# First Shape Theorem

*by Sophia*

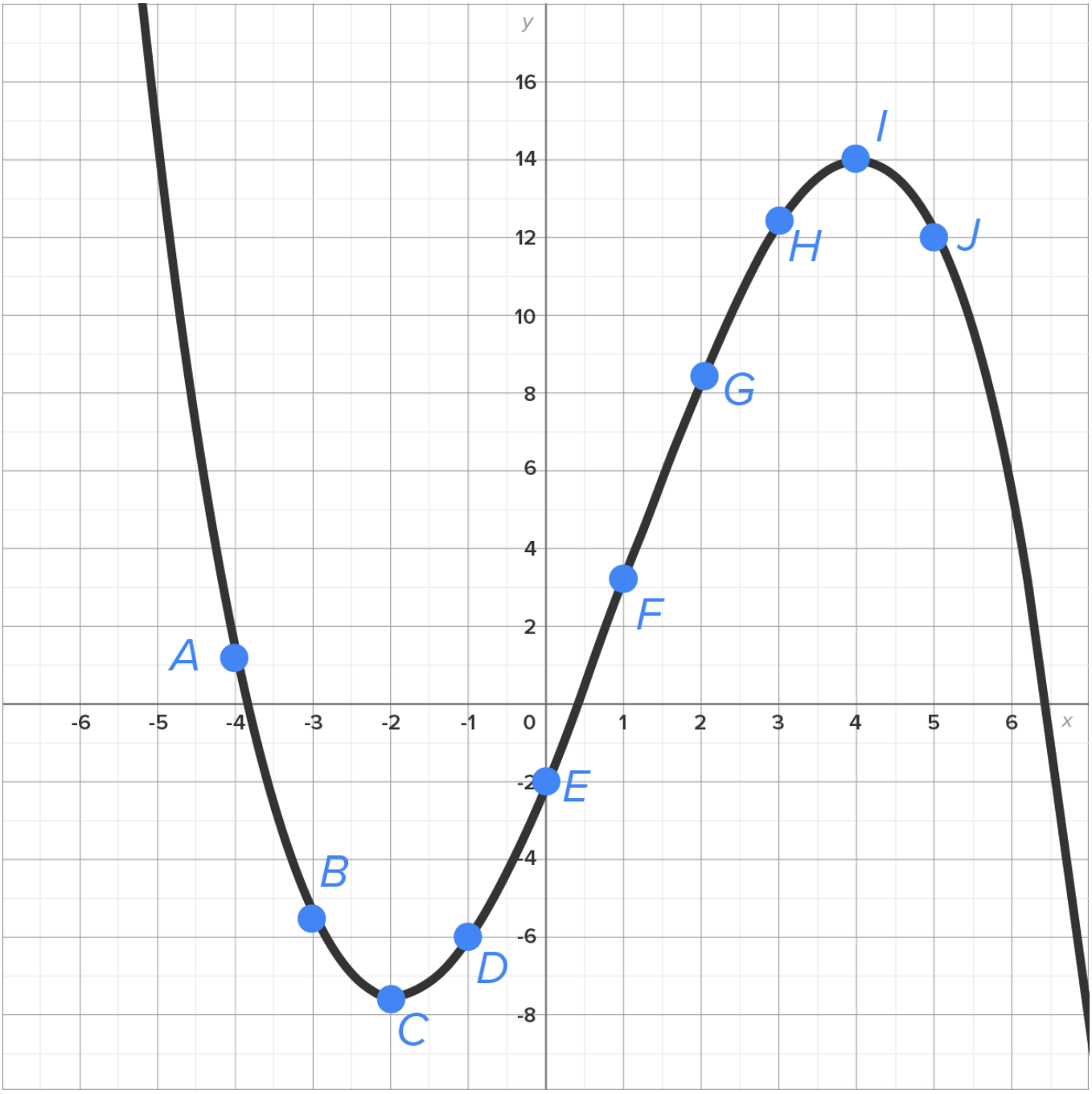
WHAT'S COVERED

In this lesson, you will use properties of a function $f(x)$ to sketch the graph of its derivative, $f'(x)$. Specifically, this lesson will cover:


1. What $f'(x)$ Tells Us About the Graph of $y = f(x)$
2. Using Slope to Graph $y = f'(x)$ Given $y = f(x)$


## 1. What $f'(x)$ Tells Us About the Graph of $y = f(x)$

Consider the graph of a function $y = f(x)$, shown below.




Note that the graph is decreasing at points *A*, *B*, and *J*. Notice also that the slopes of the tangent lines at each of these points are negative.

Note that the graph increases at points *D*, *E*, *F*, *G*, and *H*. Notice also that the slopes of the tangent lines at each of these points are positive.

Finally, points *C* and *I* are local maximum/minimum points. Notice also that the slope of the tangent line at each of these points is zero.

This leads to a very useful link between the behavior of $f(x)$ and the value of $f'(x)$.

☆ BIG IDEA

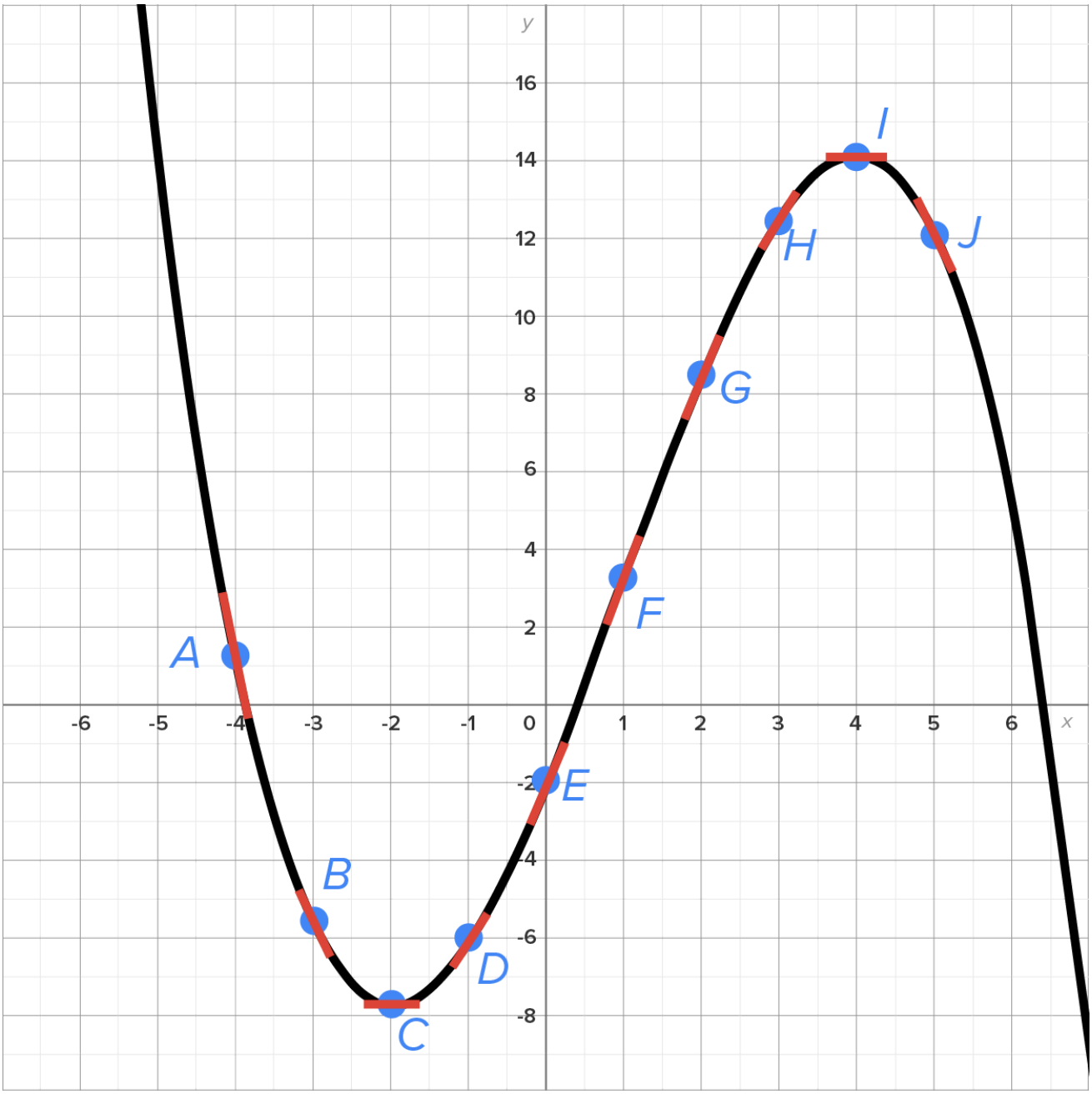
If $f(x)$ is increasing at $x = a$, then $f'(a) > 0$.

If $f(x)$ is decreasing at $x = a$, then $f'(a) < 0$.

## 2. Using Slope to Graph $y = f'(x)$ Given $y = f(x)$

Given what we know about $f'(x)$ when $f(x)$ is increasing or decreasing, we can get a rough sketch of the graph of $f'(x)$ when given the graph of $f(x)$.

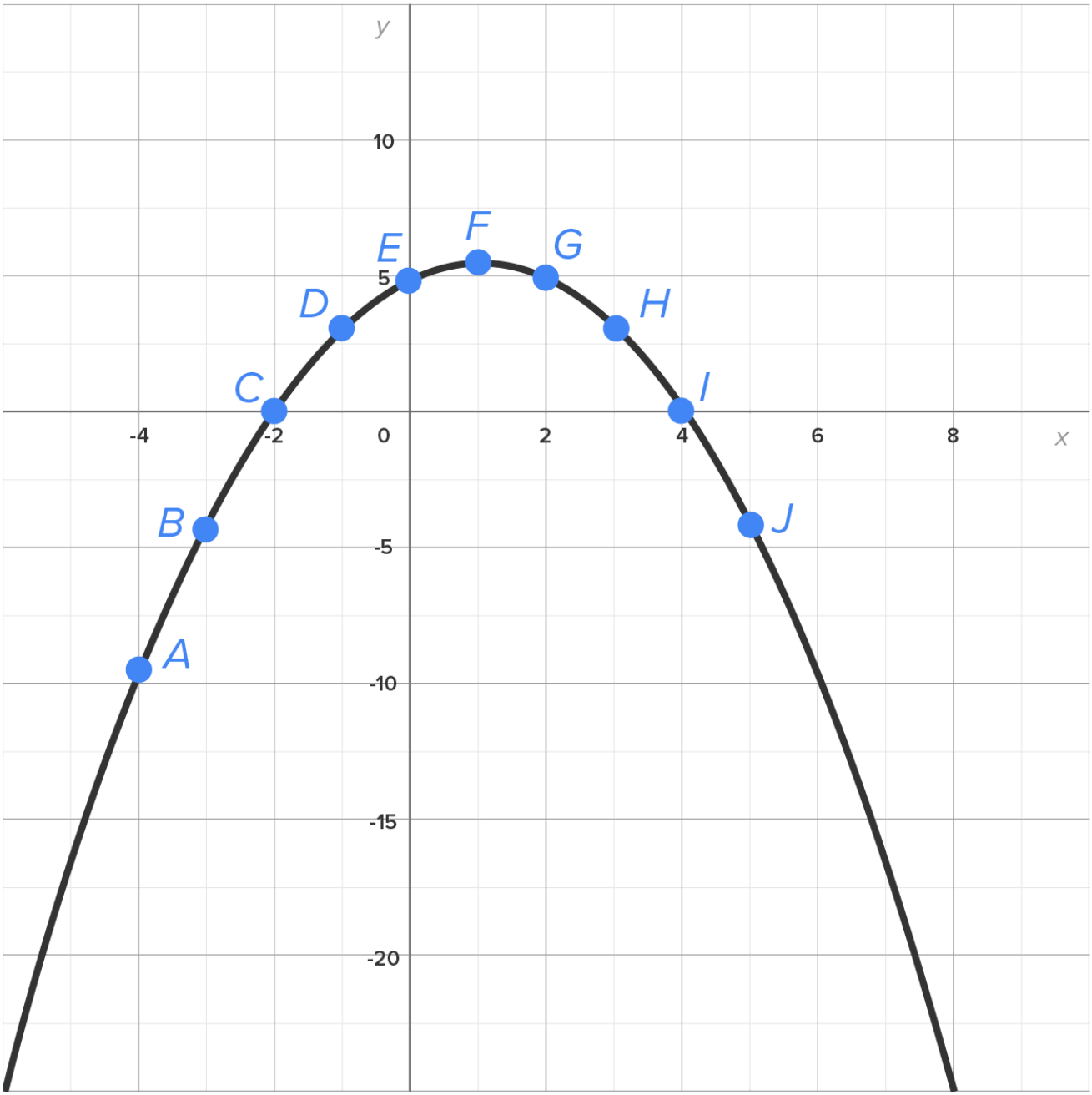
EXAMPLE Consider the graph of $y = f(x)$ shown below with tangent line segments at points *A* through *J*. Notice also the local minimum at point *C* and the local maximum at point *I*.



The behavior of $f'(x)$ can be summarized in the following table at each point. Remember that $m_{tan}$ is the value of $f'(x)$ at any point.

| Point | Value of $f'(x)$ |
|---|---|
| A | $f'(x) < 0$ |
| B | $f'(x) < 0$, but the value of $f'(x)$ is larger than its value at *A* |
| C | $f'(x) = 0$ (horizontal tangent line) |
| D | $f'(x) > 0$ |
| E | $f'(x) > 0$, but its value is noticeably greater than the slope at point *D* |
| F | $f'(x) > 0$, but its value is slightly greater than the slope at point *E* |
| G | $f'(x) > 0$, but its value is slightly less than the slope at point *F* |
| H | $f'(x) > 0$, but its value is noticeably less than the slope at point *G* |
| I | $f'(x) = 0$ (horizontal tangent line) |
| J | $f'(x) < 0$ |

The graph of the derivative is shown here. Note that the points *A* through *J* have the same x-coordinates as those marked on the graph of $f(x)$.




WATCH

In this video, we'll sketch the derivative of a function given its graph.

WATCH

In this next video, we'll sketch the derivative of a function given its graph.

## SUMMARY

In this lesson, you learned about a useful link between the behavior of $f(x)$ and the value of $f'(x)$. Specifically, **given the graph of $y = f(x)$**, it is possible to sketch **the graph of $y = f'(x)$** by **using slopes** of

the tangent lines at given points and their respective behavior.