Equations of Tangent Lines

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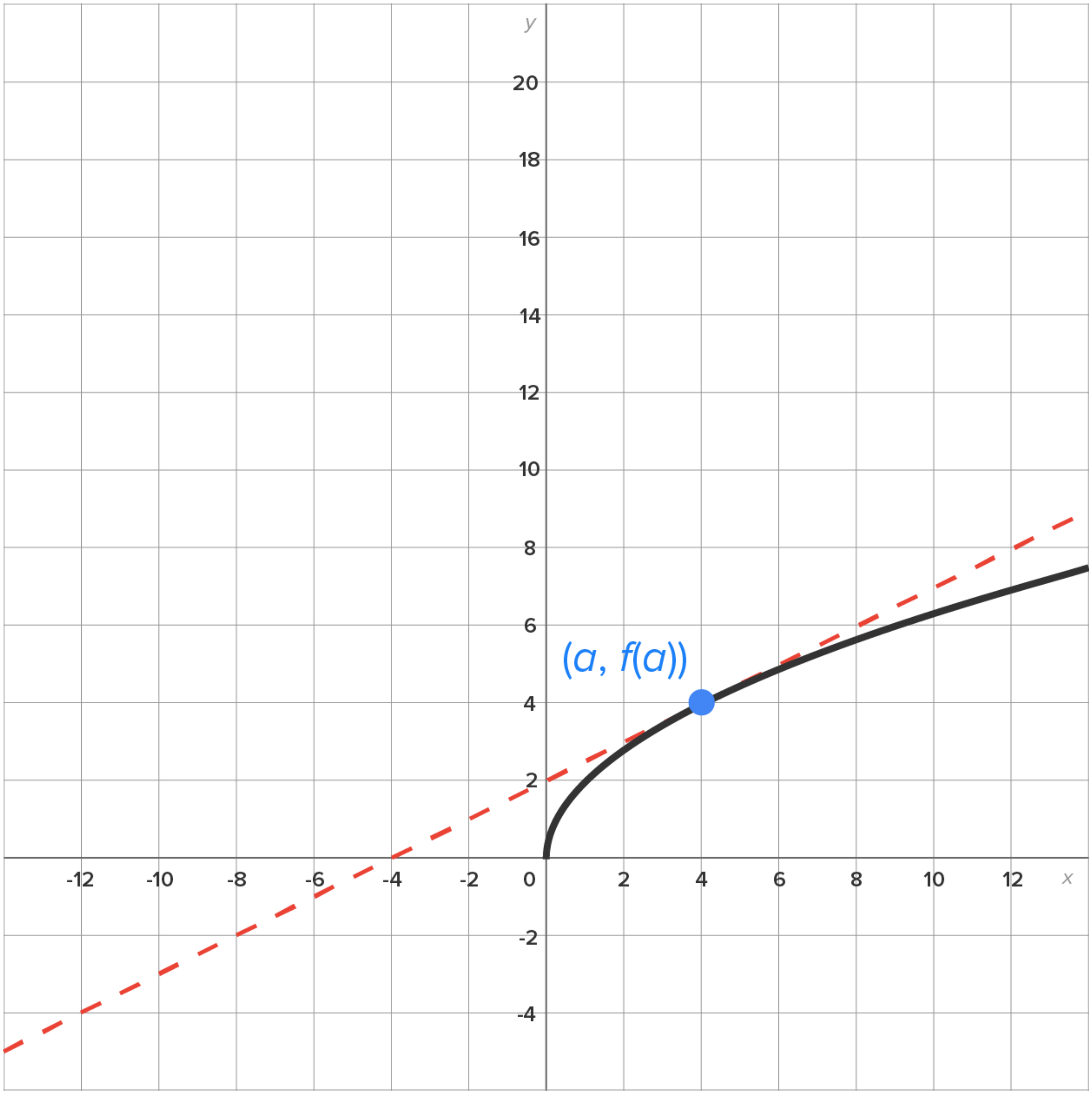
WHAT'S COVERED

In this lesson, you will use derivative rules to write the equation of a tangent line to a function $f(x)$. Specifically, this lesson will cover:

1. Writing the Equation of a Tangent Line at a Specific Point
2. Different Types of Functions
   - 2a. Power Functions ($y = x^n$)
   - 2b. $y = \sin x$ and $y = \cos x$

1. Writing the Equation of a Tangent Line at a Specific Point

Shown here is the graph of some function $y = f(x)$ and its tangent line at $(a, f(a))$.



Recall from Unit 1 that writing the equation of a line requires two things:

- The slope of the line
- A point on the line

Given a function $y = f(x)$, this information is known at $x = a$:

- The slope of the line is $f'(a)$.
- A point on the line is $(a, f(a))$.

For now, let's assume that $f'(a)$ is defined, meaning that the tangent line is nonvertical.
Now, use the point-slope form to write the equation of the tangent line:

$y-y_1=m(x-x_1)$	Use the point-slope form.
$y-f(a)=f'(a)(x-a)$	$(x_1, y_1)=(a, f(a)),\ m=f'(a)$
$y=f(a)+f'(a)(x-a)$	Add $f(a)$ to both sides to solve for y.

FORMULA TO KNOW

Equation of a Tangent Line to $y=f(x)$ at $x=a$

$$y=f(a)+f'(a)(x-a)$$

2. Different Types of Functions

Now, let's focus on the mechanics required to write tangent lines for different types of functions.

2a. Power Functions ($y=x^n$)

EXAMPLE Write the equation of the line tangent to $f(x)=x^3$ when $x=2$.

First, the line is tangent to the graph at the point $(2, f(2))$, or (2, 8). The derivative is $f'(x)=3x^2$. Then, the slope of the tangent line is $f'(2)=3(2)^2=12$.

Now, use the tangent line formula:

$y=f(a)+f'(a)(x-a)$	Use the equation of a tangent line.
$y=f(2)+f'(2)(x-2)$	$a=2$
$y=8+12(x-2)$	$f(2)=8$ and $f'(2)=12$
$y=8+12x-24$	Distribute.
$y=12x-16$	Combine like terms.

In conclusion, the equation of the tangent line is $y=12x-16$.

EXAMPLE Write the equation of the line tangent to $f(x)=\frac{1}{x^2}$ when $x=1$. The line is tangent to the graph at the point $(1, f(1))$, or (1, 1).

First, rewrite $f(x)=\frac{1}{x^2}$ with a single exponent: $f(x)=x^{-2}$. By the power rule, $f'(x)=-2x^{-3}=\frac{-2}{x^3}$. Then, the slope of the tangent line is $f'(1)=\frac{-2}{(1)^3}=-2$.

Now, use the tangent line formula:

$y = f(a) + f'(a)(x - a)$	Use the equation of a tangent line.
$y = f(1) + f'(1)(x - 1)$	$a = 1$
$y = 1 - 2(x - 1)$	$f(1) = 1$ and $f'(1) = -2$
$y = 1 - 2x + 2$	Distribute.
$y = -2x + 3$	Combine like terms.

In conclusion, the equation of the tangent line is $y = -2x + 3$.

TRY IT

Consider the function $f(x) = x^{3/2}$

Write the equation of the line tangent to the graph of this function at $x = 4$. +

The equation of the tangent line is $y = 3x - 4$. Here is why:

First, find $f'(x) = \frac{3}{2}x^{1/2}$.

Then, $f'(4) = \frac{3}{2}(4)^{1/2} = \frac{3}{2}(2) = 3$.

At this point, we have the slope of the line.

Since $f(4) = 4^{3/2} = 8$, the line contains the point $(4, 8)$.

Now, use point-slope form to write the equation of the line, then solve for *y*:

$$y - 8 = 3(x - 4)$$
$$y - 8 = 3x - 12$$
$$y = 3x - 4$$

2b. $y = \sin x$ and $y = \cos x$

Let's look at an example involving a trigonometric function.

EXAMPLE Write the equation of the line tangent to the graph of $f(x)=\cos x$ at the point $\left(\frac{\pi}{2}, 0\right)$.

First, recall that $f'(x) = -\sin x$. Then, the slope of the tangent line is $f'\left(\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$.

Now, use the tangent line formula:

$y=f(a)+f'(a)(x-a)$	Use the equation of a tangent line.
$y=f\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)\left(x-\frac{\pi}{2}\right)$	$a=\frac{\pi}{2}$
$y=0+(-1)\left(x-\frac{\pi}{2}\right)$	$f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=-1$
$y=-x+\frac{\pi}{2}$	Distribute and simplify.

Thus, the equation of the tangent line is $y=-x+\frac{\pi}{2}$.

SUMMARY

In this lesson, you learned how to **write the equation of the tangent line at a specific point**, noting that this equation can be found for a function $f(x)$ at $x=a$ as long as $f'(a)$ is defined. You also learned how to write tangent lines for **different types of functions**, such as **power functions** ($y=x^n$) and trigonometric functions ($y=\sin x$ **and** $y=\cos x$). This is a gateway for a wider variety of applications that will be discussed later in this chapter once we learn how to find derivatives of more functions.

FORMULAS TO KNOW

Equation of a Tangent Line to y = f(x) at x = a

$y=f(a)+f'(a)(x-a)$