What Is a Maximum or a Minimum?

by Sophia

:≡ **WHAT'S COVERED**

In this lesson, you will learn about the different kinds of maximum and minimum points on a graph of a function. Specifically, this lesson will cover:

1. **Definitions of Global and Local Maximum and Minimum Points**
2. **Finding Global and Local Maximum and Minimum Points**

1. Definitions of Global and Local Maximum and Minimum Points

One of the main uses of derivatives is to find minimum and maximum values of a function, or more simply put, **extreme values** (or **extrema**) of a function.

A function can have any one of the following:

- **Global (or Absolute) Maximum**: A function $f(x)$ has a global (or absolute) maximum at $x = a$ if $f(a) \geq f(x)$ for all x. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x = a$.

- **Global (or Absolute) Minimum**: A function $f(x)$ has a global (or absolute) minimum at $x = a$ if $f(a) \leq f(x)$ for all x. In other words, $f(a)$ is the smallest value of a function $f(x)$, and occurs when $x = a$.

- **Local (or Relative) Maximum**: A function $f(x)$ has a local (or relative) maximum at $x = a$ if $f(a) \geq f(x)$ for all x close to $x = a$. In other words, $f(a)$ is the largest value of a function $f(x)$ for values near $x = a$.

- **Local (or Relative) Minimum**: A function $f(x)$ has a local (or relative) minimum at $x = a$ if $f(a) \leq f(x)$ for all x close to $x = a$. In other words, $f(a)$ is the smallest value of a function $f(x)$ for values near $x = a$.

The graph shown here summarizes the differences between local and global extrema. Note that the second labeled point (from left to right) is both a local maximum and a global maximum because it meets both conditions: it is both the highest point on the graph and it is the highest point when compared to points immediately to the left and right.

global and local maximum

local maximum

local minimum

local minimum

global and local minimum
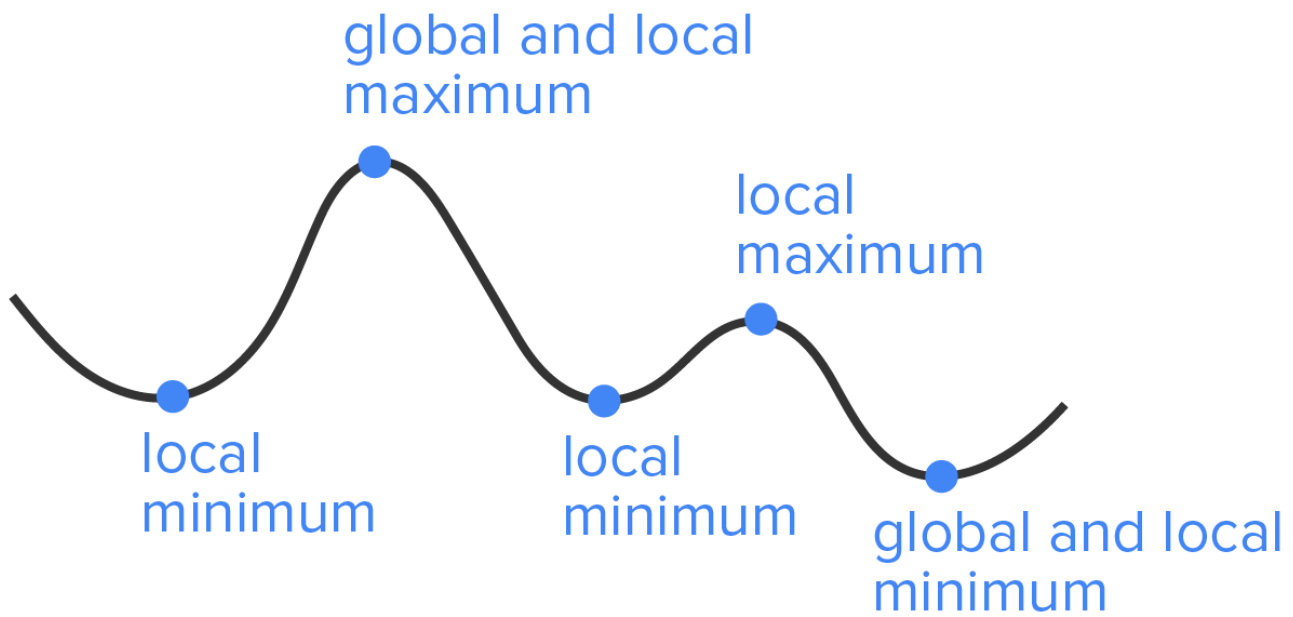
📄 **TERMS TO KNOW**

Extreme Values
The minimum or maximum values of a function.

Extrema
Another word for extreme values.

Global (or Absolute) Maximum
A function $f(x)$ has a global (or absolute) maximum at $x = a$ if $f(a) \geq f(x)$ for all x. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x = a$.

Global (or Absolute) Minimum
A function $f(x)$ has a global (or absolute) minimum at $x = a$ if $f(a) \leq f(x)$ for all x. In other words, $f(a)$ is the smallest value of a function $f(x)$, and occurs when $x = a$.

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2. Finding Global and Local Maximum and Minimum Points

↪ EXAMPLE Consider the function $f(x) = -x^4 + 10$ as shown in the graph below.
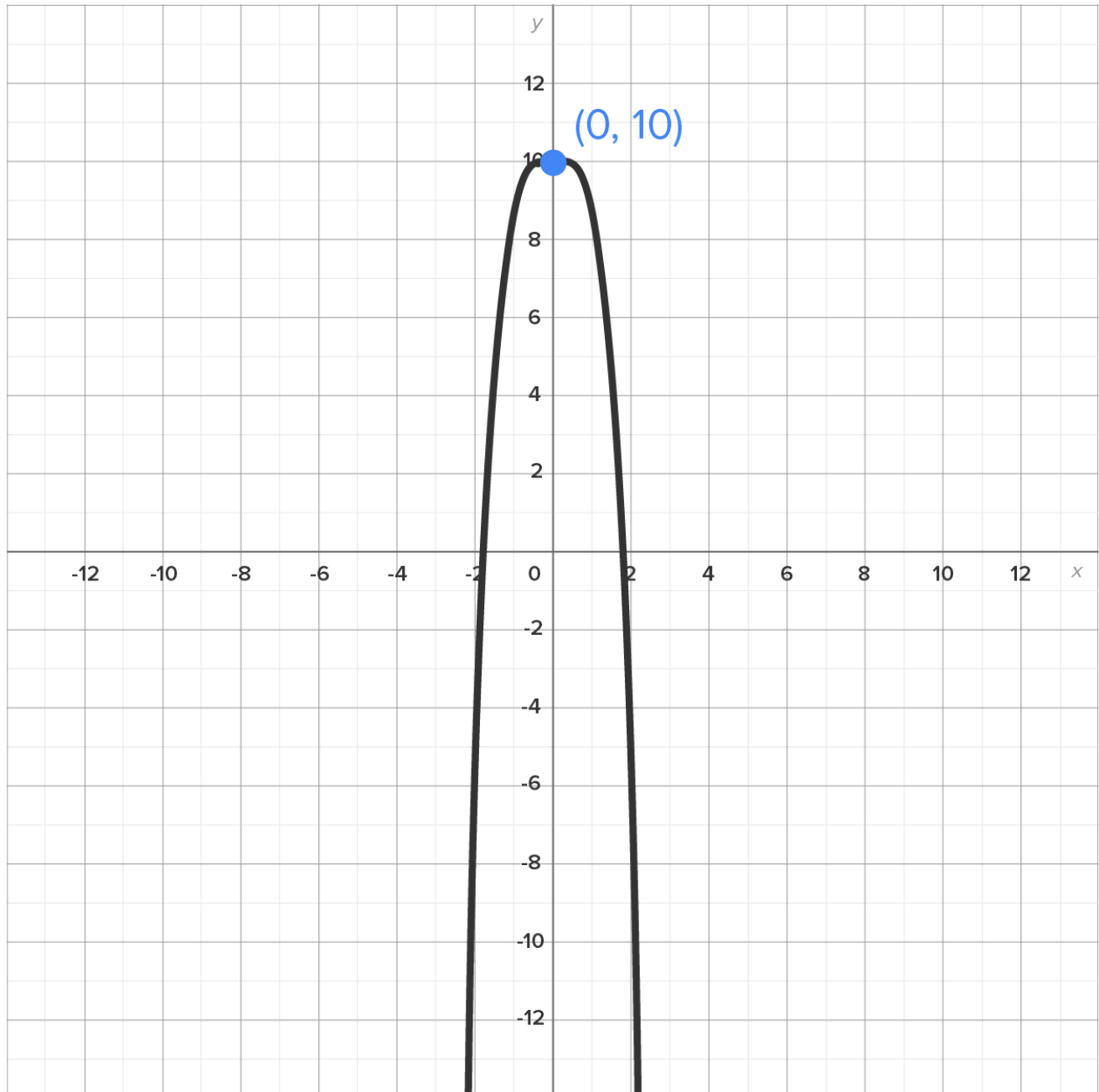
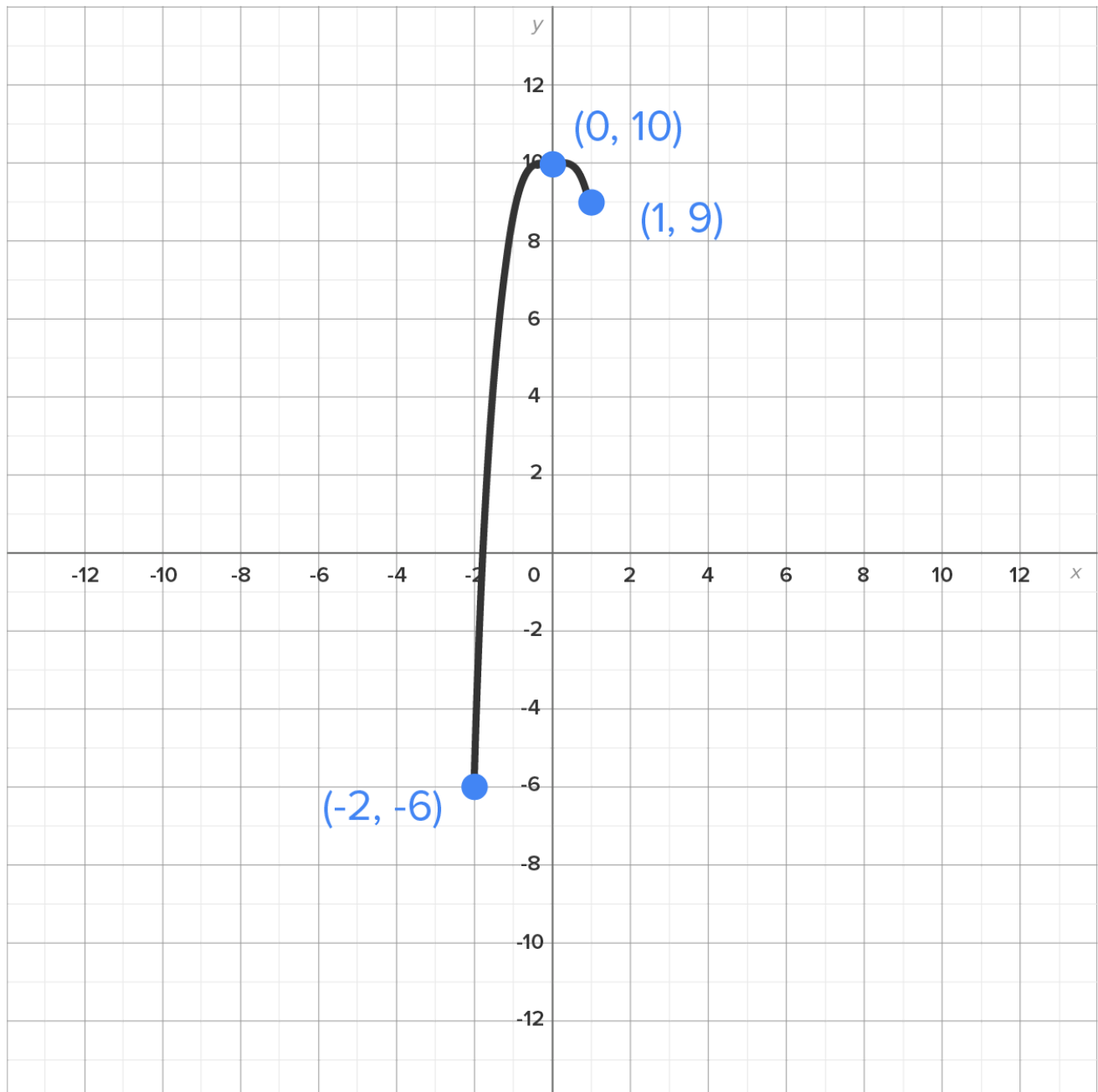
The highest point on the graph is (0, 10), while there is no lowest point. It is also the highest point when compared to other points around it.

Therefore, we say that $f(x)$ has a global maximum and local maximum at $x = 0$, and its value is 10.

There is no local or global minimum point.

⇨ EXAMPLE Now consider the function $f(x) = -x^4 + 10$, but contained on the interval $[-2, 1]$.

The highest point on the graph is (0, 10), which is also the highest point around (0, 10). Therefore, at $x = 0$, both a local and global maximum occurs and is equal to 10.

The lowest point on the graph is (-2, -6), which means that $f(x)$ has a global minimum at $x = -2$, which is equal to -6.
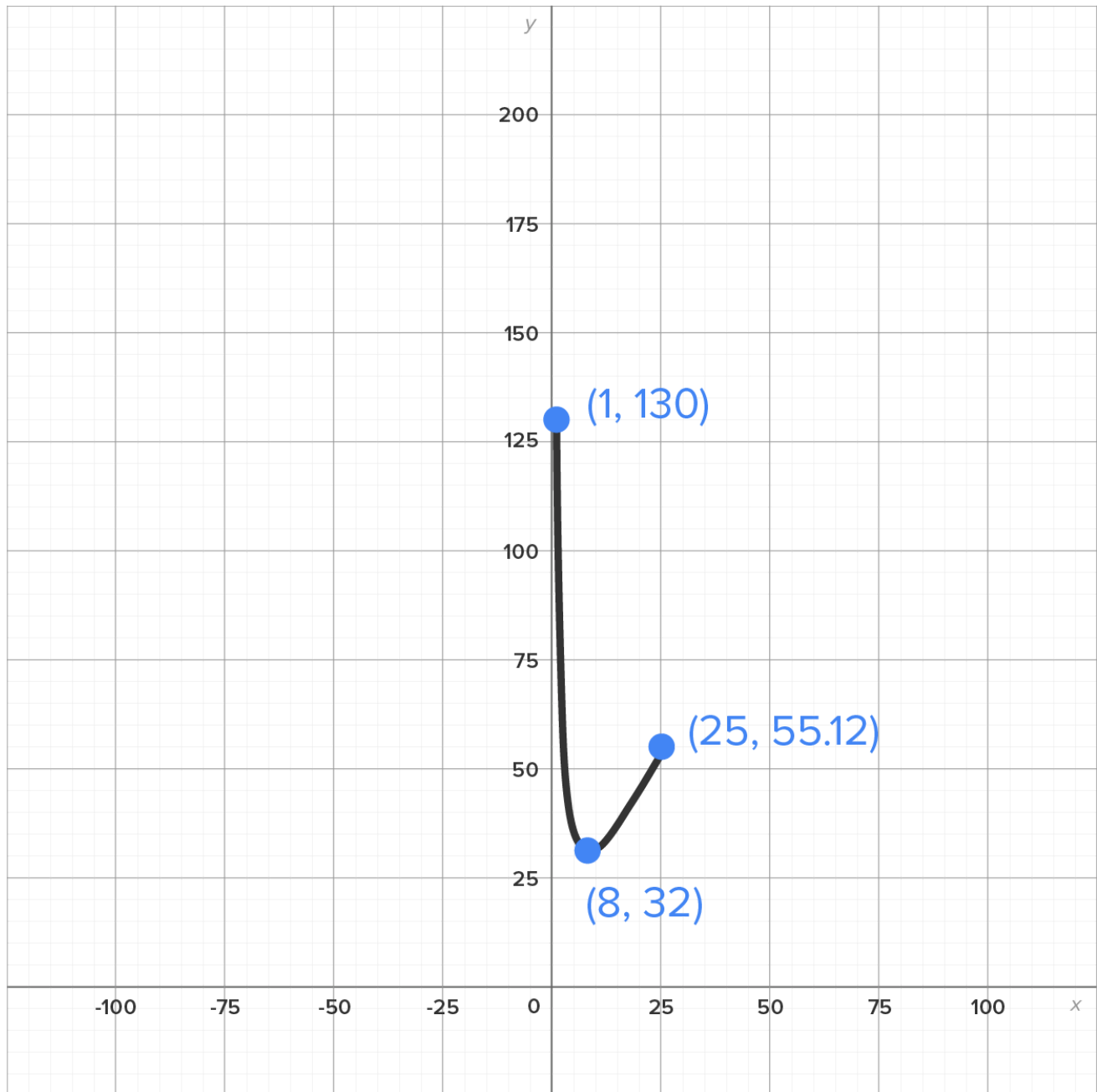
Neither (-2, -6) nor (1, 9) are considered local minimum values. This is because there is no graph on the

other side of the points to compare.

In other words, having a local extreme point at $x = a$ means that $f(a)$ is the most extreme value on an open interval containing a (both sides of $x = a$).

TRY IT

Use the graph to determine all global and local maximum and minimum values of the function.   +

The global maximum is 130 and it occurs at $x = 1$, and both a local and global minimum is 32 at $x = 8$.

The point $(1, 130)$, which is the left endpoint, is the highest point on the graph. Therefore, the global maximum value is 130. However, since $(1, 130)$ is also an endpoint, there is no local maximum value there.

The point $(8, 32)$, is the lowest point on the graph. It is also the lowest point on the graph relative to points on the left and right of it on the graph. Therefore, 32 is the global minimum value and is a local minimum value.

The point $(25, 55.12)$, which is the right endpoint, is neither the highest nor the lowest point on the graph. Since it is also an endpoint, the value 55.12 is neither a global nor local extreme value.

☑ SUMMARY

In this lesson, you learned that one of the main uses of derivatives is to find minimum and maximum values of a function. A function can have several types of extreme values, which can be identified from a graph. These **points** include: **global (or absolute) maximum**, **global (or absolute) minimum**, **local (or relative) maximum**, and **local (or relative) minimum**. Next, you explored using graphs to **find all global and local maximum and minimum values** of each respective function.

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🗎 TERMS TO KNOW

Extrema
  Another word for extreme values.

Extreme Values
  The minimum or maximum values of a function.

Global (or Absolute) Maximum
  A function $f(x)$ has a global (or absolute) maximum at $x = a$ if $f(a) \geq f(x)$ for all x. In other words, $f(a)$ is the largest value of a function $f(x)$, and occurs when $x = a$.

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Local (or Relative) Maximum

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