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Absolute Value Functions

by Sophia

WHAT'S COVERED

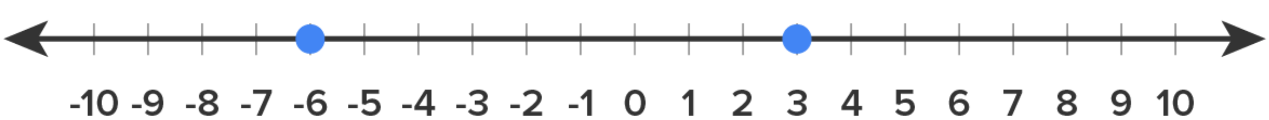
In this lesson, you will learn about absolute value functions. Specifically, this lesson will cover:

1. The Absolute Value Function

1a. The Piecewise Definition of Absolute Value

Recall that $|x|$ means "the **absolute value** of *x*", which represents the distance that a number *x* is from 0 (on the number line).

Consider the number line shown below, with the numbers 3 and -6 marked.



Since the number 3 is a distance of 3 units from 0, we say that $|3| = 3$.

Since the number -6 is a distance of 6 units from 0, we say that $|-6| = 6$.

In general, evaluating $|x|$ requires two different rules, depending on what *x* is.

- If *x* is nonnegative, then $|x|$ and *x* are the same.
- If *x* is negative, then $|x|$ is the opposite of *x* (turning a negative into a positive).

This leads to the piecewise definition for $|x|$.

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

TERM TO KNOW

Absolute Value

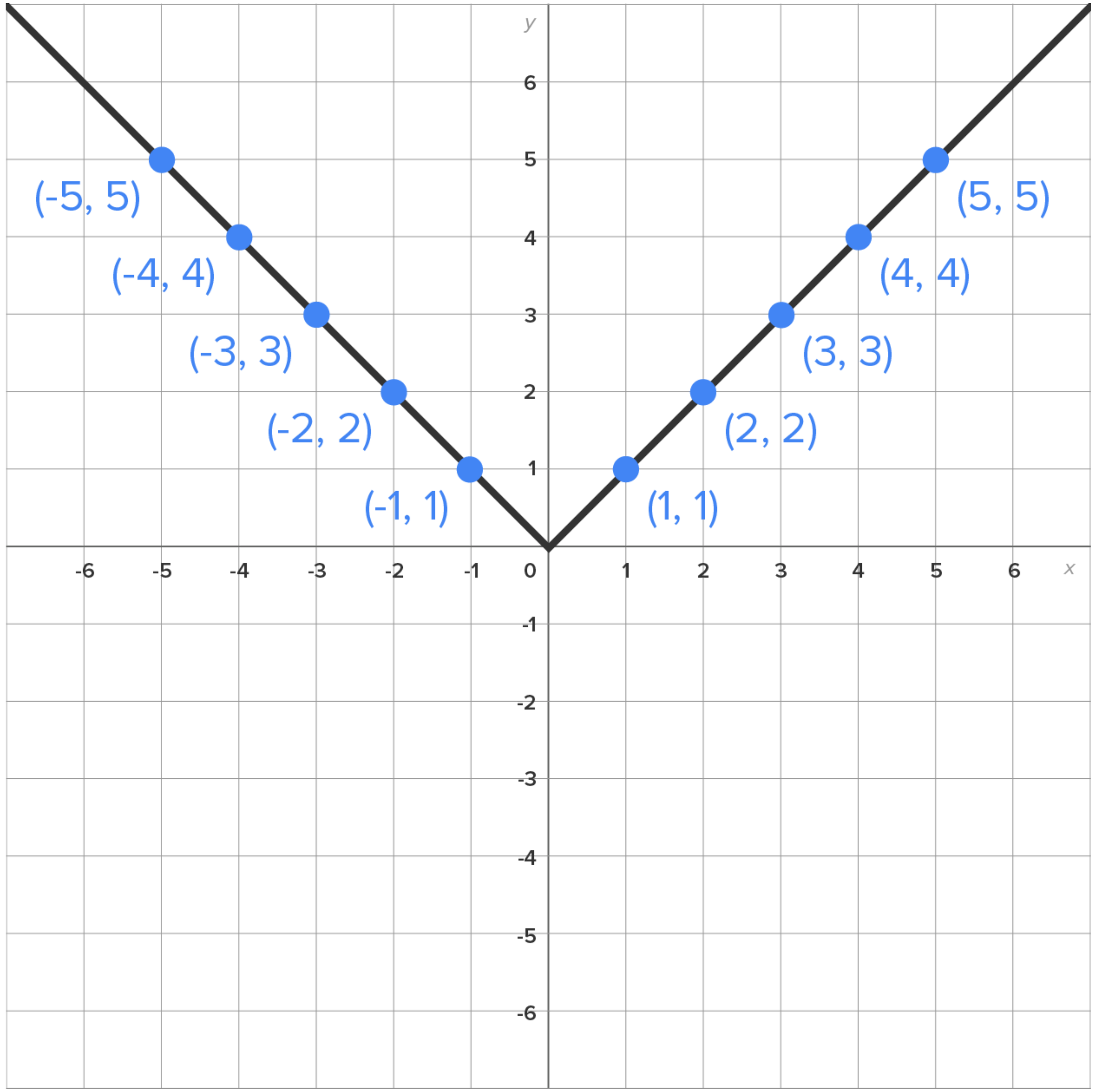
The distance that a number is from 0 on the number line.

1b. The Graph of the Basic Absolute Value Function

In the table below, you see several input-output pairs for $f(x) = |x|$.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f(x) = \|x\|$	5	4	3	2	1	0	1	2	3	4	5

Here is the resulting graph:

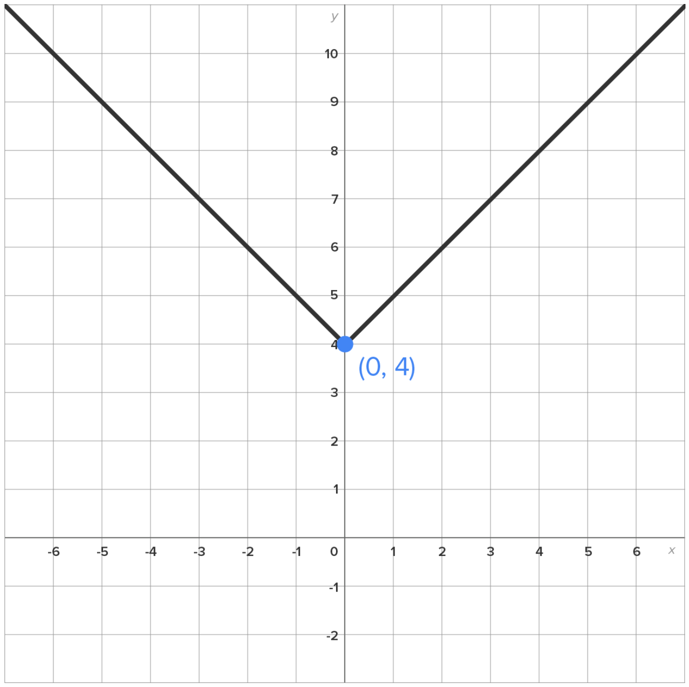
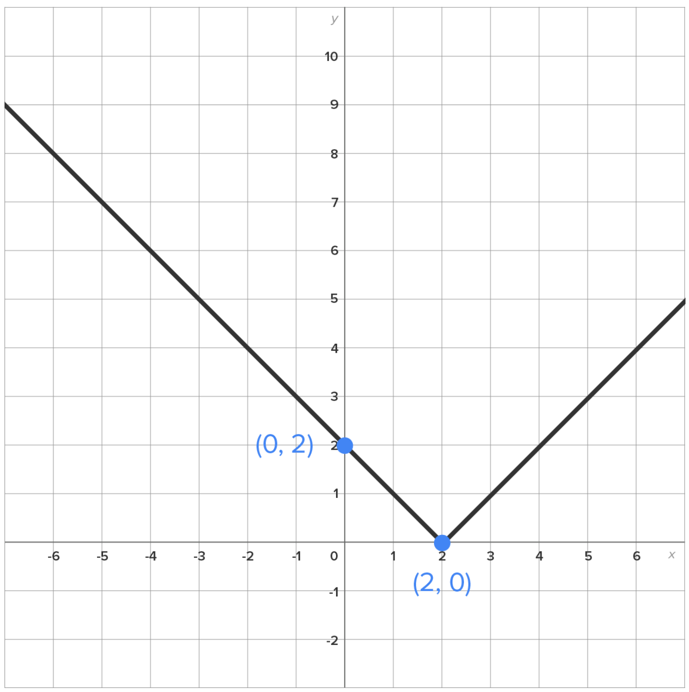


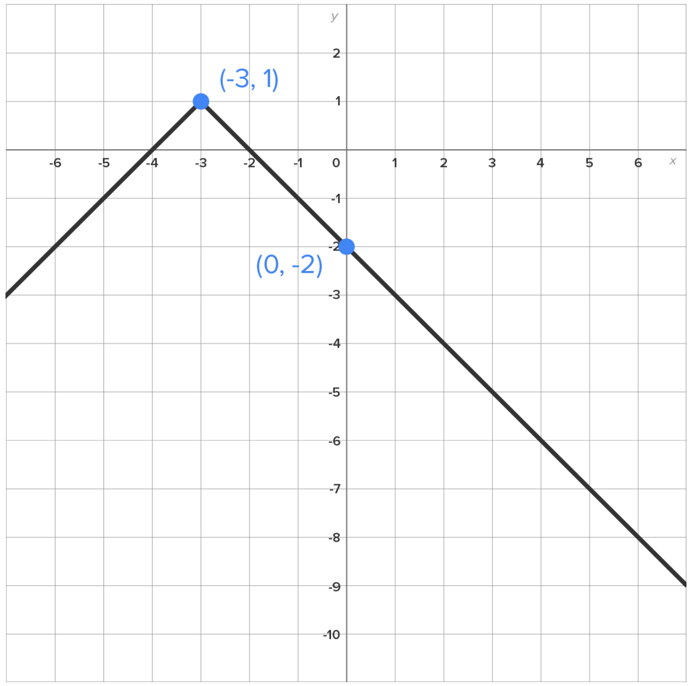
2. Graphing Absolute Value Functions

2a. Shifting, Stretching, and Reflecting the Basic Absolute Value Function

From what you learned in the “Shifting and Stretching Graphs” section, you can apply these rules to the absolute value function.

Function	Graph	Shifts and/or Stretches from $f(x) = \|x\|$

$g(x) = \|x\| + 4$		Up 4 units
$h(x) = \|x - 2\|$		Right 2 units

$j(x) = -\lvert x+3 \rvert + 1$		Left 3 units, reflected over the x-axis, then moved up 1 unit

2b. Other Absolute Value Graphs

The function $\lvert f(x) \rvert$ can be written in piecewise form by replacing "*x*" with "$f(x)$."

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \quad \rightarrow \quad |f(x)| = \begin{cases} -f(x) & \text{if } f(x) < 0 \\ f(x) & \text{if } f(x) \geq 0 \end{cases}$$

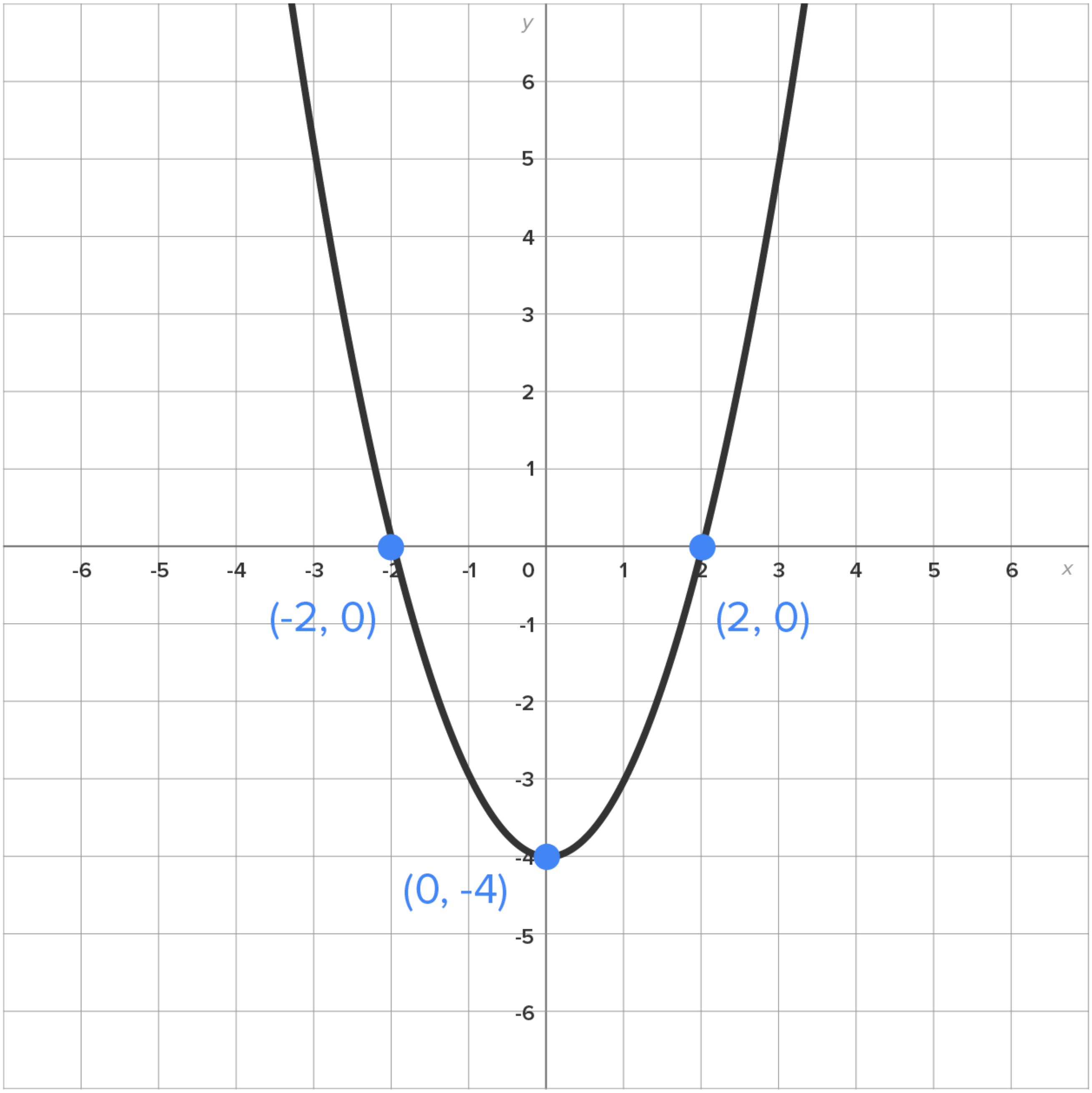
We can adapt this idea to graph a function of the form $y = \lvert f(x) \rvert$. In order to do this, think about what it means when we say $f(x) \geq 0$ and $f(x) < 0$.

If $f(x) < 0$, this really means $y < 0$, indicating that the corresponding point on the graph is below the x-axis.

If $f(x) \geq 0$, this really means $y \geq 0$, indicating that the corresponding point on the graph is on or above the x-axis.

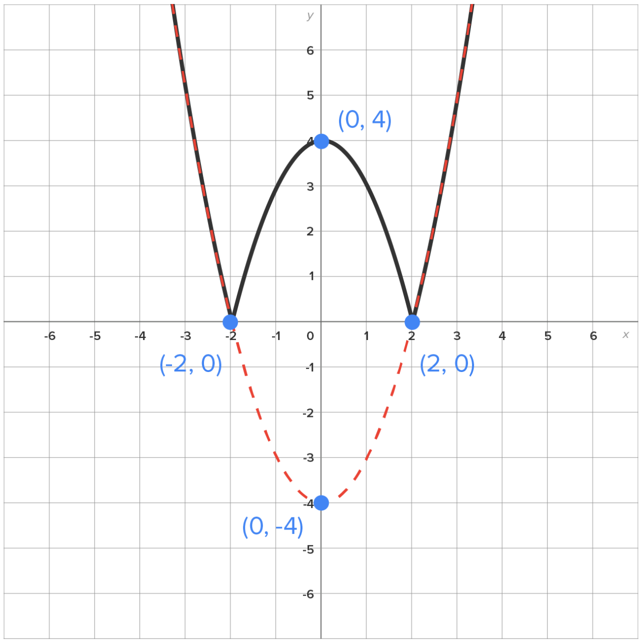
Recall from challenge 1.3.4 "Shifting and Stretching Graphs" that the graph of $y = -f(x)$ reflects the graph of $y = f(x)$ across the x-axis. Thus, if $f(x) < 0$, then the graph of $y = \lvert f(x) \rvert$ reflects over the x-axis (to the positive side). Otherwise, the graphs of $f(x)$ and $y = \lvert f(x) \rvert$ are the same.

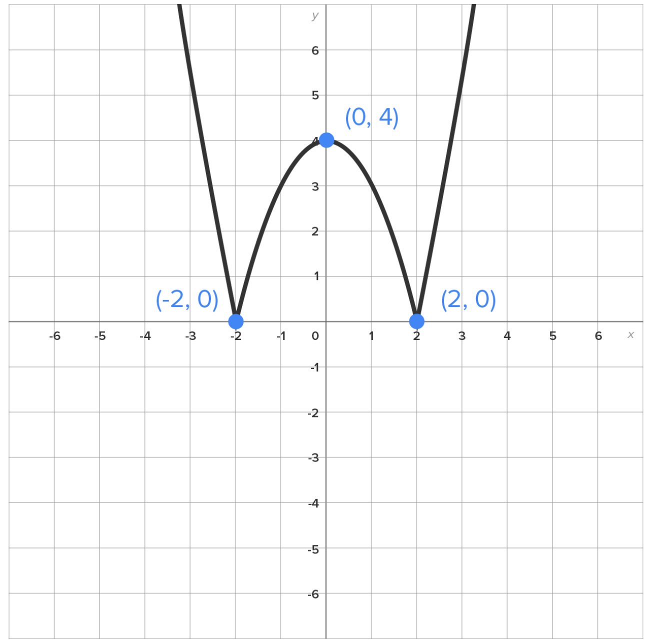
EXAMPLE The graph of $f(x) = x^2 - 4$ is shown below:



To graph $g(x) = |f(x)| = |x^2 - 4|$, notice that the graph of $f(x) = x^2 - 4$ is below the x-axis between $x = -2$ and $x = 2$. This part reflects over the x-axis, while the rest of the graph remains the same.

On the left is the graph of $g(x) = |f(x)| = |x^2 - 4|$ with the graph of $f(x)$ shown as a dashed line for comparison. On the right is the graph of $g(x) = |x^2 - 4|$.



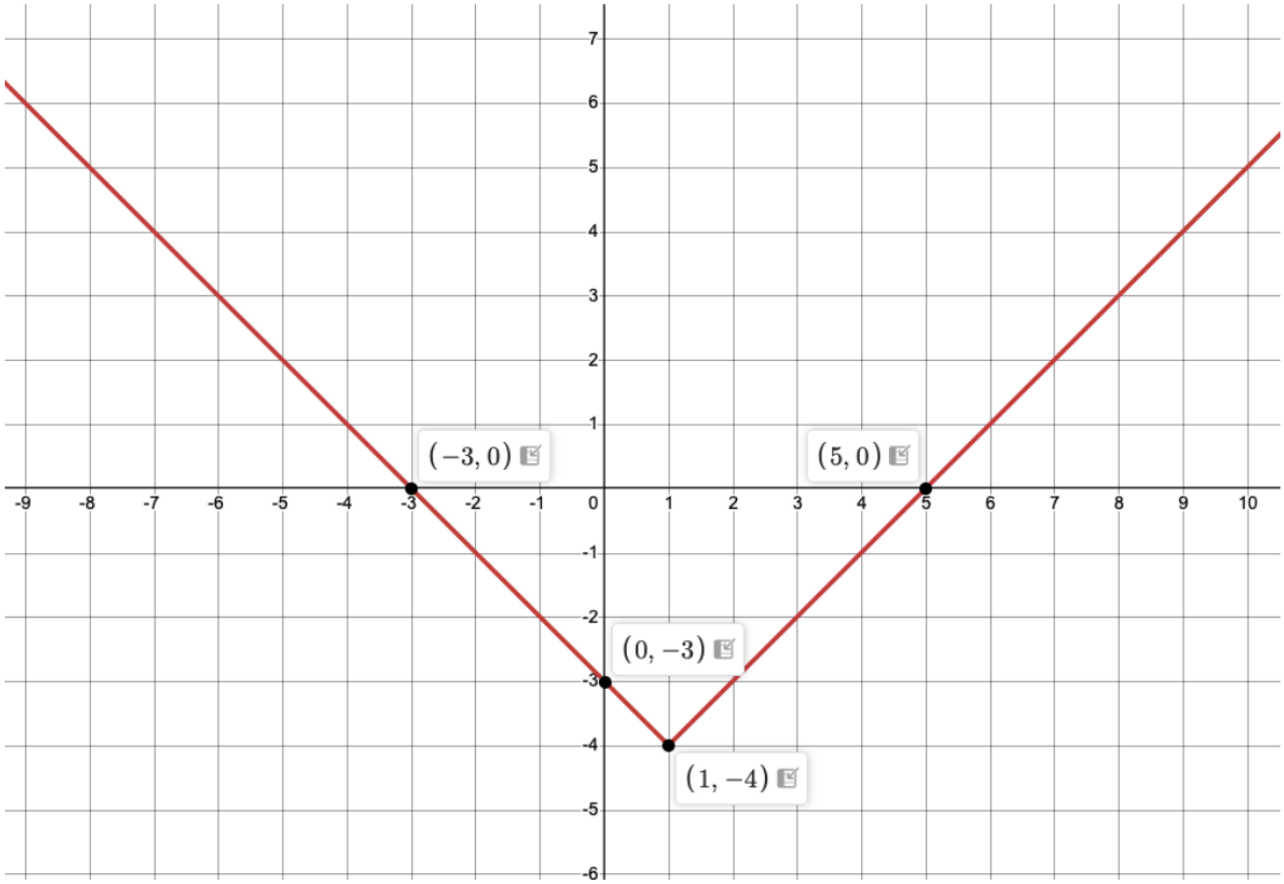


HINT

To graph $y = |f(x)|$ given the graph of $y = f(x)$, consider the following.

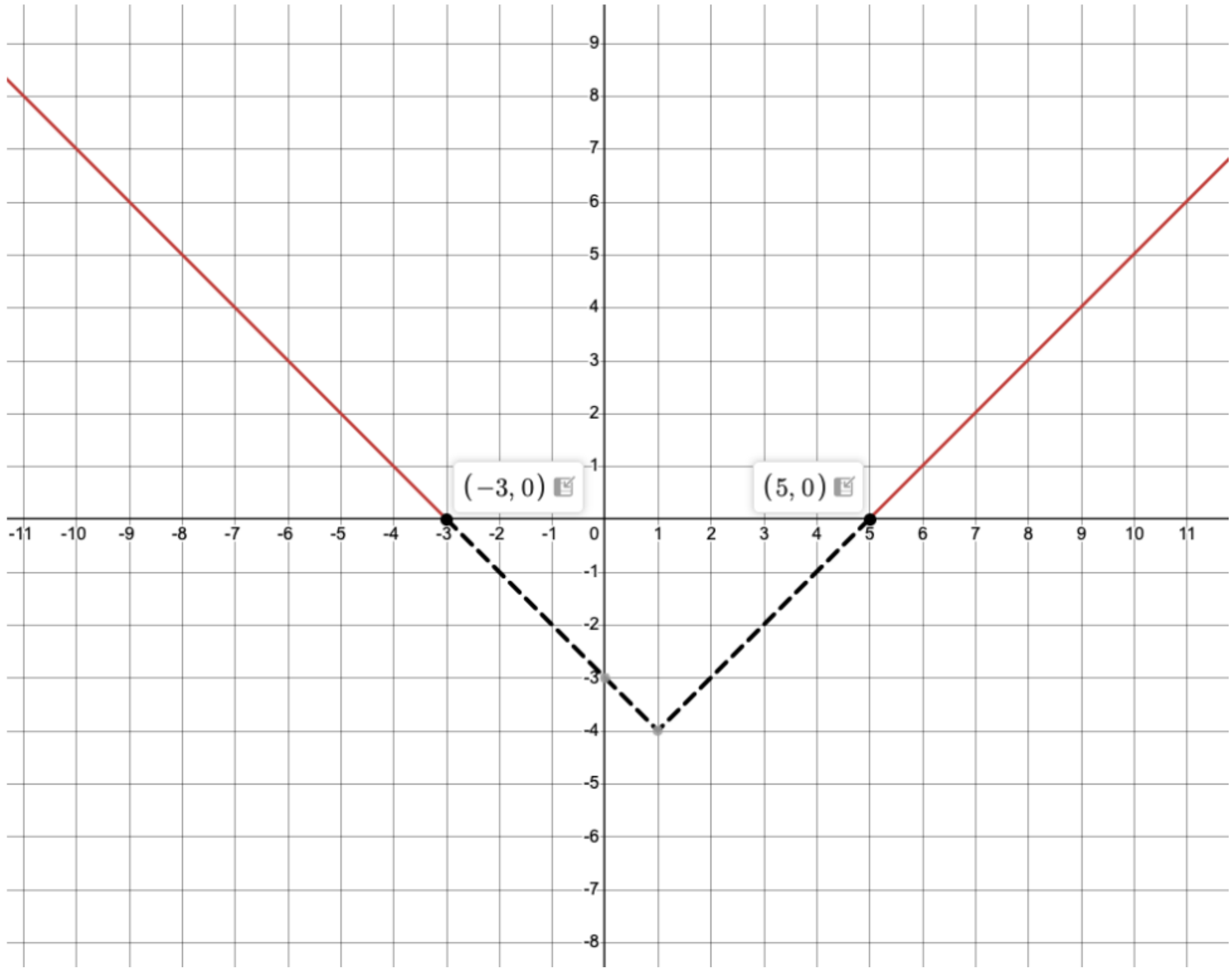
- The graphs are the same for all values of x where the graph of $y = f(x)$ is either below or above the x-axis.
- It may be helpful to locate the x-intercepts of the graph of $f(x)$ first, since these are the points where $f(x) = 0$.
- The portion lying below the x-axis gets reflected over the x-axis.

EXAMPLE Consider the graph of $y = f(x)$ shown below.



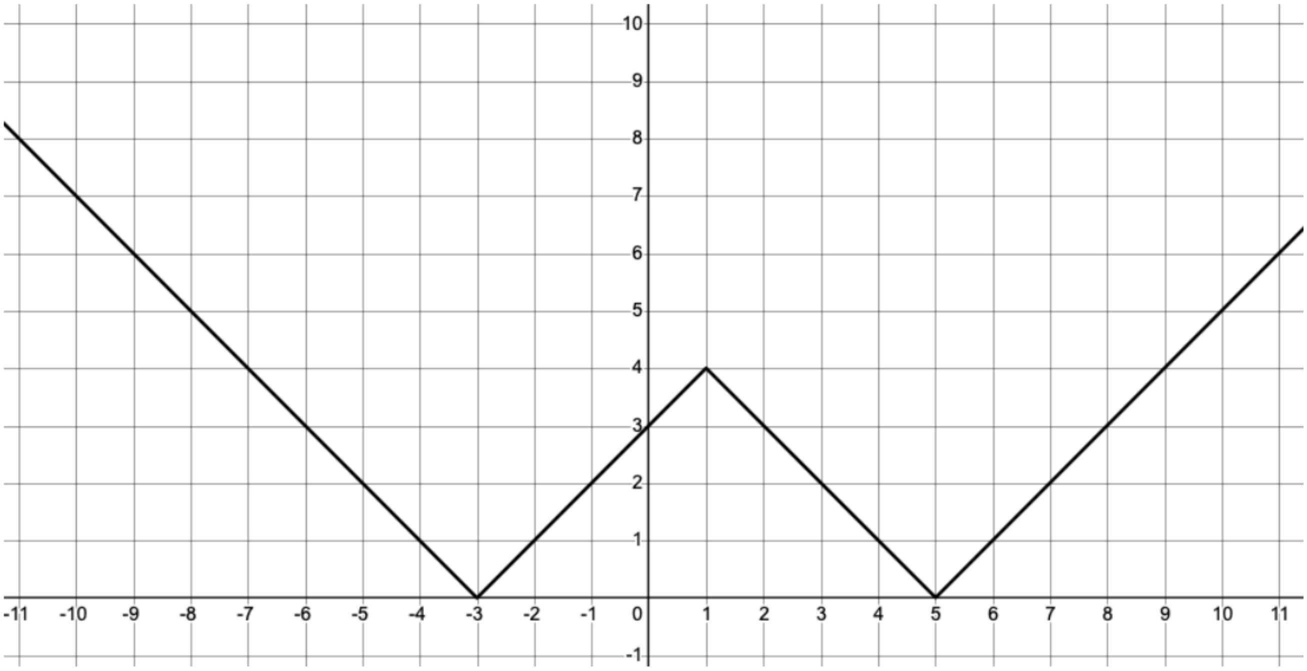
Use this graph to construct the graph of $y = |f(x)|$.

First, note that the x-intercepts are $(-3, 0)$ and $(5, 0)$. Since the graph is above the x-axis to the left of $(-3, 0)$ and to the right of $(5, 0)$, this portion of the graph will also be above the x-axis. The graph of $f(x)$ is shown again below, this time with the portion below the x-axis dashed instead of solid. This is the portion we need to focus on when graphing $y = |f(x)|$.



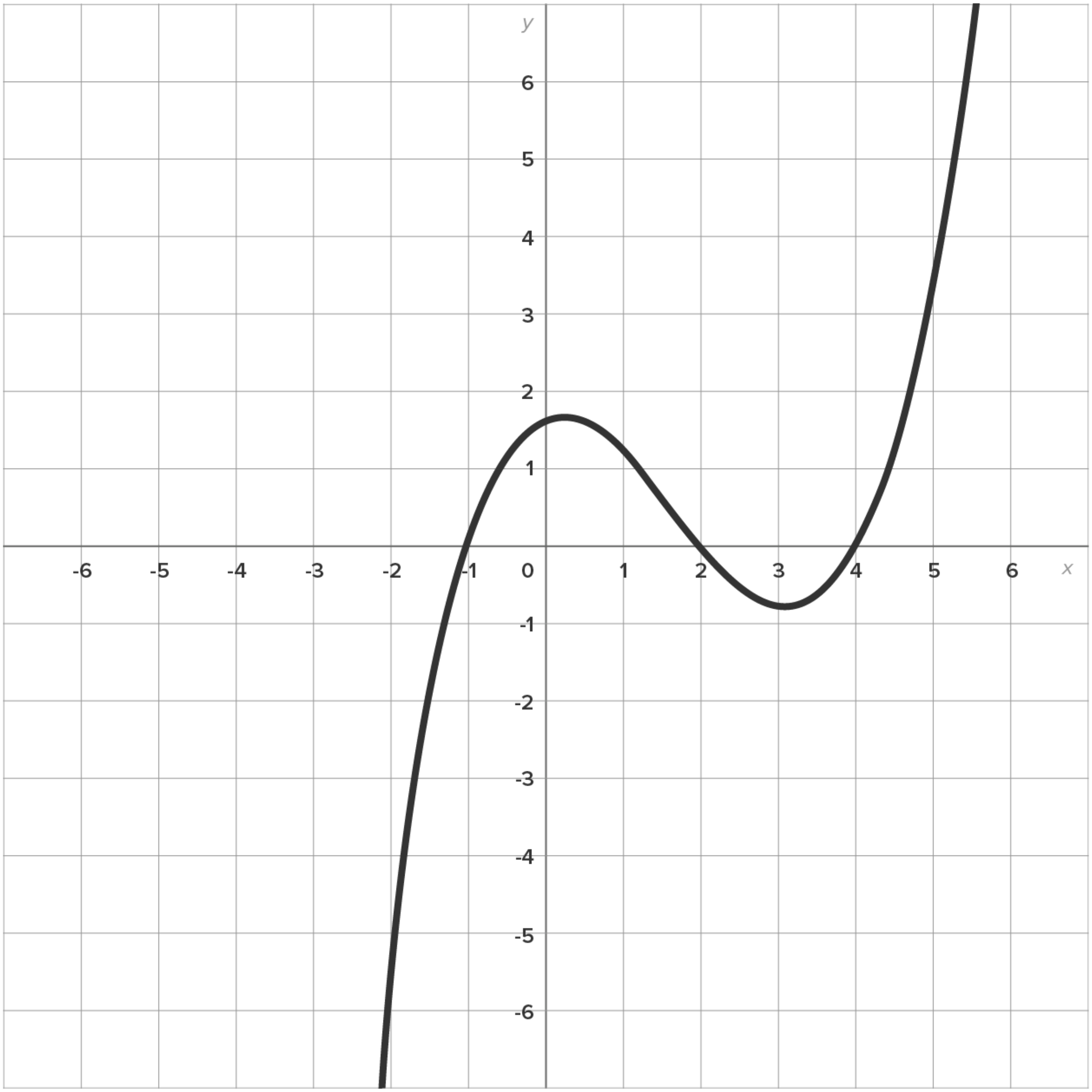
The dashed portion gets reflected over the x-axis.

This graph of $y = |f(x)|$ is shown below:



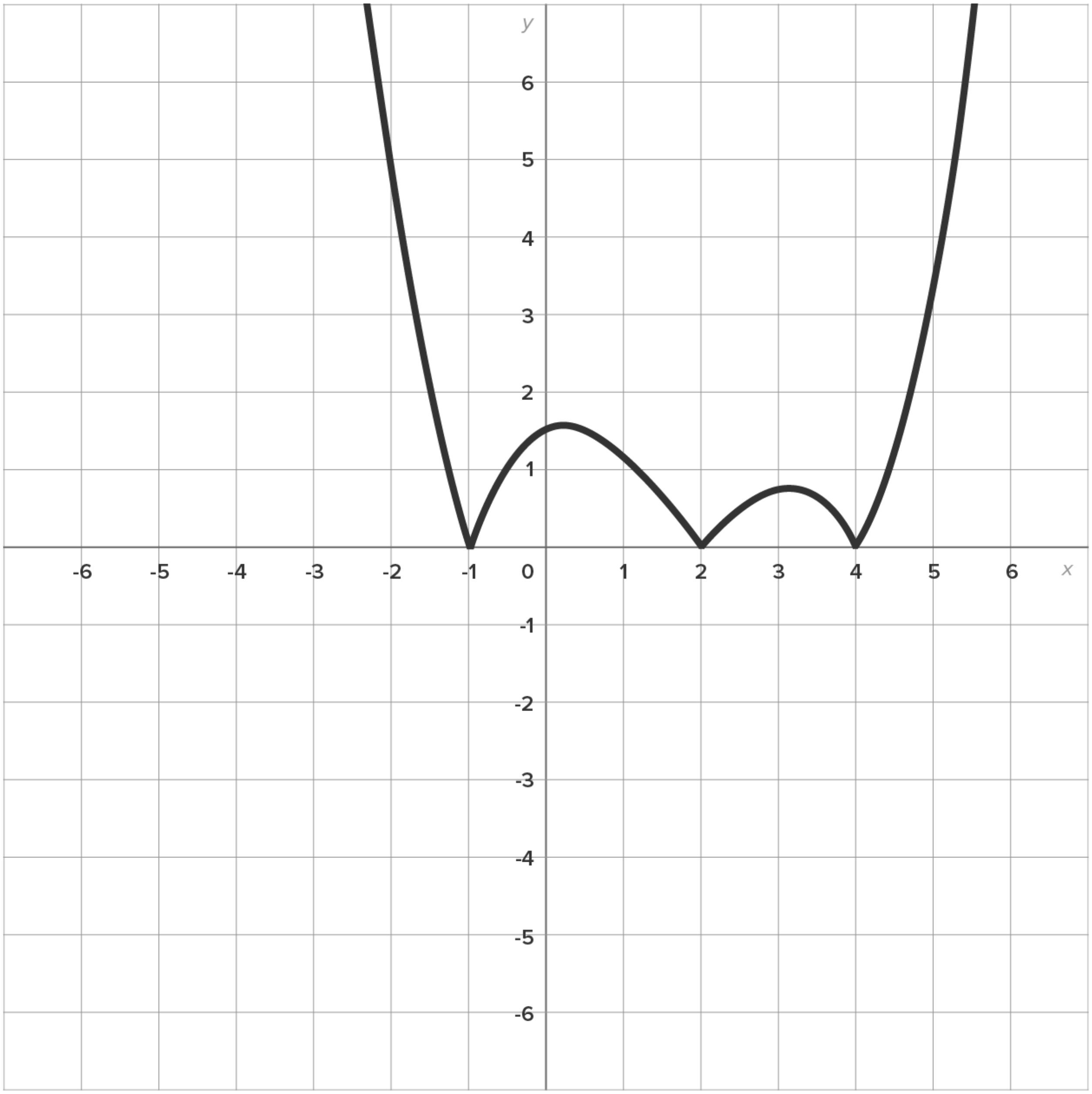
TRY IT

Consider the graph of $y = f(x)$ as shown below.



Sketch the graph of $y = |f(x)|$. +

Any portion of the graph below the x-axis is reflected over the x-axis. This means that the portion to the left of $x = -1$ and between $x = 2$ and $x = 4$ are reflected over the x-axis; while the rest of the graph remains the same. The graph below shows the result.



SUMMARY

In this lesson, you learned about **the absolute value function** of x, which represents the distance that a number x is from 0 on the number line. You learned about **the piecewise definition of absolute value**, given that in general, evaluating $|x|$ requires two different rules, depending on what x is. It's important to remember that the absolute value function may look simple on the surface, but it has a more complicated definition beyond "turning things nonnegative." You explored **the graph of the basic absolute value function**, applying rules you learned in a previous lesson to create graphs that illustrate the **shifting, stretching, and reflecting of the basic absolute value function**. Lastly, you learned about

other absolute value graphs, noting that while the basic absolute function is simply a “V” shape, graphing $y = |f(x)|$ requires more thought and care.

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TERMS TO KNOW

Absolute Value
The distance that a number is from 0 on the number line.