

Digital communication

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$$x(t) = \begin{cases} t & t \in [0, \frac{1}{2}) \\ t-1 & t \in [\frac{1}{2}, 1] \end{cases}$$

$$\begin{aligned} x(t) &= \sum_{n=-\infty}^{\infty} \langle x(t), \phi_n(t) \rangle \phi_n(t) \\ &= \sum_{n=0}^{\infty} \langle x(t), \phi_n(t) \rangle \phi_n(t) \end{aligned}$$

a)

$$\begin{aligned} * \hat{x}(n) &= \langle x(t), \phi_n(t) \rangle \\ &= \frac{1}{1} \int_{-\infty}^{\infty} x(t) \cdot \overline{\phi_n(t)} \cdot dt \\ &= 1 \int_0^1 x(t) \cdot \phi_n(t) \cdot dt \end{aligned}$$

We can observe for odd coefficients of n .

$$\hat{x}(n) = \int_0^{\frac{1}{2}} x(t) \cdot \phi_n(t) + \int_{\frac{1}{2}}^1 x(t) \cdot \phi_n(t) \cdot dt$$

$$\hat{x}(0) = \int_0^{\frac{1}{2}} t \cdot dt + \int_{\frac{1}{2}}^1 (t-1) dt$$

$$= \left[\frac{t^2}{2} \right]_0^{\frac{1}{2}} + \left[\frac{t^2}{2} - t \right]_{\frac{1}{2}}^1$$

$$= \frac{1}{8} + \left[\frac{1}{2} - 1 - \left(\frac{1}{8} - \frac{1}{2} \right) \right] = 0$$

$$\hat{x}(1) = \int_0^{\frac{1}{2}} t \cdot dt + \int_{\frac{1}{2}}^1 -t+1 \cdot dt$$

$$= \left[\frac{t^2}{2} \right]_0^{\frac{1}{2}} + \left[-\frac{t^2}{2} + t \right]_{\frac{1}{2}}^1$$

$$= \frac{1}{8} - \frac{1}{8} + 1 + \frac{1}{8} - \frac{1}{2} = \frac{1}{4}$$

observed

$$\phi_{k+1}(t) = \begin{cases} \phi_k(2t) & , t \in [0, \frac{1}{2}] \\ -\phi_k(2t-1) & , t \in [\frac{1}{2}, 1] \end{cases}$$

$$x(t) = \begin{cases} t & , t \in [0, \frac{1}{2}] \\ t-1 & , t \in [\frac{1}{2}, 1] \end{cases}$$

$$\hat{x}(0) = \int_0^{\frac{1}{2}} x(t) \phi_0(t) dt + \int_{\frac{1}{2}}^1 x(t) \phi_0(t) dt$$

$$= 0$$

$$\hat{x}(1) = \int_0^{\frac{1}{2}} x(t) \phi_1(t) dt + \int_{\frac{1}{2}}^1 x(t) \phi_1(t) dt$$

$$= \frac{1}{4}$$