Tesis de Church-Turing y Cálculo Lambda

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Alonzo Church

14 de junio de 1903 en Washington. DC.


Se graduó de Princeton en 1924 con el grado de matemáticas.

UNIQUENESS OF THE LORENTZ TRANSFORMATION.

By ALONZO CHURCH, Princeton University.

1. The object of this inquiry is to obtain a set of logically independent postulates which uniquely determine the Lorentz transformation for one dimension. For this purpose we propose the following set:

1. The required transformation expresses $\bar{x}$ and $\bar{t}$ as functions of x, t, and v, which have continuous partial derivatives with respect to x and t, where $\bar{x}$, $\bar{t}$, x, and t are real variables, and v is a parameter which may have any value less than 1 and greater than -1.

2. For no value of v is the partial derivative of $\bar{t}$ with respect to t negative for every value of x and t.

3. $dx/dt = 1$ implies that $d\bar{x}/d\bar{t} = 1$.

4. $dx/dt = v$ implies that $d\bar{x}/d\bar{t} = 0$.

5. The inverse transformation, which expresses x and t in terms of $\bar{x}$ and $\bar{t}$, is obtained from the direct transformation by replacing $\bar{x}$ by x, and $\bar{t}$ by t, and x by $\bar{x}$, and t by $\bar{t}$, and v by $-v$.

6. The transformation is unchanged if $\bar{x}$ be replaced by $-\bar{x}$, and x by $-x$, and v by $-v$.

If we suppose our units of measurement so chosen that the velocity of light is 1, the physical meaning of the last five of these postulates is as follows:

2. The transformed time, $\bar{t}$, does not flow backwards with respect to t.

3. The velocity of light is invariant.

4. The origin of the transformed coördinate system is moving along the x-axis with velocity v in the original coördinate system.

5. The resultant of two velocities with the same numerical value and with opposite directions is zero.

6. The form of the transformation is independent of our choice of a positive direction on the x-axis.

2. Let the required transformation have the form:

$$\bar{x} = \varphi(x, t, v), \qquad \bar{t} = \psi(x, t, v).$$

Then

$$\left(\psi_x \frac{dx}{dt} + \psi_t\right)\frac{d\bar{x}}{d\bar{t}} = \varphi_x \frac{dx}{dt} + \varphi_t. \tag{1}$$

Setting $dx/dt = v$ in (1), it follows from postulate 4 that

$$v\varphi_x + \varphi_t = 0. \tag{2}$$

It is a consequence of postulates 4 and 5 that $dx/dt = 0$ implies that $d\bar{x}/d\bar{t} = -v$. Therefore, setting $dx/dt = 0$ in (1), it follows that

$$\varphi_t + v\psi_t = 0. \tag{3}$$

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Integrating this with respect to t, we obtain

$$\varphi + v\psi = X, \tag{4}$$

where X is independent of t.

Setting $dx/dt = 1$ in (1), it follows from postulate 3 that

$$\psi_x + \psi_t = \varphi_x + \varphi_t. \tag{5}$$

From (2) and (3) it follows that

$$\varphi_x = \psi_t \tag{6}$$

and therefore by (5)

$$\psi_x = \varphi_t \tag{7}$$

Substituting in (2) this gives

$$v\varphi_x + \psi_x = 0$$

and therefore integrating with respect to x,

$$v\varphi + \psi = T, \tag{8}$$

where T is independent of x.

For values of v between $+1$ and -1 equations (4) and (8) can be solved simultaneously. In this way we obtain:

$$\varphi = X_1 + T_1, \qquad \psi = X_2 + T_2,$$

where X_1 and X_2 are independent of t, and T_1 and T_2 are independent of x.

From equations (6) and (7) it now follows that

$$\frac{dX_1}{dx} = \frac{dT_2}{dt}, \qquad \frac{dX_2}{dx} = \frac{dT_1}{dt}.$$

These conditions, however, can be satisfied only if dX_1/dx, dX_2/dx, dT_1/dt, and dT_2/dt are all independent of both x and t. Therefore X_1, X_2, T_1, and T_2 are of the first degree in x and t. Therefore φ and ψ are of the first degree in x and t. Let us accordingly write the transformation in the form,

$$\bar{x} = p_1x + q_1t + r_1, \qquad \bar{t} = p_2x + q_2t + r_2,$$

where p_1, p_2, q_1, q_2, r_1, and r_2 are functions of v.

Then, by (6)

$$p_1 = q_2,$$

by (7)

$$p_2 = q_1,$$

and by (3)

$$q_1 = -vq_2.$$

Therefore

$$p_2 = -vq_2.$$

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En 1925 esposó a Mary Julia Kuczinski.

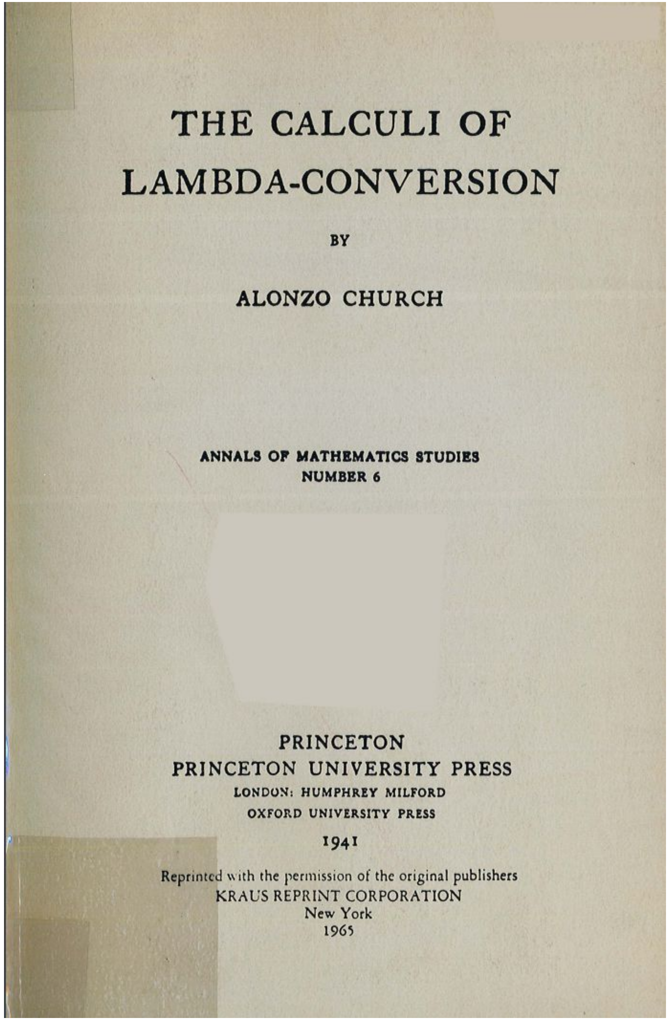
Tuvieron 3 hijos:

Alonzo Jr. (1929)

Mary Ann (1933)

Mildred (1938)

En 1941 publicó su primer libro '*The Calculi of Lambda Conversion*'



THE CALCULI OF
LAMBDA-CONVERSION

BY

ALONZO CHURCH

ANNALS OF MATHEMATICS STUDIES
NUMBER 6

PRINCETON
PRINCETON UNIVERSITY PRESS
LONDON: HUMPHREY MILFORD
OXFORD UNIVERSITY PRESS
1941

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KRAUS REPRINT CORPORATION
New York
1965

Aportes de Church en las matemáticas y lógica

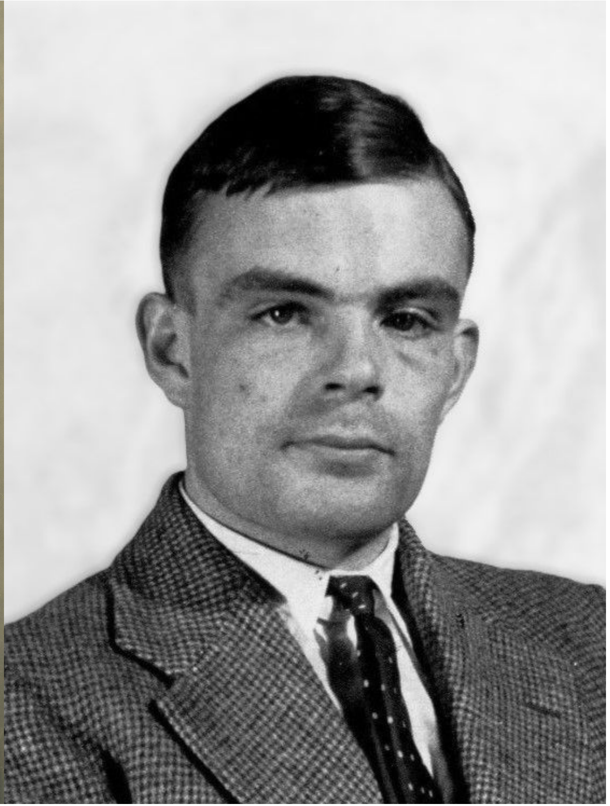
- Inventó el cálculo lambda
- Demostró que el problema Entscheidungsproblem es indecidible
- Demostró que múltiples enunciados de la aritmética de Peano son indecidibles
- Articulación de la tesis Church-Turing
- El teorema Church-Rosser

Alonzo Church

14 de junio de 1903 - 11 de agosto de 1995


Alan Turing

23 de junio de 1912 en Maida Vale, Londres



Christopher Morcom

I am sure I could not have found anywhere another companion so brilliant and yet so charming and unconceited. I regarded my interest in my work, and in such things as astronomy (to which he introduced me) as something to be shared with him and I think he felt a little the same about me ... I know I must put as much energy if not as much interest into my work as if he were alive, because that is what he would like me to do

Obtuvo el grado en matemáticas en 1934 del King’s Colllege en Cambridge.

En 1935 publicó su primer artículo

‘Equivalence of left and right almost periodicity’.

as above. With the notation Φ, Ψ explained, the matrices ξ, η are $\epsilon_{2p}\bar{\Psi}$ and $\epsilon_{2p}\bar{\Phi}$. The equation $\Phi\epsilon_{2p}\bar{\Psi}=0$ is used in Rosati's method, but not the equations $\Phi\epsilon_{2p}\bar{\Phi}=r\epsilon_{2n}$, $\Psi\epsilon_{2p}\bar{\Psi}=r\epsilon_{2p-2n}$. And, in fact, the equation for ξ, $(\Omega, \Omega\tau)\xi=0$, is satisfied also by ξX, where X is an arbitrary matrix of type $(2p-2n, 2p-2n)$; and similarly $\bar{\xi}\epsilon_{2p}\eta=0$ is satisfied not only by η but also by ηY, where Y is any matrix of type $(2n, 2n)$. The matrices X, Y may be chosen to make the identification of formulae exact in detail, and so make unnecessary the *a posteriori* verification required in the exposition of Rosati's proof in order to show that $\bar{b}\bar{m}V$, $\bar{a}\bar{k}V$ are independent sets of integrals. In the previous proof these sets arise as component parts of a precisely defined set of p independent integrals.

St. John's College,
Cambridge.

EQUIVALENCE OF LEFT AND RIGHT ALMOST PERIODICITY

A. M. TURING*.

In his paper "Almost periodic functions in a group", J. v. Neumann† has used independently the ideas of left and right periodicity. I shall show that these are equivalent.

$f(x)$ is a complex-valued function of a variable x which runs through an arbitrary group $\mathfrak{G}$. $f(x)$ is said to be right almost periodic (r.a.p.) if for each $\epsilon > 0$ we can find a finite set $b_1, \ldots, b_m$ of elements of $\mathfrak{G}$ such that to each t of $\mathfrak{G}$ there corresponds a $\mu = \mu(t)$ satisfying

$$|f(xt)-f(xb_\mu)| < \epsilon \quad \text{for all } x \in \mathfrak{G}. \tag{D}$$

The definition of left almost periodicity is obtained from this by replacing the inequality (D) by

$$|f(tx)-f(b_\mu x)| < \epsilon.$$

Suppose now that $f(x)$ is r.a.p., then to prove $f(x)$ l.a.p. it is sufficient to find, for each $\epsilon > 0$, a finite number of elements $c_1, \ldots, c_n$ of $\mathfrak{G}$ such that to each s of $\mathfrak{G}$ there corresponds a $\nu = \nu(s)$ satisfying

$$|f(sb_\pi)-f(c_\nu b_\pi)| < \epsilon \quad \text{for each } \pi; \tag{K}$$

* Received 23 April, 1935; read 25 April, 1935.
† J. v. Neumann, *Trans. American Math. Soc.*, 36 (1934), 445–492.

En 1936 publicó su primer artículo

'On Computable Numbers, with an Application to the Entscheidungsproblem'.

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

By A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

The "computable" numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbrous technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of computable numbers. According to my definition, a number is computable if its decimal can be written down by a machine.

In §§ 9, 10 I give some arguments with the intention of showing that the computable numbers include all numbers which could naturally be regarded as computable. In particular, I show that certain large classes of numbers are computable. They include, for instance, the real parts of all algebraic numbers, the real parts of the zeros of the Bessel functions, the numbers π, e, etc. The computable numbers do not, however, include all definable numbers, and an example is given of a definable number which is not computable.

Although the class of computable numbers is so great, and in many ways similar to the class of real numbers, it is nevertheless enumerable. In § 8 I examine certain arguments which would seem to prove the contrary. By the correct application of one of these arguments, conclusions are reached which are superficially similar to those of Gödel†. These results

† Gödel, "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme, I", *Monatshefte Math. Phys.*, 38 (1931), 173–198.

Tiempos de guerra

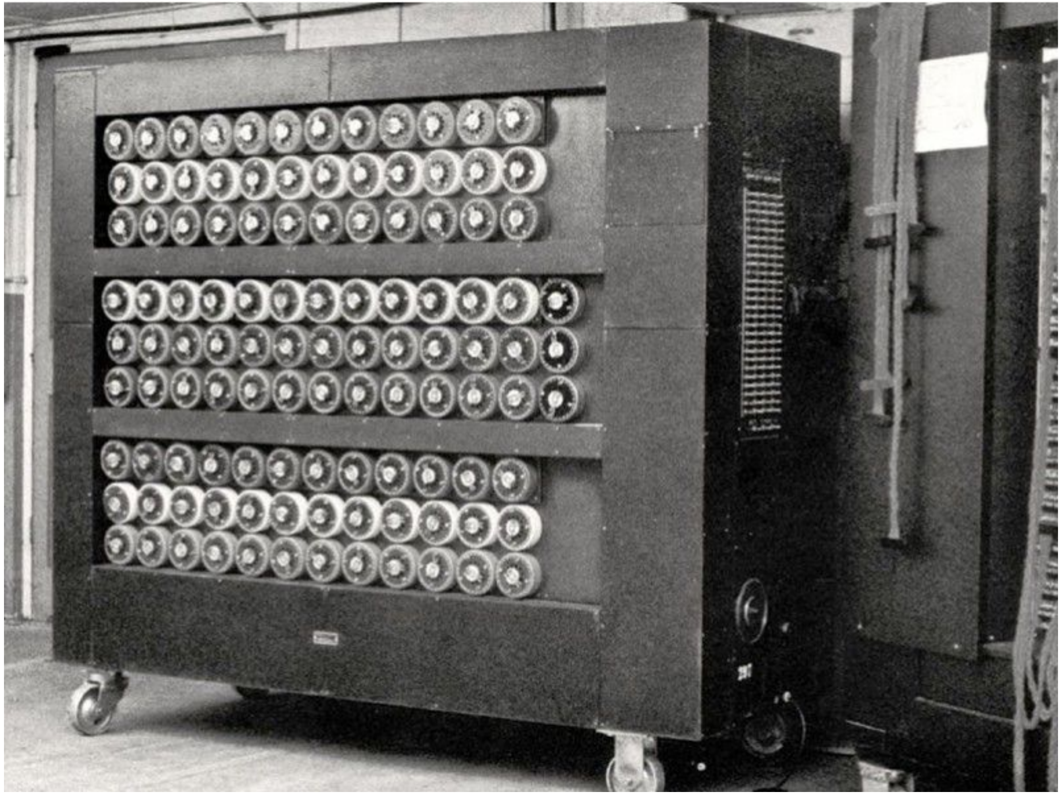
1938, Escuela de Gobierno de Códigos y Cifrados en Bletchley Park

Tiempos de guerra

1939, Inglaterra le declaró la guerra a alemania

Tiempos de guerra

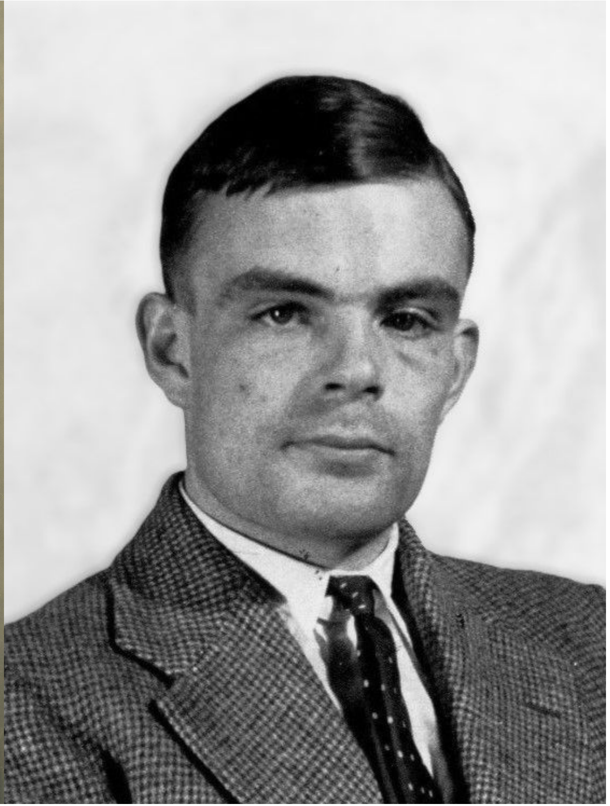
1940, The bombe



Alan Turing

23 de junio de 1912 - 8 de junio de 1954





Calculo lambda

Alonzo Church como el creador del cálculo lambda introducido en la década de 1930.

λ

¿Que es?

Función de una variable

$\lambda x.x$

$\lambda x.\lambda y.(x+y)$

$\lambda x.\lambda y.\lambda z.(x+y+z)$

Partes de un Cálculo Lambda

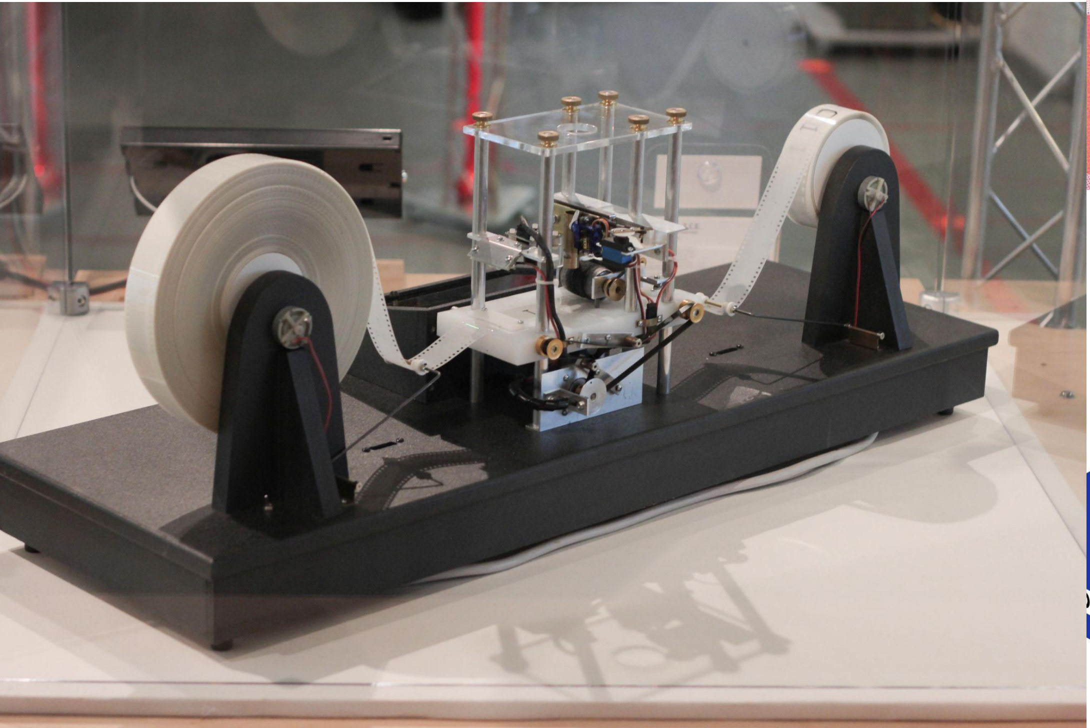
Variable, Funcion y Aplicacion

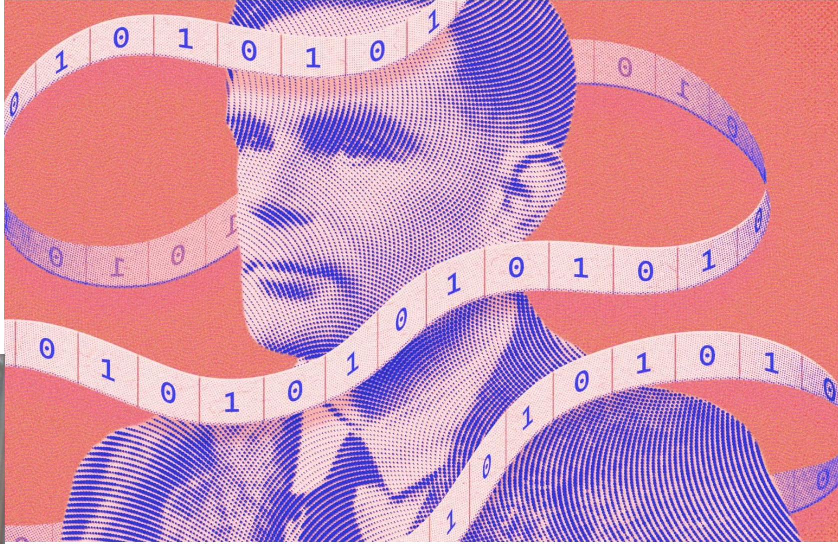
Ejemplos

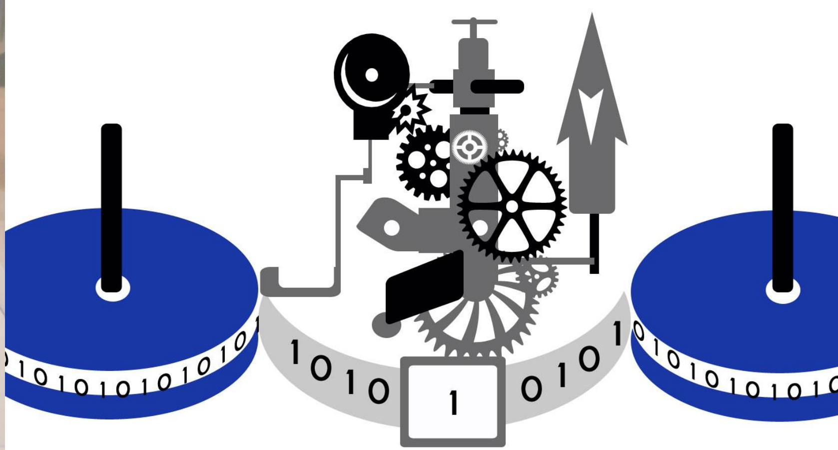
```
(λx.x)a
```

```
λx.λy.λz.(x+y+z)1 2 3
```

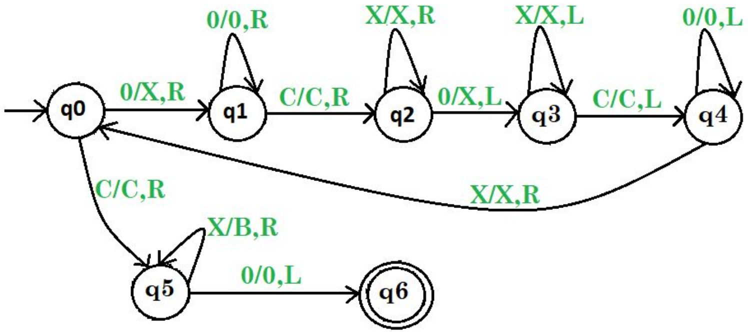
Máquinas de Turing

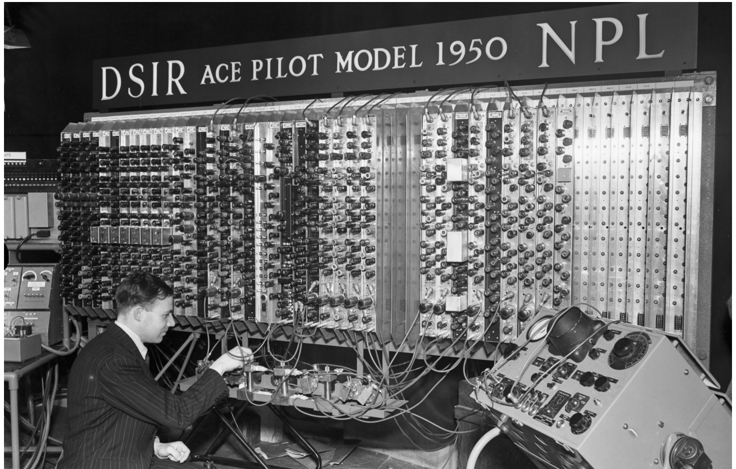






Definición Formal

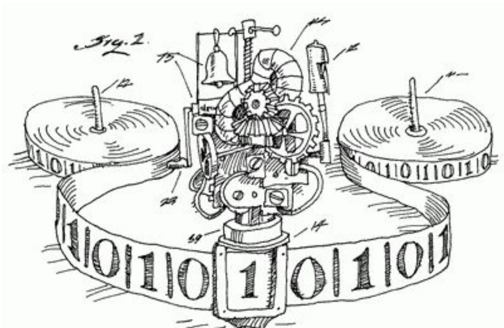




Una máquina de Turing se define formalmente como una tupla de 7:

⟨Q, q0, F, Γ, b, Σ, δ⟩

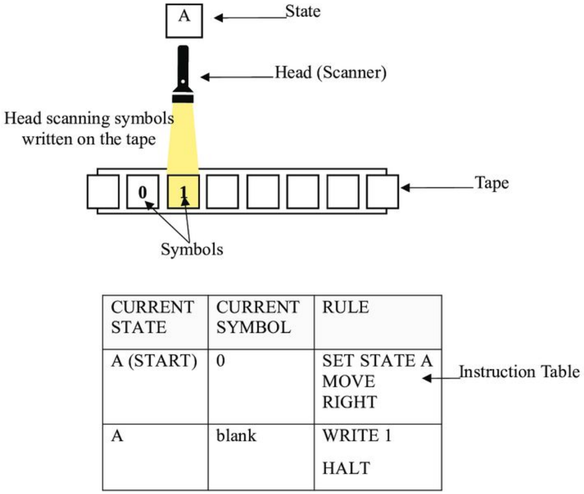
- **Q** es un conjunto finito y no vacío de estados.
- **q0 ∈ Q** es el estado inicial.
- **F ⊂ Q** es el conjunto de estados de aceptación.
- **Γ** es un conjunto finito y no vacío de símbolos de la cinta.
- **b ∈ Γ** es el símbolo en blanco.
- **Σ ⊂ Γ ∖ {b}** es un conjunto de símbolos de entrada.
- **δ : (Q ∖ F) × Γ → Q × Γ × {L, R}** es la función de transición, que es una función parcial.

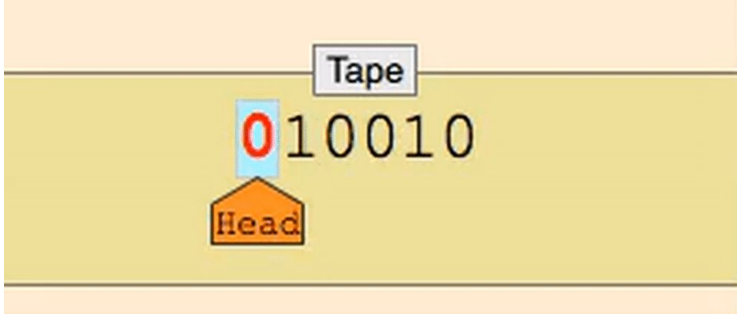


Elementos

Una máquina universal de Turing consiste en tres elementos muy sencillos:

- Una cinta, tan larga como sea necesaria, y que esté dividida en casillas. Esta misma es la que actuará como nuestra "memoria".
- Una "cabecilla", capaz de moverse por la cinta de izquierda a derecha para poder leer y escribir símbolos en ella.
- Un “programa”, el cual le dirá a la cabecilla qué es lo que debe de hacer en cada caso particular.

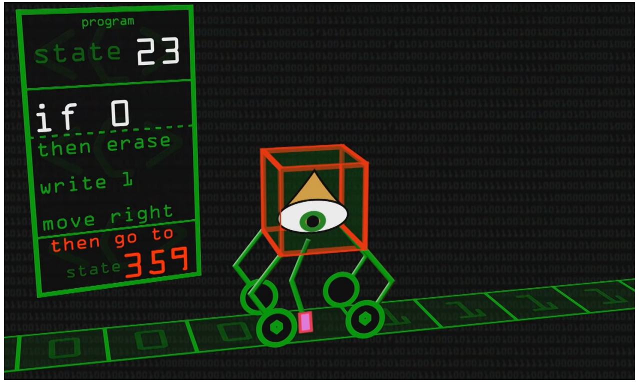


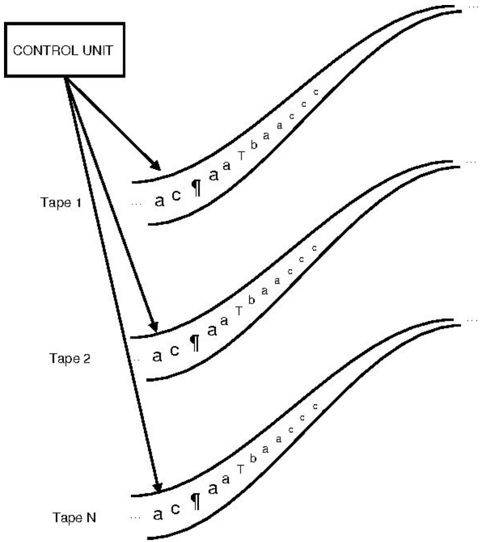


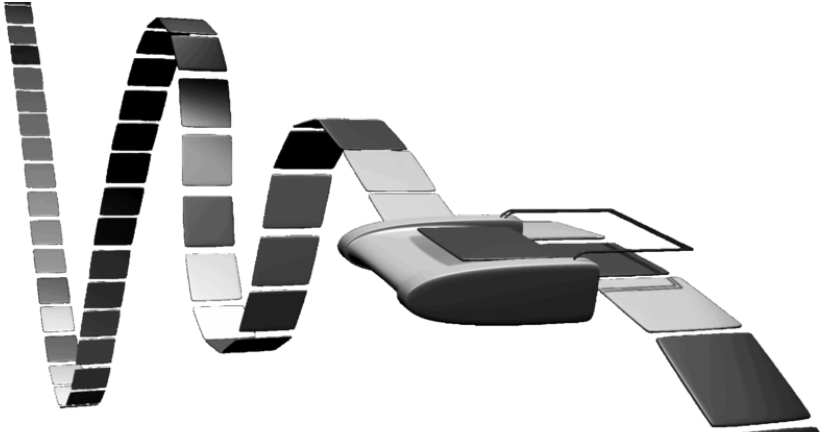
Variaciones

Existen múltiples variaciones de una máquina de Turing, entre las que se encuentran aquellas que:

- Permiten una sola cinta, o varias.
- Tienen una cinta es infinita en ambos lados.
- Únicamente pueden utilizar el alfabeto {0 , 1}, o más.
- La cinta puede quedarse en el mismo sitio o no.
- Permiten el no-determinismo.







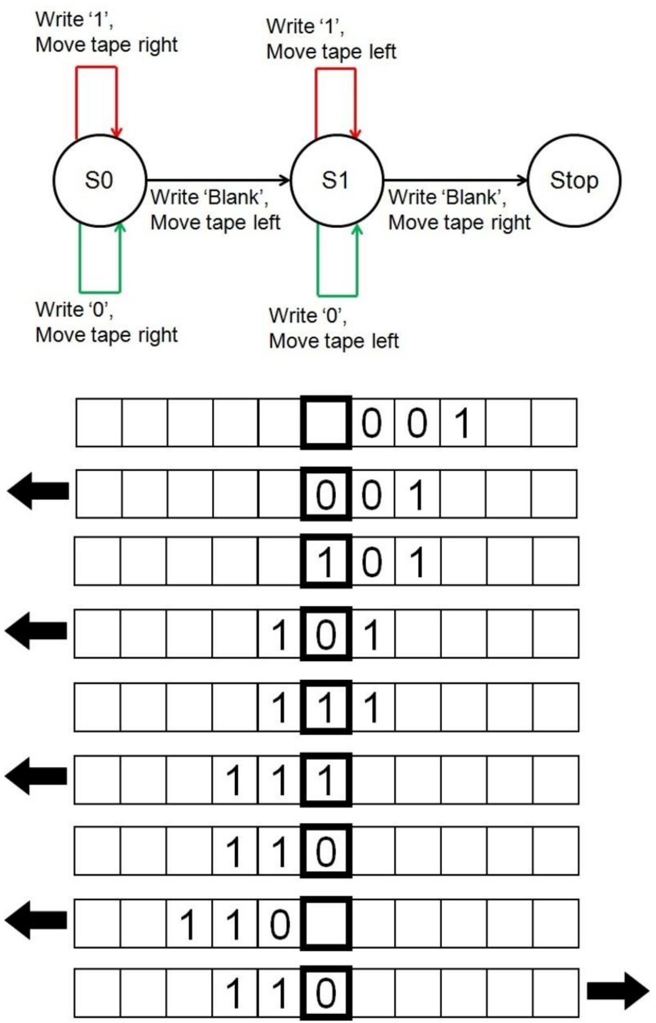
Ejemplo 1

Se hará un programa el cual sea capaz de invertir una serie de números binarios.

Ejemplo:

101110	→	*010001*
101	→	*010*
0011	→	*1100*

State	Symbol Read	Write Instruction	Move Instruction	Next State
State 0	*Blank*	*Write Blank*	*Move the head to the right*	*State 1*
	0	Write 1	Move the head to the left	State 1
	1	Write 0	Move the head to the left	State 0
State 1	*Blank*	*Write blank*	*Move the head to the left*	*Stop state*
	0	Write 1	Move the head to the right	State 1
	1	Write 0	Move the head to the right	State 1



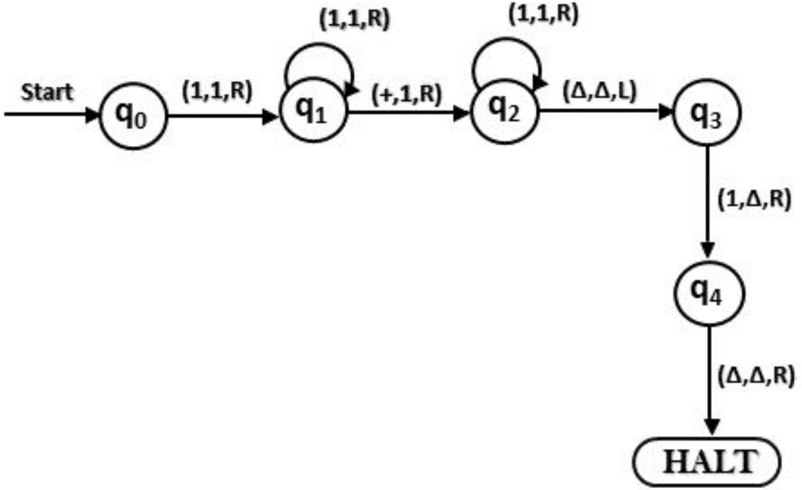
Ejemplo 2

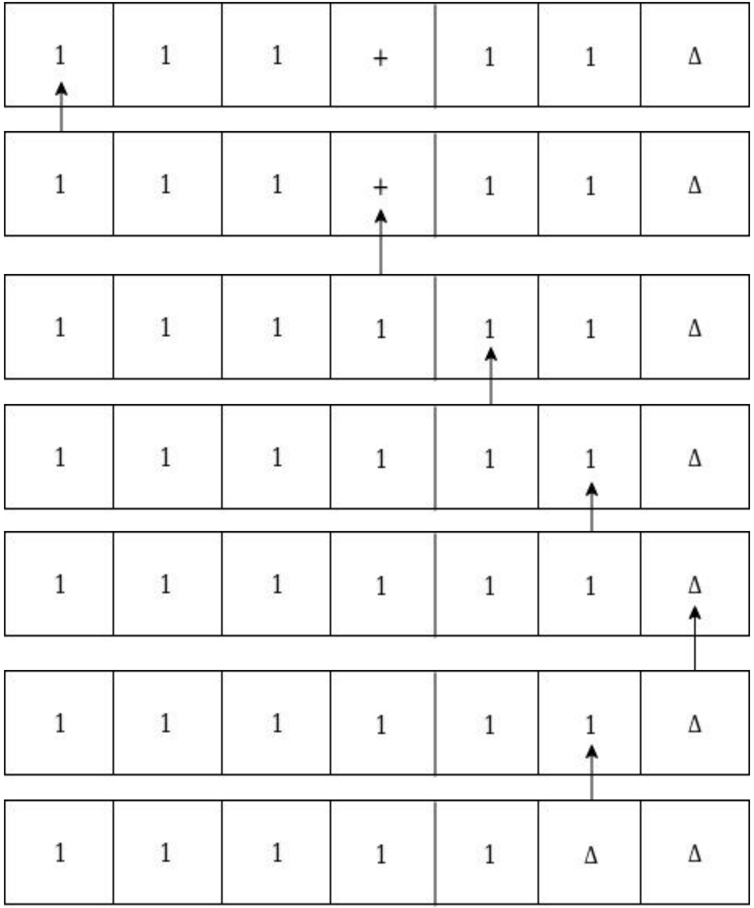
Construir una máquina de Turing (MT) para la función de adición en el sistema de números unarios.

El número unario está compuesto por un solo carácter, es decir, el número 5 se puede escribir en el sistema de números unarios como 11111. En esta máquina de Turing, vamos a realizar la adición de dos números unarios.

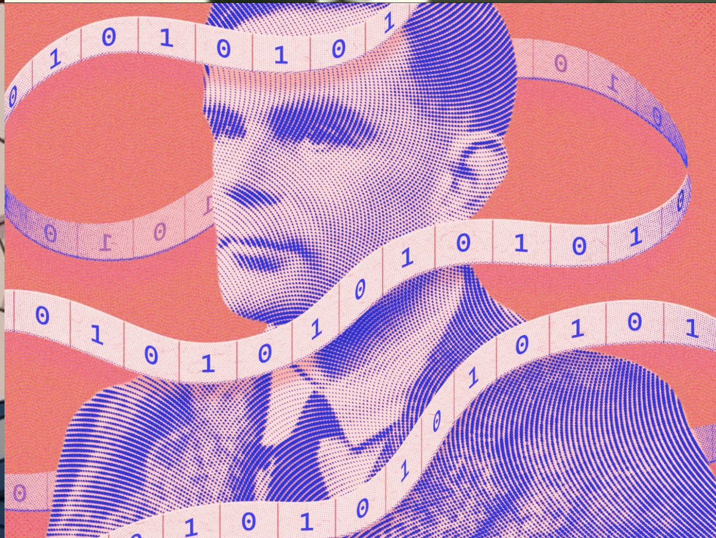
Ejemplo:

11+111 = 11111 (2 + 3)


Start
q0
(1,1,R)
q1
(1,1,R)
(+,1,R)
q2
(1,1,R)
(Δ,Δ,L)
q3
(1,Δ,R)
q4
(Δ,Δ,R)
HALT


1 1 1 + 1 1 Δ
1 1 1 + 1 1 Δ
1 1 1 1 1 1 Δ
1 1 1 1 1 1 Δ
1 1 1 1 1 1 Δ
1 1 1 1 1 1 Δ
1 1 1 1 1 Δ Δ

Tésis de Church-Turing

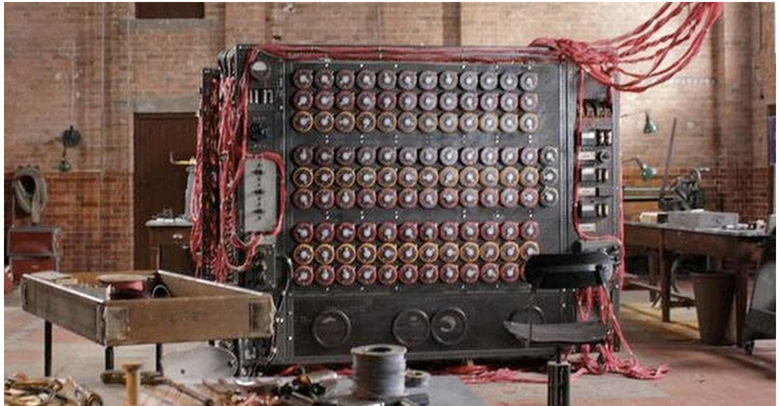


Definición

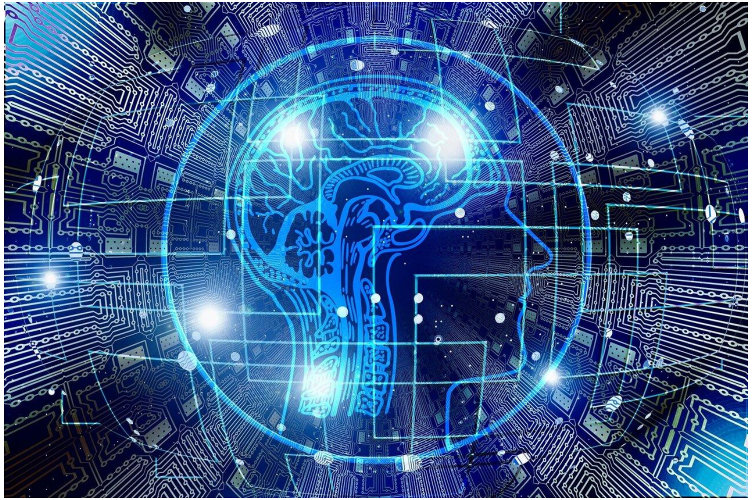
Es una hipótesis del campo de ciencias de la computación que afirma que cualquier función que pueda ser calculada por un algoritmo puede ser calculada por una máquina de Turing.

Esto es porque la máquina de Turing efectivamente captura todos los métodos concebibles de computación.

“The assumption that the intuitive notion of computable functions can be identified with partial recursive functions.”

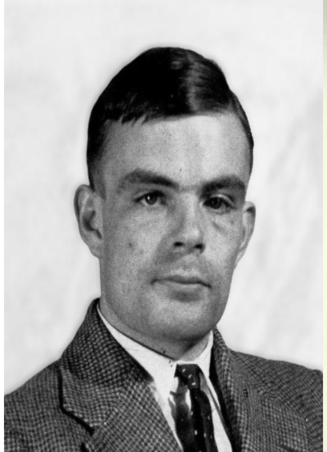


¿Se ha comprobado?



Aunque no se ha demostrado formalmente, se han encontrado muchos ejemplos de problemas que son computables según diferentes modelos (como las máquinas de Turing, el cálculo lambda, y otros) y que son equivalentes en lo que pueden resolver.

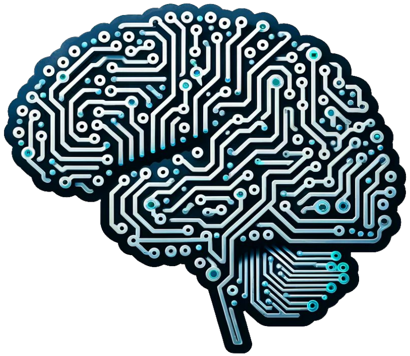
Ha resistido la prueba del tiempo en la comunidad científica y sigue siendo un pilar fundamental en la teoría de la computación

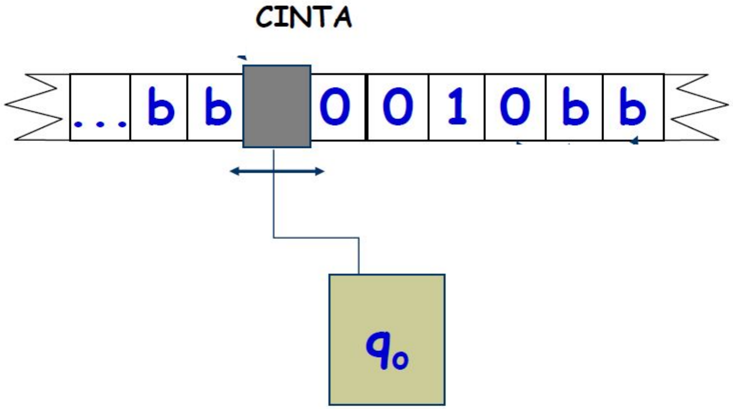


¿Qué impacto tiene la Tesis Church - Turing en la ICC?

IA, Avances tecnologicos, didacticos, algoritmos, bases de la computación, etc.

$\lambda x . x$





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