

Notes:

Definition of a limit:

Let  $f(x)$  be a function defined near the fixed number  $a$ . Then the limit of  $f(x)$  as  $a$  approaches  $x$  written  $\lim_{x \rightarrow a} f(x)$ , equals number  $L$  if the values of  $f(x)$  can be made arbitrarily close to  $L$  by having  $x$  be sufficiently close to  $a$ , but not equal to  $a$ .

Example:

- Suppose a car's given position function in meters is given by  $s(t) = t^2$  for  $0 \leq t \leq 3$ . Find the instantaneous velocity at  $t=2$  seconds ( $t=2$ ).
  - Given:
    - $s(t) = t^2$  at our position
  - Wanted:
    - Velocity at  $t=2$
    - Slope of the tangent line, so the average velocity near  $t=2$
  - Expression for avg. velocity on  $[x, 2]$ 
    - $m_{\text{sec}} = \frac{\text{rise}}{\text{run}}$
    - $\frac{s(2) - s(x)}{2 - x}$
    - $\frac{4 - x^2}{2 - x}$
  - Limit:
    - $\lim_{x \rightarrow 2} \left( \frac{4 - x^2}{2 - x} \right)$
    - Approaches 4 as  $x$  approaches 2
- Key fact:
  - When you calculate a limit, if written standard, it is a TWO SIDED LIMIT
    - So it only exists if the both go to the same value.