DSP Assignment 6 Solutions

Q1) The resolution of a 4-bit counting ADC is 0.5 volts. For an analog input of 6.6 volts, what will be the digital output of the ADC?

Ans 1)- Resolution is the smallest analog signal that can make a change in the digital output.

The output of ADC is 6.6/0.5=13.2 V .

The binary equivalent is 1110.

Q2) An analog voltage in the range 0-8 V is divided in 16 equal intervals for conversion to 4-bit digital output. Find the maximum quantization error?

Ans 2) Maximum Quantization error $=\pm\Delta/2$ where Δ is the step size.

$\Delta = (V_{max}-V_{min})$/No. Of levels = $(8 - 0)/16 = 0.5$

The answer is 0.25.

Q3) Consider an arbitrary digital filer with transfer function:

$$H(z)=\sum_{n=-\infty}^{\infty} h(n)z^{-n}.$$

Perform two-component polyphase decomposition of H(z) by grouping the even numbered samples $h_0(n)=h(2n)$ and odd numbered samples $h_1(n)=h(2n+1)$.

A) Show that $H(z)=H_0(z^2)+z^{-1}H_1(z^2)$.

B) Determine $H_0(z)$ and $H_1(z)$.

Ans 3)

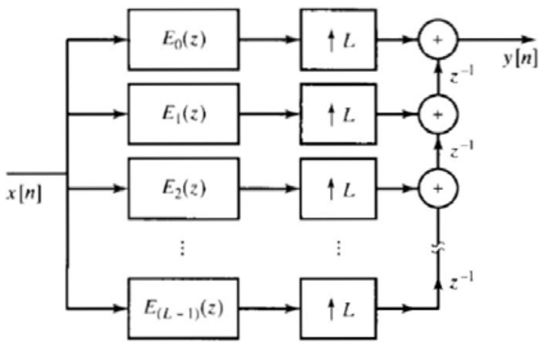
$$\begin{aligned}
H(z) &= \sum_n h(2n)z^{-2n} + \sum_n h(2n+1)z^{-2n-1} \\
&= \sum_n h(2n)(z^2)^{-n} + z^{-1}\sum_n h(2n+1)(z^2)^{-n} \\
&= H_0(z^2) + z^{-1}H_1(z^2) \\
\text{Therefore } H_0(z) &= \sum_n h(2n)z^{-n} \\
H_1(z) &= \sum_n h(2n+1)z^{-n}
\end{aligned}$$

Q4) A) Draw the efficient polyphase structure for Interpolation with I=3.

B) What is type II polyphase structure? Derive its relation with type I polyphase structure.

C) Draw the type II polyphase structure for Interpolation with I=3.

Ans 4) A)



B)

$$H(z) = \sum_{n=0}^{N-1} z^{-n} P_n(z^N)$$

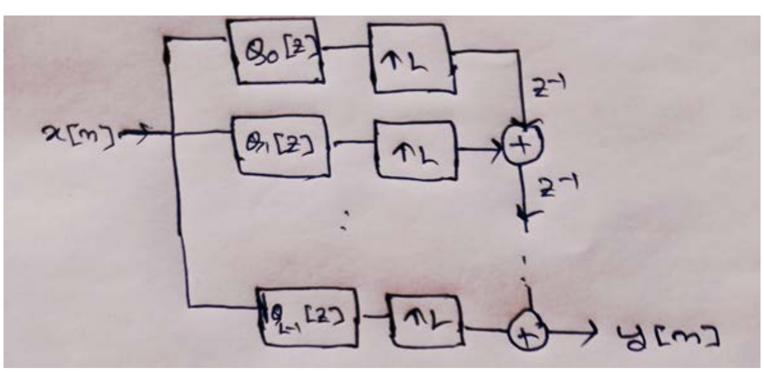
where

$$P_n(z) = \sum_{k=-\infty}^{\infty} h(kN+n) z^{-k}$$

Let $m = N-1-n$. Then

$$\begin{aligned} H(z) &= \sum_{n=0}^{N-1} z^{-(N-1-m)} P_{N-1-m}(z^N) \\ &= \sum_{n=0}^{N-1} z^{-(N-1-m)} Q_m(z^N) \end{aligned}$$

C)



Q5) Determine the signal-to-quantization noise ratio(SQNR) of a 8-bit,12-bit ,16-bit and 20-bit quantizer for a full scale sinusoidal signal.

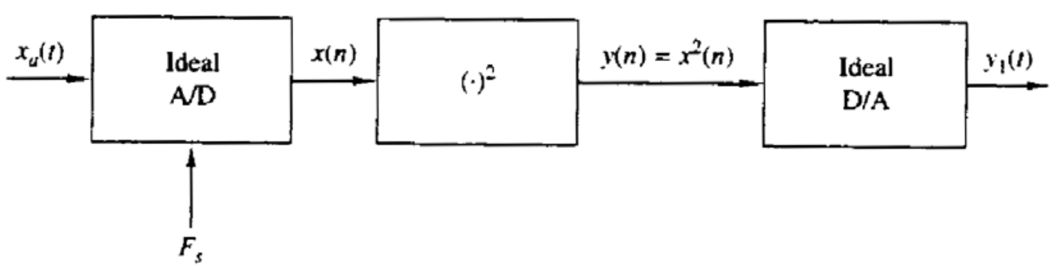
Ans 5)

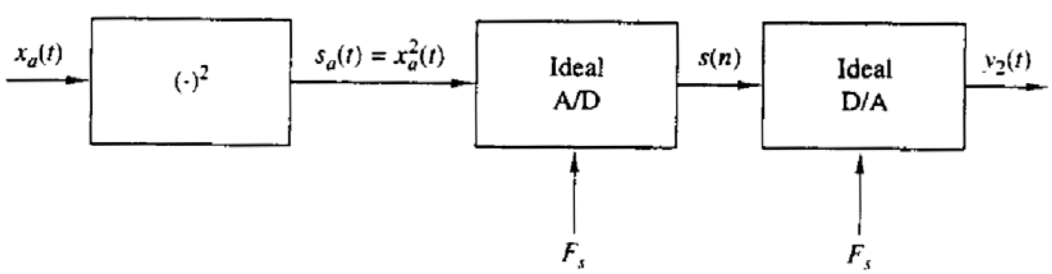
$$SQNR = \frac{P_{sig}}{P_{qnoise}} = \frac{\frac{1}{2}\left(\frac{2^B \Delta}{2}\right)^2}{\frac{\Delta^2}{12}} = 1.5 \times 2^{2B} = 6.02B + 1.76 \ \ dB$$

B (Number of Bits)	SQNR
8	50dB
12	74dB
16	98dB
20	122dB

Q6)

Determine $y_1(t)$ and $y_2(t)$ if $x_a(t) = \cos 2\pi F_0 t$, $F_0 = 20$ Hz, and $F_s = 50$ Hz or $F_s = 30$ Hz.





Ans 6)

$$
\begin{aligned}
x_a(t) &= cos 40\pi t \\
x(n) &= cos\frac{40\pi n}{50} \\
&= cos\frac{4\pi n}{5} \\
y(n) &= x^2(n) \\
&= cos^2\frac{4\pi n}{5} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{8\pi n}{5} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{2\pi n}{5} \\
y_1(t) &= \frac{1}{2} + \frac{1}{2}cos 20\pi t \\
s_a(t) &= x_a^2(t) \\
&= cos^2 40\pi t \\
&= \frac{1}{2} + \frac{1}{2}cos 80\pi t \\
s(n) &= \frac{1}{2} + \frac{1}{2}cos\frac{80\pi n}{50}
\end{aligned}
$$

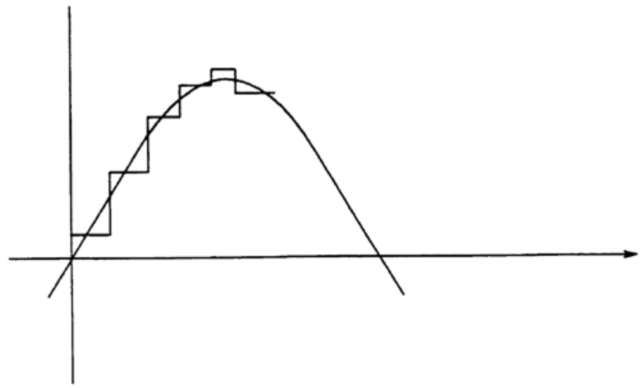
$$
\begin{aligned}
&= \frac{1}{2} + \frac{1}{2}cos\frac{8\pi n}{5} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{2\pi n}{5} \\
\text{Hence, } y_2(t) &= \frac{1}{2} + \frac{1}{2}cos 20\pi t \\
\text{For } F_s &= 30, \\
x(n) &= cos\frac{4\pi n}{3} \\
&= cos\frac{2\pi n}{3} \\
y(n) &= x^2(n) \\
&= cos^2\frac{2\pi n}{3} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{4\pi n}{3} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{2\pi n}{3} \\
y_1(t) &= \frac{1}{2} + \frac{1}{2}cos 20\pi t \\
s_a(t) &= x_a^2(t) \\
&= cos^2 40\pi t \\
&= \frac{1}{2} + \frac{1}{2}cos 80\pi t \\
s(n) &= \frac{1}{2} + \frac{1}{2}cos\frac{80\pi n}{30} \\
&= \frac{1}{2} + \frac{1}{2}cos\frac{2\pi n}{3} \\
\text{Hence, } y_2(t) &= \frac{1}{2} + \frac{1}{2}cos 20\pi t
\end{aligned}
$$

Q7)

Consider a DM coder with input $x(n) = A\cos(2\pi nF/F_s)$. What is the condition for avoiding slope overload? Illustrate this condition graphically.

Ans 7)

$$
\begin{aligned}
x(t) &= A\cos 2\pi Ft \\
\frac{dx(t)}{dt} &= -A(2\pi F)\sin 2\pi Ft \\
&= -2\pi AF\sin 2\pi Ft \\
\frac{dx(t)}{dt}\Big|\max &= 2\pi AF \le \frac{\Delta}{T} \\
\text{Hence, } \Delta &\ge 2\pi AFT \\
&= \frac{2\pi AF}{F_s}
\end{aligned}
$$



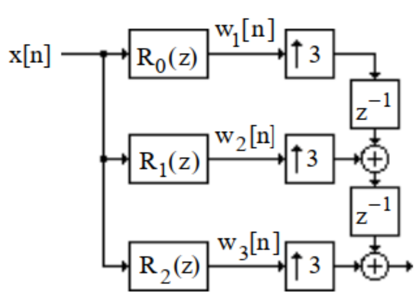
Q8) Develop a computationally efficient realisation of a factor of 3 interpolator employing a length-15 Type-II FIR filter.

Ans 8)

- A computationally efficient realization of the factor-of-3 interpolator is obtained by applying a 3-branch Type II polyphase decomposition to the interpolation filter H(z):

$$H(z) = R_2(z^3) + z^{-1}R_1(z^3) + z^{-2}R_0(z^3),$$

and then moving the up-sampler through the polyphase filters resulting in



From the above figure it follows that

$$W_3(z) = h[0]X(z) + h[3]z^{-1}X(z) + h[6]z^{-2}X(z) + h[5]z^{-3}X(z) + h[2]z^{-4}X(z),$$

$$W_1(z) = h[2]X(z) + h[5]z^{-1}X(z) + h[6]z^{-2}X(z) + h[3]z^{-3}X(z) + h[0]z^{-4}X(z), \quad \text{and}$$

$$W_2(z) = h[1]\left(X(z) + z^{-4}X(z)\right) + h[4]\left(z^{-1}X(z) + z^{-3}X(z)\right) + h[7]z^{-2}X(z).$$

A computationally efficient factor-of-3 interpolator structure based on the above equations is then as shown below: