Slope of a Tangent Line Visually

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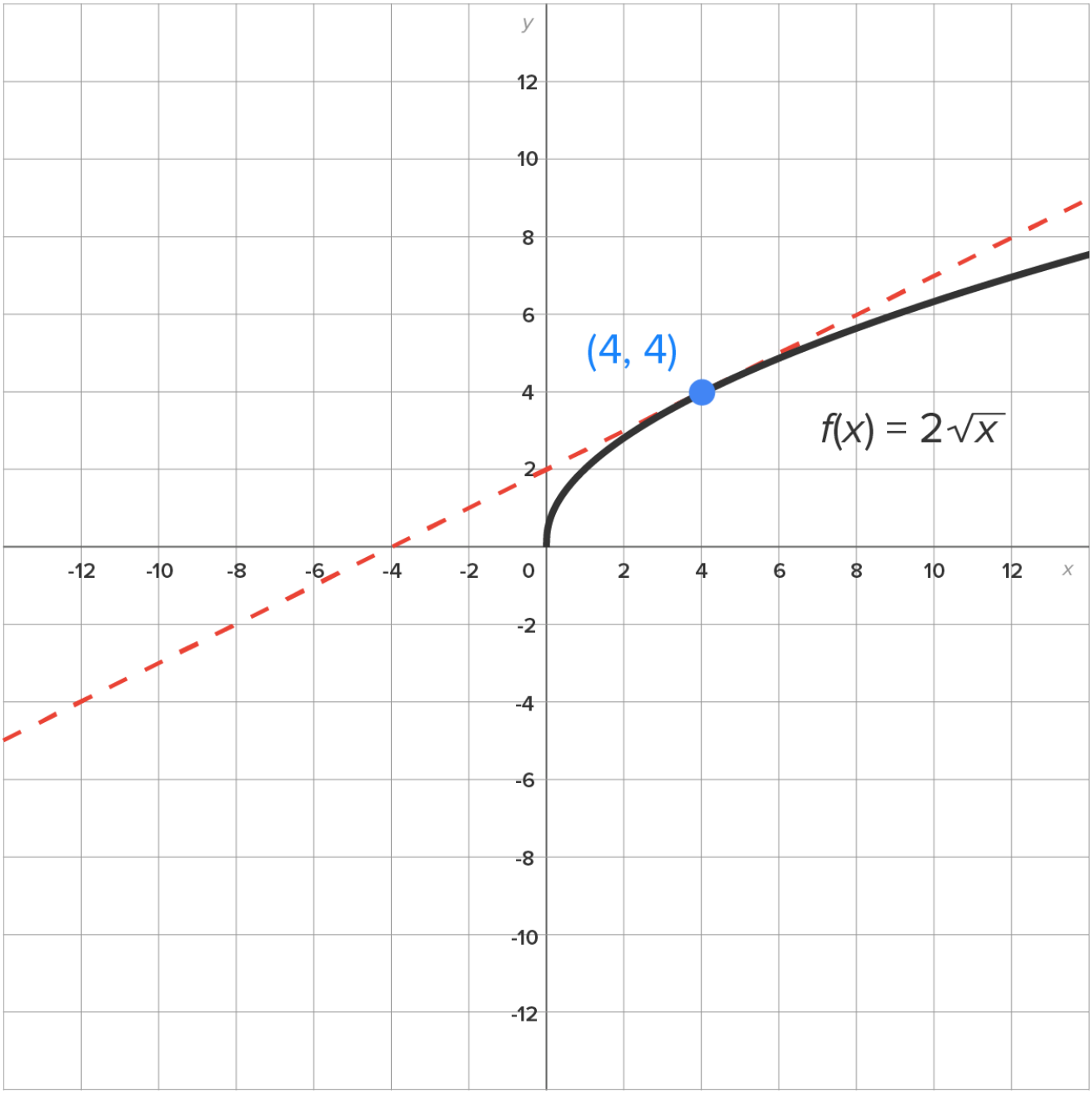
WHAT'S COVERED

In this lesson, you will learn what a tangent line is and how to estimate its slope graphically. Specifically, this lesson will cover:

1. Estimating the Slope of a Tangent Line Graphically

A **tangent line** is a line that touches a graph at one specific point (but does not cross it).

EXAMPLE The graph of $f(x) = 2\sqrt{x}$ and its tangent line at $(4, 4)$ are shown below. Use this picture to estimate the slope of the tangent line.



In order to estimate the slope of a line, two points are needed. Thus, we need another point on the line besides $(4, 4)$ to estimate the slope of this line. Inspecting closely, it looks like the point $(8, 6)$ is also contained on the line.

Thus, the slope of the tangent line is approximately $m = \frac{6-4}{8-4} = \frac{2}{4} = \frac{1}{2}$. In fact, this is the exact slope of the tangent line.

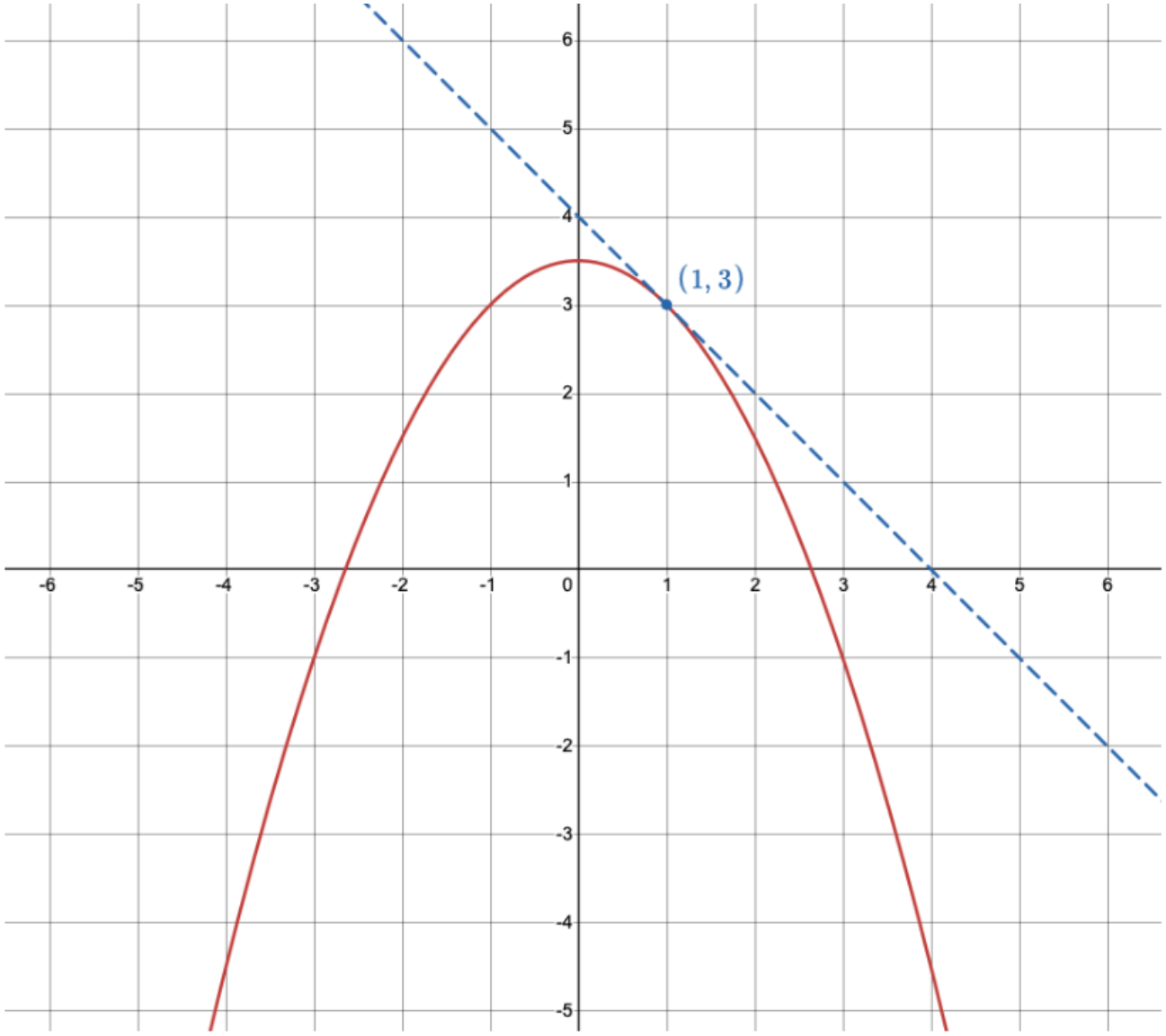
STEP BY STEP

If you are given the graph of a function and want to estimate the slope of a tangent line at $(a, f(a))$, do the following:

1. Sketch the tangent line at the point $(a, f(a))$.
2. Find another point on the line. Visually, it is easiest if the point has whole number coordinates, but sometimes this isn't possible.
3. Use the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ to compute the slope of the line.

TRY IT

Consider the graph of $f(x)$ below with its tangent line at the point $(1, 3)$.



Identify another point that is on the tangent line. +

There are several options, such as $(2, 2)$, $(3, 1)$, and $(4, 0)$.

Find the slope of the tangent line. +

Using the slope formula with the points $(1, 3)$ and $(4, 0)$, the slope is $m = \frac{0-3}{4-1} = -\frac{3}{3} = -1$.

TERM TO KNOW

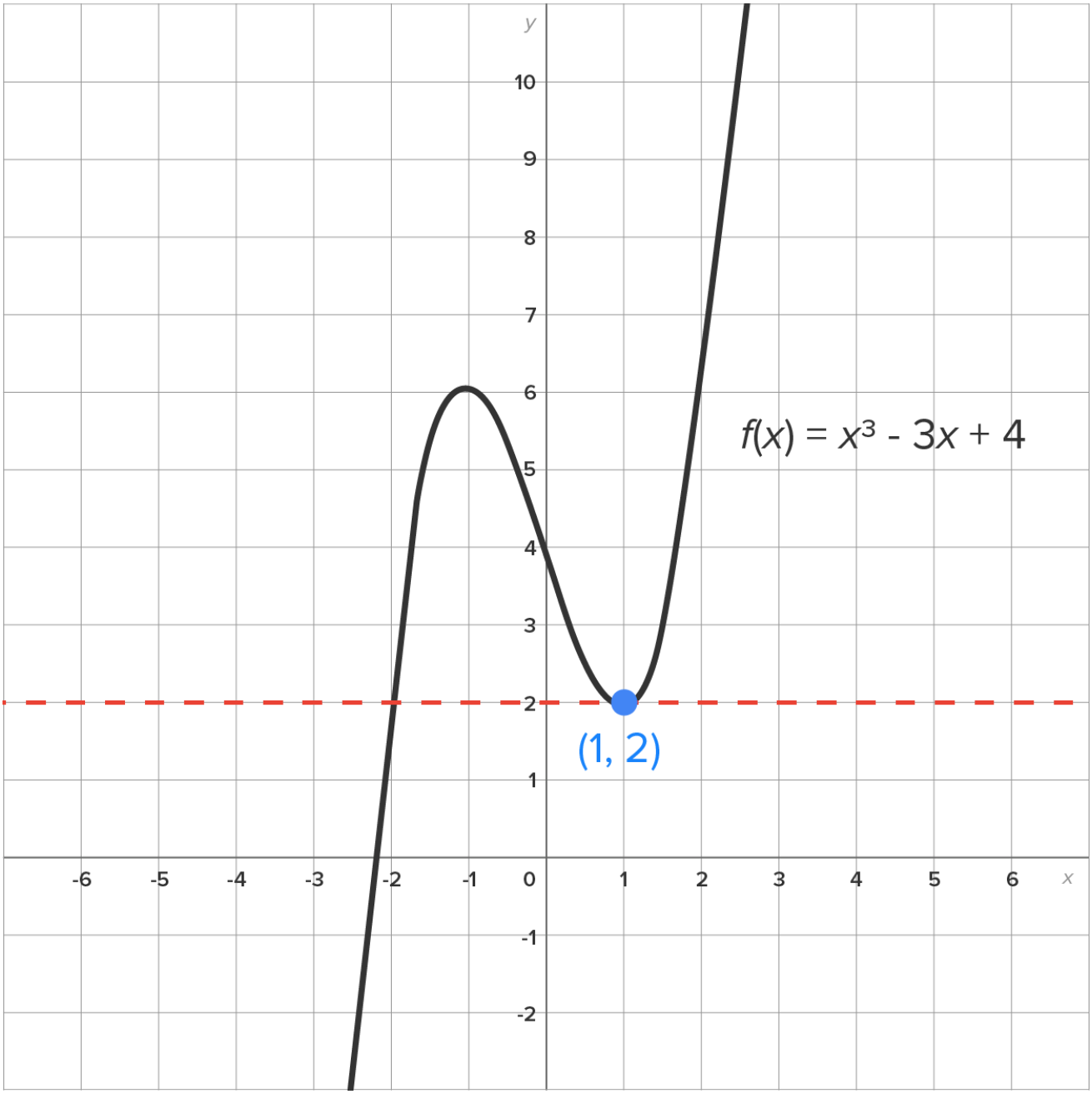
Tangent Line

A line that touches (but does not cross) the graph of a function at a specific point.

2. Horizontal Tangent Lines

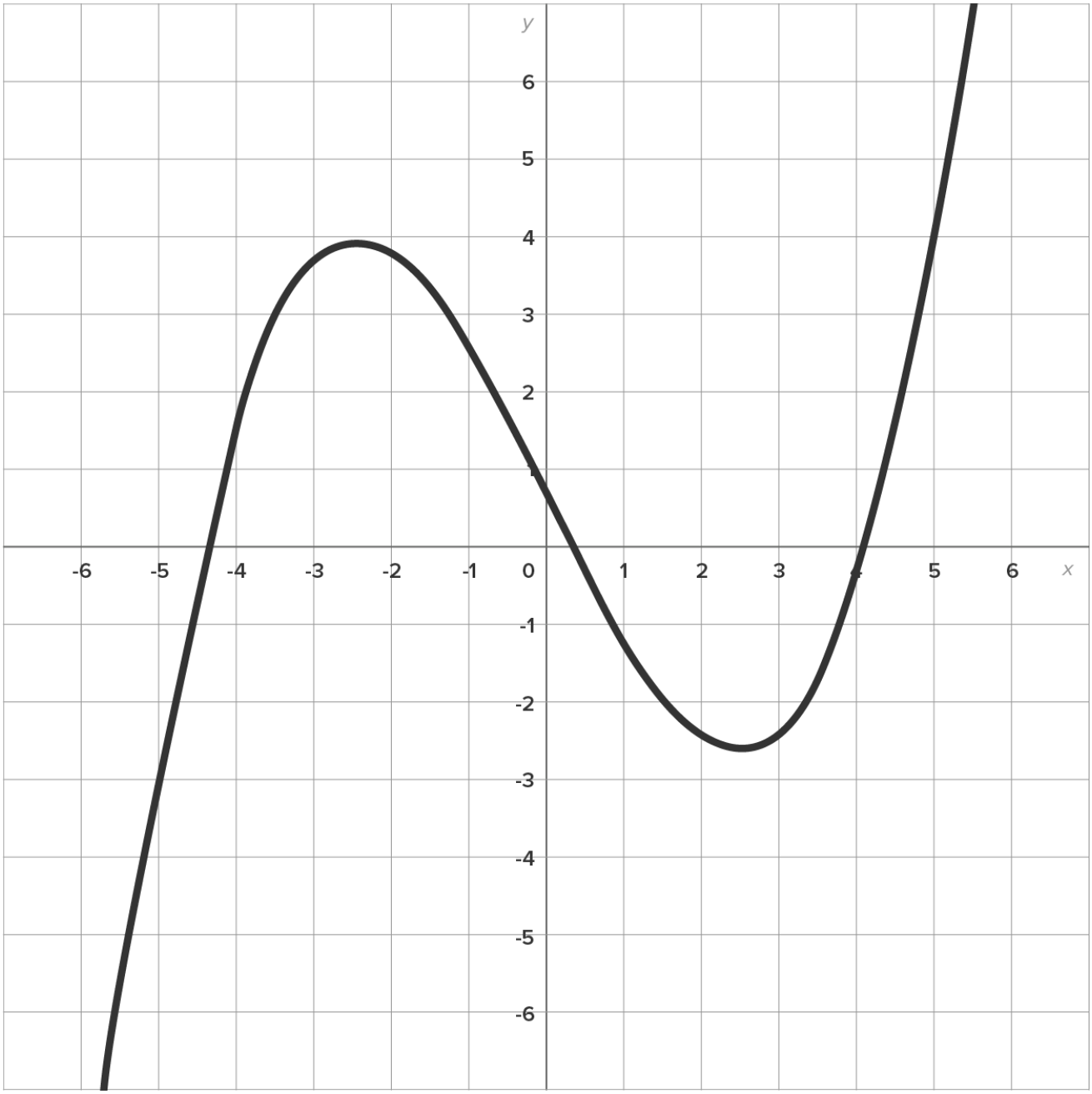
Tangent lines whose slopes are 0, also known as horizontal tangent lines, are very useful in calculus. It is important (and quite simple) to identify the places on a graph where the tangent line is horizontal.

EXAMPLE Estimate the slope of the tangent line to the curve $f(x) = x^3 - 3x + 4$ at the point $(1, 2)$. The graph of $f(x)$ and its tangent line are shown here:



The tangent line appears to be horizontal, which means its slope is zero.

EXAMPLE Estimate all values of x for which the graph of $y = f(x)$ below has a horizontal tangent line.



Looking at the graph, the values of x for which the tangent lines are horizontal are about $x = -2.5$ and $x = 2.5$.

SUMMARY

In this lesson, you learned about tangent lines, which are lines that touch (but do not cross) the graph of a function at a specific point. You learned that given the graph of a function, you can visually **estimate the slope of the tangent line graphically**. This can be accomplished by estimating another point on the tangent line. You also learned that tangent lines whose slopes are 0 are known as **horizontal tangent lines**; these are very useful in calculus.

TERMS TO KNOW

Tangent Line

A line that touches (but does not cross) the graph of a function at a specific point.