Average Rate of Change

by Sophia

WHAT'S COVERED

In this lesson, you will learn how to calculate the average rate of change of a function and how it relates to slope. Specifically, this lesson will cover:

1. Average Rate of Change

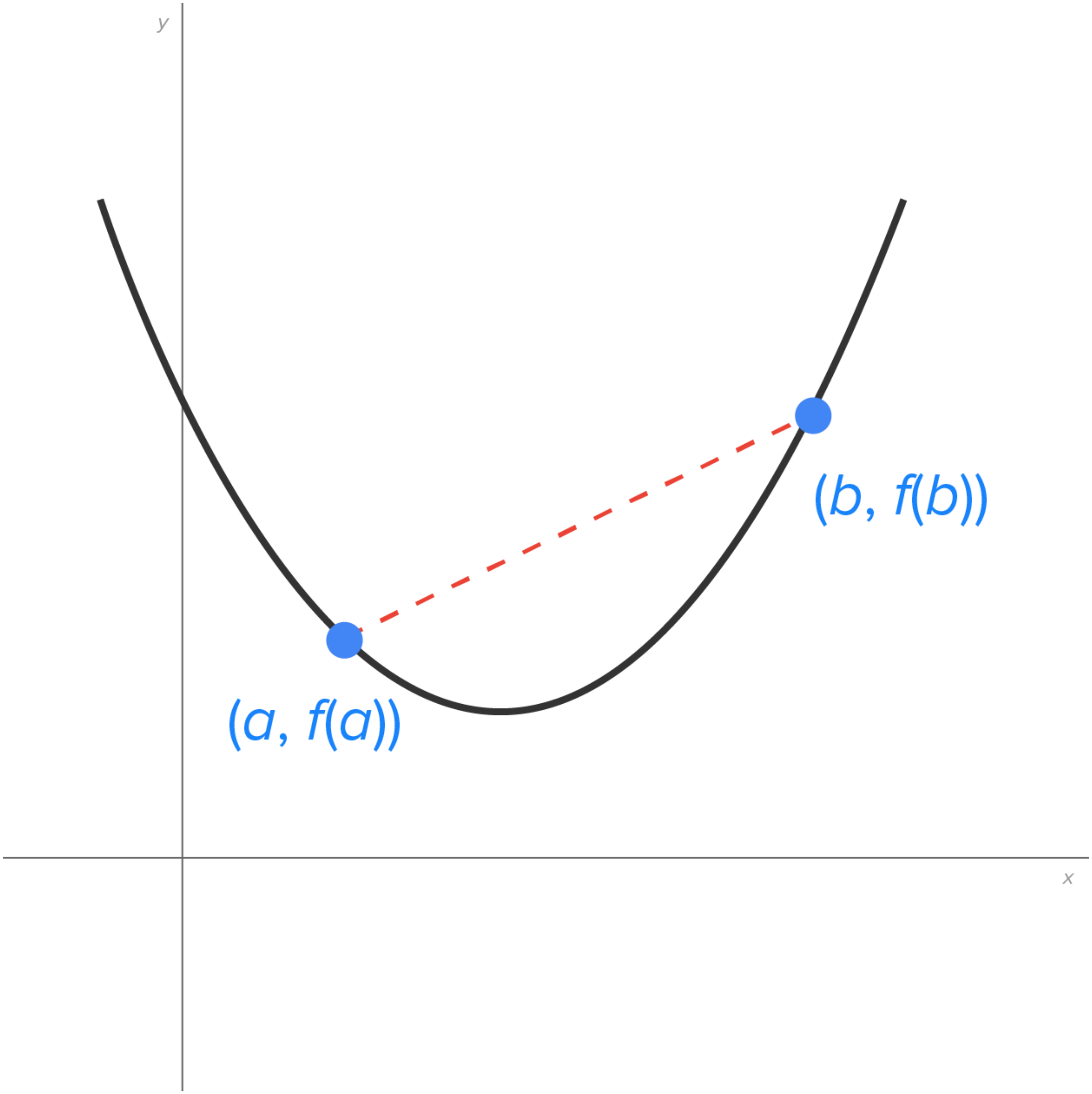
A 320-mile car ride takes 8 hours. We say that on average, the speed is 40 miles per hour. Naturally, there were times when the speed was slower (starting and stopping) and others when it was faster (highway driving). The "40 miles per hour" is an **average rate of change** over the 8 hours.

Given a function $y = f(x)$ on some interval $[a, b]$, we define the average rate of change as follows:

FORMULA TO KNOW

Average Rate of Change on the Interval [*a*, *b*]

$$\frac{\text{change in } f}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$



EXAMPLE Calculate the average rate of change of $f(x) = x^3 - 2x + 4$ on the interval $[1, 3]$.

$f(1) = 1^3 - 2(1) + 4 = 3$, $f(3) = 3^3 - 2(3) + 4 = 25$

Average rate of change $= \dfrac{25 - 3}{3 - 1} = \dfrac{22}{2} = 11$

This means that on average, the curve rises 11 units for every 1 unit of horizontal increase.

TRY IT

Consider the function $f(x) = 4\sqrt{x+5}$.

Calculate the average rate of change of this function on the interval [4, 20]. +

$f(4) = 4\sqrt{9} = 12,\ f(20) = 4\sqrt{25} = 20$

$$\text{Average rate of change} = \frac{20-12}{20-4} = \frac{8}{16} = \frac{1}{2}$$

One important application of average rate of change is **velocity**.

EXAMPLE An object is dropped off a tall building. Its height after t seconds is $h(t) = 1200 - 16t^2$ feet. What is the average rate of change of this object's height in its first 2 seconds of descent?

This translates to calculating the average rate of change from $t = 0$ to $t = 2$.

$h(0) = 1200 - 16(0)^2 = 1200,\ h(2) = 1200 - 16(2)^2 = 1136$

$$\text{Average rate of change} = \frac{1136-1200}{2-0} = \frac{-64}{2} = -32$$

Note that the unit of the average rate of change is feet/second. Thus, we can view the average rate of change as velocity. It is negative since the object is moving downward.

TERMS TO KNOW

Average Rate of Change

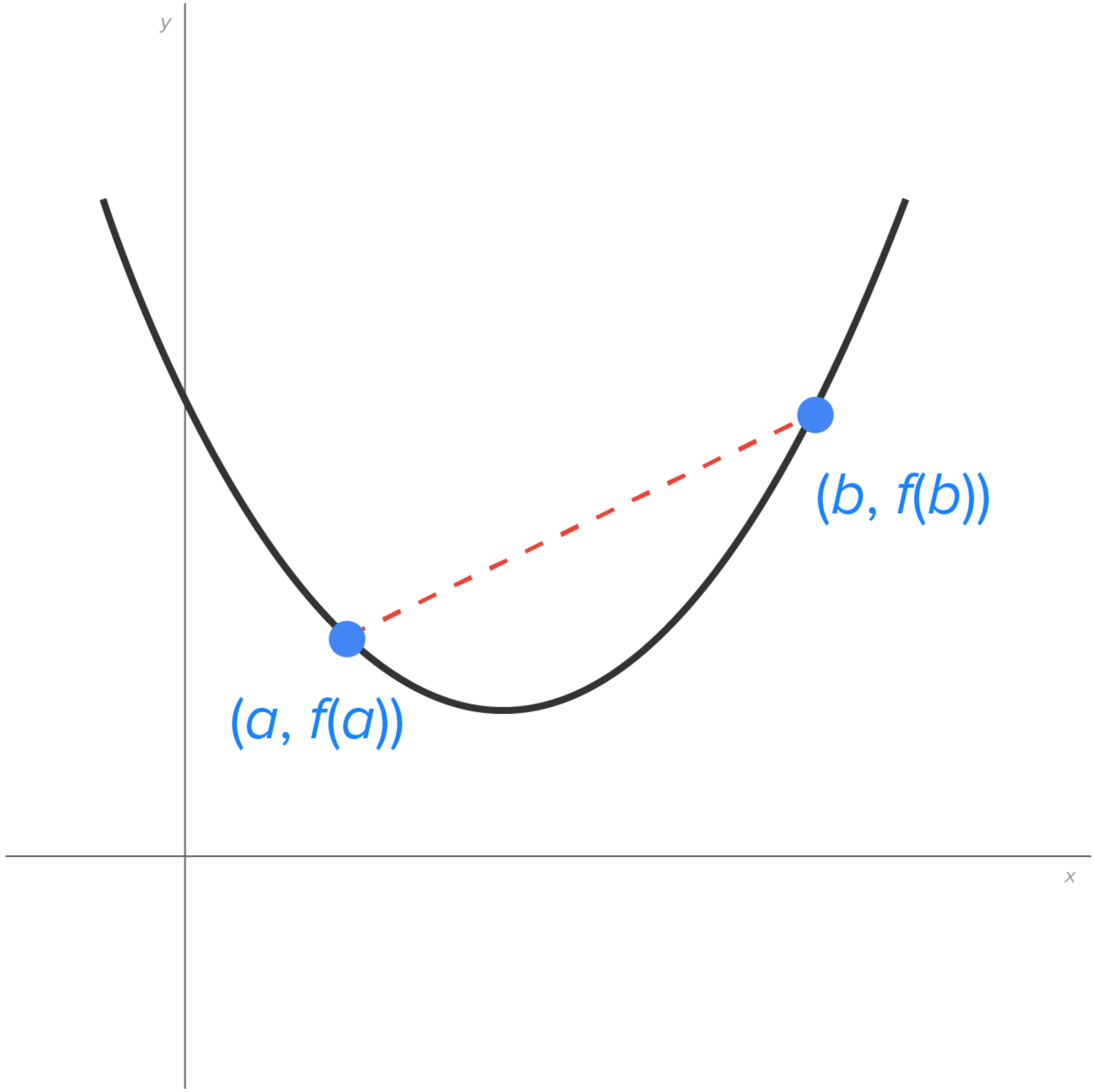
The net change divided by the length of the interval.

Velocity

The speed of some object relative to some starting point. Unlike speed, velocity can be negative.

2. Secant Lines

If you look closely at this picture, you might notice that the average rate of change is really the slope of the line connecting $(a, f(a))$ and $(b, f(b))$.



Recall that a tangent line touches the graph at one point. A secant line connects two points of the graph. The slope of the **secant line** is the average rate of change of the function between the two endpoints.

EXAMPLE Calculate the slope of the secant line of $f(x)=10-2x-x^2$ between $x=1$ and $x=4$.

$f(1)=10-2(1)-(1)^2=7,\ f(4)=10-2(4)-4^2=-14$

$$\text{Slope}=\text{Average rate of change}=\frac{-14-7}{4-1}=-\frac{21}{3}=-7$$

TRY IT

Consider the function $f(x) = 2^x$.

Find the slope of the secant line of this function between $x = 0$ and $x = 3$. +

$f(0) = 2^0 = 1, \; f(3) = 2^3 = 8$

$$\text{Slope} = \text{Average rate of change} = \frac{8-1}{3-0} = \frac{7}{3}$$

TERM TO KNOW

Secant Line

A line that contains two points of the same function.

SUMMARY

In this lesson, you learned that the **average rate of change** of a function is the net change divided by the length of the interval. This can be visualized as the slope of the line between the two points on the graph, known as the **secant line**. You also learned that one important application of average rate of change is velocity, which, unlike speed, can be negative.

TERMS TO KNOW

Average Rate of Change

The net change divided by the length of the interval.

Secant Line

A line that contains two points of the same function.

Velocity

The speed of some object relative to some starting point. Unlike speed, velocity can be negative.

FORMULAS TO KNOW

Average Rate of Change on the Interval [a, b]

$$\frac{\text{change in } f}{\text{change in } x} = \frac{f(b) - f(a)}{b - a}$$