A Machine Learning approach to distinguish between regular and chaotic motions of space debris in the geopotential approximation

Project preview

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Stardust-R WPs involved: WP3, WP8, WP1
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Programming languages: MATLAB/Python

Abstract & objectives

This activity has the aim to investigate the dynamics of a specific region of the sky around the Earth, in order to distinguish between regular and chaotic motions. The steps that allow to reach this goal are the following:

1. We start from a model problem which describes the motion of a forced pendulum. The model will be given using Hamiltonian formalism and the corresponding Hamilton's equations.

2. We integrate the equations of motion and compute its Poincare' map to understand the difference between rotational, librational, chaotic motions;

3. We consider the dataset corresponding to a particle orbiting around the Earth; like in the pendulum, the dataset describes rotational, librational, chaotic motions for different values of the elements (mean anomaly and semimajor axis). An index is assigned to distinguish between rotational, librational, chaotic motion;

4. We implement a Machine Learning algorithm to get information for a large number of initial conditions and assign the binary index to each initial condition.

Backgrounds for attendees

Dynamics sections

Attendees should be familiar with basic concepts in

- Hamiltonian dynamics,
- Integration methods,
- Chaotic motion,
- Space debris dynamics.

Any computer language can be used to integrate the equations of motion.
A good reference for space debris dynamics is the following article:

Celletti A., Gales C., "On the dynamics of space debris: 1:1 and 2:1 resonances", J. Nonlinear Science, vol. 24, n. 6, 1231-1262 (2014)

Machine learning section

Attendees should be familiar with basic concepts around deep learning and neural networks, such as:

- Supervised vs unsupervised learning. In this we'll do a classification task, which is a type of supervised learning.
- Training and validation data
- Overfitting.
- Accuracy of a classification model.
- Parameters (weights and biases) & activations in a neural network
- Batch learning, learning rate

A good resource to review all of these concepts can be found in the talk "*Deep Learning for Space Guidance*", given by Roberto Furfaro & Richard Linares during the Stardust-R Training School II [1].

[1] This material has restricted access for the Stardust-R network. If needed, contact the administrators of the workshop for getting the access code.

In practical terms, we will make use of the Python deep learning library fastai2 [2]. A good introduction to this library, and to practical deep learning in general, can be found in the first chapter of the fastai book, which is freely accessible here. In this exercise, we will work with tabular data, so attendees should make special focus on reading about that type of learning. Apart from fastai, data loading and preprocessing will be done using the pandas [3] Python library. A useful resource for getting started with the basic functionality of this library can be found in this cheatsheet.

As a development environment, any Python 3 setup is suitable for this exercise. The use of Jupyter Notebooks is recommended due to its exploratory nature for drafting code and results. In this sense, we encourage you to use services like Google Colaboratory, a free[4] Jupyter notebook environment that runs in the cloud and stores its notebooks on Google Drive or Github. It has everything pre-installed and ready-to-use, and provides easy sharing & collaborative capabilities for working in teams. A short tutorial on how to use Colab, and enable the GPU, can be found here.

Data description

Being subjected to various perturbations, space debris exhibit complex dynamics that can be described, in many cases, by reducing the general theory to simple toy models similar to forced pendulums.

Let us consider the motion of a point-mass body (space debris) under the influence of the Earth's oblateness and rotation. Using the Hamiltonian formalism, the motion is described by the Hamiltonian given by equation (3.2)[5], which is expressed in terms of the Delaunay variables $L, G, H, M, \omega, \Omega$ and the sidereal time θ, where the Delaunay actions are related to the well known orbital elements (a, e, i) through the relation (3.2)[5], M is the mean anomaly, ω is the argument of perigee and Ω is the longitude of the ascending node.

Focusing on the GEO 1:1 and MEO 2:1 resonances, which occur when $\frac{dM}{dt} - \frac{d\theta}{dt} \simeq 0$, and respectively $\frac{dM}{dt} - 2\frac{d\theta}{dt} \simeq 0$, namely at around $42164\ km$, and respectively at $26560\ km$ from the center of the Earth, the dynamics can be studied by a reduced (averaged) Hamiltonian whose secular part is given by the relation (3.9)[5] and resonant part is provided in Section 3.2.2[5], pages 1240, 1241 for the 1:1 resonance, and respectively in Section 3.2.3[5], pages 1243, 1244 for the 2:1 resonance.

[2] fastai: A Layered API for Deep Learning: https://arxiv.org/abs/2002.04688

[3] Pandas: https://pandas.pydata.org/

[4] A google account is needed to access Google Colab.

[5] A. Celletti, C. Gales, On the dynamics of space debris: 1:1 and 2:1 resonances, J. Nonlinear Science, vol. 24, n. 6, 1231--1262 (2014).

Integrating the canonical equations and the associated variational equations, we derive the Fast Lyapunov Indicator (hereafter FLI) up to time $T = 19\, years$ (see Section 5.1[5] for more details on this chaos indicator). We compute a grid of 100×100 points of the $\sigma - a$ plane, where for the 1:1 resonance the critical angle $\sigma = \lambda = M - \theta + \omega + \Omega$ ranges in the interval $[-220^o,\ 180^o]$ and the semimajor axis a spans the domain $[42120,\ 42220]\, km$, while for the 2:1 resonance the critical angle $\sigma = M - 2\theta + \omega + 2\Omega$ varies in the interval $[-200^o,\ 300^o]$ and the semimajor axis a covers the domain $[26545,\ 26590]\, km$.

The files left_fig3_index012.plt, right_fig3_index012.plt, linked to the 1:1 resonance, and respectively left_fig9_index012.plt, right_fig9_index012.plt, obtained for the 2:1 resonance, provide the data computed by integrating the canonical equations and variational equations for the following initial conditions:

a) For the 1:1 resonance: $i = 0^o$, $e = 0.005$, $\omega = 0^o$ and $\Omega = 0^o$. The data in the file left_fig3_index012.plt are obtained for a model that takes into account the effects of J_2 and J_{22}, while the data in the file right_fig3_index012.plt are computed for a model that considers the effects of all harmonics up to degree and order $n = m = 3$;
b) For the 2:1 resonance: $i = 30^o$, $\omega = 0^o$, $\Omega = 0^o$ and respectively $e = 0.1$ (left_fig9_index012.plt) and $e = 0.5$ (right_fig9_index012.plt). The model takes into accounts the effects of all harmonics up to degree and order $n = m = 4$.

The (nine) columns of these four files provide the following data: 1) the resonant angle σ (in degrees); 2) the semi-major axis a (in km); 3)-8) the value of FLI after 1 year, 2, 3, 5, 10 and 19 years, respectively; 9) a (dynamical) index with the following meaning: 0-chaotic motion, 1-regular non-resonant (rotational) motion, 2-regular resonant (librational) motion;

The values of FLI at $T = 19\, years$ (8th column) are used to make a cartography the phase space of the 1:1 and 2:1 resonances as in below Figures reproduced from Celetti and Gales (2014) (top plots of Figure 3 for the GEO 1:1 resonance, and respectively bottom plots of Figure 9 for the MEO 2:1 resonance), where darker colors denote a regular dynamics (either rotational or librational motions), while lighter colors denote chaotic behavior. The values of FLI at $T = 19\, years$ (8th column) are also used to assign the (dynamical) index of the 9th column.

Machine learning algorithms are used in connection with the values of FLI provided in the columns 3)-6). Since these columns are obtained after integrating the equations for a very short interval of time, this approach could be a very fast and powerful tool to investigate various topics in the space debris dynamics.

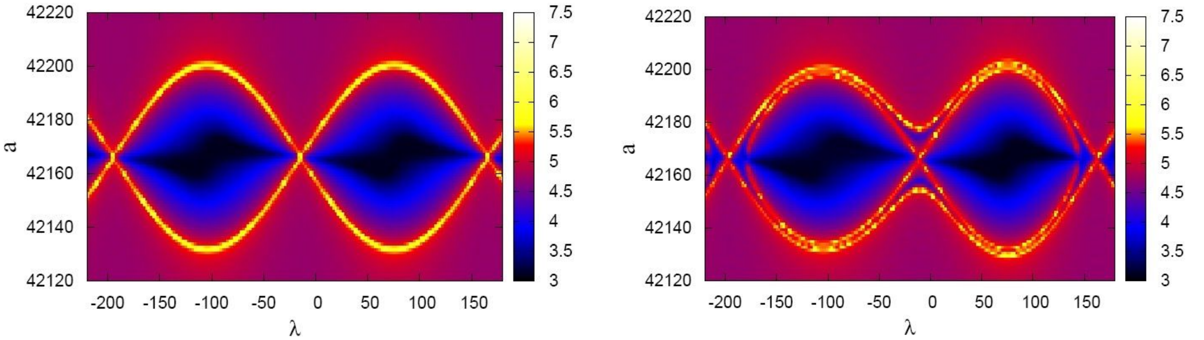

Figure 1. FLI (using Hamilton's equations) for the GEO 1:1 resonance for $e = 0.005$, $i = 0^o$, $\omega = 0^o$, $\Omega = 0^o$, under the effects of the J_2 and J_{22} (left); all harmonics up to degree and order $n = m = 3$ (right). (Reproduced from Celletti and Gales (2014), see the top plots of Figure 3.)

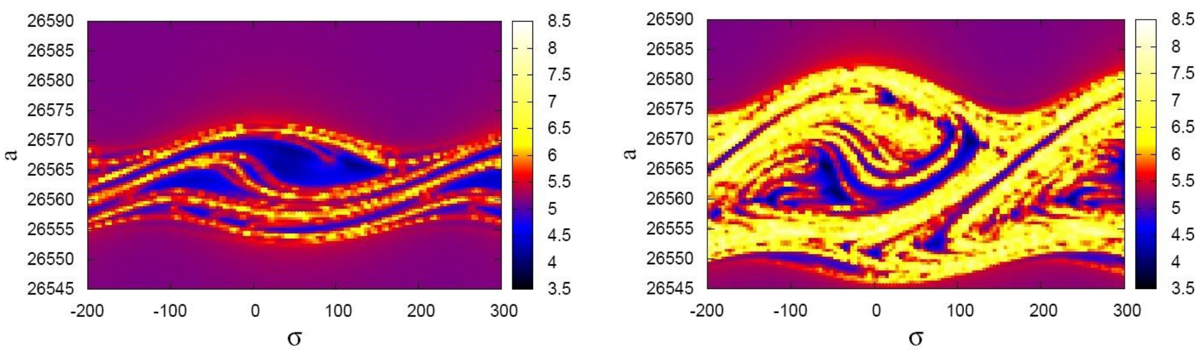

Figure 2. FLI (using Hamilton's equations) for the MEO 2:1 resonance, under the effects of all harmonics up to degree and order $n = m = 4$, for $i = 30^o$, $\omega = 0^o$, $\Omega = 0^o$: $e = 0.1$ (left); $e = 0.5$ (right). (Reproduced from Celletti and Gales (2014), see the bottom plots of Figure 9.)