AGNES STARK

OPT OUT EXAM

1.1. $$\frac{z^{11}}{z^4 \cdot z^5} \cdot \frac{z^{11}}{z^9} = \underline{\underline{z^2}}$$

1.2.

$$6^2 \cdot 3^x \cdot 2^x = 6^9$$

$$3^2 \cdot 2^2 \cdot 3^x \cdot 2^x = 3^9 \cdot 2^9$$

$$3^{2+x} \cdot 2^{2+x} = 3^9 \cdot 2^9$$

$$(3 \cdot 2)^{2+x} = (3 \cdot 2)^9$$

Function is strictly monotonic, therefore

$$\Downarrow$$

$$2+x = 9$$

$$\underline{\underline{x = 7}}$$

1.3

$$x \cdot y = 5$$

$$x^{-3} \cdot y^{-3} =$$

$$(x \cdot y)^{-3} =$$

$$(5)^{-3} = \underline{\underline{0{,}008}}$$

1.4

$$\frac{\sqrt{3^{10}}}{\sqrt{9^3}} = \frac{3^{10} \cdot 3^{\frac{1}{2}}}{9^3 \cdot 9^{\frac{1}{2}}} =$$

$$\frac{3^{10} \cdot 3^{\frac{1}{2}}}{3^3 \cdot 3^3 \cdot 3^{\frac{1}{2}} \cdot 3^{\frac{1}{2}}} = \frac{(3^{10})^{\frac{1}{2}}}{((3^2)^3)^{\frac{1}{2}}} = \frac{3^5}{3^3} =$$

$$= 3^2 = \underline{\underline{9}}$$

1.5 a.)

$x+y = y+x$ TRUE

$x(y+z) = xy + xz$ TRUE

$x^{y+z} = x^y + x^z$ FALSE

$\frac{x^y}{x^z} = x^{y-z}$ TRUE

1.6.

$$\frac{4x-10}{4} \geq 4 \quad /\cdot 4$$

$$4x - 10 \geq 16 \quad /+10$$

$$4x \geq 26 \quad /:4$$

$$\underline{\underline{x \geq 6{,}5}}$$

$\boxed{\text{x has to equal, or has to be bigger than 6,5.}}$

2.

2.2

$f(x) = 7x + 3 \qquad f(y) = 52$

~~$\langle f(52) = 7 \cdot 52 + 3 = 367 \rangle$~~

$$52 = 7y + 3 \quad /-3$$

$$49 = 7y \quad /:7$$

$$\underline{\underline{7 = y}}$$

2.3

$$10^{x^2 - 2x + 2} = 100$$

$$10^{x^2 - 2x + 2} = 10^2$$

because x^2 function is strictly monotonic

$$x^2 - 2x + 2 = 2 \quad /-2$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$\underline{\underline{x = 0}}$ or $x - 2 = 0$

$\underline{\underline{x = 2}}$

$\left.\begin{matrix} x_1 = 0 \\ x_2 = 2 \end{matrix}\right\}$ x is either 0 or 2

2.4

$$1{,}02^t = 2$$

$$\log_{1{,}02} 2 = t \qquad \frac{\log_{10} 2}{\log_{10} 1{,}02} = \underline{35{,}003 \text{ years}}$$

2.5 $\ln\left(\frac{1}{e^3}\right) = \ln 1 - \ln e^3$

$= \underbrace{\ln 1}_{0} - 3 \cdot \underbrace{\ln e}_{1}$

$= 0 - 3$

$= \underline{\underline{-3}}$

3.1 $\sum_{i=0}^{\infty} \left(\frac{1}{8^i} + 0{,}5^i \right)$

$i=0$

$a_0 = \frac{1}{8^0}$

$a_1 = \frac{1}{8^1}$ $\Big\rangle r = \frac{1}{8}$

$a_2 = \frac{1}{8^2}$

$S_1 = \frac{a_0}{1-r} = \frac{\frac{1}{8^0}}{1-\frac{1}{8}} = \frac{1}{1-\frac{1}{8}} = \frac{1}{\frac{7}{8}} = \frac{8}{7}$

$0{,}5^i$

$\frac{1}{2}^i$

$i=0$

$a_0 = \frac{1}{2}^0 = 1$

$a_1 = \frac{1}{2}^1$ $\Big\rangle r = \frac{1}{2}$

$a_2 = \frac{1}{2} \cdot \frac{1}{2}$

$S_2 = \frac{a_0}{1-r} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$

$\sum S = S_1 + S_2 = \frac{8}{7} + 2 = \underline{\underline{\frac{22}{7}}}$

3.2

$\lim_{x \to 3} \frac{x-3}{2} = \lim_{x \to 3} \frac{3-3}{2} = \lim_{x \to 3} \frac{0}{2} = \underline{\underline{0}}$

3.4

$\frac{d}{dx} \frac{x^2+3 \to f}{x+2 \to g}$ $f'(x) = \frac{f' \cdot g - g' \cdot f}{g^2} = \frac{2x \cdot (x+2) - 1 \cdot (x^2+3)}{(x+2)^2} =$

$= \frac{2x^2 \cancel{+4x} - x^3 \cancel{-3x}}{x^2+4x+4} = \underline{\underline{\frac{2x^2 + x - x^3}{x^2+4x+4}}}$

PG.3.

$= \frac{2x^2+4x-x^2-3}{x^2+4x+4} = \frac{x^2+4x-3}{x^2+4x+4}$

3.5

$$\frac{d^2}{dx^2} 4x^3 + 4$$

$f'(x) = 12x^2$

$f''(x) = \underline{\underline{24x}}$

3.6

$f(x) = \frac{1}{x}$

$\lim_{x \to 0} \frac{1}{x} = -\infty$

$\lim_{x \leftarrow 0} \frac{1}{x} = +\infty$

From two sides the limit is not the same, so it can NOT be continuous at „0". The function doesn't dirarge to the same number, so it cannot be continuous.

3.8 $f(x,y) = x^3 y^2 \quad f(2,3)$

$f(2,3) = 2^3 \cdot 3^2 = 8 \cdot 9 = \underline{\underline{72}}$

3.9 $f(x,y) = \ln(2x-y)$

$2x - y > 0$

$2x > y$ y has to be smaller than 2x

$x > \frac{y}{2}$ x has to be bigger than $\frac{y}{2}$

3.10 $\frac{\partial^2}{\partial x^2} x^5 + x^2 y^3$

$\frac{\partial}{\partial x} f(x,y) = 5x^4 + 2xy^3$ $\quad \frac{\partial^2}{\partial x^2} f = \underline{\underline{20x^3 + 2y^3}}$

[3.11 $f(x,y) = \sqrt{xy} - 0{,}25x - 0{,}25y$]

3.3 $f(x) = x^2 - 4$

$f'(x) = 2x$

$f'(-1) = \underline{\underline{-2}}$ The slope of the function in the $(-1,-3)$ point is $\underline{\underline{-2}}$.

3.12.

max x^2y^2 s.t. $x+y=5$

$x+y-5=$

$$\frac{\partial L}{\partial x} = 2x \cdot y^2 - \lambda$$

$$\frac{\partial L}{\partial y} = x^2 \cdot 2y - \lambda$$

$$\frac{\partial L}{\partial \lambda} = x+y-5$$

(1) $2xy^2 - \lambda = 0$

$2xy^2 = \lambda$

(2) $x^2 \cdot 2y - \lambda = 0$

$x^2 \cdot 2y = \lambda$

(1) = (2) $2xy^2 = x^2 \cdot 2y$ $/:2$

$xy^2 = x^2 \cdot y$ $/:x$

$y^2 = xy$ $/:y$

$y = x$

(3) $x+y-5=0$

$x+y=5$ if $x=y$ then

$\boxed{x=2{,}5 \quad y=2{,}5}$

$\lambda = 2x \cdot y^2 = 2 \cdot 2{,}5 \cdot 2{,}5^2 = \underline{\underline{31{,}25}}$

4.3

$$A = \begin{bmatrix} 3.3 & 5.1 & 4.7 \\ 2 & 6.1 & 1.23 \\ 4 & 5.76 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3.3 & 2 & 4 \\ 5.1 & 6.1 & 5.76 \\ 4.7 & 1.23 & 0 \end{bmatrix}$$

4.4

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$\det A = 2 \cdot 5 - 4 \cdot 3 = 10 - 12 = \underline{\underline{-2}}$

pg. 5.

4.1.

A.B

$$\begin{array}{cc|ccc|}
 & & 1 & 4 & 1 \\
 & & 2 & 1 & 2 \\
\hline
2 & 3 & 8 & 11 & 8 \\
4 & 1 & 6 & 17 & 6 \\
1 & 2 & 5 & 6 & 5 \\
\hline
\end{array}$$

4.2

B.A

$$\begin{array}{ccc|cc|}
 & & & 2 & 3 \\
 & & & 4 & 1 \\
 & & & 1 & 2 \\
\hline
1 & 4 & 1 & 19 & 9 \\
2 & 1 & 2 & 10 & 11 \\
\hline
\end{array}$$

2.1

$\overset{x_1}{0\,°C} \rightarrow \overset{y_1}{32\,°F}$

$\underset{x_2}{100\,°C} \rightarrow \underset{y_2}{212\,°F}$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 32 = \frac{212 - 32}{100 - 0}(x - 0)$$

$$y - 32 = 1{,}8x$$

$$y = 1{,}8x + 32$$

LINEAR FUNCTION $y = mx + t$

$y = 1{,}8x + 32$ x and y need to equal so

$$x = 1{,}8x + 32 \quad / -32$$

$$x - 32 = 1{,}8x \quad / +x$$

$$-32 = 0{,}8x$$

$\underline{\underline{-40 = x}}$ $\underline{\underline{y = -40}}$ $\underline{\underline{\text{At } -40 \text{ degrees.}}}$

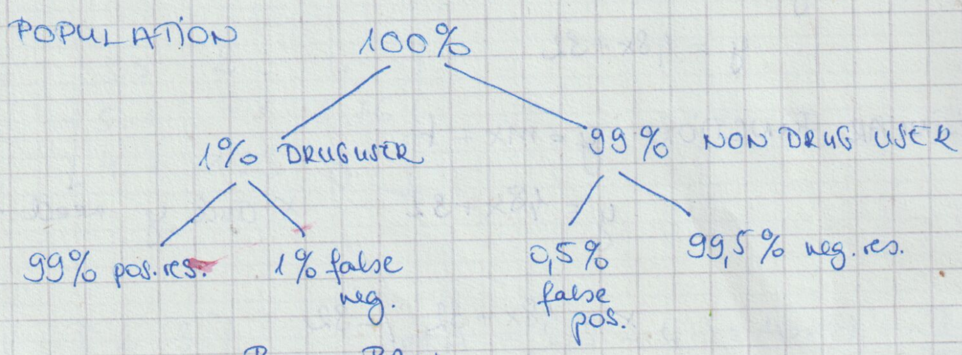
5. PROBABILITY THEORY

5.1.

$\underbrace{2}\cdot\underbrace{2}\cdot\underbrace{2}\cdot\underbrace{2} = 16$ outcomes

$\Omega = \{$(H, H, H, H);
(H, H, H, T);
(H, H, T, H);
(H, T, H, H);
(T, H, H, H);
(T, T, H, H);
(T, H, T, H);
(T, H, H, T);
(H, T, H, T);
(H, T, T, H);
(H, H, T, T);
(T, T, T, H);
(T, T, H, T);
(T, H, T, T);
(H, T, T, T);
(T, T, T, T) $\}$ ⟸ SAMPLE SET

5.2



P

Pos test : $P_{\text{druguser\&postest}} = \overset{P_{B_1}}{0{,}01} \cdot \overset{P_{A|B_1}}{0{,}99} = 0{,}0099$

$\sum_i \cancel{P(B_i|A)} = P_A$ $\quad P_{\text{nondrug postest}} = \overset{P_{B_2}}{0{,}99} \cdot \overset{P_{A|B_2}}{0{,}005} = 0{,}00495$

$\sum_i P_{pos} = 0{,}0099 + 0{,}00495 = 0{,}01485$

$\sum_i P(A|B_i) \cdot P(B_i) = P(A)$

$P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2)$

$P_{B} = \frac{0,01 \cdot 0,99}{0,01485} = \frac{0,0099}{\cancel{0,00495}\ 0,01485} \cancel{= 2\%}\ 0,667 \Rightarrow \approx 7$

Approx. 66,7 % is the probability of someone being a drug user if their test is positive.

$P(B_1/A) = \frac{P(B_1) \cdot P(A/B_1)}{P(A)}$

5.3

$\underline{\ \ } \ \ \underline{\ \ \ \ }$

$6 \cdot 6 \quad 36$

$P_{Ai} = \frac{1}{36}$

$\sum A_i = 21 \cdot 6 + 6 \cdot (21) = 252$

$\sum A_i \cdot P_{Ai} = 252 \cdot \frac{1}{36} = \underline{\underline{7}}$ The expected value is 7.

3.7 $f(x) = 3x^3 - 9x$

1.) intersections: $3x^3 - 9x = 0$

$x(3x^2 - 9) = 0$

$x = 0$

$3x^2 - 9 = 0$

$3x^2 = 9 \quad /:3$

$x^2 = 3 \quad /\sqrt{\ }$

$x = \pm\sqrt{3}$

$x_1 = -\sqrt{3}$
$x_2 = 0$
$x_3 = +\sqrt{3}$

2.) find 1st derivative $f(x) = 3x^3 - 9x$

$f'(x) = 9x^2 - 9$

$9x^2 - 9 = 0$

$9x^2 = 9 \quad /:9$

$x^2 = 1 \quad /\sqrt{\ }$

$x = \pm\sqrt{1}$ ⇒ min or max points

3.) find 2nd derivative $f''(x) = 18x$

$18x = 0$

$x = 0$

→

3.7. cont.	$-\sqrt{3} > x$	$-\sqrt{3}$	$-\sqrt{3} > x > -\sqrt{1}$	$-\sqrt{1}$	$-\sqrt{1} > x > 0$	0	$0 < x < \sqrt{1}$	$\sqrt{1}$	$\sqrt{1} < x < \sqrt{3}$	$\sqrt{3}$	$\sqrt{3} <$
$f(x)$	−	0	+	+	+	0	−	−	−	0	+
$f'(x)$	+	+	+	0	−	−	−	0	+	+	+
slope	↗	↗	↗	MAX	↘	↘	↘	MIN	↗	↗	↗
$f''(x)$	−	−	−	−	−	0	+	+	+	+	+
convexity	∩	∩	∩	∩	∩	INF.	∪	∪	∪	∪	∪

A function meets the x axis at $x = -\sqrt{3}$, $x = 0$ and at $x = \sqrt{3}$ points.

It has a maximum point at $x = -1$ and as minimum point at $x = 1$

It changes from concave to convex at $x = 0$.

3.11 $f(x,y) = \sqrt{xy} - 0{,}25x - 0{,}25y$

(1) $\partial_x f(x,y) = \frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{2}} - 0{,}25$

(2) $\partial_y f(x,y) = \frac{1}{2}x^{\frac{1}{2}}y^{-\frac{1}{2}} - 0{,}25$

(1) $\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}} - 0{,}25 = 0 \quad / \cdot 2$

$\frac{\sqrt{y}}{\sqrt{x}} - 0{,}5 = 0$

$\frac{\sqrt{y}}{\sqrt{x}} = 0{,}5$

$\sqrt{y} = \frac{1}{2}\sqrt{x}$

$y = \frac{1}{4}x \Rightarrow$ this is only true if $x = y = 0$

(2) $\frac{1}{2}\frac{\sqrt{x}}{\sqrt{y}} - 0{,}25 = 0$

$\frac{\sqrt{y}}{\sqrt{x}} - 0{,}5 = 0$

$\frac{\sqrt{x}}{\sqrt{y}} = 0{,}5$

$\sqrt{x} = \frac{1}{2}\sqrt{y}$

$x = \frac{1}{4}y \Rightarrow$ this is only true if $x = y = 0$

$u: [0;0]$

Hessian test:

$$H = \begin{bmatrix} \partial_x\partial_x f & \partial_x\partial_y f \\ \partial_y\partial_x f & \partial_y\partial_y f \end{bmatrix} = \begin{bmatrix} -\frac{1}{4}x^{-\frac{3}{2}}y^{\frac{1}{2}} & \frac{1}{4}x^{-\frac{1}{2}}y^{-\frac{1}{2}} \\ \frac{1}{4}x^{-\frac{1}{2}}y^{-\frac{1}{2}} & -\frac{1}{4}x^{\frac{1}{2}}y^{-\frac{3}{2}} \end{bmatrix}$$

$\det H(0;0) = 0$

Based on the Hessian test the function has no local minimum or maximum points.