683 Final project

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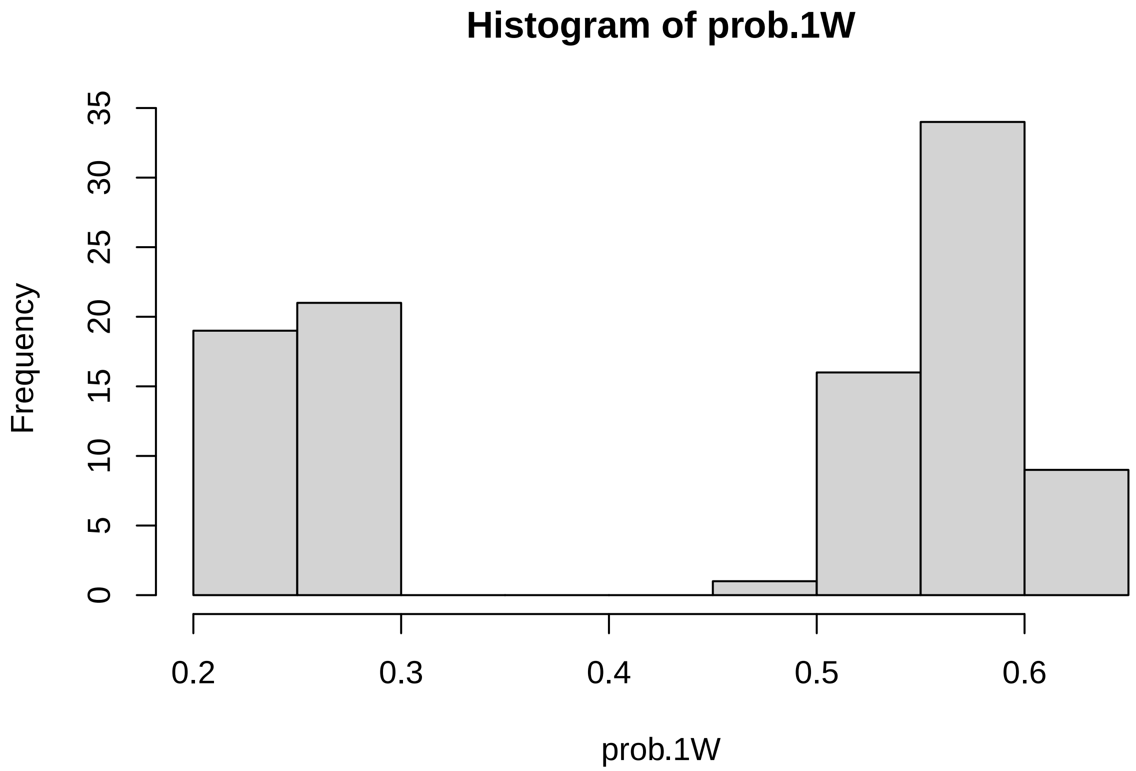
```r
set.seed(123)
# simple substitution estimator (a.k.a. parameteric G-computation)
txt <- ObsData
control <- ObsData

txt$A <- 1
control$A <- 0

g.comp.reg <- glm(Y ~ W11 + W12 + W13 + W14 + W2 + A, family="binomial", data=ObsData)
pred.txt <- predict(g.comp.reg, newdata = txt, type = "response")
pred.control <- predict(g.comp.reg, newdata = control, type = "response")
psi.hat <- mean(pred.txt - pred.control)
psi.hat
```

```
## [1] 0.01454638
```

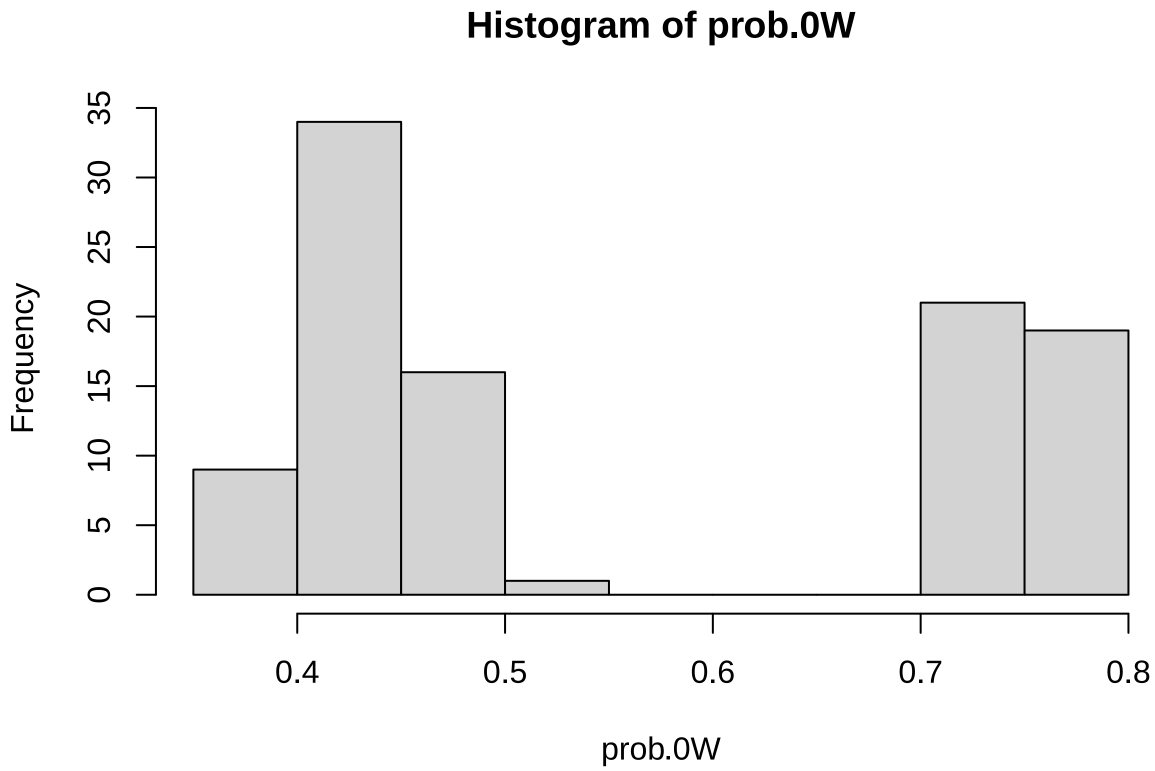
```r
# IPTW estimator
prob.AW.reg <- glm(A ~ W11 + W12 + W13 + W14, family="binomial", data=ObsData)
prob.1W <- predict(prob.AW.reg, type= "response")
prob.0W <- 1 - prob.1W

hist(prob.1W)
```



```r
summary(prob.1W)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.2216  0.2516  0.5358  0.4400  0.5727  0.6226
```

```r
hist(prob.0W)
```



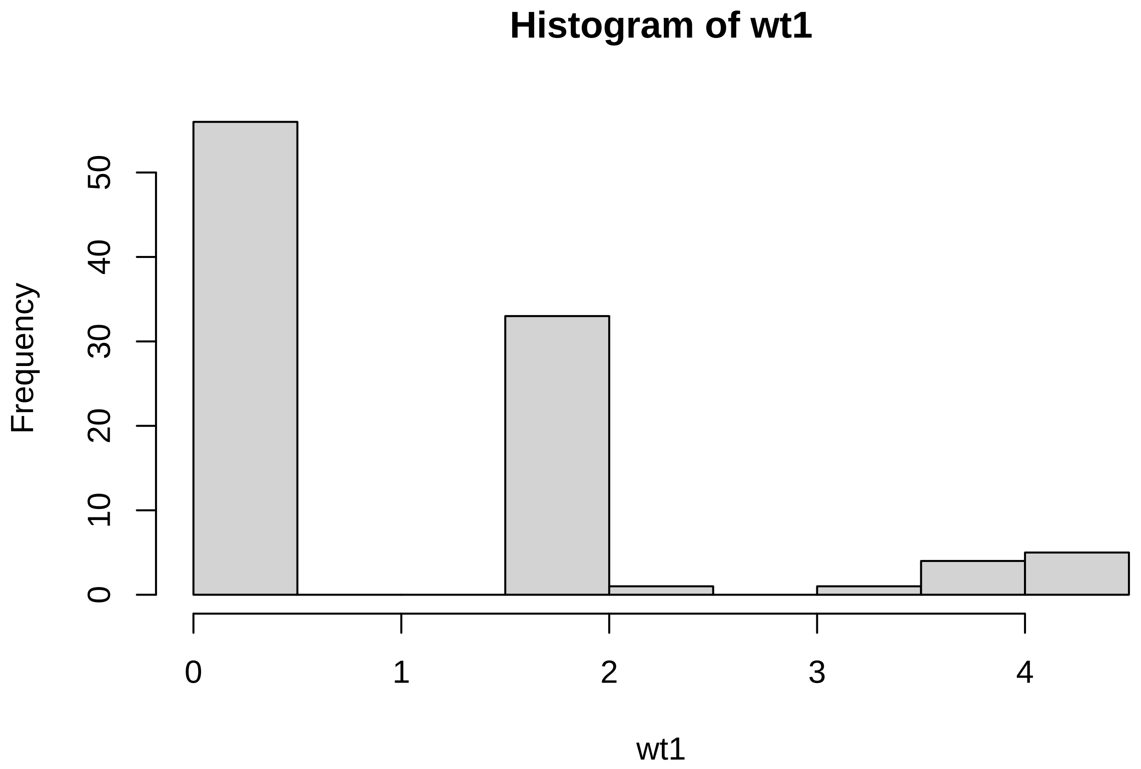
```r
summary(prob.0W)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##  0.3774  0.4273  0.4642  0.5600  0.7484  0.7784
```

```r
wt1 <- as.numeric(ObsData$A==1)/prob.1W
wt0 <- as.numeric(ObsData$A==0)/prob.0W
summary(wt1)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max. 
##   0.000   0.000   0.000   1.004   1.766   4.452
```

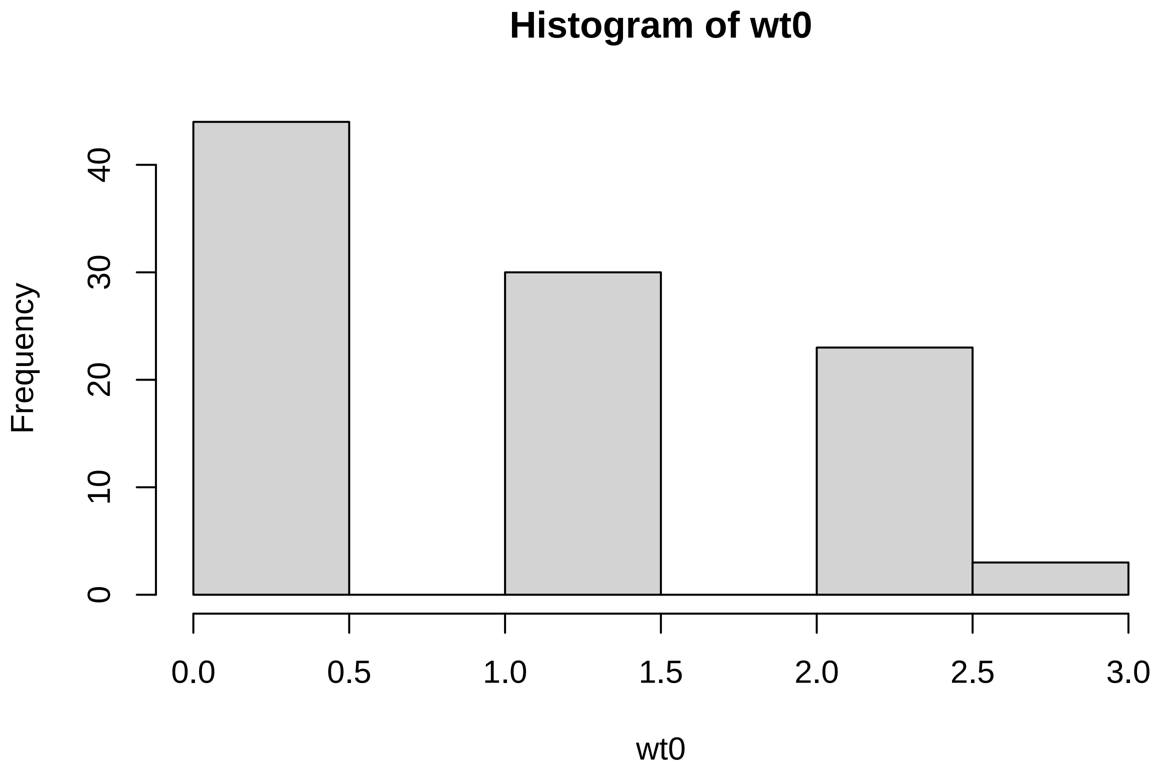
```r
hist(wt1)
```



```
summary(wt0)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   1.312   0.998   2.074   2.594
```

```
hist(wt0)
```



```
psi.iptw <- mean(wt1*ObsData$Y) - mean(wt0*ObsData$Y)
psi.iptw
```

```
## [1] -0.003532538
```

```
# Modified HT
psi.ht <- mean(wt1*ObsData$Y)/mean(wt1) - mean(wt0*ObsData$Y)/mean(wt0)
psi.ht
```

```
## [1] -0.00916455
```

```
# Unadjusted estimator
wt1.ua <- as.numeric(ObsData$A==1)/mean(ObsData$A == 1)
wt0.ua <- as.numeric(ObsData$A==0)/mean(ObsData$A == 0)
psi.unadj <- mean(wt1.ua*ObsData$Y) - mean(wt0.ua*ObsData$Y)
psi.unadj
```

```
## [1] -0.02922078
```

```
# TMLE estimator
```

SS, IPTW and TMLE estimator with super learner

```r
library("SuperLearner")
SL.library<- c(’SL.glm’, ’SL.glm.interaction’, "SL.step",
               "SL.randomForest","SL.step.forward","SL.stepAIC","SL.mean")
```

```r
run.tmle <- function(ObsData, SL.library){

  #------------------------------------------
  # Simple substitution estimator
  #------------------------------------------

  # dataframe X with baseline covariates and exposure
  X <- subset(ObsData, select=c(A, W11, W12, W13, W14,W2))

  # set the exposure=1 in X1 and the exposure=0 in X0
  X1 <- X0 <- X
  X1$A <- 1
  X0$A <- 0

  # Estimate E_0(Y|A,W) with Super Learner
  SL.outcome <- SuperLearner(Y=ObsData$Y, X=X, SL.library=SL.library,
              family="binomial", cvControl=list(V=10))

  # get the expected outcome, given the observed exposure and covariates
  expY.givenAW <- predict(SL.outcome, newdata=ObsData)$pred
  # expected outcome, given A=1 and covariates
  expY.given1W <- predict(SL.outcome, newdata=X1)$pred
  # expected outcome, given A=0 and covariates
  expY.given0W <- predict(SL.outcome, newdata=X0)$pred

  # simple substitution estimator would be
  PsiHat.SS <- mean(expY.given1W - expY.given0W)

  #------------------------------------------
  # Inverse probability of txt weighting
  #------------------------------------------

  #  Super Learner for the exposure mechanism  P_0(A=1|W)
  SL.exposure <- SuperLearner(Y=ObsData$A,
                              X=subset(ObsData, select= -c(A,Y,W2)),
                              SL.library=SL.library, family="binomial",
                              cvControl=list(V=10, stratifyCV = TRUE))

  # generate the predicted prob of being exposed, given baseline cov
  probA1.givenW <- SL.exposure$SL.predict
  # generate the predicted prob of not being exposed, given baseline cov
  probA0.givenW <- 1- probA1.givenW

  # clever covariate
  H.AW <- as.numeric(ObsData$A==1)/probA1.givenW - as.numeric(ObsData$A==0)/probA0.givenW

  # also want to evaluate the clever covariate at A=1 and A=0 for all participants
  H.1W <- 1/probA1.givenW
  H.0W <- -1/probA0.givenW
  # IPTW estimate
  PsiHat.IPTW <- mean(H.AW*ObsData$Y, na.rm = TRUE)


  #------------------------------------------
  # Targeting & TMLE
  #------------------------------------------

  # Update the initial estimator of E_0(Y|A,W)
  # run logistic regression of Y on H.AW using the logit of the esimates as offset

  expY.givenAW <- expY.givenAW - 0.000001

  logitUpdate<- glm( ObsData$Y ~ -1 +offset(qlogis(expY.givenAW)) +
                       H.AW, family='binomial')
  epsilon <- logitUpdate$coef

  # obtain the targeted estimates
  expY.givenAW.star<- plogis( qlogis(expY.givenAW)+ epsilon*H.AW )
  expY.given1W.star<- plogis( qlogis(expY.given1W)+ epsilon*H.1W )
  expY.given0W.star<- plogis( qlogis(expY.given0W)+ epsilon*H.0W )

  # TMLE point estimate
  PsiHat.TMLE<- mean(expY.given1W.star - expY.given0W.star)

  #------------------------------------------
  # Return a list withthe point estimates, targeted estimates of E_0(Y|A,W),
  # and the vector of clever covariates
  #------------------------------------------

  estimates <- data.frame(cbind(PsiHat.SS=PsiHat.SS, PsiHat.IPTW, PsiHat.TMLE))
  predictions <- data.frame(cbind(expY.givenAW.star, expY.given1W.star, expY.given0W.star))
  colnames(predictions) <- c('givenAW', 'given1W', 'given0W')
  list(estimates=estimates, predictions=predictions, H.AW=H.AW, probA1.givenW=probA1.givenW, probA0.give
}
```

```
set.seed(123)
out <- run.tmle(ObsData = ObsData, SL.library = SL.library)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

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## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: algorithm did not converge

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
est <- out$estimates
est
```

```
##     PsiHat.SS PsiHat.IPTW   PsiHat.TMLE
## 1 0.008577927 -0.07048078 -0.0001254867
```

CV Superlearner

```
X<- subset(ObsData, select= -Y )

CV.SL.out<- CV.SuperLearner(Y=ObsData$Y, X=X,
                            SL.library=SL.library, family=’binomial’,
                            cvControl = list(V = 5),
                            innerCvControl = list(list(V = 20)))

summary(CV.SL.out)
```

```
##
## Call:
## CV.SuperLearner(Y = ObsData$Y, X = X, family = "binomial", SL.library = SL.library,
##     cvControl = list(V = 5), innerCvControl = list(list(V = 20)))
##
## Risk is based on: Mean Squared Error
##
## All risk estimates are based on V =  5
##
##                Algorithm     Ave       se      Min     Max
##            Super Learner 0.10958 0.025377 0.020846 0.16398
##              Discrete SL 0.11054 0.026048 0.022500 0.16285
##               SL.glm_All 0.11662 0.025487 0.035708 0.17882
##   SL.glm.interaction_All 0.25011 0.043372 0.150000 0.45000
##              SL.step_All 0.11476 0.025447 0.026246 0.17515
##      SL.randomForest_All 0.11643 0.024761 0.032584 0.16285
##      SL.step.forward_All 0.11452 0.026196 0.022500 0.17515
##           SL.stepAIC_All 0.10819 0.025367 0.022500 0.17000
##              SL.mean_All 0.10819 0.025367 0.022500 0.17000
```

```
CV.SL.out$whichDiscrete
```

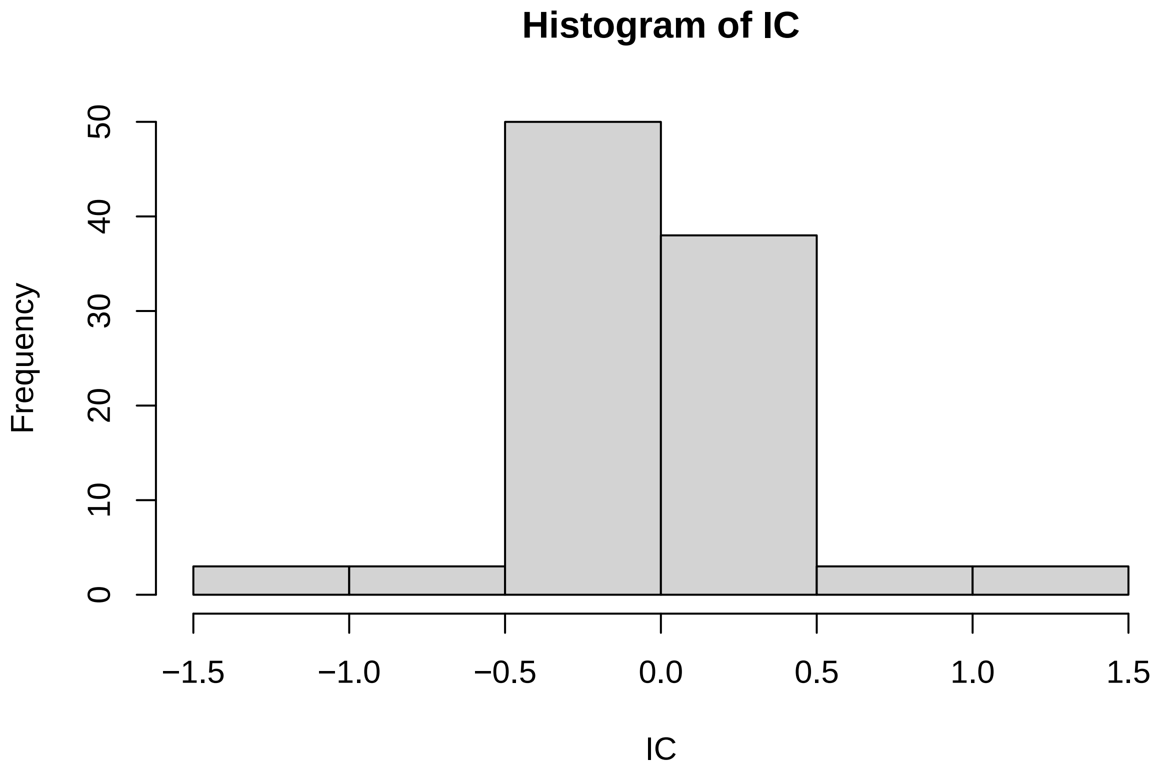
```
## $`1`
## [1] "SL.mean_All"
##
## $`2`
## [1] "SL.randomForest_All"
##
## $`3`
## [1] "SL.stepAIC_All"
##
## $`4`
## [1] "SL.randomForest_All"
##
## $`5`
## [1] "SL.mean_All"
```

Influence Curve

```
n <- nrow(ObsData)
# clever covariate
H.AW <- out$H.AW
# targeted predictions
expY.AW.star <- out$predictions[,'givenAW']
expY.1W.star <- out$predictions[,'given1W']
expY.0W.star <- out$predictions[,'given0W']
#  point estimate
PsiHat.TMLE <- est$PsiHat.TMLE

# plug-in
IC <- H.AW*(ObsData$Y - expY.AW.star) + expY.1W.star - expY.0W.star - PsiHat.TMLE
summary(IC)
```

```
##         V1
##  Min.   :-1.29683
##  1st Qu.:-0.13405
##  Median :-0.08957
##  Mean   : 0.00000
##  3rd Qu.: 0.15439
##  Max.   : 1.36008
```

```
hist(IC)
```



```
# estimate sigma^2 with the variance of the IC divided by n
varHat.IC <- var(IC)/n
varHat.IC
```

```
##             [,1]
## [1,] 0.001537978
```

```
# standard error estimate
se <- sqrt(varHat.IC)
se
```

```
##            [,1]
## [1,] 0.03921707
```

```
##### TMLE

# obtain 95% two-sided confidence intervals TMLE:
alpha <- 0.05
c(PsiHat.TMLE+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.TMLE+qnorm(alpha/2, lower.tail=F)*se)
```

```
## [1] -0.07698952  0.07673855
```

```
# calculate the pvalue tmle
2* pnorm( abs(PsiHat.TMLE /se), lower.tail=F )
```

```
##           [,1]
## [1,] 0.9974469
```

```
####### IPTW

PsiHat.IPTW <- est$PsiHat.IPTW

# obtain 95% two-sided confidence intervals TMLE:
alpha <- 0.05
c(PsiHat.IPTW+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.IPTW+qnorm(alpha/2, lower.tail=F)*se)
```

```
## [1] -0.147344817  0.006383259
```

```
# calculate the pvalue tmle
2* pnorm( abs(PsiHat.IPTW /se), lower.tail=F )
```

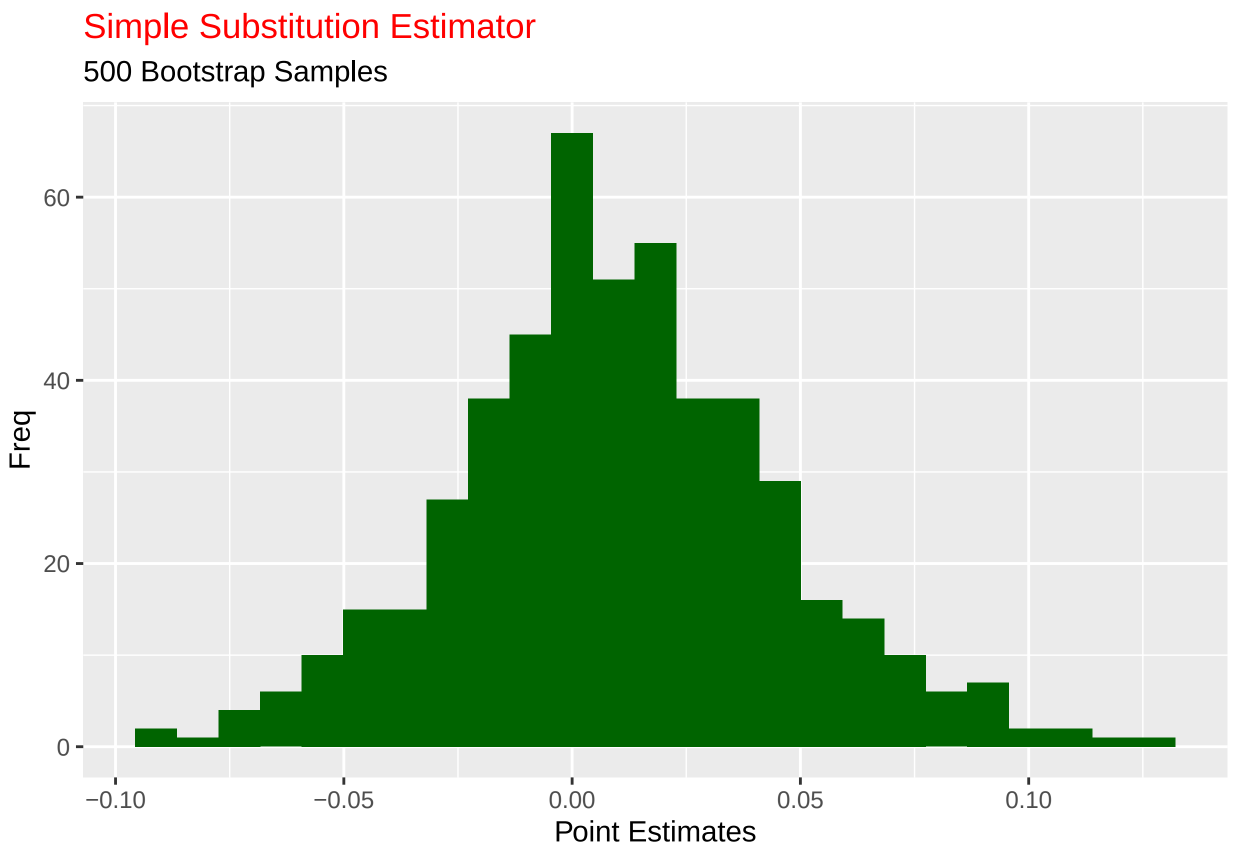
```
##            [,1]
## [1,] 0.07230441
```

Non-parametric bootstrap

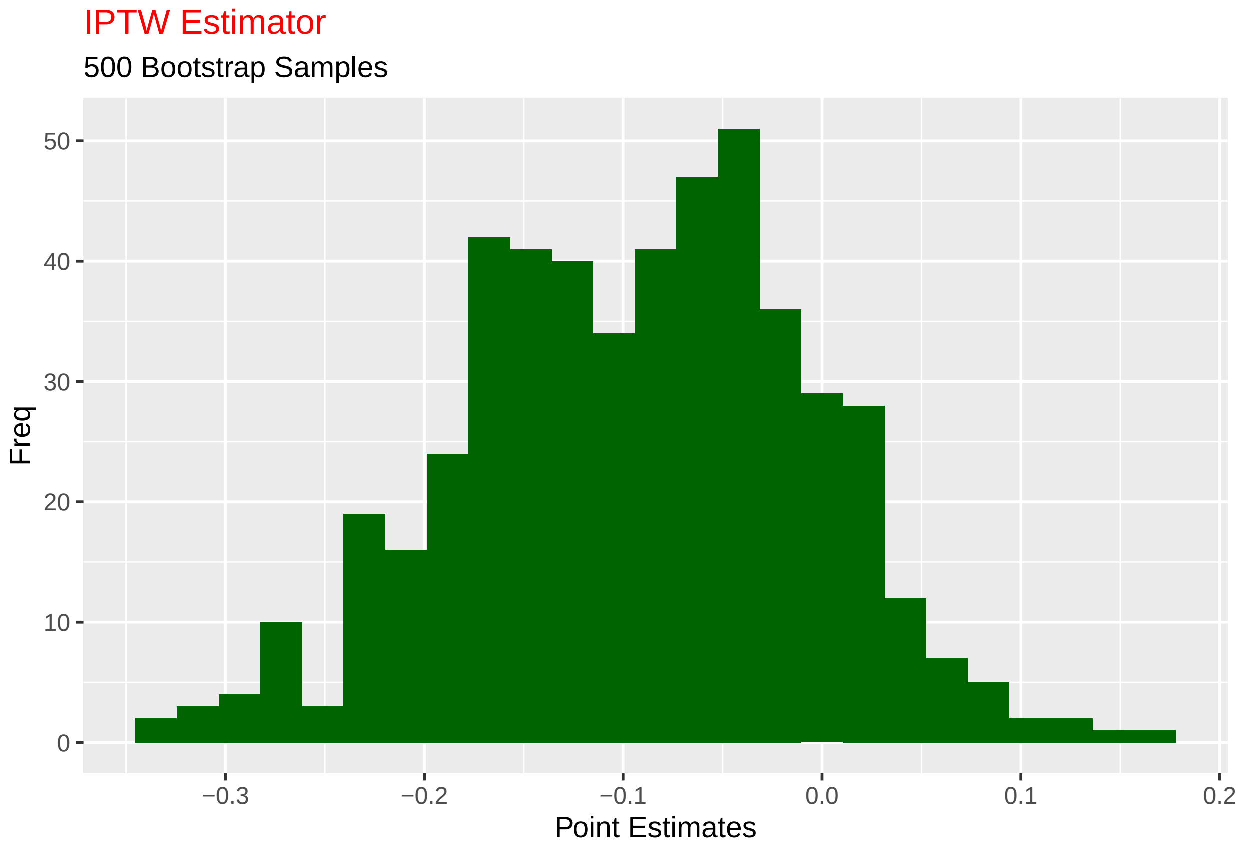
```
load(’boot_par.Rdata’)


summary(estimates)
```

```
##     SimpSubs              IPTW               TMLE
##  Min.   :-0.088893   Min.   :-0.32983   Min.   :-0.402617
##  1st Qu.:-0.012123   1st Qu.:-0.15514   1st Qu.:-0.122351
##  Median : 0.007485   Median :-0.08800   Median : 0.038971
##  Mean   : 0.010020   Mean   :-0.09301   Mean   : 0.008726
##  3rd Qu.: 0.032206   3rd Qu.:-0.03223   3rd Qu.: 0.127827
##  Max.   : 0.129803   Max.   : 0.17234   Max.   : 0.376132
##                                         NA’s   :9
```

```
ggplot(mapping = aes(estimates[,1]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="Simple Substitution Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```

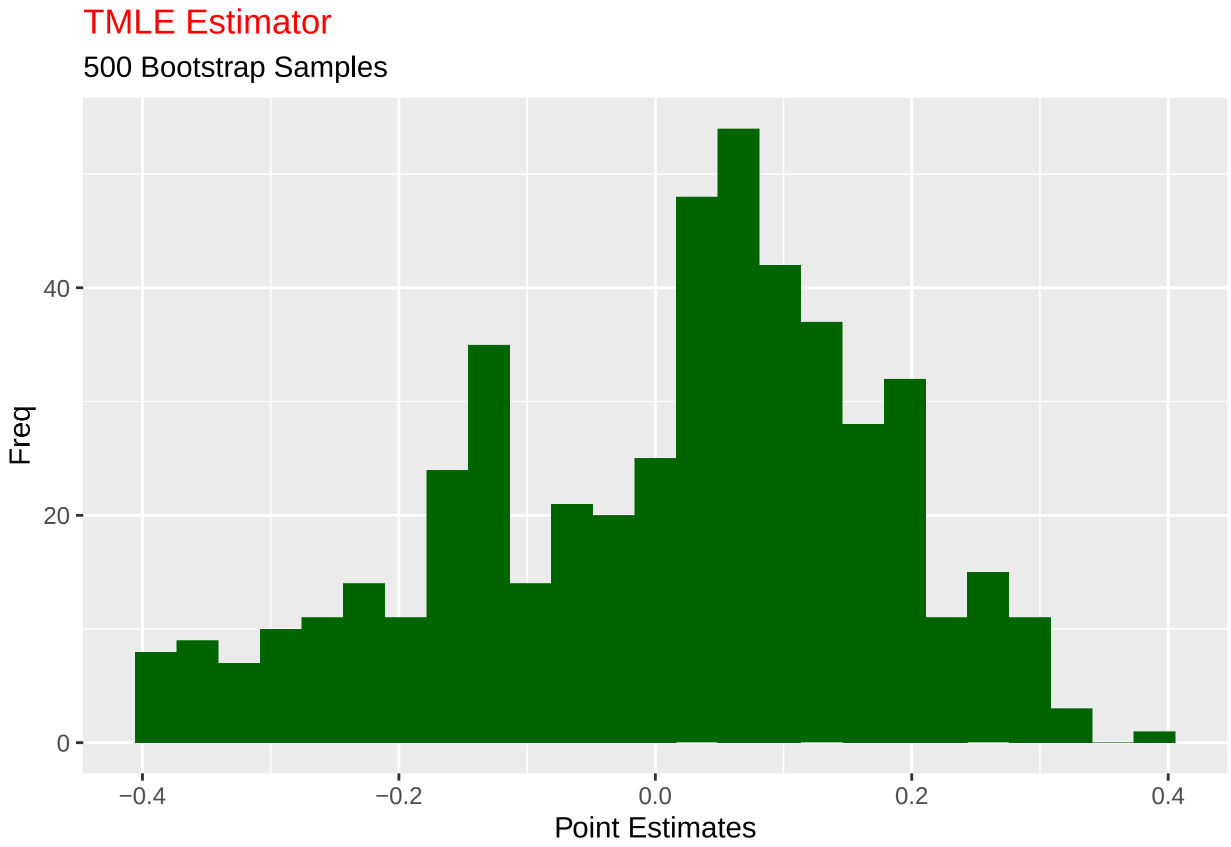


```
ggplot(mapping = aes(estimates[,2]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="IPTW Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```



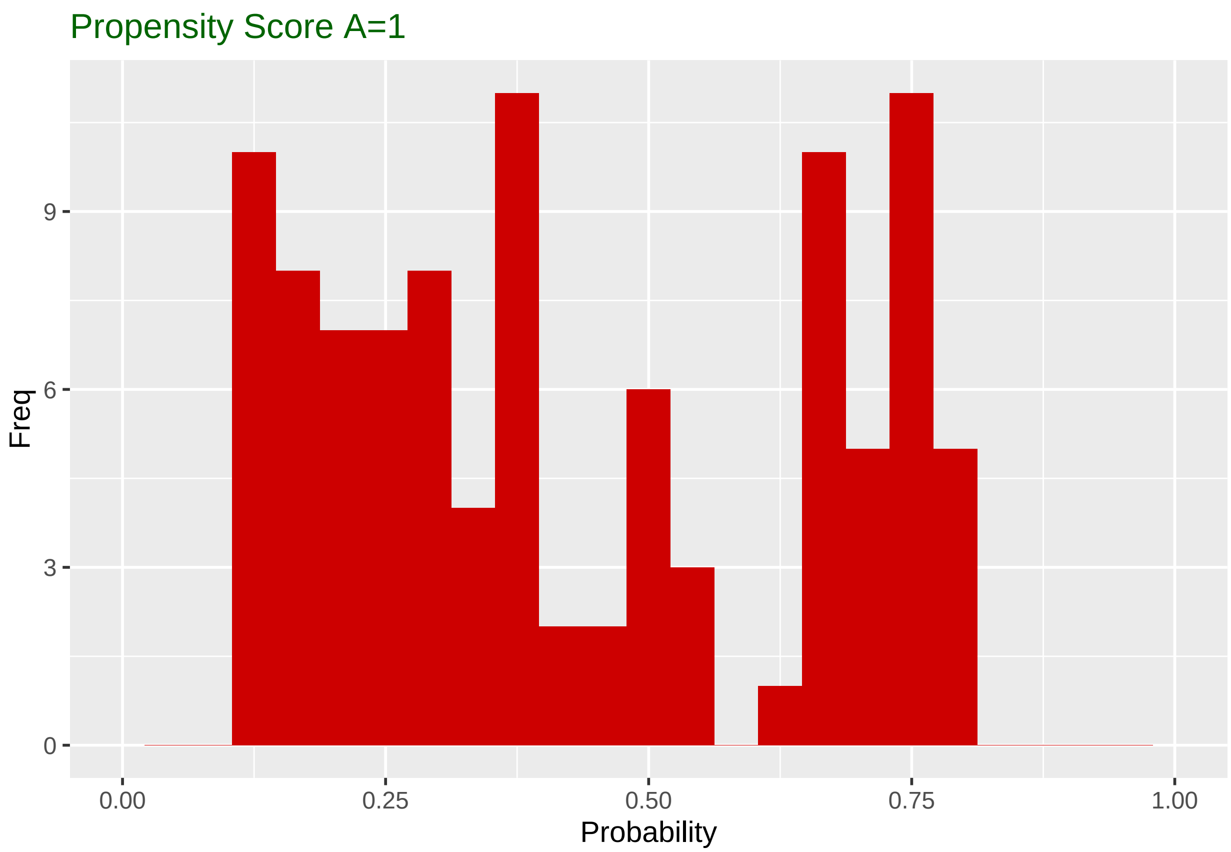
```
ggplot(mapping = aes(estimates[,3]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="TMLE Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```

```
## Warning: Removed 9 rows containing non-finite values (stat_bin).
```



```
ggplot(mapping = aes(out$probA1.givenW))+
  geom_histogram(fill="red3",bins = 25)+
  xlab("Probability")+
  ylab("Freq")+
  labs(title="Propensity Score A=1")+
  theme(plot.title = element_text(colour = "dark green"))+
  xlim(0,1)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```



```
#---------------------------------
# 95% Confidence intervals assuming a normal dist & via quantiles
#---------------------------------
create.CI <- function(pt, boot, alpha=0.05){
  Zquant <- qnorm(alpha/2, lower.tail=F)
  CI.normal <- c(pt - Zquant*sd(boot,na.rm = TRUE),
                 pt + Zquant*sd(boot,na.rm = TRUE) )
  CI.quant  <- quantile(boot, prob=c(0.025,0.975) ,na.rm=TRUE)
  out <- data.frame(rbind(CI.normal, CI.quant))
  colnames(out) <- c('CI.lo', 'CI.hi')
  out
}
```

```
# IMPORTANT - POINT OF CONFUSION FOR PAST STUDENTS
# The point estimate 'pt' is from the original dataset

# Simple Subs - note the bias because of misspecified regression? Will it converge fast enough?
est$PsiHat.SS
```

```
## [1] 0.008577927
```

```
create.CI(pt=est$PsiHat.SS, boot=estimates[,"SimpSubs"])
```

```
##                   CI.lo      CI.hi
## CI.normal -0.06110947 0.07826532
## CI.quant  -0.05987893 0.08625440
```

```
# IPTW
est$PsiHat.IPTW
```

```
## [1] -0.07048078
```

```
create.CI(pt=est$PsiHat.IPTW, boot=estimates[,"IPTW"])
```

```
##                CI.lo     CI.hi
## CI.normal -0.2417312 0.1007696
## CI.quant  -0.2721505 0.0668197
```

```
# TMLE
est$PsiHat.TMLE
```

```
## [1] -0.0001254867
```

```
create.CI(pt=est$PsiHat.TMLE, boot=estimates[,"TMLE"])
```

```
##                CI.lo     CI.hi
## CI.normal -0.3248202 0.3245693
## CI.quant  -0.3615892 0.2772986
```

```
# Compare to IC estimate
c(PsiHat.TMLE+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.TMLE+qnorm(alpha/2, lower.tail=F)*se)
```

```
## [1] -0.07698952  0.07673855
```