

# Shifting and Stretching Graphs

by Sophia



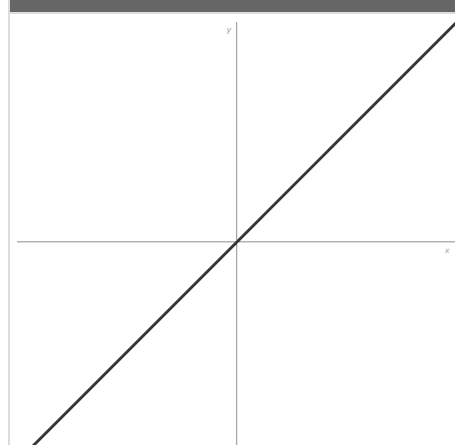
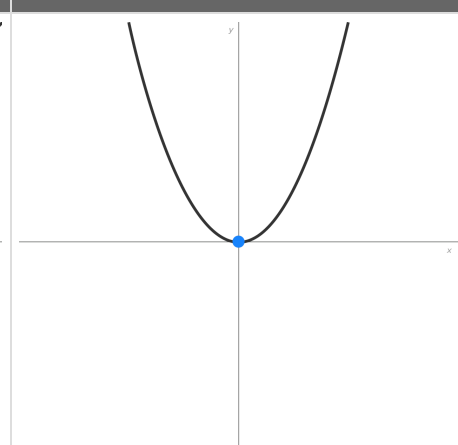
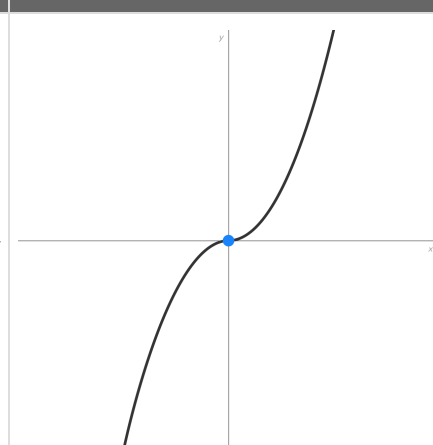
## WHAT'S COVERED

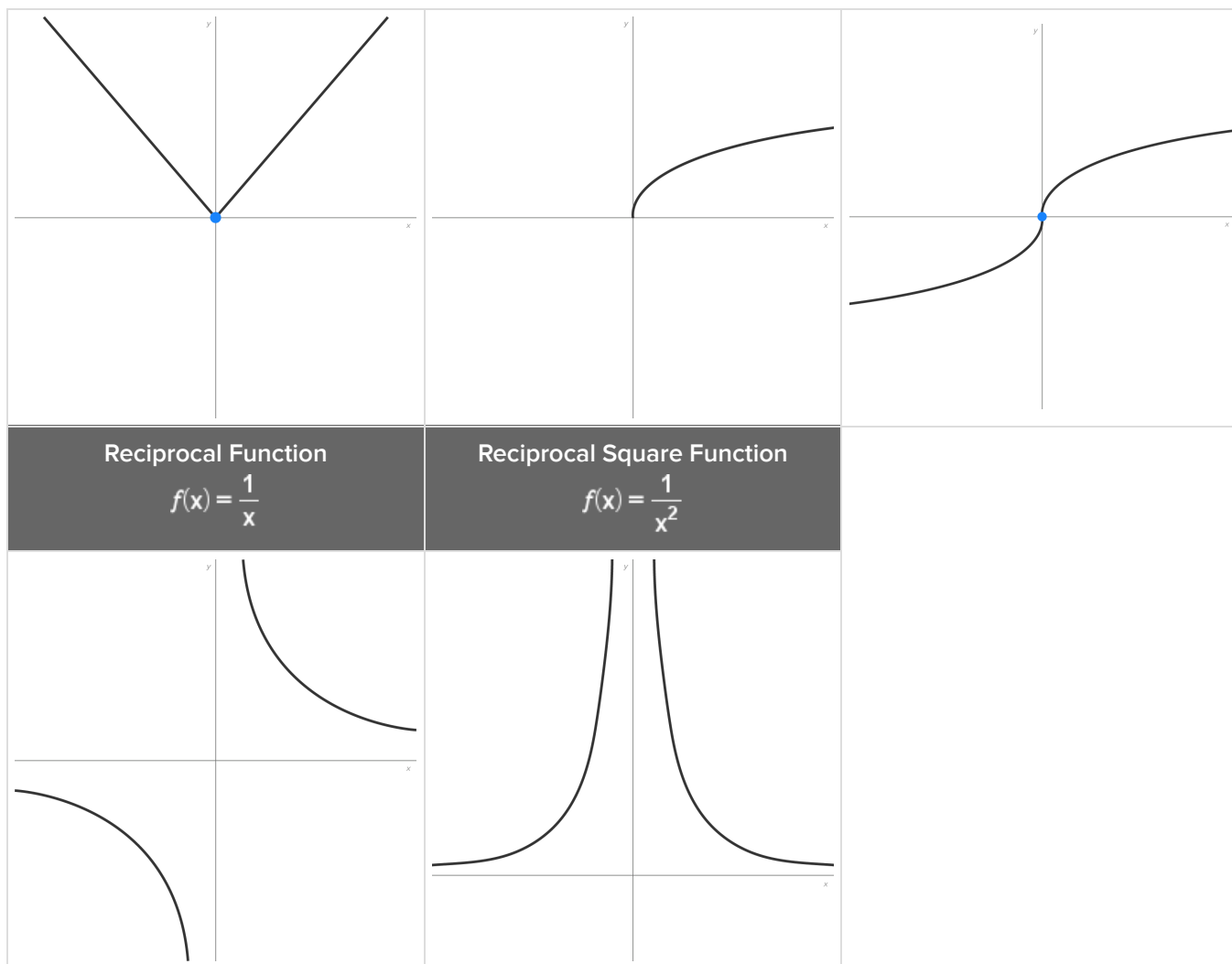
In this lesson, you will investigate how translations affect the graph of a function. Specifically, this lesson will cover:

1. Commonly Used Basic Functions and Their Graphs
2. Applying Basic Translations to  $y = f(x)$
3. Applying Several Translations to  $y = f(x)$

## 1. Commonly Used Basic Functions and Their Graphs

Here are the most commonly used graphs that are encountered in a typical algebra course. There are others as well, which will be investigated in future challenges.

Linear Function $f(x) = x$	Quadratic Function $f(x) = x^2$	Cubic Function $f(x) = x^3$
		
Absolute Value Function $f(x) =  x $	Square Root Function $f(x) = \sqrt{x}$	Cube Root Function $f(x) = \sqrt[3]{x}$

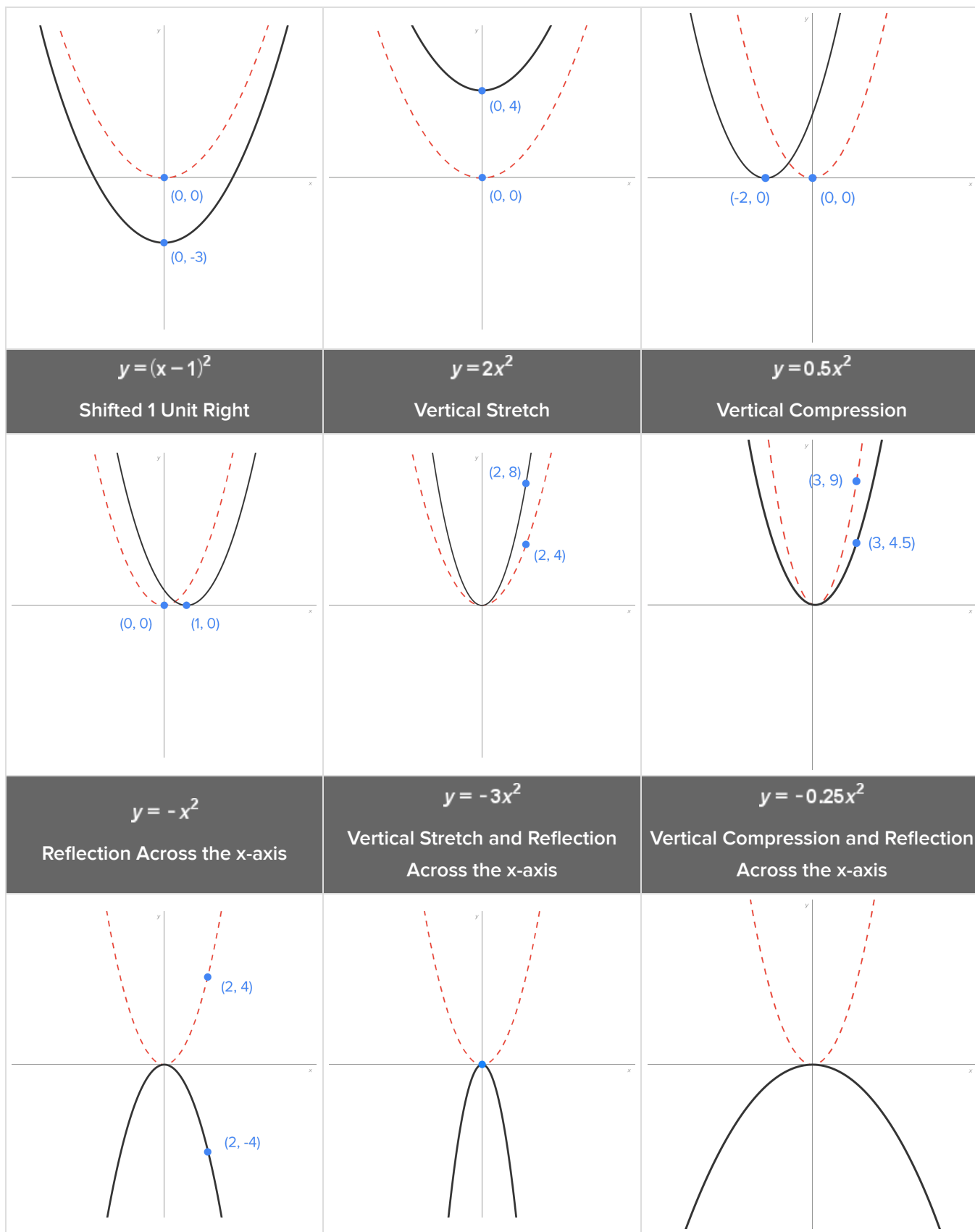


In this challenge, we will investigate translations to an equation and how they affect the graph of said equation.

## 2. Applying Basic Translations to $y = f(x)$

Shown below are several functions which are translations of  $f(x) = x^2$ . In each graph, the graph of  $f(x) = x^2$  is shown with a dashed line.

$y = x^2 - 3$ Shifted Down 3 Units	$y = x^2 + 4$ Shifted Up 4 Units	$y = (x + 2)^2$ Shifted Left 2 Units
---------------------------------------	-------------------------------------	---



Given the graph of  $y = f(x)$  and a positive constant  $k$ :

- The graph of  $y = f(x) + k$  shifts the graph up  $k$  units.
- The graph of  $y = f(x) - k$  shifts the graph down  $k$  units.
- The graph of  $y = f(x - k)$  shifts the graph right  $k$  units.
- The graph of  $y = f(x + k)$  shifts the graph left  $k$  units.
- The graph of  $y = a \cdot f(x)$  is a **vertical stretch** if  $|a| > 1$  and a **vertical compression** if  $|a| < 1$ . Also, if  $a$  is negative, the graph also reflects over the x-axis.



#### TERMS TO KNOW

##### Vertical Compression

A translation that makes all y-values of a graph smaller in magnitude, pulling a graph toward the x-axis. This is represented by  $y = a \cdot f(x)$ , where  $|a| < 1$ .

##### Vertical Stretch

A translation that makes all y-values of a graph larger in magnitude, pulling a graph toward the y-axis. This is represented by  $y = a \cdot f(x)$ , where  $|a| > 1$ .

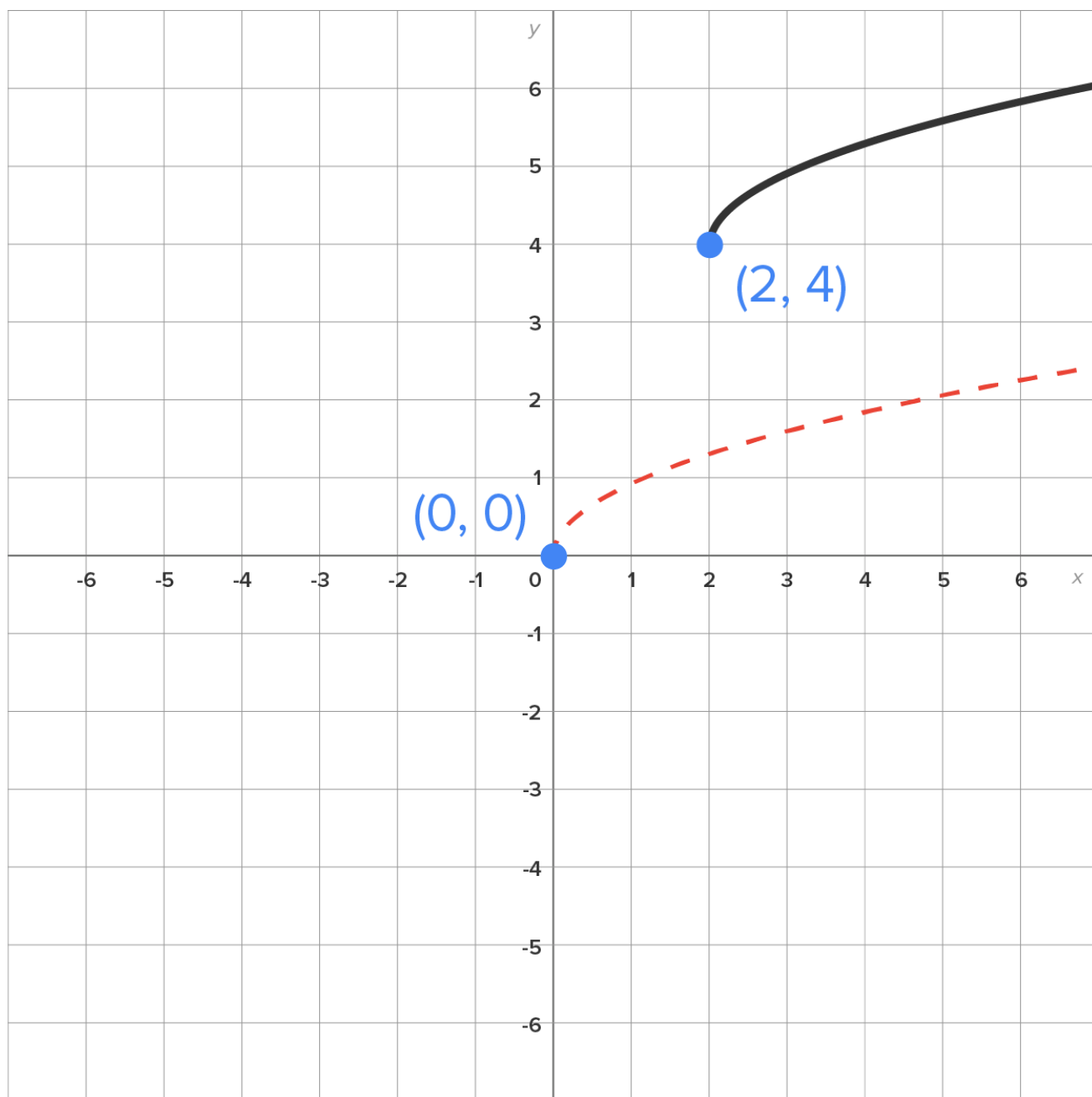
## 3. Applying Several Translations to $y = f(x)$

Given the translations discussed in the previous section, it is possible to apply several to one function.

⇒ **EXAMPLE** Consider the function  $g(x) = \sqrt{x-2} + 4$ , which is related to the function  $f(x) = \sqrt{x}$ .

- There is an “ $x - 2$ ” under the radical where the “ $x$ ” is in the base function, indicating that the graph is moved to the right two units.
- The “ $+ 4$ ” outside of the radical indicates that the graph is shifted up 4 units.

The graphs of  $f(x)$  and  $g(x)$  are shown on the same axes below. The graph of  $f(x)$  is dashed to show its relationship to  $g(x)$ .

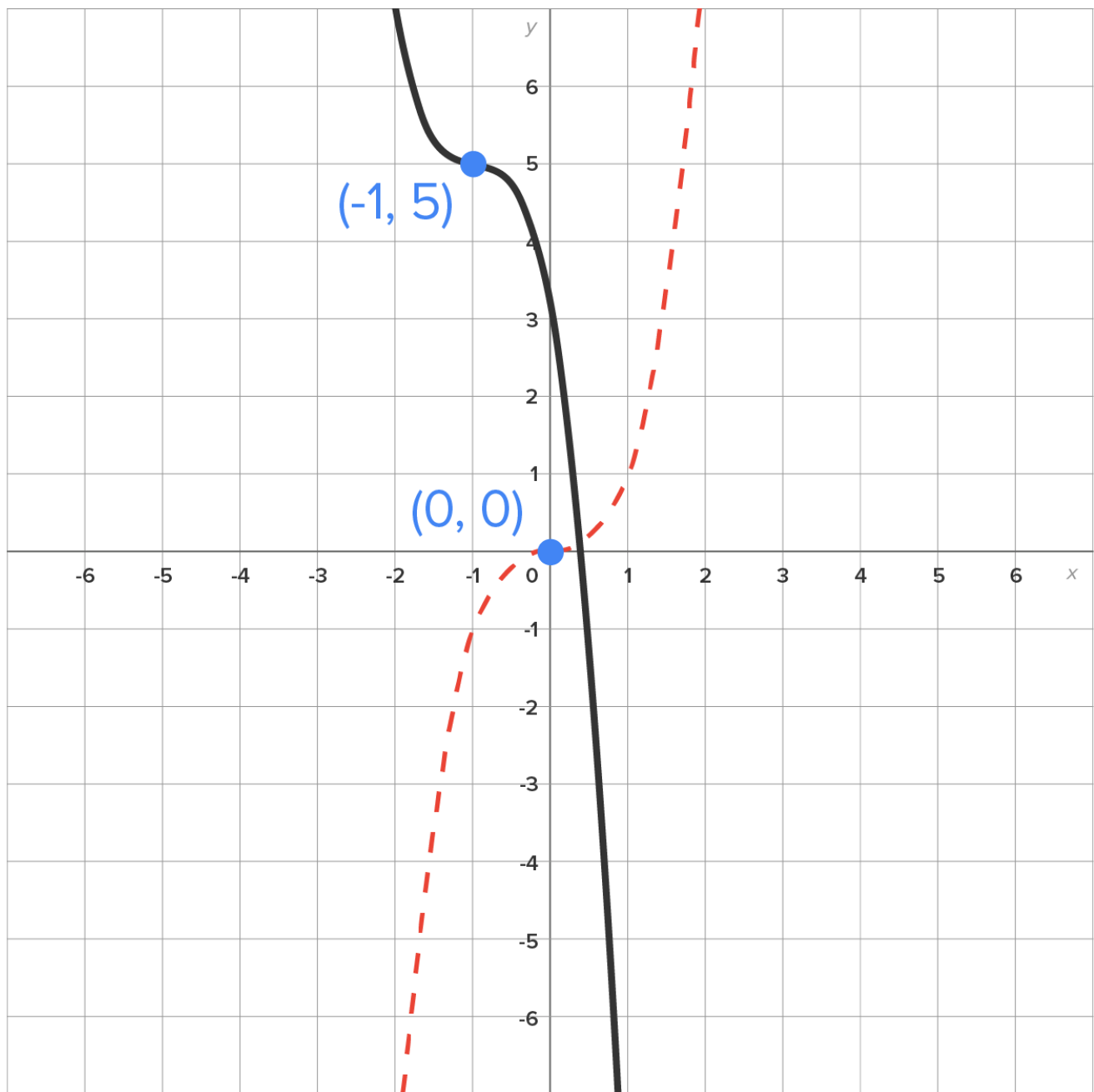


The graph of  $g(x)$  is obtained by moving the graph of  $f(x)$  to the right 2 units and up 4 units.

⇒ **EXAMPLE** Describe the sequence of transformations that are required to graph  $g(x) = -2(x+1)^3 + 5$  based on  $f(x) = x^3$ .

- The “ $x + 1$ ” tells us that the graph is shifted to the left by 1 unit.
- The “ $-2$ ” multiplied to the cubed term tells us that the graph is reflected around the x-axis and stretched vertically (since  $2 > 1$ ).
- The “ $+ 5$ ” tells us that the graph is then shifted up five units.

The graph is shown here, with  $f(x) = x^3$  dashed.



### BIG IDEA

The order in which translations are applied only matters when there is a vertical shift. Here is the order in which translations should be applied:

1. Horizontal Translations
2. Vertical Compressions/Stretches/Reflections
3. Vertical Translations



### TRY IT

Consider the equation  $g(x) = 0.75(x + 4)^2 - 3$ .

### Identify the basic function.



The basic function is  $f(x) = x^2$ .

### List the sequence of translations required to graph $g(x)$ based on the basic function.



The graph is shifted to the left by 4 units, vertically compressed by a factor of 0.75, and shifted down 3 units.



## SUMMARY

In this lesson, you began by exploring **commonly used basic functions and their graphs**. You investigated several types of translations to an equation and how they affect the graph of said equation, **applying basic translations to  $y = f(x)$**  to shift, stretch, compress, and reflect its graph. You also learned how to graph a function  $g(x)$  by **applying several translations to  $y = f(x)$** , noting the order in which translations should be applied.

Source: THIS TUTORIAL HAS BEEN ADAPTED FROM CHAPTER 0 OF "CONTEMPORARY CALCULUS" BY DALE HOFFMAN. ACCESS FOR FREE AT [WWW.CONTEMPORARYCALCULUS.COM](http://WWW.CONTEMPORARYCALCULUS.COM). LICENSE: **CREATIVE COMMONS ATTRIBUTION 3.0 UNITED STATES**.



## TERMS TO KNOW

### Vertical Compression

A translation that makes all y-values of a graph smaller in magnitude, pulling a graph toward the x-axis. This is represented by  $y = a \cdot f(x)$ , where  $|a| < 1$ .

### Vertical Stretch

A translation that makes all y-values of a graph larger in magnitude, pulling a graph toward the y-axis. This is represented by  $y = a \cdot f(x)$ , where  $|a| > 1$ .