# UNIT-3

# IMAGE RESTORATION

## 3.1 IMAGE RESTORATION

Restoration improves image in some predefined sense. It is an objective process. Restoration attempts to reconstruct an image that has been degraded by using a priori knowledge of the degradation phenomenon. These techniques are oriented toward modeling the degradation and then applying the inverse process in order to recover the original image.

Image Restoration refers to a class of methods that aim to remove or reduce the degradations that have occurred while the digital image was being obtained.
All natural images when displayed have gone through some sort of degradation:

a) During display mode
b) Acquisition mode, or
c) Processing mode

The degradations may be due to

a) Sensor noise
b) Blur due to camera mis focus
c) Relative object-camera motion
d) Random atmospheric turbulence
e) Others

### 3.1.1 A Model of Image Restoration Process

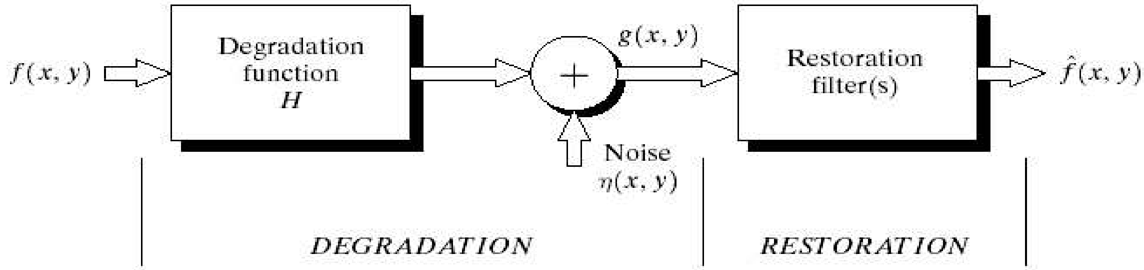
Degradation process operates on a degradation function that operates on an input image with an additive noise term.
Input image is represented by using the notation f(x,y), noise term can be represented as η(x,y).These two terms when combined gives the result as g(x,y).
If we are given g(x,y), some knowledge about the degradation function H or J and some knowledge about the additive noise teem η(x,y), the objective of restoration is to obtain an estimate *f'(x,y)* of the original image. We want the estimate to be as close as possible to the original image. The more we know about h and η , the closer f(x,y) will be to f'(x,y).
If it is a linear position invariant process, then degraded image is given in the spatial domain by

$$g(x,y)=f(x,y)*h(x,y)+\eta(x,y)$$

h(x,y) is spatial representation of degradation function and symbol * represents convolution.
In frequency domain we may write this equation as

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

The terms in the capital letters are the Fourier Transform of the corresponding terms in the spatial domain.




The image restoration process can be achieved by inversing the image degradation process, i.e.,

$$\hat{G}(u,v) = \frac{F(u,v) - N(u,v)}{H(u,v)} = \frac{F(u,v)}{\hat{H}(u,v)}$$

where $1/\hat{H}(u,v)$ is the inverse filter, and $\hat{G}(u,v)$ is the recovered image. Although the concept is relatively simple, the actual implementation is difficult to achieve, as one requires prior knowledge or identifications of the unknown degradation function $h(x,y)$ and the unknown noise source $n(x,y)$.

In the following sections, common noise models and method of estimating the degradation function are presented.

### 3.1.2 <u>Noise Models</u>

The principal source of noise in digital images arises during image acquisition and /or transmission. The performance of imaging sensors is affected by a variety of factors, such as environmental conditions during image acquisition and by the quality of the sensing elements themselves. Images are corrupted during transmission principally due to interference in the channels used for transmission. Since main sources of noise presented in digital images are resulted from atmospheric disturbance and image sensor circuitry, following assumptions can be made:

- The noise model is spatial invariant, i.e., independent of spatial location.
- The noise model is uncorrelated with the object function.

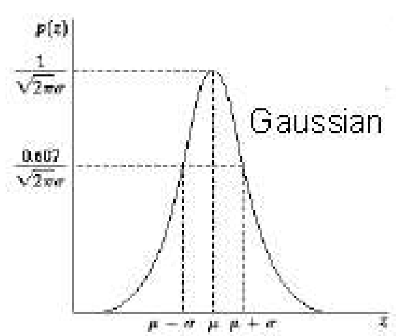
#### I. <u>Gaussian Noise</u>

These noise models are used frequently in practices because of its tractability in both spatial and frequency domain.

The PDF of Gaussian random variable, z is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$$

z= gray level
μ= mean of average value of z
σ= standard deviation



## II. Rayleigh Noise

Unlike Gaussian distribution, the Rayleigh distribution is no symmetric. It is given by the formula.

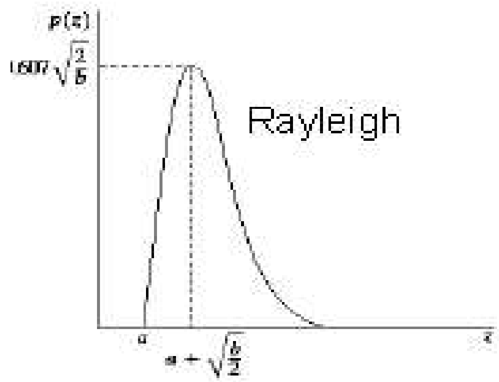
$$p(z) = \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b}}, \quad \text{for } z \geq a$$

The mean and variance of this density

Mean/variance

$$\mu = a + \sqrt{\pi b / 4}$$

$$\sigma^2 = \frac{b(4-\pi)}{4}$$




It is displaced from the origin and skewed towards the right.

## III. Erlang (gamma) Noise

The PDF of Erlang noise is given by

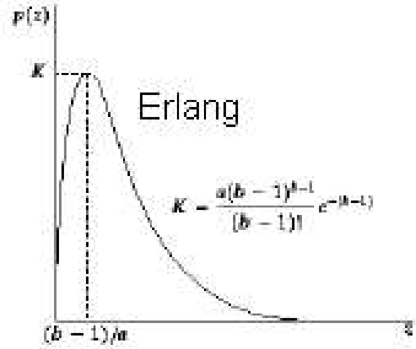
$$p(z) = \frac{a^b z^{b-1}}{(b-a)!}e^{-az}, \quad \text{for } z \geq 0$$

The mean and variance of this noise is

Mean/variance

$$\mu = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$



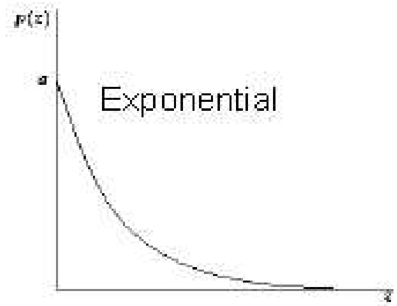

Its shape is similar to Rayleigh disruption.
This equation is referred to as gamma density it is correct only when the denominator is the gamma function.

## IV. Exponential Noise

Exponential distribution has an exponential shape.
The PDF of exponential noise is given as

$$p(z) = ae^{-az}, \quad \text{for } z \geq 0$$

Where a>0
It is a special case of Erlang with b=1




## V. Uniform Noise

The PDF of uniform noise is given by

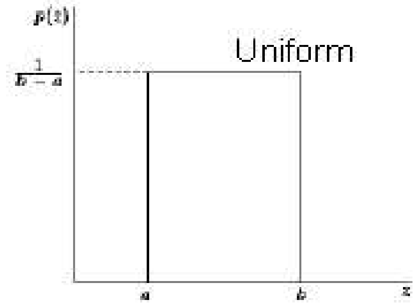
$$p(z) = \begin{cases} \dfrac{1}{(b-a)} & a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean of this density function is given by

Mean/variance

$$\mu = \frac{a+b}{2}$$

$$\sigma^2 = \frac{(b-a)^2}{12}$$

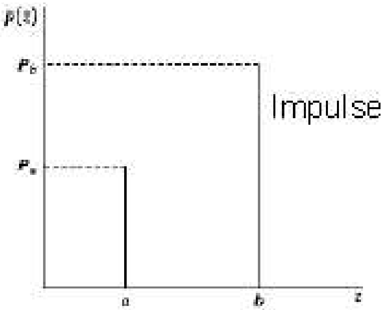



**VI. Impulse (Salt and Pepper )Noise**

In this case, the noise is signal dependent, and is multiplied to the image.
The PDF of bipolar (impulse) noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

If b>a, gray level b will appear as a light dot in image.
Level a will appear like a dark dot.




### 3.1.3 Restoration In the Presence of Noise Only-Spatial Filtering

When the only degradation present in an image is noise, i.e.

$$g(x,y)= f(x,y)+ \eta(x,y)$$
or
$$G(u,v)= F(u,v)+ N(u,v)$$

The noise terms are unknown so subtracting them from g(x,y) or G(u,v) is not a realistic approach. In the case of periodic noise it is possible to estimate N(u,v) from the spectrum G(u,v). So N(u,v) can be subtracted from G(u,v) to obtain an estimate of original image. Spatial filtering can be done when only additive noise is present.
The following techniques can be used to reduce the noise effect:

#### 3.1.3.1 Mean Filter

##### 3.1.3.1.1 Arithmetic Mean Filter

It is the simplest mean filter. Let $S_{xy}$ represents the set of coordinates in the sub image of size m*n centered at point (x,y). The arithmetic mean filter computes the average value of the corrupted image g(x,y) in the area defined by $S_{xy}$. The value of the restored image f at any point (x,y) is the arithmetic mean computed using the pixels in the region defined by $S_{xy}$.

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t)$$

This operation can be using a convolution mask in which all coefficients have value 1/mn
A mean filter smoothes local variations in image Noise is reduced as a result of blurring. For every pixel in the image, the pixel value is replaced by the mean value of its neighboring pixels $(N \times M)$ with a weight $w_k = 1/(NM)$. This will resulted in a smoothing effect in the image.

3.1.3.1.2 Geometric mean filter

An image restored using a geometric mean filter is given by the expression

$$\hat{f}(x,y) = \left( \prod_{(s,t) \in S_{xy}} g(s,t) \right)^{1/mn}$$

Here, each restored pixel is given by the product of the pixel in the subimage window, raised to the power 1/mn. A geometric mean filters but it to loose image details in the process.

3.1.3.1.3 Harmonic mean filter

The harmonic mean filtering operation is given by the expression

$$\hat{f}(x,y) = \sum_{(s,t) \in S_{xy}} g(s,t)^{Q+1} \Big/ \sum_{(s,t) \in S_{xy}} g(s,t)^{Q}$$

The harmonic mean filter works well for salt noise but fails for pepper noise. It does well with Gaussian noise also.

3.1.3.1.4 Order statistics filter

Order statistics filters are spatial filters whose response is based on ordering the pixel contained in the image area encompassed by the filter.
The response of the filter at any point is determined by the ranking result.

3.1.3.1.4.1 Median filter

It is the best order statistic filter; it replaces the value of a pixel by the median of gray levels in the Neighborhood of the pixel.

$$\hat{f}(x,y) = \underset{(s,t) \in S_{xy}}{\mathbf{median}} \{g(s,t)\}$$

The original of the pixel is included in the computation of the median of the filter are quite possible because for certain types of random noise, the provide excellent noise reduction capabilities with considerably less blurring then smoothing filters of similar size. These are effective for bipolar and unipolor impulse noise.

3.1.3.1.4.1 Max and Min Filters

Using the l00th percentile of ranked set of numbers is called the max filter and is given by the equation

$$\hat{f}(x,y) = \max_{(s,t) \in S_{xy}} \{g(s,t)\}$$

It is used for finding the brightest point in an image. Pepper noise in the image has very low values, it is reduced by max filter using the max selection process in the sublimated area sky.

The $0^{th}$ percentile filter is min filter

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

This filter is useful for flinging the darkest point in image. Also, it reduces salt noise of the min operation.

a. Midpoint Filter

The midpoint filter simply computes the midpoint between the maximum and minimum values in the area encompassed by the filter

$$\hat{f}(x,y) = \left( \max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min_{(s,t)\in S_{xy}} \{g(s,t)\} \right) \Big/ 2$$

It comeliness the order statistics and averaging .This filter works best for randomly distributed noise like Gaussian or uniform noise.

### 3.1.4 Periodic Noise By Frequency Domain Filtering

These types of filters are used for this purpose-

#### 3.1.4.1 Band Reject Filters

It removes a band of frequencies about the origin of the Fourier transformer.

##### 3.1.4.1.1 Ideal Band reject Filter

An ideal band reject filter is given by the expression

$$H(u,v) = \begin{cases} 1 & \text{if} \quad D(u,v) < D_0 - W/2 \\ 0 & \text{if} \quad D_0 - W/2 \le D(u,v) \le D_0 + W/2 \\ 1 & \text{if} \quad D(u,v) > D_0 + W/2 \end{cases}$$

D(u,v)- the distance from the origin of the centered frequency rectangle.
W- the width of the band
Do- the radial center of the frequency rectangle.

##### 3.1.4.1.2 Butterworth Band reject Filter

$$H(u,v) = 1 \Big/ \left[ 1 + \left( \frac{D(u,v)W}{D^2(u,v) - D_0^2} \right)^{2n} \right]$$

##### 3.1.4.1.3 Gaussian Band reject Filter

$$H(u,v) = 1 - \exp\left[ -\frac{1}{2} \left( \frac{D^2(u,v) - D_0^2}{D(u,v)W} \right)^2 \right]$$

These filters are mostly used when the location of noise component in the frequency domain is known. Sinusoidal noise can be easily removed by using these kinds of filters because it shows two impulses that are mirror images of each other about the origin. Of the frequency transform.

### 3.1.4.2 Band Pass Filters

The function of a band pass filter is opposite to that of a band reject filter It allows a specific frequency band of the image to be passed and blocks the rest of frequencies.
The transfer function of a band pass filter can be obtained from a corresponding band reject filter with transfer function $H_{br}(u,v)$ by using the equation-

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

These filters cannot be applied directly on an image because it may remove too much details of an image but these are effective in isolating the effect of an image of selected frequency bands.

## 3.1.5 **Notch Filters**

This type of filters rejects frequencies I predefined in neighborhood above a centre frequency
These filters are symmetric about origin in the Fourier transform the transfer function of ideal notch reject filter of radius do with centre at (…) and by symmetry at (……) is

$$H(u,v) = \begin{cases} 0 & \textbf{if} \quad D_1(u,v) \le D_0 \textbf{ or } D_2(u,v) \le D_0 \\ 1 & \textbf{otherwise} \end{cases}$$

Where

$$D_1(u,v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$
$$D_2(u,v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

Butterworth notch reject filter of order n is given by

$$H(u,v) = 1 - \exp\left[ -\frac{1}{2} \left( \frac{D_1(u,v) D_2(u,v)}{D_0^2} \right) \right]$$

A Gaussian notch reject filter has the fauna

$$H(u,v) = 1 \Bigg/ \left[ 1 + \left( \frac{D_0^2}{D_1(u,v) D_2(u,v)} \right)^n \right]$$

These filter become high pass rather than suppress. The frequencies contained in the notch areas.
These filters will perform exactly the opposite function as the notch reject filter.
The transfer function of this filter may be given as

$$Hnp(u,v) = 1 - Hnr(u,v)$$

$H_{np}(u,v)$- transfer function of the pass filter
$H_{nr}(u,v)$- transfer function of a notch reject filter

### 3.1.6 **Minimum Mean Square Error (Wiener) Filtering**

This filter incorporates both degradation function and statistical behavior of noise into the restoration process.
The main concept behind this approach is that the images and noise are considered as random variables and the objective is to find an estimate f of the uncorrupted image f such that the mean sequence error between then is minimized.

$$\hat{f}(x) = \sum_{s=-\infty}^{\infty} h_W(x-s)g(s),$$

This error measure is given by

$$e^2 = E\{[f(x) - \hat{f}(x)]^2\} = \min$$

Where e( ) is the expected value of the argument
Assuming that the noise and the image are uncorrelated (means zero average value) one or other has zero mean values
The minimum error function of the above expression is given in the frequency …….. is given by the expression.

$$H_W(u,v) = \frac{H^*(u,v)S_{ff}(u,v)}{|H(u,v)|^2 S_{ff}(u,v) + S_{nn}(u,v)} = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_{nn}(u,v)/S_{ff}(u,v)}$$

Product of a complex quantity with its conjugate is equal to the magnitude of …… complex quantity squared. This result is known as wiener Filter The filter was named so because of the name of its inventor N Wiener. The term in the bracket is known as minimum mean square error filter or least square error filter.
H*(u,v)-degradation function .
H*(u,v)-complex conjugate of H(u,v)
H(u,v) H(u,v)
Sn(u,v)=IN(u,v)I$^2$- power spectrum of the noise
Sf(u,v)=IF(u,v)$^2$- power spectrum of the underrated image
H(u,v)=Fourier transformer of the degraded function
G(u,v)=Fourier transformer of the degraded image
The restored image in the spatial domain is given by the inverse Fourier transformed of the frequency domain estimate F(u,v).
Mean square error in statistical form can be approveiment by the function

$$H_W(u,v) = \frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}$$

### 3.1.7 **Inverse Filtering**

It is a process of restoring an image degraded by a degradation function H. This function can be obtained by any method.
The simplest approach to restoration is direct, inverse filtering.
Inverse filtering provides an estimate F(u,v) of the transform of the original image simply by during the transform of the degraded image G(u,v) by the degradation function.

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

It shows an interesting result that even if we know the depredation function we cannot recover the underrated image exactly because N(u,v) is not known .
If the degradation value has zero or very small values then the ratio N(u,v)/H(u,v) could easily dominate the estimate F(u,v).