Indefinite Integrals of Exponential Functions

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WHAT'S COVERED

In this lesson, you will find antiderivatives of exponential functions and incorporate them into the antiderivatives we already know (powers and trigonometric functions). Specifically, this lesson will cover:

1. Antiderivatives of Exponential Functions

Recall that $D[e^x] = e^x$ and $D[a^x] = a^x \cdot \ln a$, assuming $a > 0$. This leads to the following antiderivative formulas:

FORMULA TO KNOW

Antiderivatives of Exponential Functions

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

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2. Antiderivatives of Functions Containing Exponential Functions

Let's get into some examples.

EXAMPLE Find the indefinite integral: $\int(4e^x - 2x + 1)dx$

Note that the same properties can be used.

$\int(4e^x - 2x + 1)dx$	Start with the original expression.
$= 4\int e^x dx - 2\int x dx + \int 1 dx$	Use the sum/difference properties and the constant multiple rule.
$= 4(e^x) - 2\left(\frac{x^2}{2}\right) + x + C$	Apply exponential and power rules.
$= 4e^x - x^2 + x + C$	Simplify.

Thus, $\int(4e^x - 2x + 1)dx = 4e^x - x^2 + x + C$.

EXAMPLE Find the indefinite integral: $\int\left(3^x - \frac{2}{3}\sin x\right)dx$

$\int\left(3^x - \frac{2}{3}\sin x\right)dx$	Start with the original expression.
$= \int 3^x dx - \frac{2}{3}\int \sin x dx$	Use the sum/difference properties and the constant multiple rule.
$= \frac{3^x}{\ln 3} - \frac{2}{3}(-\cos x) + C$	Apply formulas for $\int a^x dx$ and $\int \sin x dx$.
$= \frac{3^x}{\ln 3} + \frac{2}{3}\cos x + C$	Simplify.

Thus, $\int\left(3^x - \frac{2}{3}\sin x\right)dx = \frac{3^x}{\ln 3} + \frac{2}{3}\cos x + C$.

TRY IT

Consider $\int(x^2 - 5e^x)dx$.

Find the indefinite integral. +

$$\int(x^2 - 5e^x)dx = \int x^2 dx - \int 5e^x dx$$
$$= \frac{1}{3}x^3 - 5e^x + C$$

TRY IT

Consider $\int\left(10^x - \frac{6}{\sqrt{x}}\right)dx$.

Find the indefinite integral. +

Start by splitting up the integrals and rewriting the radical as an exponent:

$$\int\left(10^x - \frac{6}{\sqrt{x}}\right)dx = \int 10^x dx - \int 6x^{-1/2}dx$$

Now apply the antiderivative rules, then simplify:

$$= \frac{10^x}{\ln 10} - 6\left(\frac{1}{\left(\frac{1}{2}\right)}x^{1/2}\right) + C = \frac{10^x}{\ln 10} - 12x^{1/2} + C$$

Thus, $\int\left(10^x - \frac{6}{\sqrt{x}}\right)dx = \frac{10^x}{\ln 10} - 12x^{1/2} + C$.

In radical form, this is written $\frac{10^x}{\ln 10} - 12\sqrt{x} + C$.

SUMMARY

In this lesson, you learned the formula for **antiderivatives of exponential functions**, expanding your abilities to find derivatives to include finding **antiderivatives of functions containing exponential functions**.

FORMULAS TO KNOW

Antiderivatives of Exponential Functions

$$\int e^x dx = e^x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$