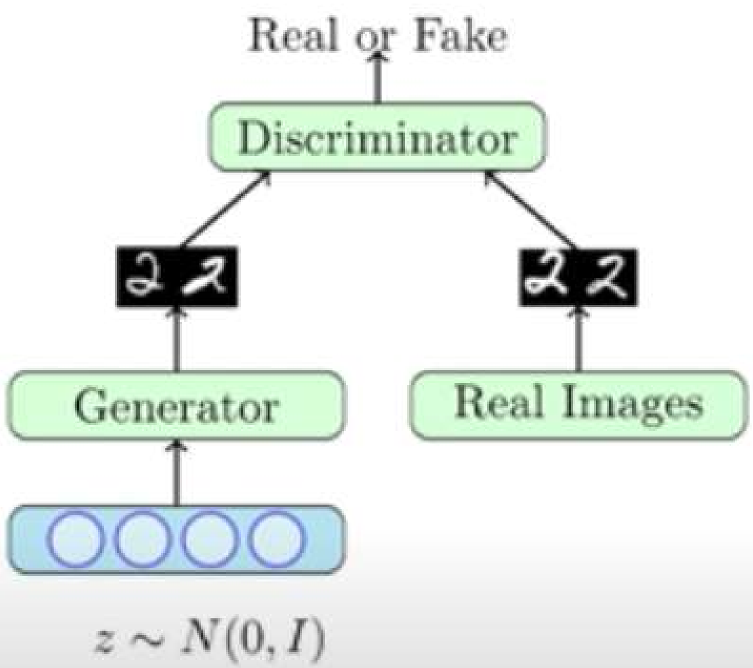


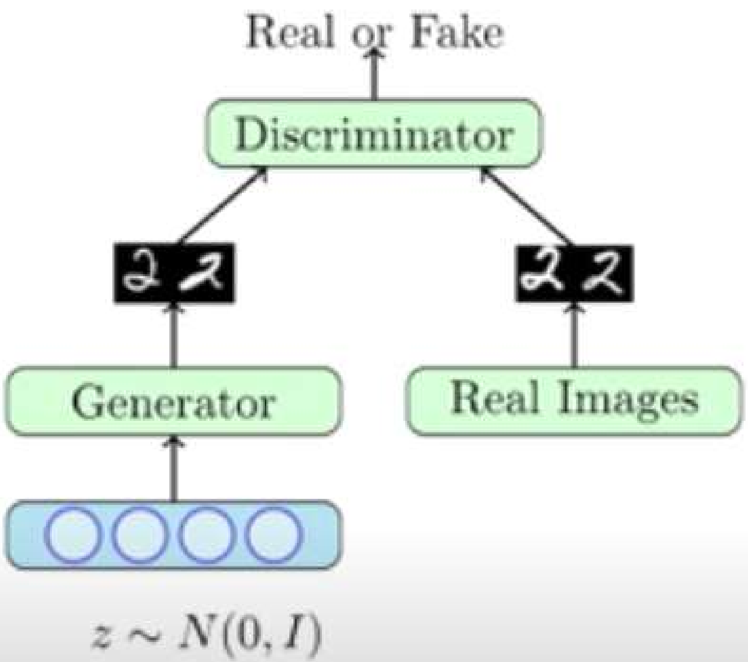
- So let's look at the full picture
- Let G_ϕ be the generator and D_θ be the discriminator (ϕ and θ are the parameters of G and D, respectively)
- We have a neural network based generator which takes as input a noise vector $z \sim N(0, I)$ and produces $G_\phi(z) = X$
- We have a neural network based discriminator which could take as input a real X or a generated $X = G_\phi(z)$ and classify the input as real/fake

$D(X) \rightarrow$ Real

$D(G_\phi(z)) \rightarrow$ generated



- What should be the objective function of the overall network?
- Let's look at the objective function of the generator first
- Given an image generated by the generator as $G_\phi(z)$ the discriminator assigns a score $D_\theta(G_\phi(z))$ to it
- This score will be between 0 and 1 and will tell us the probability of the image being real or fake
- For a given z, the generator would want to maximize $\log D_\theta(G_\phi(z))$ (log likelihood) or minimize $\log(1 - D_\theta(G_\phi(z)))$



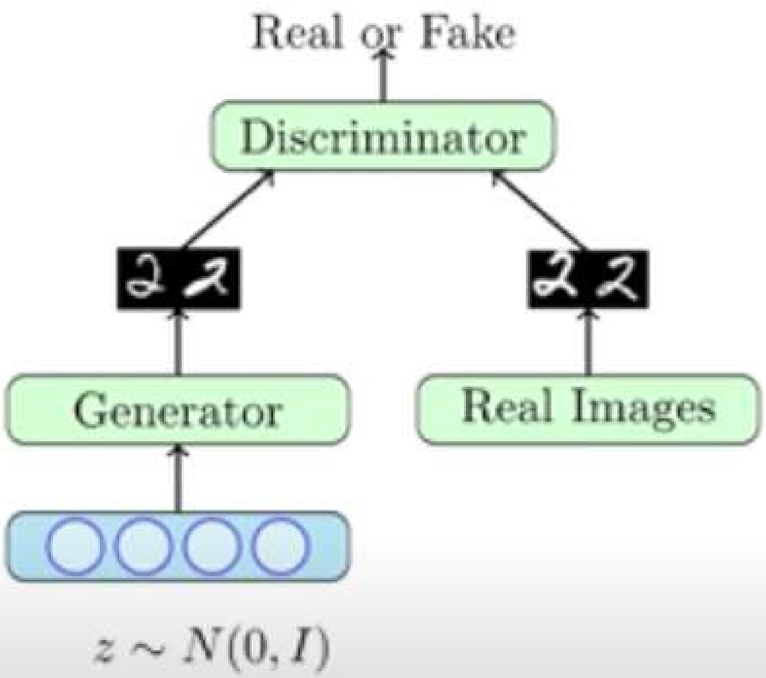
- This is just for a single z and the generator would like to do this for all possible values of z,
- For example, if z was discrete and drawn from a uniform distribution (*i.e.*, $p(z) = \frac{1}{N}\ \forall z$) then the generator's objective function would be

$$\min_{\phi} \sum_{i=1}^{N} \frac{1}{N} \log(1 - D_\theta(G_\phi(z)))$$

- However, in our case, z is continuous and not uniform ($z \sim N(0, I)$) so the equivalent objective function would be

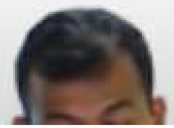
$$\min_{\phi} \int p(z) \log(1 - D_\theta(G_\phi(z)))$$

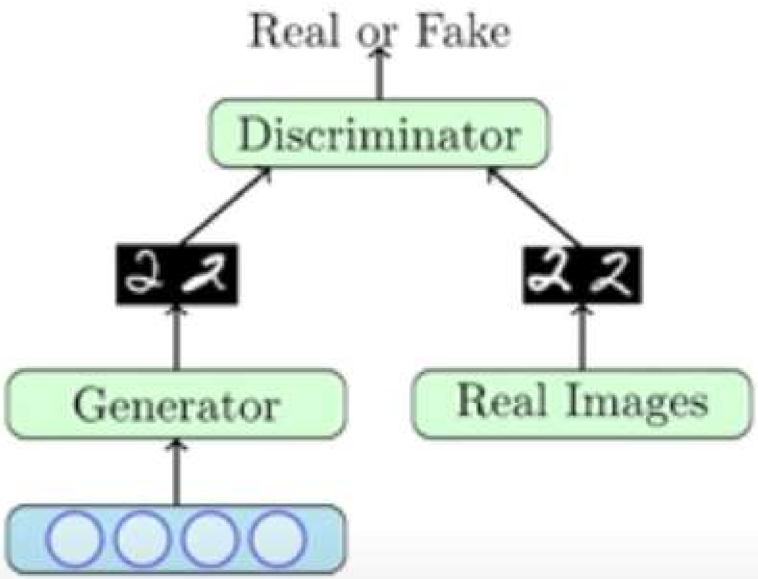
$$\min_{\phi} E_{z \sim p(z)}[\log(1 - D_\theta(G_\phi(z)))]$$



- Now let's look at the discriminator
- The task of the discriminator is to assign a high score to real images and a low score to fake images
- And it should do this for all possible real images and all possible fake images
- In other words, it should try to maximize the following objective function

$$\max_{\theta} E_{x \sim p_{data}}[\log D_\theta(x)] + E_{z \sim p(z)}[\log(1 - D_\theta(G_\phi(z)))]$$

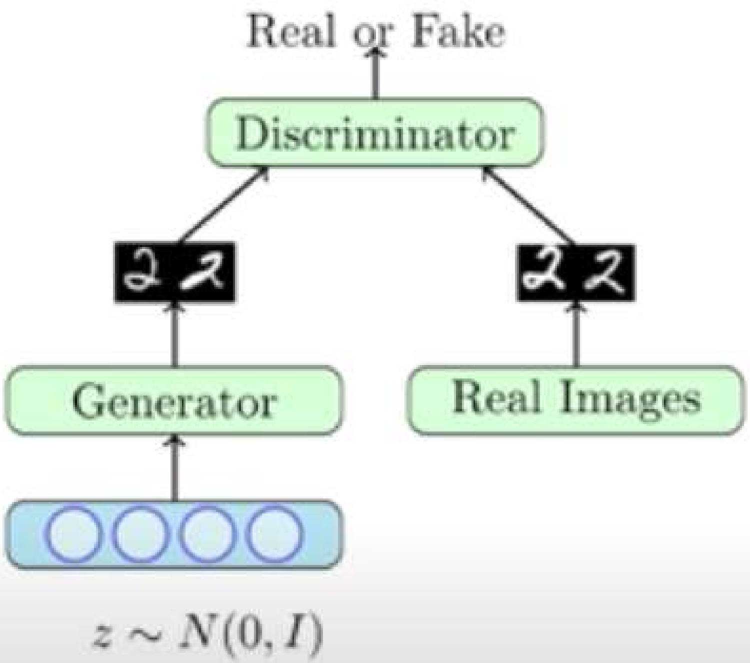




- If we put the objectives of the generator and discriminator together we get a minimax game

$$\min_{\phi} \max_{\theta} \; [\mathbb{E}_{x \sim p_{data}} \log D_\theta(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D_\theta(G_\phi(z)))]$$

- The first term in the objective is only w.r.t. the parameters of the discriminator (θ)
- The second term in the objective is w.r.t. the parameters of the generator (ϕ) as well as the discriminator (θ)
- The discriminator wants to maximize the second term whereas the generator wants to minimize it (hence it is a two-player game)



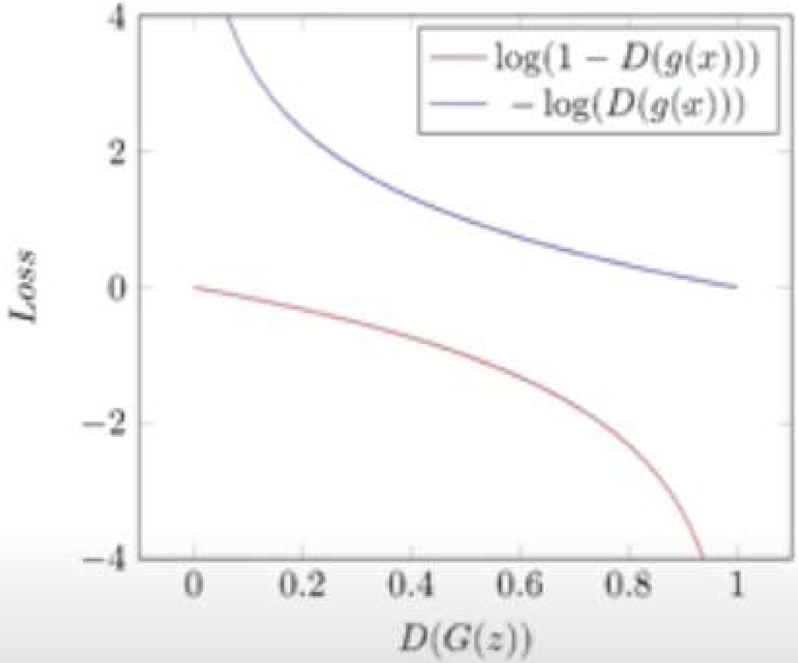
- So the overall training proceeds by alternating between these two step
- **Step 1:** Gradient Ascent on Discriminator

$$\max_{\theta} \left[\mathbb{E}_{x\sim p_{data}} \log D_\theta(x) + \mathbb{E}_{z\sim p(z)} \log(1 - D_\theta(G_\phi(z)))\right]$$

- **Step 2:** Gradient Descent on Generator

$$\min_{\phi} \; \mathbb{E}_{z\sim p(z)} \log(1 - D_\theta(G_\phi(z)))$$

- In practice, the above generator objective does not work well and we use a slightly modified objective
- Let us see why



- When the sample is likely fake, we want to give a feedback to the generator (using gradients)
- However, in this region where $D(G(z))$ is close to 0, the curve of the loss function is very flat and the gradient would be close to 0
- Trick: Instead of minimizing the likelihood of the discriminator being correct, maximize the likelihood of the discriminator being wrong
- In effect, the objective remains the same but the gradient signal becomes better

With that we are now ready to see the full algorithm for training GANs

procedure GAN TRAINING
    for number of training iterations **do**
        for k steps **do**
            • Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, ..., \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$
            • Sample minibatch of m examples $\{\mathbf{x}^{(1)}, ..., \mathbf{x}^{(m)}\}$ from data generating distribution $p_{data}(\mathbf{x})$
            • Update the discriminator by ascending its stochastic gradient:

$$\nabla_\theta \frac{1}{m} \sum_{i=1}^{m} \left[\log D_\theta\left(x^{(i)}\right) + \log\left(1 - D_\theta\left(G_\phi\left(z^{(i)}\right)\right)\right) \right]$$

        end for
        • Sample minibatch of m noise samples $\{\mathbf{z}^{(1)}, ..., \mathbf{z}^{(m)}\}$ from noise prior $p_g(\mathbf{z})$
        • Update the generator by ascending its stochastic gradient

$$\nabla_\phi \frac{1}{m} \sum_{i=1}^{m} \left[\log\left(D_\theta\left(G_\phi\left(z^{(i)}\right)\right)\right) \right]$$

    end for
end procedure