Derivatives and Graphs

by Sophia

WHAT'S COVERED

In this lesson, you will explore a visual way to estimate the slope of the tangent line of a function $y = f(x)$. Specifically, this lesson will cover:

1. Estimating the Slope of a Tangent Line Graphically
2. The Derivative

1. Estimating the Slope of a Tangent Line Graphically

When you see the title of this challenge, it might sound familiar. That is because we explored how to estimate the slope of a tangent line in a previous challenge! In this challenge though, we will review those skills as well as introduce new notation and definitions to help go further into calculus.

Recall the following formula for slope:

FORMULA TO KNOW

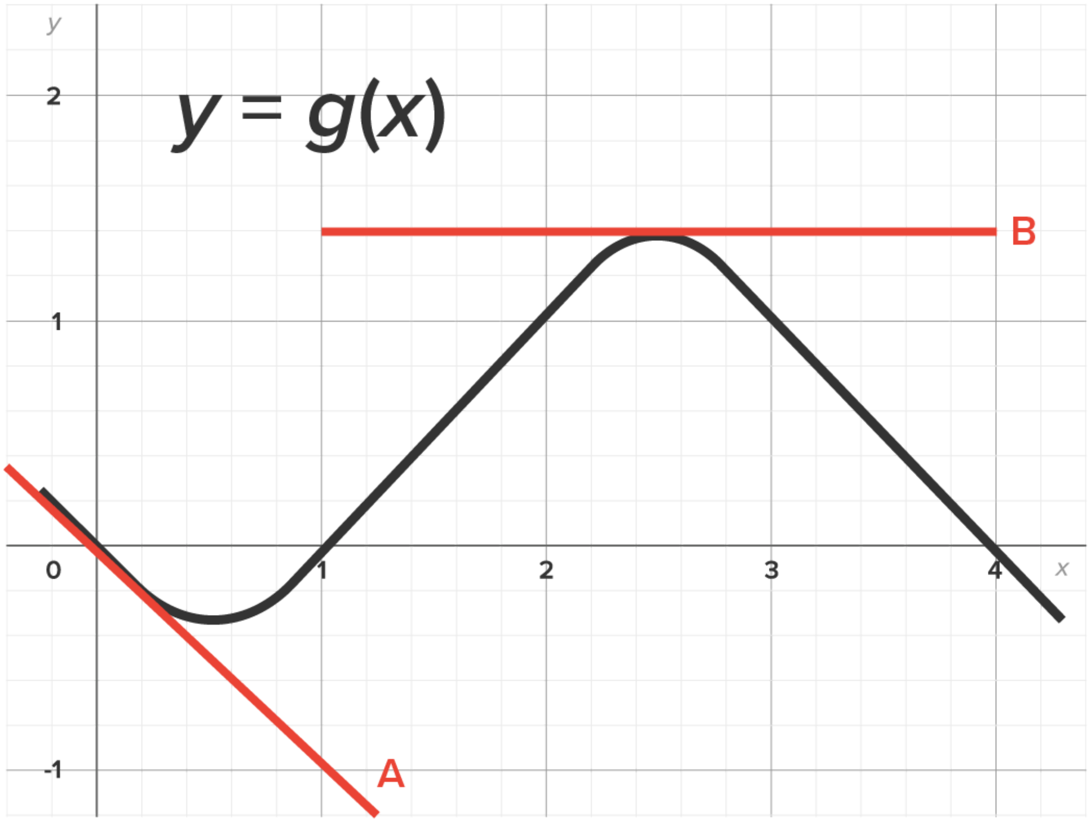
Slope of the Line Passing Through the Points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

HINT

The slope of the tangent line is often represented using $m_{\tan}$.

EXAMPLE Consider the function $y = g(x)$ whose graph is shown below.



The line marked A is the tangent line to the graph at $x=0$. By the sketch, it passes through the points (0, 0) and (1, -1). Using the slope formula $m=\frac{y_2-y_1}{x_2-x_1}$, the approximate slope is $m_{\tan}=\frac{-1-0}{1-0}=-1$.

The line marked B is tangent to the graph at $x=2.5$. Since this line appears to be horizontal, $m_{\tan}=0$.

2. The Derivative

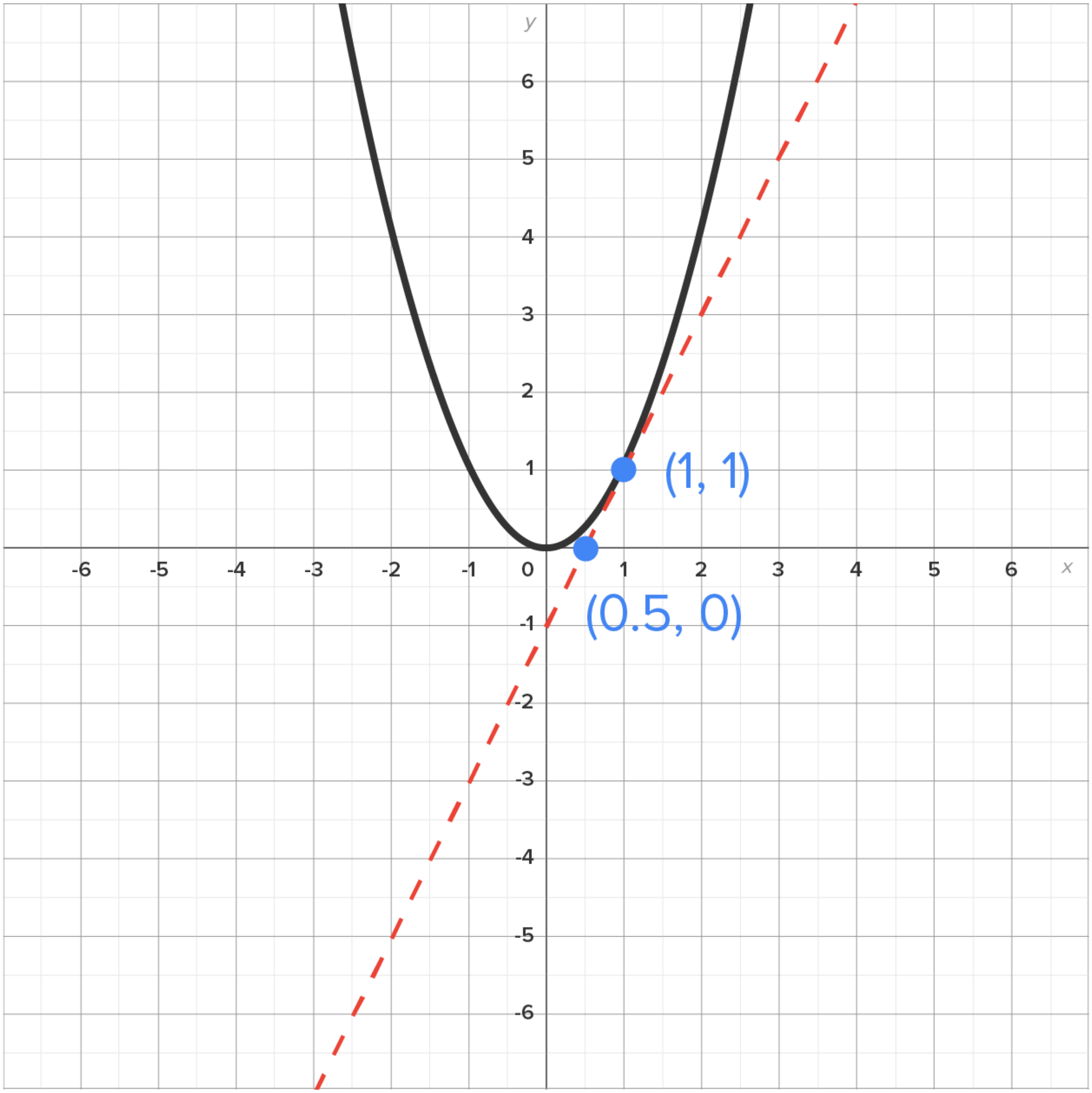
In the previous example, note how the slope of the tangent line changes as *x* changes. This tells us that the slope of the tangent line itself is a function of *x*. Instead of saying "the slope of a tangent line function," we have a more elegant name for this important aspect of a function.

BIG IDEA

The slope of the tangent line at a point on the function is equal to the **derivative** of the function at the same point.

Now, let's look at a more familiar function and the slopes of tangent lines at certain points.

EXAMPLE Consider the function $f(x)=x^2$. Let's graphically estimate the derivative (slope of the tangent line) of $f(x)$ when $x=1$.



The figure shows the graph of $f(x)=x^2$ (solid) and the tangent line (dashed) when $x=1$, which touches the graph at the point (1, 1).

Note that the tangent line when $x=1$ also passes through (0.5, 0). This means the slope of this line is 2.

So, in conclusion, the slope of the tangent line to $f(x)=x^2$ when $x=1$ is 2. Another way to say this is "the derivative of $f(x)$ when $x=1$ is 2."

TRY IT

Using the graph of $f(x) = x^2$ from the previous example, estimate the derivative of $f(x)$ when $x = -2$.

Estimate the derivative. +

The slope should be -4. The tangent line at (-2, 4) should also pass through (-1, 0).

TERM TO KNOW

Derivative

The slope of the tangent line to the graph of a function at a point is also known as the derivative of the function at that point.

SUMMARY

In this lesson, you learned that the slope of the tangent line is also known as the **derivative**, which can be represented using $m_{\tan}$. You learned how to **estimate the slope of a tangent line** (derivative) graphically by drawing a tangent line at a given point, determining another point on the tangent line, then computing the slope using the formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

TERMS TO KNOW

Derivative

The slope of the tangent line to the graph of a function at a point is also known as the derivative of the function at that point.

FORMULAS TO KNOW

Slope of the Line Passing Through the Points (x_1, y_1) and (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$