Tutorial

Subject : RF and Microwave Engineering

(EC31005)

Q1. A terminated transmission line with Z_0 = 60 Ω has a reflection co-efficient at the load of $0.4\angle 60^0$

a) What is the load impedance?

b) What is the reflection coefficient 0.3λ away from the load?

c) What is the input impedance at this point?

ANSWER:

(a)

$$Z_O = 60\Omega$$

$$\Gamma_L = 0.4\angle 60^o$$

$$\Gamma_L = \frac{Z_L - Z_O}{Z_L + Z_O} = 0.2 + j0.346$$ Load reflection coefficient

$$Z_L = (66.43 + j54.7)\Omega$$ Load Impedance

(b) The line reflection coefficient is

$$\Gamma_L(l) = \Gamma_L e^{-j2\beta l} = 0.4\angle -156^o$$

(c)

$$\tan\beta l = -3.07$$

$$Z_{in} = Z_O \frac{Z_L + jZ_O \tan\beta l}{Z_O + jZ_L \tan\beta l} = (26.65 - j10.24)\Omega$$

Q2. A lossless transmission line is terminated with a 100 Ω load. If the SWR on the line is 1.5, find the two possible values for the characteristic impedance of the line.

ANSWER:

$$|\Gamma| = \frac{(S-1)}{(S+1)} = \frac{0.5}{2.5} = 0.2$$

SWR is denoted as S

$$|\Gamma| = \left|\frac{Z_L - Z_O}{Z_L + Z_O}\right| = \left|\frac{100 - Z_O}{100 + Z_O}\right|$$

$$\Gamma = \pm 0.2$$

Either

$$\frac{100 - Z_O}{100 + Z_O} = 0.2 \Rightarrow Z_O = 100 * \left(\frac{1 - \Gamma}{1 + \Gamma}\right) = 66.7\Omega$$

or

$$\frac{100 - Z_O}{100 + Z_O} = -0.2 \Rightarrow Z_O = 100 * \left(\frac{1 - \Gamma}{1 + \Gamma}\right) = 150\Omega$$

Q3. A radio transmitter is connected to an antenna having an impedance 80+ j40 Ω with a 50 Ω coaxial cable. If the 50 Ω transmitter can deliver 30 W when connected to a 50 Ω load, how much power is delivered to the antenna?

Given: load impedance Z_L=80+j40 Ω,

Coaxial cable impedance Z_0=50 Ω,

Now the reflection coefficient is given by, $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

Substituting the values of Z_L and Z_0, $\Gamma = \frac{80 + j40 - 50}{80 + j40 + 50}$ = 0.296+j0.215

And $|\Gamma|$= 0.366

Now given that the transmitter can deliver 30 W when connected to a 50 Ω load i.e. the matched load condition, It means the Incident power = 30 W.

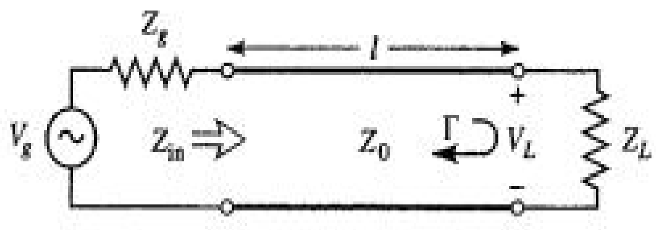
Hence the power delivered to the antenna(Load) = $P_{incident}$ - $P_{reflected}$ = $P_{incident}$ $[1- |\Gamma|^2]$.............(i)

Substituting the values of $P_{incident}$ and $|\Gamma|$ in equation (i),

P_{load}= $30[1-0.366^2]$= 25.98 W

Hence Power delivered to the Antenna is = 25.98 W.

Q4. The transmission line circuit in the accompanying figure has V_g =15 V rms, Z_g = 75 Ω, Z_0 = 75 Ω, Z_L = 60− j40 Ω, and l = 0.7λ. Compute the power delivered to the load using three different techniques:



a. Find Γ and compute

$$P_L = \left({V_g}^2/_4\right)\frac{1}{Z_0}(1-|\Gamma|^2)$$

b. Find Z_{in} and compute

$$P_L = \frac{(Vg)^2}{(Zg+Zin)2}\mathrm{Re}\{Z_{in}\}$$

c. Find V_L and compute $P_L = |V_L/Z_L|^2\,\mathrm{Re}\{Z_L\}$

Given: load impedance Z_L=60-j40 Ω,

Transmission line characteristic impedance Z_0 = 75 Ω,

Now the reflection coefficient is given by,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Substituting the values of Z_L and Z_0, $\Gamma = \frac{60-j40-75}{60-j40+75}$ = -0.021-j0.302

and |Γ|= 0.3027

contd.

a. Substituting the values of V_g and $|\Gamma|$ to compute P_L,

Hence $PL = (\frac{15^2}{4})\frac{1}{75}(1\text{-}0.3027^2)$ = **0.681 W.**

b. As the input impedance of a transmission line is given as,

$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZL \tan\beta l}$ ………………………………………..(i), Where $\beta = 2\frac{\pi}{\lambda}$

Substituting the values of Z_0, Z_L, and l in equation (i),

$Z_{in} = 75 \frac{60+j190.8}{198.1+j184.7}$ = 48.2+j27.3 Ω

Substituting the values of V_g and Z_g and Z_{in} to compute P_L,

Hence $P_L = \frac{(15)^2}{(75+48.2+j27.3)2}$ (48.2)= **0.681 W**.

c. As Voltage at a point a distance x from the load is given by,

$V(x) = V^+(e^{-j\beta x} + \Gamma e^{+j\beta x})$ where $V^+ = \frac{V_g}{2}$ = 7.5 V

Hence the voltage at load i.e. at x=0,

$V_L = V(0) = v^+(1+\Gamma)$………………………………………………..(ii) contd.

Substituting the values of V^+, Γ in equation (ii) to compute V_L,

Hence $V_L = 7.5(1-0.021-j0.302) = 7.3425-j2.265$

And $|V_L| = 7.68$

Substituting the value of V_L, to compute $P_{L,}$

Hence $P_L = |(7.3425-j2.265)/(60-j40)|2\ Re\{60-j40\}$

$= |7.68/72.1|^2(60)$

$= \mathbf{0.681\ W}$

Q5. For a purely reactive load impedance of the form Z_L = j X, show that the reflection coefficient magnitude $|\Gamma|$ is always unity. Assume that the characteristic impedance Z_0 is real.

Ans: Given,

Z_L=jX

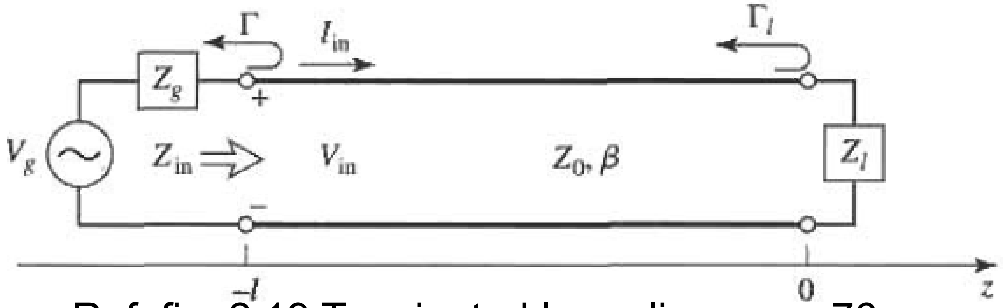
The value of reflection co-efficient is:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$\Gamma^* = \frac{-jX - Z_0}{-jX + Z_0}$$

$$|\Gamma|^2 = \Gamma\Gamma^* = \frac{jX - Z_0}{jX + Z_0} \times \frac{-jX - Z_0}{-jX + Z_0} = \frac{X^2 - jZ_0 X + jZ_0 X + Z_0{}^2}{X^2 - jZ_0 X + jZ_0 X + Z_0{}^2} = 1$$

Q6. A 50 Ω transmission line is matched to a 10 V source and feeds a load Z_L = 100 Ω. If the line is 2.3λ long and has an attenuation constant α = 0.5 dB/λ, find the powers that are delivered by the source, lost in the line, and delivered to the load
(Additional βl= 108^0)



Ref: fig. 2.19 Terminated lossy line page 76 D.M. Pozar 4th edition

Ans:

$Z_0 = Z_g = 50\ \Omega, V_g = 10V, Z_L = 100\Omega, l$=2.3 λ, α=0.5dB/λ

α=0.5dB/λ=0.0576neper/λ (Since 1Neper=8.68dB)

Since the generator is matched to the line, reflection coefficient seen looking into source $\Gamma_g = 0$ in the formula

$V_0{}^+ = V_g \frac{Z_0}{Z_0+Z_g} \frac{e^{-\gamma l}}{1-\Gamma_L \Gamma_g e^{-2\gamma l}}$ (eq 2.71)

where $V_0{}^+$ is the incident voltage referenced at z = 0

Thus, $V_0{}^+ = \frac{V_g}{2} e^{-\gamma l}$

$\boldsymbol{\gamma} l$= (α+jβ) l = 0.1325 + j108 Ω

Thus,

$V_0{}^{+} = \frac{10}{2} e^{-\alpha l}$ = 4.38V

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50}$ = 0.33 (Γ is reflection coefficient at load)

$\Gamma(l) = \Gamma e^{-2\gamma l}$ = reflection coefficient at distance *l* from load

Power delivered to line $P_{in} = \frac{|V_0{}^{+}|^2}{2Z_0}[1-\Gamma(l)^2]\, e^{2\alpha l} = \frac{4.38^2}{100}[e^{2(0.1325)} - 0.333^2 .\, e^{-2(0.1325)}]$ = 0.2337W (eq 2.92)

Power delivered to load $P_L = \frac{|V_0{}^{+}|^2}{2Z_0}[1-|\Gamma|^2] = \frac{4.38^2}{100}[1-0.333^2]$ =**0.1706W** (eq 2.93)

$P_{loss} = P_{in} - P_L$ = 0.2337-0.1706= **0.0631W** (eq 2.94)

Input impedance is

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \boldsymbol{\gamma} l}{Z_0 + Z_L \tanh \boldsymbol{\gamma} l} = 50\, \frac{100+50(0.845+j2.19)}{50+100(0.845+j2.19)} = 32.5 - j12.4\ \Omega$$

Input current is

$I_{in} = \frac{V_g}{R_g+Z_{in}} = \frac{10}{82.5-j12.4} = 0.1199,\ 8.5^0$ A

Generator power is

$P_s = \frac{V_g}{2}|I_{in}|$=5(0.1199)=**0.6W**

Power lost in R_g =

$P_{R_g} = \frac{1}{2}|I_{in}|^2\ R_g = \frac{1}{2}|0.1199|^2 50 = 0.3594\ W$

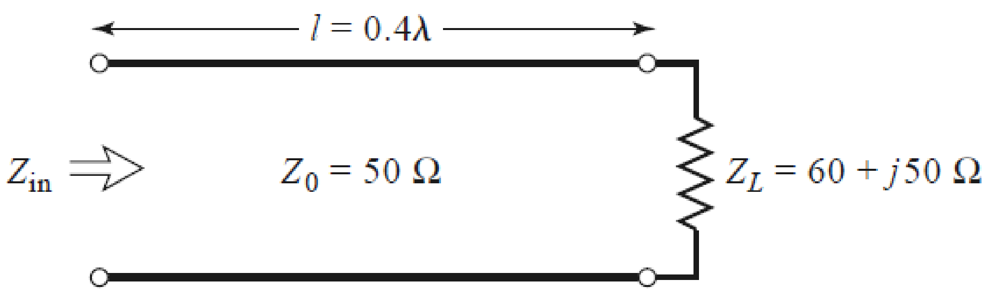
Cross Check

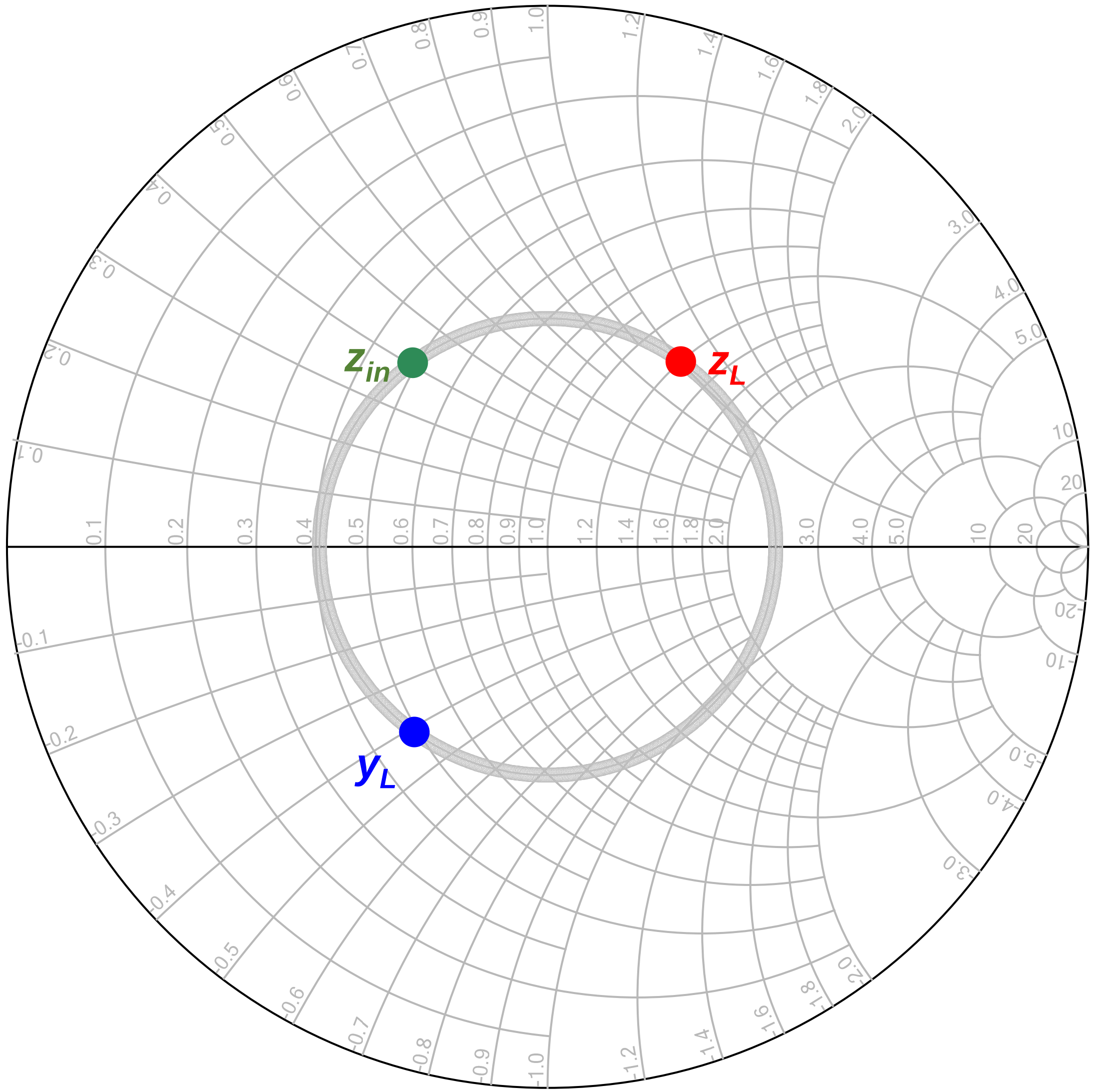
$P_L + P_{loss} + P_{R_g}$=0.1706+0.0631+0.3594=0.5931W= P_s

$P_{in} + P_{R_g}$=0.2337+0.3594=0.5931W= P_s

Q7. Use the Smith chart to find the following quantities for the transmission line circuit shown in the accompanying figure:

(a) The SWR on the line.
(b) The reflection coefficient at the load.
(c) The load admittance.
(d) The input impedance of the line.
(e) The distance from the load to the first voltage minimum.
(f) The distance from the load to the first voltage maximum.





First, mark normalized load impedance (z_L) in the Smith Chart

$$z_L = Z_L / 50 = 1.2 + j1$$

(a) Draw a circle through z_L.
This will give SWR = 2.46.

(b) Measure the angle and read the magnitude

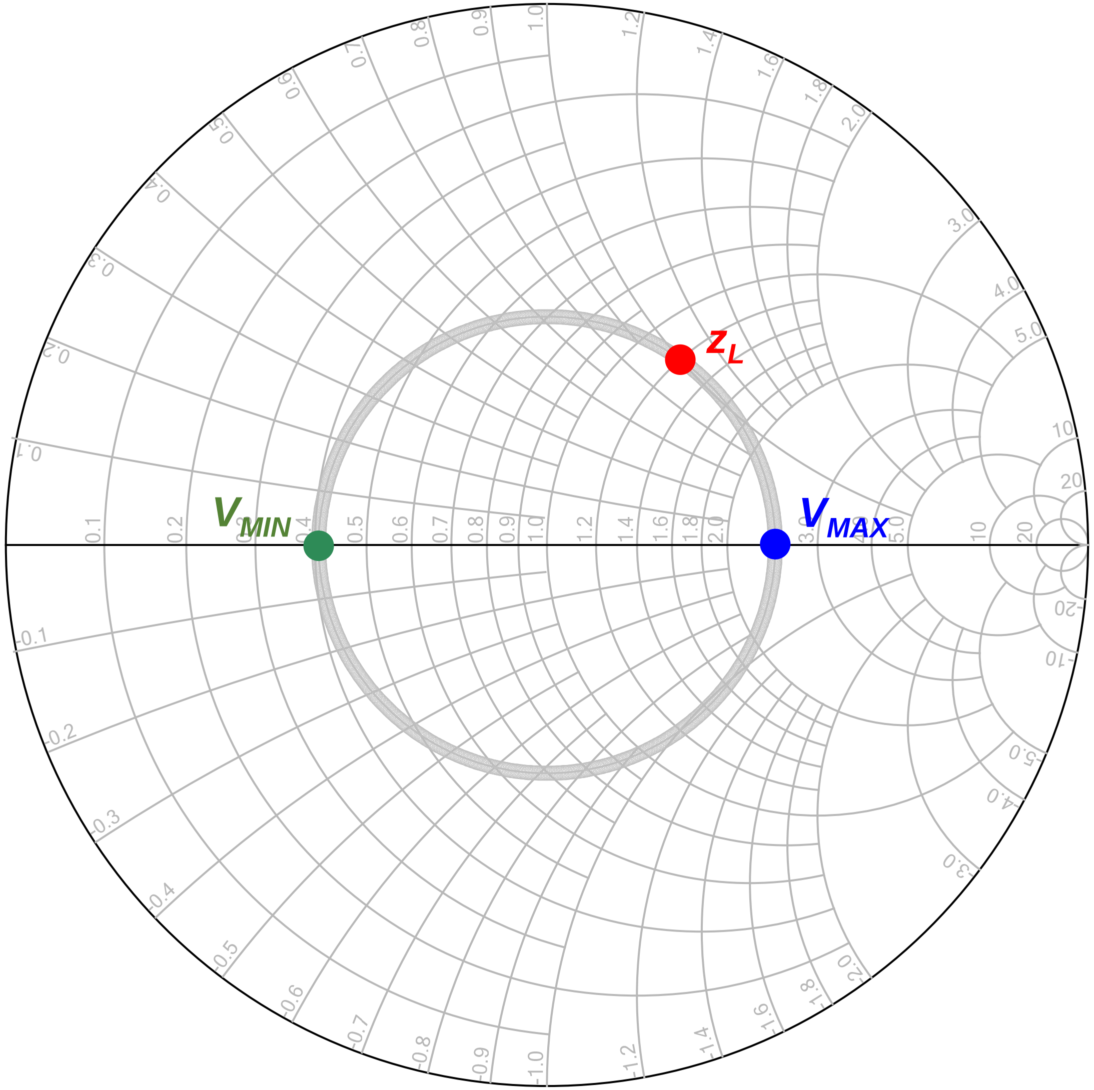
$$\Gamma = 0.422\angle 54^{\circ}$$

(c) Move 180^0 along VSWR circle to get y_L .

$$Y_L = (0.492 - j0.412) / 50 = (9.84 - j8.2) \text{ mS}$$

(d) Traverse in the clock-wise direction along constant VSWR circle by 0.4 λ to get z_{in}

$$Zin = (24.5 + j20.3)\,\Omega$$



(e) $l_{MIN} = 0.325\lambda$

(f) $l_{MAX} = 0.075\lambda$

At V_{MAX} location, the resistance will be the highest and reactive part is 0.
At V_{MIN} location, the resistance will be the lowest and reactive part is 0. Observe that, it is 0.25 λ more distant.
For both these cases, you have to traverse along VSWR circle in clock-wise direction.

Q8. Use the Smith chart to find the shortest lengths of a short-circuited 75 Ω line to give the following input impedance:

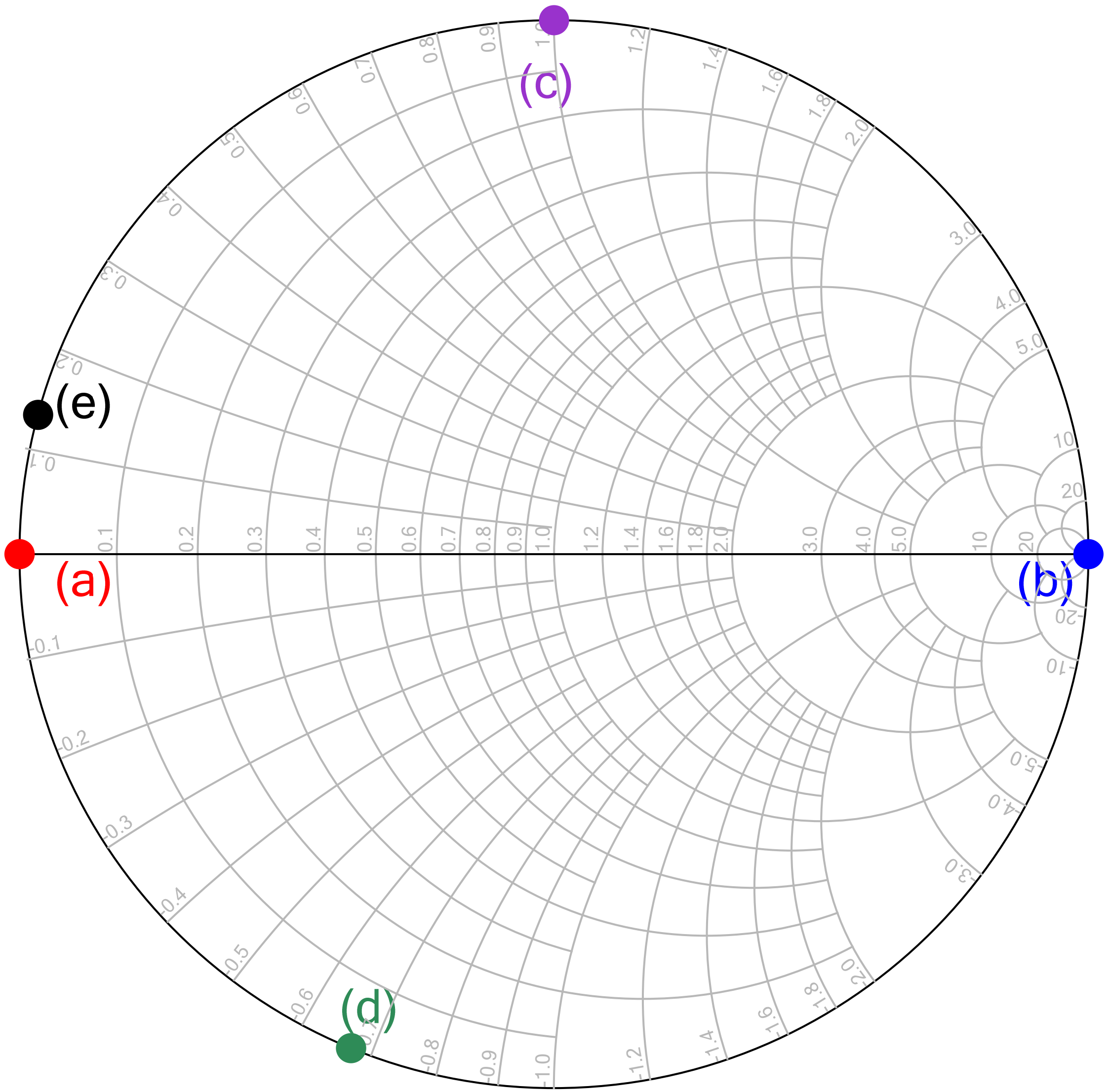
(a) $Z_{in} = 0$.

(b) $Z_{in} = \infty$.

(c) $Z_{in} = j75$.

(d) $Z_{in} = -j50$.

(e) $Z_{in} = j10$



(a) Zin = 0.

(b) Zin =∞.

(c) Zin = j75.

(d) Zin = −j50.

(e) Zin = j10

(a) L = 0 or 0.5 λ

(b) L = 0.25 λ

(c) L = 0.125 λ

(d) L = 0.406 λ

(e) L = 0.021 λ

You have to traverse in the clock-wise direction along constant VSWR circle which is the outermost circle for all the cases.
For, (c), (d) and (e) just mark the normalized input impedance, which is,

$$z_{in} = Z_{in} / 75$$

and calculate the distance from the load (here, S.C point or (a) is the load) in the clock-wise direction.