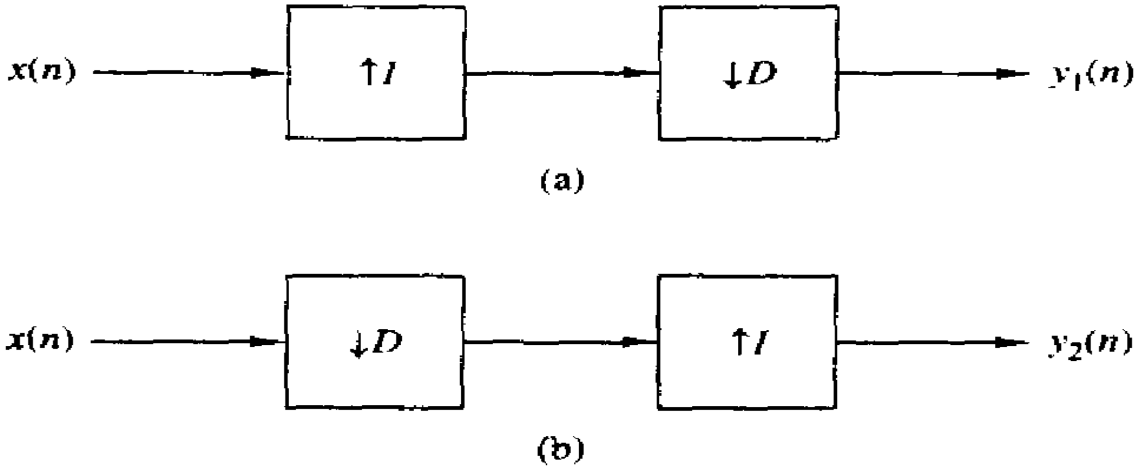
DSP Tutorial-5

Discussion

Q1) Considering the two different ways of cascading a decimator with an interpolator shown in figure.



a) If D = I, show that output of the two configuration are different i.e. the systems are not identical.

b) Show that the two systems are identical if and only if D and I are relatively prime number.

Ans-a)

$$x(n) = \{x_0, x_1, x_2, \ldots\}$$

$D = I = 2$. Decimation first

$$\begin{aligned} z_2(n) &= \{x_0, x_2, x_4, \ldots\} \\ y_2(n) &= \{x_0, 0, x_2, 0, x_4, 0, \ldots\} \end{aligned}$$

Interpolation first

$$\begin{aligned} z_1(n) &= \{x_0, 0, x_1, 0, x_2, 0, \ldots\} \\ y_1(n) &= \{x_0, x_1, x_2, \ldots\} \\ \text{so } y_2(n) &\neq y_1(n) \end{aligned}$$

(b) suppose $D = dk$ and $I = ik$ and d, i are relatively prime.

$$
\begin{aligned}
x(n) &= \{x_0, x_1, x_2, \ldots\} \\
\text{Decimation first} & \\
z_2(n) &= \{x_0, x_{dk}, x_{2dk}, \ldots\} \\
y_2(n) &= \left\{ x_0, \underbrace{0, \ldots, 0}_{ik-1}, x_{dk}, \underbrace{0, \ldots, 0}_{ik-1}, x_{2dk}, \ldots \right\} \\
\text{Interpolation first} & \\
z_1(n) &= \left\{ x_0, \underbrace{0, \ldots, 0}_{ik-1}, x_1, \underbrace{0, \ldots, 0}_{ik-1}, x_2, \underbrace{0, \ldots, 0}_{ik-1}, \ldots \right\} \\
y_1(n) &= \left\{ x_0, \underbrace{0, \ldots, 0}_{d-1}, x_d, \underbrace{0, \ldots, 0}_{d-1}, \ldots \right\}
\end{aligned}
$$

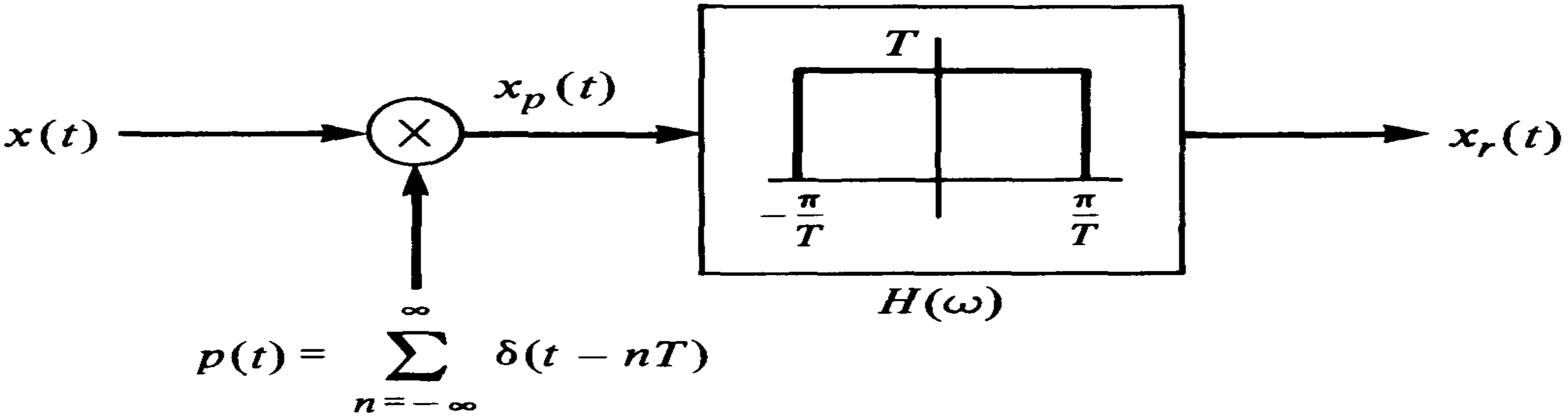
Thus $y_2(n) = y_1(n)$ iff $d = dk$ or $k = 1$ which means that D and I are relatively prime.

Q.2. Consider the following system. The sampling period $T = 1\text{ms}$ and $x(t) = \cos(2\pi f_0 t+\theta)$. Determine $x_r(t)$ for each of the following:

a) $f_0 = 250\text{Hz}$, $\theta = \pi/4$

b) $f_0 = 750\text{Hz}$, $\theta = \pi/2$

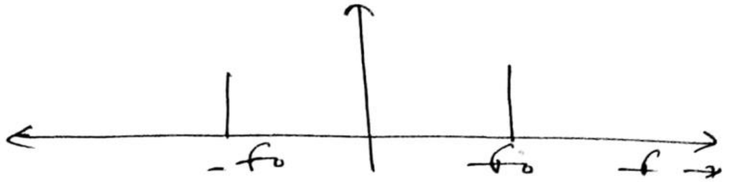
c) $f_0 = 500\text{Hz}$, $\theta = \pi/2$



Ans-

$\rightarrow x(t) = \cos(2\pi f_0 t + \theta)$

$$\Rightarrow X(f) = \frac{\delta(f-f_0) + \delta(f+f_0)}{2}$$

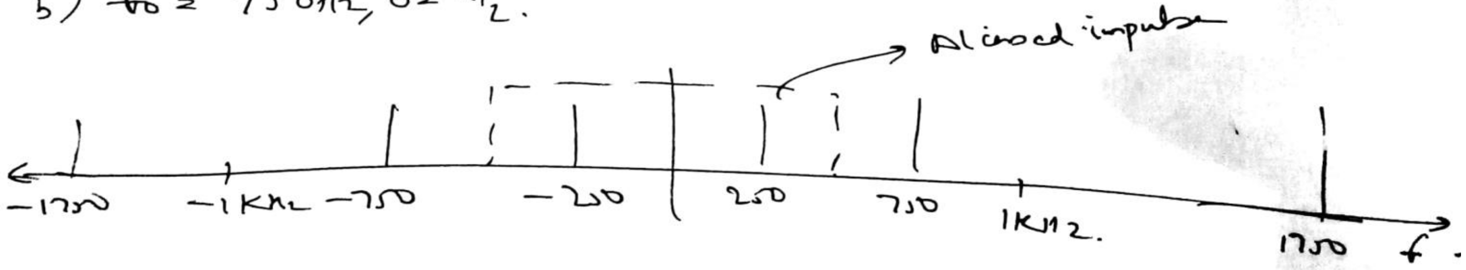


a) $f_0 = 250\,Hz$, $\theta = \pi/4$. ; $T = 1\,ms \Rightarrow f_s = 1\,KHz$.

$X_p(f) =$

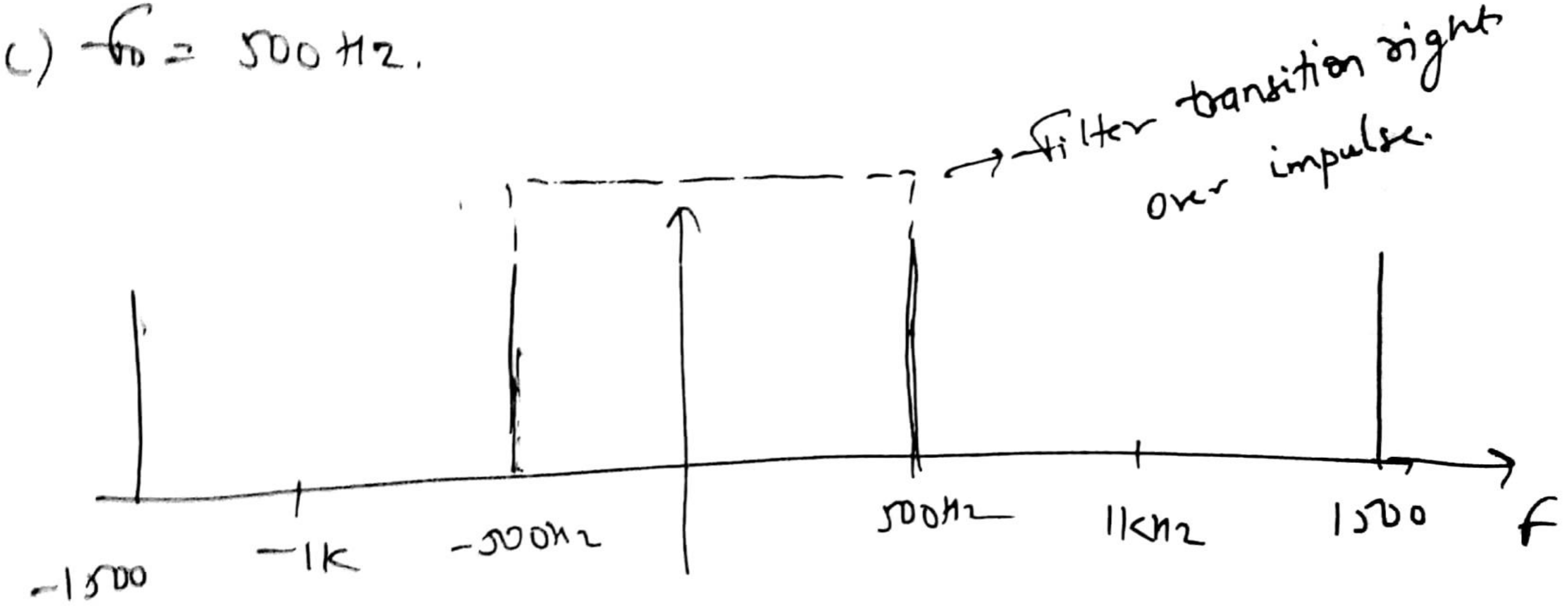
$$\Rightarrow x_r(t) = \cancel{\cos(250 \cdot 2\pi t)}$$
$$\cos(2\pi \cdot 250 t + \pi/4)$$

b) $f_0 = 750\,Hz$, $\theta = \pi/2$.



$$x_r(t) = \cos(2\pi \cdot 250 \cdot t + \pi/2)$$

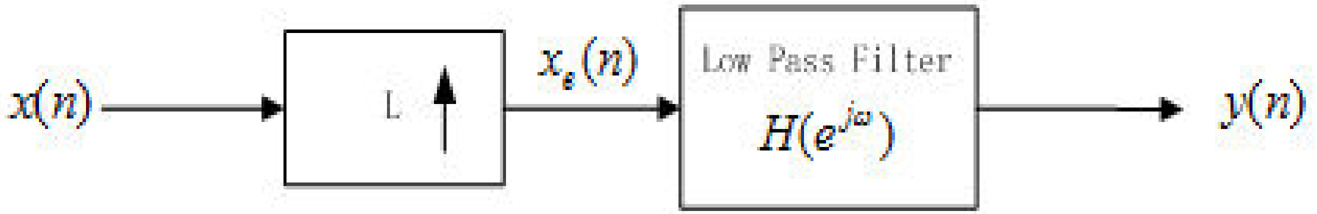
c) $f_0 = 500\,Hz$.



$x_r(t) = 0$ [As filter transition is over frequency response of $x(t)$].

Question 10 min

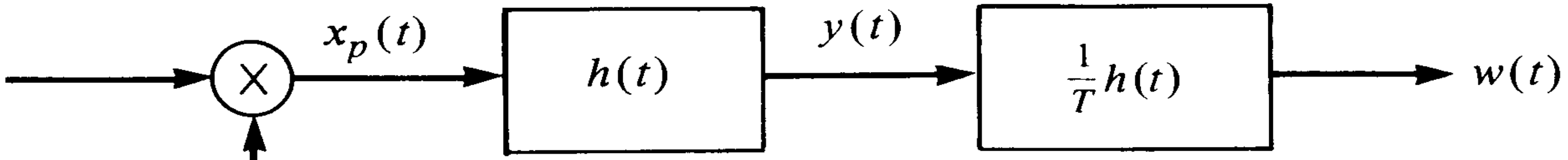
Q1)The system diagram is as follows.



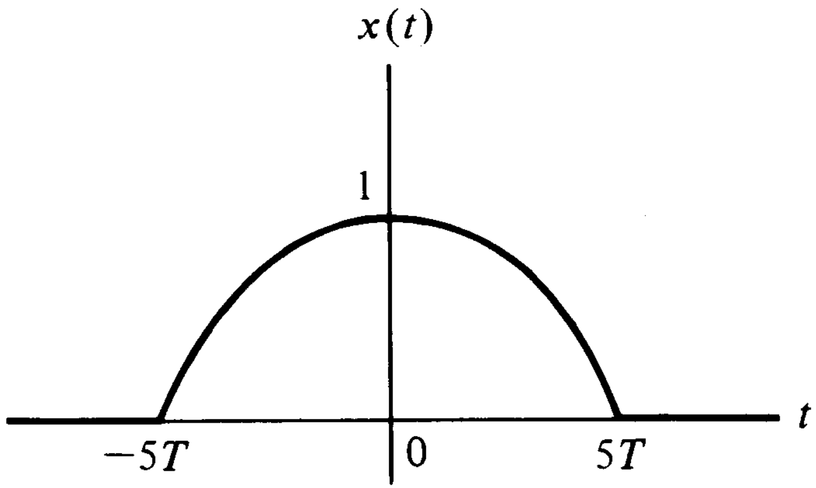
In order to get rid of aliasing, what is the cut-off frequency of the low pass filter? Explain your answer

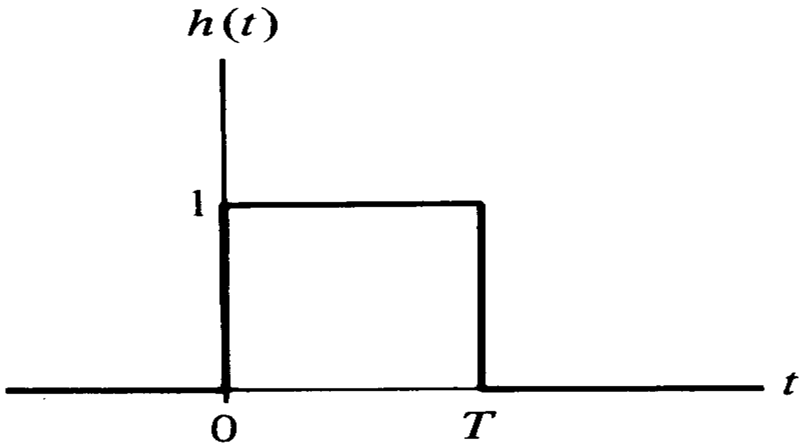
10 min

Q.2. Consider the following system given in the figure where input is x(t). Here, p(t) is an impulse train. Sketch $x_p(t)$, y(t) and w(t).



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$





10 mins

Q3) Consider the signal $x(n) = a^n u(n)$, $|a| < 1$.

(a) Determine the spectrum $X(\omega)$.

(b) The signal $x(n)$ is applied to a decimator that reduces the rate by a factor of 2. Determine the output spectrum.

(c) Show that the spectrum in part (b) is simply the Fourier transform of $x(2n)$.

Ans 1)

Let $X(e^{j\omega}), X_e(e^{j\omega}), Y(e^{j\omega})$ is the DTFT of $x(n), x_e(n), y(n)$.

Then $X(e^{j\omega})$ has a period of 2π.

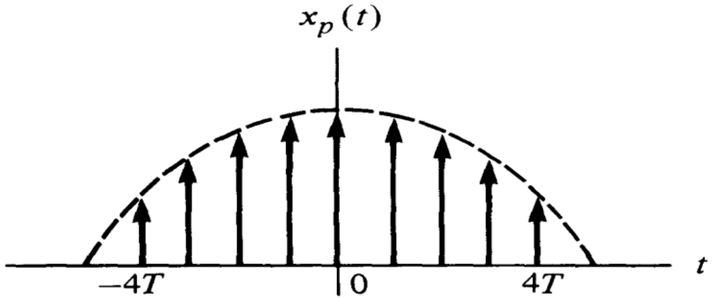
After the up sampling, all the original information of $x(n)$ will be contained in the interval $[-\pi/L, \pi/L]$.

New aliases occur in the interval of $[-\pi, -\pi/L], [\pi/L, \pi]$ which need to be filtered out.

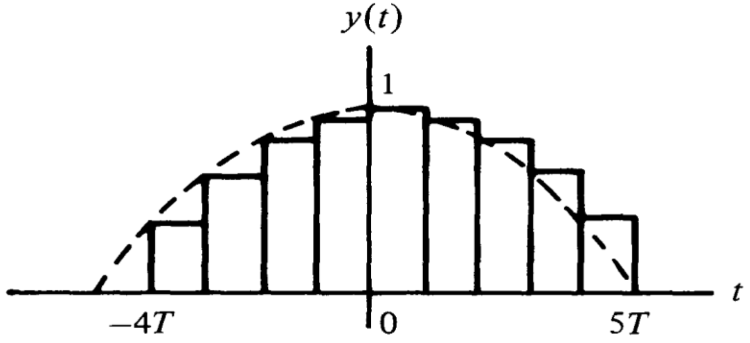
Thus, the cut-off frequency of the LP filter is π/L.

Ans 2)

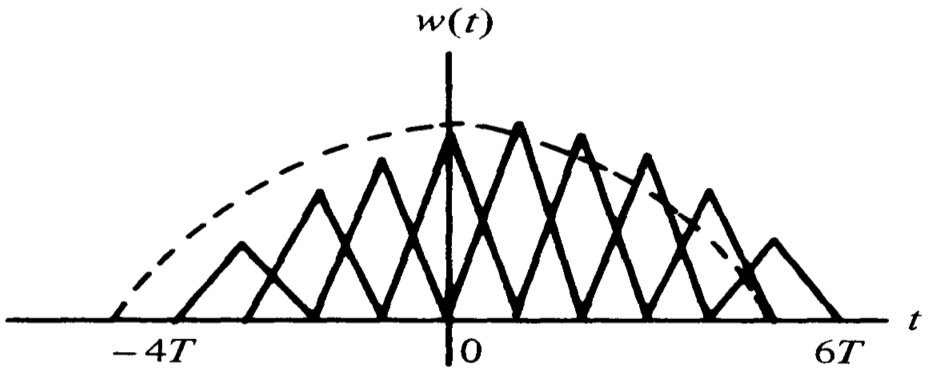
The signal $x_p(t)$ can be drawn as,



Now, $y(t) = x_p(t)*h(t)$. Hence, we can draw y(t) as,

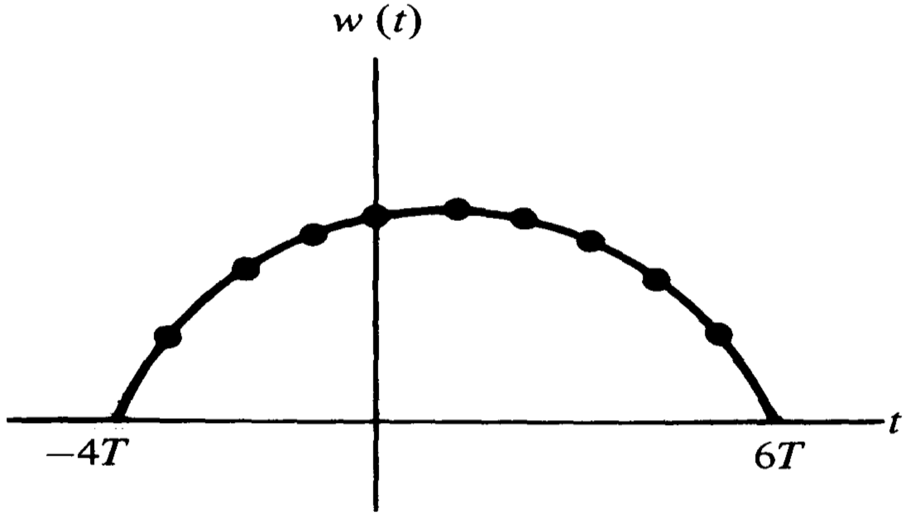


We know, the convolution of two rectangular signals is a triangular signal. The signal y(t) contains rectangular patches of similar width. Also, the system 1/T.h(t) is also a rectangular signal in time. Hence, the output w(t) can be drawn as



Hence, the output w(t) would contain superimposed triangular impulses.

We note that this superposition is actually linear interpolation between samples of x(t). Therefore our final output signal w(t) would be linear interpolation of xp(t), shifted by T.



Ans 3)

(a) $X(w) = \frac{1}{(1-ae^{-jw})}$

(b) After decimation $Y(w') = \frac{1}{2}X(\frac{w'}{2}) = \frac{1}{2(1-ae^{-\frac{jw'}{2}})}$

(c)

$$\begin{aligned} \text{DTFT}\ \{x(2n)\} &= \sum_n x(2n)e^{-jw2n} \\ &= \sum_n x(2n)e^{-jw'n} \\ &= Y(w') \end{aligned}$$