速度Verlet算法的计算公式如下

$$\boldsymbol{r}_i^{(n+1)} = \boldsymbol{r}_i^{(n)} + h\boldsymbol{v}_i^{(n)} + \boldsymbol{F}_i^{(n)}h^2/2m$$
$$\boldsymbol{v}_i^{(n+1)} = \boldsymbol{v}_i^{(n)} + h(\boldsymbol{F}_i^{(n+1)} + \boldsymbol{F}_i^{(n)})/2m$$

用速度Verlet算法计算一维谐振子

$$\begin{cases} F(x) = -kx \\ x(t=0) = 1.0 \\ v(t=0) = 0.0 \end{cases}$$

取$k = 1, m = 1, h = 0.1$

In [1]:

```
import numpy as np

def F(x):
    return -x

r0 = 1.0
v0 = 0.0

h = 0.1

r_list = [r0]
v_list = [v0]
F_list = [F(r0)]

t_tot = 20.0

N = int(t_tot/h)

for i in range(N):
    r_list.append(r_list[i] + h*v_list[i] + F_list[i]*h*h/2)
    F_list.append(F(r_list[i+1]))
    v_list.append(v_list[i] + h*(F_list[i+1] + F_list[i])/2)
```

Verlet速度算法的结果与解析解$r(t) = \cos(t)$比较

In [2]:

```
import matplotlib.pyplot as plt

t = np.linspace(0, t_tot, len(r_list))
x = np.cos(t)

plt.figure(figsize=(10,5), dpi=100)
plt.xlabel('t')
plt.ylabel('r')
plt.plot(t, x, color='blue', label='$\cos{(t)}$')
plt.scatter(t, r_list, color='red', label='Velocity Verlet')
plt.legend()
plt.show()
```