1] Two-year American Put option

Given: → Current stock Price (S_0) = Rs 150

→ Strike Price (K) = Rs 170

→ Stock price movement for year 1 : $u_1 = 1.10$ & $d_1 = 0.90$

year 2 : $u_2 = 1.15$ & $d_2 = 0.85$

→ Risk-free interest rate (r) = 0.06

→ $P_1 = \frac{e^{0.06} - d_1}{u_1 - d_1} = \frac{e^{0.06} - 0.9}{1.1 - 0.9} = \frac{1.06 - 0.9}{0.2} \approx 0.8$

$P_2 = \frac{e^{0.06} - d_2}{u_2 - d_2} = \frac{1.06 - 0.85}{0.3} \approx 0.709$

→ $S_0 = 150$

$S_{1u} = 150 \times 1.1 = 165$, $S_{1d} = 150 \times 0.9 = 135$

$S_{2uu} = 165 \times 1.15 = 189.75$, $S_{2dd} = 135 \times 0.85 = 114.75$

$S_{2ud} = S_{2du} = 165 \times 0.85 = 140.25$

→ Intrinsic Values ;-

$P_{2uu} = \max(170 - 189.75, 0) = 0$

$P_{2ud} = P_{2du} = \max(170 - 140.25, 0) = 29.75$

$P_{2dd} = \max(170 - 144.75, 0) = 55.25$

→ At S_{1u} : $P_{1u} = \max(170 - 165, e^{-0.06} \times (0.8 \times 0 + 0.191 \times 29.75)) = 8.21$

At S_{1d} : $P_{1d} = \max(170 - 135, e^{-0.06} \times (0.8 \times 29.75 + 0.191 \times 55.25)) = 35$

$P_0 = e^{-0.06} \times (0.8 \times 8.21 + 0.191 \times 35) \approx 12.23$

∴ Price of 2-yr American put option is Rs 12.23.

2) Two-Year European Call option

Given: $S_0 = Rs\ 50$

$K = Rs\ 50$

$u = 1.2$, $d = 0.8$

$r = 0.06$

$$\rightarrow p = \frac{e^{0.06} - d}{u - d} \approx \frac{1.06 - 0.8}{0.4} \approx 0.65$$

$\rightarrow S_0 = 50$

$S_{1u} = 50 \times 1.2 = 60$, $S_{1d} = 50 \times 0.8 = 40$

$S_{2uu} = 60 \times 1.2 = 72$, $S_{2dd} = 40 \times 0.8 = 32$

$S_{2ud} = S_{2du} = 60 \times 0.8 = 48$

$\rightarrow C_{2uu} = \max(72 - 50, 0) = 22$

$C_{2ud} = C_{2du} = \max(48 - 50, 0) = 0$

$C_{2dd} = \max(32 - 50, 0) = 0$

$\rightarrow$ At S_{1u}: $C_{1u} = e^{-0.06} \times (0.65 \times 22 + 0.34 \times 0) \approx 13.59$

At S_{1d}: $C_{1d} = e^{-0.06} \times (0.65 \times 0 + 0.34 \times 0) = 0$

$$\rightarrow C_0 = e^{-0.06} \times (0.65 \times 13.59 + 0.34 \times 0) \approx 8.39$$

$\therefore$ Price of the 2-yr European call option is Rs 8.39.