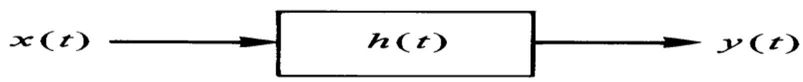
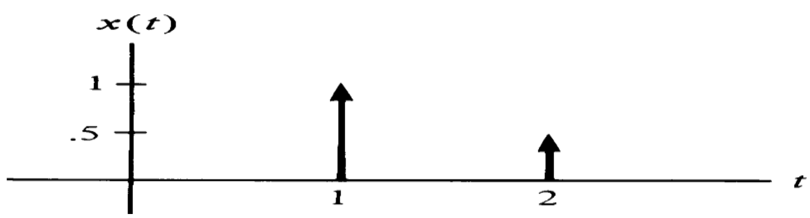
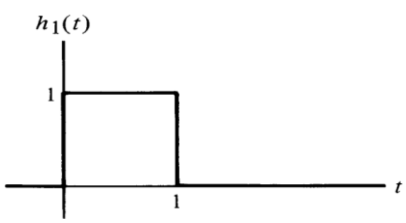
ASSIGNMENT 5

Q1) Consider the signal, x(t) = δ(t-1) and 0.5 δ(t-2). The signal x(t) is interpolated with different h(t) (h1(t), h2(t) and h3(t)) as shown below. Sketch y(t) for the following:

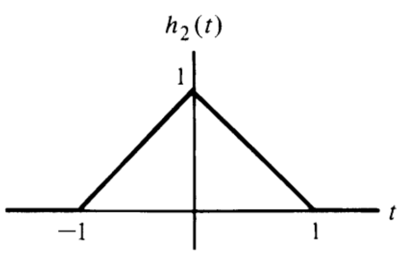




a)



b)



c)

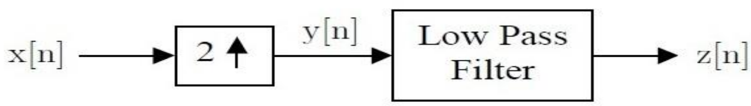
$$h(t) = \frac{\sin(\pi t)}{\pi t}$$

Q2) The following system shows an interpolator with discrete input x[n]. Assume that the low pass filter has frequency response $H(e^{j\omega})=2\text{rect}(\omega/\pi)$ for $|\omega|<\pi$. Compute the z[n] for the following inputs.

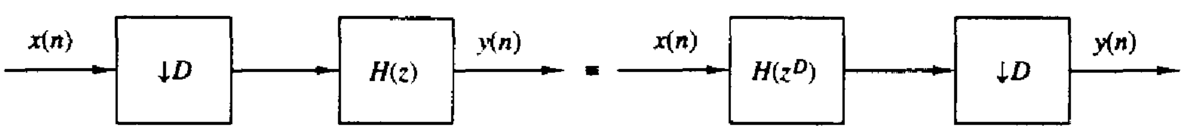
a. $x[n]=\delta[n-1]$

b. $x[n]= 1$

c. $x[n]= \cos(n\pi/4)$



Q3) Prove the equivalence of the two decimator configuration.



Q4)

Consider an arbitrary digital filter with transfer function

$$H(z) = \sum_{n=-\infty}^{\infty} h(n)z^{-n}$$

(a) Perform a two-component polyphase decomposition of $H(z)$ by grouping the even-numbered samples $h_0(n) = h(2n)$ and the odd-numbered samples $h_1(n) =$

$h(2n+1)$. Thus show that $H(z)$ can be expressed as

$$H(z) = H_0(z^2) + z^{-1}H_1(z^2)$$

and determine $H_0(z)$ and $H_1(z)$.

(b) Generalize the result in part (a) by showing that $H(z)$ can be decomposed into an D-component polyphase filter structure with transfer function

$$H(z) = \sum_{k=0}^{D-1} z^{-k} H_k(z^D)$$

Determine $H_k(z)$.

Q5)

Let $x_c(t)$ be a real-valued continuous-time signal with highest frequency $2\pi(250)$ radians/second. Furthermore, let $y_c(t) = x_c(t - 1/1000)$.

(a) If $x[n] = x_c(n/500)$, is it theoretically possible to recover $x_c(t)$ from $x[n]$? Justify your answer.

(b) If $y[n] = y_c(n/500)$, is it theoretically possible to recover $y_c(t)$ from $y[n]$? Justify your answer.

(c) Is it possible to obtain $y[n]$ from $x[n]$ using the system in Figure 1? If so, determine $H_1(e^{j\omega})$.

(d) It is also possible to obtain $y[n]$ from $x[n]$ without any upsampling or downsampling using a single LTI system with frequency response $H_2(e^{j\omega})$. Determine $H_2(e^{j\omega})$.

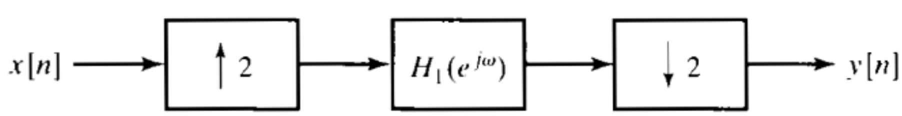


Figure 1

Q6)

Consider the system shown in Figure 2 . For each of the following input signals $x[n]$, indicate whether the output $x_r[n] = x[n]$.

(a) $x[n] = \cos(\pi n/4)$

(b) $x[n] = \cos(\pi n/2)$

(c)

$$x[n] = \left[\frac{\sin(\pi n/8)}{\pi n}\right]^2$$

Hint: Use the modulation property of the Fourier transform to find $X(e^{j\omega})$.

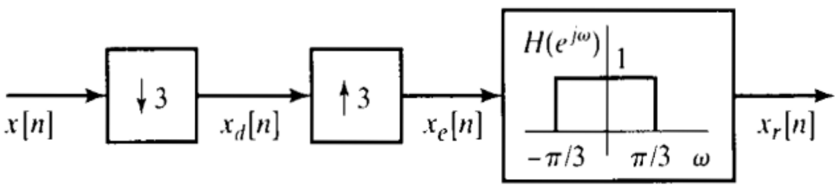


Figure 2

Q7) Which process has a block diagram as shown in the figure below? Give justification.