Ballistics: Application of Conversation Laws

Cameron Stark
Barbara De Figueiredo
Cristian D. Abades
Zachary Chuang

PS-253 Section 12, Benedict Pineyro
Department of Physical Sciences,
Embry-Riddle Aeronautical University, Daytona Beach, Fl.
September, 2018

To determine the velocity of the projectile we employed two methods, the first being a ballistics pendulum and the other being a spring firing system from a constant height. The first method we performed 5 trials where we calculated the change in initial height with the final height after launch, the velocity we calculated came out to 6.23 m/s. The second method we performed 5 trials where we calculated the distance traveled from a fixed vertical height, the velocity we calculated came out to 6.13 m/s. The percent difference of these two methods velocities came out to 1.69%, which makes sense, because the values found were calculated and measured differently.

Section I: Background

By performing this experiment, we got to find velocity by employing two different methods which allowed us to explore and understand what uncertainty in measurements could be and how it could affect experiments trying to find the same answer. The experiments' objective was to take multiple measurements and apply them to the equations of the conservation of energy and momentum, in order to find the velocity of the ball at launch. The experiments provides an example of what engineers and scientists do in real life where when creating a new product or prototype, it is tested in multiple ways of checking the same variable in order to account for any discrepancy or error that may have been involved in an experiment and that the expected results match the experimental results in all situations.

Section II: Theory and Procedure

The experiment that was preformed made use of spring-loaded fire mechanism that was fired by use of a trigger where the projectile (in this case a solid metal ball) was attached to a firing rod. The entire mechanism was attached to a solid base, which was attached to the surface of the table. The base also had a pendulum set where the ball would get caught in it and would travel up a rigid surface and get caught at the apex of the pendulum motion. The experiment was only concerned with the velocity of the ball so the spring is not of concern. In order to make use of the measurements we acquire in the first method (ballistics pendulum) we must make use of the following equation [1]. This equation came from the combination of the conservation of energy and conservation of momentum.

$$V = \frac{m+M}{m}\sqrt{2g\Delta h} \quad (1)\ [1]$$

This equation is derived from the equations of conservation of momentum, which states that momentum (p) is equal to product of mass (m) and velocity (v).

$$\vec{p} = m\vec{v} = (m+M)\vec{v} = \vec{p}\ (2)\ [1]$$

For this method we must break up the mass variable into m (for the mass of the ball) and M (for the mass of the pendulum) since the pendulum is an active part of the experiment. To perform this first method of attaining the velocity of the ball, we must measure the initial resting height of the pendulum for the platform, and then fire the ball into the ballistics pendulum for five trials. Once fired we must measure the new height that the pendulum stops at on the arresting platform. Once we have five final height measurements, subtract each by the initial to get the change in height, and then apply it to equation 1. To find the velocity of the ball using the second method, we use the following equation.

$$V = x\sqrt{\frac{g}{2y}}\ (3)\ [1]$$

To obtain the velocity by this method, we need to know some constants such as the vertical height (y) of the ball will fall which will be measured from the floor to the platform it is being launched from, and the value we will be measuring will be the horizontal distance the ball will travel which is (x). Fire the ball, and measure the distance traveled for five trials. Once five trials have been reached apply equation 3 to each horizontal distance and the average the five velocities.

Section III: Results

The Measurements and results of the in total 10 trials, 5 for method 1 and 5 for method 2 are as follows. We found

the constants for the experiment to be the following

Variable	*Value*
m (mass of ball)	0.0686kg
M (mass of pendulum	0.276kg
y (vertical distance)	0.895m
h (initial height)	0.0575m
g (gravity)	9.8 m/s^2 [1]

The Following are the results and calculations for the method 1:

Method 1 Measurements

Trial	*Height Measured m*
1	0.133
2	0.137
3	0.137
4	0.14
5	0.133

Once we have measure for 5 trials we take the initial height and subtract it from the measured height.

$$h_f - h_i = \Delta h \ (7)$$

Equation	*Δh*
$0.133 - 0.0575$	0.0755
$0.137 - 0.0575$	0.0795
$0.137 - 0.0575$	0.0795
$0.14 - 0.0575$	0.0825
$0.133 - 0.0575$	0.075

The above calculations are of the 5 changes in height of the five trials. Next steps were to find the standard deviation of the changes and then add that to the average change in height

The next steps are to find the initial velocity for each trial and add those up, this is done with equation 1.

Velocity #	*Velocity m/s*
1	6.118
2	6.277
3	6.277
4	6.395
5	6.118

From these values we can extrapolate the average calculated velocity for method 1:

Method 2 Measurements

To begin method 2, we need to first find a calculated (expected) horizontal travel distance with the calculated velocity method 1. To do this we will take equation 3 and solve for x.

Our expected horizontal range based on our calculations is 2.688 meters, so we will place a target around that point to measure how accurate our calculations were to the actual velocity.

Expected	*Actual*	*Difference*
2.688	2.59	0.099
2.688	2.63	0.059
2.688	2.57	0.119
2.688	2.59	0.099
2.688	2.61	0.079

The next steps are to take the measured horizontal distances and calculate the velocities by applying it to equation 3 for each trial.

Velocity #	*Velocity*
1	6.060
2	6.154
3	6.013
4	6.060
5	6.106

After we find all of the velocities we find the average velocity, then the standard deviations of the velocities then add those 2 values to get the total mean velocity.

From this point all we need to find is the percent difference between the two calculated velocities with equation 6

The percent different in the two calculated velocities is 1.69% showing that both methods arrived at almost the value my measuring different values.

Section IV: References

[1] Department of Physical Sciences. “Ballistics: Application of Conservation Laws.” Daytona Beach: Embry-Riddle Aeronautical University, 2016. PDF file.

Section V: Calculations

$$\Delta h_{avg} = \frac{.075+.0795+.0795+.0825+.075}{5}\,[1]$$

$$\Delta h_{avg} = 0.07984$$

$$V_1 = \frac{0.0686 + 0.276}{0.0686}\sqrt{2 * 9.8 * 0.0755}$$

$$v_{1\,avg} = 6.237\ \frac{m}{s^2}$$

$$x = \frac{v}{\sqrt{\frac{g}{2y}}}$$

$$x = \frac{6.237}{\sqrt{\frac{9.8}{2(.895)}}}$$

$$x = 2.688\,m$$

$$\sigma = \sqrt{\frac{.075+.0795+.0795+.0825+.075}{5}}\,[1]$$

$$\sigma = 0.001341614$$

$$v = 2.59\sqrt{\frac{9.8}{2\,(.895)}} = 6.060$$

$$\%diff = \frac{(6.237 - 6.1322)}{\frac{6.237 + 6.1322}{2}}\ x\ 100$$

$$\%diff = 1.69\%$$

$$v_{avg} = 6.079$$

$$\sigma = \sqrt{\frac{6.060 + 6.154 + 6.0130 + 6.060 + 6.106}{5}}$$

$$\sigma = 0.0533$$

$$v_{tot} = 6.1322$$

Pendulum Launch	Height from Platform to Screw cm	Change in Height cm	Pendulum Initial Velocity
1	13.3	0.0755	6.117817125
2	13.7	0.0795	6.277787043
3	13.7	0.0795	6.277787043
4	14	0.0825	6.395139013
5	13.3	0.0755	6.117817125
	Average Heigth in Meters	0.0785	6.23726947
	Standard Deviation of Height	0.001341641	
	Total Mean Change in Height	0.079841641	

Range Launch	Horizontal Distance Traveled M	Diffence in Expected to actual	Velocity based experimental x and y
1	2.59	0.098754928	6.060190642
2	2.63	0.058754928	6.15378432
3	2.57	0.118754928	6.013393803
4	2.59	0.098754928	6.060190642
5	2.61	0.078754928	6.106987481
		Average Velocity	6.078909377
		Standard Deviation of Velocity	0.053356606
		Total Mean Velocity	6.132265983

Mass of Ball kg	0.0686
Vertical Distance Platform to Floor m	0.895
Mass of Pendulum g	0.2764
Height from platform to Screw at rest cm	5.75
Pendulum Velocty m/s^2	6.291261564
Esitmation of Distance m	2.688754928

Percent Difference Between the two methods	1.697775758