

Course: CEC300

Department of Electrical, Computer, Software, and Systems Engineering

FLORIDA
ARIZONA
WORLDWIDE

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Navigation

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Precision vs Accuracy

- Precision and accuracy are not the same.
- Precision refers to how small an area coordinates can be defined or plotted.
 - GPS lat/long coordinates can be defined to 1/10 of a second.
- Accuracy refers to how closely a GPS receiver can calculate its position relative to its true location.
 - GPS accuracy can vary from a few millimeters to several meters.

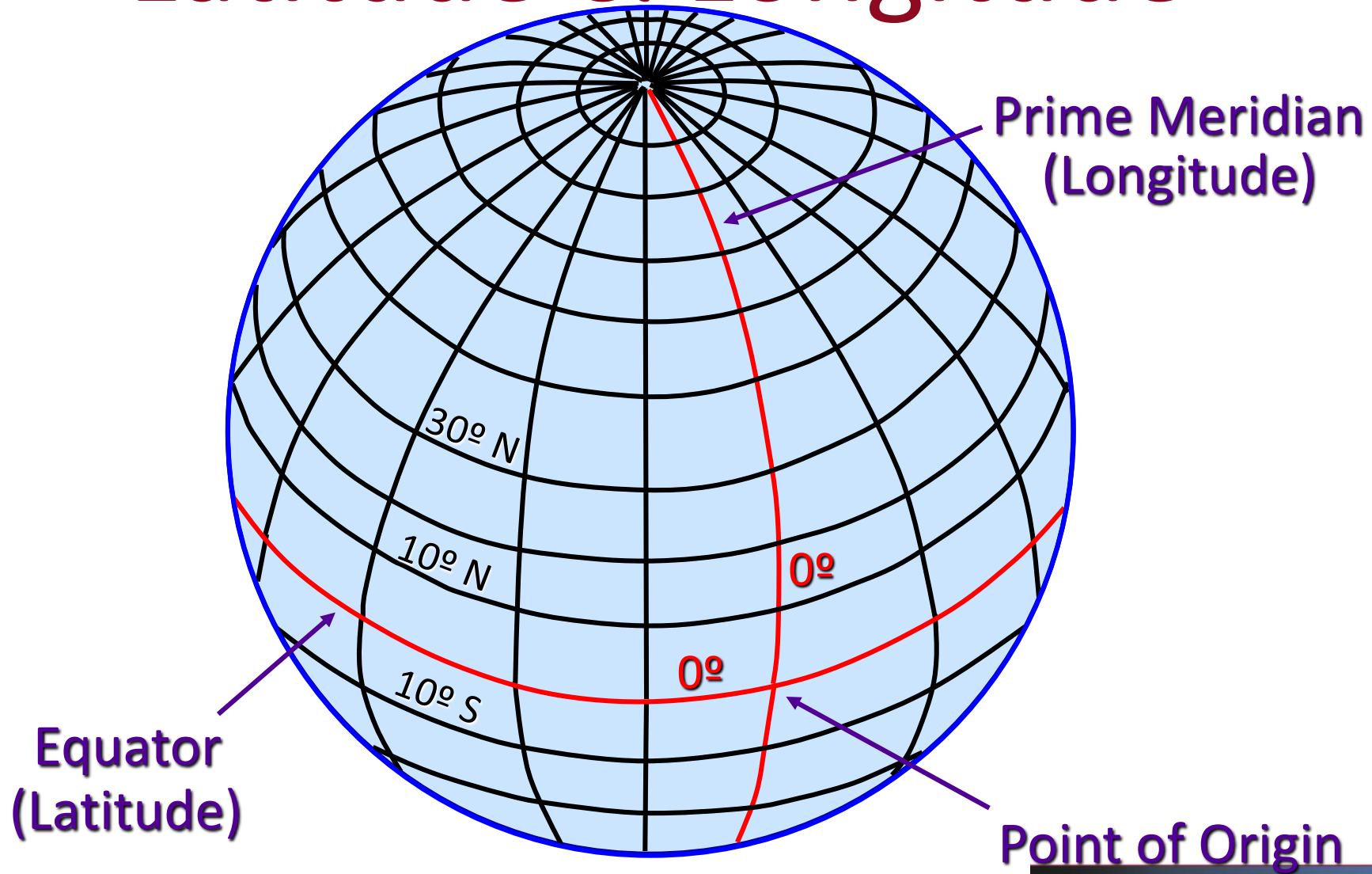
Latitude & Longitude



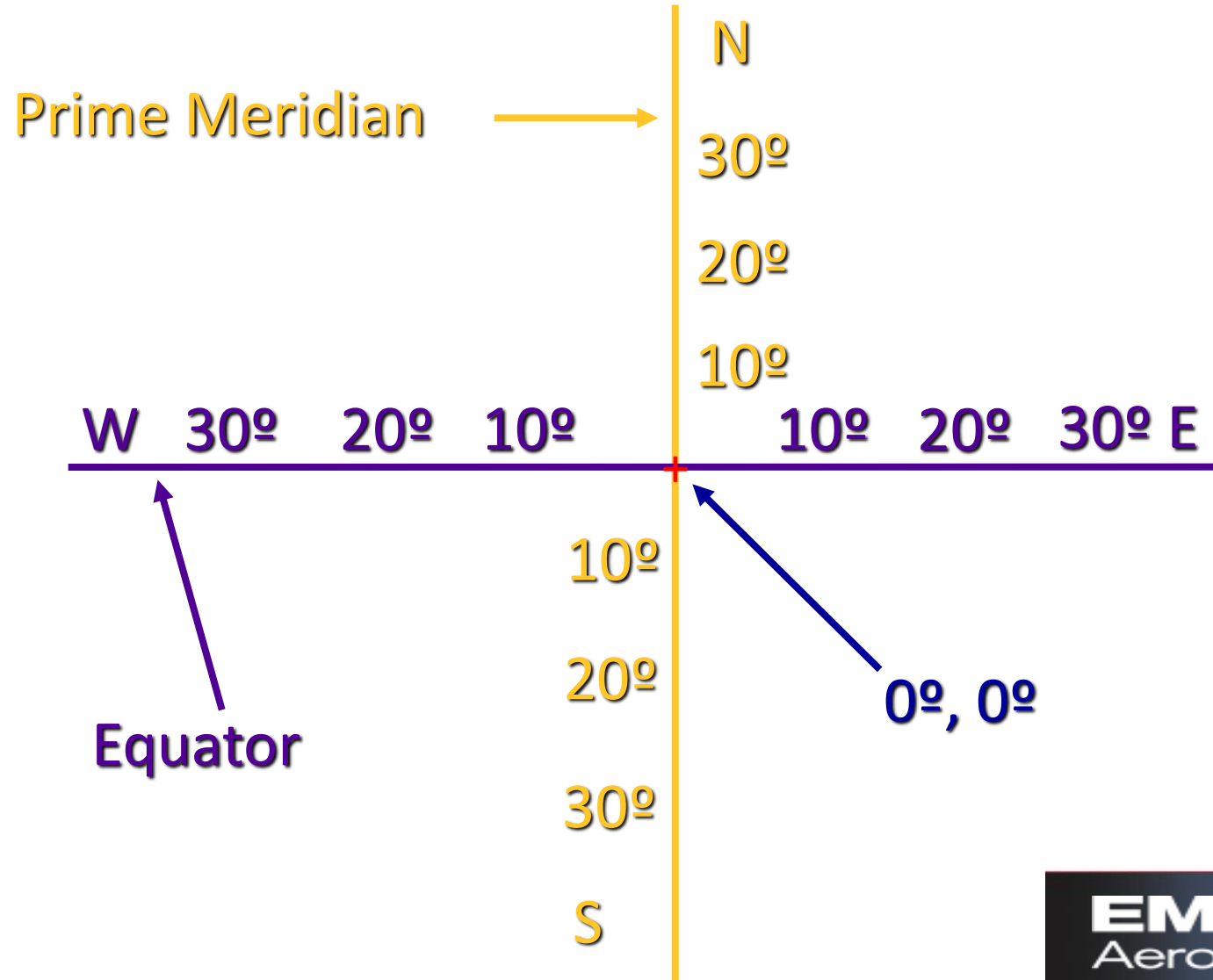
Latitude & Longitude

- A geographic (spherical) coordinate system.
- Angular coordinates are perfectly suited to the ellipsoidal shape of the earth.
 - Coordinates are expressed in degrees, minutes and seconds or decimal degrees
- Position coordinates are based on an angular distance from a known reference point.
 - That reference point is where the Prime Meridian and equator intersect.
- Lat/long is the predominant coordinate system used for nautical and aeronautical navigation.

Latitude & Longitude



Latitude & Longitude



Latitude

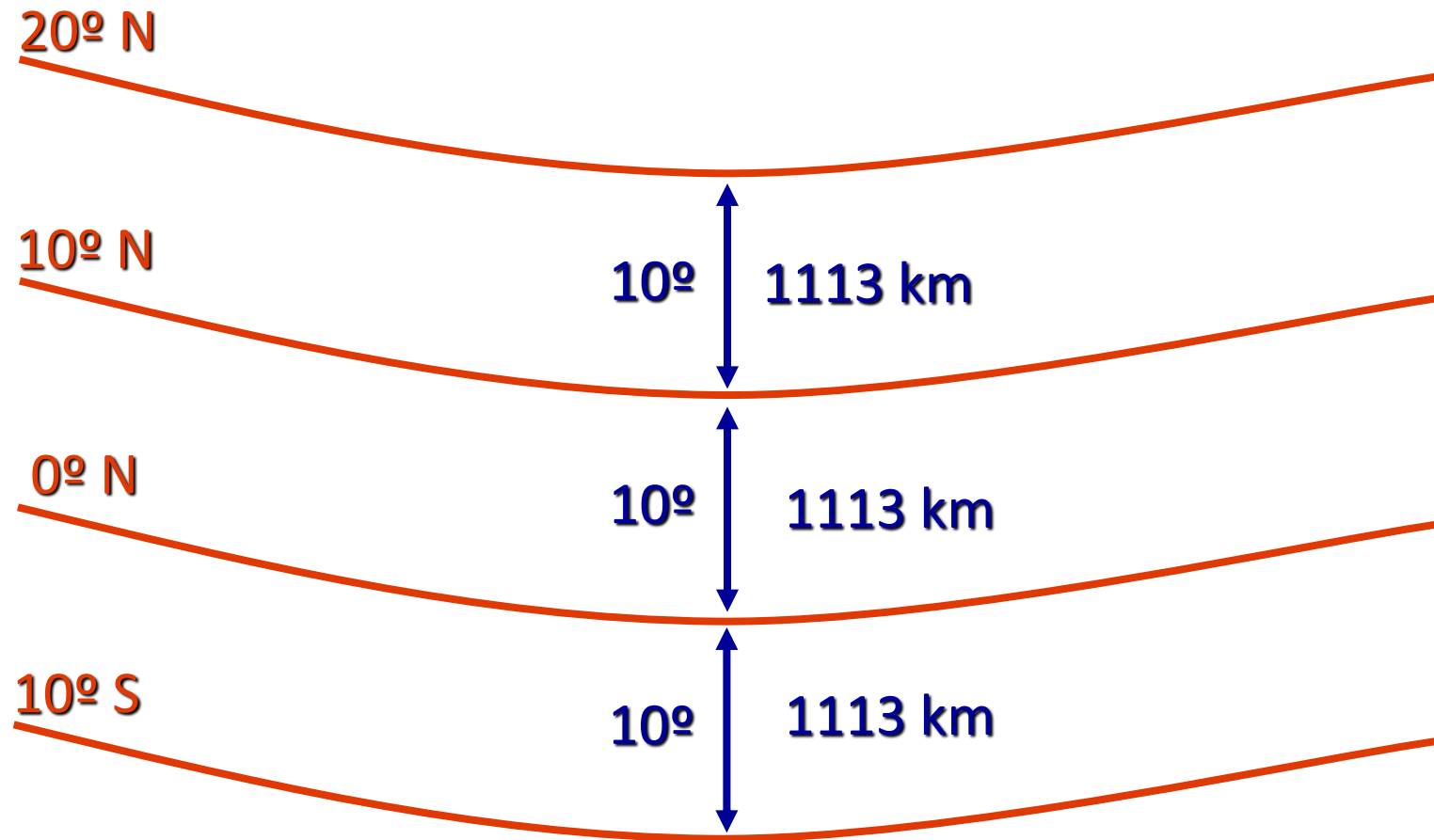
- Latitude is comprised of parallels, which are equally spaced circles around the earth paralleling the equator.
- Parallels are designated by their angle north or south of the equator (10° , 20° , etc) .
- The equator is 0° latitude, and the north and south poles are at 90° angles from the equator.
- The linear distance between parallel (latitude) lines never changes, regardless of their position on the earth.

The earth

The circumference of Earth at the equator is **about 40,075 km**
but from pole-to-pole — Earth circumference is only **40,008 km**

One degree of latitude = $40008 \text{ km} / 360 = 111.3 \text{ km}$

Parallels of Latitude



Longitude

- Longitude is comprised of meridians that form one-half of a circle, or plane.
- Meridians are designated by their angle west or east of the prime meridian.
- The prime meridian is designated 0° and extends from the north pole to the south pole through Greenwich, England.
 - Meridians are angled, and do not parallel each other.
- The linear distance between one degree of longitude at the equator is approximately 69 statute miles.
- The linear distance between one degree of longitude at the arctic circle is only about 26 statute miles.

One degree of longitude depends on the location on earth.

At the equator, one degree of longitude = $40075/360 = 111.3\text{km}$

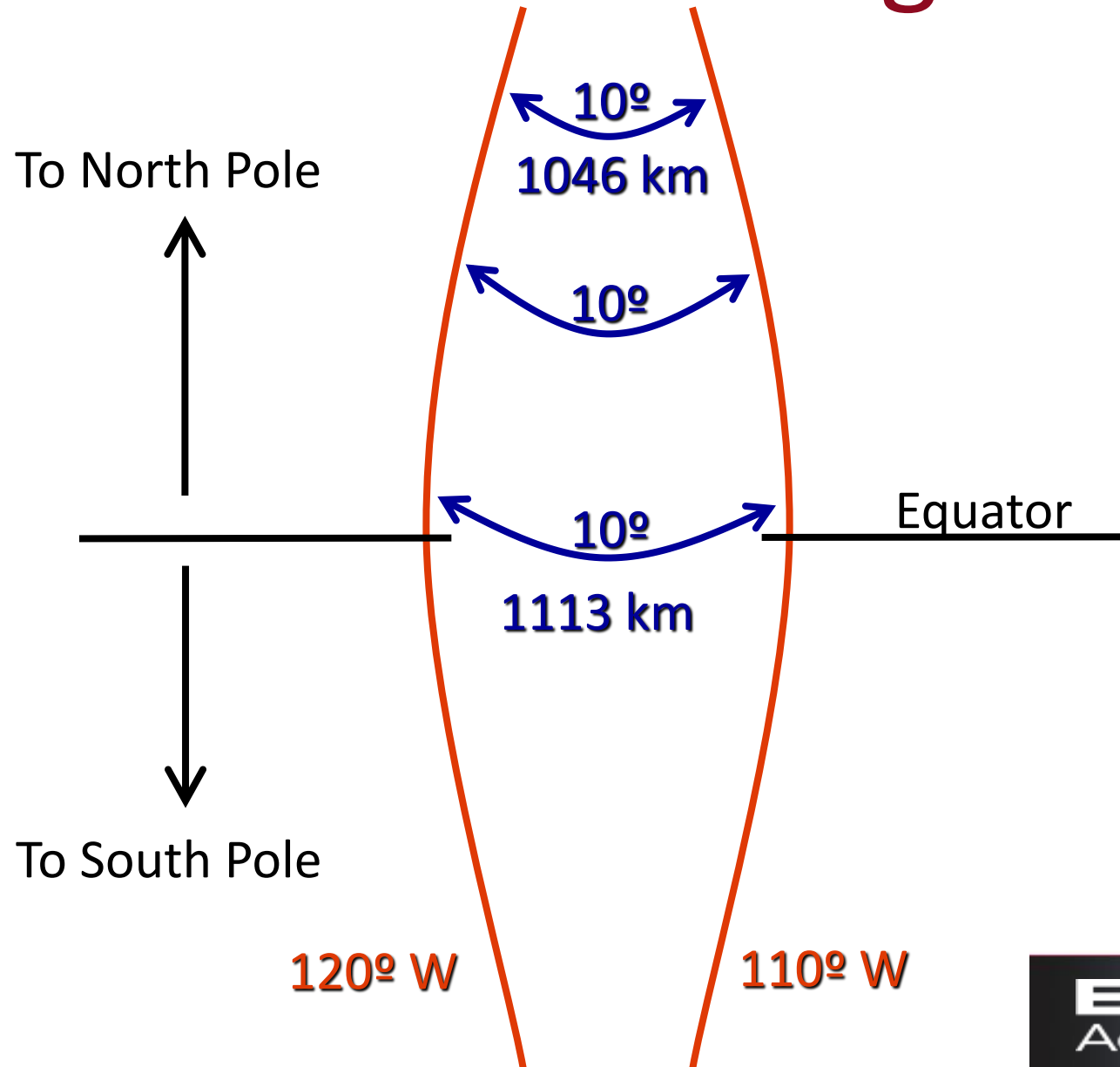
But the distance varies with latitude

One degree of longitude = $111.3\text{km} \cos(\text{latitude})$

At 20 degrees of latitude, one degree of longitude = $111.3\text{km} \cos(20 \text{ deg})$
= 104.6km

At 70 degrees of latitude, one degree of longitude = $111.3\text{km} \cos(70 \text{ deg})$
= 38km

Meridians of Longitude



Decimal degrees and Degrees, minutes seconds

There are 60 minutes in a degree

There are 60 seconds in a minute

There are 3600 seconds in a degree

$45^{\circ}10'30''$

$$=45 + 10/60 + 30/3600 = 45 + 0.167 + .008 = 45.175$$

But remember 1 degree is 111.3km so one second is 1/3600 of a degree is
.039km or 39 meters.

We will need more digits of precision for 1 meter accuracy

Navigation

I start at 29.3000 , -81.1000

I travel at heading south for 100 meters.

What is my new location?

1 degree of latitude = 111.3km

$100\text{m}/111.3\text{km} = 0.898 \times 10^{-4}$ degrees

New location is $29.3000 - 0.000898, -81.1000 = 29.2991, -81.1000$

I start at 29.3000 , -81.1000

I travel at heading west for 100 meters.

What is my new location?

1 degree of longitude = $111.3\text{km} * \cos(29.3) = 97.1\text{km}$

$100\text{m}/97.1\text{km} = 1.03 \times 10^{-3}$ degrees

New location is 29.3000,-81.1000-.00103 = 29.3, -81.1010

I start at 29.3000 , -81.1000
I travel at heading north east (45 degrees) for 100 meters.
What is my new location?
Traveled $100\text{m} \cdot \cos(45) = 70.7\text{m}$ east
Traveled $100\text{m} \cdot \sin(45) = 70.7\text{m}$ north
1 degree of latitude = 111.3km
1 degree of longitude = 97.1km
 $70.7\text{m} / 111.3\text{km} = 6.35 \times 10^{-4}$ degrees north
 $70.7\text{m} / 97.1\text{km} = 7.28 \times 10^{-4}$ degrees east
New location is $29.3000 + .000635, -81.1000 + .000728$
 $= 29.300635, -81.09927$

Navigation using the accelerometer

In this example we will assume we have a quadrotor flying in heading hold.

Accelerations in the x-axis result in motion only on the x-axis

Accelerations in the y-axis result in motion only on the y-axis

Start at 0,0 with an initial velocity of 0 and an initial acceleration of 0

At $t=0.1$ seconds the accelerometer reads $0.5g$ in the x direction and $1g$ in the y direction.

Calculating the velocity for the time interval

Suppose the vehicle is at rest and is subjected to an acceleration of $1g$ for one second.

The acceleration is 9.8m/s^2 .

After one second the vehicle velocity will have increased by 9.8 m/s .

The acceleration will produce a linear increase in velocity from the starting velocity to the final velocity

Calculate the velocity for the time interval

The velocity for any time interval can be given by

$$V_{\text{new}} = V_{\text{old}} + a * dt$$

In this case,

$$V_x(.1) = V_x(0) + a_x(0) * dt$$

$$V_x(.1) = 0 + 0.5 * 9.8 * 0.1 = 0.49 \text{ m/s}$$

$$V_y(.1) = 0 + 1.0 * 9.8 * 0.1 = 0.98 \text{ m/s}$$

(dt= 0.1 s)

Calculating the position

The vehicle velocity was not constant. It increased linearly over the time interval.

To compute the distance traveled we can use the average velocity.

$$D_{\text{new}} = D_{\text{old}} + 0.5 * (v(\text{new}) + v(\text{old})) * dt$$

$$D_x(.1) = 0 + 0.5 * 0.49 * 0.1 = .0095 \text{ m}$$

$$D_y(.1) = 0 + 0.5 * 0.98 * .1 = 0.019 \text{ m}$$

Suppose the accelerations stay the same

$$V_x(.2) = V_x(.1) + a_x(.2) * dt = 0.49 + 0.5 * 9.8 * .1 = 0.98 \text{ m/s}$$

$$V_y(.2) = V_y(.1) + a_y(.1) * dt = 0.98 + 1.0 * 9.8 * .1 = 1.96 \text{ m/s}$$

$$D_x(.2) = D_x(0.1) + 0.5 * (V_x(.2) + V_x(.1)) * dt = 0.0095 + 0.5 * (0.98 + 0.49) * 0.1 = 0.083 \text{ m}$$

$$D_y(.2) = D_y(0.1) + 0.5 * (V_y(.2) + V_y(.1)) * dt = 0.019 + 0.5 * (1.96 + 0.98) * 0.1 = 0.166 \text{ m}$$