Indefinite Integrals of Trigonometric Functions

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WHAT'S COVERED

In this lesson, you will use the derivative rules for trigonometric functions to establish antiderivative rules for some trigonometric functions, then use these rules in combination with the power rule to find antiderivatives. Specifically, this lesson will cover:

1. Derivative Rules and Associated Antiderivative Rules

Recall the derivative rules for the six trigonometric functions:

- $D[\sin x] = \cos x$
- $D[\cos x] = -\sin x$
- $D[\tan x] = \sec^2 x$
- $D[\csc x] = -\csc x \cot x$
- $D[\sec x] = \sec x \tan x$
- $D[\cot x] = -\csc^2 x$

These lead to the following antiderivative rules:

FORMULA TO KNOW

Antiderivative of sin*x*

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$$\int \sin x\,dx = -\cos x + C$$

Antiderivative of cos*x*

$$\int \cos x\,dx = \sin x + C$$

Antiderivative of sec²*x*

$$\int \sec^2 x\,dx = \tan x + C$$

Antiderivative of csc²*x*

$$\int \csc^2 x\,dx = -\cot x + C$$

Antiderivative of sec*x*tan*x*

$$\int \sec x \tan x\,dx = \sec x + C$$

Antiderivative of csc*x*cot*x*

$$\int \csc x \cot x\,dx = -\csc x + C$$

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2. Indefinite Integrals Involving Trigonometric Functions

Let’s use these new rules to find some antiderivatives:

EXAMPLE Find the indefinite integral: $\int (4\cos t - 3\sin t)dt$

$\int (4\cos t - 3\sin t)dt$	Start with the original expression.
$= 4\int \cos t\,dt - 3\int \sin t\,dt$	Use the sum/difference and constant multiples rules.
$= 4(\sin t) - 3(-\cos t) + C$	Use antiderivative formulas for sin*t* and cos*t*.
$= 4\sin t + 3\cos t + C$	Simplify.

In conclusion, $\int (4\cos t - 3\sin t)dt = 4\sin t + 3\cos t + C$.

TRY IT

Consider $\int(2\sec x\tan x+3\csc^2 x)dx$.

Find the indefinite integral. +

$$\begin{aligned}\int(2\sec x\tan x+3\csc^2 x)dx&=2\int\sec x\tan x\,dx+3\int\csc^2 x\,dx\\&=2\sec x+3(-\cot x)+C\\&=2\sec x-3\cot x+C\end{aligned}$$

Here are some practice problems that also require the rules you've already learned.

TRY IT

Consider $\int(-2\sin x-3x)dx$.

Find the indefinite integral. +

$$\begin{aligned}\int(-2\sin x-3x)dx&=\int-2\sin x\,dx-\int 3x\,dx\\&=-2(-\cos x)-3\left(\frac{1}{2}\right)x^2+C\\&=2\cos x-\frac{3}{2}x^2+C\end{aligned}$$

TRY IT

Consider $\int\left(\frac{2}{t^3}-\sec t\tan t\right)dt$.

Find the indefinite integral. +

$$\begin{aligned}\int\left(\frac{2}{t^3}-\sec t\tan t\right)dt&=\int\frac{2}{t^3}dt-\int\sec t\tan t\,dt\\&=\int 2t^{-3}dt-\int\sec t\tan t\,dt\\&=2\left(\frac{1}{-2}\right)t^{-2}-\sec t+C\\&=-t^{-2}-\sec t+C\\&=-\frac{1}{t^2}-\sec t+C\end{aligned}$$

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3. Using Trigonometric Identities to Find Indefinite Integrals

Consider the indefinite integral $\int(\sin^2\theta+\cos^2\theta)d\theta$. According to rules we know so far, we are unable to find $\int\sin^2\theta d\theta$ or $\int\cos^2\theta d\theta$. However, you may recall the identity $\sin^2\theta+\cos^2\theta=1$. This means that $\int(\sin^2\theta+\cos^2\theta)d\theta=\int 1d\theta=\theta+C$.

Thus, sometimes it is possible to rewrite the integrand using a (trigonometric) identity so that an indefinite integral formula can be used.

EXAMPLE Find the indefinite integral: $\int\tan^2 xdx$

While $\tan^2 x$ doesn't have an integration formula, we can apply a trigonometric identity.

$\int\tan^2 xdx$	Start with the original expression.
$=\int(\sec^2 x-1)dx$	Use the identity $1+\tan^2 x=\sec^2 x$.
$=\int\sec^2 xdx-\int 1dx$	Apply the difference property.
$=\tan x-x+C$	Use formulas for $\int\sec^2 xdx$ and $\int 1dx$.

Thus, $\int\tan^2 xdx=\tan x-x+C$.

THINK ABOUT IT

Notice that the antiderivatives of $\sin x$ and $\cos x$ are known, but the other four are not. It turns out that the antiderivatives of the other four trigonometric functions require more advanced techniques that are covered later in this challenge. This speaks to a larger idea that the processes for derivatives and antiderivatives can be similar, or they can be quite different.

Here is a derivative to think about for a later challenge.

- $f(x)=\ln(\sec x)$
- $f'(x)=\frac{1}{\sec x}\cdot\sec x\tan x=\tan x$, using $D[\ln u]=\frac{1}{u}\cdot u'$

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The significance is that we now have a function whose derivative is tan*x*. In other words, $\int \tan x dx = \ln|\sec x| + C$. You do not need to know this (yet)—this is just something to think about!

SUMMARY

In this lesson, you reviewed the **derivative rules** for the six trigonometric functions and learned about their **associated antiderivative rules**. Then, you were able to apply these rules to find **indefinite integrals (antiderivatives) involving trigonometric functions**, expanding on the antiderivatives that you are able to find. You also learned that it is possible to rewrite the integrand **using a trigonometric identity to find indefinite integrals**.

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FORMULAS TO KNOW

Antiderivative of cos x

$$\int \cos x dx = \sin x + C$$

Antiderivative of csc x cot x

$$\int \csc x \cot x dx = -\csc x + C$$

Antiderivative of csc^2 x

$$\int \csc^2 x dx = -\cot x + C$$

Antiderivative of sec x tan x

$$\int \sec x \tan x dx = \sec x + C$$

Antiderivative of sec^2 x

$$\int \sec^2 x dx = \tan x + C$$

Antiderivative of sin x

$$\int \sin x dx = -\cos x + C$$