UNIT-4
MORPHOLOGICAL IMAGE PROCESSING

4.1 MORPHOLOGICAL IMAGE PROCESSING

4.1.1 Introduction

The word morphology refers to the scientific branch that deals the forms and structures of animals/plants. Morphology in image processing is a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries and skeletons. Furthermore, the morphological operations can be used for filtering, thinning and pruning.
This is middle level of image processing technique in which the input is image but the output is attributes extracted meaning from an image. The language of the Morphology comes from the set theory, where image objects can be represented by sets. For example an image object containing black pixels can be considered a set of black pixels in 2D space of Z^2**,** where each elements of the set is a tuple (2-D vector) whose coordinates are the (x,y) coordinates are the coordinates of white pixel in an image.
Gary scale images can be represented as sets whose components are in Z^3 two components of each elements of the set refers to the coordinates of a pixel and the third correspond to the discrete intensity value.

4.1.2 Basics Of Set Theory

Let A be set in Z^2 and a= (a1, a2) then

a is an element of A : $a| \in A$

If a is not an element of a then $a \notin A$

If every element of set A is also an element of set B, the A said be a subset of B Written as

$$A \subseteq B$$

The union of A and B is the collection of all elements that are in one both set. It is represented as

$$C = A \bigcup B$$

The intersection of the sets A and B is the set element belonging to both A and B is represented as

$$D = A \bigcap B$$

If these are no common elements in A and B, then the sets are called disjoint sets represented as

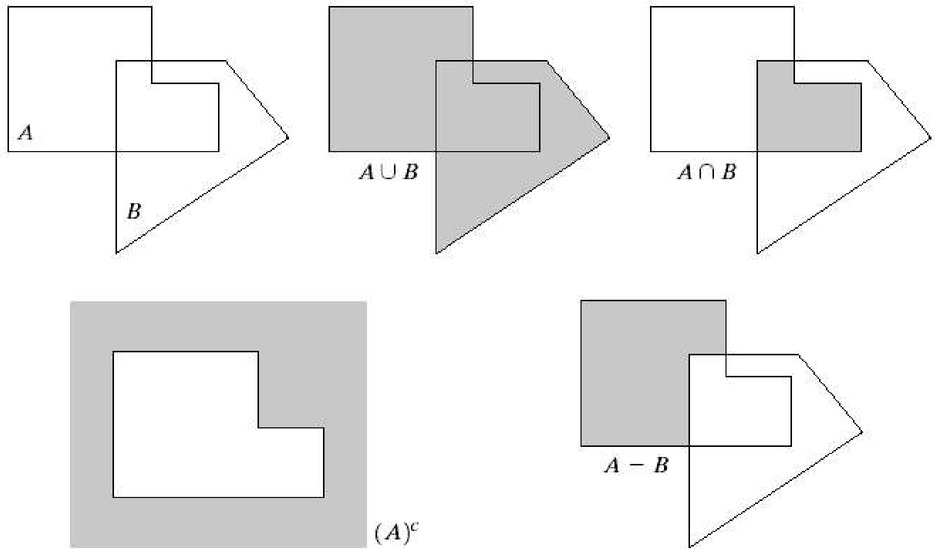
$$A \bigcap B = \varnothing$$

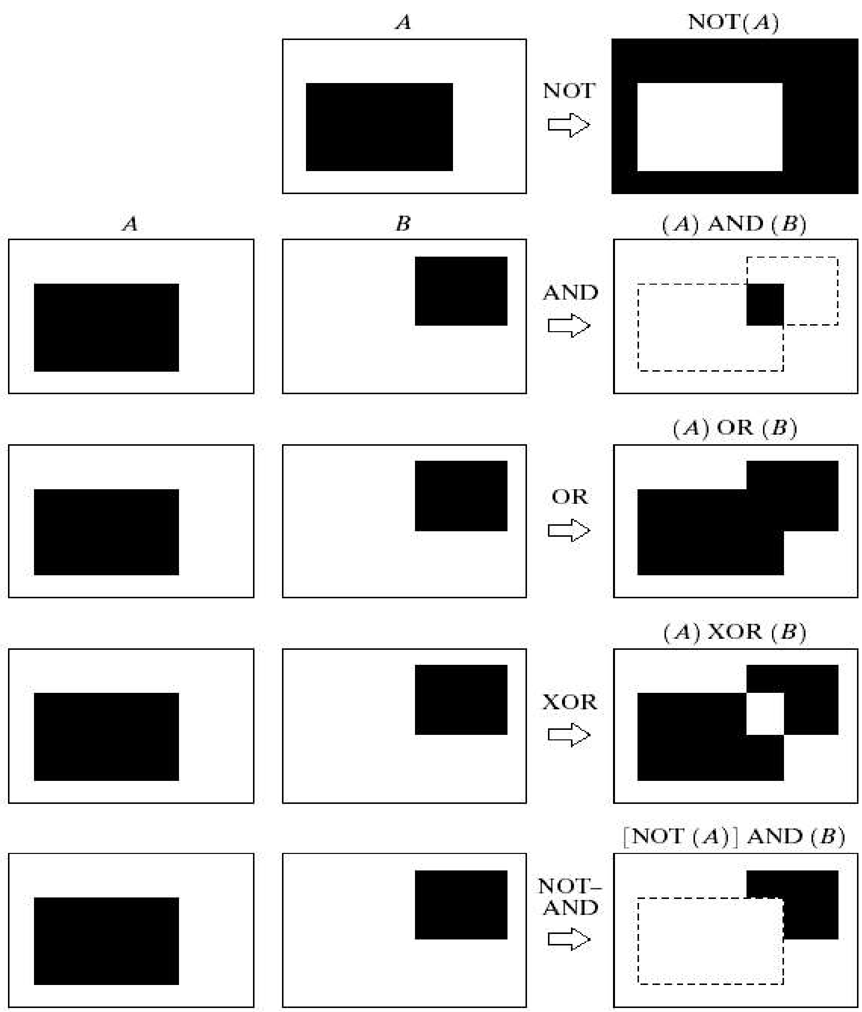
Ø is the name of the set with no members
The complements of a sets a is the set of elements in the image not contained A

$$A^c = \left\{ \omega \middle| \omega \notin A \right\}$$

The difference of two sets A and B is denoted by

$$A - B = \left\{ \omega \middle| \omega \in A, \omega \notin B \right\} = A \bigcap B^c$$


A
B
A ∪ B
A ∩ B
(A)^c
A − B


A
NOT(A)
NOT
A
B
(A) AND (B)
AND
(A) OR (B)
OR
(A) XOR (B)
XOR
[NOT (A)] AND (B)
NOT–
AND

The reflection of two set B, denoted by B is defined as

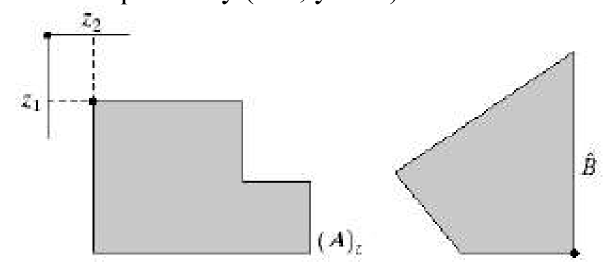
$$\hat{B} = \{\omega | \omega = -b, \ for\, b \in B\}$$

If B is the set of pixel representing an object in an image. Then A is simply the set of points in A whose (x,y) coordinates have been replaced by (z)

The translation of a set B by a point z= (z1,z2) denoted (B)z is defined as

$$(A)_z = \{\omega | \omega = a + z, \ for\, a \in A\}$$

If B is the set of set of pixel representing as object in an image Then (B)z is the set of points in B whose (x,y) coordinates have been replaced by (x+z,.y+z…)



Translation of A by z.   Reflection of B

4.1.3 **<u>Erosion & Dilation</u>**

Dilation and erosion are the two fundamental operations used in morphological image processing. Almost all morphological algorithms depend on these two operations:

4.1.3.1 <u>Dilation</u>

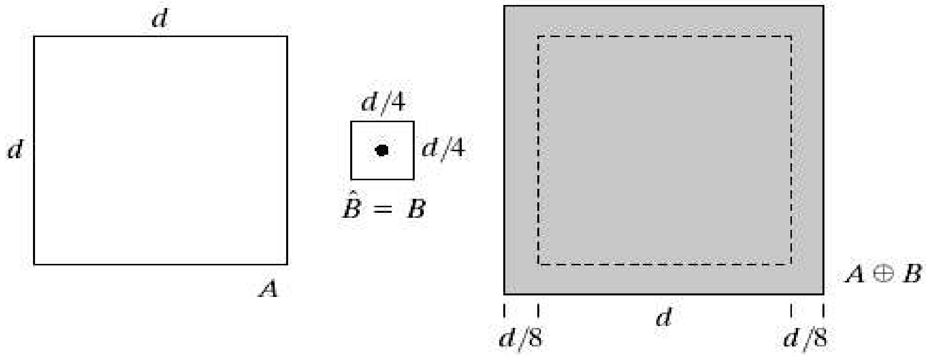
With A and B as set inZ^2 the dilation of A by B is defined as

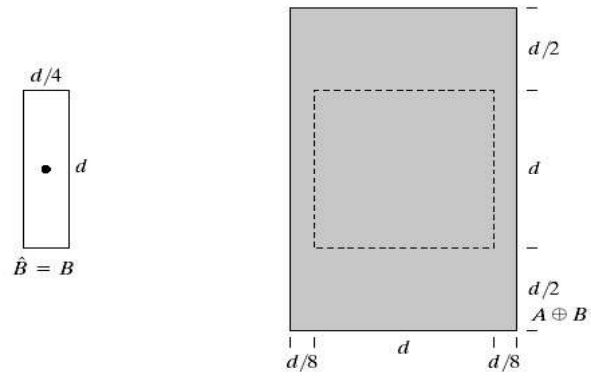
$$A \oplus B = \{z | (\hat{B})_z \cap A \neq \varnothing\}$$

Obtaining the reflection of B about the origin and shifting this reflection by z. The dilation is then the set of all displacements Z such that B and A overlap atheist by elements.
The equation may be rewritten as

$$A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$$

The set B is referred to as the structuring element in the dilation. This structuring element may be through of as a convolution mask.
Because the basic operation of flipping B about its origin and then successively displacing it so that it slides over the image A is analogue to the convolution process.





The structuring element and its refection are equal because it is symmetric with respect to the origin. The dashed line shows the boundary constitute beyond which any further displacement by z would cause the intersection of B and A to be empty.
Therefore all the points inside this boundary constitute the dilation of A to B. dilation has an advantage over low pass filtering that morphological method results directly in a binary image and convert it into a gray scale image which would require a pass with a thresh holding function to convert it back to binary form.

4.1.3.2 Erosion

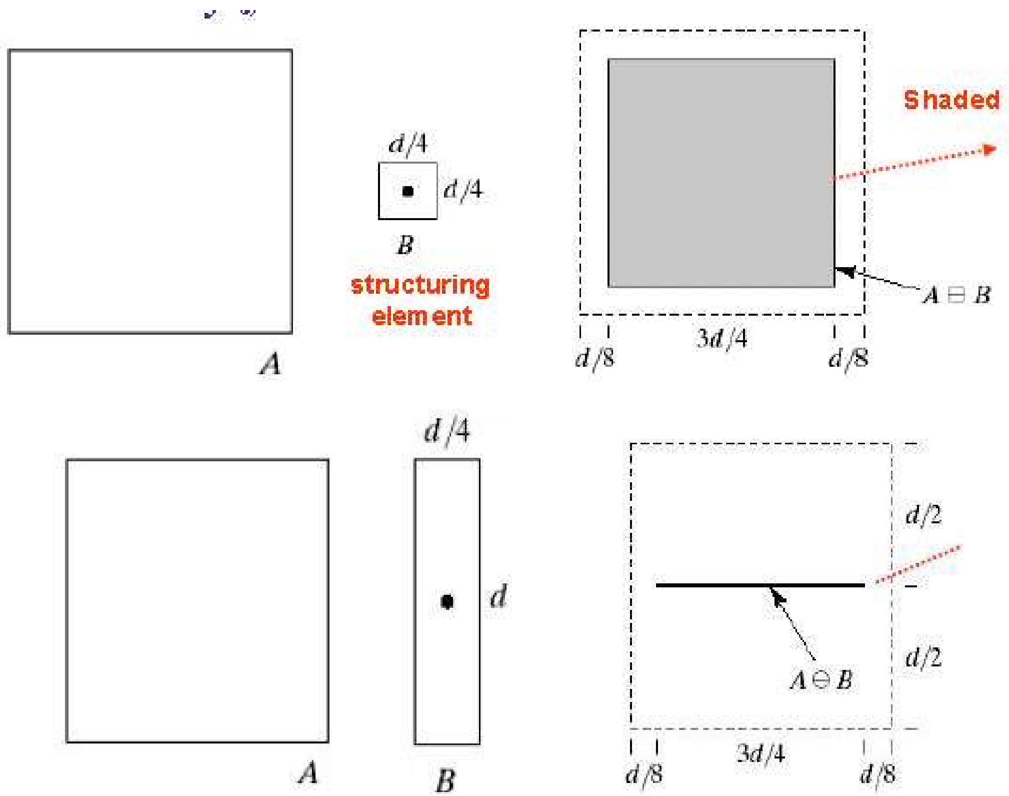
Erosion shrinks an image object. The basic effect of erosion is to erode away the boundaries of for ground pixel thus area of foreground pixel shrinks to size and holes within those areas become larger.
Mathematically, erosion of sets A by sets B is a set of all points x such that B translated by x is still contained in A.

$$A \ominus B = \{ z \mid (B)_z \subseteq A \}$$

Characteristics

- ⇨ It generally decreases the size of objects and removes small anomalies by subtracting objects with a radius smaller than the structuring element.
- ⇨ With gray scale images erosion reduces the brightness of bright objects on a dark back ground by taking the neighborhood minimum when passing the structuring element over the image.
- ⇨ With erosion binary images it completely removes objects smaller than the structuring element and removes perimeter pixels from larger image objects. For sets A and B in Z^2 the erosion of A by B denoted AøB is defined as

Erosion of A by B is the set of all points z such that B translated by z is contained in A
The boundary of the shaded region shows the limit beyond which further displacement of the origin of B would cause this set to cease being completed contained in A.



The boundary of shaded region shows the limit beyond which further displacement of the origin of B would cause this set to cease being completely contained in A.
Dilation and erosion are duals of each other with respect to set complementation and reflection,

4.1.4 Structuring Elements

These are also called kernel. It consists of a pattern specified as the coordinates of a number of discrete points suitable to some origin. All the techniques probe an image with this small shape or temples. It generally consists of a matrix of o's and 1's. Typically it is much smaller than the image being processed. The center pixel of the structuring elements is called the origin and it identifies the pixel of the interest of the pixel being processed. The pixels in the structuring elements containing 1's define the neighborhood of the structuring element.

It differs from the input image coordinates set in that it is normally much smaller. And its coordinate's origin is often not in a corner so that some coordinate element will have negative value.

The structuring element is positioned at all positions in the image and it is the compared with the corresponding neighborhood of pixels. Two main characteristics that are directly related to structuring elements.

(i) <u>Shape</u>

The element may be ball or line: convex a ring. By choosing particular structuring elements. One sets a way of differentially some objects from others according to their shape or spatial orientation.

(ii) <u>Size</u>

The structuring element can be a 3x3 or a 21x21 square.

4.1.5 <u>Opening & Closing</u>

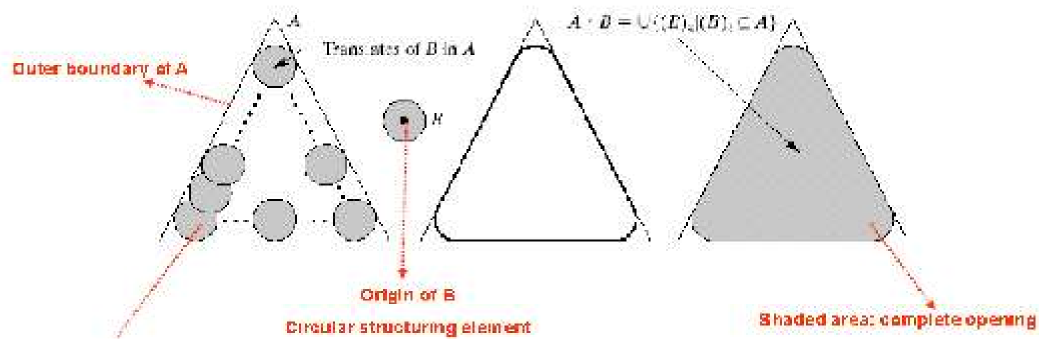
4.1.5.1 <u>Opening</u>

The process of erosion followed by dilation is called opening. It has the effect of eliminating small and thin objects, breaking the objects at thin points and smoothing the boundaries/contours of the objects.
Given set A and the structuring element B. opening of a set A by structuring element B is defined as

$$A \circ B = (A \ominus B) \oplus B$$

The opening of A by the structuring element B is obtained by taking the union of all translates of B that fit into A.
The opening operation can also be expressed by the following formula:

$$A \circ B = \bigcup \{ B_z \mid (B_z) \subseteq A \}$$



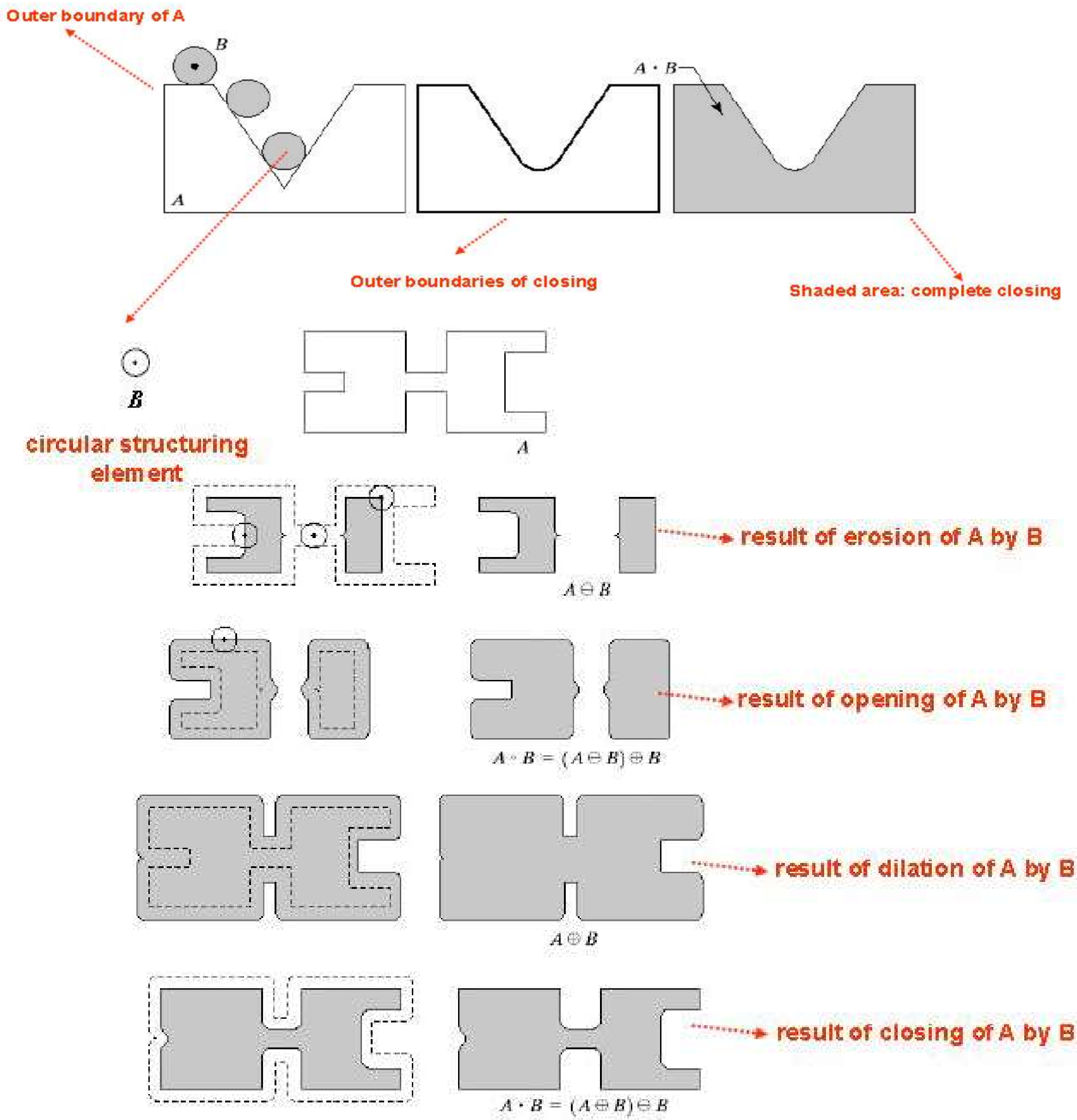
4.1.5.2 <u>Closing</u>

The process of dilation followed by erosion is called closing. It has the effect of filling small and thin holes, connecting nearby objects and smoothing the boundaries/contours of the objects.
Given set A and the structuring element B. Closing of A by structuring element B is defined by:

$$A \bullet B = (A \oplus B) \ominus B$$

The closing has a similar geometric interpretation except that we roll B on the outside of the boundary.
The opening operation can also be expressed by the following formula:

$$A \bullet B = \bigcup \{ (B_z) \mid (B_z) \cap A \neq \varnothing \}$$

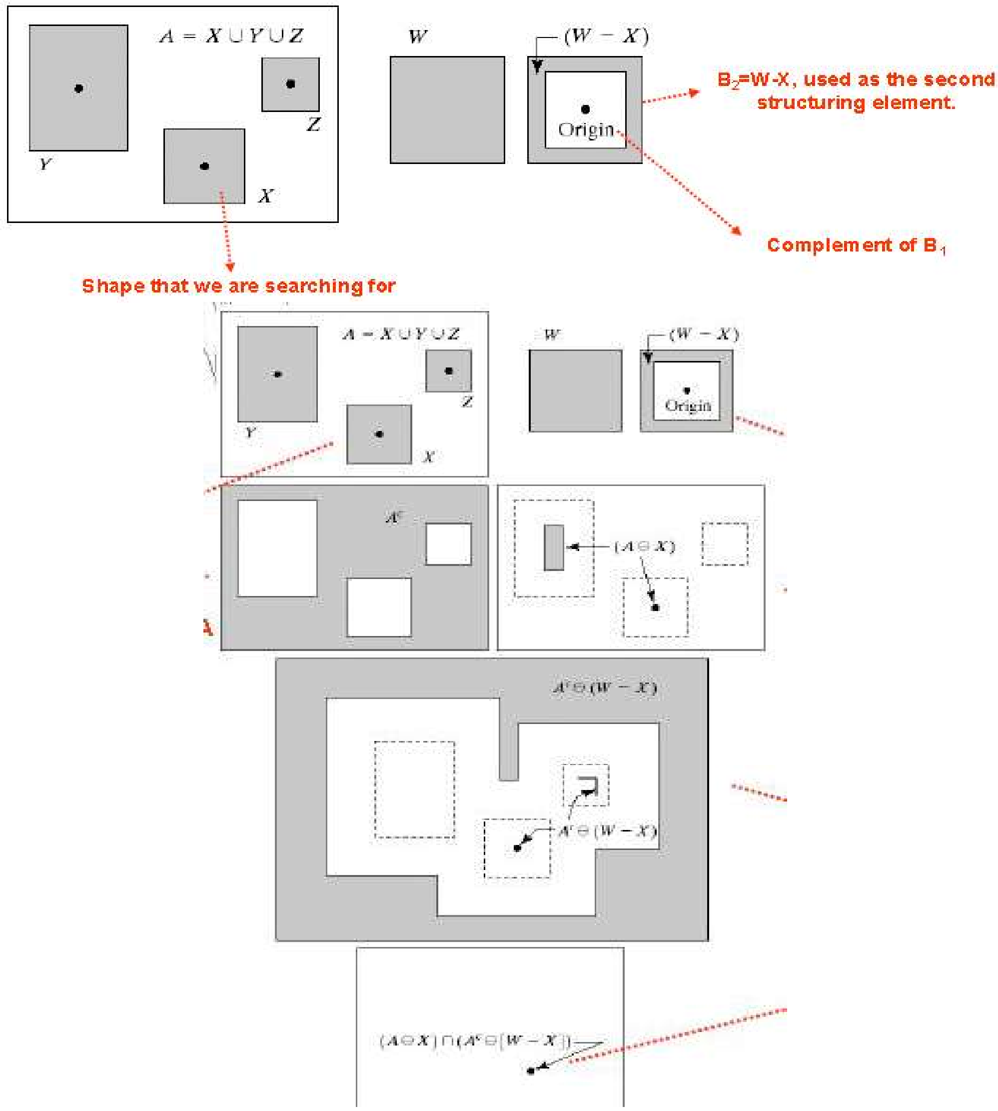


4.1.6 <u>Hit or Miss Transformation (Template Matching)</u>

Hit-or-miss transform can be used for shape detection/ Template matching.
Given the shape as the structuring element B1 the Hit-or-miss transform is defined by:

$$A \circledast B = (A \ominus B_1) \bigcap (A^c \ominus B_2)$$

Where B2 =W-X and B1=X. W is the window enclosing B1. Windowing is used to isolate the structuring element/object.



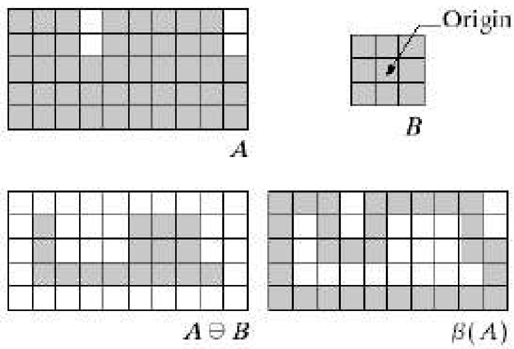
$A = X \cup Y \cup Z$
Y
Z
X
W
$(W - X)$
Origin
B_2=W-X, used as the second structuring element.
Complement of B_1
Shape that we are searching for
A^c
$(A \ominus X)$
$A^c \ominus (W - X)$
$(A \ominus X) \cap (A^c \ominus (W - X))$

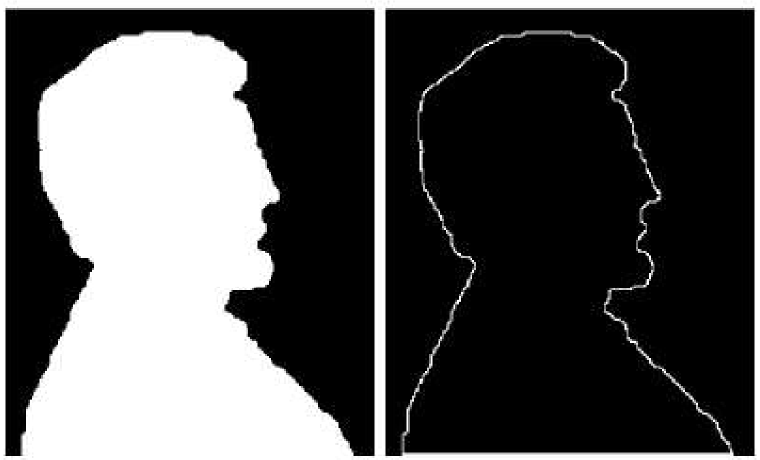
4.1.7 **Morphological Algorithms**

4.1.7.1 Boundary Extraction

The boundaries/edges of a region/shape can be extracted by first applying erosion on A by B and subtracting the eroded A from A.

$$\beta(A) = A - (A \ominus B)$$





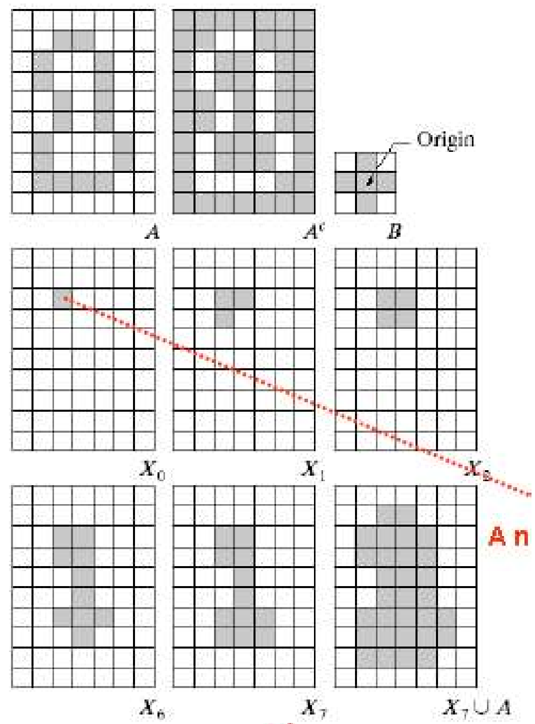
4.1.7.2 Region Filling

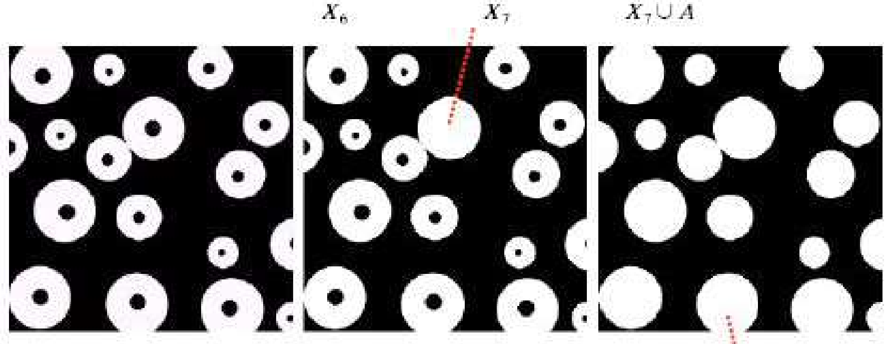
Region filling can be performed by using the following definition. Given a symmetric structuring element B, one of the non-boundary pixels (Xk) is consecutively diluted and its intersection with the complement of A is taken as follows:

$$X_k = (X_{k-1} \oplus B) \cap A^c \qquad \begin{array}{r} k = 1,2,3,\ldots \\ \textit{terminates when } X_k = X_{k-1} \\ X_0 = 1 \; (\textit{inner pixel}) \end{array}$$

Following consecutive dilations and their intersection with the complement of A, finally resulting set is the filled inner boundary region and its union with A gives the filled region F(A).

$$F(A) = X_k \cup A$$





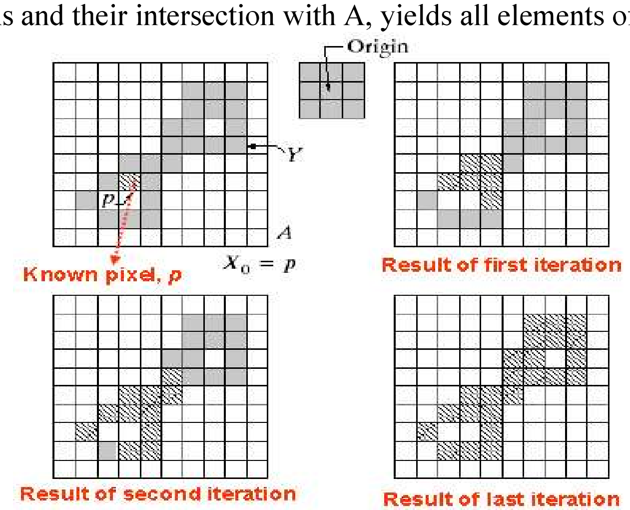
4.1.7.3 Connected Component Extraction

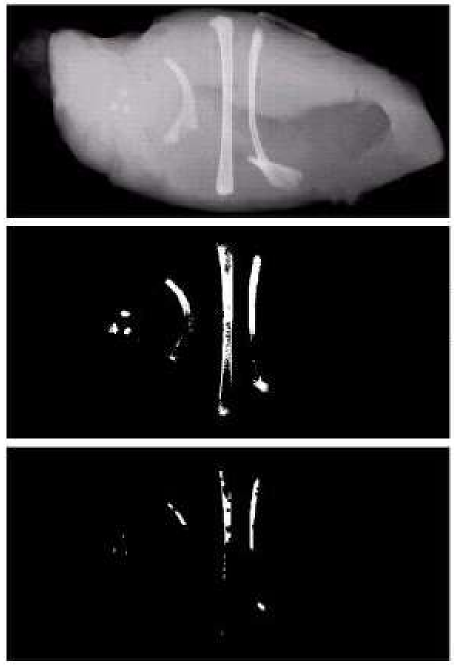
The following iterative expression can be used to determine all the pixels in component Y which is in A.

$$X_k = (X_{k-1} \oplus B) \cap A$$

X0=1 corresponds to one of the pixels on the component Y. Note that one of the pixel locations on the component must be known.

Consecutive dilations and their intersection with A, yields all elements of component Y.





4.1.7.4 Thinning & Thickening

4.1.7.4.1 Thinning

Thinning of A by the structuring element B is defined by:

$$A \otimes B = A - (A \circledast B)$$

hit-or-miss transform/template matching

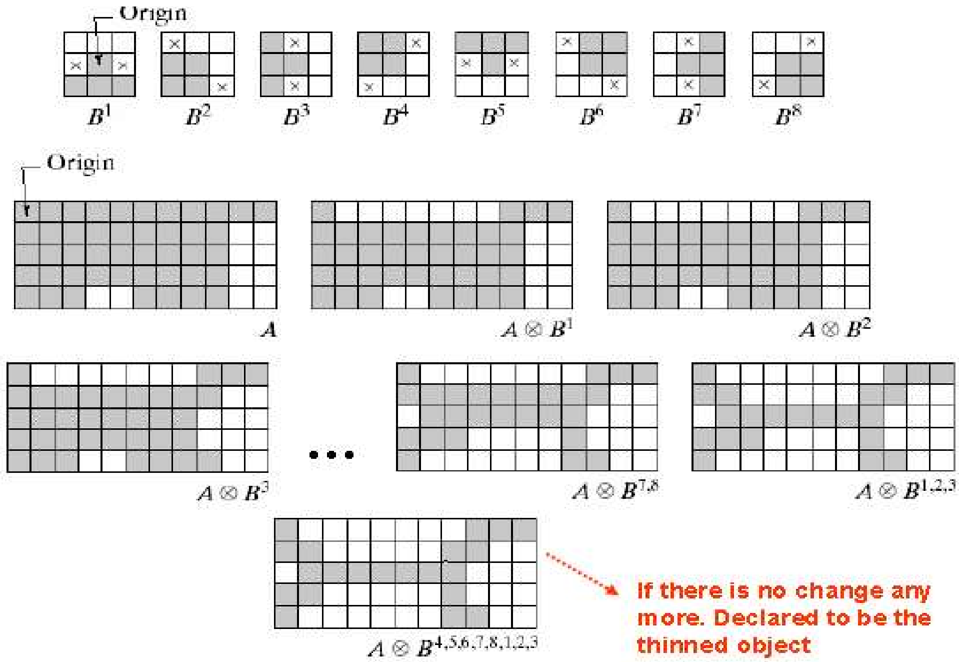
Note that we are only interested in pattern matching of B in A, so no background operation is required of the hit-miss-transform.

$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$

The structuring element B consists of a sequence of structuring elements, where Bi is the rotated version of Bi-1. Each structuring elements helps thinning in one direction. If there are 4 structuring elements thinning is performed from 4 directions separated by 90o. If 8 structuring elements are used the thinning is performed in 8 directions separated by 45o.
The process is to thin A by one pass with B1, then the result with one pass of B2, and continue until A is thinned with one pass of Bn.

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

The following sets of structuring elements are used for thinning operation.



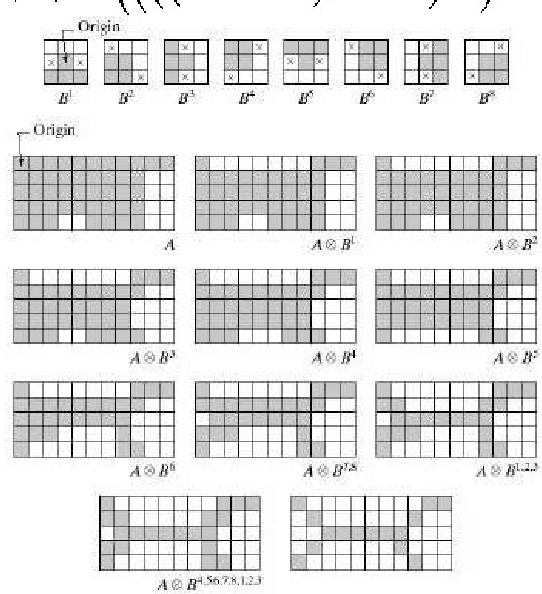
4.1.7.4.2 <u>Thickening</u>

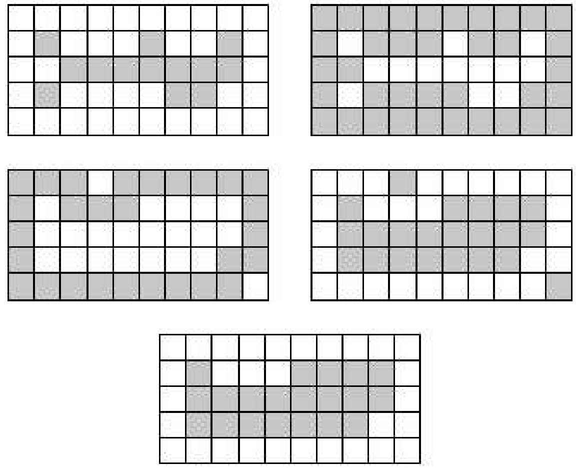
Thinning and is defined by the expression

$$A \odot B = A \cup (A \circledast B)$$

As in thinning thickening can be defined as a sequential operation;

$$A \odot \{B\} = \left(\left(\left(\left(A \odot B^1\right) \odot B^2\right) \ldots\right) \odot B^n\right)$$





4.1.7.5 <u>Skeletons</u>

$$S(A) = \bigcup_{k=0}^{k} S_k(A)$$

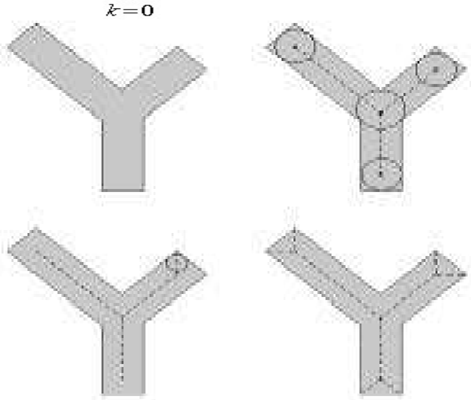
$$S_k(A) = (A \Theta kB) - (A \Theta kB) \circ B$$

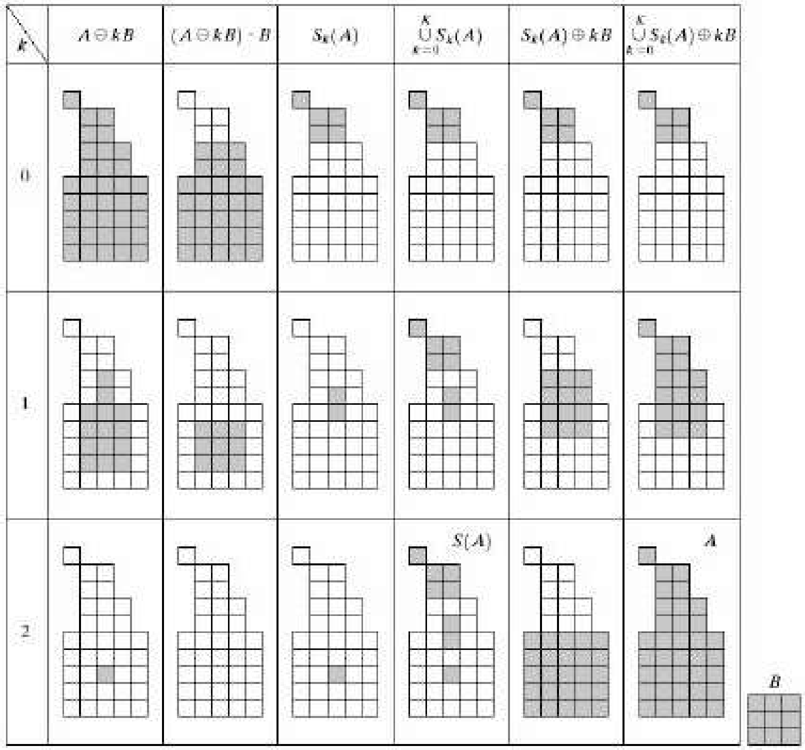
$$(A \Theta kB) = (.....(A \Theta B) \Theta B) \Theta .....) \Theta B$$

$$K = \max \left\{ k \,\middle|\, (A \Theta kB) \neq \varnothing \right\}$$

A can be reconstructed from these subsets by using the equation

$$A = \bigcup_{k=0}^{k} (S_k(A) \oplus kB)$$





4.1.7.6 Pruning

Pruning methods are an essential complement to thinning and skeleton zing algorithms because these procedures tend to leave parasitic components that need to be “cleaned up”.