DSP Assignment - 2

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1. $u[n] = u_e[n] + u_o[n]$

$$u_e[n] = \begin{cases} \frac{1}{2}, & n \neq 0 \\ 1, & n = 0 \end{cases}$$

$$u_e[n] = \frac{1}{2} + \frac{\delta[n]}{2}$$

$$u_o[n] = \begin{cases} -\frac{1}{2}, & n < 0 \\ 0, & n = 0 \\ \frac{1}{2}, & n > 0 \end{cases}$$

$$* \; u_e[n] = \frac{1}{2} + \frac{\delta[n]}{2} \xrightarrow{\mathcal{F}} \pi \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k) + \frac{1}{2}$$

$$* \; \frac{\delta[n]}{2} + \frac{\delta[n-1]}{2} = u_o[n] - u_o[n-1]$$

$$\downarrow$$

$$\frac{1 + e^{-j\omega}}{2} = U_o(e^{j\omega})\left[1 - e^{-j\omega}\right]$$

$$U_o(e^{j\omega}) = \frac{1 + e^{-j\omega}}{2(1 - e^{-j\omega})} = \frac{1}{1 - e^{-j\omega}} - \frac{1}{2}$$

$$U(e^{j\omega}) = U_e(e^{j\omega}) + U_o(e^{j\omega})$$

$$U(e^{j\omega}) = \pi \sum_{k=-\infty}^{\infty} \delta(\omega + 2\pi k) + \frac{1}{1-e^{-j\omega}}$$

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2. $y[n] + \frac{1}{4} y[n-1] - \frac{1}{8} y[n-2] = x[n] - x[n-1]$

impulse response =>

$$h[n] + \frac{1}{4} h[n-1] - \frac{1}{8} h[n-2] = \delta[n] - \delta[n-1]$$

$$\Big\downarrow \mathcal{F}$$

$$H(e^{j\omega}) + \frac{e^{-j\omega}}{4} H(e^{j\omega}) - \frac{e^{-2j\omega}}{8} H(e^{j\omega}) = 1 - e^{-j\omega}$$

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 + \frac{1 \times e^{-j\omega}}{4} - \frac{1}{8} \times e^{-2j\omega}}$$

$$= \frac{1 - e^{-j\omega}}{\left(1 - \frac{e^{-j\omega}}{4}\right)\left(1 + \frac{1}{2} e^{-j\omega}\right)}$$

$e^{-j\omega}$

(a)

$$H(e^{j\omega}) = \frac{8(1-e^{-j\omega})}{(4-e^{-j\omega})(2+e^{-j\omega})}$$

$$= \frac{A}{4-e^{-j\omega}} + \frac{B}{2+e^{-j\omega}}$$

$$\frac{8(1-e^{-j\omega})}{(4-e^{-j\omega})(2+e^{-j\omega})} = \frac{A}{4-e^{-j\omega}} + \frac{B}{2+e^{-j\omega}}$$

$$A = \frac{8\times(-3)}{6} = -4$$

$$B = \frac{8(3)}{6} = 4$$

$$H(e^{j\omega}) = \frac{-4}{(4-e^{-j\omega})} + \frac{4}{(2+e^{-j\omega})}$$

$$\Rightarrow H\{e^{j\omega}\} = \frac{-1}{(1-\frac{1}{4}e^{-j\omega})} + \frac{2}{1+\left(\frac{-1}{2}\right)e^{-j\omega}}$$

$$\Rightarrow h[n] = -\left(\frac{1}{4}\right)^n u[n] + 2\times\left(\frac{-1}{2}\right)^n u[n]$$

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(b)

For $\omega = 0$

$|H(e^{j\omega})| = 0$ $\quad$ $\angle H(e^{j\omega}) = 0$

For $\omega = \frac{\pi}{4}$

$|H(e^{j\omega})| = 0.6498$ $\quad$ $\angle H(e^{j\omega}) = 70.02^\circ$

For $\omega = -\frac{\pi}{4}$

$|H(e^{j\omega})| = 0.6498$ $\quad$ $\angle H(e^{j\omega}) = -70.02^\circ$

For $\omega = \frac{9\pi}{4}$

$|H(e^{j\omega})| = 0.6498$ $\quad$ $\angle H(e^{j\omega}) = 70.02^\circ$

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3)

$$C = \frac{\sum_{n=-\infty}^{\infty} n\, x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$$

we know

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x[n] \longrightarrow ①$$

$$\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} x[n]\,(-jn)\, e^{-j\omega n}$$

$$j\,\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n}$$

$$j\,\frac{dX(\omega)}{d\omega}\Big|_{\omega=0} = \sum_{n=-\infty}^{\infty} n\, x[n] \longrightarrow ②$$

from 1 & 2

$$\boxed{C = \frac{j\,\frac{dX(\omega)}{d\omega}}{X(\omega)}\Bigg|_{\omega=0}}$$

4) $x\{n\} = \left(\frac{1}{4}\right)^n u\{n\}$

$y\{n\} = x^2\{n\}$

$= \left(\frac{1}{4}\right)^{2n} (u\{n\})^2$

$= \left(\frac{1}{16}\right)^n u\{n\}$

$$Y(e^{j\omega}) = \frac{1}{1 - \frac{1}{16} e^{-j\omega}}$$

$$Y(e^{j0}) = \frac{1}{1 - \frac{1}{16}} = 16/15$$

5) $x\{n\} = \{-3, 4, \underset{\uparrow}{-5}, 4, 3\}$

a) $$X(\omega) = \sum_{n=-\infty}^{\infty} x\{n\} e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x\{n\} e^{0} = \sum_{n=-2}^{2} x\{n\}$$

$= 3$

B) $$x\{n\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$x\{0\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) . d\omega$$

$$\therefore \int_{-\pi}^{\pi} X(\omega) d\omega = 2\pi \times x\{0\} = -10\pi$$

c> $X(\pi) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-jn\pi}$

$$e^{-jn\pi} = \begin{cases} -1 & n = \text{odd} \\ 1 & n = \text{even} \end{cases}$$

$$X(\pi) = \sum_{n=-2}^{2} x[n]\, e^{-jn\pi}$$

$= (-3)\times 1 + 4\times(-1) + (-5)\times 1 + 4\times(-1)$

$+ 3\times(1)$

$= -13$

D> $\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega = \sum_{n=-\infty} |x[n]|^2$

$$\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega = 2\pi\,(9+16+25+16+9)$$

$= 2\pi\,(75)$

$= 150\pi$

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6)

$$y[n] = h[n] * x[n]$$

$$h[n] = \alpha^n u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-\alpha e^{-j\omega}} \quad |\alpha|<1$$

$$x[n] = \beta^n u[n] \xrightarrow{\mathcal{F}} \frac{1}{1-\beta e^{-j\omega}} \quad |\beta|<1$$

$$h[n] * x[n] \longrightarrow H(e^{j\omega}) \cdot X(e^{j\omega})$$

$$Y(e^{j\omega}) = H(e^{j\omega}) \cdot X(e^{j\omega})$$

$$= \frac{1}{(1-\alpha e^{-j\omega})(1-\beta e^{-j\omega})}$$

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7.

$$r\{n\} = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

$$R(e^{j\omega}) = \sum_{n=-\infty}^{\infty} r\{n\} e^{-j\omega n}$$

$$= \sum_{n=0}^{M} 1 \cdot e^{-j\omega n}$$

$\Rightarrow$ GP series with $a = e^{-j\omega}$

$$= \frac{1 - (e^{-j\omega})^{(M+1)}}{1 - e^{-j\omega}}$$

$$R(e^{j\omega}) = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

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8. $x[n]$ is real

we have

$$X[e^{j\omega}] = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \longrightarrow ①$$

let

$$Y[e^{j\omega}] = X[e^{j3\omega}] = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} \longrightarrow ②$$

from ①

$$X[e^{j3\omega}] = \sum_{n=-\infty}^{\infty} x[n] e^{-j3\omega n}$$

put $3n = t \Rightarrow n = \frac{t}{3}$

$$= \sum_{t=-\infty}^{\infty} x\left[\frac{t}{3}\right] e^{-j\omega t} \longrightarrow ③$$

from ② & ③

$$y[n] = x\left[\frac{n}{3}\right]$$

$$y[n] = \begin{cases} x\left[\frac{n}{3}\right] & \text{when } \frac{n}{3} \text{ is an integer} \\ 0 & \end{cases}$$