Assignment 3

04/02/2021

1. Determine the coefficients of FIR filter using rectangular window with following

$$H_d(e^{jw}) = \begin{cases} e^{-j3w} & -\pi/4 \le w \le +\pi/4 \\ 0 & \pi/4 \le w \le \pi \end{cases}$$

   Length of the filter is 7.

2. Let $h_d[n]$, $-\infty<n<\infty$, denotes the impulse response samples of a zero-phase filter with frequency response $H_d(\exp\{j\omega\})$. It is known that the frequency response $H_t(\exp\{j\omega\})$ of the zero-phase FIR filter $h_t[n]$, $-M \le n \le M$, obtained by multiplying $h_d[n]$ with a rectangular window $w_R[n]$, $-M \le n \le M$, has the least integral-squared error $\boldsymbol{\Phi}\_R$ defined in the following equation.

$$\Phi_R = \sum_{n=-\infty}^{\infty} \left| h_t[n] - h_d[n] \right|^2$$

   Let $\boldsymbol{\Phi}\_Hann$ denote the integral-squared error if a length 2M+1 Hann window is used to develop FIR filter. Determine an expression for the excess error

$$\boldsymbol{\Phi}\_excess = \boldsymbol{\Phi}\_R - \boldsymbol{\Phi}\_Hann$$

3. Repeat above problem if a Hamming window is used instead.
4. Determine the coefficients {h(n)} of a linear-phase FIR filter of length M =15 which has a symmetric unit sample response and a frequency response that satisfies the condition

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & k = 0,1,2,3 \\ 0 & k = 4,5,6,7 \end{cases}$$

5. Design a 5-tap FIR band reject filter with a lower cutoff frequency of 2,000 Hz, an upper cutoff frequency of 2,400 Hz, and a sampling rate of 8,000 Hz using the Hamming window method. Determine the transfer function.
6. What are the filter coefficients for a 3-tap FIR low pass filter with a cutoff frequency of 800 Hz and a sampling rate of 8,000 Hz using Hamming window? Also determine the transfer function and difference equation of the designed FIR system.
7. A Hilbert Transform is a filter with frequency response $H_d(w)=-j\ \mathrm{sign}(w)$
   A] Roughly plot the magnitude and phase plot of the filter.
   B] Determine $h_d(n)$.
8. A Digital Filter is defined by the difference equation $y[n] = 0.99*y[n-1] + x[n]$.
   A] Determine the filter transfer function.
   B] What is filter impulse function?
   C] Is it LPF or HPF? (hint: check pole-zero plot)