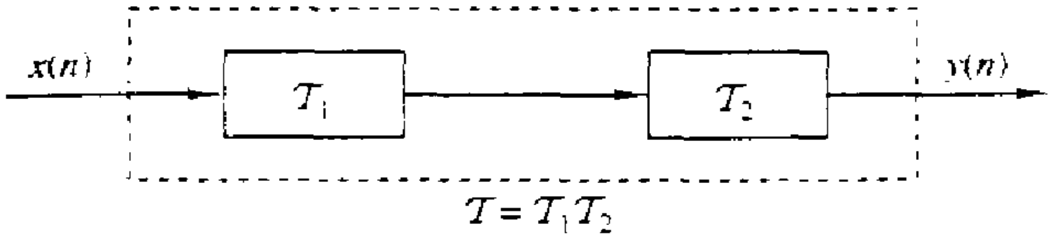
DSP Assignment 1

Q.1. Two discrete time systems *T1* and *T2* are cascaded to form a new system *T*.



If *T1* and *T2* are non-linear, then *T* is non-linear (True/False). Prove it?

A.1.

False. For example, consider

$$\mathcal{T}_1 : y(n) = x(n) + b \text{ and}$$

$$\mathcal{T}_2 : y(n) = x(n) - b, \text{ where } b \neq 0.$$

Then,

$$\mathcal{T}[x(n)] = \mathcal{T}_2[\mathcal{T}_1[x(n)]] = \mathcal{T}_2[x(n) + b] = x(n).$$

Hence $\mathcal{T}$ is linear.

Q.2. Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of energies (powers) of its even and odd components.

A.2.

First, we prove that

$$\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) = 0$$

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &= \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m) \\
&= -\sum_{m=-\infty}^{\infty} x_e(m)x_o(m) \\
&= -\sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
&= 0
\end{aligned}$$

Then,

$$\begin{aligned}
\sum_{n=-\infty}^{\infty} x^2(n) &= \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2 \\
&= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n) \\
&= E_e + E_o
\end{aligned}$$

Q.3. Show that average power Px of a real valued sequence x[n] is given by the sum of the average powers, P_ev and P_odd, of the even and odd parts of x[n] respectively.

A.3.

① given $x[n] \Rightarrow$ real valued sequence

$$\Rightarrow \cancel{x_{e}} \; x_{even}[n] = \frac{x[n] + x[-n]}{2}$$

$$x_{odd}[n] = \frac{x[n] \cancel{+} - x[-n]}{2}$$

} respective ~~add~~ even & odd parts of $x[n]$

average power of $x[n]$ = $P_{avg} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]$

evaluating $\Rightarrow x_{even}^2[n] + x_{odd}^2[n]$

$$= \frac{1}{4}\left[(x[n]+x[-n])^2 + (x[n] - x[-n])^2\right]$$

$$\Rightarrow x_{even}^2[n] + x_{odd}^2[n] = \frac{1}{2}\left[x^2[n] + x^2[-n]\right]$$

Sum of average powers of even and odd signals (parts)

$$P_{even} + P_{odd} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_{even}^2[n] + x_{odd}^2[n]$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \cancel{2}\, \frac{1}{2}\left[x^2[n] + x^2[-n]\right]$$

but $\sum_{n=-N}^{N} x^2[n] = \sum_{n=-N}^{N} x^2[-n]$

$$\Rightarrow P_{even} + P_{odd} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N} \frac{\cancel{2}\, x^2[n]}{\cancel{2}}$$

$$\boxed{P_{even} + P_{odd} = P_{avg}}$$

Q.4. A discrete time signal g[n] is given by g[n] = u[n+3] - u[n-5].

a. What is the energy of the signal g[n]?

b. What is the power of the signal g[n]?

A.4.

Q2) $g(n) = u(n+3) - u(n-5)$

$u(n+3) = 1$ for $-3 \leq n \leq \infty$

$u(n-5) = 1$ for $5 \leq n \leq \infty$

$\therefore g(n) = 1$ for $-3 \leq n \leq 4$

$\therefore$ a) Energy of $g(n) = \sum_{n=-3}^{4} |g(n)|^2$

$= \sum_{n=-3}^{4} 1$

$= 8$ (Ans.)

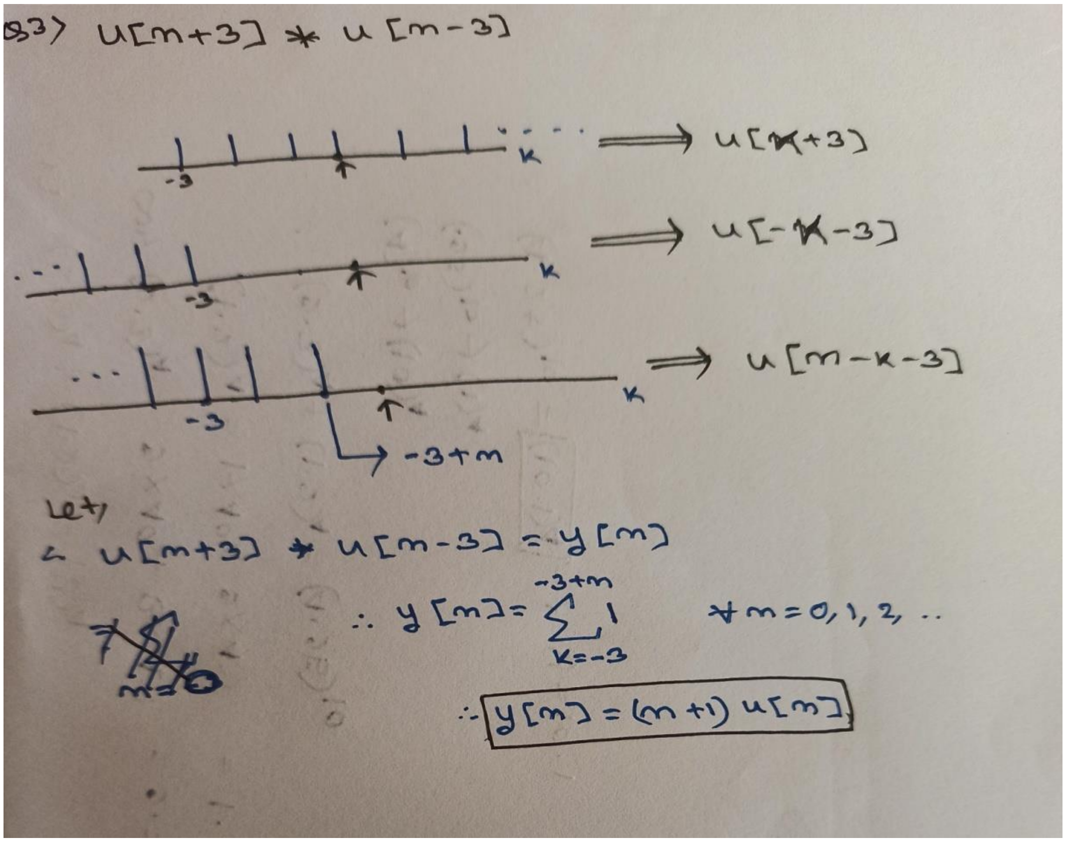
b) $g(n) \rightarrow$ finite duration signal

$g(2n) \rightarrow$ finite duration

$\hookrightarrow$ Energy signal $\rightarrow$ $\boxed{\text{Power zero}}$

Q.5. Find out the convolution: **u[n+3]*u[n-3]**.

A.5.



Q.6. The impulse response of a discrete LTI system is given by $\boldsymbol{h}(\boldsymbol{n})$ = $(\mathbf{0.5})\boldsymbol{n}\ \boldsymbol{u}(\boldsymbol{n})$ the input of which is $\boldsymbol{x}(\boldsymbol{n}) = 2\boldsymbol{\delta}(\boldsymbol{n})+\boldsymbol{\delta}(\boldsymbol{n}-\mathbf{3})$. Find the output at $\boldsymbol{n} = \mathbf{1}$ and $\boldsymbol{n} = \mathbf{4}$.

A.6.

y(n) = x(n) * h(n) = $(2\delta[n]+\delta[n-3])$ * $((0.5)^n u[n])$

y(n)=$2(0.5)^n\ (n) + (0.5)^{n-3}\ u[n-3]$ **[utilizing sifting property]**

y(1) = $2*(0.5)^1\ u[1] + 0 = 1$

y(4) = $2*(0.5)^4\ u[4] + (0.5)^{4-3}\ u[1] = (0.5)^3 + (0.5) = 5/8$

Q.7. Consider a discrete-time system:

$$y[n] = \prod_{k=-\infty}^{n} e^{x[k]}$$

Is this a linear or a non-linear system?

A.7.

We have, $y_1[n] = \prod_{k=-\infty}^{n} e^{x_1[k]}$ and $y_2[n] = \prod_{k=-\infty}^{n} e^{x_2[k]}$

So, $$y_1[n] + y_2[n] = \prod_{k=-\infty}^{n} e^{x_1[k]} + \prod_{k=-\infty}^{n} e^{x_2[k]}$$

$$= e^{\sum_{k=-\infty}^{n} x_1[k]} + e^{\sum_{k=-\infty}^{n} x_2[k]}$$

To prove superposition, $y_3[n] = \prod_{k=-\infty}^{n} e^{x_1[k]+x_2[k]}$

$$\Rightarrow y_3[n] = \prod_{k=-\infty}^{n} e^{x_1[k]}.e^{x_2[k]} \quad \Rightarrow y_3[n] = e^{\sum_{k=-\infty}^{n} x_1[k] + \sum_{k=-\infty}^{n} x_2[k]}$$

$$y_3[n] \neq y_1[n] + y_2[n]$$

Therefore, the system is non-linear.

Q.8. Let **g[n] = x1[n] * x2[n] * x3[n]** and **h[n] = x1[n-N1] * x2[n-N2] * x3[n-N3]**. Express **h[n]** in terms of **g[n]**. (Note * indicates convolution operation)

A.8.

(2) given $g[n] = x_1(n) * x_2(n) * x_3[n]$

$h[n] = x_1[n-N_1] * x_2[n-N_2] * x_3[n-N_3]$

as we know any discrete signal can be written as

$$x(n) = \sum_{k=-\infty}^{\infty} x[k] \cdot \delta(n-k)$$

↳ ~~sum of impulses~~

$\Rightarrow \quad x(n) = x(n) * \delta(n)$

by shift invariant property of Convolution sum

$$x[n-N] = x(n) * \delta[n-N]$$
$$= x(n-N) * \delta[n]$$

where "N" is constant (some)

$\Rightarrow \quad h(n) = x_1(n-N_1) * x_2(n-N_2) * x_3[n-N_3]$

shift - invariant property

$h(n) = x_1(n) * \delta(n-N_1) * x_2(n) * \delta[n-N_2] * x_3(n) * \delta(n-N_3)$

by commutative property

i.e. $x[n] * h(n) = h(n) * x(n)$

$\Rightarrow h(n) = \underbrace{x_1(n) * x_2(n) * x_3(n)}_{g(n)} * \delta[n-N_1] * \delta[n-N_2] * \delta(n-N_3]$

$= g(n) * \underbrace{\delta[n-N_1] * \delta(n-N_2) * \delta(n-N_3]}_{\delta[n-(N_1+N_2+N_3)]}$

$\Rightarrow \boxed{h(n) = g[n-(N_1+N_2+N_3)]}$

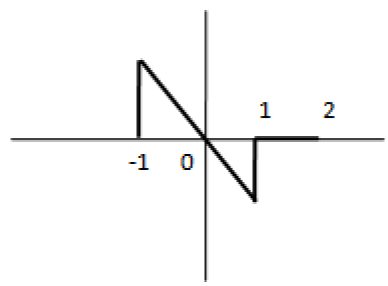
Q.9 Consider the following signal:

$$x(t) = u(t-1) + r(t-1) - r(t+1) + u(t+1); \ -1 \leq t \leq 2$$

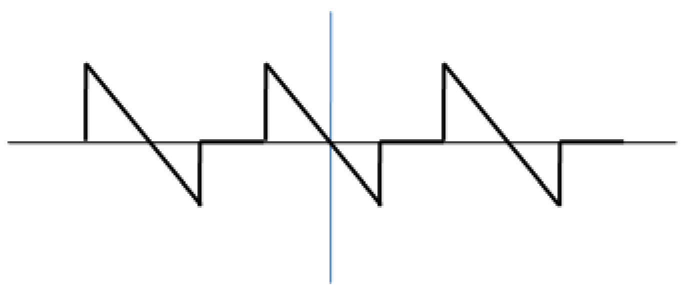
Sketch $x(t-kT)$, where T=3 and $k \in (-\infty, \infty)$

A.9.

Given x(t),



Hence, **x(t-kT)** for different values of k is,



Q.10. Find whether the following systems are causal:-

a. y[n] = x[n] - x[n+1]

b. y[n] = x[-n]

A.10.

a. y[n] = x[n] - x[n+1] is **non causal** because the output for all time depends on future value of the input.

b. $y[n] = x[-n]$ the output for any positive value of n, say n_o, depends on input value at $n = -n_o$ which are past values.

But for negative value of n, say $-n_o$, the output depends on input values at $n = n_o$ which are future values.

So the system is **non causal.**