DSP Tutorial

Week 1

Time limit: 15 min

1. a. Compute the convolution: y(n) = x(n)*h(n) where,

   $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ for cases **a = b** and **a ≠ b.**

   b. Let $\boldsymbol{y(n)} = \boldsymbol{x(n)} * \boldsymbol{h(n)}$. Then show that

   $\boldsymbol{x(n - n1) * h(n - n2) = y(n - n1 - n2)}$

Time limit: 25 min

2. a. Comment on the periodicity of the following signals:

I. $\sin(\pi^2 n)$

II. $\sin(\pi \frac{62n}{10})$

III. $\cos(\pi \frac{30n}{105})$

b. A continuous-time sinusoidal signal Xa(t) = cos(**Ω**t) is sampled at t = nT, -∞<n<∞, generating the discrete-time sequence x[n] = Xa(nT) = cos(**Ω**nT). For what values of "T" is x[n] a periodic sequence? What is the fundamental period of x[n] if **Ω** = 30 radians and T = π/6 seconds?

c. A continuous time signal x(t) = cos(2πt) is sampled in every T seconds. The sampled signal is denoted by x[n].

I. For T = 0.13 sec, what will be the period of x[n]?

II. What is the limiting value of T (upper or lower) for successful reconstruction of x(t) from x[n]?

A.1. a.

$$y(n) = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k) = b^n \sum_{k=0}^{n} (ab^{-1})^k$$

$$y(n) = \begin{cases} \frac{b^{n+1} - a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1) u(n), & a = b \end{cases}$$

A.1. b.

1. **To Prove: x[n-n1] * h[n-n2] = y[n-n1-n2]**

From convolution definition,

y[n] = x[n] * h[n] = $\sum_{k=-\infty}^{\infty}$ x[k] h[n-k] …..(i)

x[n-n1] * h[n-n2] = $\sum_{k=-\infty}^{\infty}$ x[k-n1] h[n-k-n2] ……..(ii)

let k-n1 = p then k = p+n1, from eq(ii)

x[n-n1] * h[n-n2] = $\sum_{p=-\infty}^{\infty}$ x[p] h[n-n1-n2-p] = y[n-n1-n2], (from eq(i) by replacing n by n-n1-n2)

Therefore, x[n-n1] * h[n-n2] = y[n-n1-n2].

A.2. a.

1) $\sin(\pi^2 n)$

$\sin[\pi^2 (n+N)]$.

$= \sin(\pi^2 n + \pi^2 N)$

Let the signal be periodic. Hence,

$\pi^2 N = 2\pi k$

$\Rightarrow N = \frac{2k}{\pi}$.

Since $\frac{2k}{\pi}$ is not an integer for any integer value of k,
$\sin(\pi^2 n)$ is not periodic.

2) $\sin\left(\pi \frac{62n}{10}\right)$.

$\Rightarrow N = \frac{2\pi}{62\pi} \times 10k = \frac{20k}{62} = \frac{10k}{31}$

For $k=31$, $N=10$

3) $\cos\left(\pi \frac{30n}{105}\right)$

$\Rightarrow \cos\left(\frac{\pi 30n}{105} + \frac{\pi 30N}{105}\right)$

$\Rightarrow 2\pi k = \frac{30\pi N}{105}$

$\Rightarrow N = \frac{105 \cdot k}{15} = 7k$

For $k=1$, $N=7$.

A.2. b.

given $x_a(t) = \cos(\Omega t)$

$t = nT$

$\Rightarrow \quad x[n] = x_a(nT) = \cos(\Omega nT)$

For $x[n]$ to be periodic

$$x[n] = x[n+N], \quad \boxed{\begin{array}{l} N \neq 0 \\ N \in \mathbb{Z} \text{ (integer)} \end{array}}$$

$\Rightarrow \quad \cos(\Omega nT) = \cos(\Omega nT + \Omega NT)$

$\Rightarrow \quad \Omega NT = 2k\pi, \quad k \in \mathbb{Z}$ (integers)

$\Omega N T = 2k\pi \Rightarrow$

$$\Rightarrow \boxed{\frac{\Omega T}{2\pi} = \frac{k}{N}}$$ Condition for periodicity

i.e. for $x(n)$ to be periodic $\frac{\Omega T}{2\pi}$ must be a rational number.

$\Rightarrow T = \frac{k}{N} \cdot \frac{2\pi}{\Omega}$ gives the periodic sequence where $\frac{k}{N}$ is rational

↳ given $\Omega = 30$ radians, $T = \frac{\pi}{6}$

$$\Rightarrow \frac{\Omega T}{2\pi} = \frac{\overset{5}{\cancel{30}} \times \cancel{\pi}}{2\cancel{\pi} \times \cancel{6}} = \frac{5}{2} = \frac{k}{N}$$

$\frac{5}{2}$ is rational number ~~with~~ of lowest terms

$$\Rightarrow \boxed{\text{fundamental period}, N = 2}$$

A.2. c.

Q1) $x(t) = \cos(2\pi t)$

$x(n) = \cos(2\pi nT)$

a)

$\therefore$ Let, $x(n)$ has periodicity with N samples

$\therefore x(n+N) = x(n)$ for each n

$\therefore \cos(2\pi(n+N)T) = \cos(2\pi nT)$

or, $\cos(2\pi nT + \underbrace{2\pi NT}) = \cos(2\pi nT)$

$\boxed{2\pi NT = 2\pi K}$ (K int)

$\therefore N = \frac{K}{T}$

$= \frac{K}{.13}$

$= \frac{100 \times K}{13}$

$\therefore$ for, $K = 13$, $\boxed{N = 100}$

b) $x(t) = \cos(2\pi t)$

$\hookrightarrow f = f_{max} = 1\,Hz$ $\therefore f_s > 2\,Hz$

$\therefore$ ~~fs~~. $\therefore T < \frac{1}{2}\,s$ $\therefore \boxed{T < .5 \text{ sec}}$