Other Asymptotes As x Approaches ∞ and -∞

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WHAT'S COVERED

In this lesson, you will investigate other types of asymptotes that are neither horizontal nor vertical. Specifically, this lesson will cover:

1. Slant (Oblique) Asymptotes
2. Other Nonlinear Asymptotes

BEFORE YOU START

Previously, we focused only on rational functions in which the degree of polynomial in the numerator is less than or equal to the degree of the polynomial in the denominator, which results in the rational function having a horizontal asymptote.

When the degree of the numerator is larger, one of two things happen:

- If the numerator has degree that is exactly one greater than the degree of the denominator, then there is no horizontal asymptote, but there is a slant (also called oblique) asymptote.
- If the numerator has degree that is greater than the denominator by more than one, then there is no horizontal or slant asymptote, but there is a nonlinear asymptote.

When the degree of the numerator is larger than the degree of the denominator, long division can be used to rewrite the expression. This is illustrated in the video below with the function $f(x) = \frac{2x^3 - 5x^2 + 4x + 3}{x - 2}$.

1. Slant (Oblique) Asymptotes

When a rational function $f(x)$ doesn't have a horizontal asymptote, it could have a **slant asymptote**, which is a slanted line that the graph of $f(x)$ approaches as $x \to \pm\infty$.

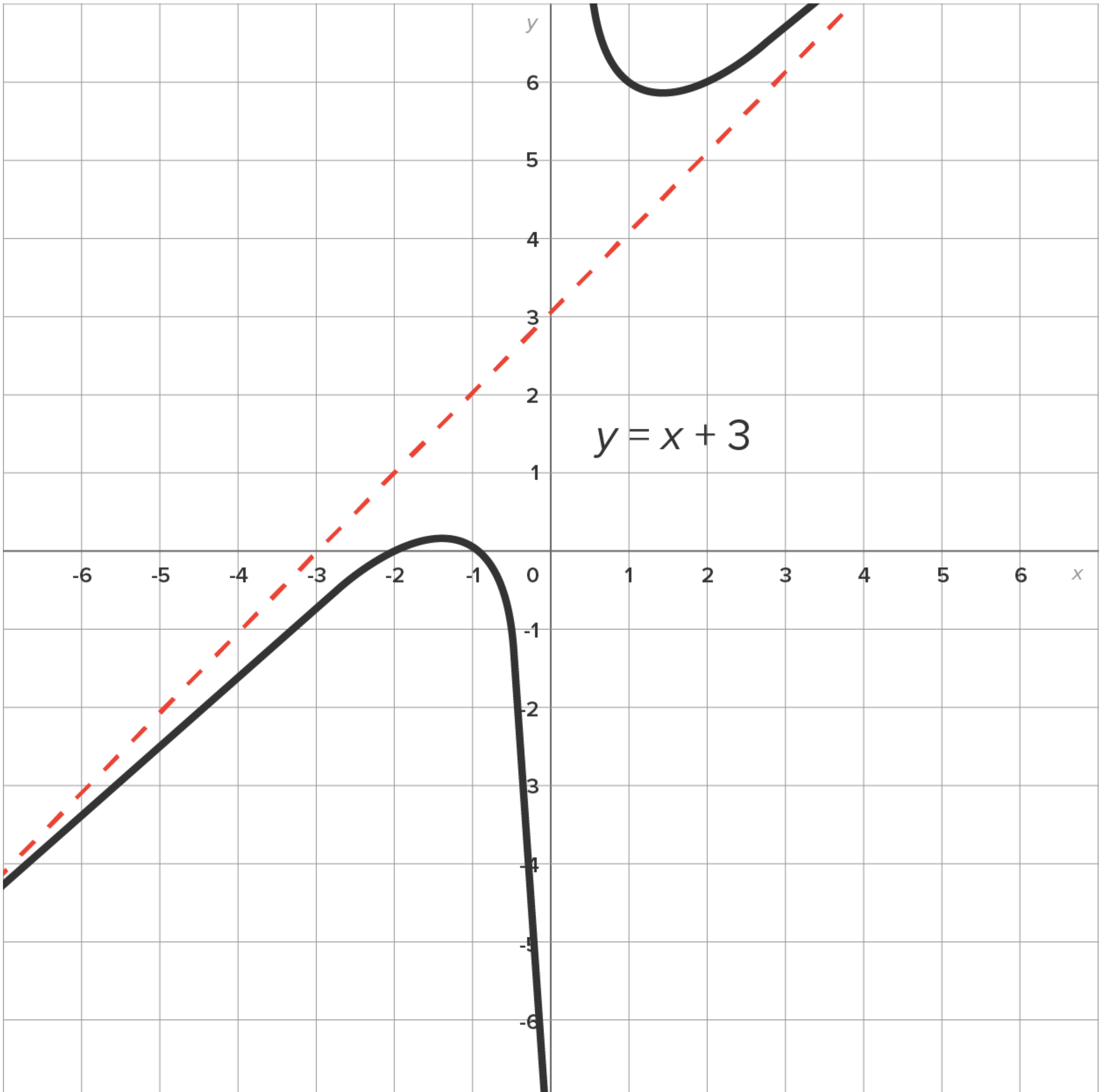
To see how to recognize a slant asymptote, let's look at this first example.

➦ EXAMPLE Consider the function $f(x) = \frac{x^2 + 3x + 2}{x}$.

Performing the division, we have $f(x) = \frac{x^2}{x} + \frac{3x}{x} + \frac{2}{x} = x + 3 + \frac{2}{x}$.

As $x \to \pm\infty$, $\frac{2}{x} \to 0$, which means the graph of $f(x)$ gets closer to the graph of $y = x + 3$. Thus, the slant asymptote is $y = x + 3$.

The graph of $f(x)$ along with its slant asymptote (dashed) is shown in the figure. Note how the graph approaches its slant asymptote as $x \to \pm\infty$.



TRY IT

Consider the function $f(x) = \dfrac{3x^2 + 5x + 2}{x + 2}$.

Identify the slant asymptote of the graph of this function.

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Performing the long division, we have:

$$\begin{array}{r} 3x-1 \\ x+2\overline{)3x^2+5x+2} \\ \underline{-3x^2-6x} \\ -x+2 \\ \underline{x+2} \\ 4 \end{array}$$

Therefore, $f(x) = \dfrac{3x^2+5x+2}{x+2} = 3x-1+\dfrac{4}{x+2}$

The graph of $f(x)$ has the slant asymptote $y=3x-1$ since $\dfrac{4}{x+2} \to 0$ as $x \to \pm\infty$.

TERM TO KNOW

Slant (Oblique) Asymptote

The slanted line that a graph approaches as $x \to \pm\infty$.

2. Other Nonlinear Asymptotes

A **nonlinear asymptote** is the curve that a graph approaches as $x \to \pm\infty$.

EXAMPLE Consider the function $f(x) = \dfrac{x^3+1}{x-2}$.

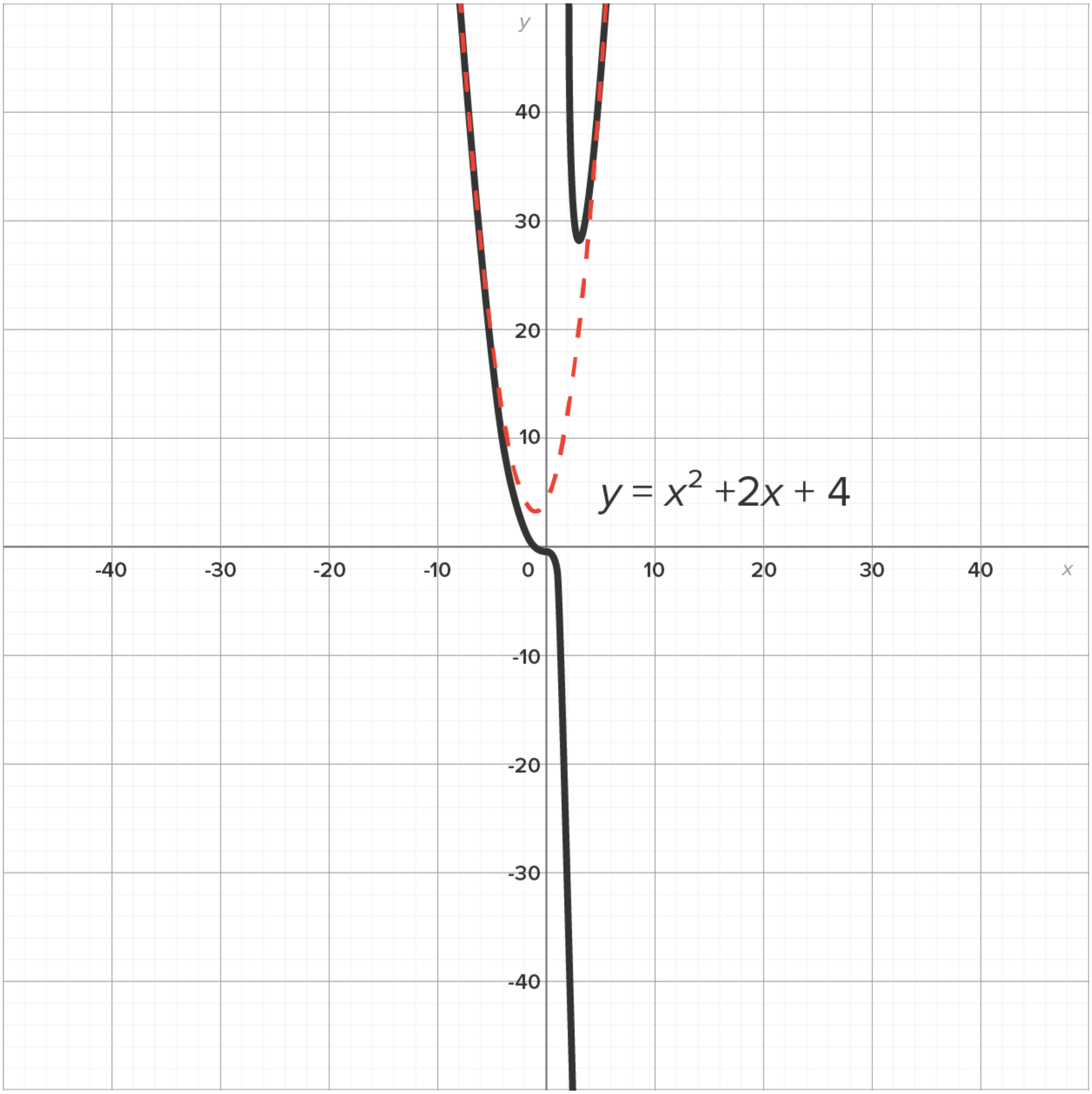
To set up the long division, first rewrite x^3+1 as x^3+0x^2+0x+1

Next, perform the long division:

$$\begin{array}{r} x^2+2x+4 \\ x-2\overline{)x^3+0x^2+0x+1} \\ \underline{-x^3+2x^2} \\ 2x^2 \\ \underline{-2x^2+4x} \\ 4x+1 \\ \underline{-4x+8} \\ 9 \end{array}$$

Therefore, $f(x) = x^2+2x+4+\dfrac{9}{x-2}$

Since $\dfrac{9}{x-2} \to 0$ as $x \to \pm\infty$, the graph of $f(x)$ has a nonlinear asymptote $y=x^2+2x+4$. The graph of $f(x)$ along with the nonlinear asymptote (dashed) is shown in the figure.



TRY IT

Consider the function $f(x) = \dfrac{x^4 + x^2 + x}{x^2 + 1}$.

Write the equation of the nonlinear asymptote of the function.

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First, perform the long division:

$$\begin{array}{r} x^2 \\ x^2+1\overline{)x^4+x^2+x} \\ \underline{-x^4-x^2} \\ x \end{array}$$

Therefore, $f(x) = x^2 + \dfrac{x}{x^2+1}$

Since $\dfrac{x}{x^2+1} \to 0$ as $x \to \infty$, the nonlinear asymptote is $y = x^2$

TERM TO KNOW

Nonlinear Asymptote

The curve that a graph approaches as $x \to \pm\infty$.

SUMMARY

In this lesson, you learned that when a rational function doesn't have a horizontal asymptote, it could have either a **slant (oblique) asymptote**, which is a slanted line that the graph of $f(x)$ approaches as $x \to \pm\infty$, or a **nonlinear asymptote**, which is the curve that a graph approaches as $x \to \pm\infty$.

TERMS TO KNOW

Nonlinear Asymptote

The curve that a graph approaches as $x \to \pm\infty$.

Slant (Oblique) Asymptote

The slanted line that a graph approaches as $x \to \pm\infty$.