1.自由落体运动

求解微分方程

In[∘]:=
```
sol = DSolve[{x'[t] == v[t], v'[t] == -g - b/m v[t], x[0] == h, v[0] == v0},
  {x[t], v[t]}, t] // ExpandAll
```
求解微分方程　展开全部

Out[∘]= $\left\{\left\{v[t]\to -\frac{g\,m}{b}+\frac{e^{-\frac{b\,t}{m}}\,g\,m}{b}+e^{-\frac{b\,t}{m}}\,v0,\ x[t]\to h+\frac{g\,m^2}{b^2}-\frac{e^{-\frac{b\,t}{m}}\,g\,m^2}{b^2}-\frac{g\,m\,t}{b}+\frac{m\,v0}{b}-\frac{e^{-\frac{b\,t}{m}}\,m\,v0}{b}\right\}\right\}$

终端速度

In[∘]:=
```
First[Flatten[sol]] /. b t/m → Infinity
```
第一个　压平　无穷大

Out[∘]= $v[t]\to -\frac{g\,m}{b}$

In[∘]:=
```
Flatten[sol][[1]] /. b t/m → Infinity
```
压平　无穷大

Out[∘]= $v[t]\to -\frac{g\,m}{b}$

单位选取

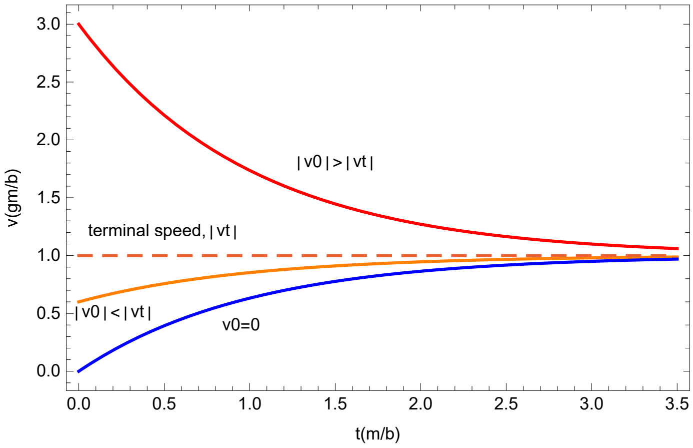
In[∘]:=
```
vv[t_] = (v[t] /. Flatten[sol][[1]]) //. {g m/b → 1, b/m → 1}
```
压平

Out[∘]= $-1+e^{-t}+e^{-t}\,v0$

速度图像

In[∘]:=
```
Plot[Evaluate[{Abs[vv[t]] /. v0 → -3, Abs[vv[t]] /. v0 → -0.6, Abs[vv[t]] /. v0 → 0 , 1}],
  {t, 0, 3.5}, PlotRange → All, Frame → True, FrameLabel → {"t(m/b)", "v(gm/b)"},
  PlotStyle → {{Red}, {Orange}, {Blue}, {Dashing[{0.02}]}},
  Prolog → {Text["v0=0", {0.95, 0.4}], Text["|v0|<|vt|", {0.2, 0.5}],
    Text["|v0|>|vt|", {1.5, 1.8}], Text["terminal speed,|vt|", {0.5, 1.2}]}]
```
绘图 计算 绘对值 绘对值 绘对值 绘制范围 全部 边框 真 边框标签 绘图样式 红色 橙色 蓝色 虚线线段配置 绘制主⋯ 文本 文本 文本 文本

Out[∘]=



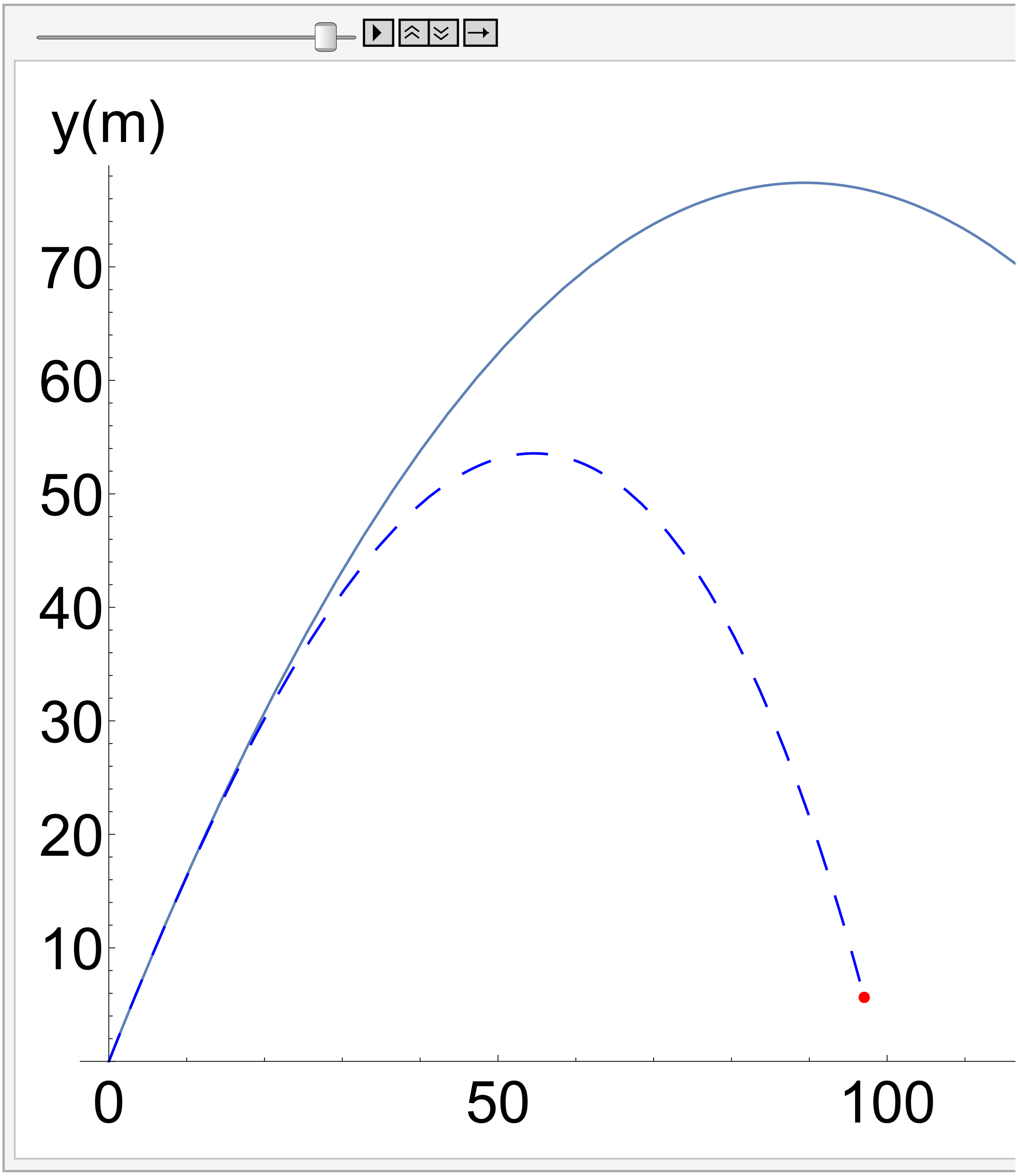
2.抛物运动

In[1]:=
```
ClearAll["Global`*"]
清除全部
projectile[v0_ /; 0 < v0 ≤ 80, θ0_ /; 30 < θ0 ≤ 85] := Module[
                                                      模块
  {k = 5.2 × 10^-3, g = 9.81, vx0, vy0,
   x, y, R, H, tmax, T, pathWithoutAirResistance, sol, tab},
  vx0 = v0 Cos[θ0 / 180 * Pi] // N;
           余弦             圆周率 数值运算
  vy0 = v0 Sin[θ0 / 180 * Pi] // N;
           正弦             圆周率 数值运算
  R = (2 vx0 vy0)/g;
  H = vy0^2/(2 g);
  T = (2 vy0)/g;
  pathWithoutAirResistance = Plot[vy0/vx0 x - 1/2 g/vx0^2 x^2, {x, 0, R}, PlotRange → {0, H}];
                             绘图                                         绘制范围

  sol = NDSolve[
        数值求解微分方程组
    {x''[t] == -k Sqrt[x'[t]^2 + y'[t]^2] x'[t], y''[t] == -k Sqrt[x'[t]^2 + y'[t]^2] y'[t] - g,
     x[0] == 0, y[0] == 0, x'[0] == vx0, y'[0] == vy0}, {x, y}, {t, 0, T}];
  x[t_] = x[t] /. sol[[1, 1]];
  y[t_] = y[t] /. sol[[1, 2]];
  tmax = t /. FindRoot[y[t], {t, T}];
              求根
  tab = Table[
        表格
    Show[
    显示
     pathWithoutAirResistance,
     Graphics[{AbsolutePointSize[7], Red, Point[{x[(tmax / 32) i], y[(tmax / 32) i]}]}],
               绝对点大小               红色  点
     ParametricPlot[{x[t], y[t]}, {t, 0, (tmax / 32) i + 0.0001},
     绘制参数图
      PlotStyle → {Blue, Dashing[{0.02, 0.02}]}], PlotRange →
      绘图样式      蓝色   虚线线段配置                绘制范围
      {{-0.01, R}, {-0.01, 1.02 H}}, ImageSize → 1000, LabelStyle → {FontSize → 36},
                                      图像尺寸           标签样式       字体大小
     AxesLabel → {"x(m)", "y(m)", AspectRatio → Automatic}], {i, 0, 32}];
                                  宽高比         自动
  Export[NotebookDirectory[] <> "projectile.gif", tab];
  导出    当前笔记本的目录
  ListAnimate[tab]
  列表帧动画
 ];
```

In[5]:=
```
projectile[45, 60]
```

Out[5]=



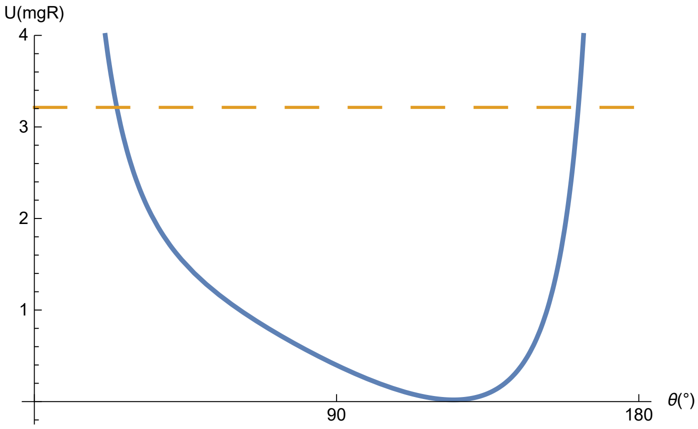
3. 钟摆运动

有效势

In[∘]:=
```
effectivePotential[θ0_ /; 0 ≤ θ0 ≤ 180, θdot0_, φdot0_, opts___Rule] :=
 Module[{pφ},
 模块
  pφ = Sin[θ0 / 180 Pi]^2 φdot0;
  Plot[{1/2 pφ^2/Sin[θ / 180 Pi]^2 + Cos[θ / 180 Pi],
                                    余弦        圆周率
    1/2 (θdot0^2 + pφ^2/Sin[θ0 / 180 Pi]^2) + Cos[θ0 / 180 Pi]}, {θ, 0 + 10^-3, 180 - 10^-3},
                                               余弦         圆周率
   Ticks → {{0, 90, 180}, Automatic}, AxesLabel → {"θ(°)", "U(mgR)"},
   刻度                   自动          坐标轴标签
   PlotStyle → {{Thickness[0.0075]}, {Dashing[{0.05, 0.05}]}}, opts]
   绘图样式      粗细                   虚线线段配置
 ]
```

In[∘]:=
```
effectivePotential[135, 2.5, 1.5 × 2^(1/4), PlotRange → {-0.25, 4}]
                                            绘制范围
```

Out[∘]=



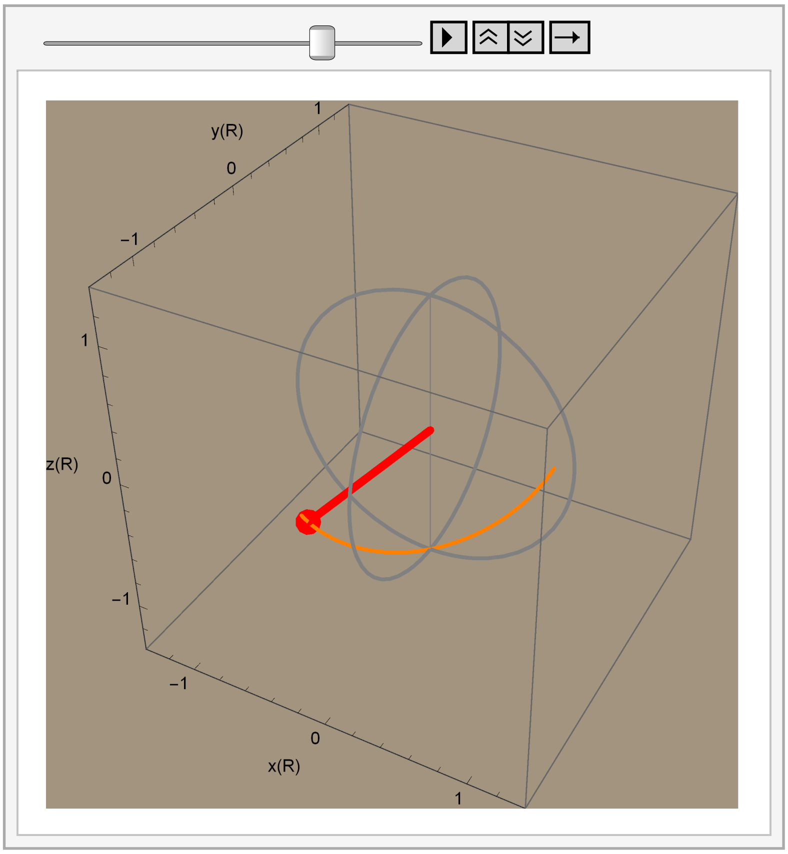
运动方程

In[∘]:=
```
ClearAll["Global`*"];
清除全部
sphericalPendulum[θ0_ /; 0 ≤ θ0 ≤ 180, θdot0_ /; Abs[θdot0] < 10, φ0_ /; 0 ≤ φ0 ≤ 360,
                                                 绝对值
  φdot0_ /; Abs[φdot0] < 10, tmax_ /; 1 ≤ tmax ≤ 60, nv_Integer: 999, opts___Rule] :=
             绝对值
 Module[{n = nv, pφ, sol, θ, φ, x, y, z, sphere},
 模块
  If[n > 1,
  如果
   If[n == 999, n = Round[3 tmax]];
   如果              舍入
```

```
    pφ = (Sin[θ0 °]^2 φdot0) // N;
                              数值运算
    sol = NDSolve[{θ''[t] - Cos[θ[t]]/Sin[θ[t]]^3 pφ^2 - Sin[θ[t]] == 0, φ'[t] == pφ/Sin[θ[t]]^2,
          数值求解微分方程组                          正弦
       θ[0] == θ0 °, θ'[0] == θdot0, φ[0] == φ0 °}, {θ, φ}, {t, 0, tmax + 0.01},
      MaxSteps → 6000];
      最多步数
    θ = θ /. sol[[1, 1]];
    φ = φ /. sol[[1, 2]];
    x[t_] := Sin[θ[t]] Cos[φ[t]];
             正弦       余弦
    y[t_] := Sin[θ[t]] Sin[φ[t]];
             正弦       正弦
    z[t_] := Cos[θ[t]];
             余弦
    sphere = ParametricPlot3D[{{0, Sin[t], Cos[t]}, {Sin[t], 0, Cos[t]}}, {t, 0, 2 Pi},
             绘制三维参数图       正弦    余弦       正弦       余弦                 圆周率
      PlotStyle → {LineColor → Gray}];
      绘图样式                 灰色
    tab = Table[Show[sphere,
          表格  显示
       Graphics3D[{Gray, Line[{{0, 0, -1}, {0, 0, 1}}]}],
                   灰色  线段
       ParametricPlot3D[{x[t], y[t], z[t]}, {t, 0, (tmax / (n - 1)) i + 0.0001},
       绘制三维参数图
        PlotStyle → {LineColor → Orange},
                                 橙色
        PlotPoints → (25 + Round[12 tmax / (n - 1) i])],
                           舍入
       Graphics3D[{Thickness[0.0125], Red, Line[{{0, 0, 0},
                   粗细               红色  线段
           {x[(tmax / (n - 1)) i], y[(tmax / (n - 1)) i], z[(tmax / (n - 1)) i]}}],
         PointSize[0.04], Red, Point[{x[(tmax / (n - 1)) i],
                          红色  点
           y[(tmax / (n - 1)) i], z[(tmax / (n - 1)) i]}]}],
       PlotRange → {{-1.25, 1.25}, {-1.25, 1.25}, {-1.25, 1.25}},
       绘制范围
       BoxRatios → {1, 1, 1},
       边界框比例
       opts,
       AxesLabel → {"x(R)", "y(R)", "z(R)"}], {i, 0, n - 1}];
       坐标轴标签
    Export[NotebookDirectory[] <> "sphericalPendulum.gif", tab];
    导出   当前笔记本的目录
    ListAnimate[tab]
    列表帧动画
   ]
  ]

In[•]:= sphericalPendulum[120, 0, 45, 0, 13.6, Background → RGBColor[0.640004, 0.580004, 0.5]]
                                                 背景色       RGB颜色
```

Out[ ]=



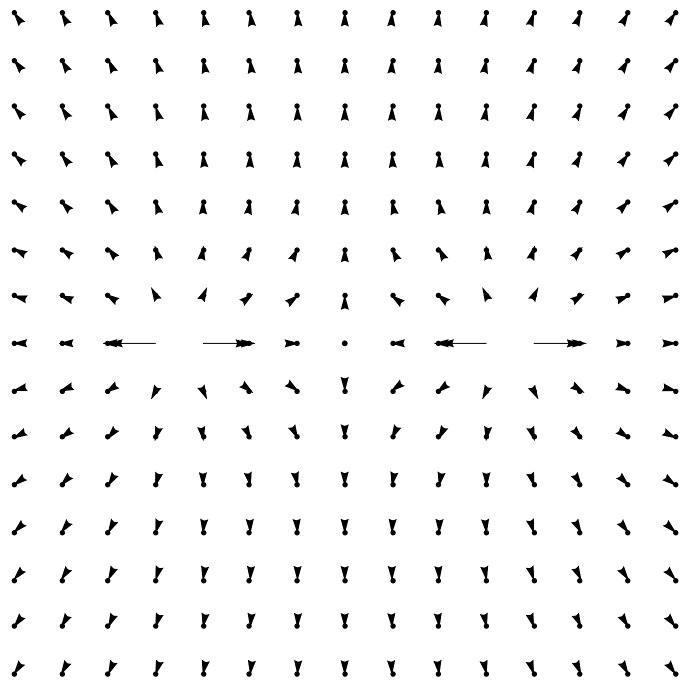
4. 电场

矢量图

In[ ]:=
```
Needs["VectorFieldPlots`"]
```
需要

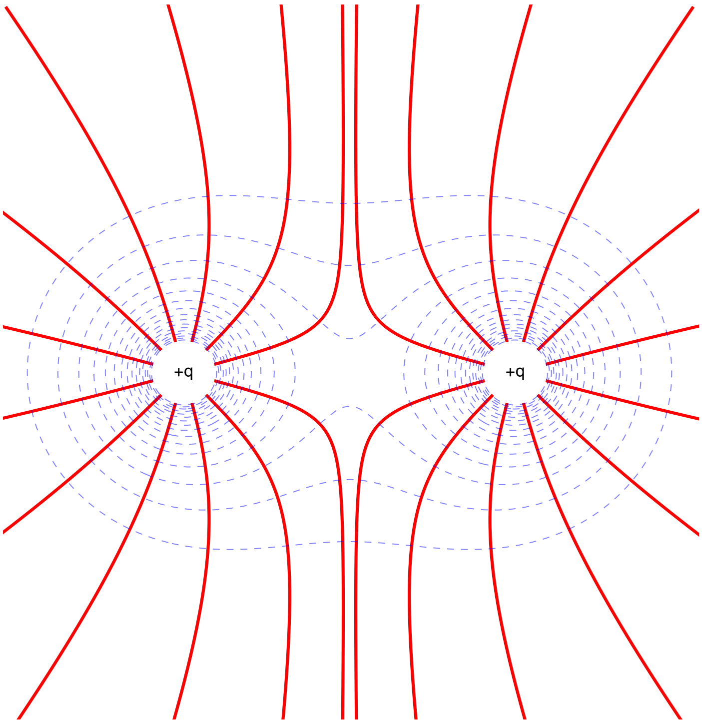
$$\text{GradientFieldPlot}\left[-\frac{1}{\sqrt{(x+1)^2+y^2}}-\frac{1}{\sqrt{(x-1)^2+y^2}},\ \{x,-2,2\},\ \{y,-2,2\}\right]$$

Out[ ]=



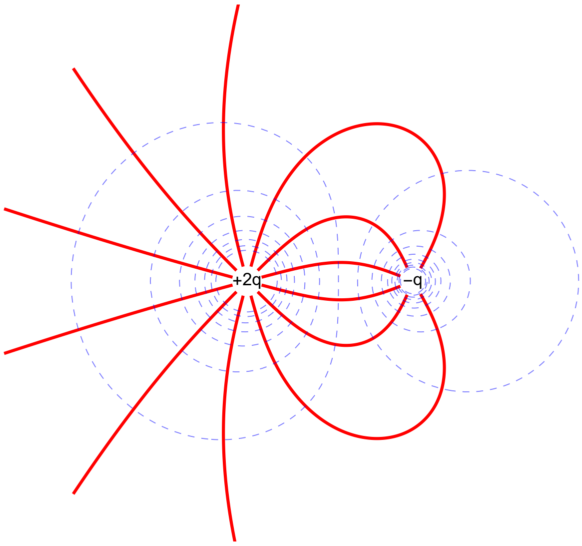
电场线与等势线

```
In[•]:= φ[x_, y_] := 1/Sqrt[(x + 1)^2 + y^2] + 1/Sqrt[(x - 1)^2 + y^2];
EF[x_, y_] := Evaluate[-∇{x,y} φ[x, y]];
               计算
ℰ[x_, y_] := Norm[EF[x, y]];
              模
f[{x_, y_}] := {x, y} + 0.02 EF[x, y]/ℰ[x, y];
x0[n_] := N[-1 + 0.2 Cos[n π/12]];
          数值运算   余弦
y0[n_] := N[0.2 Sin[n π/12]];
          数…  正弦
g[{x_, y_}] :=
  FixedPointList[f, {x, y}, SameTest → ((Norm[#2] > 3) || (#2[[1]] > 0) &)];
  固定点列表                相同检验       模
g /@ Table[{x0[n], y0[n]}, {n, 1, 11, 2}];
     表格
coordinates1 =
  Join[%, % /. {x_, y_} → {x, -y}, % /. {x_, y_} → {-x, y}, % /. {x_, y_} → {-x, -y}];
  连接
SetOptions[ListLinePlot, PlotStyle → Red];
设置选项    绘制点集的线条  绘图样式      红色
Show[
显示
 ListLinePlot /@ coordinates1,
 绘制点集的线条
 ContourPlot[φ[x, y], {x, -2, 2}, {y, -2, 2},
 绘制等高线
  ContourShading → False,
  等高线阴影        假
  PlotRange → {1.1, 6.0},
  绘制范围
  Contours → 16,
  等高线
  PlotPoints → 50,
  绘图点
  Frame → False,
  边框    假
  ContourStyle → {{Blue, Dashing[{0.01, 0.01}]}}],
  等高线样式        蓝色  虚线线段配置
 Graphics[{Text["+q", {-1, 0}], Text["+q", {1, 0}]}],
 图形       文本                 文本
 AspectRatio → Automatic,
 宽高比         自动
 PlotRange → {{-2, 2}, {-2, 2}},
 绘制范围
 Axes → False
 坐标轴  假
]
```

Out[ ]=


```
In[•]:= φ[x_, y_] := 2/Sqrt[(x + 1)^2 + y^2] + -1/Sqrt[(x - 1)^2 + y^2];
EF[x_, y_] := Evaluate[-∇{x,y} φ[x, y]];
                计算
ℰ[x_, y_] := Norm[EF[x, y]];
             模
f[{x_, y_}] := {x, y} + 0.02 EF[x, y]/ℰ[x, y];
x0[n_] := N[-1 + 0.2 Cos[n π/12]];
          数值运算   余弦
y0[n_] := N[0.2 Sin[n π/12]];
          数…  正弦
g[{x_, y_}] := FixedPointList[f, {x, y},
               固定点列表
  SameTest → ((Norm[#2] > 4) || EuclideanDistance[#2, {1, 0}] ≤ 0.2 &)];
  相同检验      模                欧几里得距离
g /@ Table[{x0[n], y0[n]}, {n, 1, 11, 2}];
     表格
coordinates2 = Join[%, % /. {x_, y_} → {x, -y}];
               连接
SetOptions[ListLinePlot, PlotStyle → Red];
设置选项    绘制点集的线条  绘图样式    红色
Show[
显示
 ListLinePlot /@ coordinates2,
 绘制点集的线条
 ContourPlot[φ[x, y], {x, -4, 4}, {y, -3, 3},
 绘制等高线
  ContourShading → False,
  等高线阴影       假
  PlotRange → {-6, 6.0},
  绘制范围
  Contours → 16,
  等高线
  PlotPoints → 50,
  绘图点
  Frame → False,
  边框    假
  ContourStyle → {{Blue, Dashing[{0.01, 0.01}]}}],
  等高线样式       蓝色  虚线线段配置
 Graphics[{Text["+2q", {-1, 0}], Text["-q", {1, 0}]}],
 图形       文本                  文本
 AspectRatio → Automatic,
 宽高比        自动
 PlotRange → {{-4, 4}, {-3, 3}},
 绘制范围
 Axes → False
 坐标轴  假
]
```

Out[∘]=



5. 带电粒子在电磁场下运动

In[∘]:=
```
ClearAll["Global`*"];
清除全部
r[t_] := {x[t], y[t], z[t]};
EF := {0, E0, 0};
BF := {0, 0, B0};
Thread /@
逐项作用
   {m r''[t] == q EF + q r'[t] × BF, r[0] == {0, 0, 0}, r'[0] == {vx0, vy0, vz0}} // Flatten;
                                                                                    压平
sol = DSolve[%, {x[t], y[t], z[t]}, t] //. {B0 → m ω / q, E0 → vd B0} // Flatten //
      求解微分方程                                                          压平
     Simplify // ExpandAll;
     化简        展开全部
eqn = sol /. Rule → Equal
             规则    恒等
```

Out[∘]= $\left\{x[t] == t\,vd + \frac{vy0}{\omega} - \frac{vy0\,Cos[t\,\omega]}{\omega} - \frac{vd\,Sin[t\,\omega]}{\omega} + \frac{vx0\,Sin[t\,\omega]}{\omega},\ y[t] == \frac{vd}{\omega} - \frac{vx0}{\omega} - \frac{vd\,Cos[t\,\omega]}{\omega} + \frac{vx0\,Cos[t\,\omega]}{\omega} + \frac{vy0\,Sin[t\,\omega]}{\omega},\ z[t] == t\,vz0\right\}$

In[∘]:=
```
Map[(# - (t vd + vy0 / ω))^2 &, eqn[[1]]];
映射
Map[(# - (vd / ω - vx0 / ω))^2 &, eqn[[2]]];
映射
Thread[Plus[%%, %], Equal];
逐项作用 加              恒等
MapAt[Simplify, %, 2]
作用于 化简
```

Out[∘]= $\left(-t\,vd - \frac{vy0}{\omega} + x[t]\right)^2 + \left(-\frac{vd}{\omega} + \frac{vx0}{\omega} + y[t]\right)^2 == \frac{vd^2 - 2\,vd\,vx0 + vx0^2 + vy0^2}{\omega^2}$

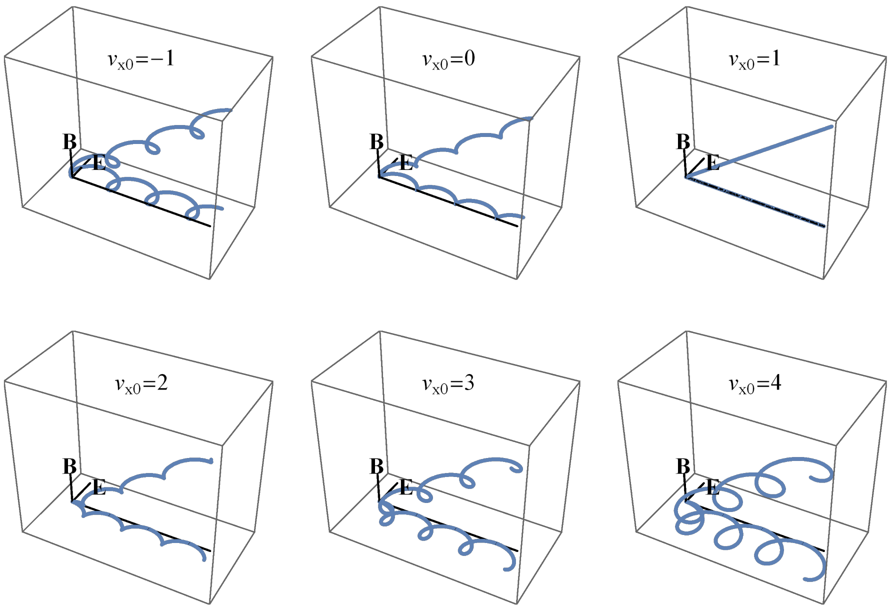
In[∘]:=
```
{x[t_], y[t_], z[t_]} = {x[t], y[t], z[t]} /. sol;
```

```
In[•]:= g[i_] := (vx0 = i;
          Show[
          显示
           ParametricPlot3D[{x[t], y[t], 0}, {t, 0, 22}],
           绘制三维参数图
           ParametricPlot3D[r[t], {t, 0, 22}],
           绘制三维参数图
           Graphics3D[{Thickness[0.008],
           三维图形    粗细
             Line[{{0, 0, 0}, {0, 0, 5}}],
             线段
             Line[{{0, 0, 0}, {0, 5, 0}}],
             线段
             Line[{{0, 0, 0}, {22, 0, 0}}],
             线段
             Text[Style["E", FontFamily → "Times", Bold, 12], {1.3, 5.5, 0}],
             文本 样式   …  字体系列      乘    粗体
             Text[Style["B", FontFamily → "Times", Bold, 12], {0, 0, 7}],
             文本 样式      字体系列      乘    粗体
             Text[Style["vx0=" <> ToString[vx0],
             文本 样式          转换为字符串
               FontFamily → "Times", 12], Scaled[{0.4, 0.9, 0.85}]]
               字体系列      乘           比例坐标
            }],
           Axes → False, PlotRange → {{-2, 24}, {-7, 7}, {0, 22}}, ImageSize → 140]
           坐标轴 假    绘制范围                                   图像尺寸
         );
```

In[•]:=
```
{ω, vd, vz0, vy0} = {1, 1, 0.75, 0};
Print["\nω = ", ω, " vd = ", vd, " vz0 = ", vz0, " vy0 = ", vy0, "\n"];
```
打印
```
Grid[Partition[Table[g[i], {i, -1, 4}], 3], Spacings → {1.5, 2.0}]
```
格子 划分 表格 间隔

ω = 1 v_d = 1 v_{z0} = 0.75 v_{y0} = 0

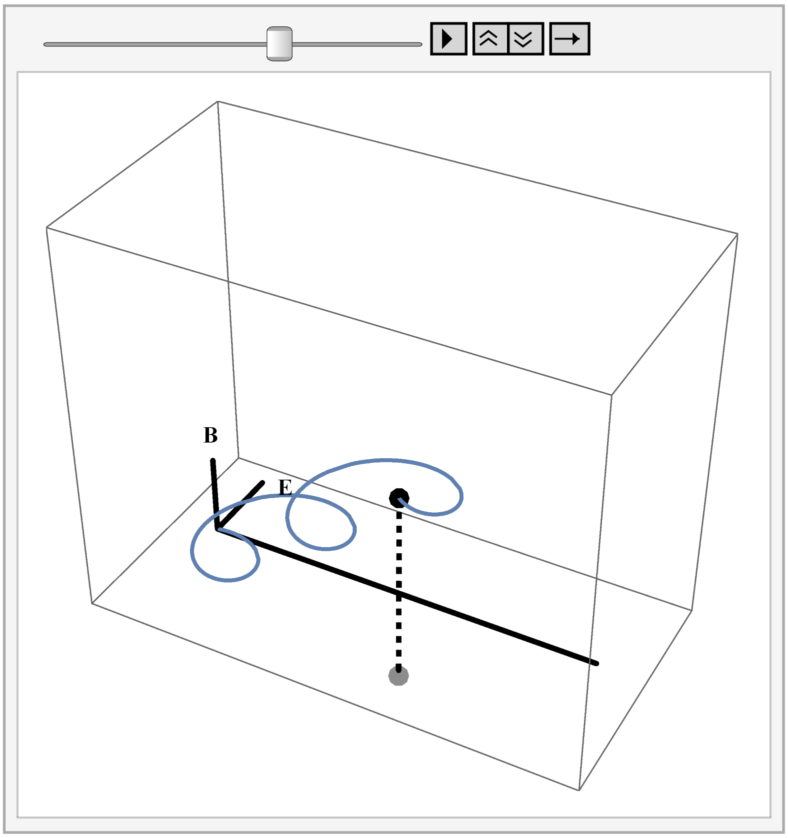
Out[•]=



In[•]:=
```
motion[parms_] := ({ω, vd, vx0, vy0, vz0} = parms;
  ListAnimate[
  列表帧动画
   Table[Show[
   表格   显示
     ParametricPlot3D[r[t], {t, 0, i + 0.0001}],
     绘制三维参数图
     Graphics3D[{Thickness[0.008],
     三维图形     粗细
       Line[{{0, 0, 0}, {0, 0, 5}}],
       线段
       Line[{{0, 0, 0}, {0, 5, 0}}],
       线段
       Line[{{0, 0, 0}, {24, 0, 0}}],
       线段
       Text[Style["E", FontFamily → "Times", Bold, 12], {1.3, 5.5, 0}],
       文本 样式   … 字体系列    乘   粗体
       Text[Style["B", FontFamily → "Times", Bold, 12], {0, 0, 7}],
       文本 样式     字体系列    乘   粗体
       {Dashing[{0.01, 0.01}], Line[{r[i], {x[i], y[i], 0}}]},
        虚线线段配置          线段
       {PointSize[0.03], Point[r[i]], GrayLevel[0.55], Point[{x[i], y[i], 0}]}
        点的大小         点           灰度级           点
      }],
     Axes → False, PlotRange → {{-2, 25}, {-7, 7}, {0, 22}}
     坐标轴  假      绘制范围
    ], {i, 0, 25}]
  ])
```

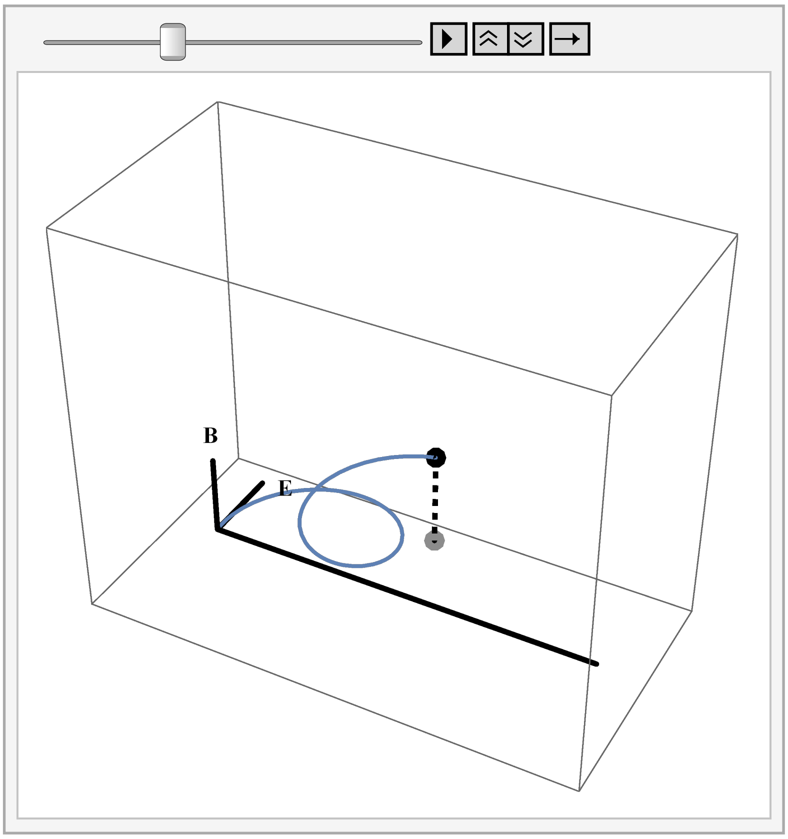
In[∘]:= `motion[{1, 1, 4, 0, 0.75}]`

Out[∘]=



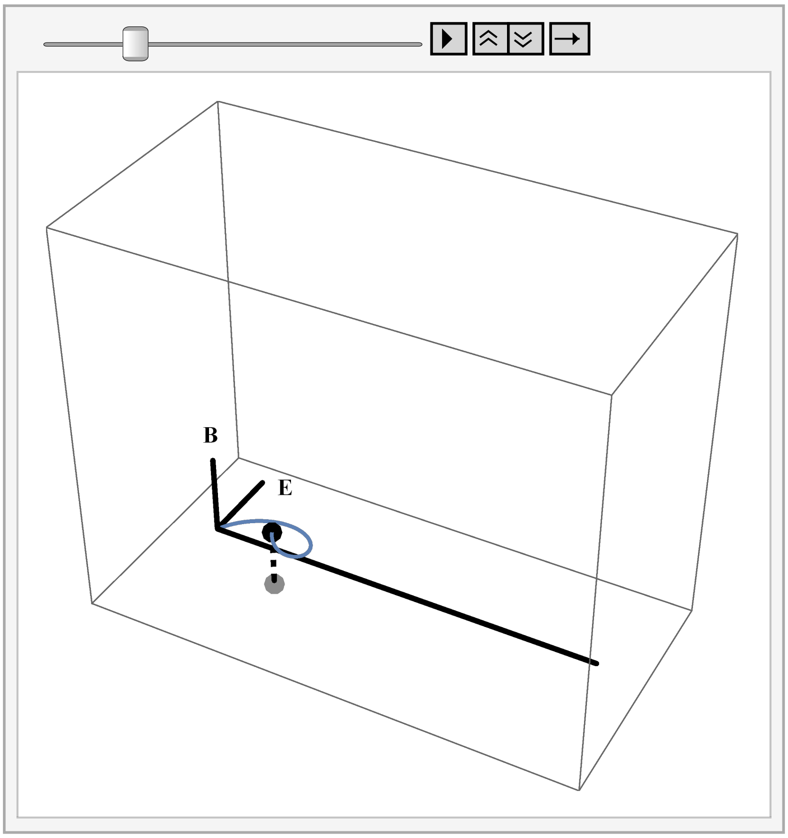
In[∘]:= `motion[{1, 1, 0, 4, 0.75}]`

Out[∘]=



In[∘]:=
```
motion[{1, 1, 2, 2, 0.75}]
```

Out[∘]=

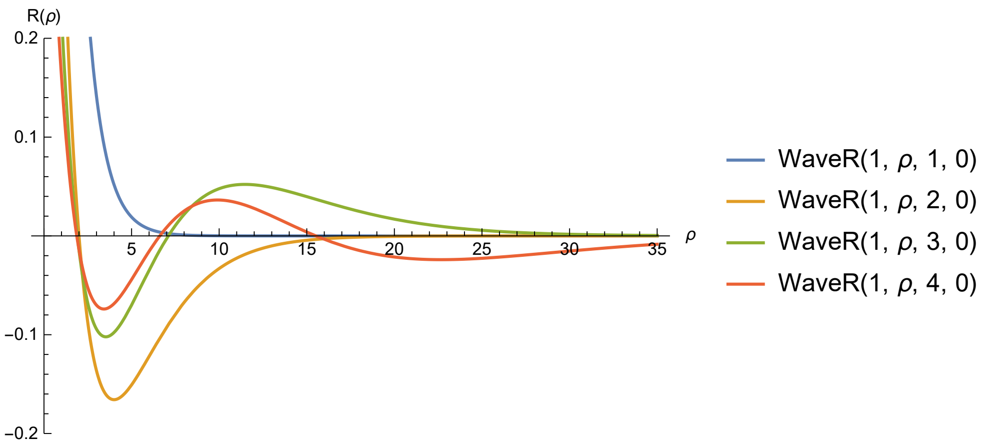


6. 粒子在中心力场中的运动

In[∘]:=
```
Get[NotebookDirectory[] <> "CoulombPotential.wl"]
```
⌊… ⌊当前笔记本的目录

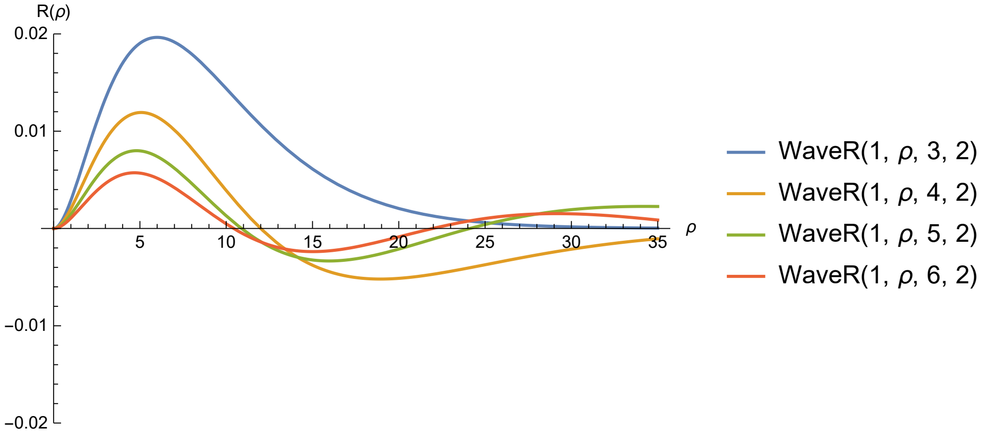
In[∘]:=
```
Plot[{WaveR[1, ρ, 1, 0], WaveR[1, ρ, 2, 0], WaveR[1, ρ, 3, 0], WaveR[1, ρ, 4, 0]},
 {ρ, 0, 35}, AxesLabel → {"ρ", "R(ρ)"}, Prolog → Thickness[0.001],
 PlotRange → {-0.2, 0.2}, PlotLegends → "Expressions"]
```
⌊绘图 ⌊坐标轴标签 ⌊绘制主… ⌊粗细 ⌊绘制范围 ⌊绘图的图例

Out[∘]=



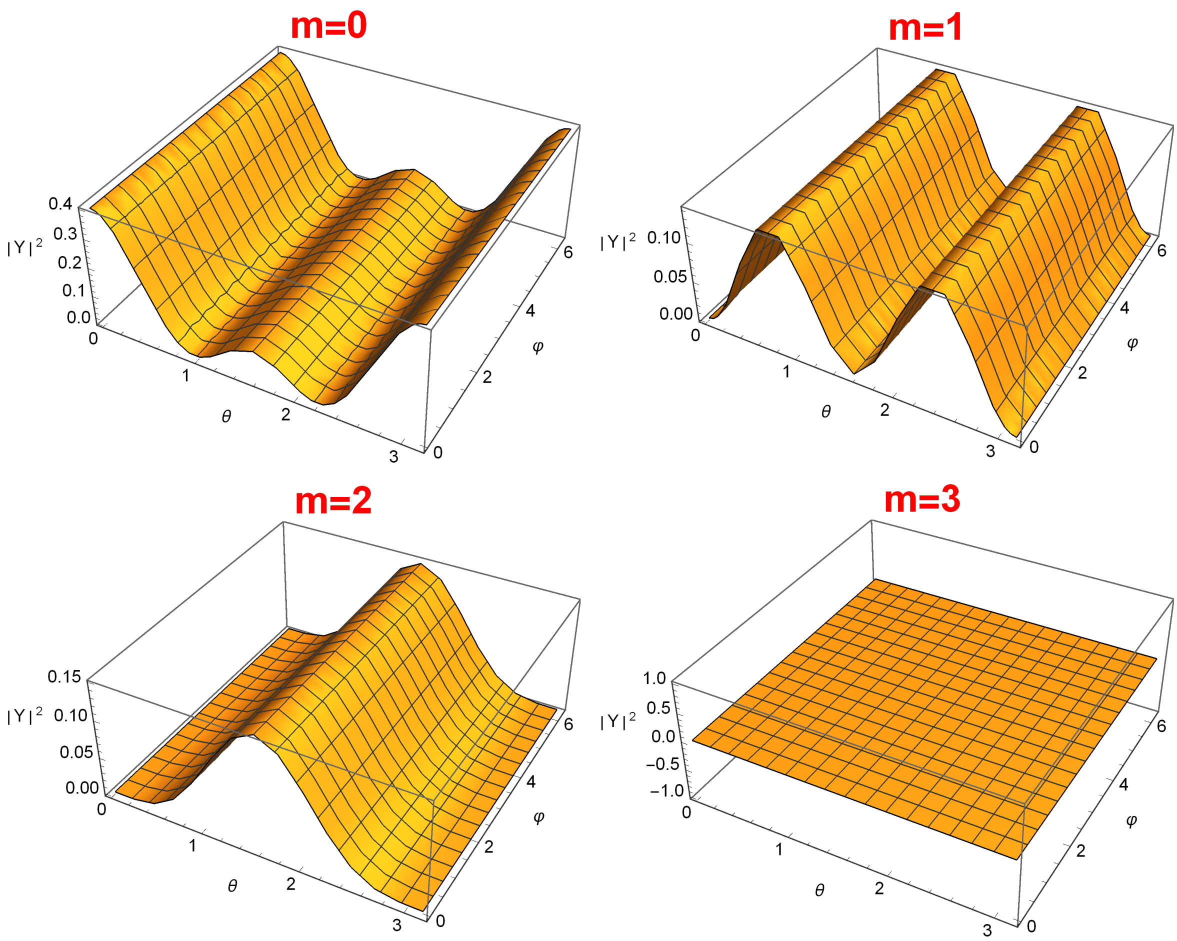
In[∘]:=
```
Plot[{WaveR[1, ρ, 3, 2], WaveR[1, ρ, 4, 2], WaveR[1, ρ, 5, 2], WaveR[1, ρ, 6, 2]},
绘图
 {ρ, 0, 35}, AxesLabel → {"ρ", "R(ρ)"}, Prolog → Thickness[0.001],
              坐标轴标签                    绘制主…   粗细
 PlotRange → {-0.02, 0.02}, PlotLegends → "Expressions"]
 绘制范围                    绘图的图例
```

Out[∘]=



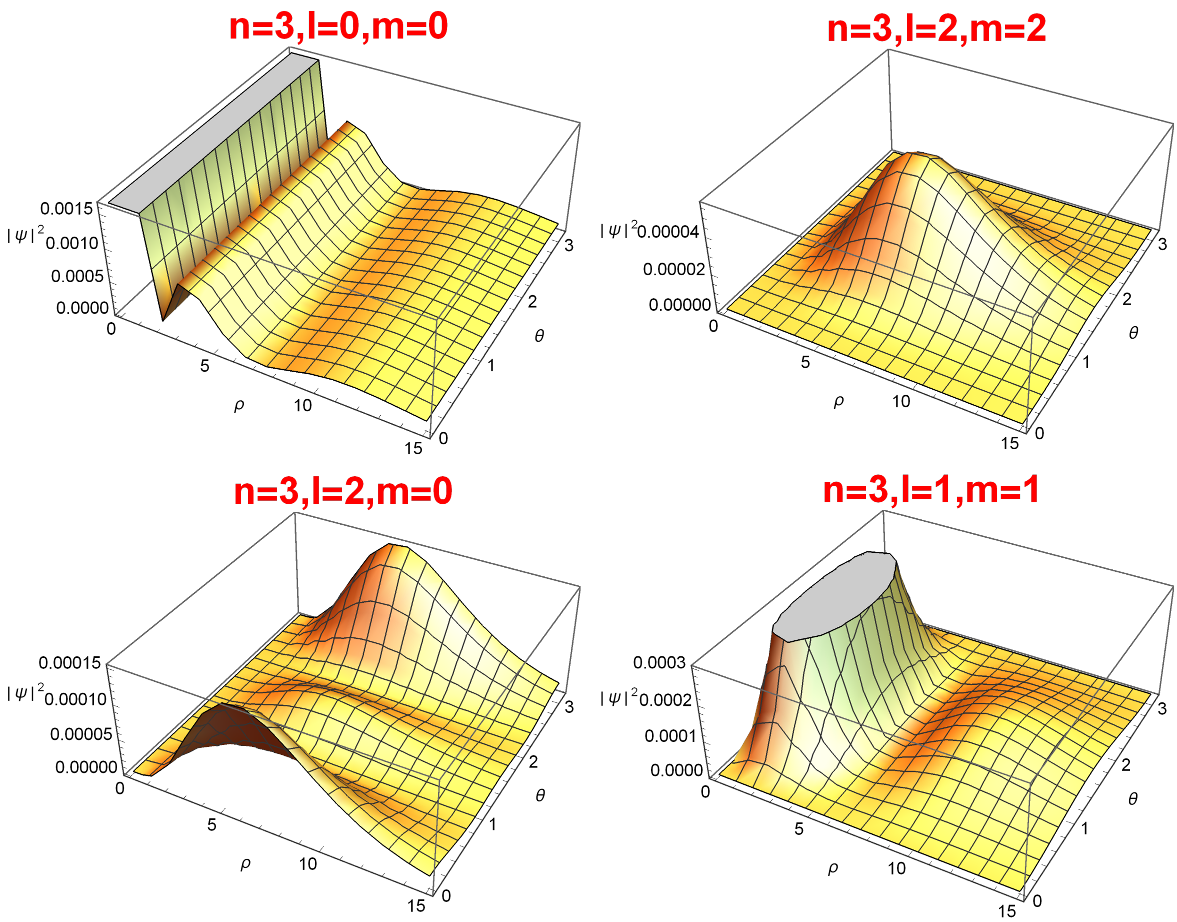
In[•]:=
```
Grid[Partition[Table[Plot3D[Abs[WaveA[theta, phi, 2, i]]^2,
格子   划分      表格   绘制…   绝对值
    {theta, 0, Pi}, {phi, 0, 2 Pi}, AxesLabel → {"θ", "φ", "|Y|²"}, PlotLabel →
                圆周率          …      坐标轴标签                          绘图标签
     Style["m=" <> ToString[i], Red, Bold, 20], ImageSize → 300], {i, 0, 3}], 2]]
     样式          转换为字符串    红色  粗体        图像尺寸
```

Out[•]=



In[ ]:=
```
Grid[Partition[
格子  划分
  Table[Plot3D[Abs[WaveF[1, ρ, theta, Pi / 2, num[[1]], num[[2]], num[[3]]]]^2,
  表格  绘制⋯  绝对值              圆周率
    {ρ, 0, 15}, {theta, 0, Pi}, AxesLabel → {"ρ", "θ", "|ψ|2"}, Lighting → True,
                             ⋯  坐标轴标签                        光照       真
    PlotLabel → Style["n=" <> ToString[num[[1]]] <> ",l=" <> ToString[num[[2]]] <>
    绘图标签    样式           转换为字符串                  转换为字符串
       ",m=" <> ToString[num[[3]]], Red, Bold, 20], ImageSize → 300],
                转换为字符串            红色 粗体       图像尺寸
   {num, {{3, 0, 0}, {3, 2, 2}, {3, 2, 0}, {3, 1, 1}}}], 2]]
```

Out[ ]=



7. 非相对论性薛定谔方程本征能量限

变分法

$$\hat{H} = -\frac{\nabla^2}{2\mu} + a r^n$$

选取氢原子基态波函数作为试探函数，计算期望值

```
ψ[r_, λ_] = λ^(3/2)/Sqrt[π] e^(-λ r);
g[r_, λ_] = ∂_{r,r} ψ[r, λ] + 2/r ∂_r ψ[r, λ];
(*T*) - 1/(2 μ) 4 π ∫_0^∞ ψ[r, λ] × g[r, λ] r^2 dr
(*V*) 4 π ∫_0^∞ ψ[r, λ] a r^n ψ[r, λ] r^2 dr
```

Out[∘]= ConditionalExpression$\left[\frac{\lambda^2}{2\mu}, \mathrm{Re}[\lambda] > 0\right]$

Out[∘]= ConditionalExpression$\left[2^{-1-n}\, a\, \lambda^{-n}\, \mathrm{Gamma}[3+n], \mathrm{Re}[n] > -3\ \&\&\ \mathrm{Re}[\lambda] > 0\right]$

所以

$$E(\lambda) = \frac{\lambda^2}{2\mu} + 2^{-1-n} a \lambda^{-n} \Gamma(3+n) = \frac{\lambda^2}{2\mu} + \frac{a}{2}\frac{\Gamma(n+3)}{(2\lambda)^n}$$

求最小值

```
In[∘]:= 𝔼[λ_] := λ^2/(2 μ) + a/2 Gamma[n + 3]/(2 λ)^n;
Solve[∂_λ 𝔼[λ] == 0, λ]
解方程
𝔼[λ] /. %[[1]]
```

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[∘]= $\left\{\left\{\lambda \to \left(2^{-1-n}\, a\, n\, \mu\, \mathrm{Gamma}[3+n]\right)^{\frac{1}{2+n}}\right\}\right\}$

Out[∘]= $\frac{\left(2^{-1-n}\, a\, n\, \mu\, \mathrm{Gamma}[3+n]\right)^{\frac{2}{2+n}}}{2\mu} + 2^{-1-n}\, a\, \mathrm{Gamma}[3+n] \left(\left(2^{-1-n}\, a\, n\, \mu\, \mathrm{Gamma}[3+n]\right)^{\frac{1}{2+n}}\right)^{-n}$

线性势

验证正交性

In[∘]:=

$$\text{psi0}[\lambda\_, r\_] := \frac{\lambda^{\frac{3}{2}}}{\sqrt{\pi}} e^{-\lambda r};$$

$$\text{psi1}[\lambda\_, r\_] := \frac{\lambda^{\frac{3}{2}}}{\sqrt{3\pi}} (3 - 2\lambda r)\, e^{-\lambda r};$$

$$4\pi \int_0^\infty \text{psi0}[\lambda, r] \times \text{psi1}[\lambda, r]\, r^2\, dr$$

Out[∘]= ConditionalExpression[0, Re[λ] > 0]

哈密顿量期望矩阵

In[∘]:=

$$\text{psi}[\lambda\_, r\_] := \begin{pmatrix} \text{psi0}[\lambda, r] \\ \text{psi1}[\lambda, r] \end{pmatrix};$$

"定义径向哈密顿算子";

$$H[\text{psi}\_] := -\frac{1}{2\mu}\left(\partial_{r,r}\text{psi} + \frac{2}{r}\partial_r \text{psi}\right) + a\, r\, \text{psi};$$

$$\text{psi}[\lambda, r].H\,/@\,\text{psi}[\lambda, r]^{T} \text{ // Simplify // MatrixForm}$$

化简 矩阵格式

$$\text{HM}[\lambda\_, a\_, \mu\_] = \text{Simplify}\left[4\pi \int_0^\infty \%\, r^2\, dr \text{ // ExpandAll}, \lambda > 0\right];$$

化简 展开全部

HM[λ, a, μ] // MatrixForm

矩阵格式

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{e^{-2 r \lambda} \lambda^3 \left(2\lambda - r\lambda^2 + 2 a r^2 \mu\right)}{2\pi r \mu} & \frac{e^{-2 r \lambda} \lambda^3 \left(-11 r \lambda^2 + 2 r^2 \lambda^3 + 6 a r^2 \mu + \lambda\left(10 - 4 a r^3 \mu\right)\right)}{2\sqrt{3}\,\pi r \mu} \\ \frac{e^{-2 r \lambda} \lambda^3 (-3 + 2 r \lambda)\left(-2\lambda + r\lambda^2 - 2 a r^2 \mu\right)}{2\sqrt{3}\,\pi r \mu} & \frac{e^{-2 r \lambda} \lambda^3 (-3 + 2 r \lambda)\left(11 r \lambda^2 - 2 r^2 \lambda^3 - 6 a r^2 \mu + 2\lambda\left(-5 + 2 a r^3 \mu\right)\right)}{6\pi r \mu} \end{pmatrix}$$

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{\lambda^3 + 3 a \mu}{2\lambda\mu} & \frac{2\lambda^3 - 3 a \mu}{2\sqrt{3}\,\lambda\mu} \\ \frac{2\lambda^3 - 3 a \mu}{2\sqrt{3}\,\lambda\mu} & \frac{5a}{2\lambda} + \frac{7\lambda^2}{6\mu} \end{pmatrix}$$

求矩阵本征值和能量本征值估计

In[∘]:= Eigenvalues[HM[λ, a, μ]]

特征值

Out[∘]=

$$\left\{\frac{5\lambda^3 + 12 a \mu - 2\sqrt{4\lambda^6 - 6 a \lambda^3 \mu + 9 a^2 \mu^2}}{6\lambda\mu}, \frac{5\lambda^3 + 12 a \mu + 2\sqrt{4\lambda^6 - 6 a \lambda^3 \mu + 9 a^2 \mu^2}}{6\lambda\mu}\right\}$$

In[∘]:=
```
Eigenvalues[HM[1, 1, 1/2]]
特征值

% // N
数值运算
```

Out[∘]= $\left\{\frac{1}{3}\left(11+\sqrt{13}\right), \frac{1}{3}\left(11-\sqrt{13}\right)\right\}$

Out[∘]= {4.86852, 2.46482}

基态

In[∘]:=
```
FindMinimum[Eigenvalues[HM[λ, 1, 1/2]][[1]], {λ, 0.5}]
求极小值和…   特征值
```

Out[∘]= {2.35344, {λ → 1.4561}}