Tutorial 3

04/02/2021

Discussion

- **Example :**Q1:: Suppose, we want to design a low pass filter with pass band up to 2 kHz and stop band from 2.5 kHz. The sampling frequency is 10 kHz. If we take a [A] rectangular window and [B] hamming window for FIR design, what will be the order of the filter (in each case)?
- **Ans1:** $\omega_c = 2\pi f_c T_s$

→ Pass band : $\omega_p = 2\pi \times \frac{2}{10} = \frac{2\pi}{5}$

Stop band : $\omega_s = 2\pi \times \frac{2.5}{10} = \frac{\pi}{2}$

→ So, $\Delta\omega = \omega_s - \omega_p$

$= \pi/2 - 2\pi/5$

$= \pi/10$ (Transition width)

→ For windowing, main Lobe width ≈ transition width

[A] For Rectangular window : $\frac{4\pi}{(M+1)} = \frac{\pi}{10}$

$\Rightarrow (M+1) = 40$

$\Rightarrow M = 39$

$\therefore$ Filter order $(N) \geq (M-1)$

$\Rightarrow N \geq 38$

[B] For Hamming window : $\frac{8\pi}{M} = \frac{\pi}{10}$

$\Rightarrow M = 80$

$\therefore$ Filter order $(N) \geq 79$

Discussion

- **Example :**Q2:: Find the impulse response of a FIR filter using rectangular window with the desired frequency response given as

$$H_d(e^{jw}) = \begin{cases} e^{-j2w} & |w| \le w_c \\ 0 & \text{otherwise} \end{cases}$$

Length of the filter is 5 and w_c is 1 rad/sample.

➢ **Ans2**: Taking I.F.T of $H_d(e^{jw})$,

$$h_d(n) = \frac{1}{2\Pi} \int_{-\Pi}^{\Pi} \left(H_d(e^{j\omega}) \cdot e^{j\omega n} \right) d\omega$$

$$h_d(n) = \frac{\sin(n-2)\,\omega_c}{(n-2)\pi}$$

Considering a rectangular window,

$$w_r(n) = \begin{cases} 1 & n=0 \text{ to } 4 \\ 0 & \text{otherwise.} \end{cases}$$

Response of filter is $h(n)=h_d(n).w_r(n)$

Therefore, $h(n) = \dfrac{\sin(n-2)}{(n-2)\pi}$ for $0 \le n \le 4$

Questions:

20 minutes

Q.1. Design an FIR linear phase digital filter approximating the ideal frequency response:

$$H_d(\omega) = \begin{cases} 1, & for\ |\omega| \le \frac{\pi}{6} \\ 0, for\ \frac{\pi}{6} < |\omega| \le \pi \end{cases}$$

Determine the impulse response of a 25-tap filter based on the window method with a rectangular window.

[Hint: Do not forget to calculate the delay appropriately of the frequency response.]

Q.2. Determine the frequency response of FIR filter defined by $y(n) = 0.25x(n) + x(n-2) + 0.25x(n-4)$. Calculate the phase delay and group delay.

[Hint:
Phase delay $= \tau = \frac{-\theta(\omega)}{\omega}$

Group delay $= -\frac{d(\theta(\omega))}{d\omega}$]

Q.3. The ideal analog differentiator is described by:

$$y_a(t) = \frac{dx_a(t)}{dt}$$

Where $x_a(t)$ is input and $y_a(t)$ is the output. If $x_a(t)$ is given by $e^{j2\pi Ft}$, determine and sketch the frequency response of the system.

Find the frequency response the system given by $y[n] = x[n] - x[n-1]$ and compare it with the frequency response of $h_a(t)$.

Solution

- Q.1. Design an FIR linear phase digital filter approximating the ideal frequency response:

$$H_d(\omega) = \begin{cases} 1, \ \ for\ |\omega| \le \frac{\pi}{6} \\ 0, for\ \frac{\pi}{6} < |\omega| \le \pi \end{cases}$$

Determine the impulse response of a 25-tap filter based on the window method with a rectangular window

- **Ans1:** To obtain the desired length of 25, a delay of $\frac{25-1}{2} = 12$ is incorporated into $H_d(w)$. Hence,

$$\begin{aligned} H_d(w) &= 1e^{-j12w}, \quad 0 \le |w| \le \frac{\pi}{6} \\ &= 0, \quad \text{otherwise} \\ h_d(n) &= \frac{1}{2\pi}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} H_d(w)e^{-jwn}dw \\ &= \frac{sin\frac{\pi}{6}(n-12)}{\pi(n-12)} \\ \text{Then, } h(n) &= h_d(n)w(n) \end{aligned}$$

where $w(n)$ is a rectangular window of length $N = 25$.

Solution

- Q.2. Determine the frequency response of FIR filter defined by $y(n) = 0.25x(n) + x(n-2) + 0.25x(n-4)$. Calculate the phase delay and group delay.

- **Ans2:**

$$y(n) = 0.25x(n) + x(n-2) + 0.25x(n-4)$$

$$Y(e^{j\omega}) = 0.25X(e^{j\omega}) + e^{-2j\omega}X(e^{j\omega}) + 0.25e^{-4j\omega}X(e^{j\omega})$$

$$Y(e^{j\omega}) = X(e^{j\omega})(0.25 + e^{-2j\omega} + 0.25e^{-4j\omega})$$

$$H(e^{j\omega}) = (0.25 + e^{-2j\omega} + 0.25e^{-4j\omega})$$

$$H(e^{j\omega}) = e^{-2j\omega}(0.25e^{2j\omega} + 1 + 0.25e^{-2j\omega})$$

$$H(e^{j\omega}) = e^{-2j\omega}(1 + 0.5cos2\omega)$$

Hence, $|H(e^{j\omega})| = (1 + cos2\omega)$ and $\angle H(e^{j\omega}) =$ -2ω.

$$\text{Phase delay} = \tau = \frac{-\theta(\omega)}{\omega} = 2$$

$$\text{Group delay} = -\frac{d(\theta(\omega))}{d\omega} = 2$$

Solution

Q.3. The ideal analog differentiator is described by:

$$y_a(t) = \frac{dx_a(t)}{dt}$$

Where $x_a(t)$ is input and $y_a(t)$ is the output. If $x_a(t)$ is given by $e^{j2\pi Ft}$, determine and sketch the frequency response of the system.

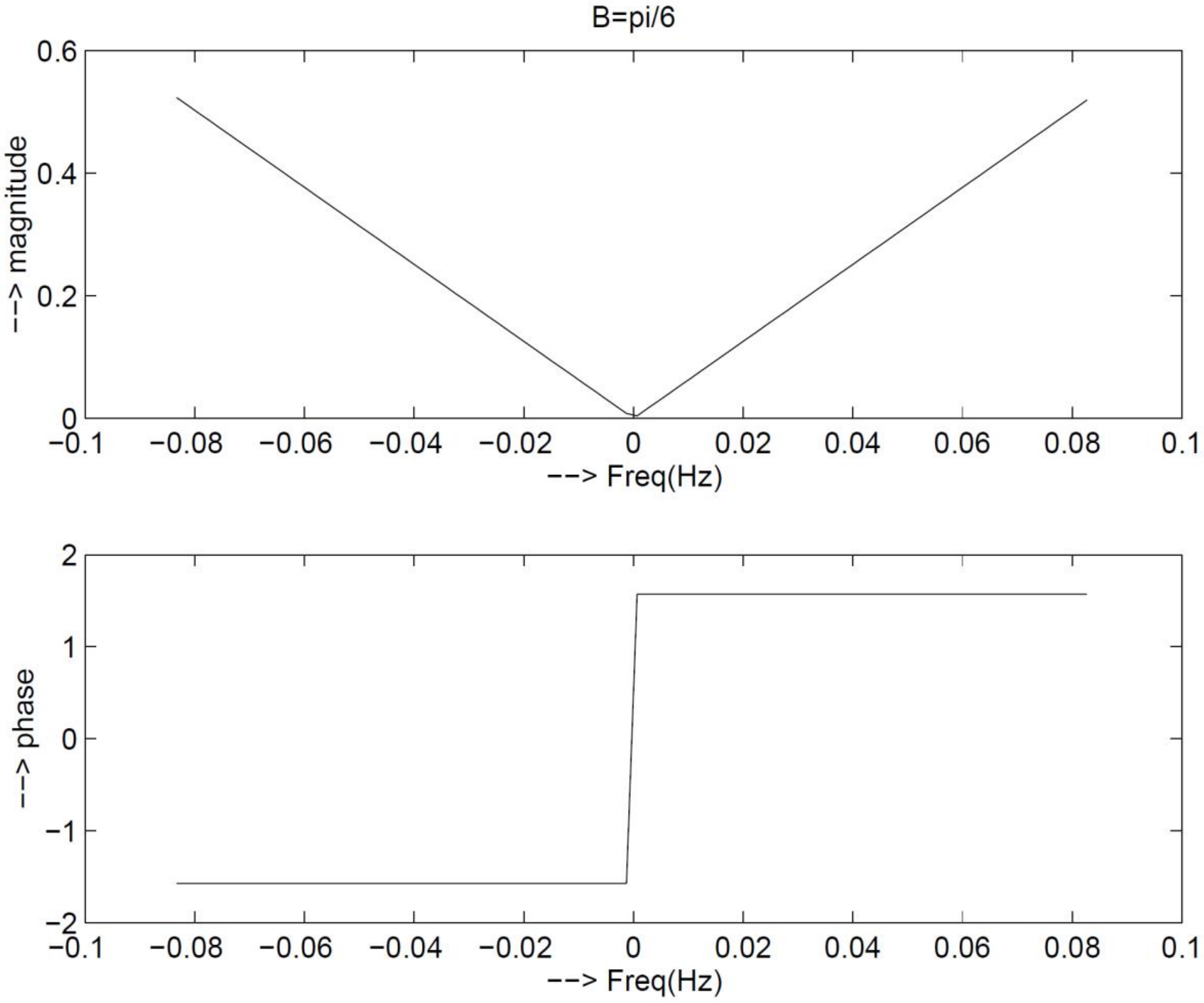
Find the frequency response the system given by $y[n] = x[n] - x[n-1]$ and compare it with the frequency response of $h_a(t)$.

- **Ans.3.** To obtain frequency response of a differentiator,

$$\begin{aligned} y_a(t) &= \frac{dx_a(t)}{dt} \\ &= \frac{d}{dt}[e^{j2\pi Ft}] \\ &= j2\pi F e^{j2\pi Ft} \\ \text{Hence, } H(F) &= j2\pi F \end{aligned}$$

$$\begin{aligned} |H(F)| &= 2\pi F \\ \angle H(F) &= \frac{\pi}{2}, \qquad F > 0 \\ &= -\frac{\pi}{2}, \qquad F < 0 \end{aligned}$$

Hence, the frequency response plot is given as,

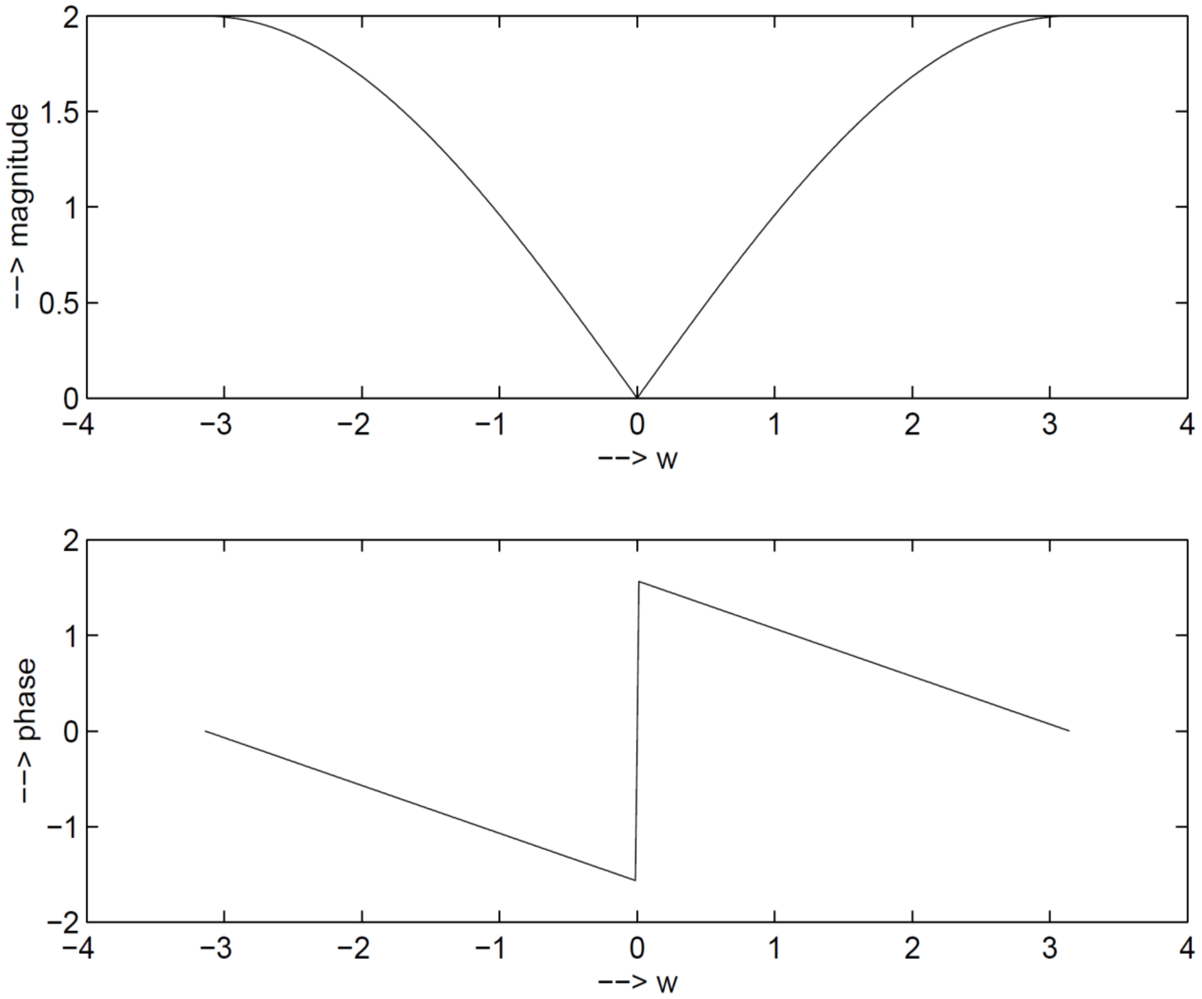


Frequency response of differentiator

Next, for y[n]=x[n]-x[n-1], we have,

$$\begin{aligned}
y(n) &= x(n) - x(n-1) \\
H(z) &= 1 - z^{-1} \\
H(w) &= 1 - e^{-jw} \\
&= e^{-j\frac{w}{2}}(2jsin\frac{w}{2}) \\
|H(w)| &= 2|sin\frac{w}{2}| \\
\angle H(w) &= \frac{\pi}{2} - \frac{w}{2}
\end{aligned}$$

The frequency response of the difference equation is given as,



Frequency response of the difference equation

Note that for small w, $sin\frac{w}{2} \approx \frac{w}{2}$ and $H(w) \approx jwe^{-j\frac{w}{2}}$, which is a suitable approximation to the differentiator