26/2/20

Bernoulli trials : $P(H) = 0.7$
$P(T) = 0.3$

let $n = 10$

$P(HH \cdots H) = (0.7)^{10}$ – seems to be event with highest probability
– but atypical event
– least possibility of occurrence

→ $X_1, X_2, \cdots X_n$: $\frac{1}{n} \sum X_i \longrightarrow \mathbb{E}(X)$

consider log(pmf) – $\log(p(x_1)), \cdots, \log(p(x_n))$

apply LLN on $\log(p(x_i))$

$$\frac{1}{n} \sum_{i=1}^{n} \log(p(x_i)) \longrightarrow \mathbb{E}(\log(p(x_i))) = -H(X)$$

$$\frac{1}{n} \sum_{i=1}^{n} \log(p(x_i)) = \frac{1}{n} \log\left(\prod p(x_i)\right)$$

$$= \frac{1}{n} \log(p(x_1, x_2, \cdots x_n)) \longrightarrow -H(X)$$

$$p(x_1, x_2, \cdots x_n) \longrightarrow 2^{-nH(X)}$$

i.e., for a small ϵ

$$2^{-n(H(X)+\epsilon)} \leq p(x_1, x_2, \cdots x_n) \leq 2^{-n(H(X)-\epsilon)}$$

as $\epsilon \to 0$: $p(x_1, x_2, \cdots x_n) = 2^{-nH(X)}$

$H(X) = -0.7 \log(0.7) - 0.3 \log(0.3) = 0.88.$

$2^{-nH(X)} = 2^{-10 \times .88} = 2^{-8.8} = 2.24 \times 10^{-3}$

Cardinality of such set $\rightarrow 2^{nH(X)} = 2^{8.8}$

$|A_\epsilon| \longrightarrow 445.72$

$\therefore$ 7H, 3T will be most likely events – Typical event.

<u>Universal</u> <u>Coding</u> (Ziv-Lempel)

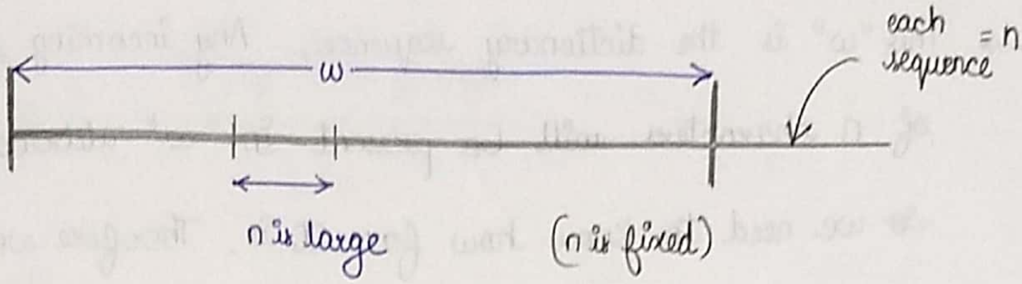
$X = \{a_1, a_2, \cdots a_M\}$ steady state have been reached

$\longrightarrow$ Stationary

$H(X|S) = H_\infty(X)$

normally $\lceil \log_2 M \rceil$ bits are required to represent M symbols $(a_1, a_2, \cdots a_M)$ $\equiv$ trivial coding

$\rightarrow$ For w characters taking together: $w \lceil \log_2 M \rceil$ (trivially)



$2^{-nH_\infty(X)}$ probability for each sequence (n symbol sequence)

$\Rightarrow$ there are $2^{nH_\infty(X)}$ such sequences.

$\quad\hookrightarrow$ each is equiprobable.

$n \cdot 2^{nH_\infty(X)}$ – Sequence of characters this long

– all $2^{nH_\infty(X)}$ (n symbol) sequences occur typically in this sequence.

<u>Eg</u>: $M = 64$, $\lceil \log M \rceil = 6$

$n = 10$, $H_\infty(X) = 3$

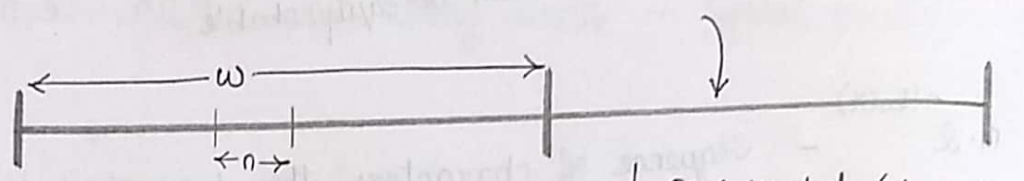
10×2^{30} – we can see all sequences of 10 bits long, where each one is equiprobable, in this much length string.

$$\boxed{w \sim 10 \times 2^{30}}$$

$$\therefore \boxed{w \sim n \times 2^{nH_\infty(X)}}$$

→ This "w" is the dictionary sequence. Any incoming sequence of n characters will be present in "w" aldready. So we need to know how far it is. Therefore we encode this distance.

→ Simply we are storing / encoding the pointers of n-length sequences in "w".

→ 

n is fixed

$L >> w$.

→ Do trivial coding for dictionary string (w):

$$w\lceil \log_2 M\rceil = \text{number of bits required for } w$$

→ # of new symbols = L

# of n-symbol sequences = $\frac{L}{n}$

# of bits for coding distance = $\log_2 w$

$$\frac{L}{n}\log w = \text{number of bits required for } L$$

→ Total number of bits $(w+L)$ = $w\lceil \log_2 w\rceil + \frac{L}{n}\log_2 w$

no. of bits per symbol $= \dfrac{\omega\lceil \log M \rceil + \frac{L}{n}\log \omega}{L+\omega}$

$\because L >> \omega$

$\approx 0 + \frac{1}{n}\log \omega = \frac{1}{n}\log \omega$

$= \frac{1}{n}\log\left(n \cdot 2^{nH_\infty(X)}\right)$

$= \frac{1}{n}\left[\log n + nH_\infty(X)\right]$

assymptotically $(n \to \infty)$

$\approx H_\infty(X)$

27/2/20

$X = \{a\ b\ c\ d\ e\ f\ g\ h\}$; $\lceil \log_2 M \rceil = 3$

$H_0(X) = 1$, $n = 4$, $w \sim n \cdot 2^{n H_0(X)} = 4 \times 2^4 = 64$.

- Markov chain.

← w : 64 char

38

aacaabcadfaadeacbabfggaaghadeabeaabbefgghahahbc--
deacdfgha|

* Trivial coding : with one extra bit

a - 1000
b - 1001
c - 1010
d - 1011
e - 1100
f - 1101
g - 1110
h - 1111

w: $\underset{a}{\underline{1000}}$ $\underset{a}{\underline{1000}}$ $\underset{c}{\underline{1010}}$ $\underset{a}{\underline{1000}}$ - (4×64 bits) ≡ 4w bits. dictionary

→ abfggadfhad(~~abbggh ahaaca~~)hh bcdeacd : dfgha abfgga

n=6 ← max, l=38 (abfgga); n=2, l=5 (df); n=3, l=31 (had); n=8, l=12 (bcdeacd); n=5, l=13 (dfgha abfgga)

hh ↳ this has not occur in w.

Run length

$n=6,\ l=38 \equiv \underset{l}{\underline{000000100110}}\ \underset{n}{\underline{000110}} \equiv$ abfgga

$n=2,\ l=5 \equiv \underset{l}{\underline{000101}}\ \underset{n}{\underline{0010}} \equiv$ df

$n=3,\ l=31 \equiv$

$\underline{000000\ 100110}$ $\underline{000110}$ $\underline{000101}$ $\underline{0010}$ $\underline{0000111111}$ $\underline{0011}$ $\underset{h}{\underline{1111}}$

$\underline{00001100}$ $\underline{00001000}$

* if there is no match, we go with defined trivial coding
* we can differentiate b/w trivial coding and run length-w matching by first bit.

First bit : 0 – match in w
1 – trivial coding

—— w |——sequence——→

* --- dfgha abfgga df had hh bcdeacd dfgha abfgga

$l = 5$

$n = 13$ (there is no upper bound

$\underset{l}{\underline{000101}}$ $\underset{n}{\underline{00001011}}$

→ Markov Chain: $\begin{bmatrix} & \\ & \end{bmatrix}_{8\times 8}$

→ ergodic.

eg:

$$\begin{array}{c} a \\ b \\ c \\ d \end{array}\begin{bmatrix} 0.25 & 0.30 & 0.15 & 0.30 \\ 0.50 & 0.20 & 0.10 & 0.20 \\ 0.10 & 0.40 & 0.40 & 0.10 \\ 0.60 & 0.10 & 0.15 & 0.15 \end{bmatrix}$$

$u = rnd[0,1]$

$u \in [0, 0.25) \Rightarrow$ choose $x(0) = a$

$u \in [0.25, 0.5) \Rightarrow$ choose $x(0) = b$

$u \in [0.5, 0.75) \Rightarrow$ choose $x(0) = c$

$u \in [0.75, 1.0] \Rightarrow$ choose $x(0) = d$

if $x(0) = a$

$u(1) = rnd[0,1]$

$u(1) \in [0, 0.25) \Rightarrow x(1) = a$

$u(1) \in [0.25, 0.55) \Rightarrow x(1) = b$

$u(1) \in [0.55, 0.70) \Rightarrow x(1) = c$

$u(1) \in [0.7, 1.0] \Rightarrow x(1) = d$

if $x(0) = b$

$u(1) = rnd[0,1]$

$u(1) \in [0, 0.5) \Rightarrow x(1) = a$

$u(1) \in [0.5, 0.7) \Rightarrow x(1) = b$

$u(1) \in [0.7, 0.8) \Rightarrow x(1) = c$

$u(1) \in [0.8, 1.0] \Rightarrow x(1) = d$

Assignment : 8x8 markov chain

$H_\infty = 1$

$n \sim 6$

$\omega = 6 \cdot 2^6 \sim 1000$

$L = 1000\,000$

1. Markov chain coding
2. Z-L encoding
3. Z-L decoding

$\left.\begin{array}{l}\text{①}\\\text{②}\\\text{③}\end{array}\right\} \rightarrow$ 3 parts of code

→ Bernouli Random Variable : $P(H) = p$, $P(T) = q$. , n trials.

typical event : np heads nq tails

$$(p)^{np} \cdot (q)^{nq}$$

$$= 2^{\log_2 (p^{np} \cdot q^{nq})}$$

$$= 2^{n[p \log_2 p + q \log_2 q]}$$

$$= 2^{-nH}$$

28/9/20

Random Analog Source → Sampling → Quantization → Binary Coding (Symbols)

Binary Coding → (pmf, Symbols) → Source Coding → Random Bit String

0101, 1001, ...

Channel — Bandpass

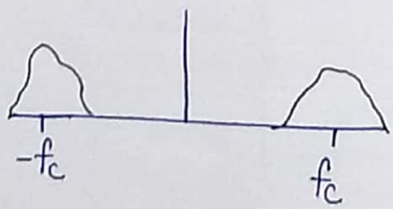
Rx — Reciever

* Analog Communication :

Baseband Analog Signal $x(t)$ → Modulation → Channel

$$y(t) = R\left[x(t) \cdot e^{j2\pi f_c t} \right]$$

* Channel : Bandpass



- Attenuation
- Group delay - Phase delay } → Distortion equalization
- Multipath
- Noise.

→ $x(t) = \sum_{k \in \mathbb{Z}} x_k \phi_k(t)$, $x_k = \langle x(t), \phi_k(t) \rangle$

→ Take $\begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}_{4\times1}$ vector $\xrightarrow{\text{Bijective Map}} \mathbb{R}$

(2⁴ vectors possible)

$$\begin{tikzcd}
& x_k \arrow[r, "\text{synthesis}"] & x(t) \arrow[r, "\text{mod}"] & y(t) \arrow[d, "\text{channel}"] \\
\widehat{\begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}}_{4\times1} & \hat{x}_k \arrow[l, "\text{Back map}"'] & \hat{x}(t) \arrow[l, "\text{inner product}"] & \hat{y}(t) \arrow[l, "\text{demod}"']
\end{tikzcd}$$

→ Mostly random analog source signal is redundant, so modulating completely is not necessary. We just want to modulate the changes occurred i.e., modulating only INFORMATION. Now there is no redundancy.

→ Total 4 conversions : I/P → A/D → D/A → A/D → D/A → O/P
(In digital communication)

→ Digital modulation : mapping, synthesis, modulation

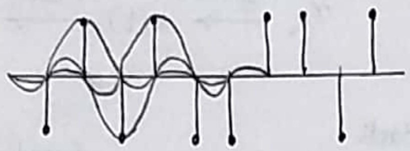
→ 0101001101   1 bit mapping $0 \rightarrow -5$ $a_0 = -5$

$1 \rightarrow +5$ $a_1 = +5$

$A = \{a_0, a_1\}$; $x_K \in A$

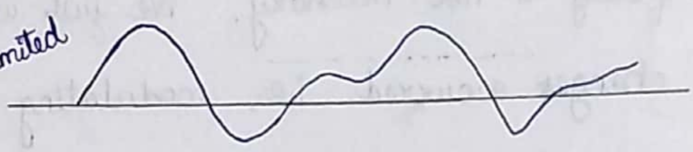
T- time for every bit

Bit rate = $1/T$



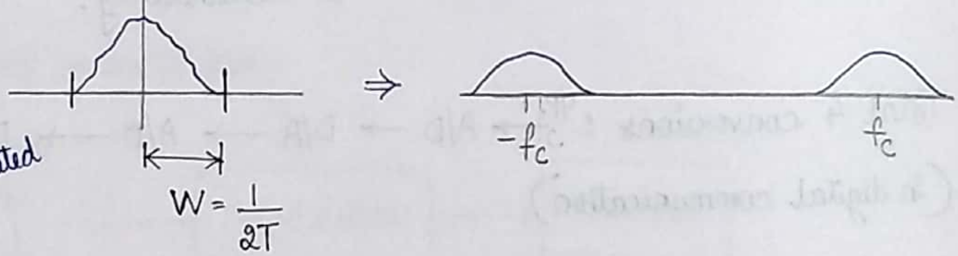
$$x(t) = \sum_{K \in \mathbb{Z}} x_K \cdot \phi_K(t) = \sum x_K \cdot \text{Sinc}\left(\frac{t}{T} - K\right)$$

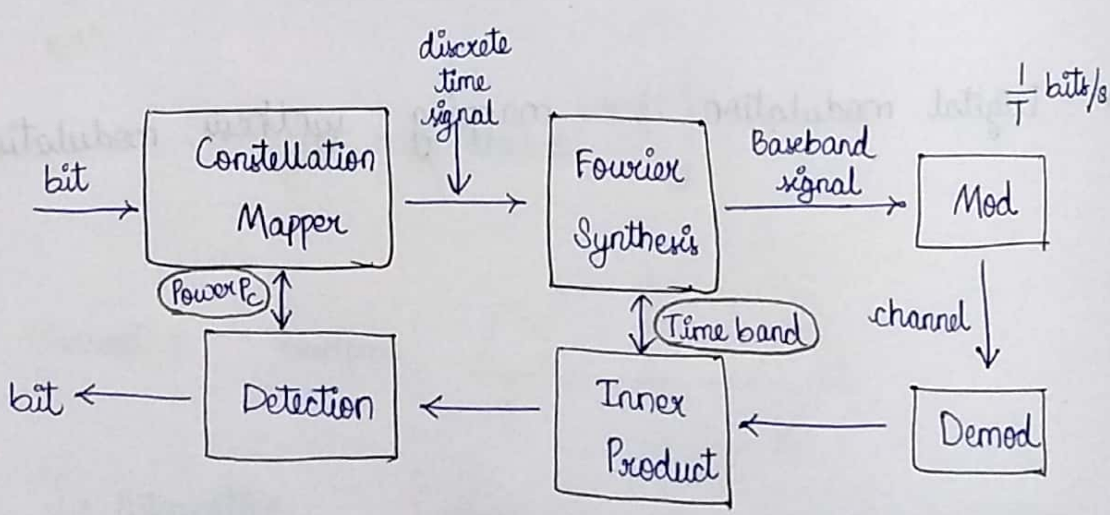
not timelimited

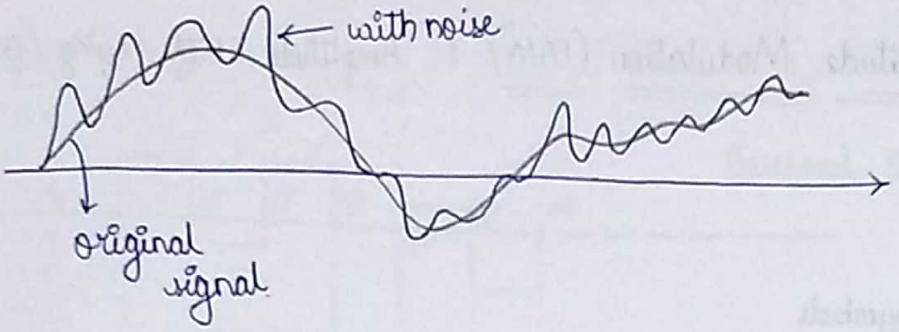


Fourier Synthesis

absolute ~~time~~ bandlimited



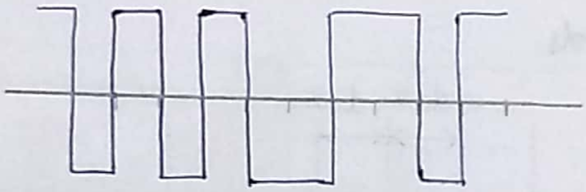


→ 

we get a distribution of values around +5 & -5 due to noise.

* There is a loss of power.

→ choosing $\phi_K(t)$ as rectangular pulse



timelimited

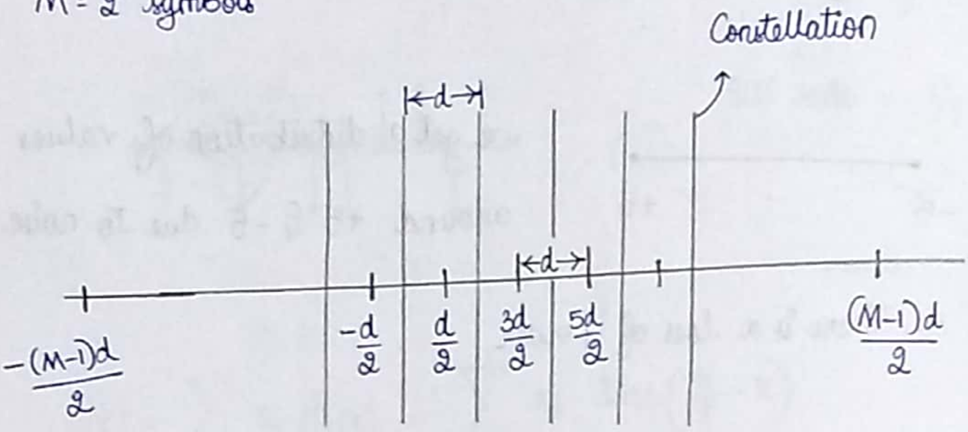
but infinite bandwidth

* Sinc & Rect are two extremes of mapping
* So we take approximately timelimited & bandlimited

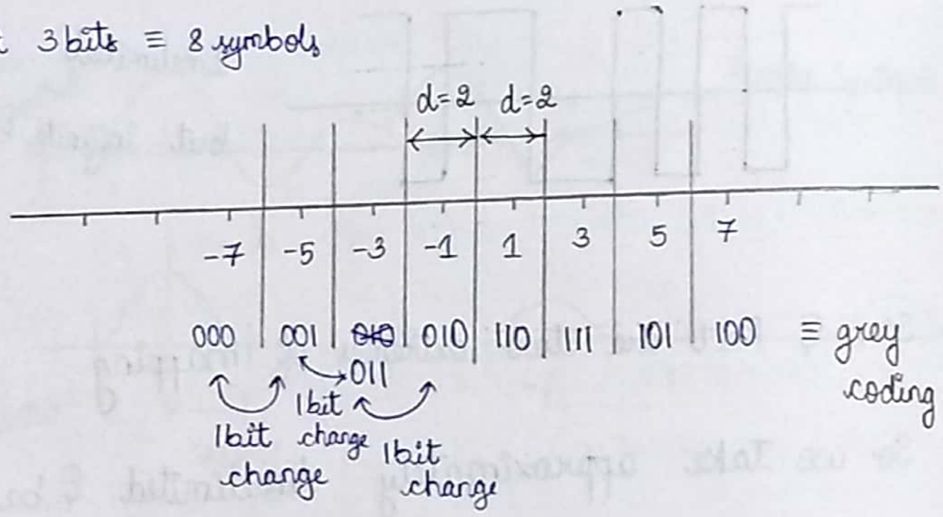
Pulse Amplitude Modulation (PAM) : Amplitude Shift Keying (ASK)

b bits

$M = 2^b$ symbols



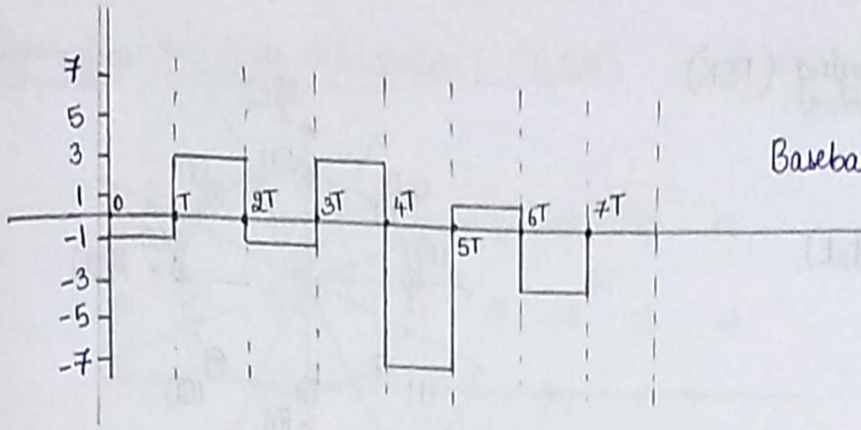
→ let 3 bits ≡ 8 symbols



010	111	010	111	000	110	011
−1	3	−1	3	−7	1	−3

$$\phi_k(t) = u(t-kT) - u(t-(k+1)T)$$

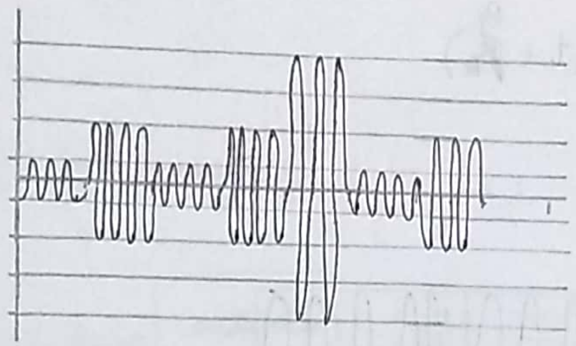
$$x(t) = \sum x_k \phi_k(t)$$



Baseband Signal

$$\rightarrow \quad y(t) = \mathcal{R}\left[x(t) \cdot e^{2\pi f_c t} \right]$$

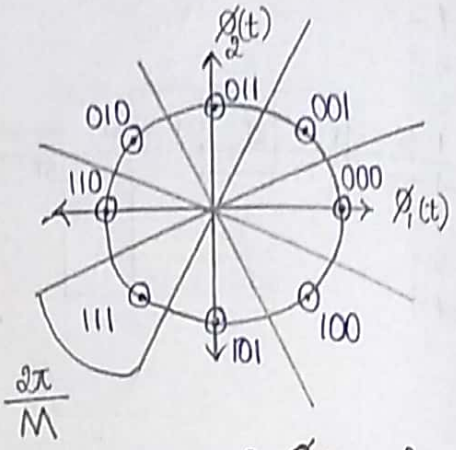
$$= \sum_k x_k \phi_k(t) \cos(2\pi f_c t)$$



$y(t)$

→ Read about Power consumption from book

Phase Shift Keying (PSK)



$M = 2^P = 8$ (let)

$P = 3$ (let)

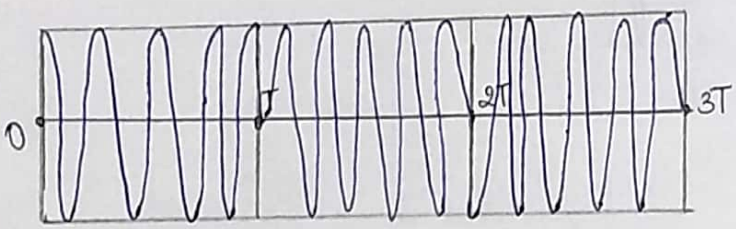
$\theta_k / \cancel{\phi_k(t)} = \frac{2\pi}{M} k$

$k = 0, 1, \dots M-1$

$$y(t) = \mathcal{R}\left[A e^{j\overset{\theta_k}{\cancel{\phi_k}}} \cdot e^{j 2\pi f_c t} \right]$$

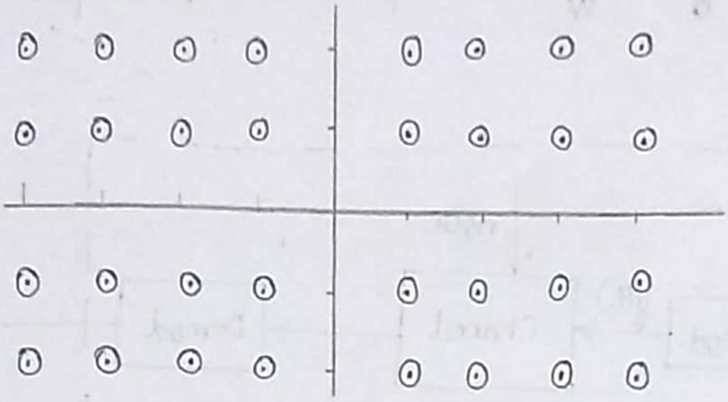
$$= A \cos \overset{\theta_k}{\cancel{\phi_k}} \cdot \cos(2\pi f_c t) - A \sin \overset{\theta_k}{\cancel{\phi_k}} \cdot \sin(2\pi f_c t)$$

$$= A \cos(2\pi f_c t + \overset{\theta_k}{\cancel{\phi_k}})$$



→ Read about Power Consumption

Quadrature Amplitude Modulation : (QAM)   2 Dimensional

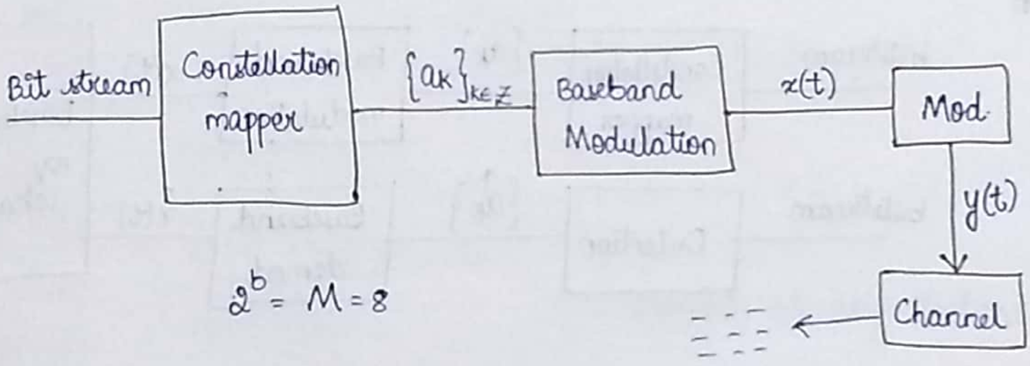


M=32

$$y(t) = A_{k_i} \cos(2\pi f_c t) + A_{k_q} \sin(2\pi f_c t)$$

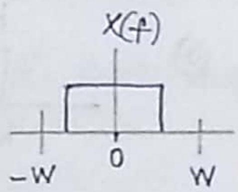
4/3/20

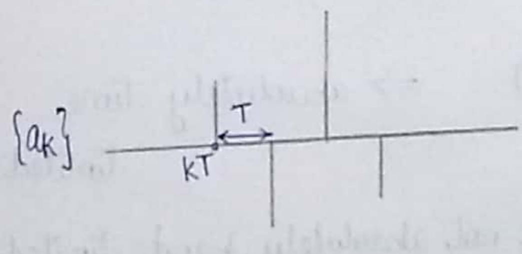
PAM : (ASK)



$2^b = M = 8$

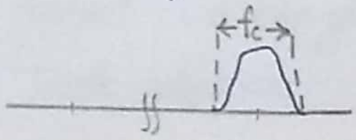
$x(t) \equiv$



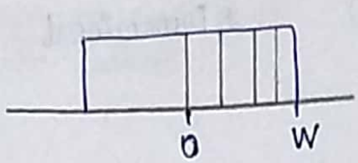


{AM, DSB-SC, SSB, PM, FM}

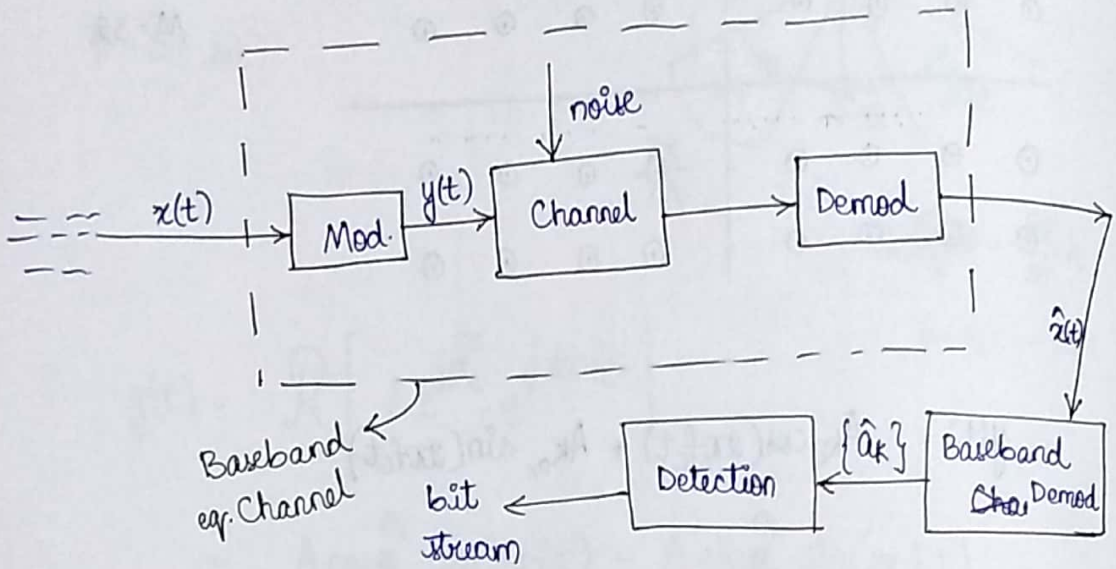
$y(t) \equiv$



FSK :

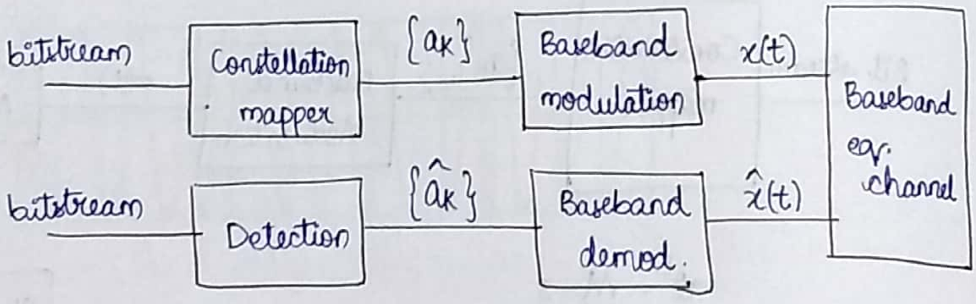


→ Splitting the band and each part is dealt separately.

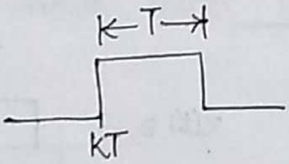


→ This mappings are not linear.

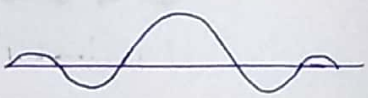
*



→ ⓐ



carrier →



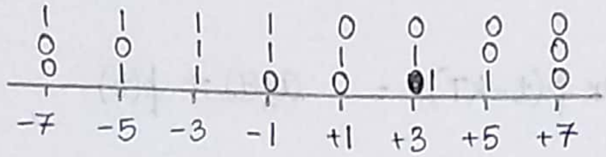
$$x(t) = \sum_{k \in \mathbb{Z}} a_k \, u(t-kT)$$

⇒ absolutely time limited

⇒ but not absolutely band limited

→ Sampling at any instant in a time window ≡ T always gives ~~a defined value~~ back sequence.

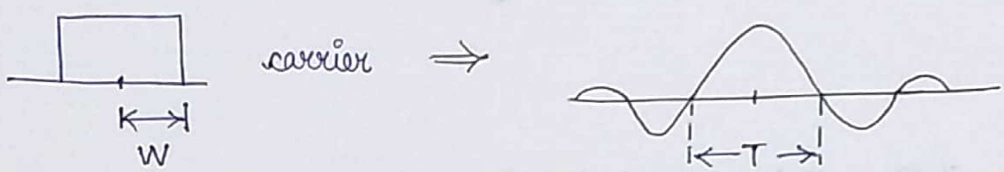
* $T = 1ms \equiv$ clk is 1KHz



3 kilobits per sec

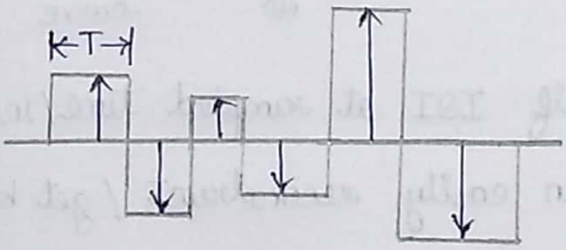
(V)

→ ⓑ



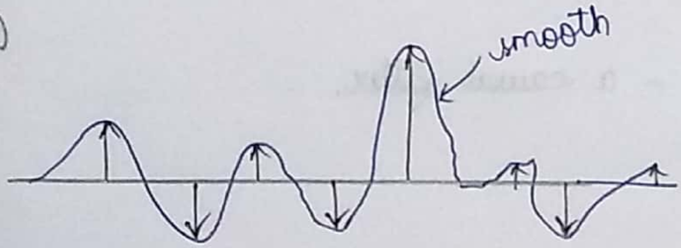
→ Here we need to sample at exact instant kT, then only we get back sequence

(a)



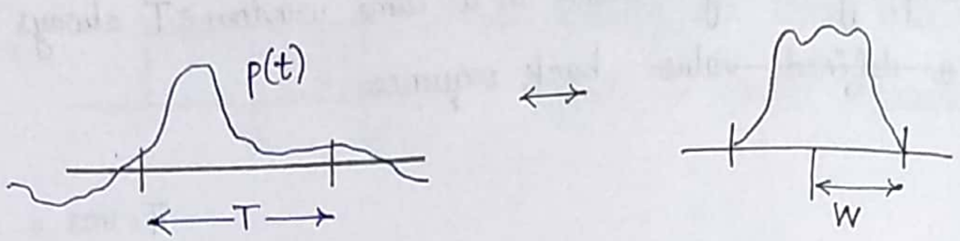
Sample at any instant in time window T, we can get back sequence

(b)



Sample at exact instant, we can get back sequence

→



$$x(t) = \sum_{k \in \mathbb{Z}} a_k \, p(t-kT) = a_\delta(t) * p(t)$$

where $$a_\delta(t) = \sum_{k \in \mathbb{Z}} a_k \cdot \delta(t-kT)$$

$$x(t) \equiv \text{filtering of } a_\delta(t) \text{ with } P(f).$$

ISI : Inter Symbol Interference

→ at an instant, the signal value is not completly due to that symbol, it is sum of effects of symbols in that time window.

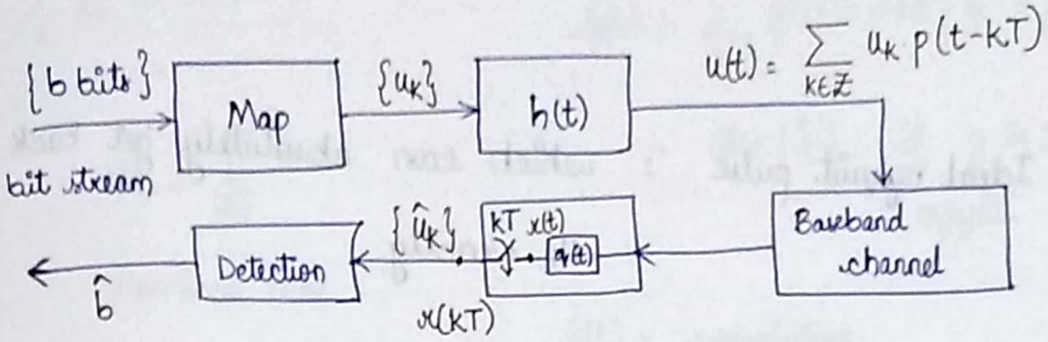
→ This is also possible if $p(t) \equiv \text{Sinc}(t)$
an ideal nyquist curve

→ Now while sampling, if ISI at sampled time/instant is "zero" then we can easily reconstruct / get back the sequence.

Sinc – a causal filter.

11/3/20

Nyquist Criterion / Matched Filter :



Linear Baseband Decoding.

$$r(t) = q(t) * u(t) = \int_{-\infty}^{\infty} u(\tau) \cdot q(t-\tau) \cdot d\tau$$

$$= \int_{-\infty}^{\infty} \left[\sum_{k \in \mathbb{Z}} u_k \, p(\tau - kT) \right] \cdot q(t-\tau) \cdot d\tau$$

$$= \sum_{k \in \mathbb{Z}} u_k \int_{-\infty}^{\infty} p(\tau - kT) \cdot q(t-\tau) \cdot d\tau .$$

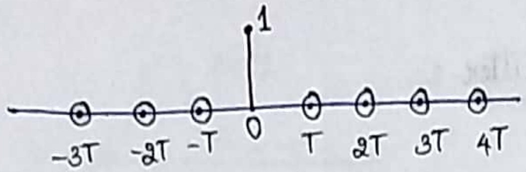
provided $\quad g = p * q = \int_{-\infty}^{\infty} p(\tau) \cdot q(t-\tau) \cdot d\tau \in L_2$

$$= \sum_{k \in \mathbb{Z}} u_k \cdot g(t-kT)$$

$$r(kT) = \sum_{m \in \mathbb{Z}} u_k \, g((k-m)T) = u_k \text{ iff } g(kT) = \delta_k$$

Scanned with CamScanner

Restriction is on Function values at nT;

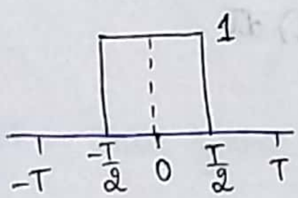


≡ Ideal nyquist pulse : which can absolutely get back u_k exactly.

→ in this case we say h and q matched.

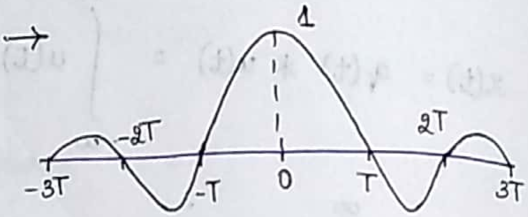
→ we can recover the signal without ISI

→



Ideal nyquist curve

→



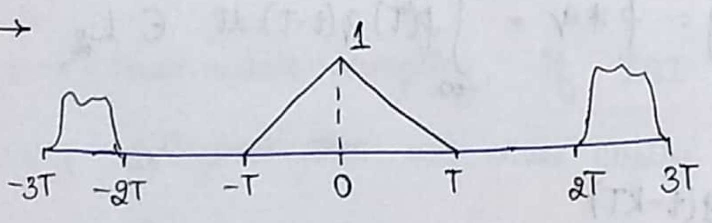
ideal nyquist curve

→ infinite B/W

→ synchronization issue

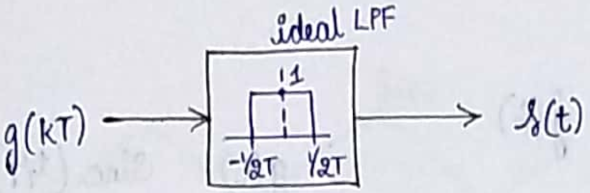
→ dies down very slowly.

→



ideal nyquist curve

→ can have differentiability issues



$= \sum_k g_k \, \delta(t-kT)$

$$s(t) = \sum_k g(kT) \cdot \mathrm{sinc}\left(\frac{t}{T} - k\right)$$

$$= \mathrm{sinc}\left(\frac{t}{T}\right) \quad \text{if } g \text{ is ideal nyquist.}$$

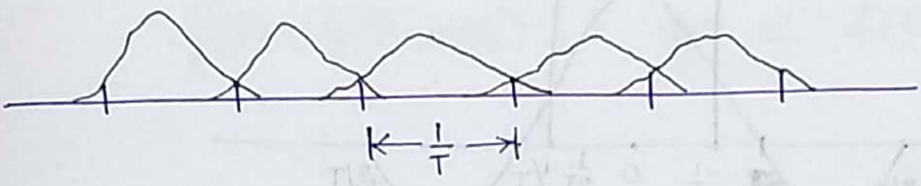
$W_b = \frac{1}{2T}$

nominal B/W

$s(t) =$ convolution

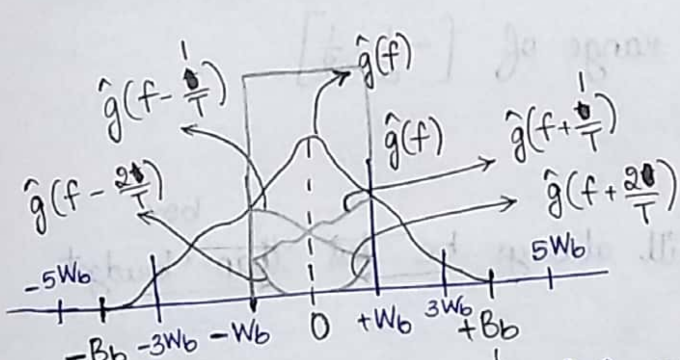
$$\sum g(kT) * \mathrm{sinc}\left(\frac{t}{T}\right)$$

$$\left[\sum_m \hat{g}\left(f + \frac{m}{T}\right)\right] \cdot \Pi(fT) = T\,\Pi(fT)$$



* L_2 convergence :
$$\int_{-\infty}^{\infty} \left[\sum_m \hat{g}\left(f+\frac{m}{T}\right) \cdot \Pi(fT) - T\,\Pi(fT) \right]^2 df = 0$$

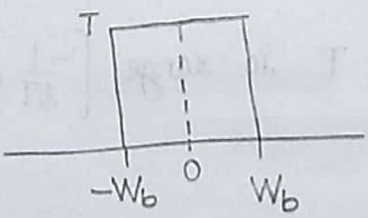
→ aliasing is happening.

→ 

$W_b = \frac{1}{2T}$

→ Design B/W $\equiv$ $\hat{g}(f) = 0, \ |f| > B_b$

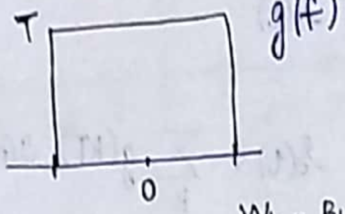
⇓



$\equiv T\,\Pi(fT)$

budget B/W. : box budget

→



$g(t) = \text{Sinc}(t/T)$

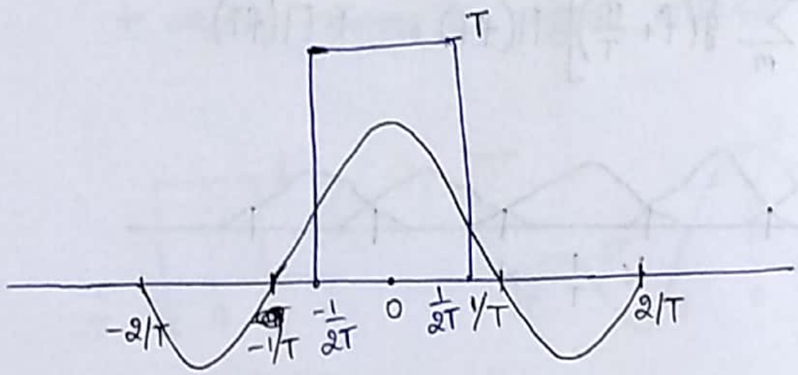
$\Rightarrow$ very much nyquist

$W_b = B_b = \frac{1}{2T}$

nominal B/W = design B/W

Entire budget B/W is used.
box budget

→



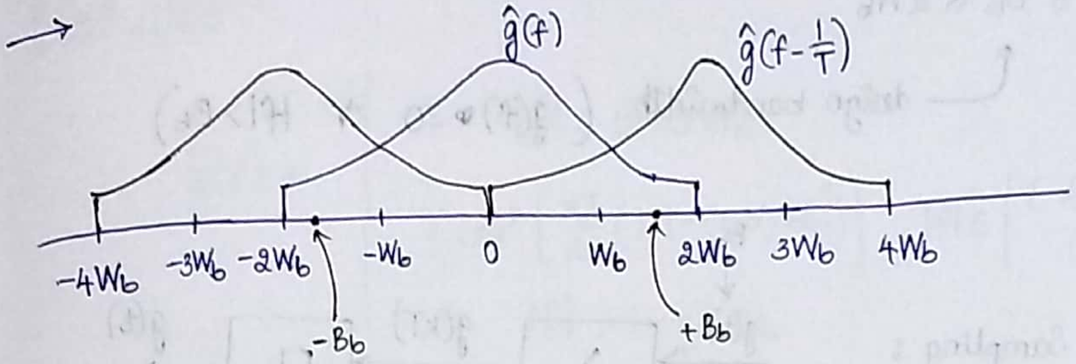
check whether $\text{Sinc}(t) + \text{Sinc}(t-1) + \text{Sinc}(t-2) + ---$

$$= \sum_{n=-5}^{5} \text{Sinc}(t+n)$$

in range of $\left[-\frac{1}{2}, \frac{1}{2}\right]$

Even here Sum will always be less than box budget provided

$$\sum_{m \in \mathbb{Z}} \text{Sinc}\left(\frac{t}{T} - m\right) \leq T \text{ in range } \left[\frac{-1}{2T}, \frac{1}{2T}\right]$$



$$\sum_{m=0,-1} \hat{g}(f+2mW_b)\ \Pi(fT) = T\Pi(fT)$$

$W_b < B_b \leq 2W_b$

maximum number of overlaps = 2

$$\hat{g}(f) + \hat{g}(f-2W_b) = T \quad \text{for all} \quad f \in [0, W_b]$$

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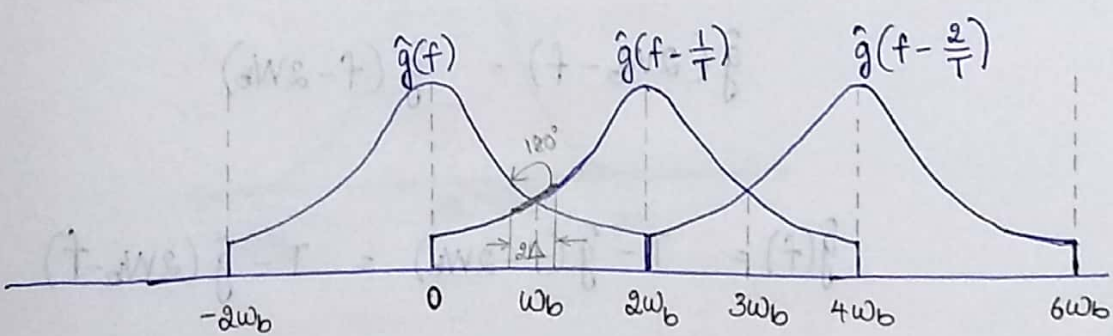
Nyquist criterion:

$$\text{l.i.m} \sum_{m\in\mathbb{Z}} \hat{g}\left(t+\frac{m}{T}\right) \cdot \Pi(fT) = T\Pi(fT)$$

limit in mean square (L_2)

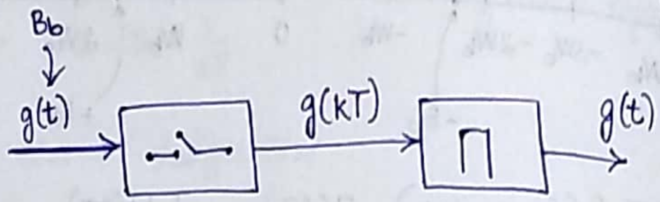


$\frac{1}{2T} = W_b \leftarrow$ nominal bandwidth

$W_b \leq B_b \leq 2W_b$

design bandwidth $\left(\hat{g}(f) = 0 \quad \forall \; |f| > B_b \right)$

→ In Sampling :



$$W_b = \frac{1}{2T} \geq B_b \Rightarrow \text{no aliasing}$$

→ But here we are doing exact opposite ; that is we are using aliasing.

$$W_b \leq B_b \leq 2W_b$$

$$\therefore \quad \hat{g}(f) + \hat{g}(f - 2W_b) = T \quad \forall \; f \in [0, W_b]$$

~~we know~~

we take $g(t)$ – real & even $\Rightarrow$ $\hat{g}(f)$ – real & even

$$\hat{g}(2W_b - f) = \hat{g}(f - 2W_b)$$

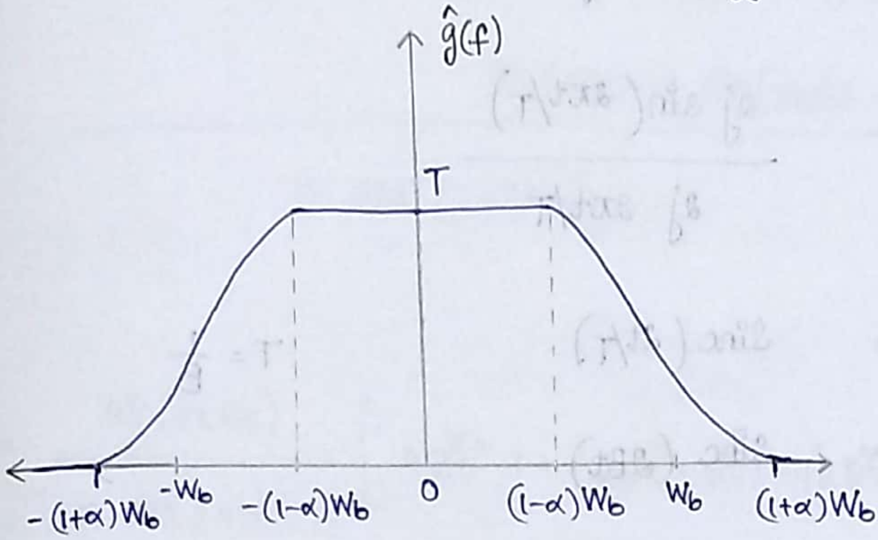
$$\hat{g}(f) = T - \hat{g}(f - 2W_b) = T - \hat{g}(2W_b - f)$$

let $W_b - f = \Delta$

$$\boxed{\hat{g}(W_b - \Delta) = T - \hat{g}(W_b + \Delta)}$$ Rotational symmetry

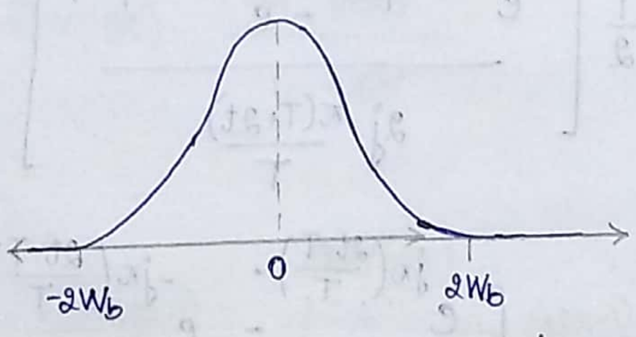
Raised Cosine :

$$\hat{g}(f) = \begin{cases} T, & |f| \leq (1-\alpha)W_b \\ T\cos^2\left[\frac{\pi T}{2\alpha}\left(|f| - (1-\alpha)W_b\right)\right], & |f| \in \left[(1-\alpha)W_b, (1+\alpha)W_b\right] \\ 0, & |f| \geq (1+\underbrace{\alpha}_{\text{roll off}})W_b \end{cases}$$



* consider $\alpha = 1$: $\hat{g}(f) = T\cos^2\left(\frac{\pi T}{2} f\right)$

$$= \frac{T}{2}\left(1 + \cos(\pi T f)\right)$$



$$g(t) = \int_{-\infty}^{\infty} \hat{g}(f) \cdot e^{j2\pi f t} \, df$$

$$= \frac{T}{2}\int_{-2W_0}^{2W_0} [1+\cos(\pi f T)]\, e^{j2\pi f t}\, df$$

$$1^{st} \text{ term} = \frac{T}{2}\cdot \frac{e^{j2\pi t/T} - e^{-j2\pi t/T}}{j2\pi t}$$

$$= \frac{2j \sin(2\pi t/T)}{2j \cdot 2\pi t/T}$$

$$= \text{Sinc}(2t/T) \qquad T = \frac{1}{B}$$

$$= \text{Sinc}(2Bt)$$

$$2^{nd} \text{ term} = \frac{T}{2}\int_{-1/T}^{1/T} \left(\frac{e^{j\pi f T} + e^{-j\pi f T}}{2}\right) e^{j2\pi f t}\, df$$

$$= \frac{1}{2}\left[\frac{e^{j\pi\left(\frac{T+2t}{T}\right)} - e^{-j\pi\left(\frac{T+2t}{T}\right)}}{2j\, \frac{\pi(T+2t)}{T}}\right]$$

$$+ \frac{1}{2}\left[\frac{e^{j\pi\left(\frac{2t-T}{T}\right)} - e^{-j\pi\left(\frac{2t-T}{T}\right)}}{2j\, \pi\left(\frac{2t-T}{T}\right)}\right]$$

$$= \frac{1}{2}\text{Sinc}(2Bt+1) + \frac{1}{2}\text{Sinc}(2Bt-1)$$

$$\therefore \quad g(t) = \text{Sinc}(2Bt) + \frac{1}{2}\text{Sinc}(2Bt+1) + \frac{1}{2}\text{Sinc}(2Bt-1)$$

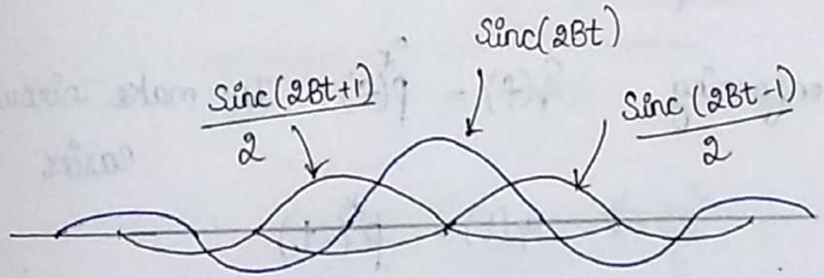
$$2W_0 = B = \frac{1}{T}$$

$$g(t) = \frac{(4B^2t^2-1)\sin(2\pi Bt) + Bt(2Bt-1)\sin(2\pi Bt+\pi) - Bt(2Bt+1)\sin(2\pi Bt-\pi)}{\pi\, 2Bt\,(4B^2t^2-1)}$$

$$= \frac{\sin(2\pi Bt)}{\pi\, 2Bt\,(4B^2t^2-1)}\left[4B^2t^2-1-(2B^2t^2+Bt)-(2B^2t^2-Bt)\right]$$

$$= \frac{\sin(2\pi Bt)}{\pi\, 2Bt\,(1-4B^2t^2)} = \frac{2\sin(\pi Bt)\cos(\pi Bt)}{2\,\pi Bt\,(1-4B^2t^2)}$$

$$= \text{Sinc}(Bt)\cdot\frac{\cos(\pi Bt)}{1-4B^2t^2}$$



for general α : $\quad g_\alpha(t) = \dfrac{\text{sinc}(Bt)\cos(\pi\alpha Bt)}{1-4B^2\alpha^2t^2}$

$g(t)$ is strictly bandlimited but not ~~to~~ timelimited.

We need to show :

$u(t) = \sum u_k\, p(t-kT)$

$u_k = \langle u(t), p(t-kT)\rangle$ $\quad \equiv$ equivalent to $\quad u(t) * q(t) \underline{\;kT\;}$

[multiplying and integration] $\quad$ filtering and sampling

high analog cost $\quad$ low analog cost.

Let

$\rightarrow \quad \hat{g}(f) = \hat{p}(f)\cdot\hat{q}(f)$

$\hat{p}(f) \leftarrow$ Complex

we are enforcing $\hat{q}(f) = \hat{p}^*(f)$ to make circuit easier

$\Rightarrow \quad q(t) = p^*(-t)$

$\hat{g}(f) = |\hat{p}(f)|^2$

$$g(kT) = \int_{-\infty}^{\infty} p(\tau) \cdot q(kT-\tau) \cdot d\tau$$

$$= \int_{-\infty}^{\infty} p(\tau) \cdot p^*(\tau - kT) \cdot d\tau$$

$$= \langle p(t), p(t-kT) \rangle = \underline{\underline{\delta_k}}$$

↳ Nyquist condition

$\therefore \{ p(t-kT) \}$ set is orthonormal basis.

Nyquist condition $\Rightarrow$ orthonormal basis.

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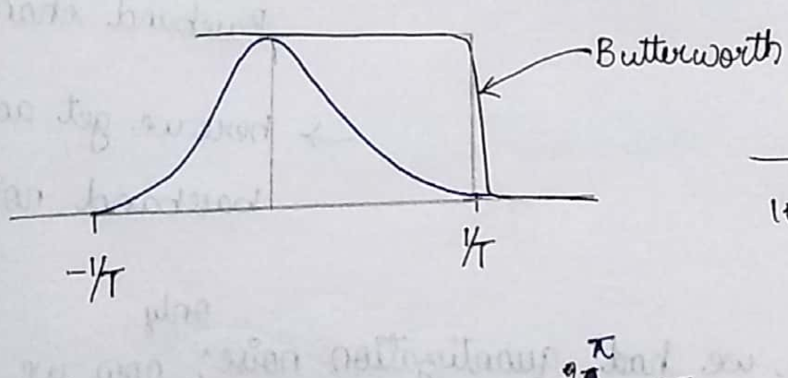
$$\hat{g}(f) = \frac{T}{2}\left(1 + \cos\left(\frac{T\omega}{2}\right)\right) = \frac{T}{2}\left(1 + \cos(\pi T f)\right)$$

realization can be done by FIR / IIR



$$\frac{1}{1 + \left(\frac{\omega}{2B}\right)^{2n}}$$

$$\frac{2\pi}{B} = T$$

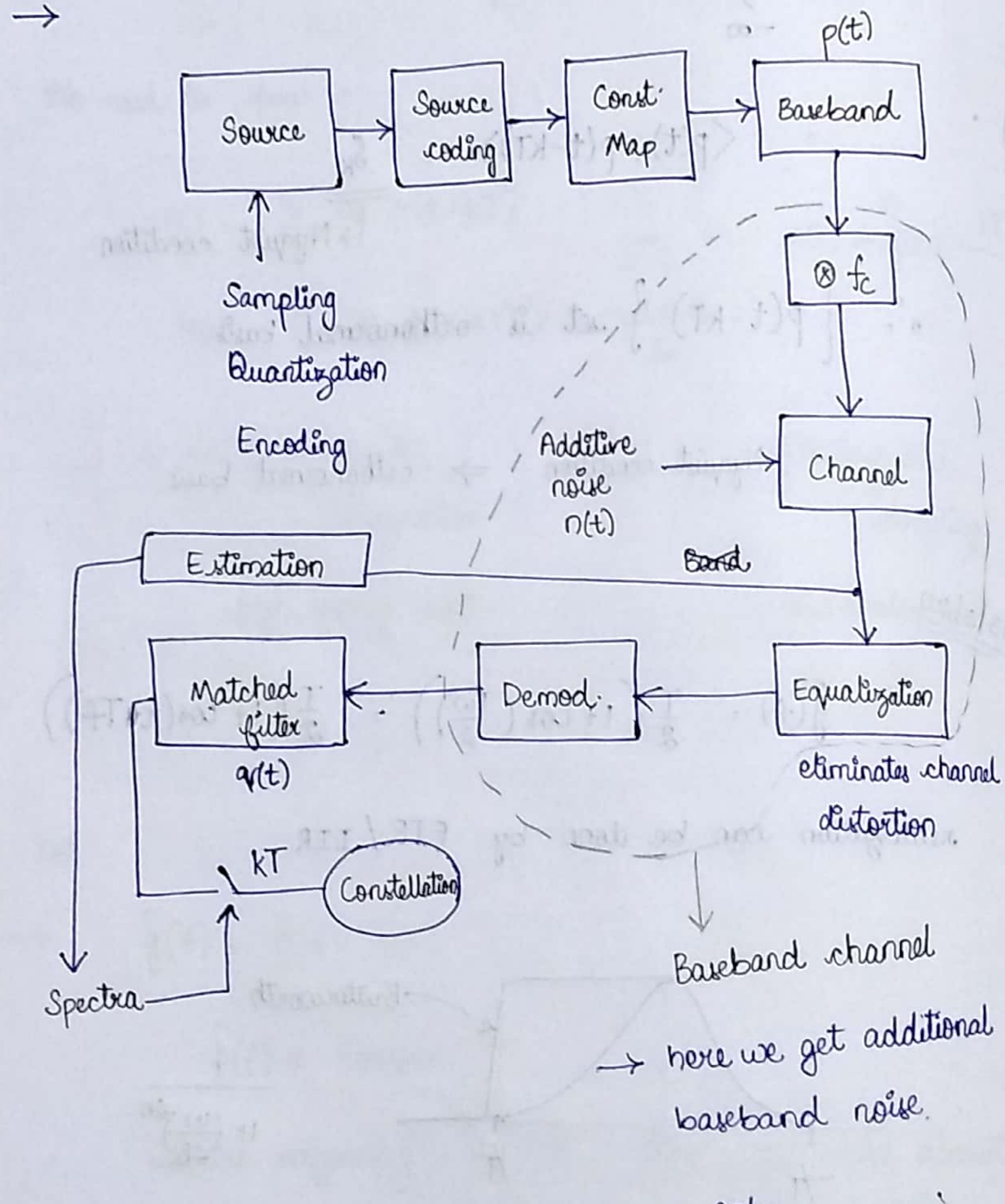
$$\cancel{\hat{g}(f) = \frac{1}{1 + \left(\frac{\omega}{2B}\right)^{2n}} \Big[1 + 1 - \frac{\pi\omega}{B}}$$

READ in PROAKIS :

Modulation : (i) Map : energy & prob. of error

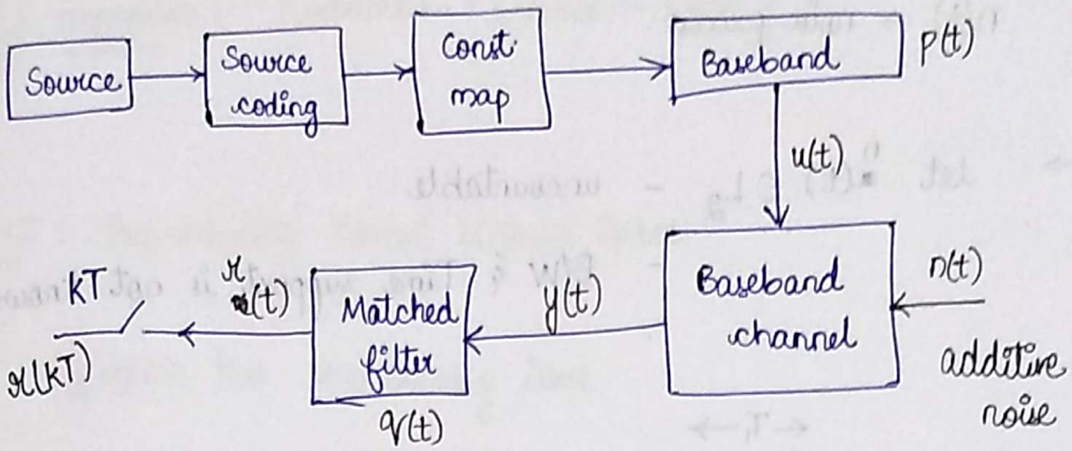
(ii) Continuous phase modulation

→



→ Until now, we had only quantization noise; now we have another noise ≡ Baseband noise.

nyquist ↳ $g(t) = p(t) * q(t)$ (without $n(t)$)



$$u(t) = \sum_k u_k \cdot p(t-kT)$$

$$y(t) = u(t) + n(t) = \sum_k u_k \cdot p(t-kT) + n(t)$$

$$r(t) = y(t) * q(t) = \sum_k u_k\, g(t-kT) + \int n(\tau)\, q(t-\tau)\, d\tau$$

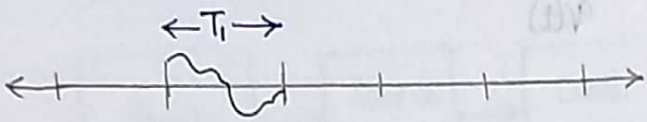
$$\boxed{r(kT) = u_k + n_k}$$

where $n_k = \int n(\tau) \cdot q(kT-\tau) \cdot d\tau$.

$n(t) \rightarrow$ noise process.

$\rightarrow$ let $\overset{n}{x}(t) \in L_2$ - uncountable

- B/W & Time support is not known.



$n(t) \quad , \quad t \in T_1$

$$n(t) = \sum_{k \in \mathbb{Z}} n_k \phi_k(t) \quad , \quad t \in T_1$$

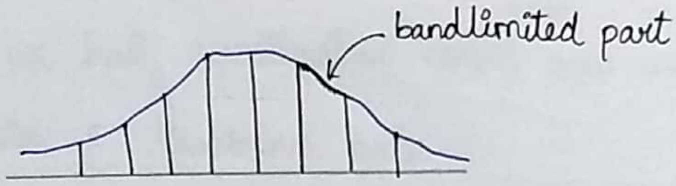
$$n_k = \langle n(t), \phi_k(t) \rangle = \int_{T_1} (\quad)(\quad).$$

$n_k \rightarrow$ random variable

in each truncated window, we apply F.S in that window. We get a set of countable coefficients in each window.

This way uncountable $\Rightarrow$ countable

$\rightarrow$ Similarly



each such part has an inverse in time domain.

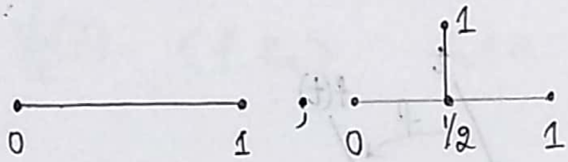
[KKL expansion : Koshambi - Karhunen - Loeve.]

RKHS : Reproducing Kernel Hilbert Space.

→ L_2 space has smoothening issue.

$$\|f-g\|_2 = 0$$

let



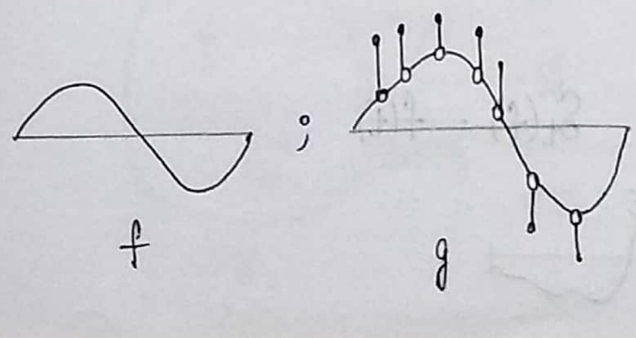
$f(t) = 0 \;\forall\, t = [0,1]$ $\qquad$ $g(t) = 0 \;\forall\, t = [0,\tfrac{1}{2}) \cup (\tfrac{1}{2}, 1]$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 1 \;,\, t = \tfrac{1}{2}$

$\|f-g\|_2 = 0$. : L_2 distance is zero but both functions are not same.

∴ L_2 convergence is not point wise convergence; in case of spikes involved

also



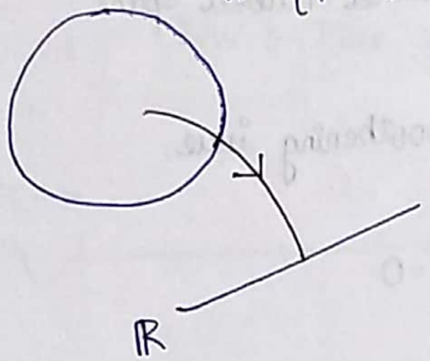
$$\|f-g\|_2 = 0$$

→ So we need a smooth Hilbert space.

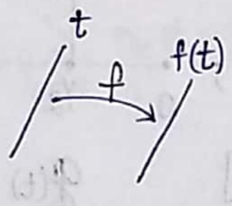
Functional :

$$H = \{ f : E \to \mathbb{R} \} \quad \|f\|_2 < \infty$$

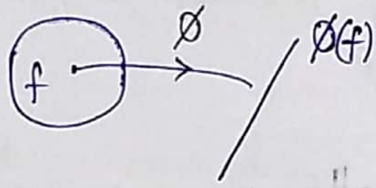
$E \subset \mathbb{R}$



function :

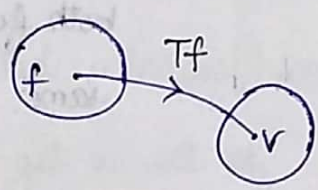


functional :



∞ no. of functionals can be defined on H.

operator :

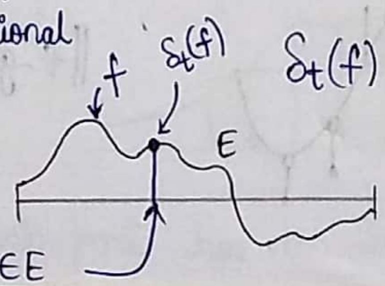


<u>Dirac</u> <u>Functional</u> :

$$\delta_t : H \longrightarrow \mathbb{R}$$

functional

$$\delta_t(f) = f(t)$$



eg: $\displaystyle\int_E \delta(\tau - t)\cdot f(\tau)\cdot d\tau = f(t)$

$\delta(t) * f = f(t)$

↑ this particular dirac delta functional is not continuous

<u>Riesz representation theorem</u>:

Any linear functional (continuous) ϕ_x on f can be written as inner product.

$$\phi_x(f) = \langle f, K_x \rangle \qquad K_x \in H.$$

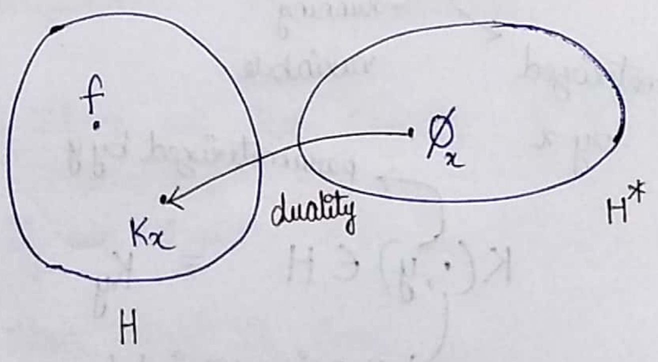
linear: $\phi(\alpha f + \beta g) = \alpha\phi(f) + \beta\phi(g)$

continuity : $\|f - g\|_2 < \epsilon \Rightarrow |\phi(f) - \phi(g)| < \delta$

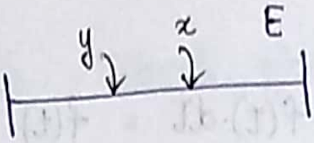
if we use continuous dirac functional and shift it along $(-\infty, \infty)$, L_2 convergence will imply point wise convergence.

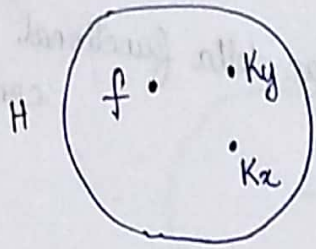
* for smooth functions : L_2 convergence $\not\Rightarrow$ Point wise convergence.

*



→ $E \in \mathbb{R}$





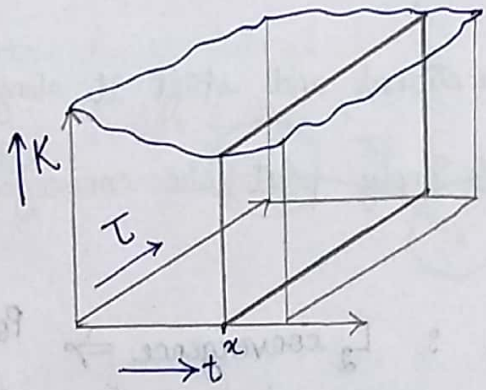
$$\phi_y(K_x) = \langle K_x, K_y \rangle$$

$$= K(x,y)$$ → kernel

$K : E \times E \to \mathbb{R}$ (function)

~~K_x → fix x in K~~

→



$K(t,\tau) : E \times E \to \mathbb{R}$

$K(\cdot,\cdot)$ ← Kernal

$K(x,y) \in \mathbb{R}$

$t = x \Rightarrow K(x,\cdot) \in H \equiv K_x$

parameterized by x.

running variable

$K(\cdot,y) \in H \equiv K_y$

parameterized by y

running variable

→ The space of functions where dirac functional is continuous is known as RKHS.

RKHS : A subspace of L_2 where δ_t is continuous.

→ all functions need not be ~~so~~ smooth.

eg:

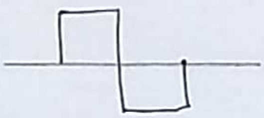
not smooth but $\in$ RKHS

zero removed

→ functions with spikes are removed.

→ also few functions which are not continuous, are also part of space.

eg:



not continuous but $\in$ RKHS.

i.e. enforcing atleast one type of continuity (either right or left) are part of RKHS.

Proving uniqueness : By contradiction.

let there are two Kernals K, K'.

if we show $K(\cdot,\cdot) = K'(\cdot,\cdot)$ for all points then proof is done.

$$\| K(\cdot,\cdot) - K'(\cdot,\cdot) \|^2 = \langle K(\cdot,\cdot) - K'(\cdot,\cdot); K(\cdot,\cdot) - K'(\cdot,\cdot) \rangle$$

$\therefore$ [Kernal is unique to RKHS.]

$$= \langle K(\cdot,\cdot), K(\cdot,\cdot) \rangle + \langle K'(\cdot,\cdot), K'(\cdot,\cdot) \rangle - \langle K'(\cdot,\cdot), K(\cdot,\cdot) \rangle$$

$$- \langle K(\cdot,\cdot), K'(\cdot,\cdot) \rangle$$