DSP Tutorial 2

Time: 15 min

[Q1]

[A] The DTFT of a signal is given by :

$$X(e^{jw}) = \frac{1}{(1-ae^{-jw})^2} \quad \text{for } |a|<1$$

Find out the discrete time signal x(n).

[B] Find discrete time Fourier transform of $x[n]=a^{|n|}$. $|a|<1$

[Q2] Given, $x(n)$ = $\sin(\omega_0 n + f)$ is the input to a LTI system with frequency response $H(e^{j\omega})$. If the output of the system is $Ax(n - n_0)$, then find the most general form of $\angle H(e^{j\omega})$.

Time: 15 min

[Q3] Given a sequence $x[n]$ such that, $x[-3] = 1$, $x[-2] = 1$, $x[-1] = 0$, $x[0] = 5$, $x[1] = 1$. Let $X(e^{j\omega})$ be the discrete-time Fourier Transform (DTFT) of $x[n]$. Find the value of the given expression:

$$\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$$

[Q4] Determine the signal x(n) having Fourier transform $X(\omega) = \cos^2 \omega$

[Q5] If the DTFT of x[n] is denoted as X[w], find out the DTFT of x[n] * $\overline{x[-n]}$. Here, "*" denotes convolution and $\overline{x}$ denotes conjugate operation.

Solution

[Q1]

[A]

→ Given, $X(e^{j\omega}) = \dfrac{1}{(1 - ae^{-j\omega})^2}$ $\quad |a| < 1$

We know, $a^n u(n) \longrightarrow \dfrac{1}{1 - ae^{-j\omega}}$

So, $a^n u(n) * a^n u(n) \longrightarrow \dfrac{1}{(1 - ae^{-j\omega})^2}$

So, $x(n) = a^n u(n) * a^n u(n)$

$$= \sum_{l=-\infty}^{\infty} a^l u(l)\, a^{(n-l)} u(n-l)$$

$$= \sum_{l=0}^{n} a^n$$

$$= a^n \sum_{l=0}^{n} 1$$

$\therefore$ $\boxed{x(n) = (n+1) a^n u(n)}$

[Q1]
[B]

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} a^{|n|} e^{-jwn}$$

$$X(e^{jw}) = \sum_{n=0}^{\infty} a^{n} e^{-jwn} + \sum_{n=-\infty}^{-1} a^{-n} e^{-jwn}$$

Substituting m=-n in the second summation, we obtain

$$X(e^{jw}) = \sum_{n=0}^{\infty} (a e^{-jw})^{n} + \sum_{m=1}^{\infty} (a e^{jw})^{m} = \frac{1}{1-a e^{-jw}} + \frac{a e^{jw}}{1-a e^{jw}} = \frac{1-a^2}{1-2a\cos\omega + a^2}$$

[Q2]

$y(n) = Ax(n-n_0)$

$\therefore Y(\omega) = Ae^{-j\omega n_0} X(\omega)$

$\therefore H(\omega) = \frac{Y(\omega)}{X(\omega)}$

$\boxed{= Ae^{-j\omega n_0}}$

$\boxed{\angle H(\omega) = -\omega n_0 \quad -\pi \leq \omega < \pi}$

periodic with period 2π

general form

$\angle H(\omega) = -\omega n_0 + 2\pi k$

k any intgl

[Q3]

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$$

For n = 0, we get,

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega.0}\,d\omega$$

$$x[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega$$

$$\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = 2\pi x[0] = 2\pi \times 5$$

[Q4]

$$
\begin{aligned}
X(w) &= cos^2(w) \\
&= (\frac{1}{2}e^{jw} + \frac{1}{2}e^{-jw})^2 \\
&= \frac{1}{4}(e^{j2w} + 2 + e^{-j2w}) \\
x(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)e^{jwn}dw \\
&= \frac{1}{8\pi}[2\pi\delta(n+2) + 4\pi\delta(n) + 2\pi\delta(n-2)] \\
&= \frac{1}{4}[\delta(n+2) + 2\delta(n) + \delta(n-2)]
\end{aligned}
$$

[Q5]

$$x[n] \longleftrightarrow X(e^{j\omega})$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$X^*(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x^*[n]\, e^{+j\omega n}$$

$$n \to -n$$

$$\sum_{n=-\infty}^{\infty} x^*[-n]\, e^{-j\omega n} = X^*(e^{j\omega})$$

$$y(n) = x(n) \circledast x^*[-n]$$

$$Y(e^{j\omega}) = X(e^{j\omega})\, X^*(e^{j\omega})$$

$$Y(e^{j\omega}) = |X(e^{j\omega})|^2$$