DSP Assignment-7 Solutions

1.

The ideal frequency response of the filter shown in Fig. 6.12. We know

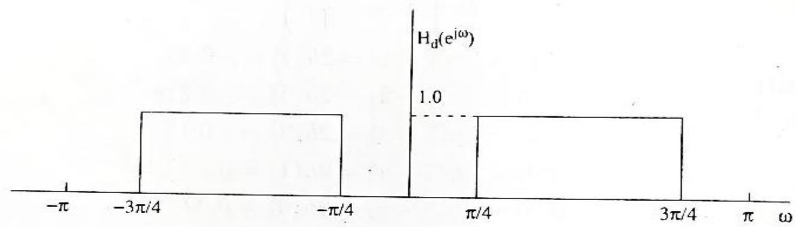


Fig. 6.12 Ideal frequency response of Bandpass filter of example 6.7.

$$\begin{aligned}
h_d(n) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n} \\
&= \frac{1}{2\pi}\left[\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{4}} e^{j\omega n}d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{j\omega n}d\omega\right] \\
&= \frac{1}{2\pi jn}\left[e^{-j\pi n/4} - e^{-j3\pi n/4} + e^{j3\pi n/4} - e^{j\pi n/4}\right] \\
&= \frac{1}{\pi n}\left[\sin\frac{3\pi}{4}n - \sin\frac{\pi}{4}n\right] \qquad -\infty \leq n \leq \infty \qquad (6.63)
\end{aligned}$$

Truncating $h_d(n)$ to 11 samples, we have

$$\begin{aligned}
h(n) &= h_d(n) \quad \text{for} \quad |n| \leq 5 \\
&= 0 \quad \text{otherwise}
\end{aligned}$$

The filter coefficients are symmetrical about $n = 0$ satisfying the condition $h(n) = h(-n)$.

For $n = 0$

$$\begin{aligned}
h(0) &= \frac{1}{2\pi}\left[\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{4}} d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\omega\right] \\
&= \frac{1}{2\pi}\left[-\frac{\pi}{4} + \frac{3\pi}{4} + \frac{3\pi}{4} - \frac{\pi}{4}\right] = \frac{1}{2} = 0.5
\end{aligned}$$

$$h(1) = h(-1) = \frac{\sin\frac{3\pi}{4} - \sin\frac{\pi}{4}}{\pi} = 0$$

$$h(2) = h(-2) = \frac{\sin\frac{3\pi}{2} - \sin\frac{\pi}{2}}{2\pi} = \frac{-2}{2\pi} = -0.3183$$

$$h(3) = h(-3) = \frac{\sin\frac{9\pi}{4} - \sin\frac{3\pi}{4}}{3\pi} = 0$$

$$h(4) = h(-4) = \frac{\sin 3\pi - \sin\pi}{4\pi} = 0$$

$$h(5) = h(-5) = \frac{\sin\frac{15\pi}{4} - \sin\frac{5\pi}{4}}{5\pi} = 0$$

The transfer function of the filter is

$$\begin{aligned} H(z) &= h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\left(z^{n} + z^{-n}\right)\right] \\ &= 0.5 - 0.3183(z^{2} + z^{-2}) \end{aligned}$$

The transfer function of the realizable filter is

$$\begin{aligned} H'(z) &= z^{-5}\left[0.5 - 0.3183(z^{2} + z^{-2})\right] \\ &= -0.3183z^{-3} + 0.5z^{-5} - 0.3183z^{-7} \end{aligned}$$

The filter coefficients of the causal filters are

$$h(0) = h(10) = h(1) = h(9) = h(2) = h(8) = h(4) = h(6) = 0$$
$$h(3) = h(7) = -0.3183$$
$$h(5) = 0.5$$

$$\overline{H}(e^{j\omega}) = \sum_{n=1}^{\frac{N-1}{2}} a(n)\cos\omega n$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.5$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = 0$$
$$a(2) = 2h(5-2) = 2h(3) = -0.6366$$
$$a(3) = 2h(5-3) = 2h(2) = 0$$
$$a(4) = 2h(5-4) = 2h(1) = 0$$
$$a(5) = 2h(5-5) = 2h(0) = 0$$

$$\overline{H}(e^{j\omega}) = 0.5 - 0.6366\cos 2\omega$$

ω (in degrees)	0	20	30	45	60	75	90
$\overline{H}(e^{j\omega})$	−0.1366	0.012	0.1817	0.5	0.818	1.05	1.1366
$\lvert H(e^{j\omega})\rvert_{dB}$	−17.3	−38.17	−14.8	−6.02	−1.74	0.4346	1.11

ω (in degrees)	105	120	135	150	160	180
$\overline{H}(e^{j\omega})$	1.05	0.818	0.5	0.1817	0.012	−0.1366
$\lvert H(e^{j\omega})\rvert_{dB}$	0.4346	−1.74	−6.02	−14.8	−38.17	−17.3

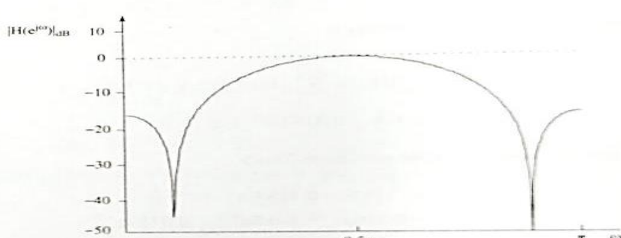


Fig. 6.13 Frequency response of Bandpass filter of example 6.7.

2.

Given $H_d(e^{j\omega}) = e^{-j3\omega}$

The frequency response is having a term $e^{-j\omega(N-1)/2}$ which gives $h(n)$ symmetrical about $n = \frac{N-1}{2} = 3$, i.e., we get a causal sequence.

We have

$$h_d(n) = \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{-j3\omega} e^{j\omega n} d\omega$$
$$= \frac{1}{2\pi}\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}} e^{j(n-3)\omega} d\omega$$
$$= \frac{\sin\frac{\pi}{4}(n-3)}{\pi(n-3)}$$

For $N = 7$ we have

$$h_d(0) = h_d(6) = 0.075$$
$$h_d(1) = h_d(5) = 0.159$$
$$h_d(2) = h_d(4) = 0.22$$
$$h_d(3) = 0.25$$

The non-causal window sequence is

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -(N-1)/2 \le n \le (N-1)/2$$
$$= 0 \quad \text{otherwise}$$

For $N = 7$

$$w_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for} \quad -3 \le n \le 3$$
$$= 0 \quad \text{otherwise}$$
$$w_{Hn}(0) = 0.5 + 0.5 = 1$$
$$w_{Hn}(-1) = w_{Hn}(1) = 0.5 + 0.5\cos\frac{\pi}{3} = 0.75$$

$$w_{Hn}(-2) = w_{Hn}(2) = 0.5 + 0.5\cos\frac{2\pi}{3} = 0.25$$

$$w_{Hn}(-3) = 0.5 + 0.5\cos\pi = 0$$

The causal window sequence can be obtained by shifting the sequence $w_{Hn}(n)$ to right by 3 samples, i.e.,

$$w_{Hn}(0) = w_{Hn}(6) = 0;\ w_{Hn}(1) = w_{Hn}(5) = 0.25$$
$$w_{Hn}(2) = w_{Hn}(4) = 0.75\ \&\ w_{Hn}(3) = 1$$

The filter coefficients using Hanning window are

$$\begin{aligned}
h(n) &= h_d(n)w_{Hn}(n) \quad \text{for} \quad 0 \le n \le 6 \\
h(0) &= h(6) = h_d(0)w_{Hn}(0) = (0.075)(0) = 0 \\
h(1) &= h(5) = h_d(1)w_{Hn}(1) = (0.159)(0.25) = 0.03975 \\
h(2) &= h(4) = h_d(2)w_{Hn}(2) = (0.22)(0.75) = 0.165 \\
h(3) &= h_d(3)w_{Hn}(3) = (0.25)(1) = 0.25
\end{aligned}$$

3.

Given

$$y(n) = 0.25x(n) + x(n-1) + 0.25x(n-2)$$

Taking Fourier transform on both sides

$$Y(e^{j\omega}) = 0.25X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega}) + 0.25e^{-2j\omega}X(e^{j\omega})$$

$$\begin{aligned}
H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} &= 0.25 + e^{-j\omega} + 0.25e^{-2j\omega} \\
&= e^{-j\omega}(0.25e^{j\omega} + 1 + 0.25e^{-j\omega}) = e^{-j\omega}(1 + 0.5\cos\omega) \\
&= e^{-j\omega}\overline{H}(e^{j\omega})
\end{aligned} \quad (6.4$$

Comparing Eq. (6.41a) with Eq. (6.25) we get $\theta(\omega) = -\omega$.

The phase delay $\tau_p = \frac{-\theta(\omega)}{\omega} = \frac{\omega}{\omega} = 1.$

The group delay $= -\frac{d\theta(\omega)}{d\omega} = \frac{-d}{d\omega}(-\omega) = 1.$

4.

Solution If $z_2 = 2$ is a zero for a linear phase filter, then $z_2' = 1/2$ is also a zero. If $z_1 = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}}$, whose $|z| = 1$ is a zero, then $z_1' = z_1^*$ is also a zero. The total number of zeros are four

$$z_1 = \frac{1}{\sqrt{2}} + \frac{j}{\sqrt{2}},\ z_1^* = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}};\ z_2 = 2;\ z_2' = \frac{1}{2}$$

$$\begin{aligned}
H(z) &= (1 - 2z^{-1})\left(1 - \frac{1}{2}z^{-1}\right)\left[1 - \left(\frac{1+j}{\sqrt{2}}\right)z^{-1}\right]\left[1 - \left(\frac{1-j}{\sqrt{2}}\right)z^{-1}\right] \\
&= \left(1 - \frac{5}{2}z^{-1} + z^{-2}\right)\left(1 - \sqrt{2}z^{-1} + z^{-2}\right)
\end{aligned}$$

5.

The ideal magnitude response with samples for the given specification is shown in Fig. 6.59.

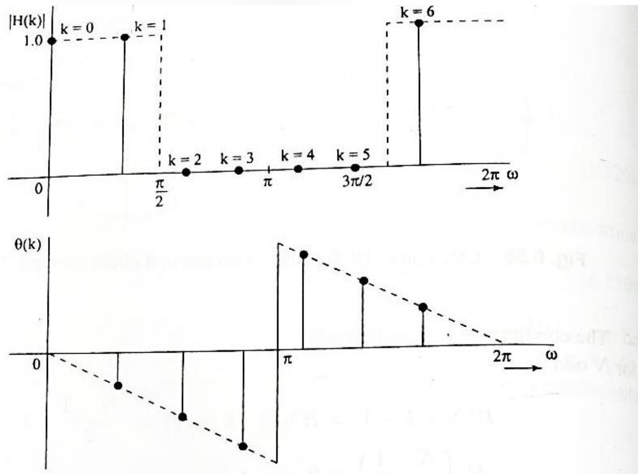


Fig. 6.59 Ideal magnitude and phase response with samples for example 6.15

Given $N = 7$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{7}} \quad k = 0, 1, 2, \ldots 6$$

From Fig. 6.59 we have

$$\begin{aligned} |H(k)| &= 1 \quad \text{for} \quad k = 0, 1, 6 \\ &= 0 \quad \text{for} \quad k = 2, 3, 4, 5 \end{aligned} \tag{6.140}$$

Using Eq.(6.126) we have

$$\begin{aligned} \theta(k) &= -\left(\frac{N-1}{N}\right)\pi k = -\frac{6}{7}\pi k \quad \text{for} \quad k = 0, 1, 2, 3 \\ &= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k = 6\pi - \frac{6\pi k}{7} = \frac{6\pi}{7}(7-k) \quad \text{for} \quad k = 4, 5, 6 \end{aligned} \tag{6.141}$$

Now the frequency response of the linear phase filter can be wirtten by substituting Eq.(6.140) and Eq.(6.141) in Eq.(6.120)

$$\begin{aligned} H(k) &= e^{-j6\pi k/7} \quad k = 0, 1 \\ &= 0 \quad \text{for} \quad k = 2, 3, 4, 5 \\ &= e^{-j6\pi(k-7)/7} \quad \text{for} \quad k = 6 \end{aligned}$$

The filter coefficients for N odd are given by

$$h(n) = \frac{1}{N}\left\{H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k)e^{j2\pi kn/7}\right]\right\} \quad n = 0, 1, \ldots N-1$$

$$= \frac{1}{7}\left\{1 + 2Re(e^{-j6\pi/7}e^{j2\pi kn/7})\right\}$$

$$= \frac{1}{7}\left\{1 + 2Re(e^{j2\pi(n-3)/7})\right\}$$

$$= \frac{1}{7}\left\{1 + 2\cos\frac{2\pi}{7}(n-3)\right\}$$

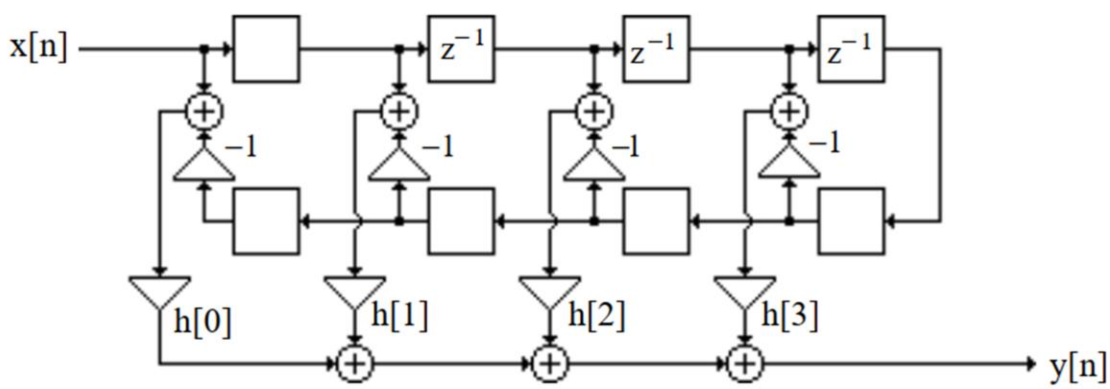
$$h(0) = h(6) = \frac{1}{7}\left(1 + 2\cos\frac{6\pi}{7}\right) = -0.11456$$

$$h(1) = h(5) = \frac{1}{7}\left(1 + 2\cos\frac{4\pi}{7}\right) = 0.07928$$

$$h(2) = h(4) = \frac{1}{7}\left(1 + 2\cos\frac{2\pi}{7}\right) = 0.321$$

$$h(3) = \frac{1}{7}(1+2) = 0.42857$$

6.



(all square boxes = z^{-1})

7. A. a) The filter is stable since its transfer function $H(z)= 1/(1-0.99z^{-1})$ has a pole z=0.99.

b) It is a low pass filter since it has a pole close to z=1 i.e. w=0.

B.

The passband ripple is given by $20 \log_{10} (1.05) = 0.42$ dB, and the attenuation in the stopband $-20 \log_{10} 0.005 = 46$ dB. The analog passband frequency is $0.3\pi F_s / 2\pi = 1.2$ kHz and the stopband $0.4\pi F_s / 2\pi = 1.6$ kHz

8.

Type 1: symmetric, odd

Type 2: symmetric, even

Type 3: antisymmetric, odd

Type 4: antisymmetric, even

The positions of zeros in the complex plane are as follows:

Type 1: Either an even number or no zeros at z = 1 and z=-1

Type 2: Either an even number or no zeros at z = 1, and an odd number of zeros at z=-1

Type 3: An odd number of zeros at z = 1 and z=-1

Type 4: An odd number of zeros at z = 1, and either an even number or no zeros at z=-1

9.

$$M = 15. H_r(\frac{2\pi k}{15}) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0.4, & k = 4 \\ 0, & k = 5, 6, 7 \end{cases}$$

$$\begin{aligned} H_r(w) &= h(\frac{M-1}{2}) + 2\sum_{n=0}^{\frac{M-3}{2}} h(n) cosw(\frac{M-1}{2} - n) \\ h(n) &= h(M-1-n) \\ h(n) &= h(14-n) \\ H_r(w) &= h(7) + 2\sum_{n=0}^{6} h(n) cosw(7-n) \end{aligned}$$

Solving the above eqn yields,

$$\begin{aligned} h(n) = \{&0.3133, -0.0181, -0.0914, 0.0122, 0.0400, -0.0019, -0.0141, 0.52, \\ &0.52, -0.0141, -0.0019, 0.0400, 0.0122, -0.0914, -0.0181, 0.3133\} \end{aligned}$$

10.

(a) There are only zeros, thus $H(z)$ is FIR.
(b)

$$\begin{aligned}
\text{Zeros: } z_1 &= -\frac{4}{3}, \\
z_2 &= -\frac{3}{4}, \\
z_{3,4} &= \frac{3}{4}e^{\pm j\frac{\pi}{3}} \\
z_{5,6} &= \frac{4}{3}e^{\pm j\frac{\pi}{3}}
\end{aligned}$$

$$\begin{aligned}
z_7 &= 1 \\
\text{Hence, } z_2 &= \frac{1}{z_1^*} \\
z_4 &= z_3^* \\
z_5 &= \frac{1}{z_3^*} \\
z_6 &= z_5^* \\
z_1 &= \frac{1}{z_7} = 1 \\
\text{and } H(z) &= z^{-6}H(z^{-1})
\end{aligned}$$

Therefore, $H(w)$ is linear phase.