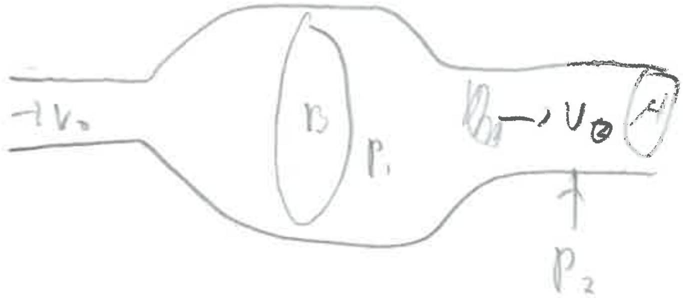
2. Placing another turbine hinders the velocity of water. This is analogous to placing a resistor in series with another one. ~~The current gets smaller will power dissipated by those resistors remain constant.~~

$p + \frac{1}{2}\rho v^2 + \rho g h$ constant

$h = 0$ since flat

~~Assume p · t~~

$p + \frac{\rho}{2}v^2 =$ constant



$V_1 B = V_2 A$

$\to V_2 = \frac{V_1 B}{A} = V_0$

$P_1 + \rho\frac{V_1^2}{2} = P_2 + \frac{V_0^2}{2}\rho$

$P_2 = P_1 + \frac{\rho}{2}(V_1^2 - V_0^2)$

$= P_1 + \frac{\rho V_0^2}{2}(\frac{A^2}{B^2} - 1)$

$= P_1 - \frac{\rho V_0^2}{2}(1 - \frac{A^2}{B^2})$

$\Delta P = P_1 - P_2$

$= \rho\frac{V_0^2}{2}(1 - \frac{A^2}{B^2})$

since $V_2 = V_0$, $\Phi_1 = \Phi_0$

$\Phi = 2\Phi_0 = A V_0^3 (1 - \frac{A^2}{B^2})e$ ✓ units.

This is consistent with conservation of energy, as long as the total pressure drop is less than ΔP, You'll increase power generated. If pressure change is equal to or larger than ΔP, I guess power generated by this system is equal to power provided by pump.

Okay.

3. $V = V_0$, across all pipes with cross sectional area A

$\Phi_n = n\Phi_0$ ✗

$\frac{\Phi_n}{\Phi_0} = n$

B.

1. $\lambda_w = \alpha V_w$

λ_w : $\frac{N}{m} = \frac{kg\, m\, s^{-2}}{m} = kg\, s^{-2}$

V_w : m/s

$\therefore \quad kg\, s^{-2} = \alpha \cdot m s^{-1}$

$\alpha = kg\, m^{-1} s^{-1}$ ✓

$v_c = \beta V_w$

clearly, β is dimensionless ✓

2. Force due to wind :

~~$dF = \lambda_w\, dt = \alpha V_w \cdot dt$~~ $\rightarrow$ $F = \lambda_w \cdot R = R \alpha V_w$

This balances with F_v

F is also in Y direction, as that's how λ_w is defined.

~~$dv(r) = dv_c(r) = d\beta v_w$~~ $\quad dv_c(r) = d\beta V_w$

~~$= \beta\, dV_w$~~ $\quad = \beta\, dV_w$

~~$\therefore \lambda R \alpha V_w = \beta\eta \frac{dv_w}{dr}$~~

~~$\int_r^R \lambda \alpha R\, dr = \beta\eta \frac{dV_w}{V_w}$~~

$R \alpha V_w = \eta \frac{dv(r)}{dr}$

$\int_r^R V_w \alpha R\, dr = \eta \int_{v(r)}^{v_c} dv$ $\quad$ I don't think V_w relates to $dv(r)$

~~$\lambda \alpha \frac{R^2 - r^2}{2} = \eta \ln \frac{v_c}{v(r)}$~~

$V_w \alpha R (R-r) = \eta (v_c - v(r))$

$= \eta (\beta V_w - v(r))$

$\eta v(r) = V_w (\eta\beta - \alpha R(R-r))$

$v(r) = \frac{V_w(\eta\beta - \alpha R(R-r))}{2\eta}$ close

~~Should~~

3. According to equation, speed varies linearly from centre to ~~wave~~ front, with speed being largest at wall front
fire

This makes sense, as all the forces are on the fire fronts and the fire front drags the fire at the centre due to viscosity.

7.

~~In degrees,~~

In degrees : $V = V_f e^{\ln 2 \cdot \frac{\theta}{10}}$

In radians : $V = V_f e^{\ln 2 \cdot \frac{\theta \cdot 18}{\pi}}$

$a = \ln 2$

This works because $e^{\ln 2 \cdot \frac{\theta}{10}} = (e^{\ln 2})^{\frac{\theta}{10}}$

$= 2^{\frac{\theta}{10}}$

8. $t_{up} = \frac{D_{up}}{V_{up}} = \frac{D}{\cos\theta \cdot 2^{\frac{18\theta}{\pi}} \cdot V_f}$

$t_{down} = \frac{D_{down}}{V_{down}} = \frac{D}{\cos\alpha \cdot 2^{-\frac{18\alpha}{\pi}} \cdot V_f}$

$t_{flat} = \frac{2D}{V_f}$

$\alpha = \theta$

$t_{total} = \frac{D}{V_f}\left(\frac{1}{\cos\theta\, 2^{k}} + \frac{1}{\cos\theta\, 2^{-k}}\right)$ where $k = \frac{\theta}{10}$

$= \frac{D}{V_f}\left(\frac{2^{k} + 2^{-k}}{\cos\theta}\right)$

$\therefore$ $t_{total} \geq t_{flat}$

when $\theta = 0$, $t_{total} = \frac{2D}{V_f} = t_{flat}$ ✗

when $\theta > 0$, $2^{k} + 2^{-k}$ increases while $\cos\theta$ decreases, so t_{total} will be even bigger.

$\therefore$ move more quickly on flat terrain

9.

C.

1. (a) ~~Magnetic~~

The first ~~magnetic~~ maxwell equation can stay the same

$\oint B \cdot ds = 0$ becomes $\oint B \cdot ds = \frac{q_{m,enc}}{\mu_0}$, q_m is magnetic "charge"

~~Consequently, the third la~~

~~I guess the third law can still be written as~~

~~$\oint E \cdot dl = -\frac{d\varphi_B}{dt}$, so it doesn't change~~

The third law changes as ~~e~~ electric field can be caused by "~~mag~~ magnetic current"

~~Moving magnetic monopoles could theoretically produce "magnetic current"~~

~~so $\oint E \cdot dl = \frac{d\varphi_B}{dt} + \frac{dq_E}{dt} \cdot \varepsilon_0$~~

The fourth law doesn't change, as it is ~~oo~~ magnetic fields produced by electricity,

(b) $I_m = \frac{dq_m}{dt}$

$J_m = \frac{dI_m}{dA}$

$\therefore [J_m] = \frac{[q_m]}{s \cdot m^2}$

$[E] = \frac{[F]}{[q]} = kgms^{-2}c^{-1}$

$[B] = \frac{[F]}{[q][v]} = kgs^{-1}c^{-1}$

Assuming symmetry

~~$F = q_m v E$~~

~~$E = \frac{F}{q}$~~

~~$\therefore F = q_m v \frac{F}{q}$~~

~~$q_m = \frac{q}{v}$~~

~~$= \cancel{c m^{-1}} s$~~

~~$\rightarrow J_m = c m^{-3}$~~

$F_q = B q_m$

$[q_m] = \frac{[F]}{[B]}$

$= \frac{kgms^{-2}}{kgs^{-1}c^{-1}}$

$= ms^{-1}c$ ✓

$[J_m] = m^{-1}s^{-2}c$ ✓

(c) $F = q_m v^\alpha B^\beta$

$kg\, m s^{-2} = m s^{-1} C\, m^\alpha s^{-\alpha}\, kg^\beta m^\beta s^{-2\beta} C^{-\beta}$ $\quad \beta = 1$

$\rightarrow kg\, m s^{-2} = m^{2+\alpha} s^{-3-\alpha} \cdot kg$

$\rightarrow \alpha = -1$

$\therefore F_m = q(B + \frac{1}{v} \times E)$

charge

Gauss's law doesn't change

$\oint B \cdot ds = \frac{q_{m,enc}}{\mu_0}$ ×

$\oint E \cdot dl = \frac{-d}{dt} \iint B \cdot ds - \frac{1}{\varepsilon_0} \iint J_m \cdot ds$ ×

Ampere's law doesn't change

Scrumptious Solenoids

2. (a) $r(\frac{H}{2}) = R$

$r(0) = r(H) = 0$

$\therefore r(z) =$

$(1 - |z - \frac{H}{2}| \cdot \frac{2}{H})^2 \cdot R$

$= (1 - |\frac{2z}{H} - 1|)^2 \cdot R$

absolute value

c)

b) $B = \cancel{I_{sol} \mu_0 n_{sol}}$

$I_{sol} \mu_0 n_{sol}$

$= I_s \cos(wt) \mu_0 n_{sol}$

$\varphi_B = AB$

$= \pi R^2 B$

$= I_s \cos(wt) \mu_0 n_{sol} \pi R^2$

$-\frac{d\varphi_B}{dt} = w I_s \sin(wt) \mu_0 n_{sol} \pi r(z)^2$

Let $w I_s \sin(wt) \mu_0 n_{sol} = k$

$\therefore -\frac{d\varphi_B}{dt} = k \pi r(z)^2$

$\cancel{= k (1-|\frac{2z}{H}-1|)^2 R^2 \cdot \pi \cdot R^2}$

$\cancel{= k (1-|\frac{2z}{H}-1|)^4 \pi R^4}$

$\varepsilon = \int d\varepsilon = \cancel{\text{integral ...}} \, dz$

$\int_0^H k 2\pi r(z) dz \cdot N$

$= k 2\pi R \int_0^H (1-|\frac{2z}{H}-1|)^2 dz \cdot N$

$= k 2\pi R \left[\int_{\frac{H}{2}}^{H} (1-|\frac{2z}{H}-1|)^2 dz + \int_0^{\frac{H}{2}} (1-|\frac{2z}{H}-1|)^2 dz\right] \cdot N$

when $z > \frac{H}{2}$

$\varepsilon = k 2\pi R \int_{\frac{H}{2}}^{H} (2-\frac{2z}{H})^2 dz$

$= 2 \cdot \frac{1}{2} k \pi R H \cdot \frac{1}{3} = \frac{1}{3} k \pi R H$

when $z < \frac{H}{2}$

$\varepsilon = k 2\pi R \int_0^{\frac{H}{2}} |\frac{2z}{H}|^2$

$= k 2\pi R \cdot \frac{H}{6}$

$= \frac{1}{3} k \pi R H$

$\therefore \varepsilon_{tot} = 2\varepsilon = k \pi R H \cdot N$

$= w I_s \sin(wt) \mu_0 n_{sol} \pi R H N \cdot \frac{2}{3}$

Dynamic Detectors

$$\frac{N_f kT}{2} = \frac{1}{2} m_e v_i^2$$

Temperature unknown

$$\frac{dv}{dt} = \frac{qEJ - mv}{mJ}$$

$$\int \frac{dv}{qEJ - mv} = \int \frac{dt}{mJ}$$. J is constant I suppose

$$\int \frac{dv}{a+bv} = \ln|a+bv| \cdot \frac{1}{b}$$

$$\therefore \ln|qEJ - mv| \cdot \frac{-1}{m} = \frac{t}{mJ} + c'$$

$$\ln|qEJ - mv| = \frac{-t}{J} + c$$

$$qEJ - mv = Ae^{\frac{-t}{J}}$$

$$v(t) = \frac{qEJ - Ae^{\frac{-t}{J}}}{m_e}$$

Assuming we know initial temperature to get thermal motion.

$$\frac{3kT}{2} = \frac{1}{2} m_e v_i^2$$

$$v_i = \sqrt{\frac{3kT}{m_e}}$$

when $t=0$, $v = v_i$

$$\frac{qEJ - A}{m_e} = \sqrt{\frac{3kT}{m_e}}$$

$$qEJ - A = \sqrt{3kTm_e}$$

$$A = qEJ - \sqrt{3kTm_e}$$

$$\therefore v(t) = \frac{qEJ - (qEJ - \sqrt{3kTm_e})e^{\frac{-t}{J}}}{m_e}$$

(b)

as $t \gg \tau$, $v(t) \to \frac{qE\tau}{m}$

thus, v_D for electrons is larger than v_D for holes,

as ~~m_e~~ $m_e < m_i$

(c)

$p = q d$

$[p] = C \cdot m$

$F = qE$

$[E] = \frac{[F]}{[q]} = kg\, m s^{-2} C^{-1}$

$\therefore [\alpha]\ C^2\, kg^{-1}\, s^2$

E $\Gamma\Omega$

1. $L = T - V$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\varphi}^2 - mgx\sin\alpha$$

$\uparrow$

minus instead of plus, small problem

~~Additionally, $J = I\dot{\varphi}$, which is not 0 else cylinder won't be rolling.~~

The main problem is that there is friction which causes ball to roll. Lagrangian mechanics fails when there is friction. That's why you get $I\ddot{\varphi} = 0$ ✗

2.

~~$dx = R\,d\varphi$~~

~~How to get $\lambda(t)$?~~

$$dx = R\,d\varphi$$

$$\therefore a_x(x,\varphi) = 1, \quad a_\varphi(x,\varphi) = R$$

$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\varphi}^2 - mgx\sin\alpha$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 1 \cdot \lambda(x)$$

$$m\ddot{x}^2 + mg\sin\alpha = \lambda(x)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = R \cdot \lambda(\varphi)$$

$$I\ddot{\varphi} = R\,\lambda(\varphi)$$

$\longrightarrow$ Part F is on next few pages

F - ANC

1.

2. $d(t) = m\ddot{z} + \gamma\dot{z} + kz$

$z = At\,e^{-\alpha t}$

$I = P_0 = Ft$

$F = \frac{P_0}{t}$

$\cancel{\frac{P_0}{t} = mAt\,\alpha^2 e^{-\alpha t} - 2\sqrt{km}\cdot\alpha At\,e^{-\alpha t} + kAt\,e^{-\alpha t}}$

$\cancel{\frac{P_0}{t} = At\,e^{-\alpha t}(m\alpha^2 - 2\sqrt{km}\,\alpha + k)}$

when $d(t) = 0$

$0 = m\alpha^2 At\,e^{-\alpha t} - 2\alpha\sqrt{km}\,e^{-\alpha t}\cdot At + k\,e^{-\alpha t}\cdot At$

$\rightarrow\ 0 = m\alpha^2 - 2\sqrt{km}\cdot\alpha + k$

$0 = \alpha^2 - 2\sqrt{\frac{k}{m}}\cdot\alpha + \frac{k}{m}$

$(\alpha - \sqrt{\frac{k}{m}})^2 = 0$

$\alpha = \sqrt{\frac{k}{m}}$, A cancels out so A doesn't matter

$\cancel{\frac{P_0}{t} = mAt\cdot\frac{k}{m}e^{-\sqrt{\frac{k}{m}}t} - 2\sqrt{km}\cdot\sqrt{\frac{k}{m}}At\,e^{-\sqrt{\frac{k}{m}}t} + kAt\,e^{-\sqrt{\frac{k}{m}}t}}$

$\cancel{= kAt\,e^{-\alpha t} - 2kAt\,e^{-\alpha t} + kAt\,e^{-\alpha t}}$

$= 0$

choose $A=1$ for simplicity and $z(t) = t\,e^{-\alpha t}$

3. $m\ddot{z}_1 + d\dot{z}_1 + kz_1 = 0$

$m\ddot{z}_2 + d\dot{z}_2 + kz_2 = 0$

$\rightarrow \quad m(\ddot{z}_1 + \ddot{z}_2) + d(\dot{z}_1 + \dot{z}_2) + k(z_1 + z_2) = 0$

thus $z_3 = z_1 + z_2$ also solution