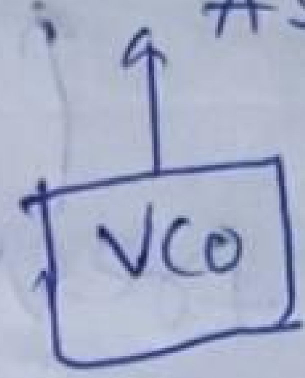
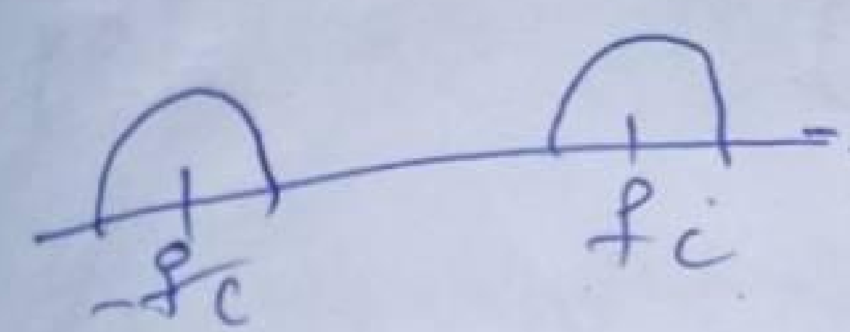
3. phase-locked loop:-

$$x(t) = A_c \cos(2\pi f_c t + \phi(t))$$

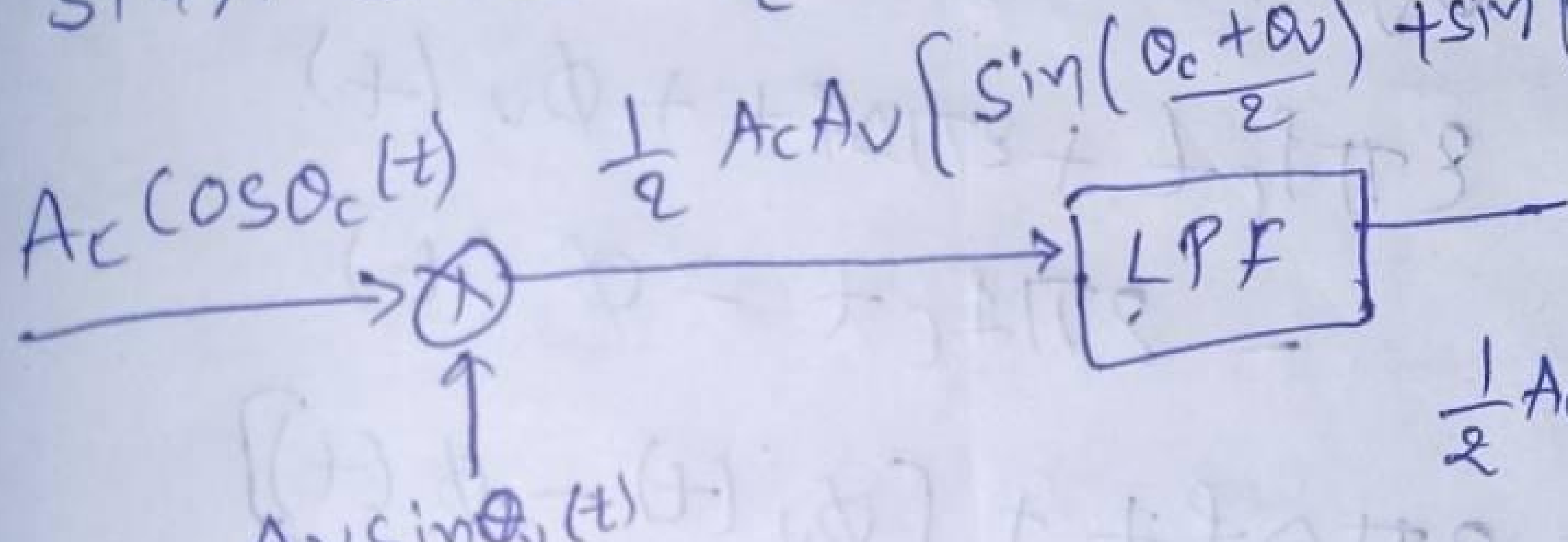
Pass band signal

$A \sin(2\pi f_c t + \phi)$





$$\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$$



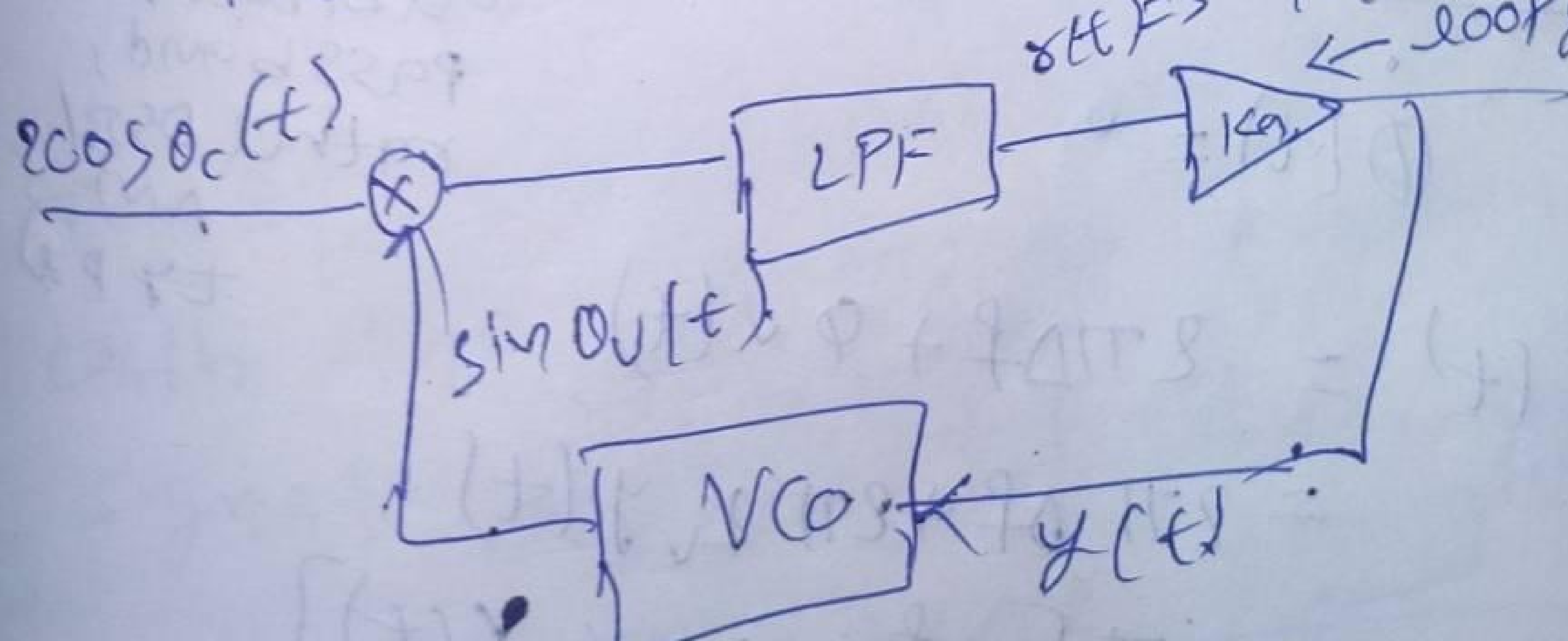
$$\frac{1}{2} A_c A_v \sin\left(\frac{\theta_v - \theta_c}{2}\right)$$

$$= \frac{1}{2} A_c A_v \sin(\epsilon(t))$$

assume: $\theta_v(t) = \theta_c(t) + 2\epsilon(t)$

$\Rightarrow \theta_c(t) = 2\pi f_c t + \phi(t)$ ← Arbitaly Passband signal.

$A_c = 2$



$$\theta_v(t) = 2\pi f_N t + \phi_v(t)$$

$f_v = f_c + \Delta f$, at $y(t) = 0$ (bias)

$$\phi_v(t) = 2\pi K_v \int_0^t y(\tau)\,d\tau$$ ← frequency modulatn

↑ frequency Deviation

$$\dot{\phi}_v(t) = 2\pi K_v y(t)$$

$$\epsilon(t) = \theta_v(t) - \theta_c(t)$$

$$= 2\pi f_c t + 2\pi\Delta f t + \phi_v(t) - 2\pi f_c t - \phi(t)$$

$$= 2\pi\Delta f t + [\phi_v(t) - \phi(t)]$$

Differentiating :-

$$\dot{\epsilon}(t) = 2\pi\Delta f + \dot{\phi}_v(t) - \dot{\phi}(t)$$

Assumptn :- Let $\phi(t) = \phi_0$ (The signal is not arbitrary passband, rather DSB/AM type)

$$\dot{\phi}(t) = 0$$

$$\dot{\epsilon}(t) = 2\pi\Delta f + \dot{\phi}_v(t)$$

$$= 2\pi\Delta f + 2\pi K_v y(t)$$

$$= 2\pi\{\Delta f + K_v . K_a Y(t)\}$$

$\dot{\epsilon}(t) = \Delta f + K.\sin\epsilon(t)$

$$\boxed{\frac{1}{2\pi K}\dot{\epsilon}(t) - \sin\epsilon(t) = \frac{\Delta f}{K}}$$

↑ Dynamics of PLL

Let us analyse steady state

At steady state,

$\epsilon(t) = \epsilon_{ss}$

$\dot{\epsilon}(t) = 0$

$-\sin\epsilon_{ss} = \frac{\Delta f}{K}$

$$\boxed{\epsilon_{ss} = -\sin^{-1}\left(\frac{\Delta f}{K}\right)}$$

$y_{ss} = K_a \sin\epsilon_{ss}.$

$= K_a \frac{\Delta f}{K} = \frac{\Delta K}{K_v}$

Vco output:

$V_{ss} = \sin(2\pi f_c t + \phi_0 - \epsilon_{ss})$.

if $\frac{\Delta f}{K} << 1$, then V_{ss} looks with the incoming passband signal $2\cos(2\pi f_c t + \phi_0)$.

$K = K_a . K_v >> \Delta f$ — loop gain

$\Rightarrow$ $\boxed{K_a >> \frac{\Delta f}{K_v}}$

If $\frac{\Delta f}{K} \approx 0$, $\epsilon_{ss} \approx 0$

After $t_0 > 0$,
$\epsilon(t)$ is small,
$\sin \epsilon(t) \approx \epsilon(t)$.

$$\frac{1}{2\pi K} \dot{\epsilon}(t) - \epsilon(t) = 0$$

$$\dot{\epsilon}(t) = \epsilon(t_0) e^{-2\pi K(t - t_0)}$$

$\frac{1}{2\pi K} \rightarrow$ time constant

As long as $\phi(t)$ varying slowly in comparision of $\frac{1}{2\pi K}$, then the PLL will work.

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* Linearised PLL loop equations in time domain

$$\theta_o = \frac{KF(p)}{p} A \theta_e + \frac{K_V}{p} V_e$$

$$= \frac{K_V}{p} \{ F(p) K_d A \theta_e + V_e)$$

$$\theta_e = \theta_i - \theta_o$$

* Transformatⁿ of signals & system into complex ~~freq.~~ domain S by means of Laplace transform

$$p = s$$

$$\theta_i(s) = L \{ V_c(t) \}$$

$$\theta_o(s) = \frac{KF(s)}{s} A \theta_e(s) + \frac{K_V}{s} V_e(s)$$

$$V_e(s) = 0$$

$$\theta_e(s) = \theta_i(s) - \theta_o(s)$$

$$\theta_o(s) = \frac{AK F(s)}{s} [\theta_i(s) - \theta_o(s)]$$

$$\left(1+\frac{AKF(s)}{s}\right)\theta_o(s) = \frac{AKF(s)}{s}\theta_i(s)$$

$$\theta_o(s) = \frac{AKF(s)}{s+AKF(s)}\theta_i(s) = H(s)\,\theta_i(s)$$

$H(s)$ is closed-loop transfer function

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