683 Final project

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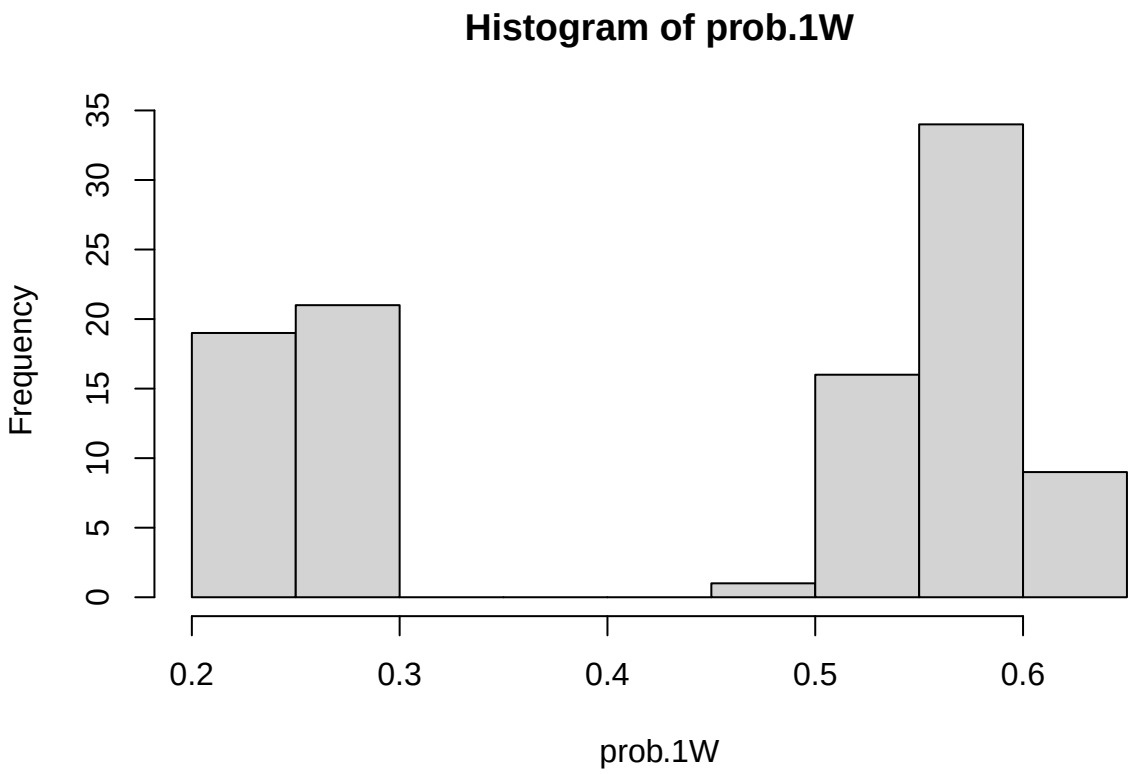
11/6/2021

```
set.seed(123)
# simple substitution estimator (a.k.a. parameteric G-computation)
txt <- ObsData
control <- ObsData

txt$A <- 1
control$A <- 0

g.comp.reg <- glm(Y ~ W11 + W12 + W13 + W14 + W2 + A, family="binomial", data=ObsData)
pred.txt <- predict(g.comp.reg, newdata = txt, type = "response")
pred.control <- predict(g.comp.reg, newdata = control, type = "response")
psi.hat <- mean(pred.txt - pred.control)
psi.hat
```

```
## [1] -0.01454638
```

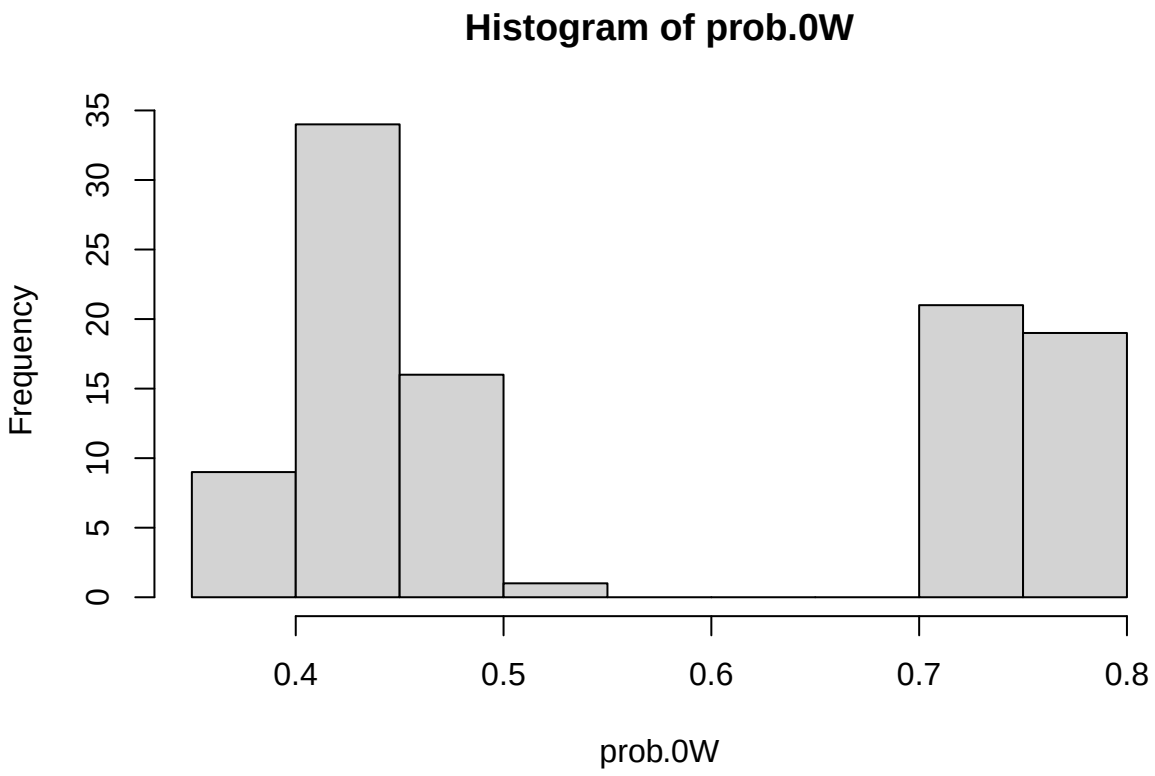
```
# IPTW estimator
prob.AW.reg <- glm(A ~ W11 + W12 + W13 + W14, family="binomial", data=ObsData)
prob.1W <- predict(prob.AW.reg, type= "response")
prob.0W <- 1 - prob.1W

hist(prob.1W)
```




```
summary(prob.1W)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.2216  0.2516  0.5358  0.4400  0.5727  0.6226
```

```
hist(prob.0W)
```



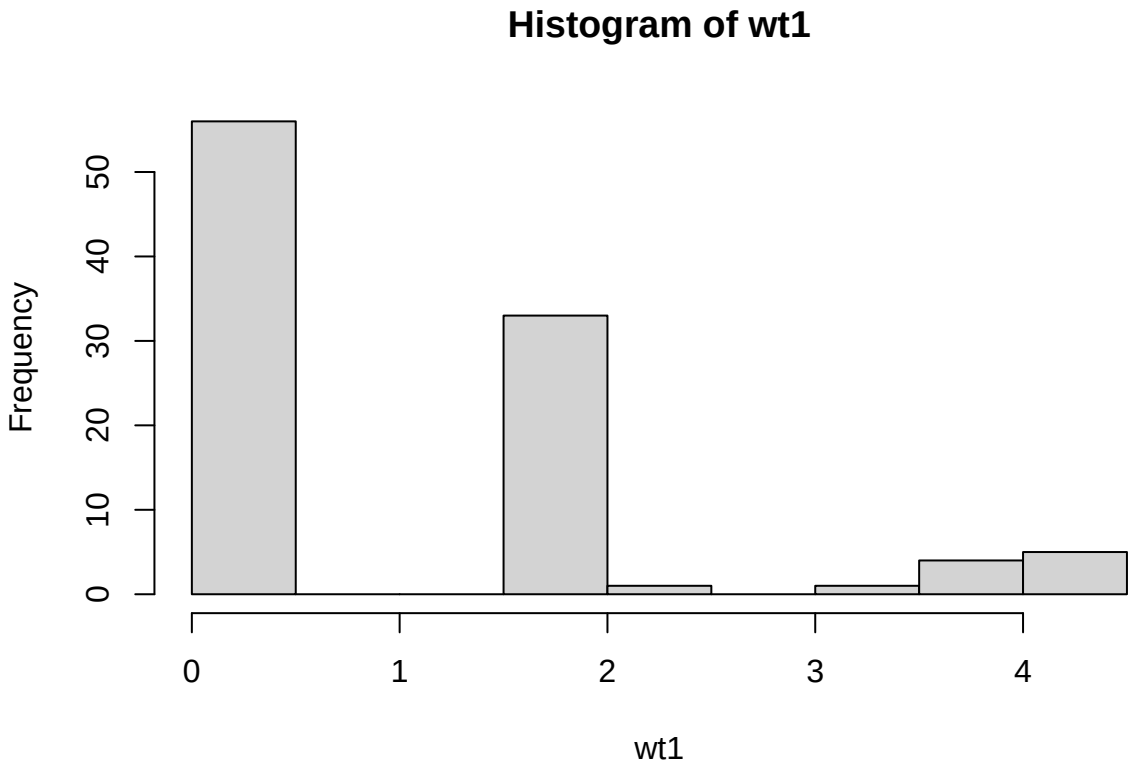

```
summary(prob.0W)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##  0.3774  0.4273  0.4642  0.5600  0.7484  0.7784
```

```
wt1 <- as.numeric(ObsData$A==1)/prob.1W
wt0 <- as.numeric(ObsData$A==0)/prob.0W
summary(wt1)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   0.000   1.004   1.766   4.452
```

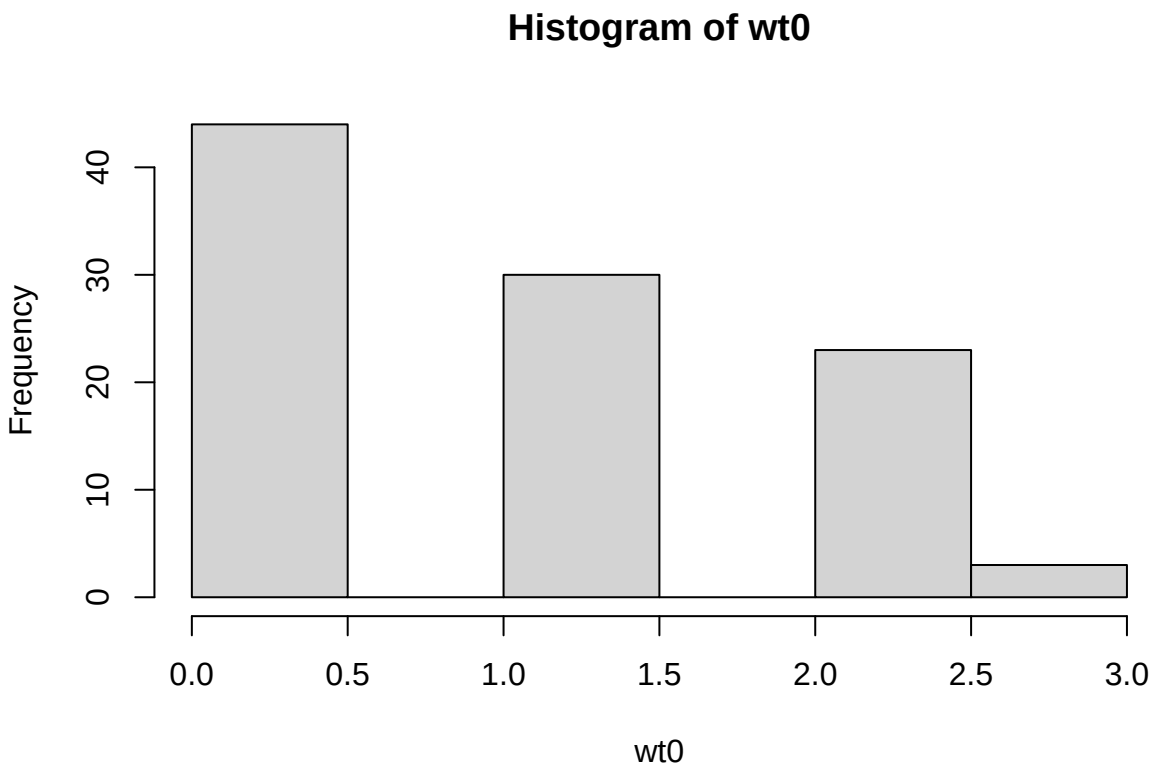
```
hist(wt1)
```




```
summary(wt0)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   0.000   0.000   1.312   0.998   2.074   2.594
```

```
hist(wt0)
```




```
psi.iptw <- mean(wt1*ObsData$Y) - mean(wt0*ObsData$Y)
psi.iptw
```

```
## [1] 0.01002469
```

```
# Modified HT
psi.ht <- mean(wt1*ObsData$Y)/mean(wt1) - mean(wt0*ObsData$Y)/mean(wt0)
psi.ht
```

```
## [1] 0.00916455
```

```
# Unadjusted estimator
wt1.ua <- as.numeric(ObsData$A==1)/mean(ObsData$A == 1)
wt0.ua <- as.numeric(ObsData$A==0)/mean(ObsData$A == 0)
psi.unadj <- mean(wt1.ua*ObsData$Y) - mean(wt0.ua*ObsData$Y)
psi.unadj
```

```
## [1] 0.02922078
```

```
# TMLE estimator
```

SS, IPTW and TMLE estimator with super learner

```r
library("SuperLearner")
SL.library<- c('SL.glm', 'SL.glm.interaction', "SL.step",
               "SL.randomForest","SL.step.forward","SL.stepAIC","SL.mean")
```

```r
run.tmle <- function(ObsData, SL.library){

  #-----------------------------------------
  # Simple substitution estimator
  #-----------------------------------------

  # dataframe X with baseline covariates and exposure
  X <- subset(ObsData, select=c(A, W11, W12, W13, W14,W2))

  # set the exposure=1 in X1 and the exposure=0 in X0
  X1 <- X0 <- X
  X1$A <- 1
  X0$A <- 0

  # Estimate E_0(Y|A,W) with Super Learner
  SL.outcome <- SuperLearner(Y=ObsData$Y, X=X, SL.library=SL.library,
              family="binomial", cvControl=list(V=10))

  # get the expected outcome, given the observed exposure and covariates
  expY.givenAW <- predict(SL.outcome, newdata=ObsData)$pred
  # expected outcome, given A=1 and covariates
  expY.given1W <- predict(SL.outcome, newdata=X1)$pred
  # expected outcome, given A=0 and covariates
  expY.given0W <- predict(SL.outcome, newdata=X0)$pred

  # simple substitution estimator would be
  PsiHat.SS <- mean(expY.given1W - expY.given0W)

  #-----------------------------------------
  # Inverse probability of txt weighting
  #-----------------------------------------

  #  Super Learner for the exposure mechanism  P_0(A=1|W)
  SL.exposure <- SuperLearner(Y=ObsData$A,
                              X=subset(ObsData, select= -c(A,Y,W2)),
                              SL.library=SL.library, family="binomial",
                              cvControl=list(V=10, stratifyCV = TRUE))

  # generate the predicted prob of being exposed, given baseline cov
  probA1.givenW <- SL.exposure$SL.predict
  # generate the predicted prob of not being exposed, given baseline cov
  probA0.givenW <- 1- probA1.givenW

  # clever covariate
  H.AW <- as.numeric(ObsData$A==1)/probA1.givenW - as.numeric(ObsData$A==0)/probA0.givenW

  # also want to evaluate the clever covariate at A=1 and A=0 for all participants
  H.1W <- 1/probA1.givenW
  H.0W <- -1/probA0.givenW
```

```
  # IPTW estimate
  PsiHat.IPTW <- mean(H.AW*ObsData$Y, na.rm = TRUE)


  #----------------------------------------
  # Targeting & TMLE
  #----------------------------------------

  # Update the initial estimator of E_0(Y|A,W)
  # run logistic regression of Y on H.AW using the logit of the esimates as offset

  expY.givenAW <- expY.givenAW - 0.000001

  logitUpdate<- glm( ObsData$Y ~ -1 +offset(qlogis(expY.givenAW)) +
                       H.AW, family='binomial')
  epsilon <- logitUpdate$coef

  # obtain the targeted estimates
  expY.givenAW.star<- plogis( qlogis(expY.givenAW)+ epsilon*H.AW )
  expY.given1W.star<- plogis( qlogis(expY.given1W)+ epsilon*H.1W )
  expY.given0W.star<- plogis( qlogis(expY.given0W)+ epsilon*H.0W )

  # TMLE point estimate
  PsiHat.TMLE<- mean(expY.given1W.star - expY.given0W.star)

  #----------------------------------------
  # Return a list withthe point estimates, targeted estimates of E_0(Y|A,W),
  # and the vector of clever covariates
  #----------------------------------------

  estimates <- data.frame(cbind(PsiHat.SS=PsiHat.SS, PsiHat.IPTW, PsiHat.TMLE))
  predictions <- data.frame(cbind(expY.givenAW.star, expY.given1W.star, expY.given0W.star))
  colnames(predictions) <- c('givenAW', 'given1W', 'given0W')
  list(estimates=estimates, predictions=predictions, H.AW=H.AW, probA1.givenW=probA1.givenW, probA0.giv
}
```

```
set.seed(123)
out <- run.tmle(ObsData = ObsData, SL.library = SL.library)
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

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## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: algorithm did not converge

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred

## Warning: glm.fit: fitted probabilities numerically 0 or 1 occurred
```

```
est <- out$estimates
est
```

```
##     PsiHat.SS PsiHat.IPTW  PsiHat.TMLE
## 1 -0.01154755 0.007405697 -0.001736848
```

CV Superlearner

```
X<- subset(ObsData, select= -Y )
CV.SL.out<- CV.SuperLearner(Y=ObsData$Y, X=X,
                            SL.library=SL.library, family='binomial',
                            cvControl = list(V = 5),
                            innerCvControl = list(list(V =20)))

summary(CV.SL.out)
```

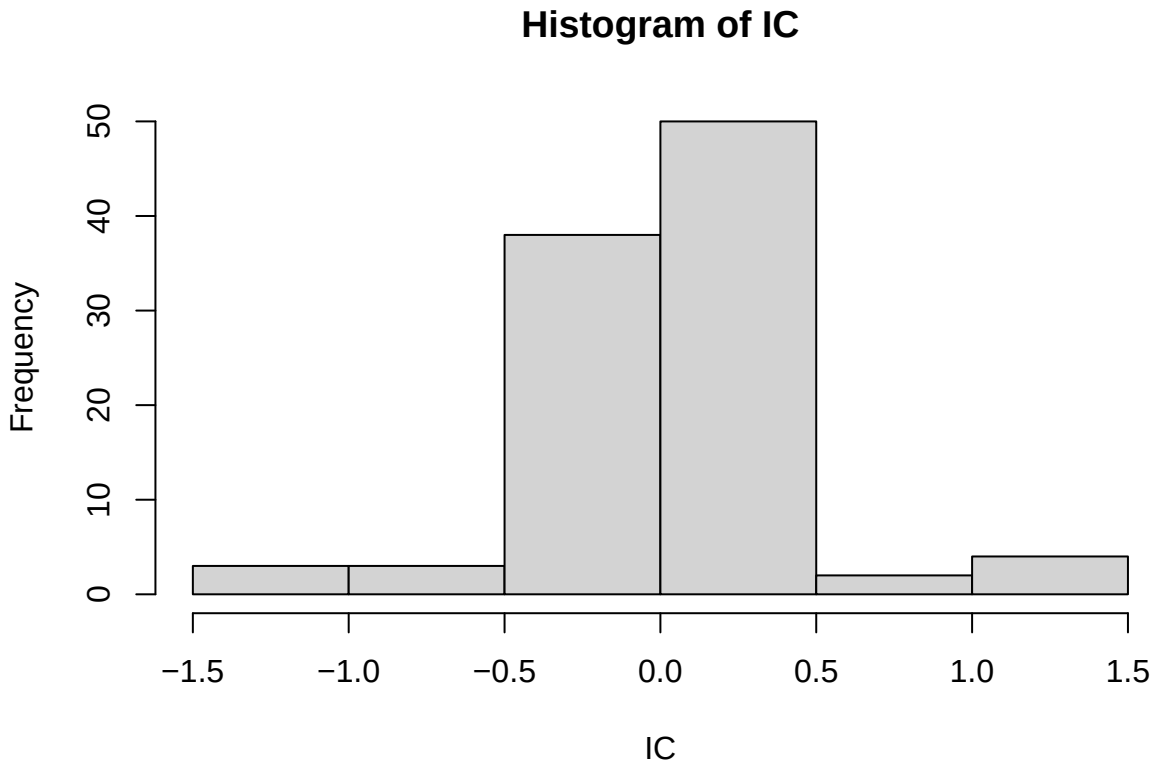
```
##
## Call:
## CV.SuperLearner(Y = ObsData$Y, X = X, family = "binomial", SL.library = SL.library,
##     cvControl = list(V = 5), innerCvControl = list(list(V = 20)))
##
## Risk is based on: Mean Squared Error
##
## All risk estimates are based on V =  5
##
##                Algorithm     Ave       se      Min     Max
##           Super Learner 0.10463 0.023276 0.070131 0.12935
##             Discrete SL 0.11212 0.025168 0.083258 0.12891
##              SL.glm_All 0.12693 0.026052 0.097734 0.16780
##  SL.glm.interaction_All 0.29806 0.045259 0.198018 0.36518
##             SL.step_All 0.11065 0.024642 0.067470 0.13037
##     SL.randomForest_All 0.10702 0.023984 0.083258 0.13574
##     SL.step.forward_All 0.10927 0.025401 0.067470 0.13037
##          SL.stepAIC_All 0.10650 0.025014 0.055156 0.12891
##             SL.mean_All 0.10650 0.025014 0.055156 0.12891
```

Influence Curve

```r
n <- nrow(ObsData)
# clever covariate
H.AW <- out$H.AW
# targeted predictions
expY.AW.star <- out$predictions[,'givenAW']
expY.1W.star <- out$predictions[,'given1W']
expY.0W.star <- out$predictions[,'given0W']
#  point estimate
PsiHat.TMLE <- est$PsiHat.TMLE

# plug-in
IC <- H.AW*(ObsData$Y - expY.AW.star) + expY.1W.star - expY.0W.star - PsiHat.TMLE
summary(IC)
```

```
##        V1
##  Min.   :-1.31959
##  1st Qu.:-0.16434
##  Median : 0.08188
##  Mean   : 0.00000
##  3rd Qu.: 0.12865
##  Max.   : 1.28855
```

```r
hist(IC)
```



```r
# estimate sigma^2 with the variance of the IC divided by n
varHat.IC <- var(IC)/n
varHat.IC
```

```
##              [,1]
## [1,] 0.001566286
```

```r
# standard error estimate
se <- sqrt(varHat.IC)
se
```

```
##            [,1]
## [1,] 0.03957634
```

```r
##### TMLE

# obtain 95% two-sided confidence intervals TMLE:
alpha <- 0.05
round(c(PsiHat.TMLE+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.TMLE+qnorm(alpha/2, lower.tail=F)*se),4)
```

```
## [1] -0.0793  0.0758
```

```r
# calculate the pvalue tmle
2* pnorm( abs(PsiHat.TMLE /se), lower.tail=F )
```

```
##           [,1]
## [1,] 0.9649953
```

```r
####### IPTW

PsiHat.IPTW <- est$PsiHat.IPTW

# obtain 95% two-sided confidence intervals:
round(c(PsiHat.IPTW+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.IPTW+qnorm(alpha/2, lower.tail=F)*se),4)
```

```
## [1] -0.0702  0.0850
```

```r
# calculate the pvalue
2* pnorm( abs(PsiHat.IPTW /se), lower.tail=F )
```

```
##           [,1]
## [1,] 0.8515631
```

```r
####### SS

PsiHat.SS <- est$PsiHat.SS

# obtain 95% two-sided confidence intervals:
round(c(PsiHat.SS+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.SS+qnorm(alpha/2, lower.tail=F)*se),4)
```

```
## [1] -0.0891  0.0660
```

```
# calculate the pvalue
2* pnorm( abs(PsiHat.SS /se), lower.tail=F )
```

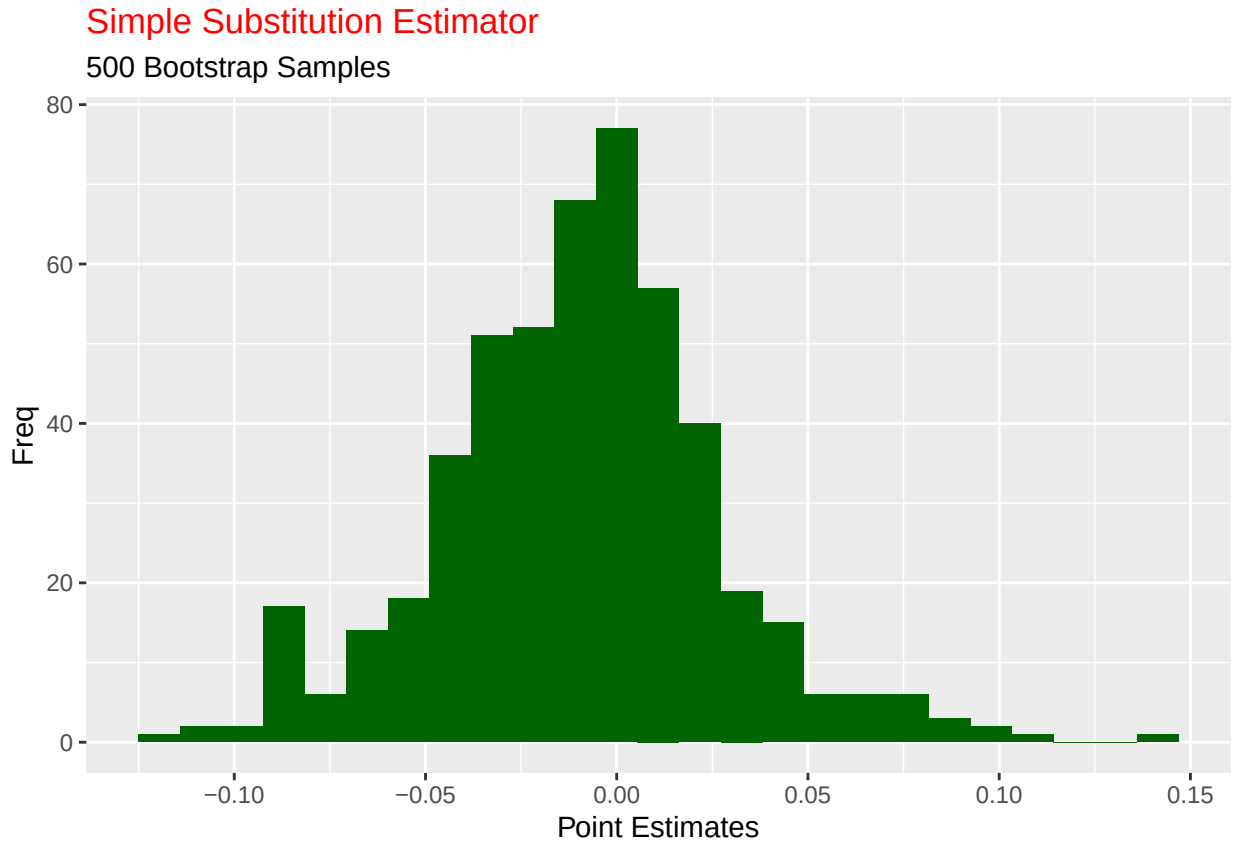
```
##           [,1]
## [1,] 0.7704555
```

Non-parametric bootstrap

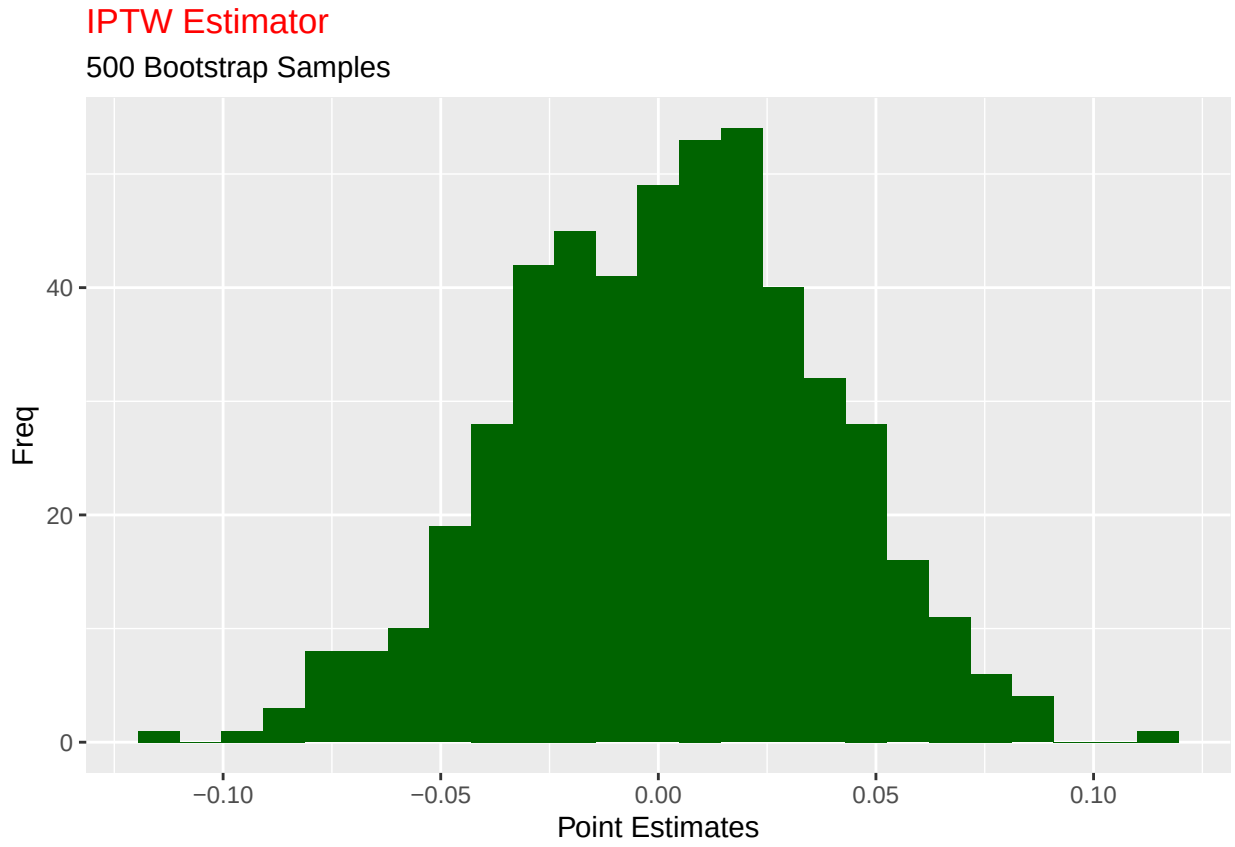
```
load('boot_par.Rdata')

colnames(estimates)<-c("SimpSubs", "IPTW", "TMLE")

summary(estimates)
```

```
##     SimpSubs              IPTW                TMLE          
##  Min.   :-0.121058   Min.   :-0.110710   Min.   :-0.35724  
##  1st Qu.:-0.031503   1st Qu.:-0.022379   1st Qu.:-0.12376  
##  Median :-0.007640   Median : 0.003216   Median :-0.02707  
##  Mean   :-0.009831   Mean   : 0.002566   Mean   : 0.01231  
##  3rd Qu.: 0.010143   3rd Qu.: 0.026642   3rd Qu.: 0.15362  
##  Max.   : 0.140009   Max.   : 0.118716   Max.   : 0.52217  
##                                          NA's   :2
```

```
ggplot(mapping = aes(estimates[,1]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="Simple Substitution Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```

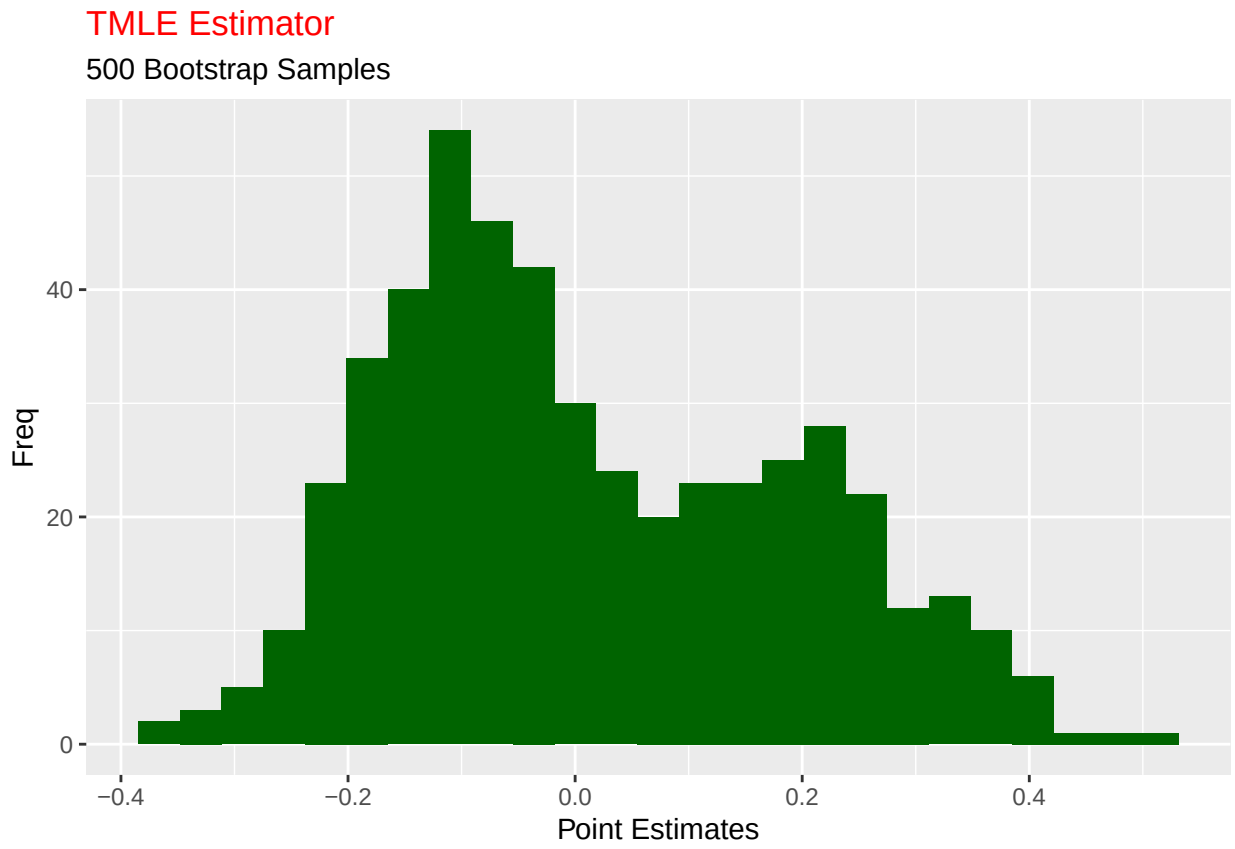



```
ggplot(mapping = aes(estimates[,2]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="IPTW Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```



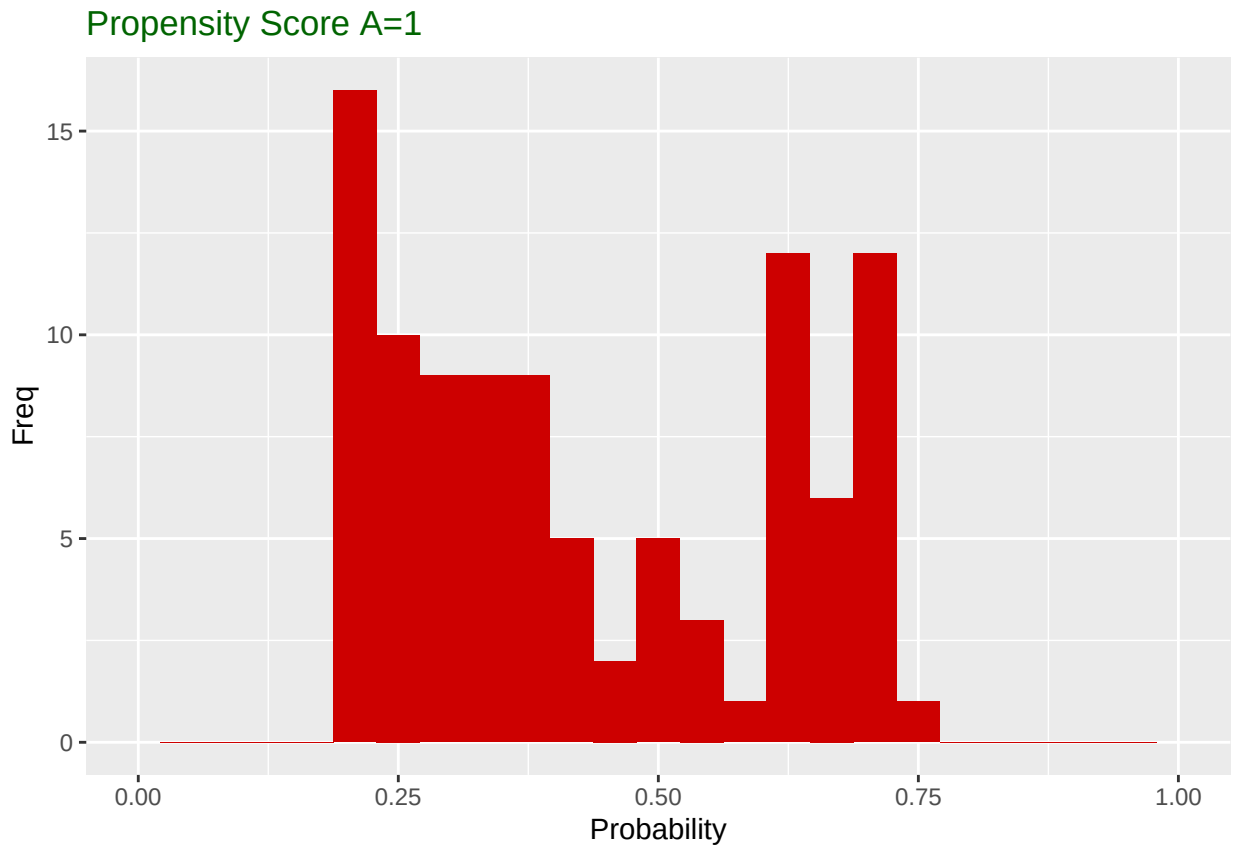

```
ggplot(mapping = aes(estimates[,3]))+
  geom_histogram(fill="dark green",bins = 25)+
  xlab("Point Estimates")+
  ylab("Freq")+
  labs(title="TMLE Estimator",
       subtitle = "500 Bootstrap Samples")+
  theme(plot.title = element_text(colour = "red"))
```

```
## Warning: Removed 2 rows containing non-finite values (stat_bin).
```




```
ggplot(mapping = aes(out$probA1.givenW))+
  geom_histogram(fill="red3",bins = 25)+
  xlab("Probability")+
  ylab("Freq")+
  labs(title="Propensity Score A=1")+
  theme(plot.title = element_text(colour = "dark green"))+
  xlim(0,1)
```

```
## Warning: Removed 2 rows containing missing values (geom_bar).
```




```
weights1 <- as.numeric(ObsData$A==1)/out$probA1.givenW
summary(weights1)
```

```
##        V1
##  Min.   :0.0000
##  1st Qu.:0.0000
##  Median :0.0000
##  Mean   :0.7332
##  3rd Qu.:1.5474
##  Max.   :2.4583
```

```
#---------------------------------
# 95% Confidence intervals assuming a normal dist & via quantiles
#---------------------------------
create.CI <- function(pt, boot, alpha=0.05){
  Zquant <- qnorm(alpha/2, lower.tail=F)
  CI.normal <- c(pt - Zquant*sd(boot,na.rm = TRUE),
                 pt + Zquant*sd(boot,na.rm = TRUE) )
  CI.quant  <- quantile(boot, prob=c(0.025,0.975) ,na.rm=TRUE)
  out <- data.frame(rbind(CI.normal, CI.quant))
  colnames(out) <- c('CI.lo', 'CI.hi')
  out
}
```

```r
# IMPORTANT - POINT OF CONFUSION FOR PAST STUDENTS
# The point estimate 'pt' is from the original dataset

# Simple Subs - note the bias because of misspecified regression? Will it converge fast enough?
est$PsiHat.SS
```

```
## [1] -0.01154755
```

```r
create.CI(pt=est$PsiHat.SS, boot=estimates[,"SimpSubs"])
```

```
##                 CI.lo      CI.hi
## CI.normal -0.08264802 0.05955292
## CI.quant  -0.08675253 0.06975379
```

```r
# IPTW
est$PsiHat.IPTW
```

```
## [1] 0.007405697
```

```r
create.CI(pt=est$PsiHat.IPTW, boot=estimates[,"IPTW"])
```

```
##                 CI.lo      CI.hi
## CI.normal -0.06291296 0.07772436
## CI.quant  -0.07220831 0.06989418
```

```r
# TMLE
est$PsiHat.TMLE
```

```
## [1] -0.001736848
```

```r
create.CI(pt=est$PsiHat.TMLE, boot=estimates[,"TMLE"])
```

```
##                CI.lo     CI.hi
## CI.normal -0.3484276 0.3449539
## CI.quant  -0.2599113 0.3679674
```

```r
# Compare to IC estimate
c(PsiHat.TMLE+qnorm(alpha/2, lower.tail=T)*se,
  PsiHat.TMLE+qnorm(alpha/2, lower.tail=F)*se)
```

```
## [1] -0.07930504  0.07583135
```