DSP Assignment-I

Harshavardhan Alimi

18EC10021

1) False

Let $\quad T_1(x(n)) = a\,x^3(n)$

& $\quad T_2(x(n)) = x^{1/3}(n)$

both are non Linear.

then.

$$T(x(n)) = T_2 T_1(x(n))$$

$$= T_2(a\,x^3(n))$$

$$= a^{1/3}\,x(n)$$

which is a linear.

So it false.

2)

To Prove,

$$\int_{-\infty}^{\infty} x^2(t).dt = \int_{-\infty}^{\infty} x_e^2(t).dt + \int_{-\infty}^{\infty} x_o^2(t).dt$$

where, $x(t)$ - real valued signal

$x_e(t)$ - even component

$x_o(t)$ - odd component.

so, we have

$$x(t) = x_e(t) + x_o(t)$$

$$\int_{-\infty}^{\infty} x^2(t).dt = \int_{-\infty}^{\infty} (x_e(t) + x_o(t))^2 . dt$$

$$= \int_{-\infty}^{\infty} x_e^2(t).dt + \int_{-\infty}^{\infty} x_o^2(t).dt + 2\int_{-\infty}^{\infty} x_e(t) x_o(t).dt$$

as, $x_e(t).\ x_o(t)$ is an odd signal

then, we have,

$$\int_{-T}^{T} x_e(t).\ x_o(t).dt = 0$$ as it is a odd signal

($\because$ area under an odd signal over symmetrical limits is zero), Hence,

$$\therefore \int_{-\infty}^{\infty} x_e(t).\ x_o(t).dt = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} x_e^2(t).dt + \int_{-\infty}^{\infty} x_o^2(t).dt$$

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3>

Let $x\{n\}$ is a real valued signal

then,

$\Rightarrow x\{n\} = x_e\{n\} + x_o\{n\}$

Power of a signal $x\{n\}$ is given by

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x\{n\}|^2$$

Let power of $x_e\{n\}$ be P_e

power of $x_o\{n\}$ be P_o

$$P_e = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_e[n]|^2$$

$$P_o = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_o\{n\}|^2$$

as, $x_e\{n\} = \dfrac{x\{n\} + x\{-n\}}{2}$

$x_o\{n\} = \dfrac{x[n] - x\{-n\}}{2}$

Let,

$$r\{n\} = \left|\frac{x\{n\} + x\{-n\}}{2}\right|^2 + \left|\frac{x[n] - x\{-n\}}{2}\right|^2$$

as $x\{n\}$ is a real valued

$$= \frac{2x\{n\}^2 + 2x\{-n\}^2 + 2x\{n\}x\{-n\} - 2x\{n\}x\{-n\}}{4}$$

$$= \frac{(x\{n\})^2 + (x\{-n\})^2}{2}$$

$$P_e + P_o = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x[n]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1}{2} \left((x[n])^2 + (x[-n])^2 \right) \right)$$

$$= \lim_{N \to \infty} \frac{1}{2} \times \frac{1}{2N+1} \left\{ \sum_{n=-N}^{N} (x[n])^2 + \sum_{n=-N}^{N} (x[-n])^2 \right\}$$

$$= \lim_{N \to \infty} \frac{1}{2} \times \frac{1}{2N+1} \left[\sum_{n=-N}^{N} (x[n])^2 + \sum_{t=-N}^{N} (x[t])^2 \right]$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (x[n])^2$$

$$= P$$

i.e., $\boxed{P_e + P_o = P}$

Hence Proved.

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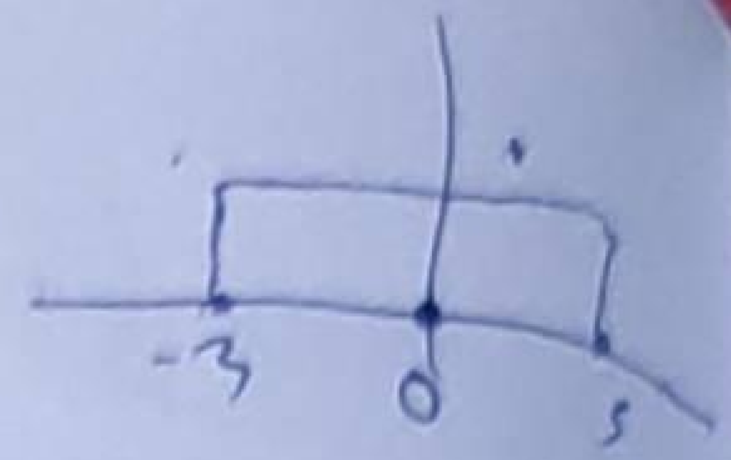
4)

$g[n] = u[n+3] - u[n-5]$



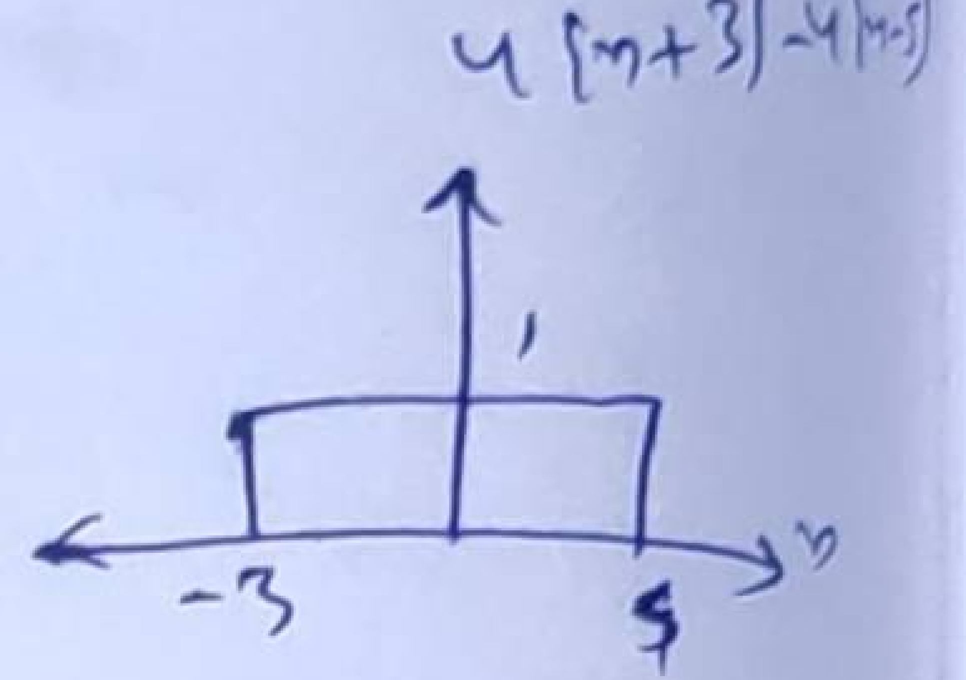
a.

$$\text{Energy} = \sum_{n=-\infty}^{\infty} |g[n]|^2$$

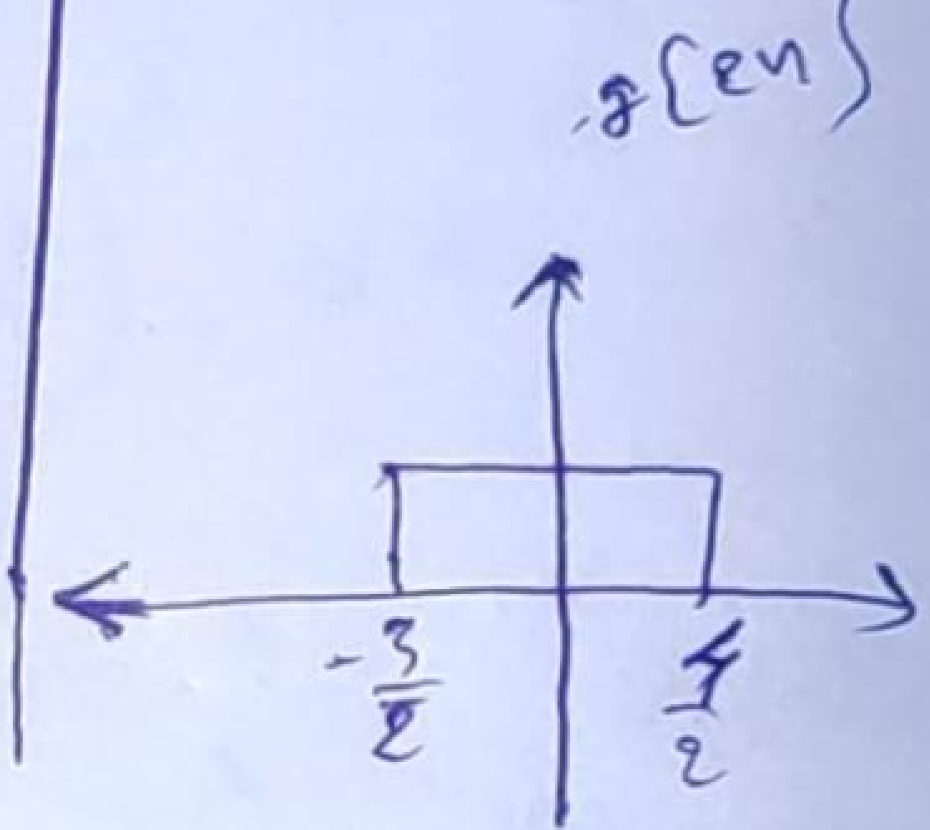
$$= \sum_{n=-\infty}^{\infty} |u[n+3] - u[n-5]|^2$$

$$= \sum_{n=-3}^{4} |1|^2$$

$$= 8.$$



$g[2n] = u[2n+3] - u[2n - 5]$



$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |g[2n]|^2$$

if $N > \frac{5}{2}$

$$= \lim_{N\to\infty} \frac{1}{2N+1} (8)$$

$$= 0$$

5) $u[n+3] * u[n-3]$

$$= \sum_{k=-\infty}^{\infty} u[k+3] \cdot u[n-k-3]$$

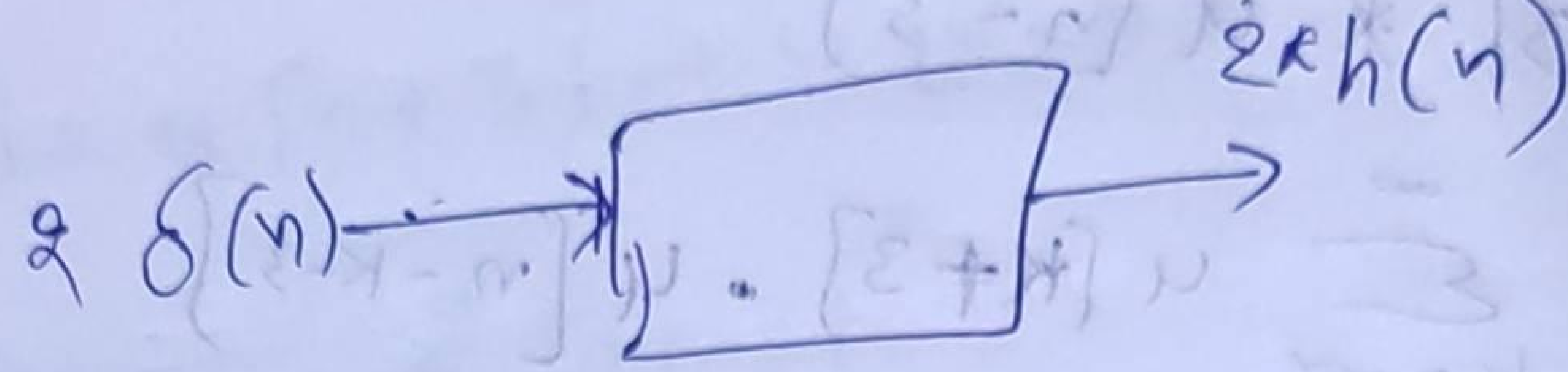
$$= \sum_{k=-3}^{n-3} (1) \qquad \text{if } n \geq 0$$

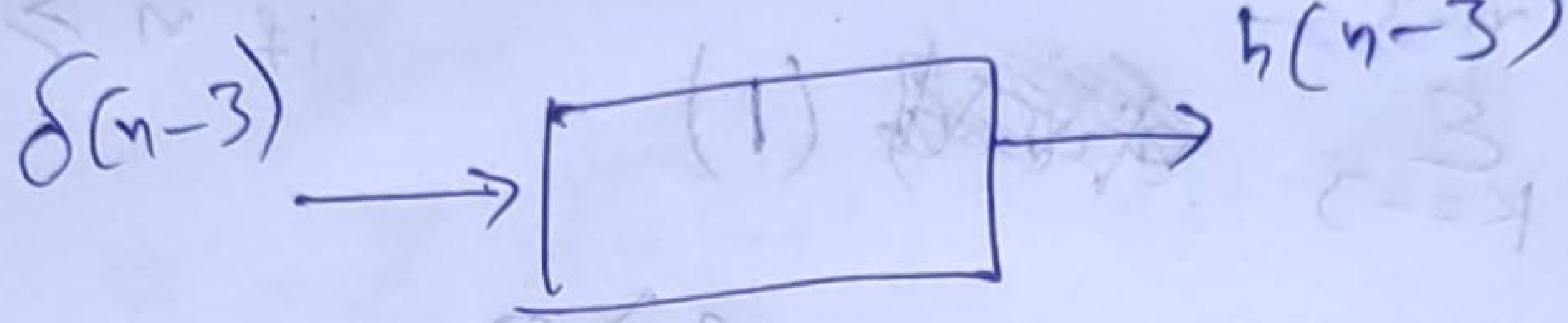
$$= n+1 \qquad n \geq 0$$

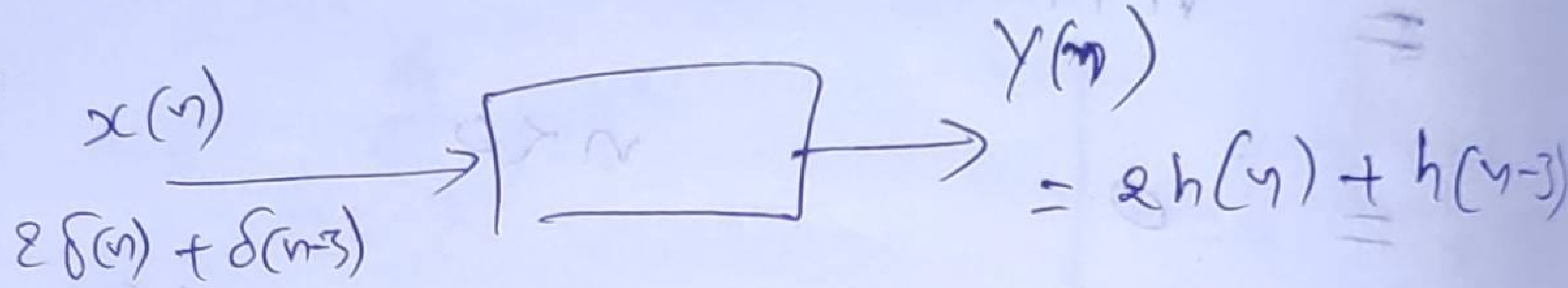
$$= 0 \qquad n < 0$$

$$u[n+3] * u[n-3] = \begin{cases} 0 & n < 0 \\ n+1 & n \geq 0 \end{cases}$$

6)







$$y(n) = 2\left[\frac{1}{2} n\, u(n)\right] + \frac{1}{2}(n-3)\, u(n-3)$$

$$y(1) = 1$$

$$y(4) = 4 + \frac{1}{2} \times 1 \times 1 = 4.5$$

7)
$$y_1\{n\} = \prod_{k=-\infty}^{n} e^{x_1\{k\}}$$

$$y_2\{n\} = \prod_{k=-\infty}^{n} e^{x_2\{k\}}$$

$$x_3\{k\} = x_1\{k\} + x_2\{k\}$$

$$y_3\{n\} = \prod_{k=-\infty}^{n} e^{x_3\{k\}}$$

$$= \prod_{k=-\infty}^{n} e^{x_1\{k\} + x_2\{k\}}$$

$$= \prod_{k=-\infty}^{n} e^{x_1\{k\}} * e^{x_2\{k\}}$$

$$= y_1\{n\} * y_2\{n\} \neq y_1\{n\} + y_2\{n\}$$

non-linear

8. Let. $h\{n\} = s\{n\} * \cancel{s\{n\}}\ x_3\{n-N_3\}$

$s\{n\} = \cancel{x}_1[n-N_1] * x_2[n-N_2]$

$= \sum_{K=-\infty}^{\infty} x_1\{K-N_1\} \cdot x_2\{n-K-N_2\}$

Put $p = K - N_1$

$K = p + N_1$

$= \sum_{p=-\infty}^{\infty} x_1\{p\} \cdot x_2\{n-N_1-N_2-K\}$

$= T\{n-N_1-N_2\}$

let

$T\{n\} = x_1\{n\} * x_2\{n\}$

$g\{n\} = T\{n\} * x_3\{n\}$

$h\{n\} = s\{n\} * x_3[n-N_3]$

$= T\{n-N_1-N_2\} * x_3\{n-N_3\}$

$= \sum_{K=-\infty}^{\infty} x_3\{K-N_3\} \cdot T\{n-K-N_1-N_2\}$

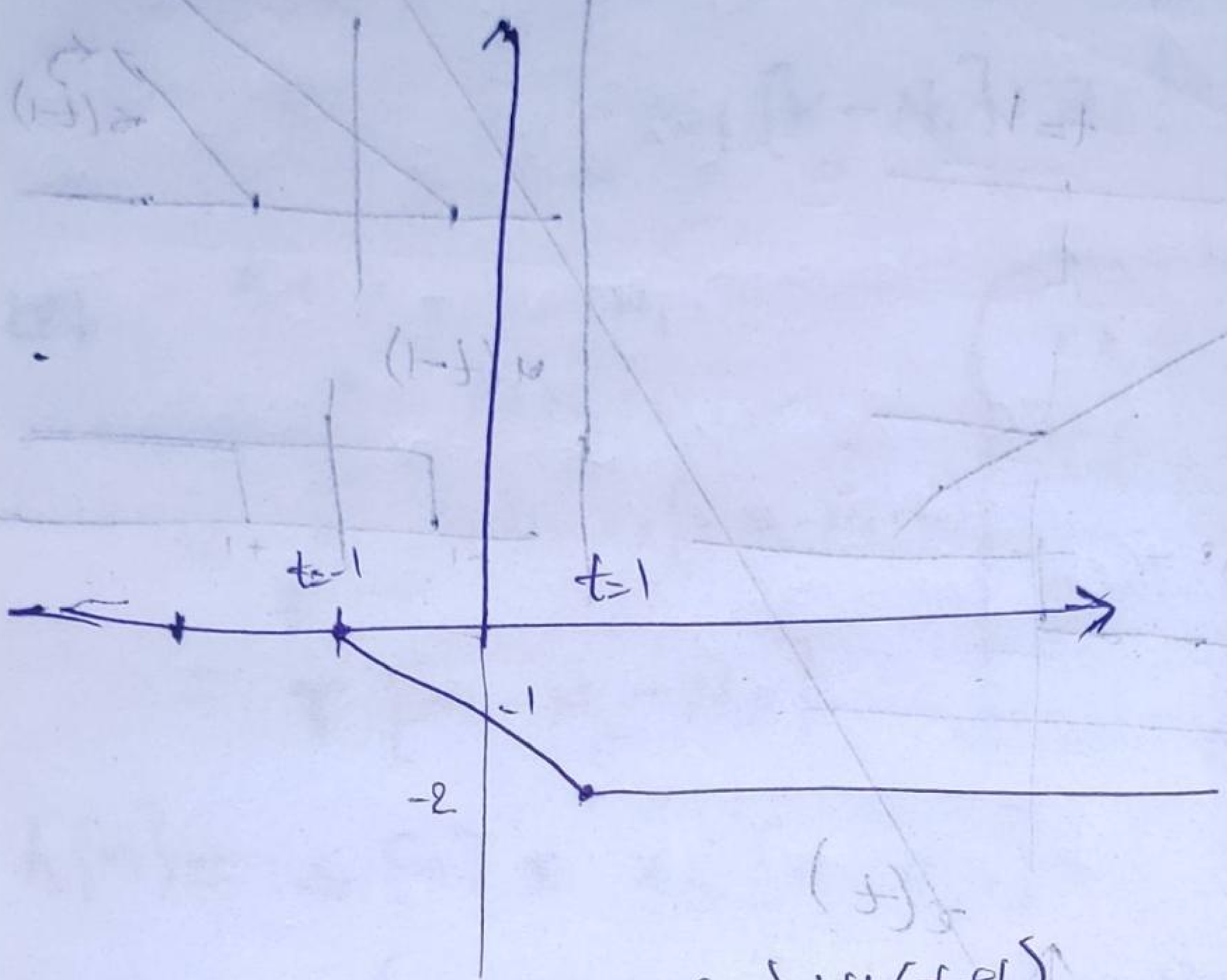
$K = p + N_3, \quad p = K - N_3$

$= \sum_{p=-\infty}^{\infty} x_3\{p\} \cdot T\{n-N_1-N_2-N_3-p\}$

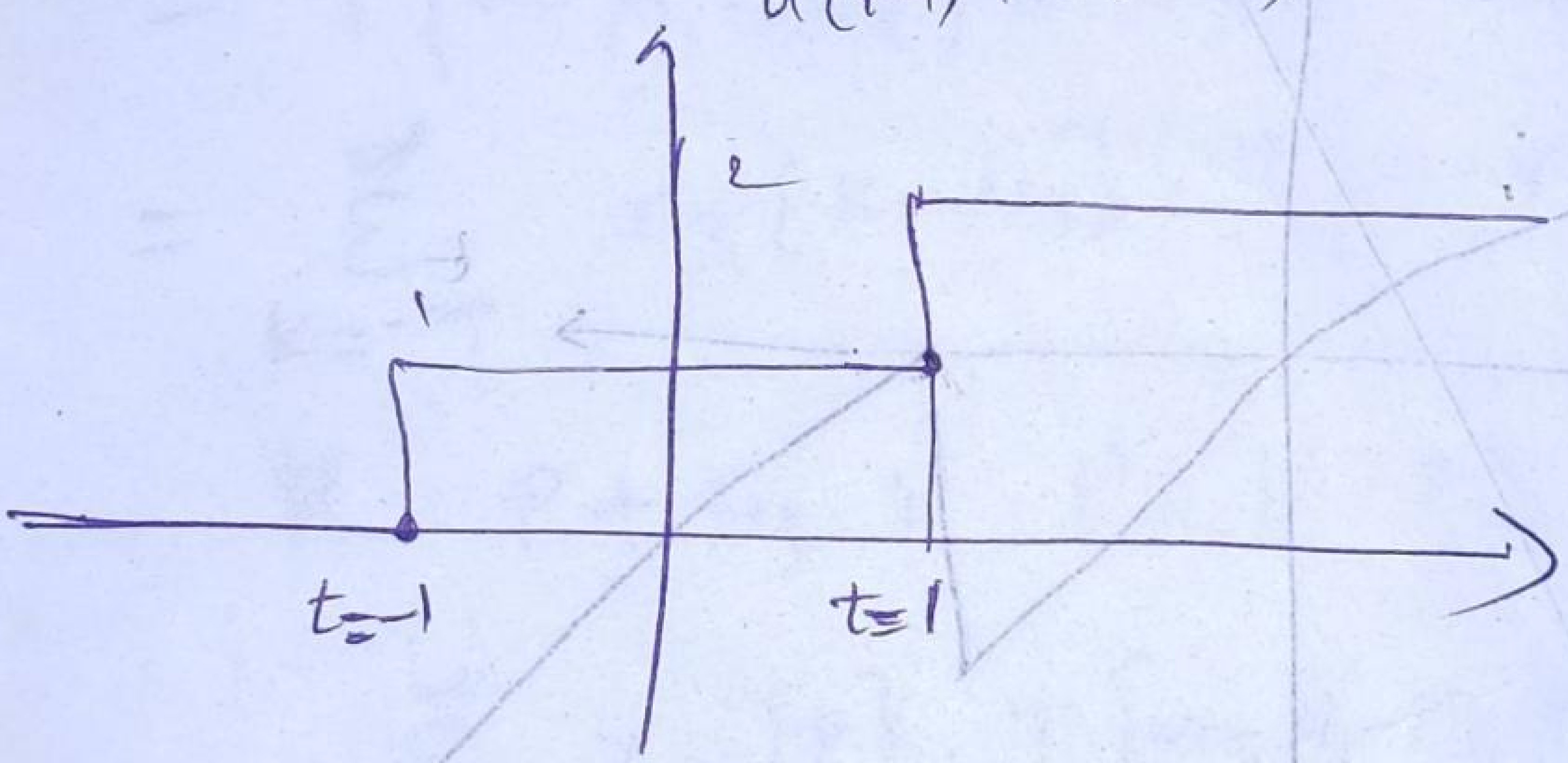
$h\{n\} = g\{n-N_1-N_2-N_3\}$ //

9. $x(t) = u(t-1) + r(t-1) - r(t+1) + u(t+1)$

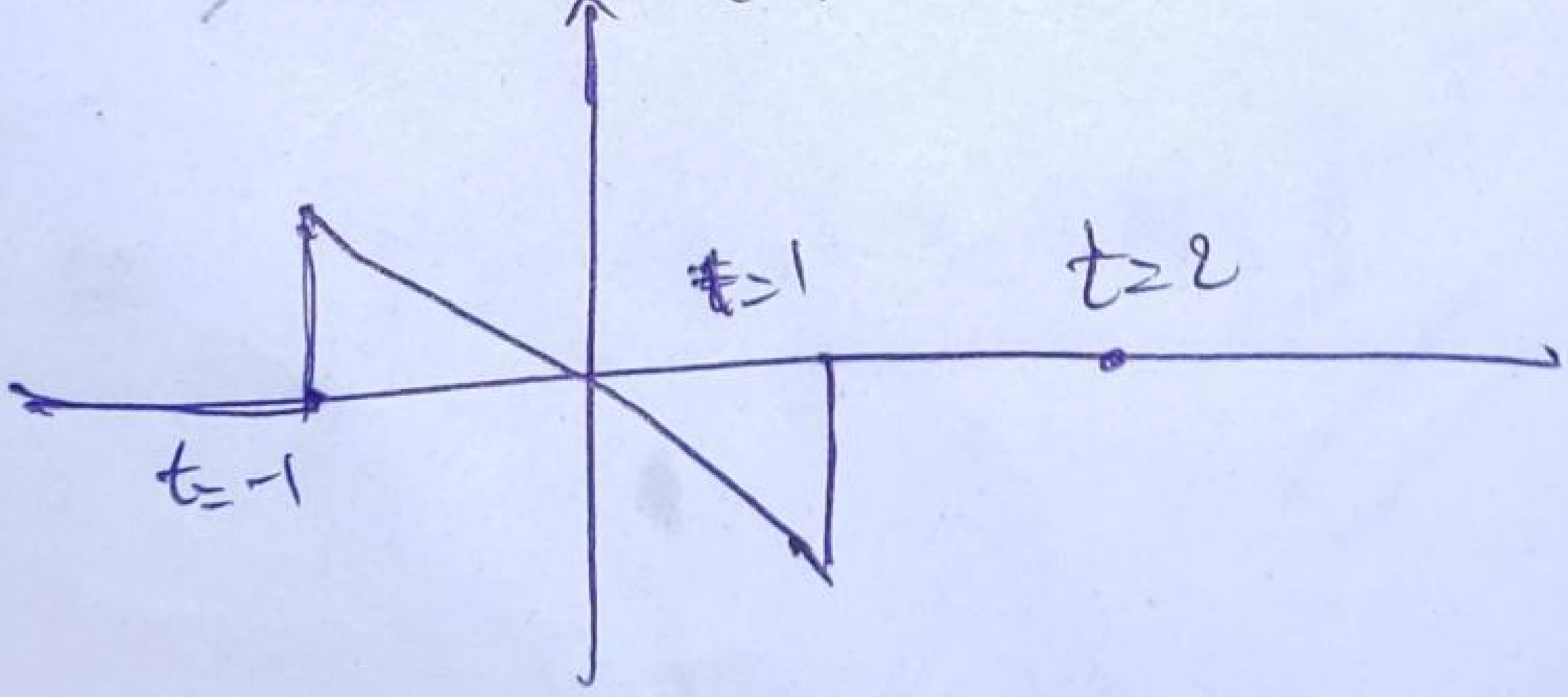
$r(t-1) - r(t+1)$

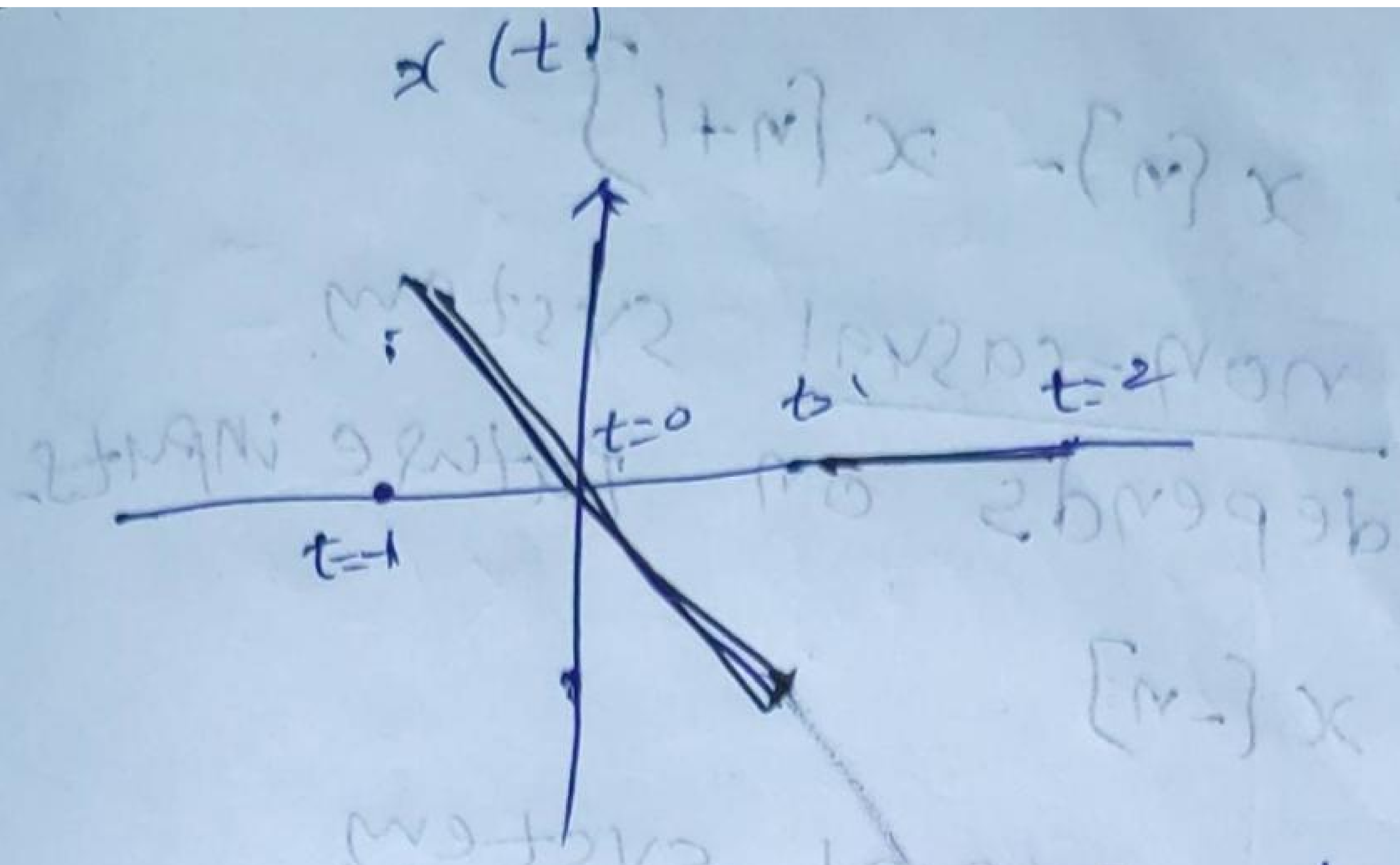


$u(t-1) + u(t+1)$



$x(t)$



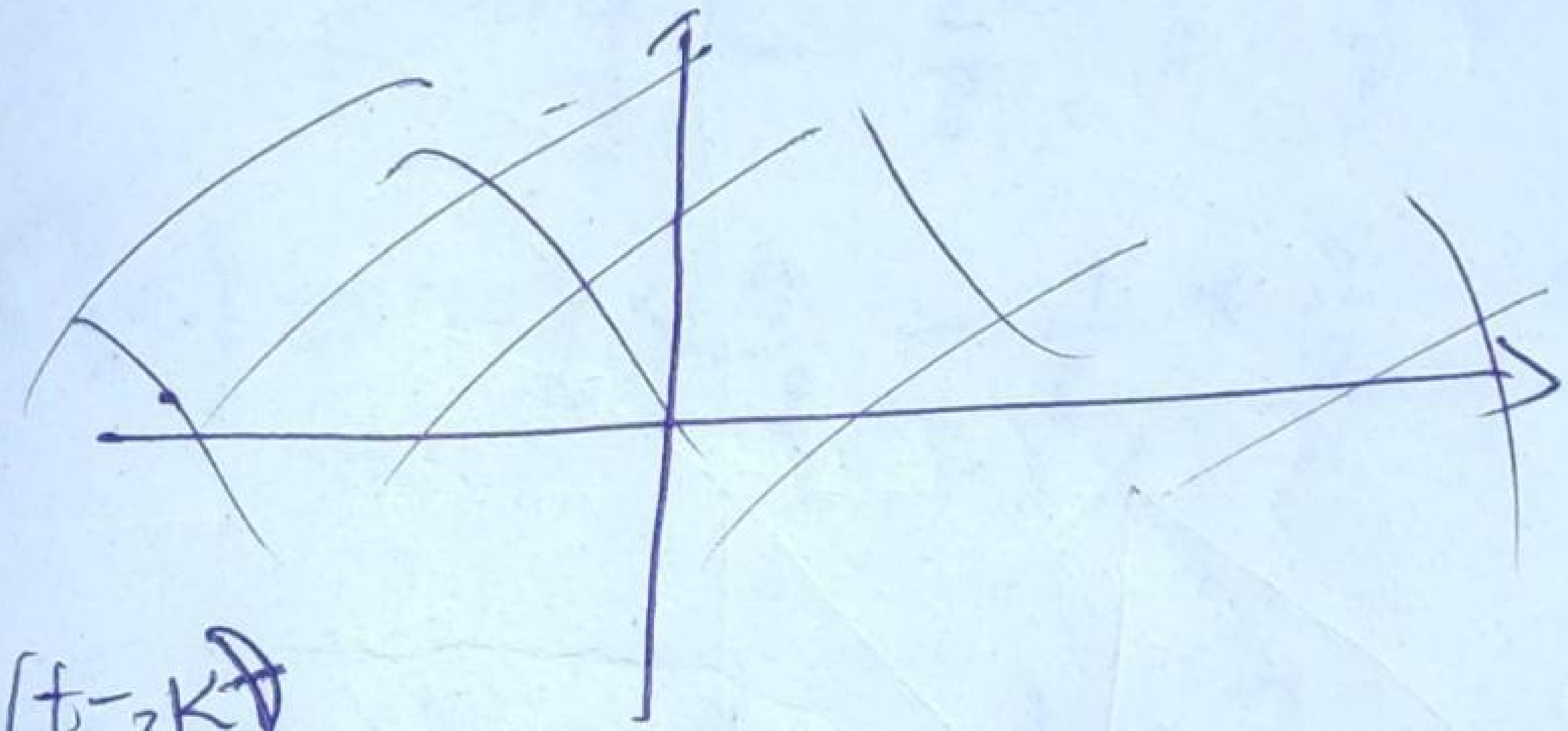


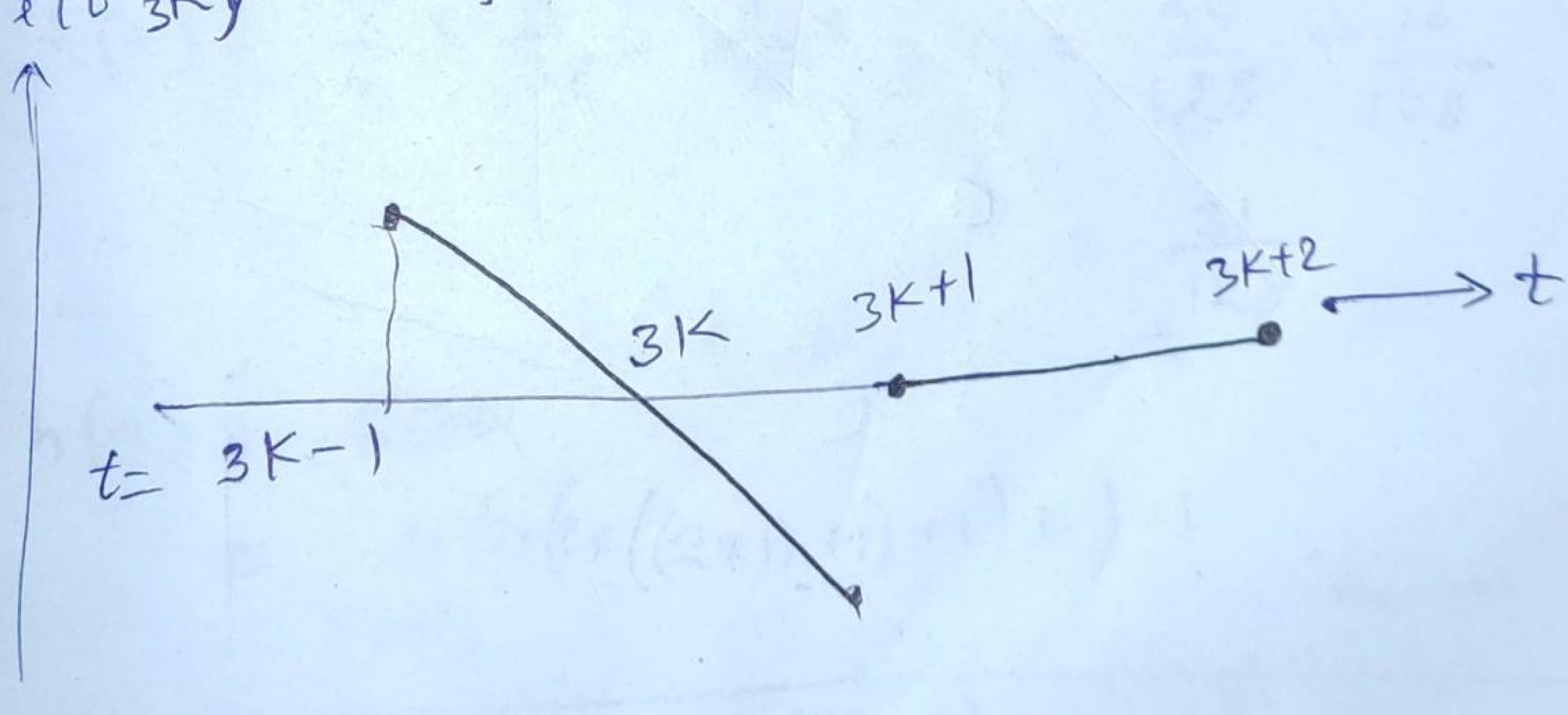
for $x(t-KT)$ $\quad T=3 \quad K\in(-\infty,\infty)$

$$+3K-1 \le t \le +3K+2$$

$$-1 \le t-3K \le 2$$

$$3K-1 \le t \le 2+3K$$





10.

a. $y[n] = x[n] - x[n+1]$

It is a non-casual system as it depends on future inputs.

b. $y[n] = x[-n]$

It is a non-casual system

consider $n = -2$ $y[-2] = x[2]$

$\Downarrow$

depends on future inputs.