Term Paper

Raised cosine Pulse

Name: Harshavardhan Alimi

Roll-No: 18EC10021

Email: alimiharshavardhan@gmail.com

I. INTRODUCTION

The **raised-cosine pulses** frequently used for pulse-shaping in digital modulation due to its ability to minimise intersymbol interference (ISI). Its name comes from the fact that the non-zero portion of the frequency spectrum of its simplest form ($\alpha = 1$) is a cosine function, raised up to sit above the frequency (f) axis.

We also use these pulses to overcome the practical difficulties encountered with the ideal Nyquist channel. To overcome difficulties we can extend the bandwidth from the minimum value (say W) to an adjustable value (say between W and 2W).

And so To get a trade off between bandwidth and ripple losses we use raised-cosine pulse.

In this term paper we analysed about raised cosine pulses,why do we need it and its uses, its mathematical representation in time and frequency domain.we discussed about the energy stored in the pulse in a bandwidth.We calculated the expression for the Hilbert transform of a train of such pulses,and also we have seen the ssb and dsb modulation of the pulse.Finally, We observed its response when passed through an ideal low pass filter and a butterworth filter.. Finally,We find out the output for the SSB and DSB SC modulation of pulse

II. RAISED-COSINE PULSE

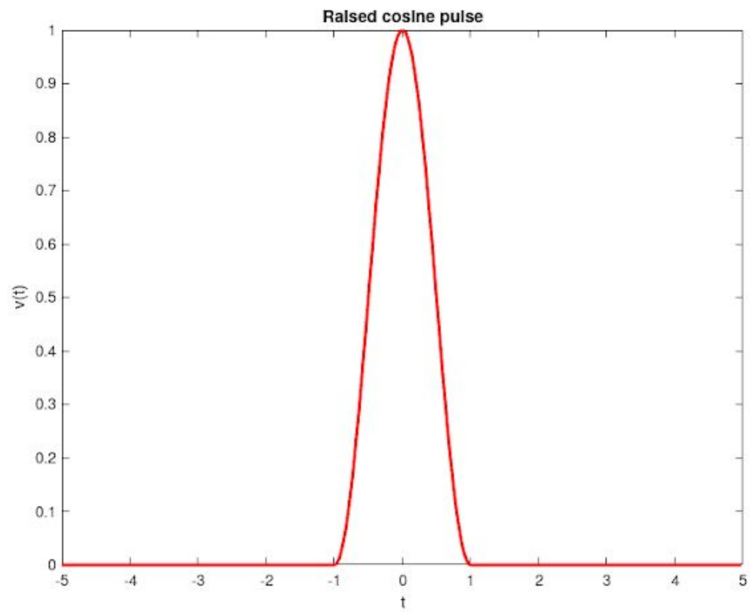
A. MATHEMATICAL REPRESENTATION OF TIME AND FREQUENCY DOMAIN

The raised-cosine pulse in the **Time domain** (impulse response) is thus as follows:

$$h(t) = \frac{A}{2}\,(1 + cos\frac{\pi t}{2})\Pi(t / 2\tau)$$

Where A denotes the maximum amplitude, 2τ denotes the time period for rectangular pulse.

Plot of raised cosine pulse in Time domain abstracted from MATLAB(with $\tau = 1$)



The **Fourier transform** of a raised-cosine pulse is defined as follows:

$$H(f) = \frac{A\tau\ sinc(2f\tau)}{1-(2f\tau)^2}$$

Derivation of Fourier Transform of Raised Cosine:

The general equation of fourier transform for h(t) is

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\, dt$$

For raised cosine $h(t) = \frac{A}{2}\,(1 + cos\frac{\pi t}{\tau})\ \Pi(t\,/\,2\tau)$

$$H(F) = \int_{-\infty}^{\infty} \frac{A}{2}(1 + cos\frac{\pi t}{T})\Pi(\frac{t}{2T})e^{j2\pi ft}dt$$

$$= \int_{-T}^{T} \frac{A}{2}(1 + cos\frac{\pi t}{T})e^{j2\pi ft}dt$$

$$= \frac{A}{2}\int_{-T}^{T} e^{j2\pi ft}dt + \frac{A}{2}\int_{-T}^{T} cos\frac{\pi t}{T}e^{j2\pi ft}dt$$

$$= \frac{A}{2}\frac{e^{j2\pi fT} - e^{-j2\pi fT}}{j2\pi f} + \frac{A}{2}\int_{-T}^{T}\frac{1}{2}(e^{j\pi t/T} + e^{-j\pi t/T})e^{j2\pi ft}dt$$

$$= \frac{A}{2}\frac{sin2\pi fT}{\pi f} + \frac{A}{2}\int_{-T}^{T}\frac{1}{2}(e^{j\pi t/T} + e^{-j\pi t/T})e^{j2\pi ft}dt$$

$$= \frac{A}{2}(\frac{sin2\pi fT}{\pi f} + \frac{sin(2\pi f + \pi/T)t}{2\pi f + \pi/T} + \frac{sin(2\pi f - \pi/T)t}{2\pi f - \pi/T})$$

$$= \frac{A}{2}(\frac{sin2\pi fT}{\pi f} - \frac{T}{\pi}sin(2\pi fT)(\frac{4fT}{(2fT)^2 - 1}))$$

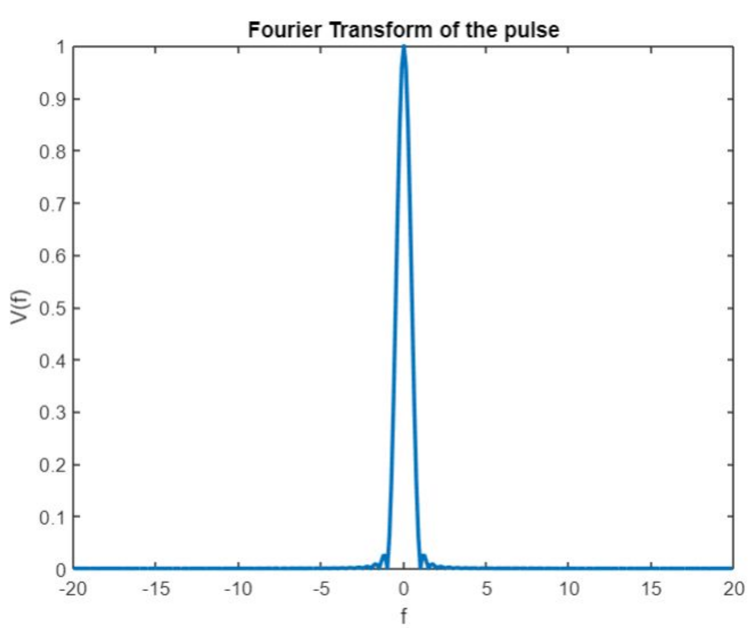
$$= \frac{Asin(2\pi fT)}{2\pi f}(\frac{(2fT)^2 - 1 - (2fT)^2}{(2fT)^2 - 1})$$

$$= \frac{AT}{1 - (2fT)^2}(\frac{sin(2fT)}{2fT})$$

$$= \frac{AT sinc(2fT)}{1 - (2fT)^2}$$

Note that H(f) is real and non-negative(i.e., its phase spectrum is zero for all frequencies) and the area under this pulse is also unity.

Plot of raised cosine pulse in **Frequency** domain abstracted from MATLAB



B. BandWidth

a. Effective Bandwidth

The formula for effective bandwidth is:

$$B = \frac{\int_{-\infty}^{+\infty} |X(f)| df}{2X(0)}$$

From the Fourier transform of the waveform:

$$X(f) = \frac{AT\mathrm{sinc}(2fT)}{1-(2fT)^2}$$

$$X(0) = AT \lim_{f\to 0} \frac{\mathrm{sinc(2fT)}}{1-(2fT)^2} = AT\frac{1}{1}\left(\text{as } \lim_{t\to 0} \mathrm{sinc}t = 1\right)$$

$$X(0) = AT$$

$$B = \frac{1}{2AT}\int_{-\infty}^{+\infty} \frac{AT\mathrm{sinc}(2fT)}{1-(2fT)^2} df$$

$$= \frac{A}{4AT}\int_{-\infty}^{+\infty} \frac{2T sin(2fT)}{(2fT)\left(1-(2fT)^2\right)} df$$

Let 2fT = x

$$B = \frac{1}{4T}\int_{-\infty}^{+\infty} \frac{sin(x)}{(x)\left(1-(x)^2\right)} dx$$

$$B = \frac{1}{8T}\int_{-\infty}^{+\infty} \frac{sin(x)}{(x)}\left(\frac{1}{1-x}+\frac{1}{1+x}\right) dx$$

$$= \frac{1}{8T}\int_{-\infty}^{+\infty} \frac{sin(x)}{(x)}\frac{1}{1-x}+\frac{sin(x)}{(x)}\frac{1}{1+x} dx$$

$$= \frac{1}{8T}\int_{-\infty}^{+\infty} \sin x\left(\frac{1}{x}+\frac{1}{1-x}+\frac{1}{x}-\frac{1}{1+x}\right) dx$$

$$= \frac{1}{8T}\left(\int_{-\infty}^{+\infty} \sin x\left(\frac{2}{x}\right)dx + \int_{-\infty}^{+\infty} \sin x\left(\frac{1}{1-x}\right)dx - \int_{-\infty}^{+\infty} \sin x\left(\frac{1}{1+x}\right)dx\right)$$

$$= \frac{1}{8T}\left(\int_{-\infty}^{+\infty} \sin x\left(\frac{2}{x}\right)dx - \int_{-\infty}^{+\infty} \sin x\left(\frac{2}{1+x}\right)dx\right)$$

$$= \frac{1}{4T}\left(\int_{-\infty}^{+\infty} \sin x\left(\frac{1}{x}\right)dx - \int_{-\infty}^{+\infty} \sin x\left(\frac{1}{1+x}\right)dx\right)$$

$$= \frac{1}{4T}\pi\,(1-\cos 1) = \frac{0.361}{T}$$

Effective Bandwidth = $\frac{0.361}{T}$

C. <u>ENERGY :</u>

A. ENERGY STORED IN (-W,W):

$$E_W = \int_{-w}^{w} \left(a \cdot \frac{1 + \cos\left(\pi \cdot \frac{t}{\tau}\right)}{2} \right)^2 dt$$

$$\frac{a^2 \left(\tau \sin\left(\frac{2\pi w}{\tau}\right) + 8\tau \sin\left(\frac{\pi w}{\tau}\right) + 6\pi w\right)}{8\pi}$$

B. ENERGY STORED IN ENTIRE PULSE:

$$Eg = \int_{-\infty}^{\infty} |\tfrac{A}{2}(1 + cos(\pi t/\tau))\pi(t/2\tau)|^2 \, dt$$

$$= \int_{-\tau}^{\tau} |\tfrac{A}{2}(1 + cos(\pi t/\tau))|^2 \, dt$$

$$= \frac{A^2}{4} \int_{-\tau}^{\tau} (1 + cos(\pi t/\tau)^2 \; + \; 2cos(\pi t/\tau) \, dt$$

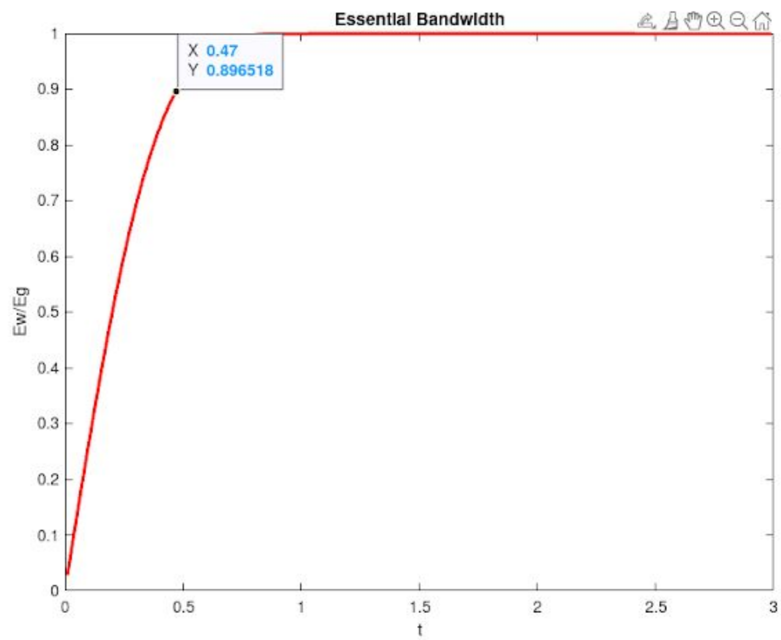
$$Eg = 3A^2\tau/4$$

C. ESSENTIAL BANDWIDTH:

-The bandwidth which stores more than the 90% of the total energy.

I.e., Ew/Eg =0.9

We can obtain that bandwidth from the plot plotted in matlab:



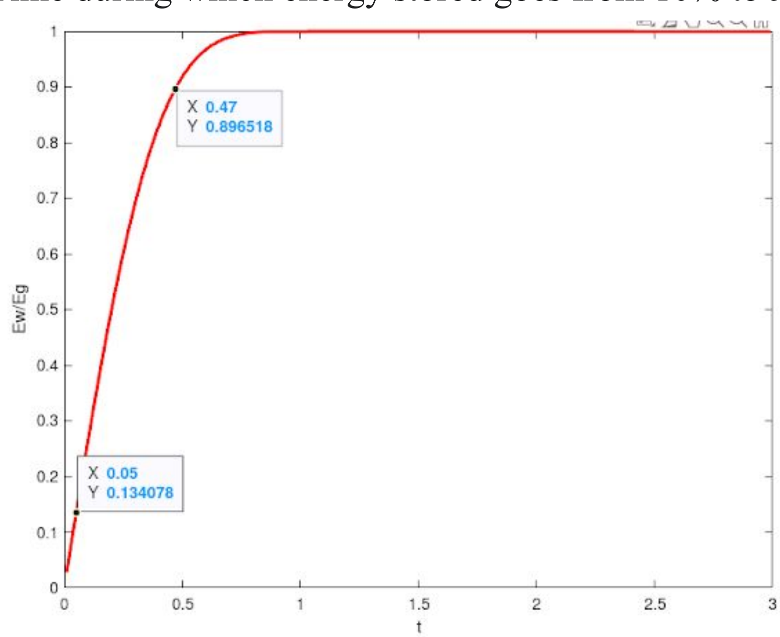
From plot we get T=0.47,

Essential Bandwidth =1/T=2.1276

D. Bandwidth required to Reproduce the actual Pulse:

B>=1/ Trise

Trise:Time during which energy stored goes from 10% to 90%.



TRISE=0.47-0.05=0.42

B>=2.38

E. BANDWIDTH REQUIRED TO DETECT A PULSE:

B>=1/ 2*TRISE

TRISE:Time during which energy stored goes from 10% to 90%.

Trise=0.42

B>=1.1905

III. HILBERT TRANSFORM OF TRAIN OF PULSES

Hilbert transform of a signal x(t) is defined as the transform in which phase angle of all components of the signal is shifted by +/- 90^o ,this linear operator can be obtained by convolving the function with the $1 / \pi t$

Hilbert transform of x(t) is represented with $\hat{x}(t)$,and it is given by

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(k)}{t-k} dk$$

The inverse Hilbert transform is given by

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(k)}{t-k} dk$$

Derivation of hilbert transform of raised cosine pulse

Here p(t) is our raised cosine pulse.

From Fourier Series Analysis, we can represent p(t) as follows:-

$$p(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n sin(n\omega_0 t)$$

where again, $b_n = 0$ and,

$$a_n = \frac{2}{T}\int_T p(t)cos(n\omega_0 t)dt = \frac{2}{2T}\int_{-T}^{T} \frac{A}{2}(1 + cos\frac{\pi t}{T})cos(n\omega_0 t)dt$$

$$= \frac{A}{T}(\frac{sin(n\omega_0 T)}{n\omega_0} + \frac{1}{2}\int_0^T (cos(\frac{\pi}{T} + n\omega_0)t + cos(\frac{\pi}{T} - n\omega_0)t)dt)$$

$$= \frac{A}{T}(\frac{sin(n\omega_0 T)}{n\omega_0} + \frac{1}{2}(\frac{sin(\frac{\pi}{T} + n\omega_0)T}{\frac{\pi}{T} + n\omega_0} + \frac{sin(\frac{\pi}{T} - n\omega_0)T}{\frac{\pi}{T} - n\omega_0}))$$

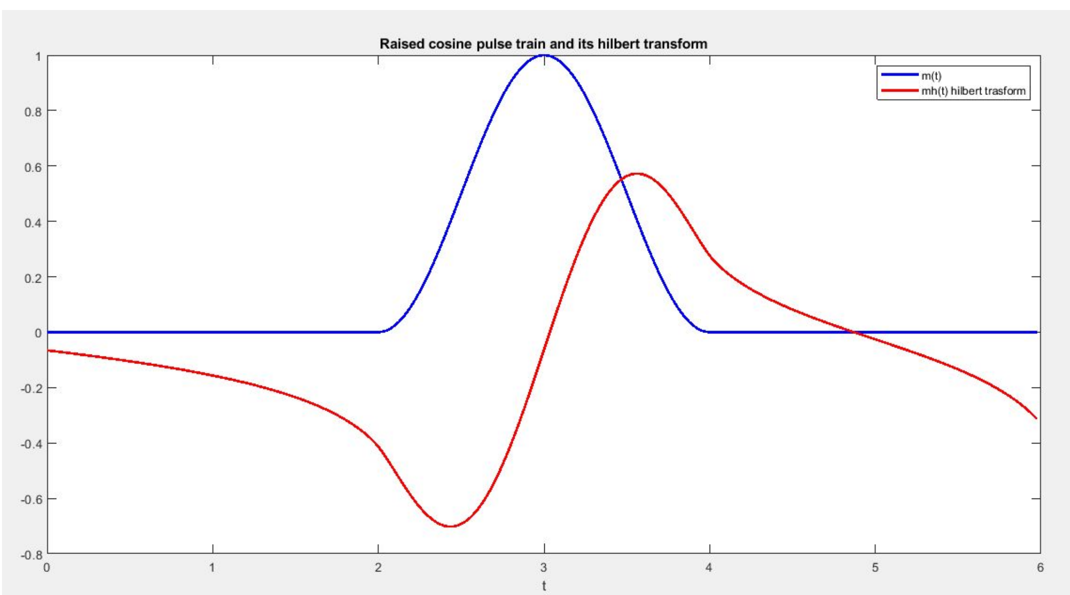
So, $$p(t) = \sum_{n=0}^{\infty} \frac{A}{T}(\frac{sin(n\omega_0 T)}{n\omega_0} + \frac{sin(n\omega_0 T)}{2}\frac{2\pi/T}{(\frac{\pi}{T})^2 - n^2\omega_0^2})cos(n\omega_0 t)$$

$$= \sum_{n=0}^{\infty} \frac{A}{T}(\frac{sin(n\omega_0 T)}{n\omega_0} + \frac{\pi sin(n\omega_0 T)}{T}(\frac{1}{(\frac{\pi}{T})^2 - n^2\omega_0^2}))cos(n\omega_0 t)$$

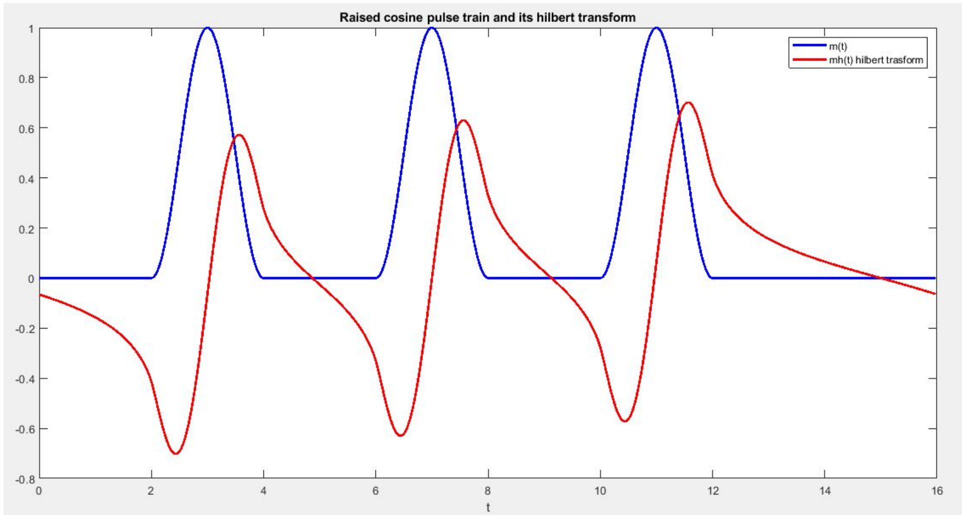
$$\implies \hat{p}(t) = \sum_{n=0}^{\infty} \frac{Asin(n\omega_0 T)}{T}(\frac{1}{n\omega_0} + \frac{\pi}{2((\frac{\pi}{T})^2 - n^2\omega_0^2)})sin(n\omega_0 t)$$

Hilbert Transform of a single Raised Cosine Pulse:

Hilbert Transform of raised cosine pulse in Time domain abstracted from MATLAB



Hilbert Transform of Train of Pulses:

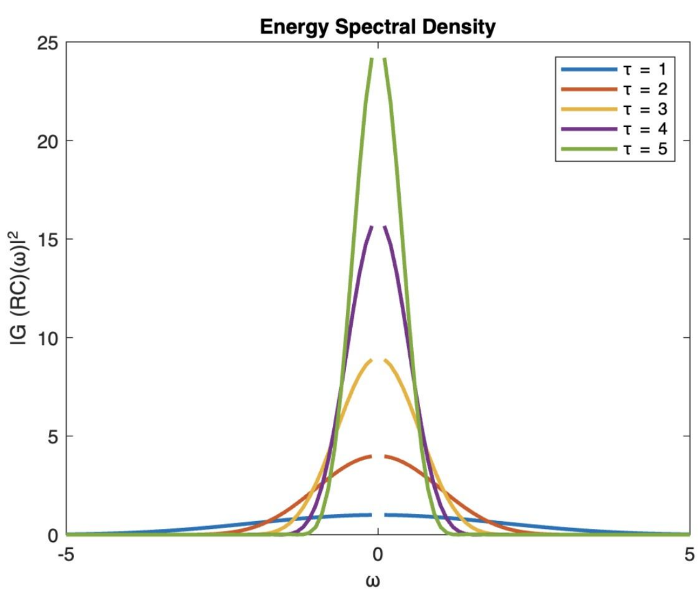


IV. SPECTRAL DENSITY:

Input spectral density is given as :

Energy Spectral Density = | H(w) |^2

$$= \frac{\pi^4 \sin^2(\tau\omega)}{(\tau^2\omega^3 - \pi^2\omega)^2}$$

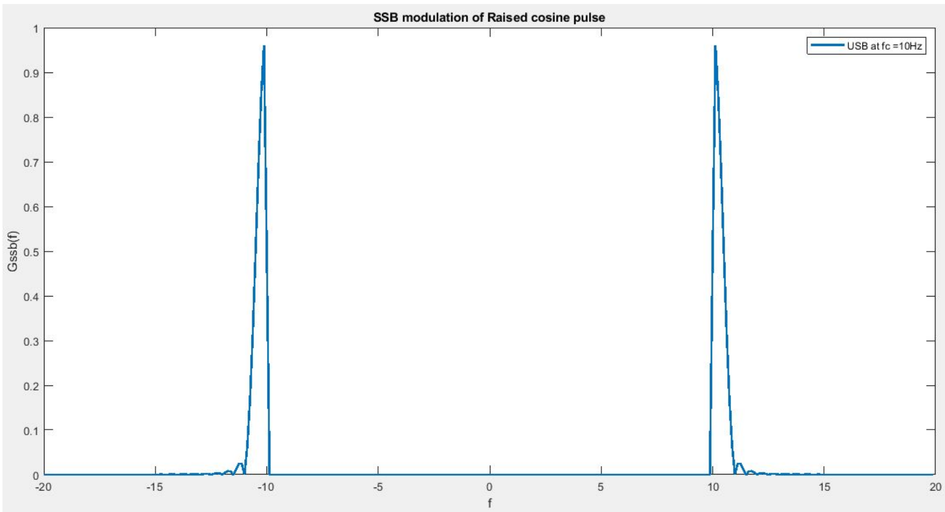


V. SSB modulation:

As it has a very high and a very low frequency content ,the Normal SSB modulation is not a very efficient way of modulation.

The output of SSB modulation is

$r(t) = g(t) \cos(2\pi fc) \mp H(g(t)) \sin(2\pi fc)$

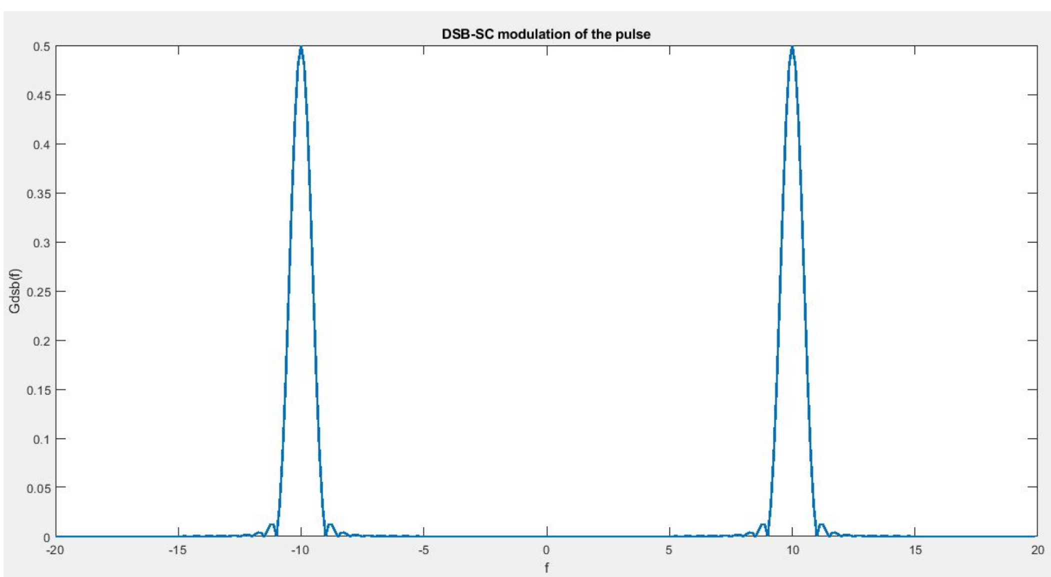


VI. DSB SC modulation:

As we can see that the average value of g(t) is non zero and DSB SC has more bandwidth,we don't usually prefer it for raised cosine pulse.

The output of DSB SC modulation is

$$r(t) = g(t) \cos(2\pi fc)$$



VII. Ideal low pass filter Pulse response:

A low-pass filter (LPF) is a filter that passes signals with a frequency lower than a selected cutoff frequency and attenu-ates signals with frequencies higher than the cutoff frequency.The exact frequency response of the filter depends on the filter design. The filter is sometimes called a high-cut filter, or treble-cut filter in audio applications.An ideal low-pass filter completely eliminates all frequen-cies above the cutoff frequency while passing those below un-changed; its frequency response is a rectangular function andis a brick-wall filter. The transition region present in practical filters does not exist in an ideal filter. An ideal low-pass filter can be realized mathematically (theoretically) by multiplying a signal by the rectangular function in the frequency domain or, equivalently, convolution with its impulse response, a sinc function, in the time domain.

The below is the low pass filter:

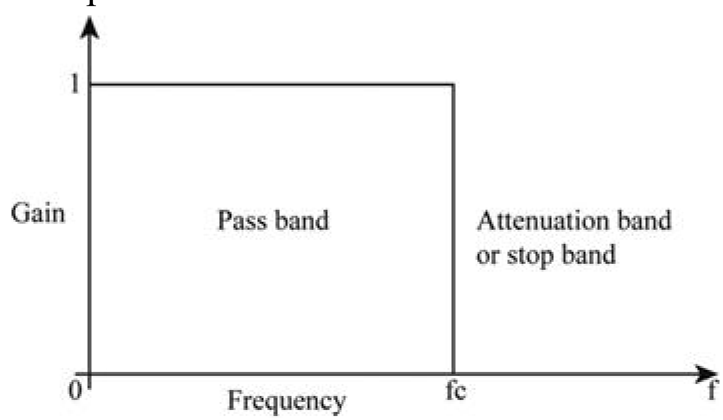
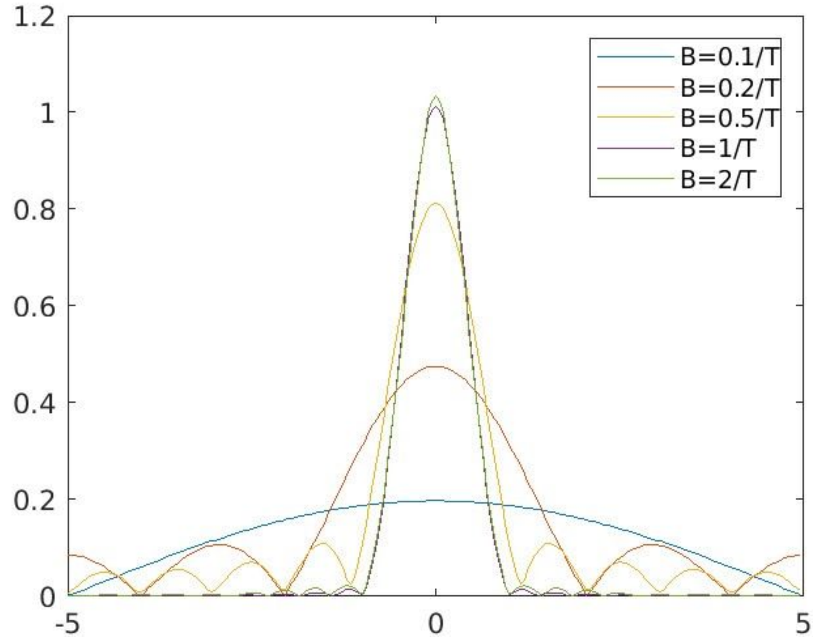


Figure 1

Plots plotted in matlab with various bandwidths-
B is the bandwidth of the low pass ideal filter.



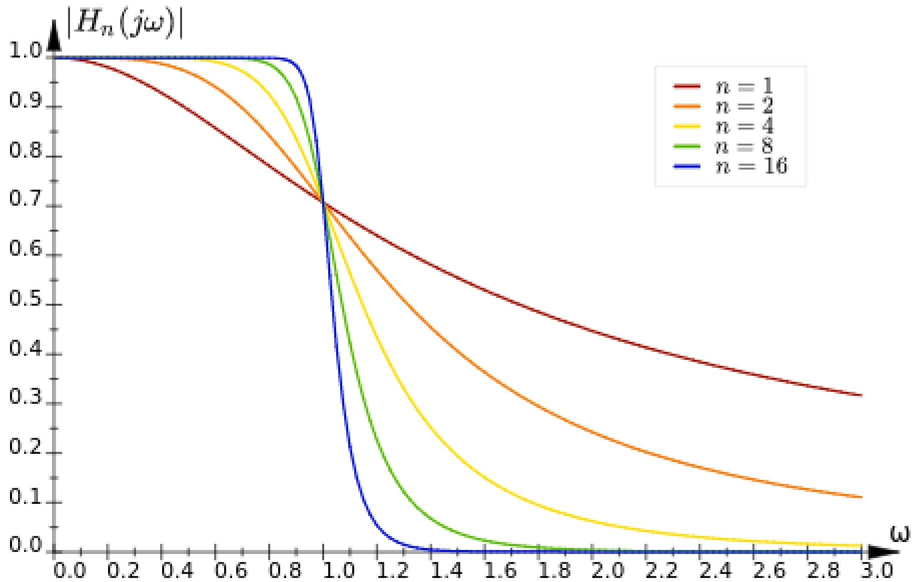
When we pass the pulse through a high bandwidth filter the whole signal will be passed since it passes most of the frequencies.

VIII. Butter worth filter Pulse response:

The signal processing filter which is having a flat frequency response in the passband can be termed as Butterworth filter and is also called as a maximally flat magnitude filter.There are various types of Butterworth filters such as low pass Butterworth filter and digital Butterworth filter.The filters are used for shaping the signal's frequency spectrum in communication systems or control systems. The corner frequency or cutoff frequency is given by the equation
fc=1/(2*pi*R*C).
Butterworth showed that a low pass filter could be designed whose cutoff frequency was normalized to 1 radian per second and whose frequency response (gain) was

$$G(\omega) = \frac{1}{\sqrt{1+\omega^{2n}}},$$

The butterworth filter:



The frequency response of raised cosine pulse when passed through butterworth filter: