DSP CT-1 Solutions

1. Consider a signal x(t)=cos(300πt) sampled at a rate of 500 per second. Write the Fourier Transform and Discrete Fourier Transform of the sampled signal and plot them. 2+2

Ans.

Given,

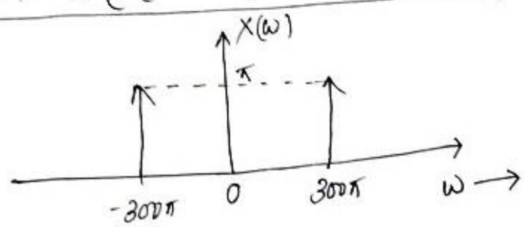
$x_c(t) = \cos(300\pi t)$

$\omega_0 = 300\pi$, $f_s = 500Hz$

$X(\omega) = FT\{x_c(t)\}$

$= FT\{\cos(300\pi t)\}$

$$\boxed{X(\omega) = \pi\left[\delta(\omega - 300\pi) + \delta(\omega + 300\pi)\right]}$$

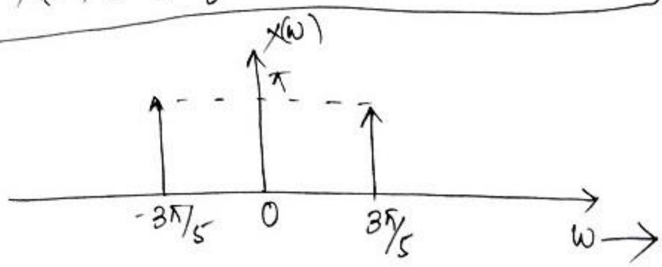


$X(e^{j\omega}) = DFT\left\{\cos 300\pi \frac{n}{F_S}\right\}$ $\quad t = nT_S = \frac{n}{F_S}$

$= DFT\left\{\cos \frac{3\pi}{5} n\right\}$ $\quad \omega_0 = \frac{3\pi}{5}$

$= \sum_{n=-\infty}^{\infty} \cos \frac{3\pi}{5} n \, e^{-j\omega n}$

$$\boxed{X(\omega) = \pi\left[\delta\left(\omega - \frac{3\pi}{5}\right) + \delta\left(\omega + \frac{3\pi}{5}\right)\right]}$$



Consider a signal (t)=cos$(600\pi$t) sampled at a rate of 1000 per second. Write the Fourier Transform and Discrete Fourier Transform of the sampled signal and plot them. 2+2

Ans.

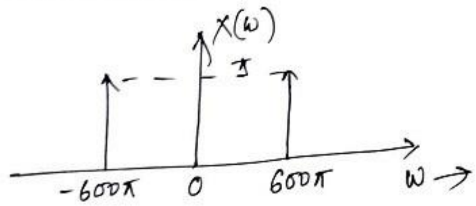
Given,

$x_c(t) = \cos(600\pi t)$

$\omega_0 = 600\pi \quad , \quad F_s = 1000 \text{ Hz}$

$X(\omega) = FT\{\cos(600\pi t)\}$

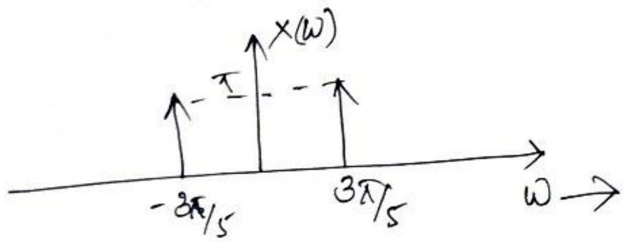
$$\boxed{X(\omega) = \pi\left[\delta(\omega - 600\pi) + \delta(\omega + 600\pi)\right]}$$



$X(\omega) = DFT\left\{\cos 600\pi \frac{n}{F_s}\right\} \qquad t = n/F_s$

$= DFT\left\{\cos \frac{3\pi}{5} n\right\} \qquad \omega_0 = \frac{3\pi}{5}$

$X(\omega) = \pi\left[\delta\left(\omega - \frac{3\pi}{5}\right) + \delta\left(\omega + \frac{3\pi}{5}\right)\right]$



2. Show if the discrete-time signal x$[n]=3\cos(0.2n+0.6)$ is periodic. 2

Show if the discrete-time signal x$[n]=2\cos(0.25n+0.5)$ is periodic.

Ans.

(2) $\boxed{\text{SET 1}}$ $x[n] = 3\cos(0.2n + 0.6)$

For $x[n]$ to be periodic or Let N to be periodic

$x[n+N] = 3\cos(0.2(n+N) + 0.6)$

$= 3\cos(0.2n + 0.6 + \underline{0.2N})$ ↳ it should be $2\pi k$

So, $0.2N = 2\pi k$

$N = \frac{2}{0.2}\pi(k)$ [k is also integer]

$N = 10\pi k$

$\frac{k}{N} = \frac{1}{10\pi}$ [k & N are integers, hence not periodic]

As, here N does not have integer values,

So, it is not Periodic $\boxed{\text{NOT PERIODIC}}$

$\boxed{\text{SET2}}$ $x[n] = 2\cos(0.25n + 0.5)$

Let N to be periodic

$x[n+N] = 2\cos(0.25(n+N) + 0.5)$

$= 2\cos(0.25n + 0.5 + \underline{0.25N})$ ↳ it should be $2\pi k$

Hence, $0.25N = 2\pi k$

$N = \frac{2\pi}{0.25} k$

$N = 8\pi(k)$ [k takes only integer values]

$\frac{k}{N} = \frac{1}{8\pi}$ [k & N are integers, hence not periodic]

∴ The signal is not periodic

$\boxed{\text{NOT PERIODIC}}$

3. For the following $x\,[n]$, $x\,[0]=\underline{6}$ (i.e. the underscore designates n = 0). Find $x\,[2n-3]$. **2**

$x[n]$={....,1,2,3,4,5,$\underline{6}$,5,4,3,2,1,0,1,2,3,4,5,6,....}

Ans. Given,

$x[n]$={....,1,2,3,4,5,$\underline{6}$,5,4,3,2,1,0,1,2,3,4,5,6,....}
$x[n$-3]={....,1,2,$\underline{3}$,4,5,6,5,4,3,2,1,0,1,2,3,4,5,6,....}

$x[2n$-3]={....,1,1,$\underline{3}$,5,5,3,1,1,3,5,....}

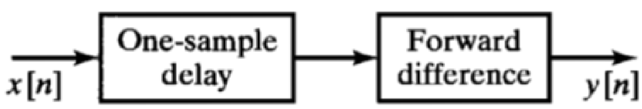
For the following $x\,[n]$, $x\,[0]=\underline{7}$ (i.e. the underscore designates n = 0). Find $x\,[3n-2]$. **2**

$x[n]$={....,1,2,3,4,5,6,$\underline{7}$,6,5,4,3,2,1,2,3,4,5,6,7,....}

Ans. Given,

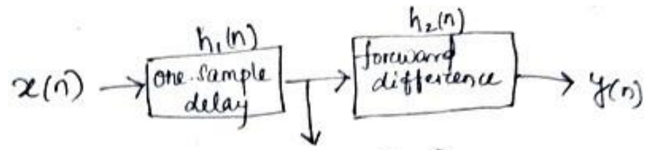
$x[n]$={....,1,2,3,4,5,6,$\underline{7}$,6,5,4,3,2,1,2,3,4,5,6,7,....}
$x[n$-2]={....,1,2,3,4,$\underline{5}$,6,7,6,5,4,3,2,1,2,3,4,5,6,7,....}
$x[3n$-2]={....,3,2,$\underline{5}$,6,3,2,5,....}

4. Find unit sample response of the system that is equivalent to the following by considering unit sample response of individual blocks. What is its physical significance? **3**



Ans. y(n)=x(n)-x(n-1)

It is a backward difference system and can act as a High pass filter.



$y_1(n) = x(n-1)$

$y_1(n) = x(n-1)$ → one-sample delay

$\therefore \boxed{y(n) = x(n) - x(n-1)}$ ↳ forward difference

$Y(z) = X(z) - z^{-1}X(z)$

$H(z) = 1 - z^{-1}$

$\boxed{H(e^{j\omega}) = 1 - e^{-j\omega}}$

$\left.\begin{array}{ll} \omega = 0, & H(e^{j\omega}) = 0 \\ \omega = \infty, & H(e^{j\omega}) = 1 \end{array}\right\}$ HPF

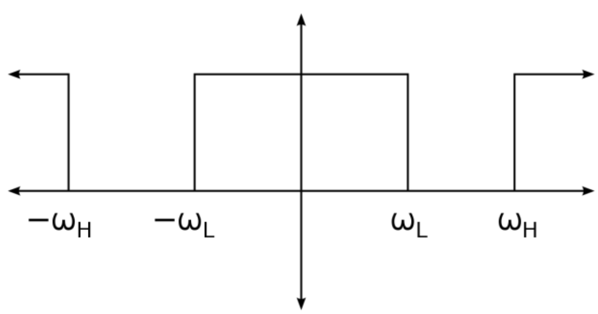
The given system will act as high Pass filter (HPF).

5. You have a LPF and a HPF block for which you can vary the individual cut-off frequencies. Show using block diagram how you can get a BSF from these. 1

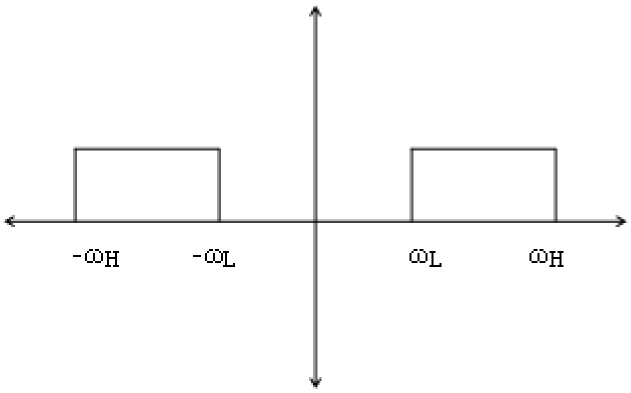
You have a LPF and a HPF block for which you can vary the individual cut-off frequencies. Show using block diagram how you can get a BPF from these. 1

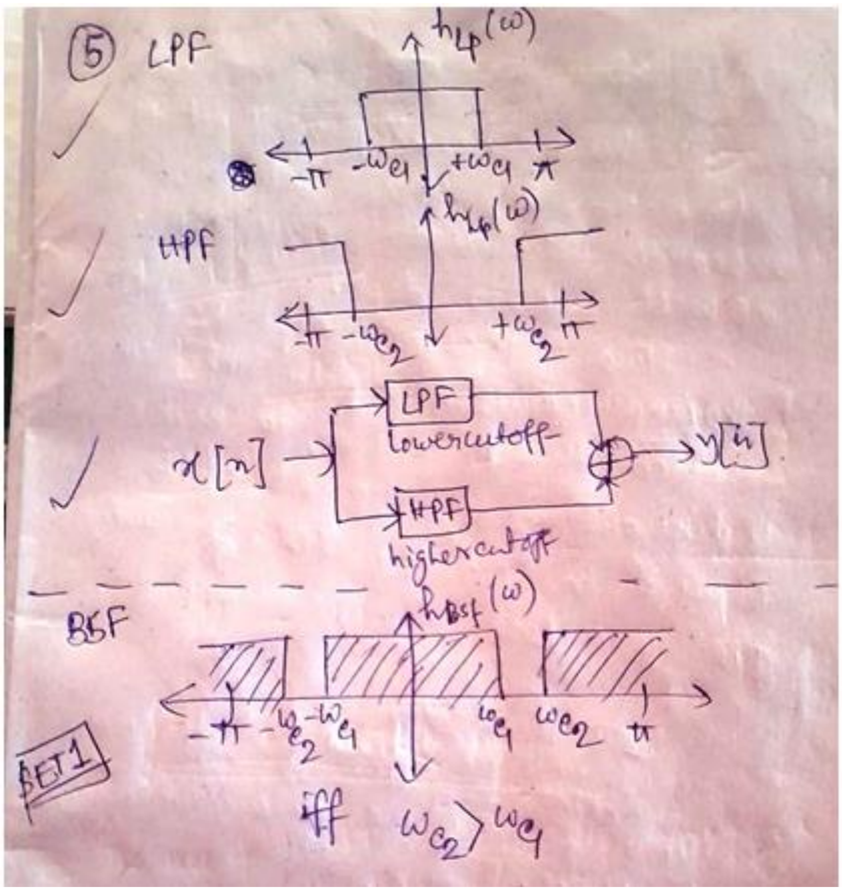
Ans.

BSF:

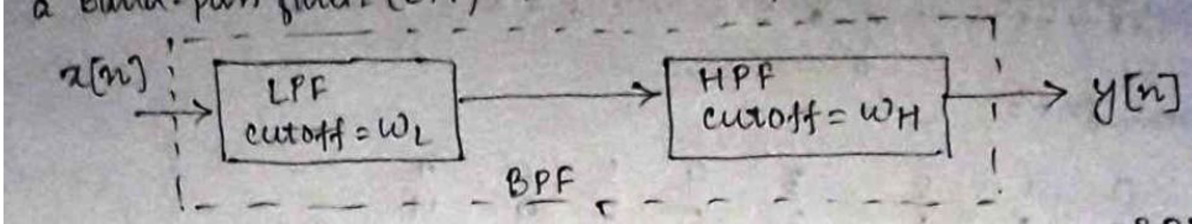


BPF:


-ωH
-ωL
ωL
ωH


(5) LPF
$h_{Lp}(\omega)$
$-\pi$ $-\omega_{c_1}$ $+\omega_{c_1}$ π
HPF
$h_{Hp}(\omega)$
$-\pi$ $-\omega_{c_2}$ $+\omega_{c_2}$ π
$x[n]$
LPF
lowercutoff
HPF
highercutoff
$y[n]$
BSF
$h_{BSf}(\omega)$
$-\pi$ $-\omega_{c_2}$ $-\omega_{c_1}$ ω_{c_1} ω_{c_2} π
SET 1
iff $\omega_{c_2} > \omega_{c_1}$

a Band-pass filter (BPF) constructed from LPF & HPF:-



- The LPF & HPF are cascaded and to have a proper BPF transfer function the necessary condition is: $\boxed{\omega_L > \omega_H}$

or, ω_L = higher cutoff of BPF
ω_H = lower " " BPF.

6. For the system defined by the following equation, find DTFT at $\omega=\pi/4$.
$[n]+{}_{14}[n-1]-{}_{18}y[n-2]=x[n]-x[n-1]$ **4**

Ans.

(a) The use of the Fourier transform simplifies the analysis of the difference equation.

$$y[n] + \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] - x[n-1],$$

$$Y(\Omega)\left(1 + \frac{1}{4}e^{-j\Omega} - \frac{1}{8}e^{-j2\Omega}\right) = X(\Omega)(1 - e^{-j\Omega}),$$

$$\frac{Y(\Omega)}{X(\Omega)} = H(\Omega) = \frac{1 - e^{-j\Omega}}{(1 + \frac{1}{2}e^{-j\Omega})(1 - \frac{1}{4}e^{-j\Omega})}$$

We want to put this in a form that is easily invertible to get the impulse response $h[n]$. Using a partial fraction expansion, we see that

$$H(\Omega) = \frac{2}{1 + \frac{1}{2}e^{-j\Omega}} + \frac{-1}{1 - \frac{1}{4}e^{-j\Omega}},$$

so

$$h[n] = 2(-\tfrac{1}{2})^n u[n] - (\tfrac{1}{4})^n u[n]$$

(b) At $\Omega = 0$, $H(\Omega) = 0$. At $\Omega = \pi/4$, $H(\Omega) = 0.65e^{j(1.22)}$. Since $h[n]$ is real, $H(\Omega) = H^*(-\Omega)$, so $H(-\Omega) = H^*(\Omega)$ and $H(-\pi/4) = 0.65e^{-j(1.22)}$. Since $H(\Omega)$ is periodic in 2π,

$$H\left(\frac{9\pi}{4}\right) = H\left(\frac{\pi}{4}\right) = 0.65e^{j(1.22)}$$

7. Using parseval's relation for DTFT evaluate the integral **4**

$$\int_{0}^{\Pi} \frac{4}{5 + 4\cos(\omega)}\, d\omega$$

Ans.

Parsevals relation $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 \, d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$

Soln

$$\left|\frac{1}{1-\alpha e^{-j\omega}}\right|^2 = \frac{1}{1+\alpha^2-2\alpha\cos\omega}$$

(a) $$\int_0^{\pi} \frac{4}{5+4\cos\omega} \, d\omega = \frac{1}{2}\int_{-\pi}^{\pi} \frac{4}{5+4\cos\omega} \, d\omega$$

even func

$$= \frac{1}{2}\int_{-\pi}^{\pi} \frac{4}{(1+2e^{-j\omega})^2}$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \left|\frac{2}{1+2e^{-j\omega}}\right|^2 d\omega$$

$$\frac{4}{5+4\cos\omega} = \frac{4}{1+4+4\cos\omega}$$

$$= \frac{4}{1+4+\cancel{4}^{2}\frac{(e^{j\omega}+e^{-j\omega})}{\cancel{2}}}$$

$$= \frac{4}{1+4+2e^{j\omega}+2e^{-j\omega}}$$

$$= \frac{4}{1+2e^{-j\omega}+4.e^{j\omega}e^{-j\omega}+2e^{j\omega}}$$

$$= \frac{4}{1(1+2e^{-j\omega})+2e^{j\omega}(1+2e^{-j\omega})}$$

$$= \frac{4}{(1+2e^{-j\omega})(1+2e^{j\omega})}$$

$$\frac{1}{2}\int_{-\pi}^{\pi} \frac{4}{5+4\cos\omega} d\omega = 2\int_{-\pi}^{\pi} \frac{1}{(1+2e^{-j\omega})} \cdot \frac{1}{(1+2e^{j\omega})} d\omega$$

$$= 2\int_{-\pi}^{\pi} X(\omega) \cdot X^*(\omega) \, d\omega = 2\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

$$x(e^{j\omega}) = \frac{1}{1+2e^{-j\omega}}$$

$$\alpha = -2 \Rightarrow |\alpha| > 1$$

$\Rightarrow x[n]$ should be anti-causal

$$x[n] = [-2]^n u[-n-1]$$

$$\int_0^{\pi} \frac{4}{5+4\cos\omega} d\omega = \frac{1}{2}\int_{-\pi}^{\pi} \frac{4}{5+4\cos\omega} d\omega$$

b

$$= 2\int_{-\pi}^{\pi} \frac{1}{5+4\cos\omega} d\omega$$

$$= 4\pi \sum_{n=-\infty}^{-1} |x[n]|^2$$

$$\int_0^{\pi} \frac{4}{5+4\cos\omega} d\omega = 4\pi \sum_{n=-\infty}^{-1} |x[n]|^2$$

$$= 4\pi \sum_{n=-\infty}^{-1} |[-2]^n|^2$$

$$= 4\pi \sum_{n=-\infty}^{-1} 4^n$$

$$= 4\pi \sum_{r=1}^{\infty} 4^{-r} = 4\pi \frac{1/4}{1-1/4}$$

$$\boxed{\int_0^{\pi} \frac{4}{5+4\cos\omega} d\omega = \frac{4\pi}{3}}$$