Time - 30 min

Marks - 8

1. Consider the functions in $\mathcal{L}^2[0,1]$.

$$\phi_0(t) = +1 \quad , t \in [0,1]$$

$$\phi_1(t) = \begin{cases} +1, & t \in [0, \frac{1}{2}) \\ -1, & t \in [\frac{1}{2}, 1] \end{cases}$$

$$\phi_2(t) = \begin{cases} +1, & t \in [0, \frac{1}{4}) \\ -1, & t \in [\frac{1}{4}, \frac{3}{4}) \\ +1, & t \in [\frac{3}{4}, 1] \end{cases}$$

$$\phi_3(t) = \begin{cases} +1, & t \in [0, \frac{1}{8}) \\ -1, & t \in [\frac{1}{8}, \frac{3}{8}) \\ +1, & t \in [\frac{3}{8}, \frac{1}{2}) \\ -1, & t \in [\frac{1}{2}, \frac{5}{8}) \\ +1, & t \in [\frac{5}{8}, 7/8) \\ -1, & t \in [7/8, 1) \end{cases}$$

⋮

and so on.

Observe, this set is a set of orthonormal basis functions in $\mathcal{L}^2[0,1]$.

Now consider the following function

$$x(t) = \begin{cases} t, & t \in [0, \frac{1}{2}) \\ t-1 & t \in [\frac{1}{2}, 1] \end{cases}.$$

a) Express $x(t)$ as a Fourier Series where the basis functions are $\{\phi_K(t)\}_{K=0}^{\infty}$.

b) What kind of convergence is achieved?

c) Compare this basis functions with complex exponentials.