Digital Signal Processing

Assignment-4

1. What is the time domain signal for $X[z] = \frac{1}{1-\frac{1}{4}z^{-2}}, |z| > \frac{1}{4}$?

2. Determine the z-transform and ROC of the following

$$x(n) = \begin{cases} (1/2)^n & n \geq 5 \\ 0 & n \leq 4 \end{cases}$$

3. Find the inverse z-transform and ROC of the given expression:

$$X(z) = \left(\frac{1-z^2}{z}\right)^2$$

4. Let, $x[n]=\delta[n-2]+\delta[n+2]$.Find the unilateral z transform of $x[n]$?
5. Find the z-transform of $x(n)= (1/5)^n\ u(n-5)$
6. Prove that Z.T. of $x(n)=n$ is $z/(z-1)^2$

   And Find

$$Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$$

7.

Consider $G(z) = \frac{P(z)}{Q(z)}$, let ρ_l be the residue of $G(z)$ at a simple pole $z = \lambda_l$, show that

$$\rho_l = -\lambda_l \frac{P(z)}{Q'(z)} \quad at \quad z = \lambda_l$$

where

$$Q'(z) = \frac{dQ(z)}{dz^{-1}}$$

8.

Consider an arbitrary digital filter with transfer function

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

Perform a two-component polyphase decomposition of $H(z)$ by grouping the even-numbered samples $h_0(n) = h(2n)$ and the odd-numbered samples $h_1(n) = h(2n+1)$. Thus show that H(z) can be expressed as,

$$H(z) = H_0(z^2) + z^{-1}H_1(z^2)$$

9.

Consider

$$G(z) = \frac{p_0 + p_1 z^{-1} .... + p_M z^{-M}}{d_0 + d_1 z^{-1} .... + d_N z^{-N}} \quad where \quad M < N$$

if $G(z)$ has only simple poles, show that

$$\frac{p_0}{d_0} = sum \quad of \quad the \quad residues \quad in \quad the \quad PFE \quad of \quad G(z)$$

Note: Partial Fraction Expansion(PFE)

10.

Determine the causal signal $x(n)$ having the z-transform

$$X(z) = \frac{1}{(1 - 2z^{-1})(1 - z^{-1})^2}$$