RF and Microwave Engineering (EC 31005)

Introduction (P1)

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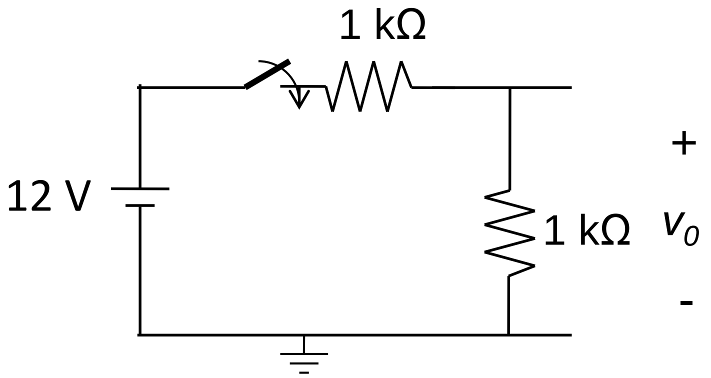
RF and Microwave Engineering (EC 31005)

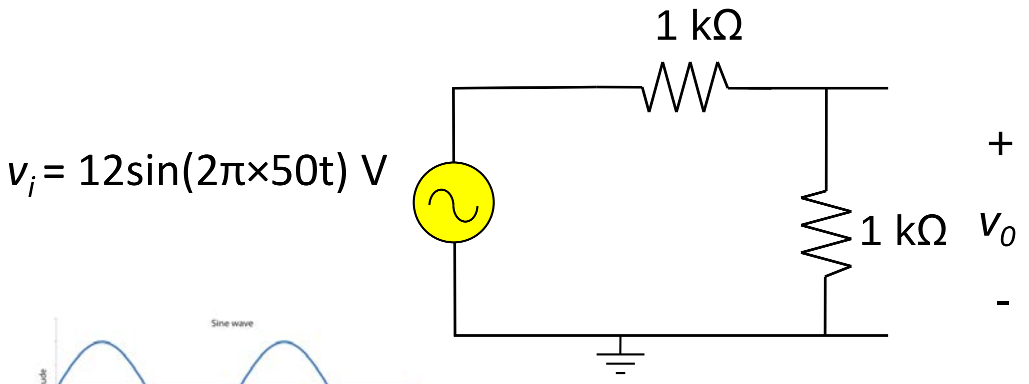
Book:
David M. Pozar, Microwave Engineering, Wiley, 2012.

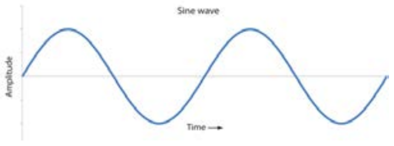
- Register in Moodle
- MCQ test in CIC (every alternate week)
- Come prepared for the lab experiments.

Why Electromagnetic theory..?



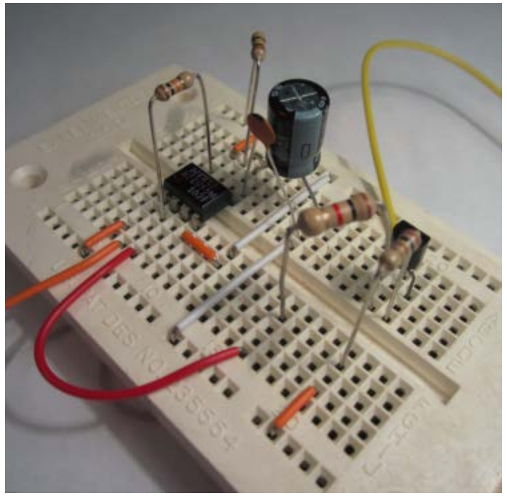
Voltage divider





Voltage divider

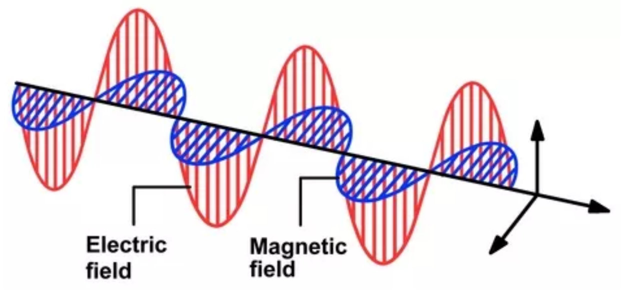
- KCL and KVL are not applicable..!
- Maxwell's equations→ electromagnetic wave.
- Electric field, magnetic field, attenuation constant α, phase constant β.
- Lumped components – different behaviour.
- Distributed form.



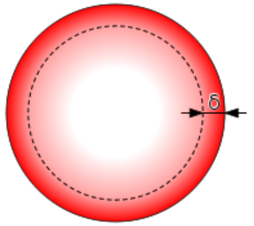
Breadboard circuit.

Why Electromagnetic theory..?

- Free space wavelength at 10 GHz = 3 cm.
- 1 cm in space @ 10 GHz → 120^o change in phase.
- High attenuation inside metal.
- Skin depth: $\delta_{cu}|_{50\ Hz} = 8.5$ mm, $\delta_{cu}|_{10\ kHz} = 660$ µm, $\delta_{cu}|_{10\ GHz} = 0.66$ µm.
- Use special guiding structures: coaxial cable, rectangular waveguide, microstrip line etc.



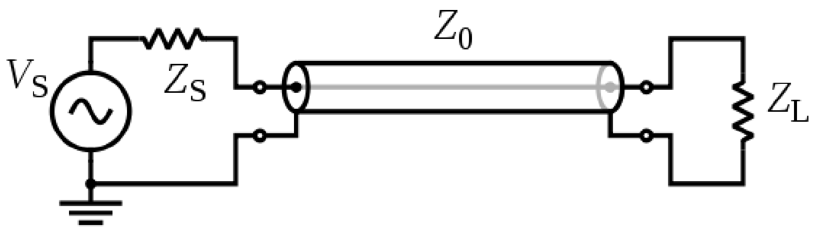
Electromagnetic wave in free space.



AC current distribution.

Problems at Microwave frequencies (3-30 GHz):

1. Phase
2. Attenuation



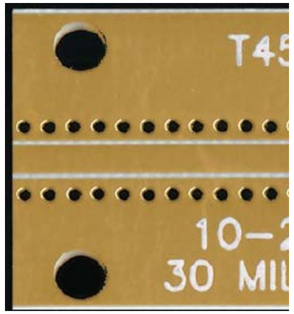
Co-axial cable, a transmission line

Different forms of guiding structures

Coaxial cable.

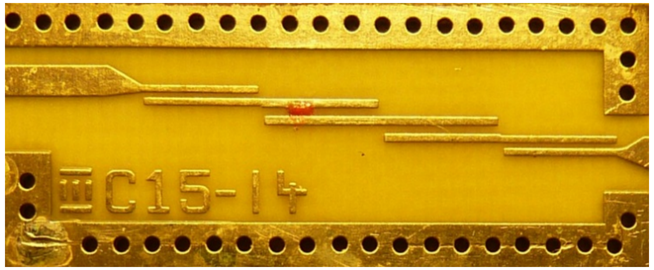
Rectangular waveguide.

Microstrip line.



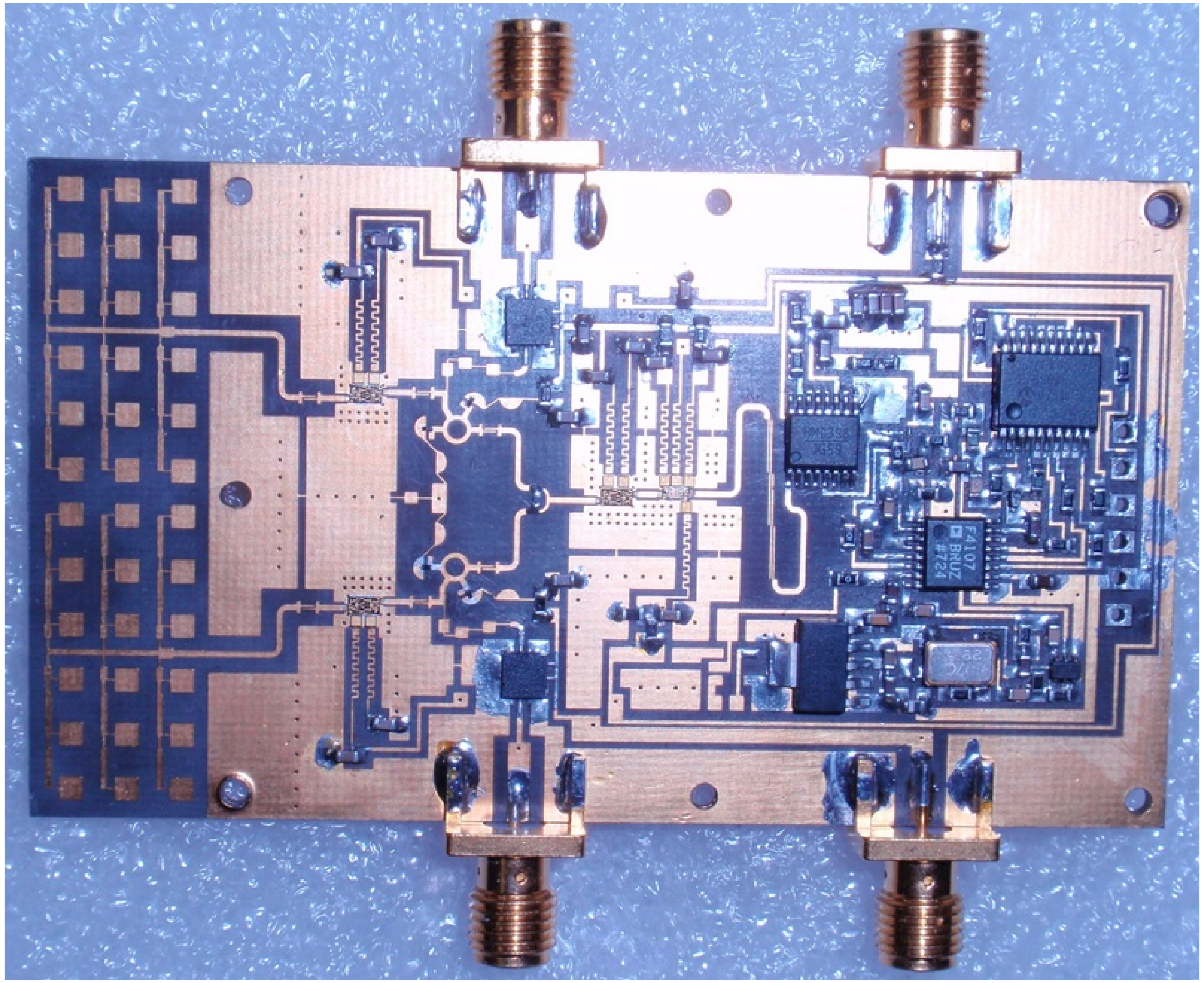
Coplanar waveguide

- Metal is to guide the wave.
- Different guiding structures is used at different modes.
- Characteristics: attenuation, power handling capability, mono-mode bandwidth → all frequency dependent terms
- Size, cost.

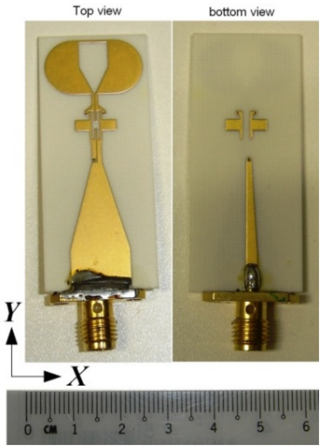


Millimeter-wave PCB based filter.

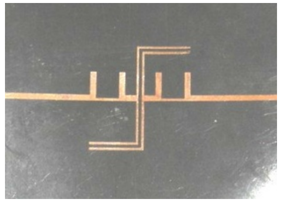
Automobile radars

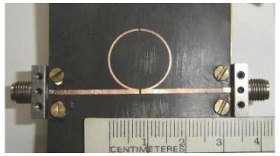


FMCW radar @ 77 GHz.

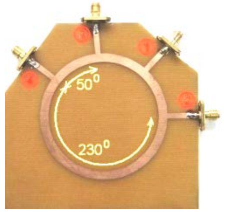


Antennas



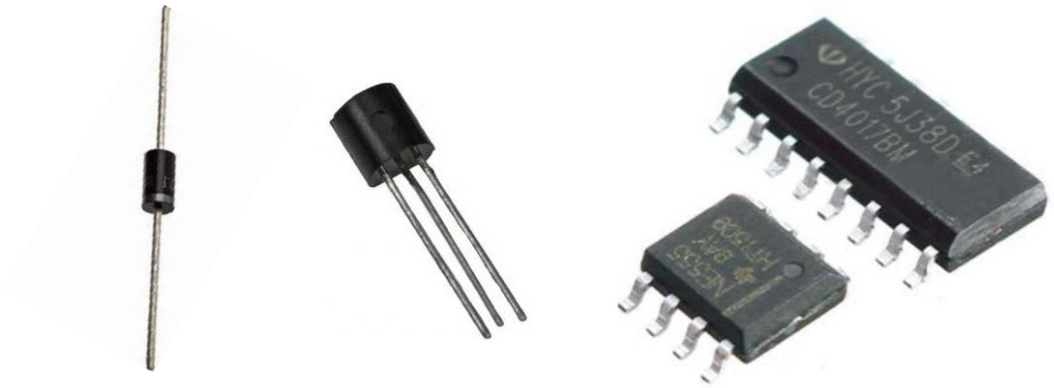


Filters

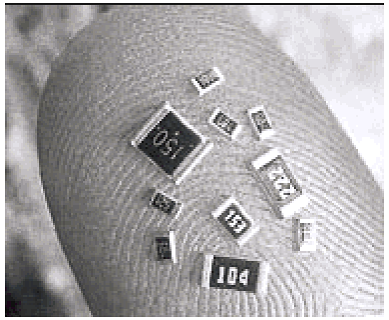


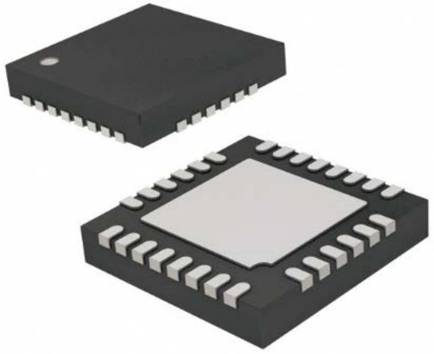
Couplers

Surface mountable devices (SMD)

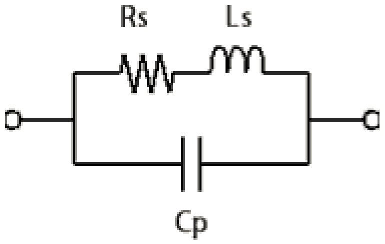


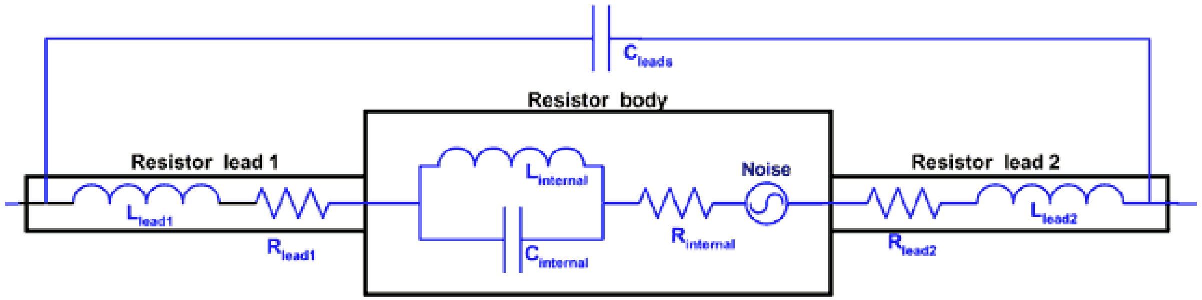
Low frequency discrete components





Microwave components





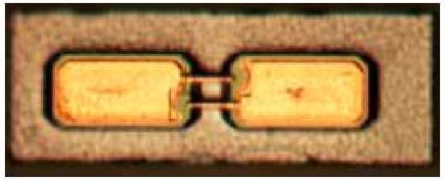
Equivalent circuit of a SMD resistor (simplified)

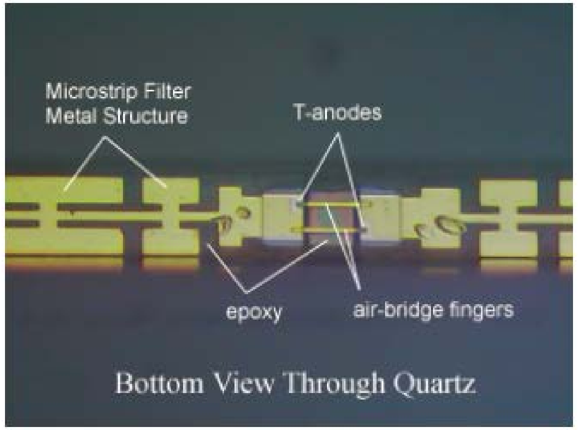
Design steps

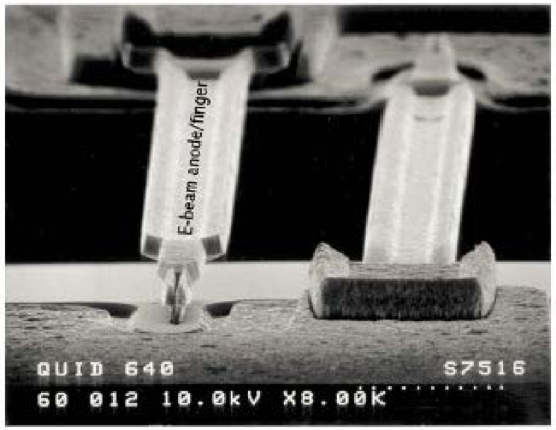
- Device model (two/ *n*-port device)
- Design of passive circuit (initial estimation → full wave electromagnetic simulation → optimization).
- Co-simulation (both active and passive).
- Optimization.

Back to back diodes
for SHP mixer

Courtesy VDI

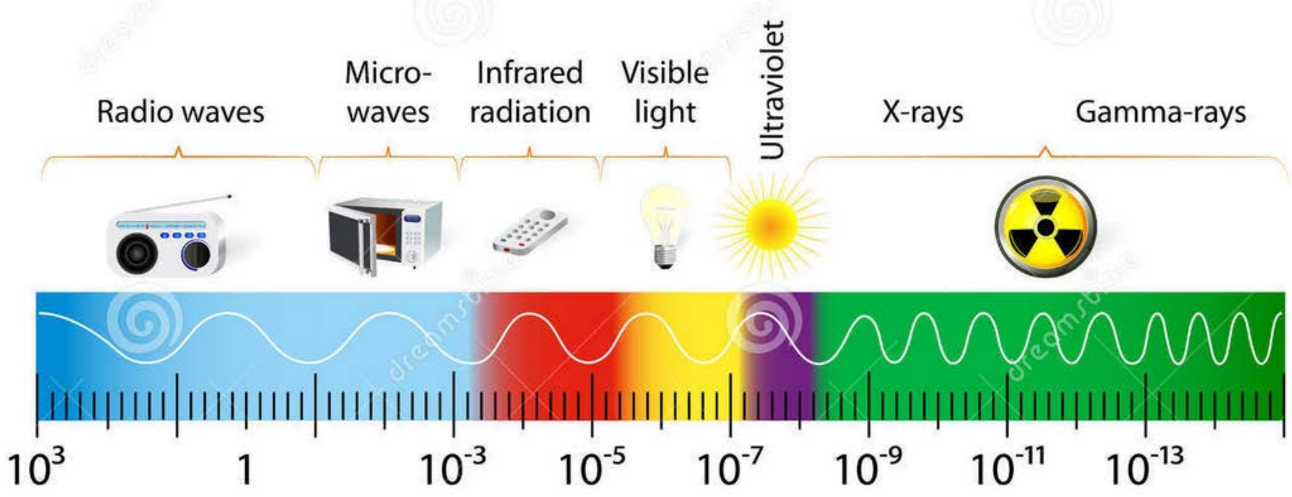


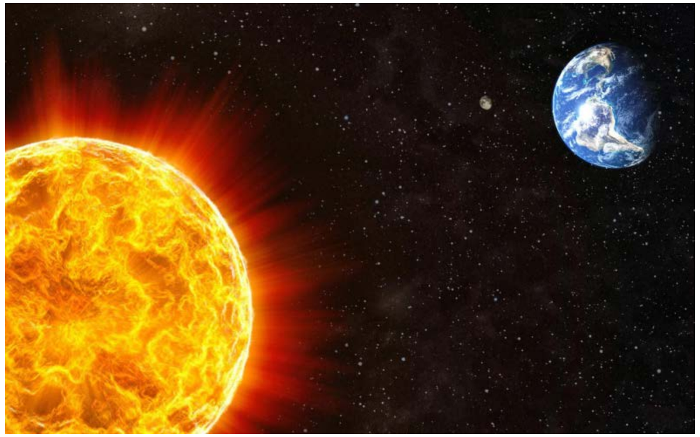




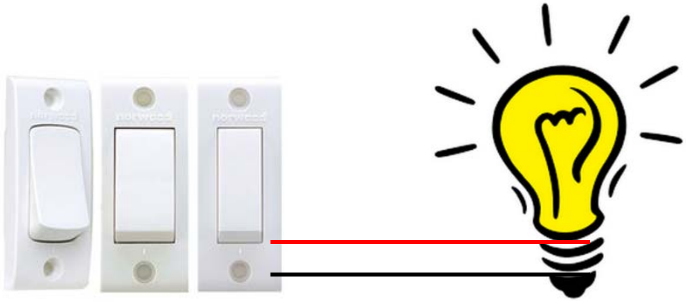
A back to back diode with integrated filter for mixer design.

Electromagnetic spectrum

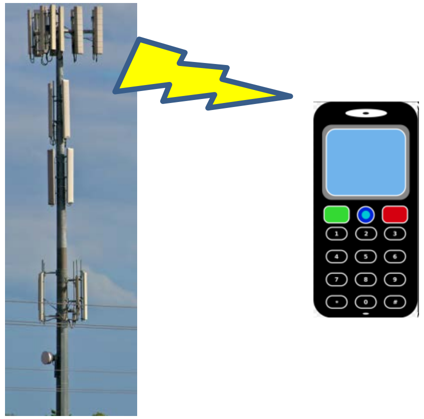




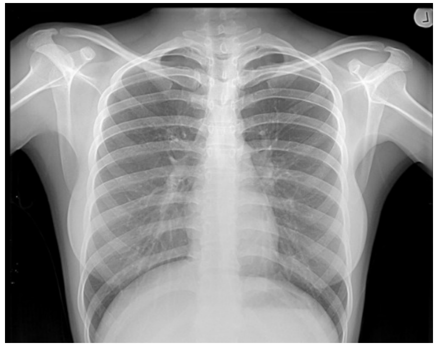
Sun is a source of electromagnetic waves.



Electromagnetic wave travels through the wire.



Personal communication



X-ray photo.

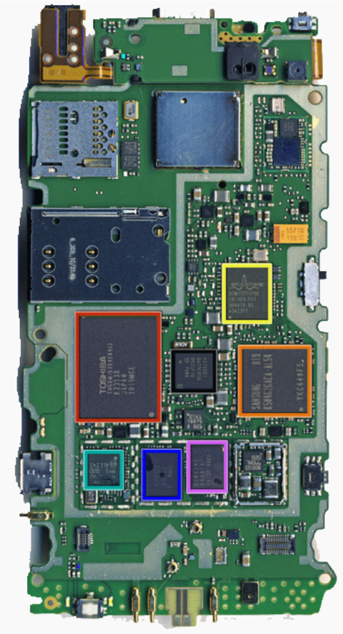
Band designation

Bands	Frequency range	Typical uses
L	1 - 2 GHz	military telemetry, **GPS, mobile phones** (GSM), amateur radio.
S	2 - 4 GHz	weather and ship radar, some communications satellites, microwave ovens, radio astronomy, mobile phones, **wireless LAN**, Bluetooth, ZigBee, GPS, amateur radio.
C	4 - 8 GHz	long-distance radio telecommunications.
X	8 - 12 GHz	satellite communications, radar, terrestrial broadband, space communications, amateur radio.
Ku	12 - 18 GHz	satellite communications.
K	18 - 26.5 GHz	radar, satellite communications, astronomical observations.
Ka	26.5 - 40 GHz	satellite communications, scanner.
Q	33 - 50 GHz	satellite communications, terrestrial microwave communications, radio astronomy.
V	50 - 75 GHz	millimetre wave radar research and other kinds of scientific research.
E	60 - 90 GHz	UHF transmissions.
W	75 - 110 GHz	satellite communications, millimeter-wave radars, military radar targeting and tracking applications, and **automobile radar.**
F	90 - 140 GHz	Radio astronomy, microwave devices/communications, wireless LAN, most modern radars, communications satellites, satellite television broadcasting, DBS, amateur radio, **passive imaging**.
D	110 - 170 GHz	radio astronomy, radio relay, remote sensing, scanner

Applications

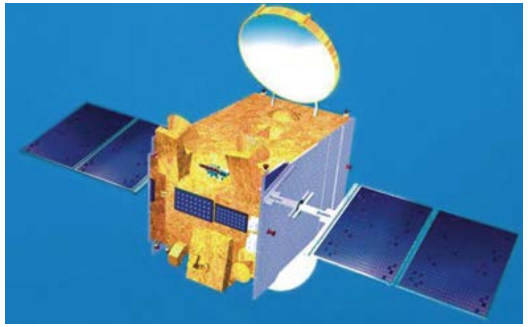
Battle field scenario.

Ship radar



Nokia phone circuit board.

Telescope: CARMA in D-array (1cm, 3 mm).



Communication satellite
(INSAT- 4D)

Application: passive imaging

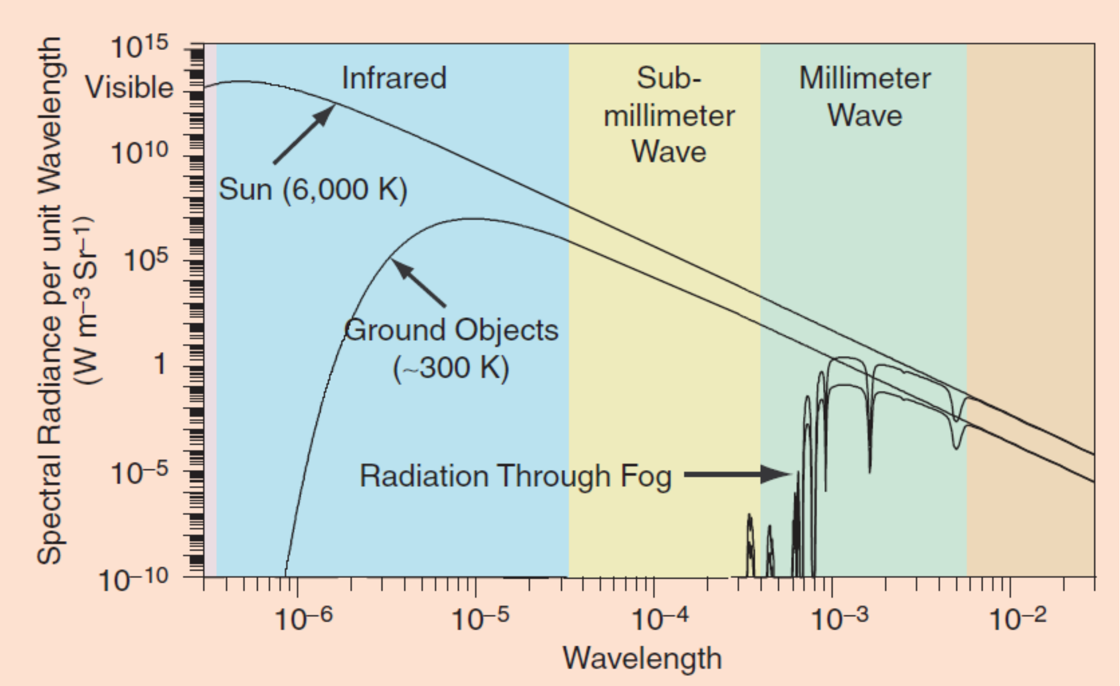
Effective radio metric temperature $T_E = T_s + T_{sc}$

Surface brightness temp T_s = Physical temperature T_0 × emissivity ε

Scattered radiometric temp T_{sc} = reflectivity ρ × radiometric temperature T_{ILLU}

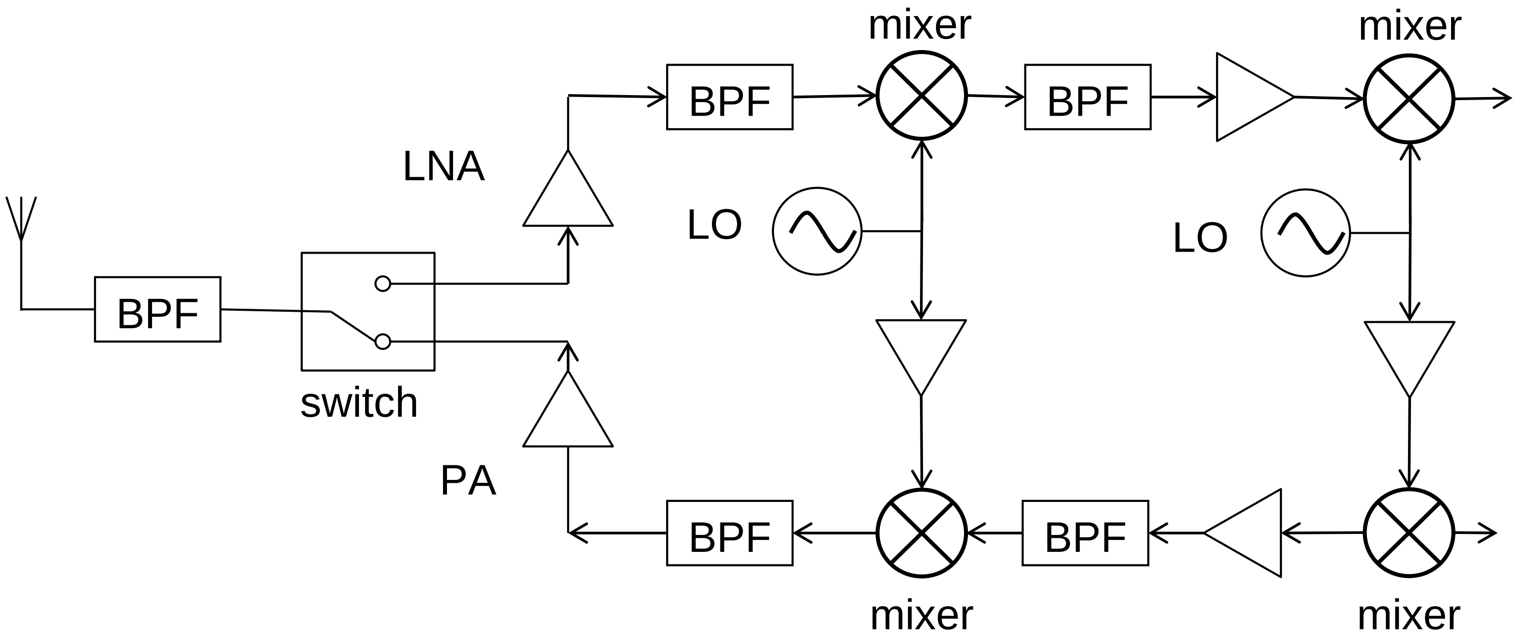
Table 1. Effective emissivity of common materials at various frequencies.

	Effective Emissivity		
Surface	44 GHz	94 GHz	140 GHz
Bare metal	0.01	0.04	0.06
Painted metal	0.03	0.10	0.12
Painted metal under canvas	0.18	0.24	0.30
Painted metal under camouflage	0.22	0.39	0.46
Dry gravel	0.88	0.92	0.96
Dry asphalt	0.89	0.91	0.94
Dry concrete	0.86	0.91	0.95
Smooth water	0.47	0.59	0.66
Rough or hard-packed dirt	1.00	1.00	1.00



Effect of 1 km fog (visibility 50 m) on blackbody radiation.

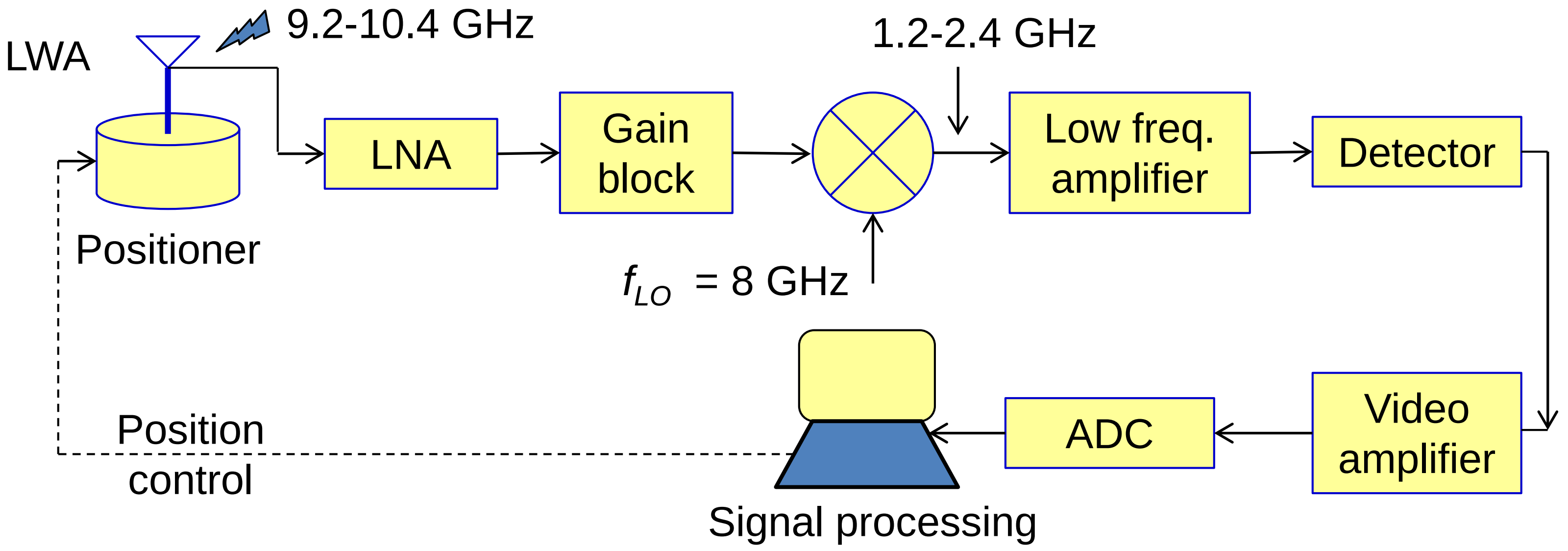
Wireless system



Block diagram of a millimeter wave super heterodyne receiver.

Wireless system

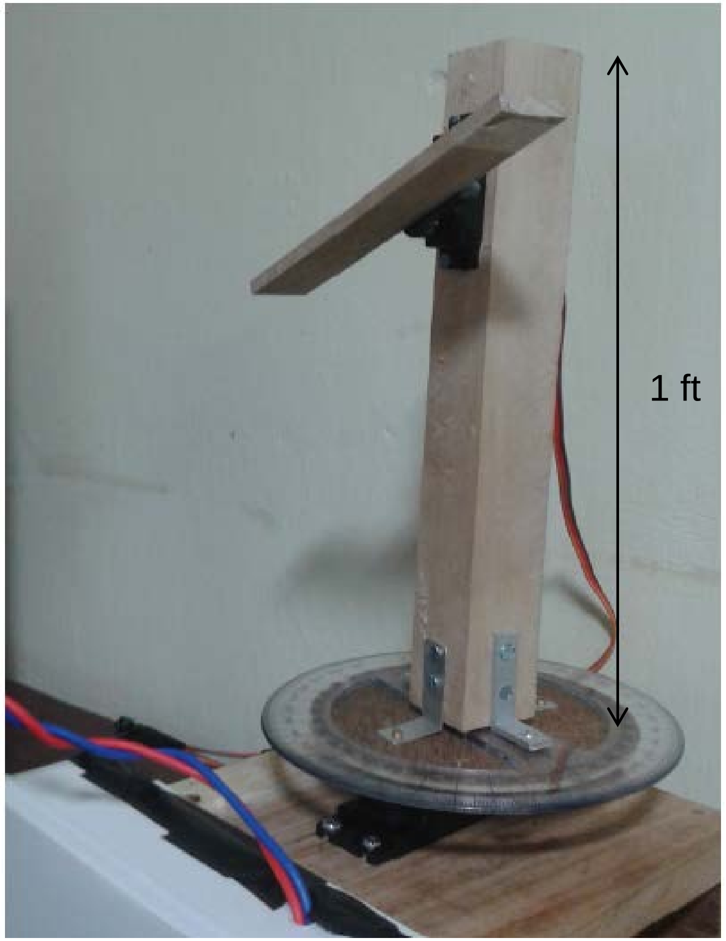
X-band imaging system @ ECE, IIT-KGP



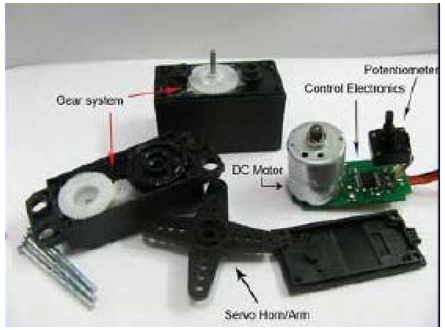
Schematic diagram of the developed imaging system.

- Technology demonstration using the available in-house instruments.
- Frequency band is arbitrarily chosen (higher frequency for better resolution, Ku-band components are expensive).
- Security application (human body).

Imaging system

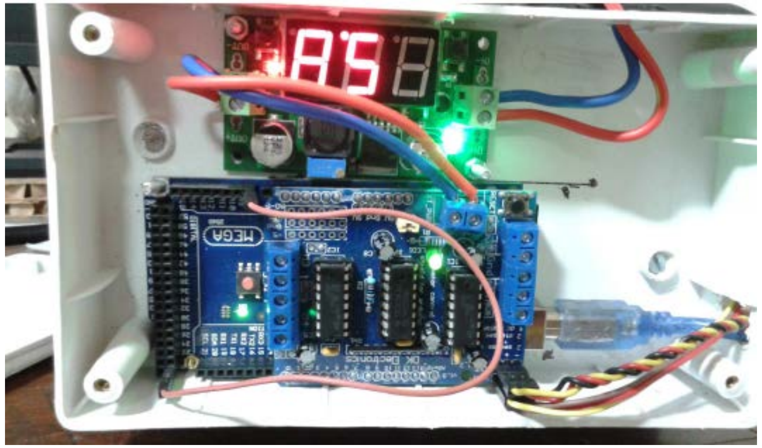


Two axes antenna positioner.

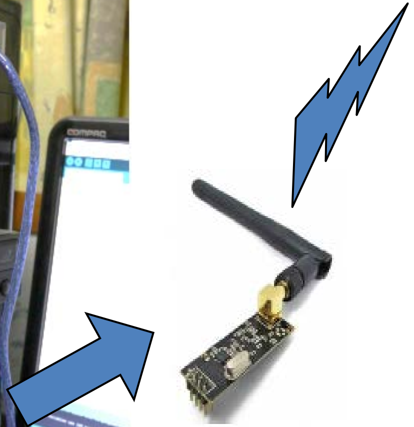


Servo motor.

Power supply.



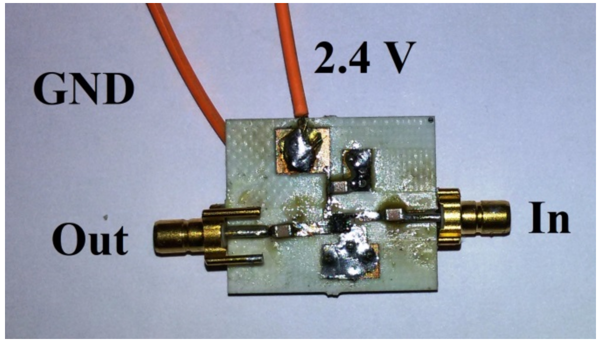
Motor control unit.



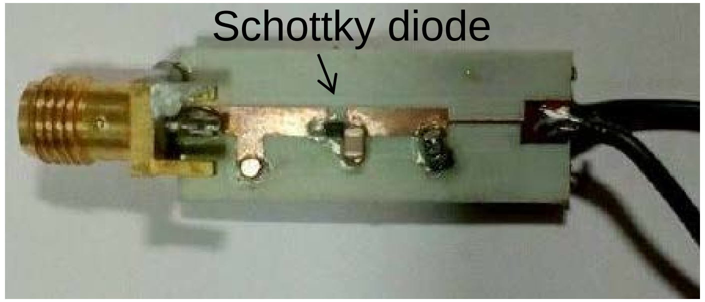
NRF24L01 Transceiver Module

Imaging system

Photograph of the LNA: VMMK-2203-BLKG.
Gain: 15 dB, NF: 2.3 dB

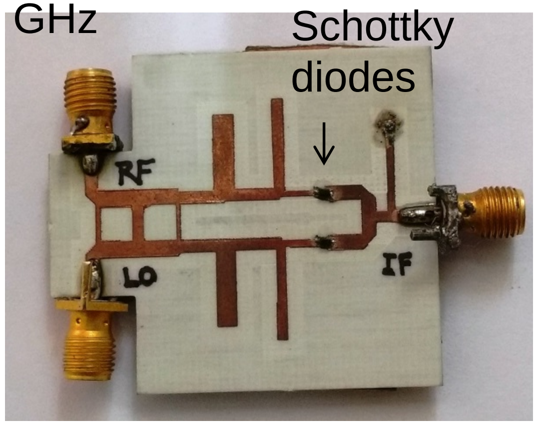


Low frequency amplifier, measured gain 22-20 dB (1-3 GHz)



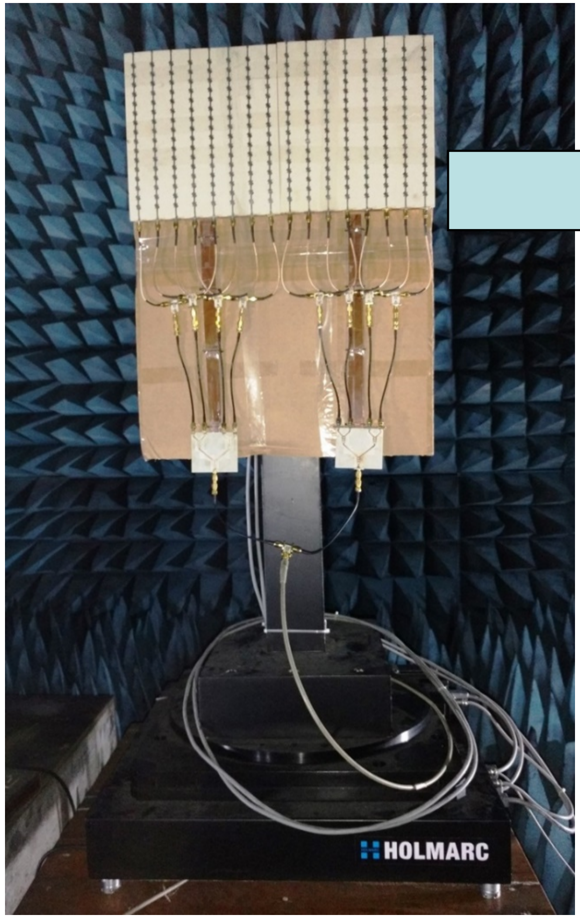
Power detector using Schottky diode

RF: 9.2-10.4 GHz

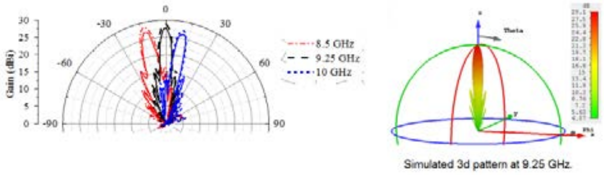


Single-balanced wideband mixer using BLC and SMS 7630-079LF diodes

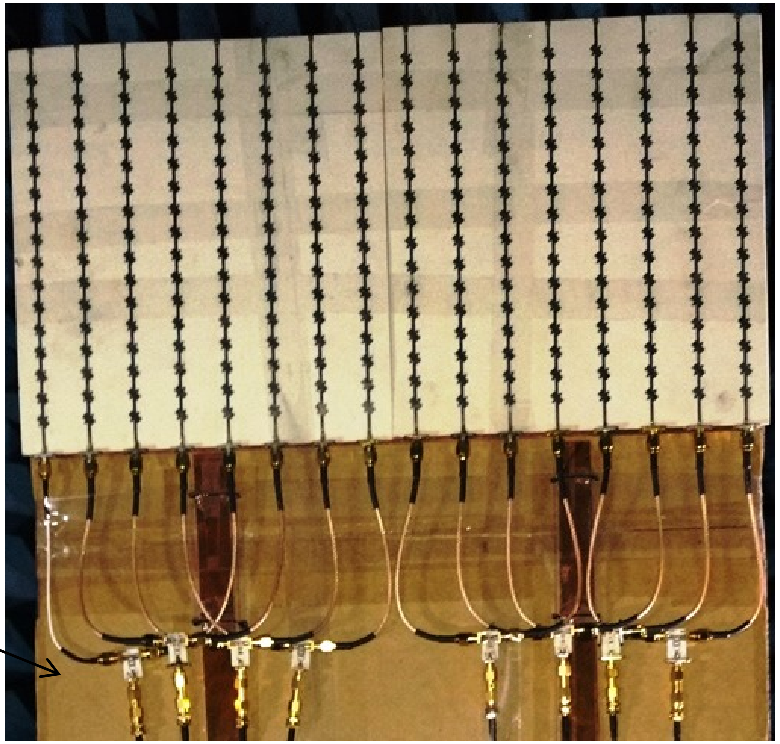
Imaging system



16 element array installed on the antenna positioner (triangular distribution).

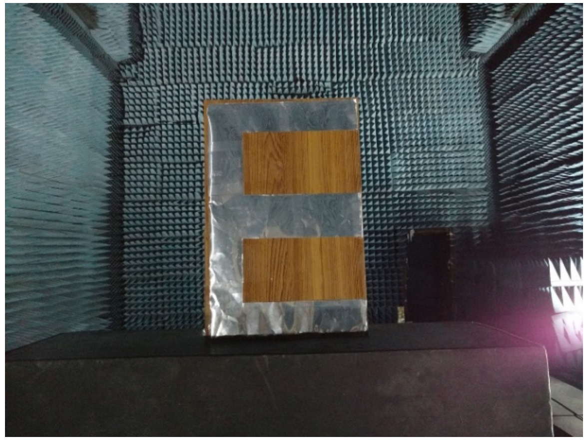


Wilkinson power dividers



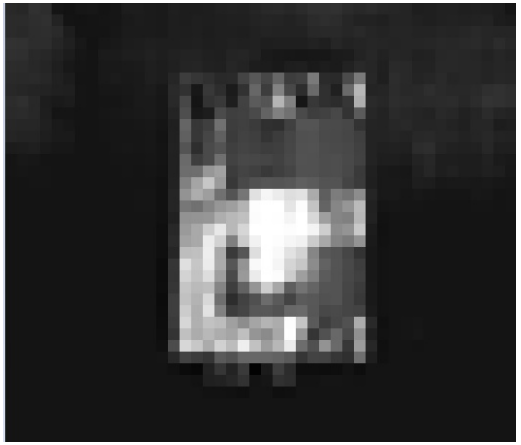
Frequency: 8-12 GHz (-22^{o} to41^{o})
Measured peak-gain: 28 dBi
3dB beam widths: 3.4^{o}/7.2^{o}

Imaging system



Wooden panel
dimension: 18’’×30’’

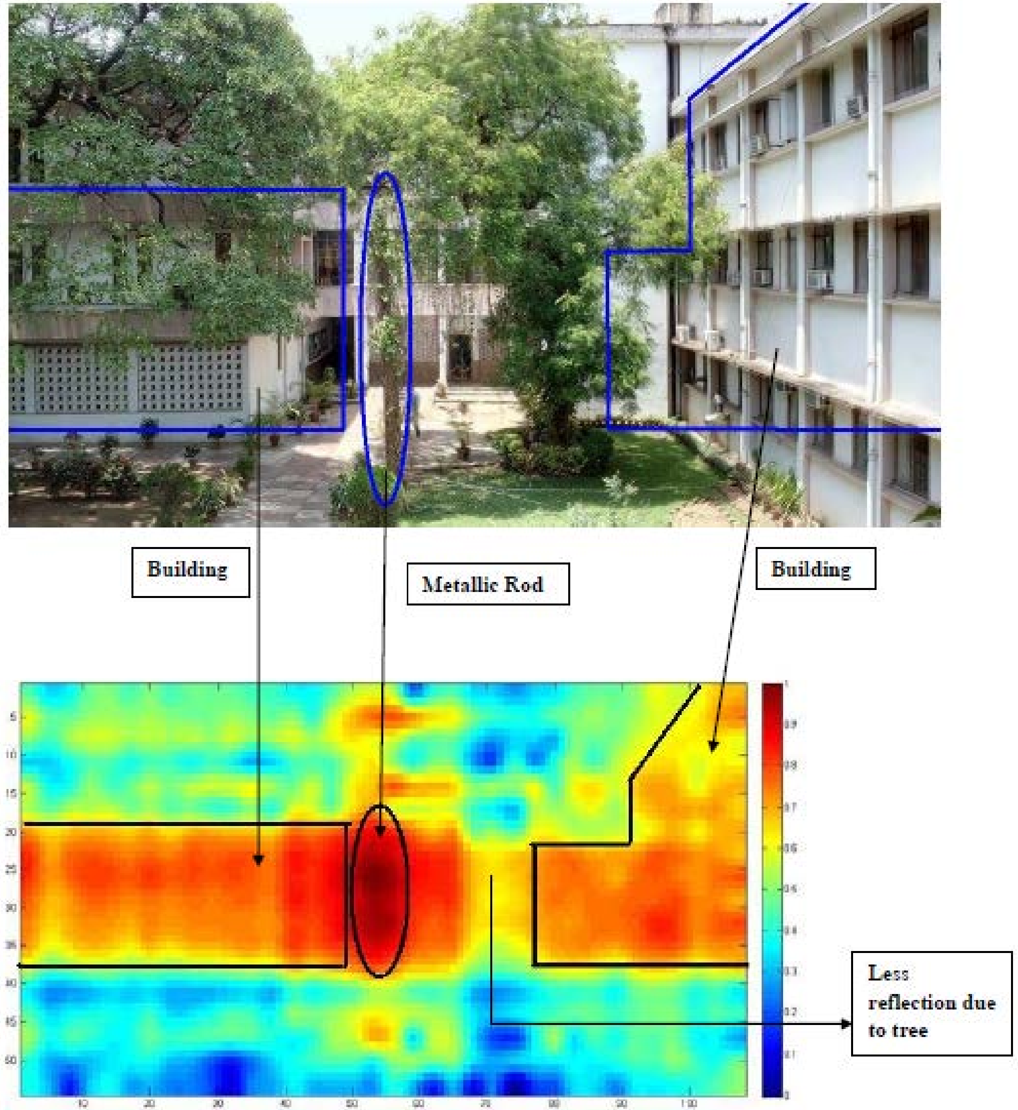
The panel covered with
paper.



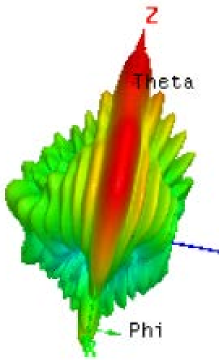
Reconstructed image after
some signal processing.

- Frequency sweep of 9.3-10.4 GHz corresponds to vertical scan -5.9^o to10.1^o.
- The target is placed 4 meter away from the receiver.
- A horn illuminates the target (radiated power 13 dBm).
- For this example, quality of the image is limited by receiver dynamic range.

Imaging system



Top view Bottom view



Frequency: 9.3-10.4 GHz (-5.9º to10.1º)
3dB beam width: 6.6-7.2º

Lab classes

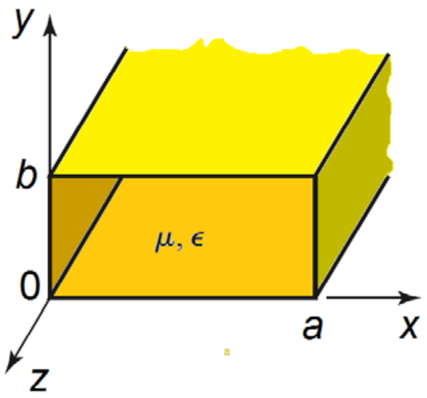
- **Simulation**

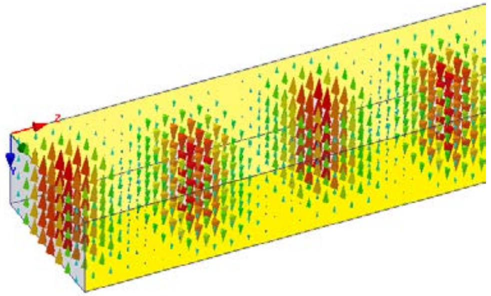
1. Studies on waveguide
2. Studies on microstrip line

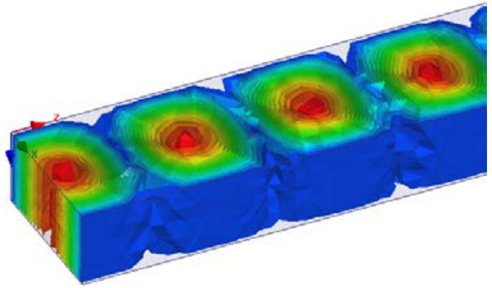
- **Experiment**

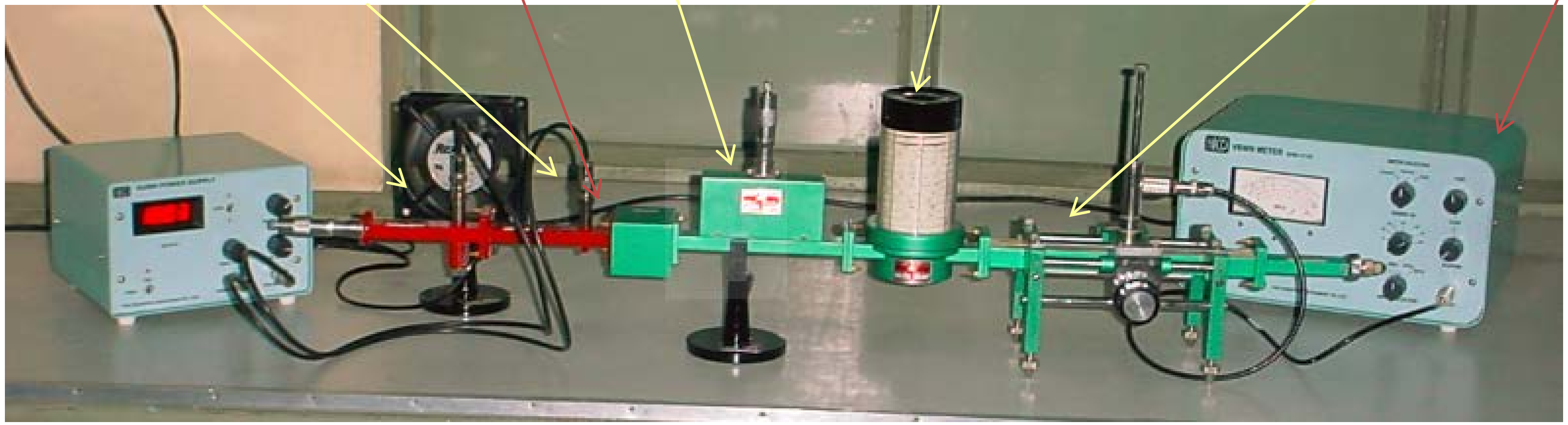
1. Waveguide bench
2. Characteristics of a Gunn diode
3. Radiation characteristics of an antenna

Waveguide bench



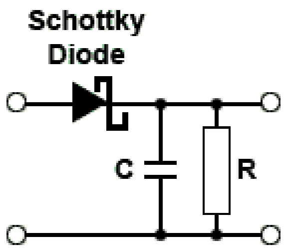

Vector Electric field distribution.


Scalar Electric field distribution.



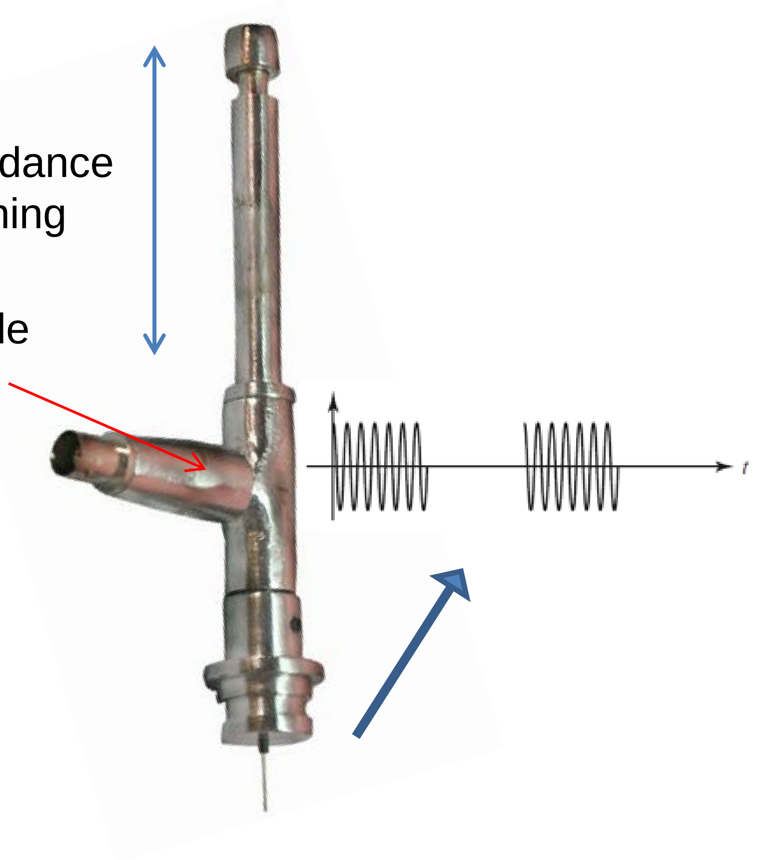
Waveguide bench

Waveguide bench

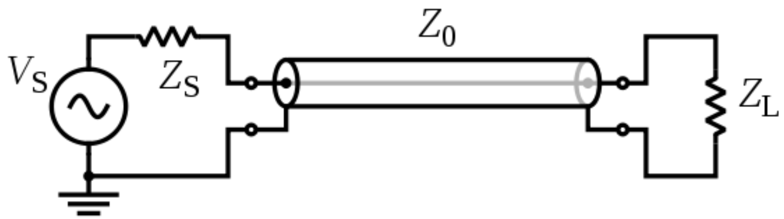


Schottky detector
(crystal detector uses
cat-whisker principle)

Slotted waveguide with
probe carriage



Probe with tuning arrangement



Co-axial cable, a transmission line.

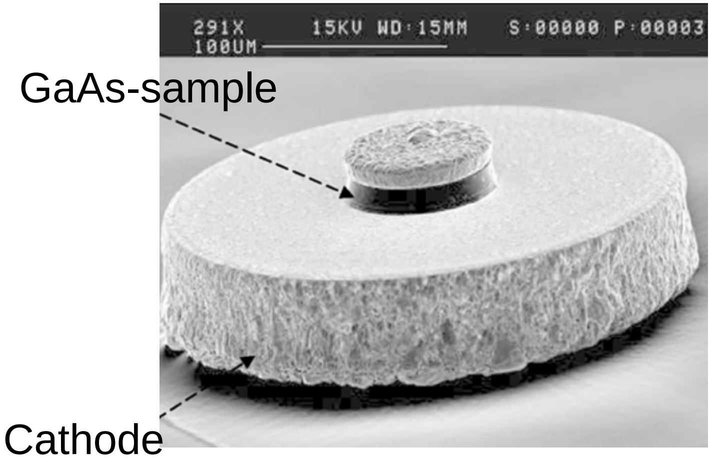
- Input impedance,

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)},$$

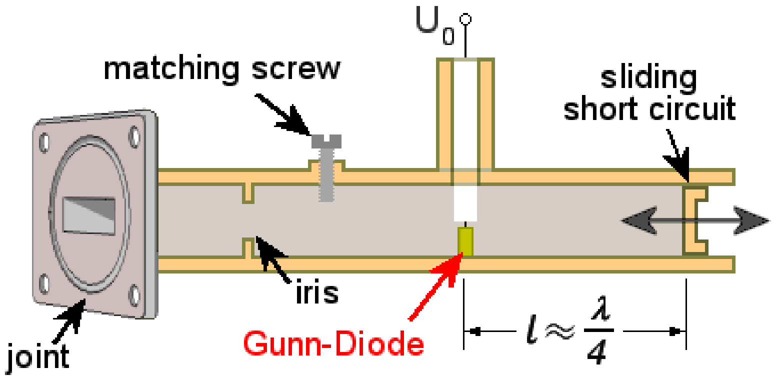
$$\text{where} \quad \beta l = \frac{2\pi}{\lambda_g} l = \theta.$$

Gunn diode

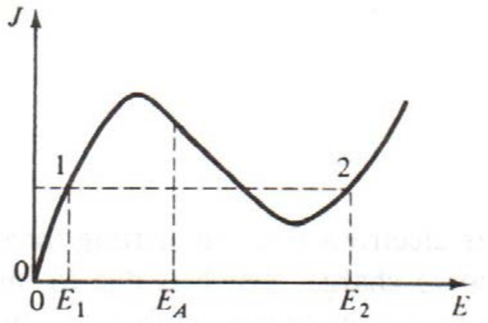
Gunn diode



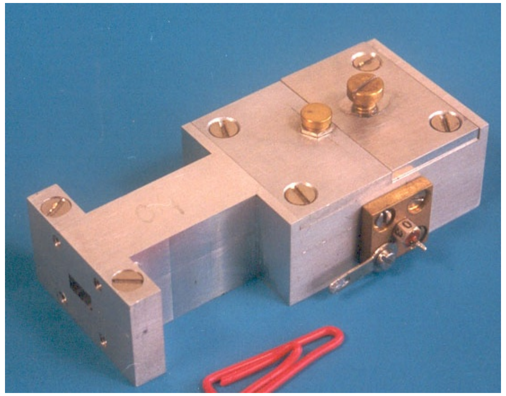
Electron microscope picture of a Gunn diode.



Internal connection of the source.



Current vs. Electric field.



Gunn diode as a mm-wave source.

J C Maxwell

J C Maxwell, *A Treatise on Electricity and Magnetism,* 1873.

- Describe electric and magnetic fields engaged in an eternal cycle of creation, destruction, and rebirth.

$$\nabla . E = \rho_v \qquad Gauss'\ Law$$

$$\nabla . B = 0 \qquad Gauss'\ Law\ \ for\ magnetism$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t} \qquad Faraday's\ law$$

$$\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t} \qquad Ampere's\ law$$

James Clerk Maxwell (1831–1879).

- Originally Maxwell represented the relationships in terms of *quaternions.* Oliver Heaviside and Josiah Willard Gibbs later developed in terms of vector calculus.
- That year, Hermann von Helmholtz sponsored a prize for the first experimental confirmation of Maxwell's predictions.

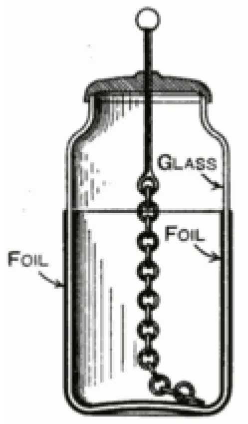
The first wireless system

- Maxwell was indeed correct (1886-1888)..!
- Oliver Lodge (University College in Liverpool), published his own confirmation one month after Hertz.

  ✓ Source..?
  ✓ Receiver..?
  ✓ Frequency..?

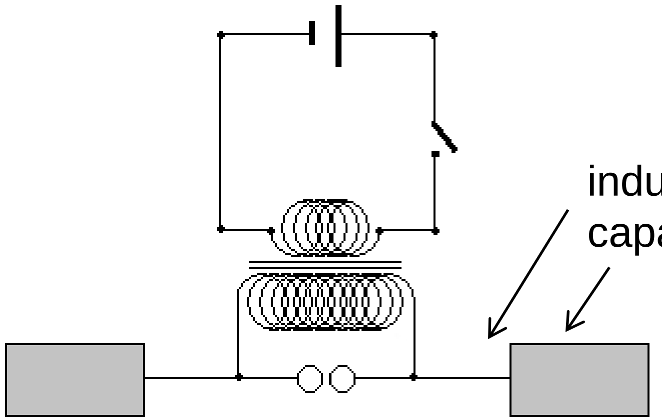
- George Francis FitzGerald, suggested in 1883 that one might use the known oscillatory spark discharge of Leyden jars (capacitors) to generate electromagnetic waves.
- Transmitter – heavily modified Leyden jar.
- A ring antenna with an integral spark gap with a micrometer screw for fine adjustment.
- Modest lab size of 12 m × 8 m → a few hundred MHz.

Heinrich Rudolf Hertz
(1857-1894)

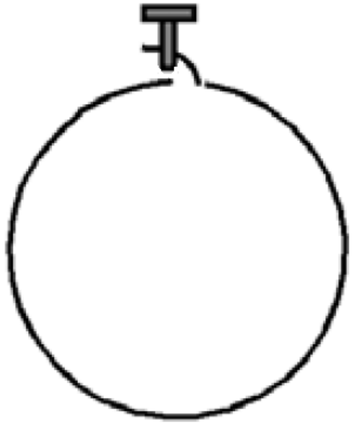


Leyden jar

The first wireless system



Spark-gap transmitter
(first dipole antenna).



Loop receiver with adjustable screw
(first loop antenna).

- Hertz's death led to the publication of a memorial tribute written by Augusto Righi (a professor in the University of Bologna) that, in turn, inspired one of his students Guglielmo Marconi to dedicate himself to developing commercial applications of wireless (successful transatlantic wireless communications on 12 December 1901).
- Frustrated by the inherent limitations of a spark-gap detector, adapted a peculiar device developed by Edouard Branly in 1890 - a *coherer as dubbed* by Lodge.

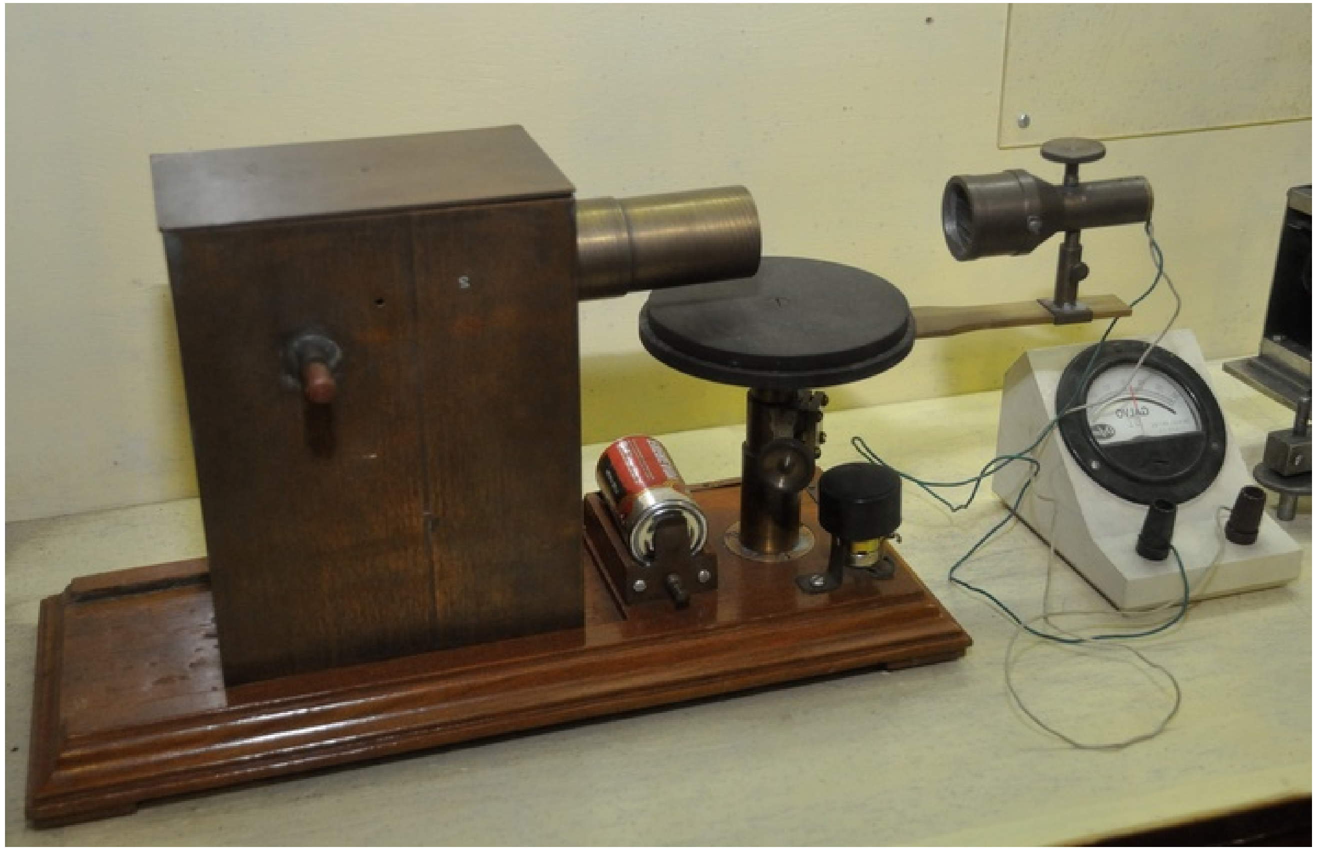
Thomas H. Lee, *Planar Microwave Engineering: A Practical Guide to Theory, Measurement, and Circuits*, Cambridge University Press.

The first wireless system

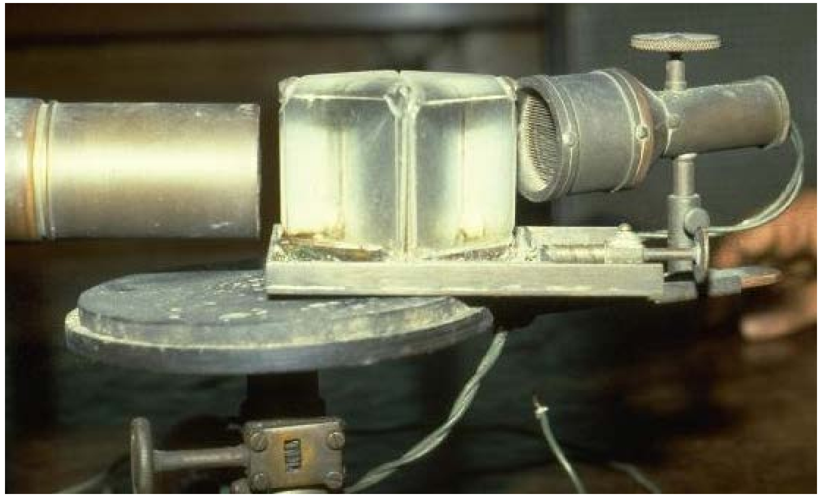
- Receiving problem was solved by an Indian scientist by inventing detector (patent filed in 1901).
- Radiation is focused on the point contact, and the resistance change that accompanies the consequent heating registers as a change in current flowing through an external circuit – bolometer.
- The first millimetre wave communication system in the world.
- In 1895, transmission and reception of 60 GHz signal (?), over 23 meters distance.
- Pioneering work: spark transmitter, coherer, dielectric lens, polarizer, horn antenna and cylindrical diffraction grating.
- 2.5 cm to 5 mm wavelength.
- Waveguide transmission would be forgotten for four decades, but Rayleigh (Bose’s teacher) had most of it worked out (including the concept of a cutoff frequency) in 1897.

Jagadish Chandra Bose (1858–1937).

First millimeter-wave system



World first millimetre-wave system.



Original set up.

Point contact detector.

Spark gap as a source.

Further improvement

Detector:

- Henry Harrison Chase Dunwoody filed the first patent application for a rectifying detector using carborundum (SiC) on 23 March 1906.
- Greenleaf Whittier Pickard (an MIT graduate) filed patent for a silicon detector on 30 August 1906.
- Galena detectors became quite popular because they are inexpensive and need no bias. Problems: difficult to maintain catwhisker wire contact and one has to find a sensitive spot in crystal surface.
- Carborundum detectors need a bias of a couple of volts, but mechanically stable (widely used in ships).

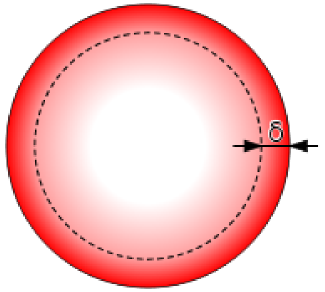
Source:

- Spark gap: wide spectrum → interference.
- William Duddell exploited the negative resistance of arc lamp to produce audio oscillations.
- Higher frequency: Valdemar Poulsen used the negative resistance associated with a glowing DC arc (hydrogen gas in a strong magnetic field) to keep an *LC* circuit in constant oscillation.
- Leonard Fuller provided the theoretical advances that allowed arc power to go up to 1-MW.

Skin depth

- The skin effect is due to opposing eddy currents induced by the changing magnetic field resulting from the alternating current.

  Current density inside metal: $J = J_S\, e^{-d/\delta}$



AC current distribution.

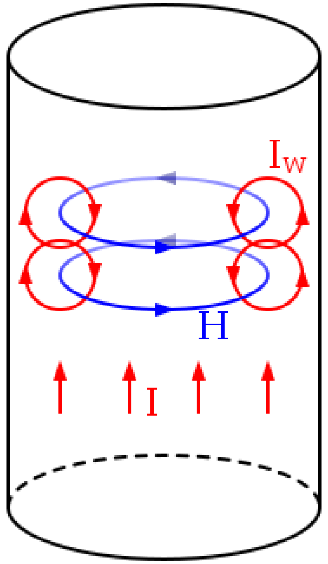
Skin depth: the depth below the surface of the conductor at which the current density has fallen to 1/e (≈ 0.37) of J_{S}.

For lower ρ, $\delta = \sqrt{\dfrac{2\rho}{\omega\mu_r\mu_0}}$

$\delta_{cu}|_{50\text{ Hz}} = 8.5$ mm, $\delta_{cu}|_{10\text{ kHz}} = 660$ µm, $\delta_{cu}|_{10\text{ GHz}} = 0.66$ µm, $\delta_{cu}|_{100\text{ GHz}} = 0.21$ µm.

- General expression:

$$\delta = \left(\frac{1}{\omega}\right)\left\{\left(\frac{\mu\epsilon}{2}\right)\left[\left(1+\left(\frac{1}{\rho\omega\epsilon}\right)^2\right)^{1/2} - 1\right]\right\}^{-1/2}$$



Opposite eddy currents.

Phase and group velocities

- The phase velocity of a wave is the rate at which the phase of the wave propagates.

$$v_p = \frac{\omega}{\beta}, \qquad \text{where } \beta = \sqrt{k^2 - k_c^2} = 2\pi/\lambda_g .$$

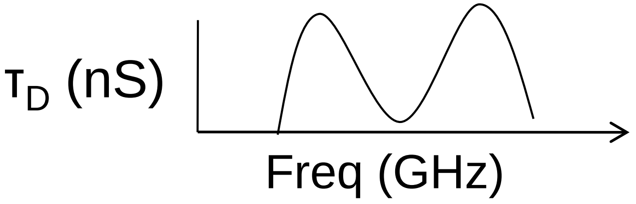
- The group velocity of a wave is the velocity with which the overall shape of the waves' amplitudes (envelope of the wave) propagates.

Group velocity can be thought of as the signal velocity (v_{en}) of the waveform (in non-absorptive medium)

$$v_g = d\omega/d\beta .$$

Group delay:

Group delay is a measure of the time delay of the amplitude envelopes of the various sinusoidal components of a signal through a device under test.



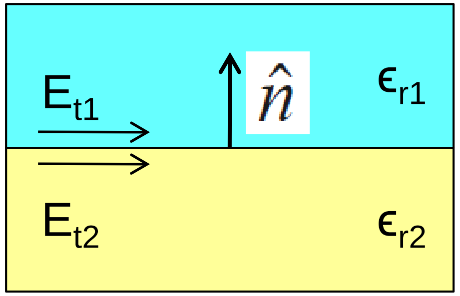
$$\tau_d = -\frac{d\varphi}{d\omega} = -\frac{d\angle S_{21}}{d\omega} .$$

Boundary conditions

Dielectric interface:

No charge or surface current density →

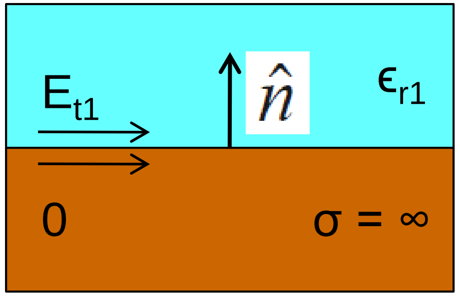
$$\hat{n}\cdot\bar{D}_1 = \hat{n}\cdot\bar{D}_2, \quad \hat{n}\times\bar{E}_1 = \hat{n}\times\bar{E}_2,$$
$$\hat{n}\cdot\bar{B}_1 = \hat{n}\cdot\bar{B}_2, \quad \hat{n}\times\bar{H}_1 = \hat{n}\times\bar{H}_2.$$



PEC interface (electric wall):

Non zero charge and surface current density →

$$\hat{n}\cdot\bar{D} = \rho_s, \qquad \hat{n}\times\bar{E} = 0,$$
$$\hat{n}\cdot\bar{B} = 0, \qquad \hat{n}\times\bar{H} = \bar{J}_s$$

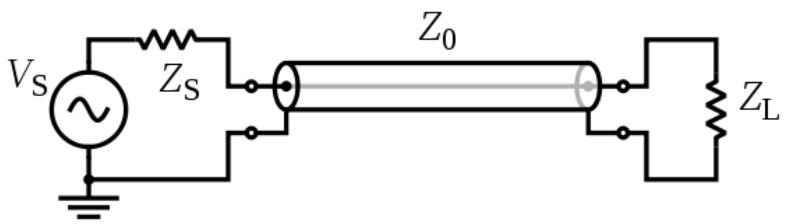


Magnetic wall interface:

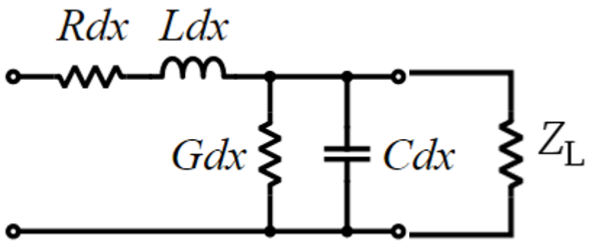
Tangential magnetic field is zero.

$$\hat{n}\cdot\bar{D} = 0, \qquad \hat{n}\times\bar{E} = -\bar{M}_s,$$
$$\hat{n}\cdot\bar{B} = 0, \qquad \hat{n}\times\bar{H} = 0$$

Terminated transmission line



Co-axial cable, a transmission line.



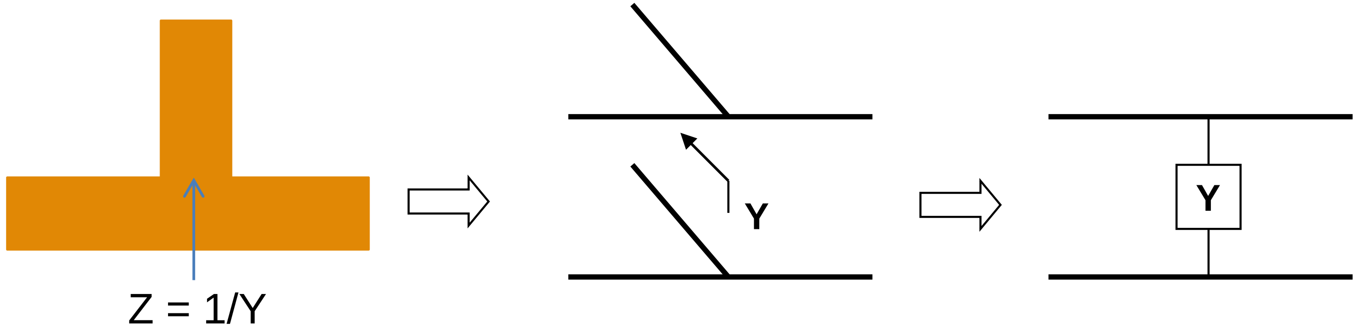
Elementary section of a tx line with termination.

- Input impedance,

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)},$$

$$\text{where} \quad \beta l = \frac{2\pi}{\lambda_g} l = \theta.$$

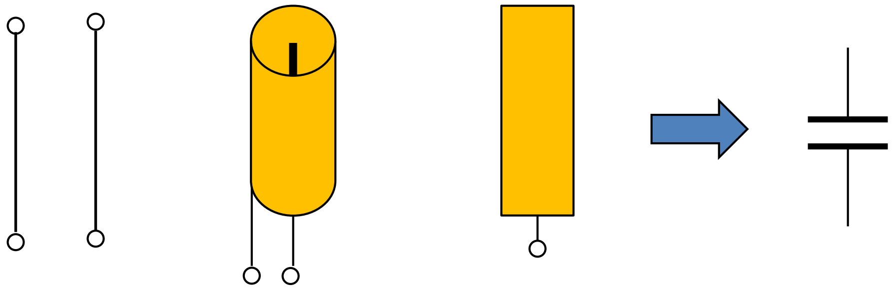
- For lossless line → $Z_0 = \sqrt{L/C}, \quad v_p = 1\big/\sqrt{LC}$.



A stub as shunt admittance.

Open- and short-stubs

Open-stub



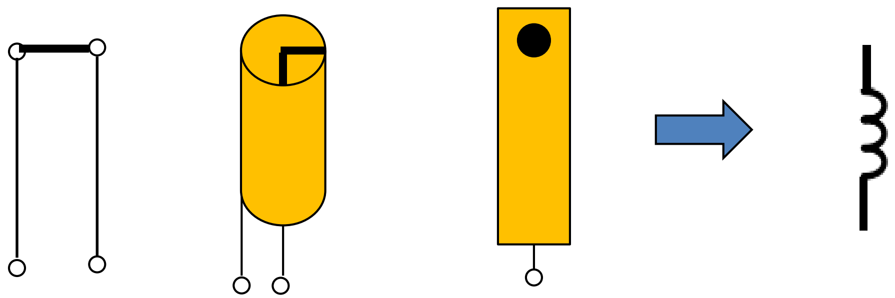
Shunt open-stub

$$Z_{in} = -jZ_0 \cot(\beta l), \quad Z_L \to \infty.$$

For $l < \lambda_g/4 \; (\theta = 90^0)$: Z_{in} is capacitive.

Open-stub, for $l < \lambda_g/4$: $C = \dfrac{1}{\omega Z_0 \cot\theta}$.

Short-stub



Shunt shorted-stub

$$Z_{in} = jZ_0 \tan(\beta l), \quad Z_L \to 0.$$

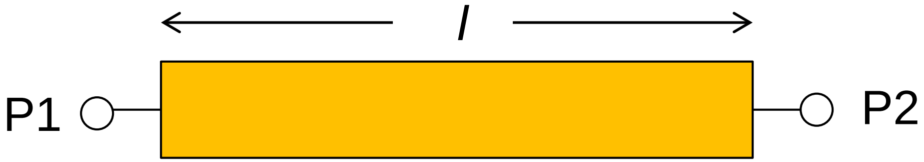
For $l < \lambda_g/4 \; (\theta = 90^0)$: Z_{in} is inductive.

Short-stub, for $l < \lambda_g/4$: $L = \dfrac{Z_0 \tan\theta}{\omega}$.

- A section of open or short transmission line can be used for impedance matching (stub matching)

Use of a section of transmission line

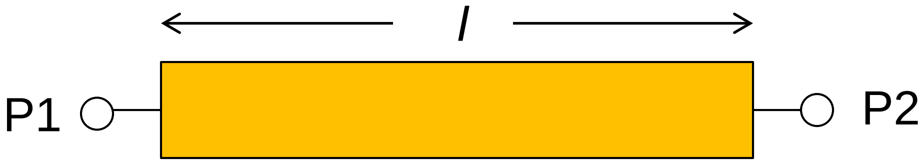
- **Phase shifter (narrowband):**



Top view of a section of microstrip line.

$$\theta = \beta l, \quad \lambda_g \approx \frac{\lambda_0}{\sqrt{\varepsilon_r}}.$$

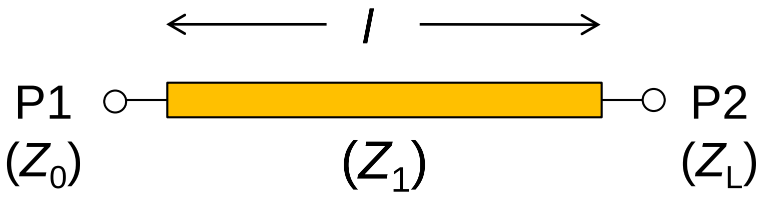
- **Delay line (wideband):**



Top view of a section of microstrip line.

$$\tau_D = l \Big/ \frac{\partial \omega}{\partial \beta} = -\frac{\partial \theta}{\partial \omega}.$$

- **Quarter wavelength transformer:**

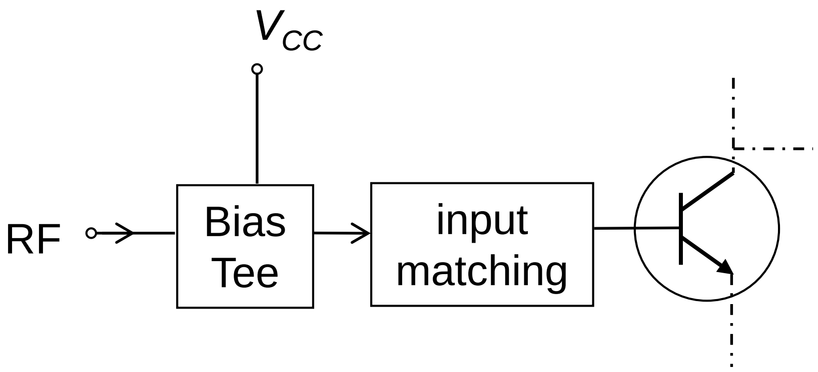


A microstrip line quarter wave transformer.

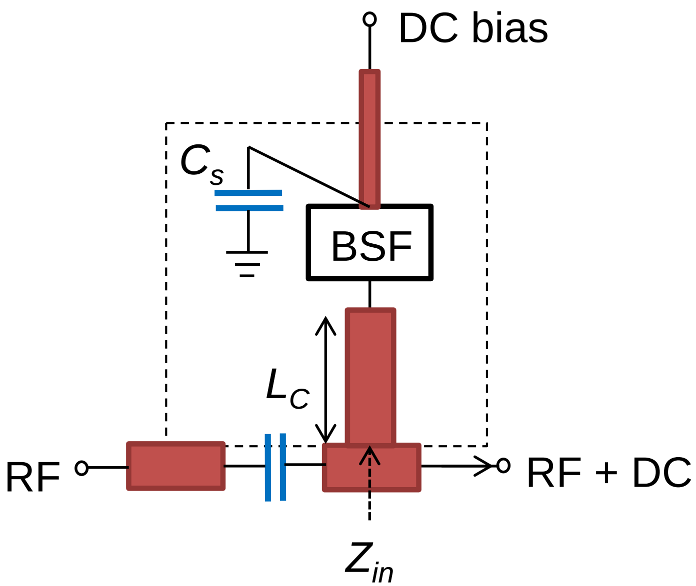
$$Z_{in} = Z_1 \frac{Z_L + jZ_1 \tan(\beta l)}{Z_1 + jZ_L \tan(\beta l)} = Z_0$$

$$\Rightarrow Z_1 = \sqrt{Z_0 Z_L}$$

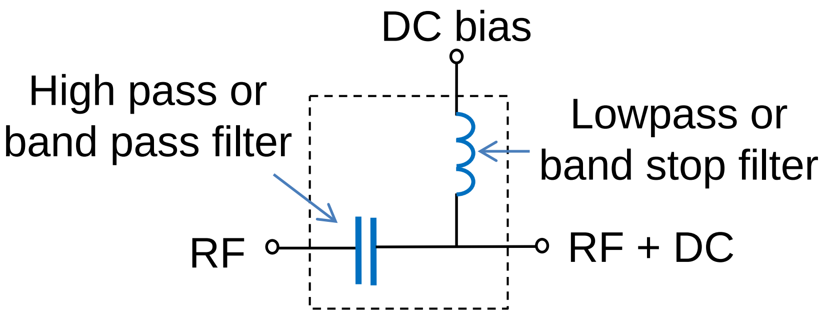
Bias tee



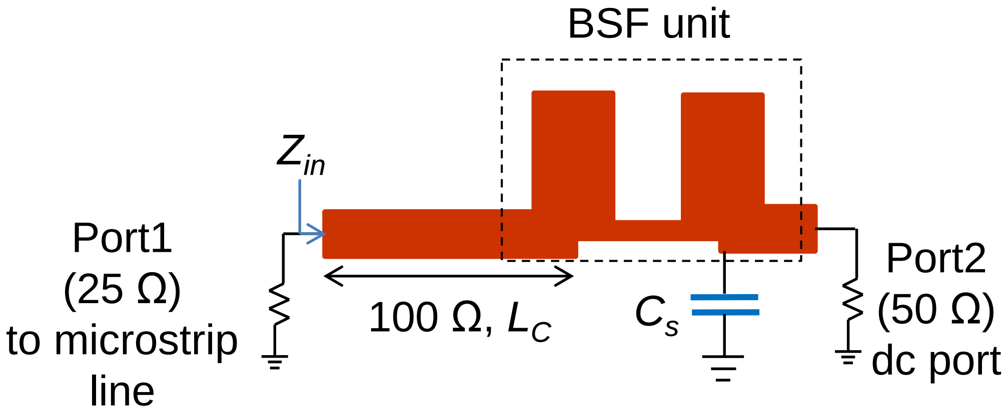
Use of a bias tee in an amplifier.



A bias tee at microwave frequencies.



A bias tee in LC form.



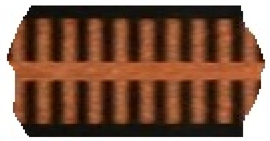
Slow wave and fast wave

Slow wave : V_p <c (speed of light in free space)

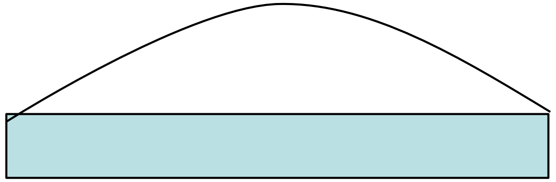
Non-radiating, radiates only at discontinuities.

Examples: helixes, dielectric slabs or rods, corrugated conductors.

At fixed f, $\lambda_g \downarrow \rightarrow V_p \downarrow \rightarrow \beta \uparrow$



Top view



L (mm)

Electrical length $\theta = \beta L$.

Fast wave : V_p >c

Radiates continuously along its length. Examples: leaky wave antennas (β the beam angle α controls the beamwidth).

At fixed f, $\lambda_g \uparrow \rightarrow V_p \uparrow \rightarrow \beta \downarrow$

Metamaterial : negative V_p

Metamaterials are not found in nature. A periodic structure with periodicity much smaller than wavelength.

- Both permittivity (ε) and permeability (μ) are negative.
- Exhibit a negative index of refraction for particular wavelengths (negative-index metamaterials).
- The time-averaged Poynting vector is antiparallel to phase velocity.
- Cherenkov radiation points the other way.

Reflection coefficient

- Reflection coeffcient $= \dfrac{\textit{reflected voltage or current}}{\textit{incident voltage or current}}$

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{-I_{ref}}{I_{inc}}.$$

- At the load end,

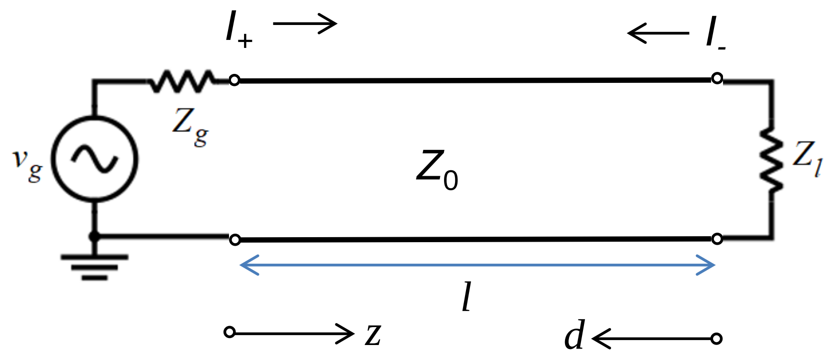
$$\Gamma_l = \frac{V_{-}e^{\gamma l}}{V_{+}e^{-\gamma l}} = \frac{Z_l - Z_0}{Z_l + Z_0} \quad \rightarrow \text{a complex number} \quad \rightarrow |\Gamma_l| e^{j\theta_l}.$$

- Generalized reflection coefficient, $\mathbf{\Gamma} \equiv \dfrac{\mathbf{V}_{-}e^{\gamma z}}{\mathbf{V}_{+}e^{-\gamma z}}$

- Reflection coefficient at a distance *d* from the load end,

$$\mathbf{\Gamma}_d = \frac{\mathbf{V}_{-}e^{\gamma(\ell - d)}}{\mathbf{V}_{+}e^{-\gamma(\ell - d)}} = \frac{\mathbf{V}_{-}e^{\gamma\ell}}{\mathbf{V}_{+}e^{-\gamma\ell}}e^{-2\gamma d} = \mathbf{\Gamma}_\ell e^{-2\gamma d} \qquad [z = l - d]$$

$$= \mathbf{\Gamma}_\ell e^{-2\alpha d} e^{-j2\beta d} = |\mathbf{\Gamma}_\ell| e^{-2\alpha d} e^{j(\theta_\ell - 2\beta d)}$$



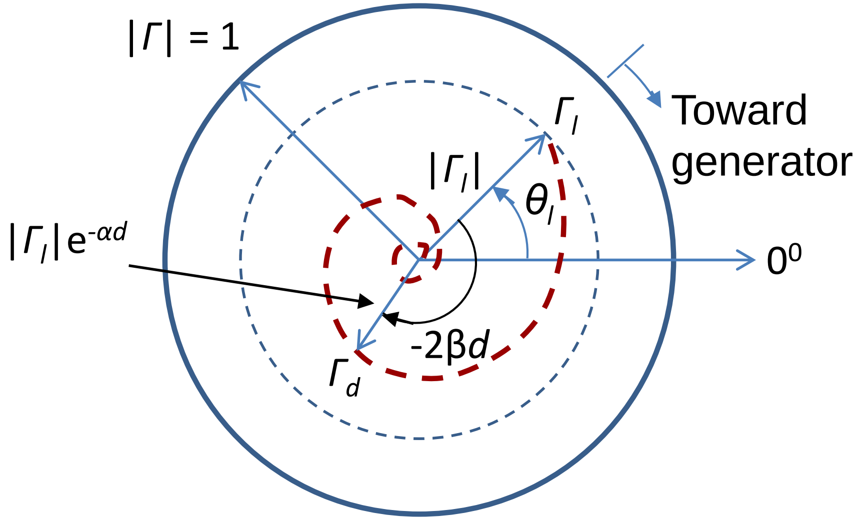
Transmission line terminated in a load.

$$\mathbf{V}_\ell = \mathbf{V}_{+}e^{-\gamma\ell} + \mathbf{V}_{-}e^{\gamma\ell}$$

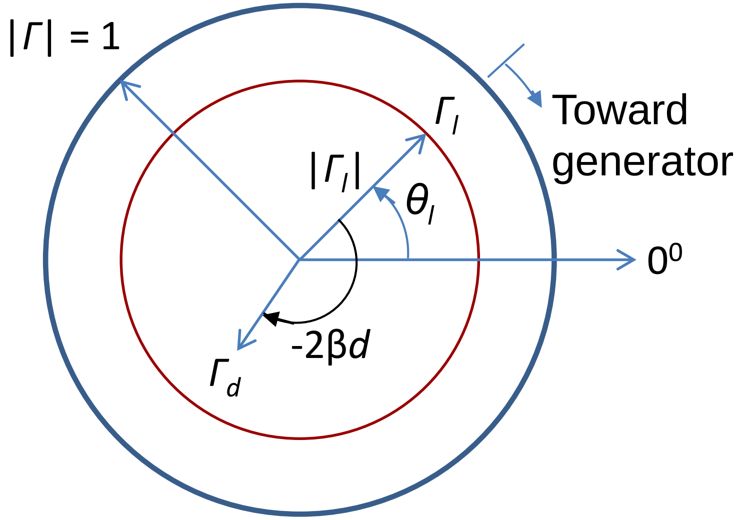
$$\mathbf{I}_\ell = \frac{1}{\mathbf{Z}_0}(\mathbf{V}_{+}e^{-\gamma\ell} - \mathbf{V}_{-}e^{\gamma\ell})$$

Return loss = $-20\log_{10}(|\Gamma|)$ dB (appositive quantity)

Reflection coefficient



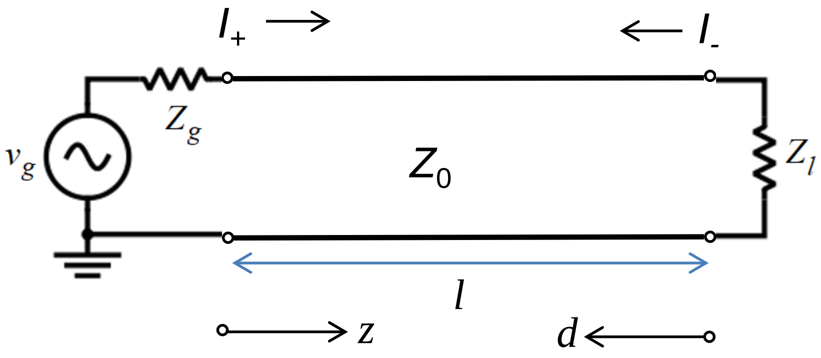
Reflection coefficient for lossy line.



Reflection coefficient for lossless line.

- Toward load, θ is positive.
- Reflection coefficient at a distance *d* from the load end,

$$\Gamma_d = \Gamma_\ell e^{-2\alpha d} e^{-j2\beta d} = |\Gamma_\ell| e^{-2\alpha d} e^{j(\theta_\ell - 2\beta d)}$$

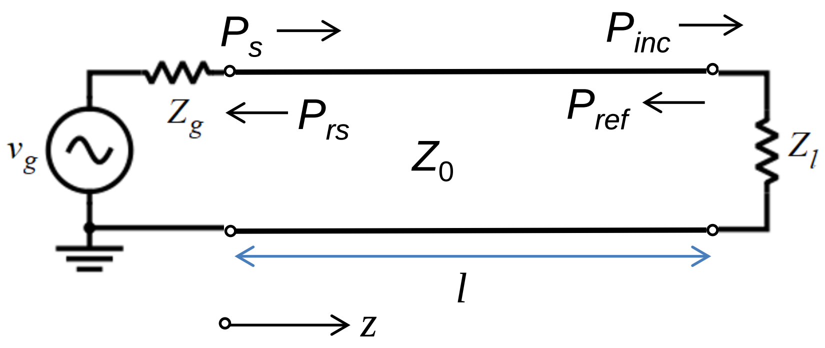


Transmission line terminated in a load.

Transmission coefficient

- Transmission coeffcient $= \dfrac{transmitted\ \ voltage\ \ or\ \ current}{incident\ \ voltage\ \ or\ \ current}$

$$T = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}}.$$



Transmission line terminated in a load.

- At the load end,

$$\mathbf{V}_+ e^{-\gamma\ell} + \mathbf{V}_- e^{\gamma\ell} = \mathbf{V}_{tr} e^{-\gamma\ell}$$

$$\frac{\mathbf{V}_+}{\mathbf{Z}_0} e^{-\gamma\ell} - \frac{\mathbf{V}_-}{\mathbf{Z}_0} e^{\gamma\ell} = \frac{\mathbf{V}_{tr}}{\mathbf{Z}_\ell} e^{-\gamma\ell} \qquad \Gamma_\ell = \frac{\mathbf{V}_- e^{\gamma\ell}}{\mathbf{V}_+ e^{-\gamma\ell}} = \frac{\mathbf{Z}_\ell - \mathbf{Z}_0}{\mathbf{Z}_\ell + \mathbf{Z}_0}$$

- Putting the value of reflection coefficient in the voltage equation,

$$\mathbf{T} = \frac{\mathbf{V}_{tr}}{\mathbf{V}_+} = \frac{2\mathbf{Z}_\ell}{\mathbf{Z}_\ell + \mathbf{Z}_0}$$

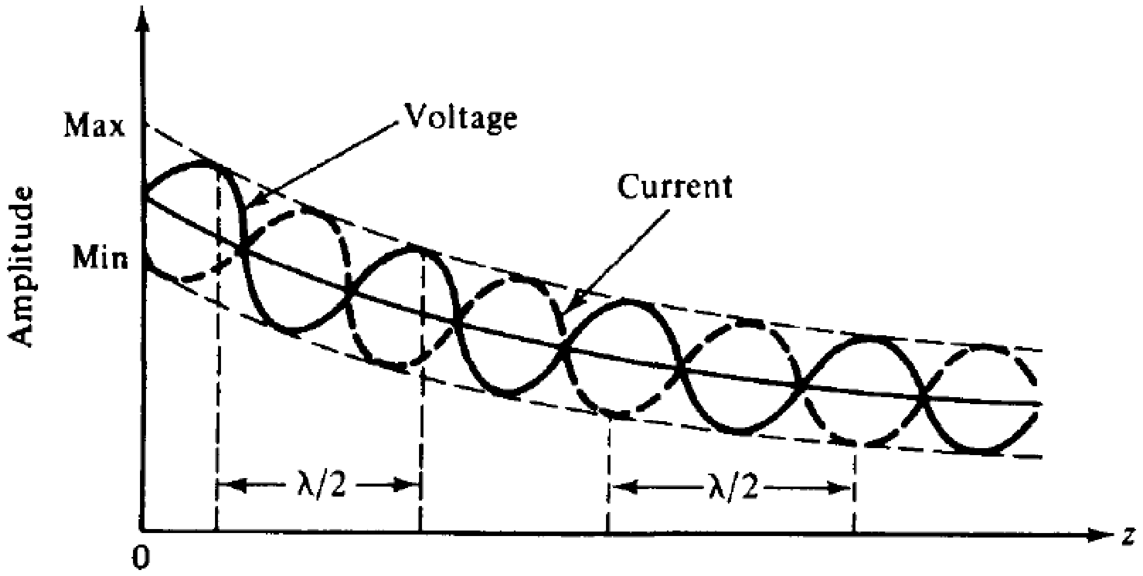
- Using the value of reflection coefficient,

$$\boxed{\mathbf{T}^2 = \frac{\mathbf{Z}_\ell}{\mathbf{Z}_0}(1 - \Gamma_\ell^2)}$$

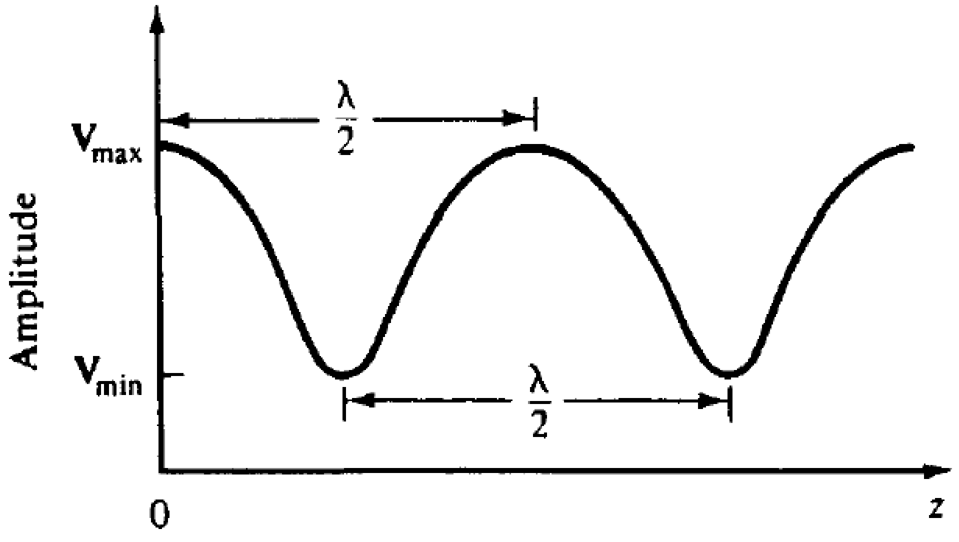
- Power carried to the load, $P_{tr} = \dfrac{(\mathbf{V}_+ e^{-\alpha\ell})^2}{2\mathbf{Z}_0} - \dfrac{(\mathbf{V}_- e^{\alpha\ell})^2}{2\mathbf{Z}_0} = \dfrac{(\mathbf{V}_{tr} e^{-\alpha\ell})^2}{2\mathbf{Z}_\ell}$

Insertion loss = -20log$_{10}$(|T|) dB (appositive quantity)

Standing wave



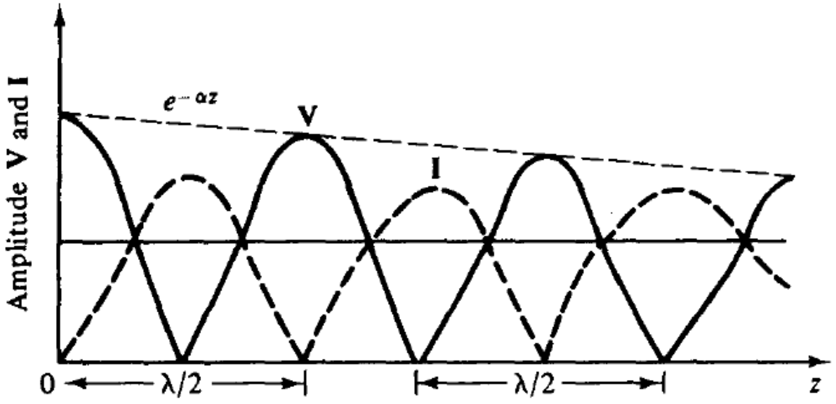
Standing wave pattern in a lossy line.



Voltage standing wave pattern in a lossless line.

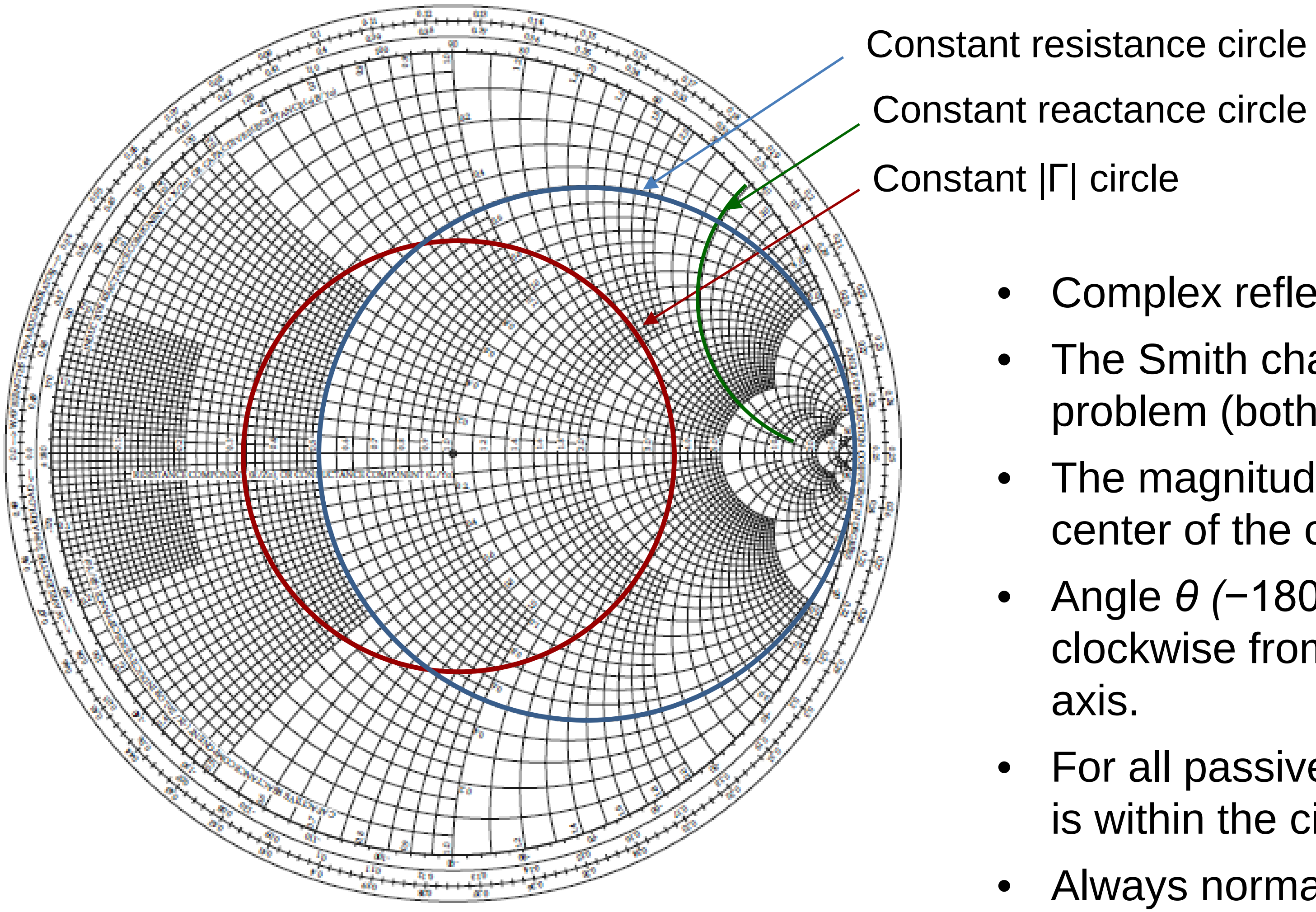
$$\text{Standing-wave ratio} \equiv \rho \equiv \frac{|\mathbf{V}_{max}|}{|\mathbf{V}_{min}|} = \frac{|\mathbf{I}_{max}|}{|\mathbf{I}_{min}|}$$

$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad |\Gamma| = \frac{\rho - 1}{\rho + 1}$$



Amplitudes of voltage and current standing waves.

Smith chart



The Smith chart

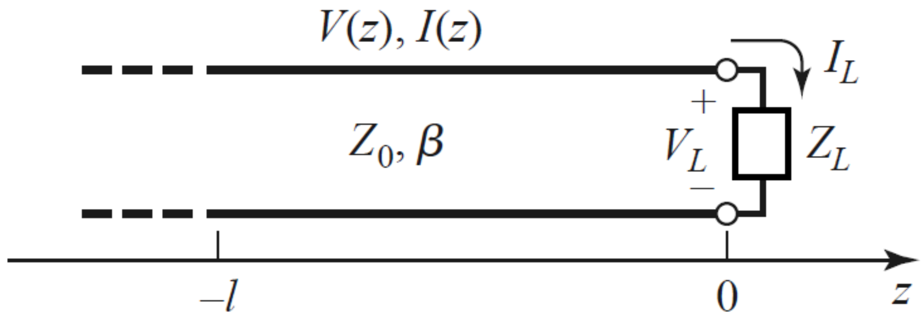
- Complex reflection coefficient: $\Gamma = |\Gamma| e^{j\theta}$
- The Smith chart is used to solve transmission line problem (both active and passive)
- The magnitude $|\Gamma|$ is plotted as a radius from the center of the chart.
- Angle θ ($-180^o \leq \theta \leq 180^o$) is measured counter-clockwise from the right-hand side of the horizontal axis.
- For all passive components, the reflection coefficient is within the circle ($|\Gamma| \leq 1$).
- Always normalized impedance is considered ($\boldsymbol{z} = Z/Z_0$).

Smith chart (lossless case)

- Reflection coefficient at the load end,

$$\Gamma = \frac{z_L - 1}{z_L + 1} = |\Gamma| e^{j\theta}$$

where $\boldsymbol{z_L} = Z_L/Z_0$, normalized load impedance.



Transmission line terminated in a load.

- Normalized impedance in term Γ at the load end,

$$z_L = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}.$$

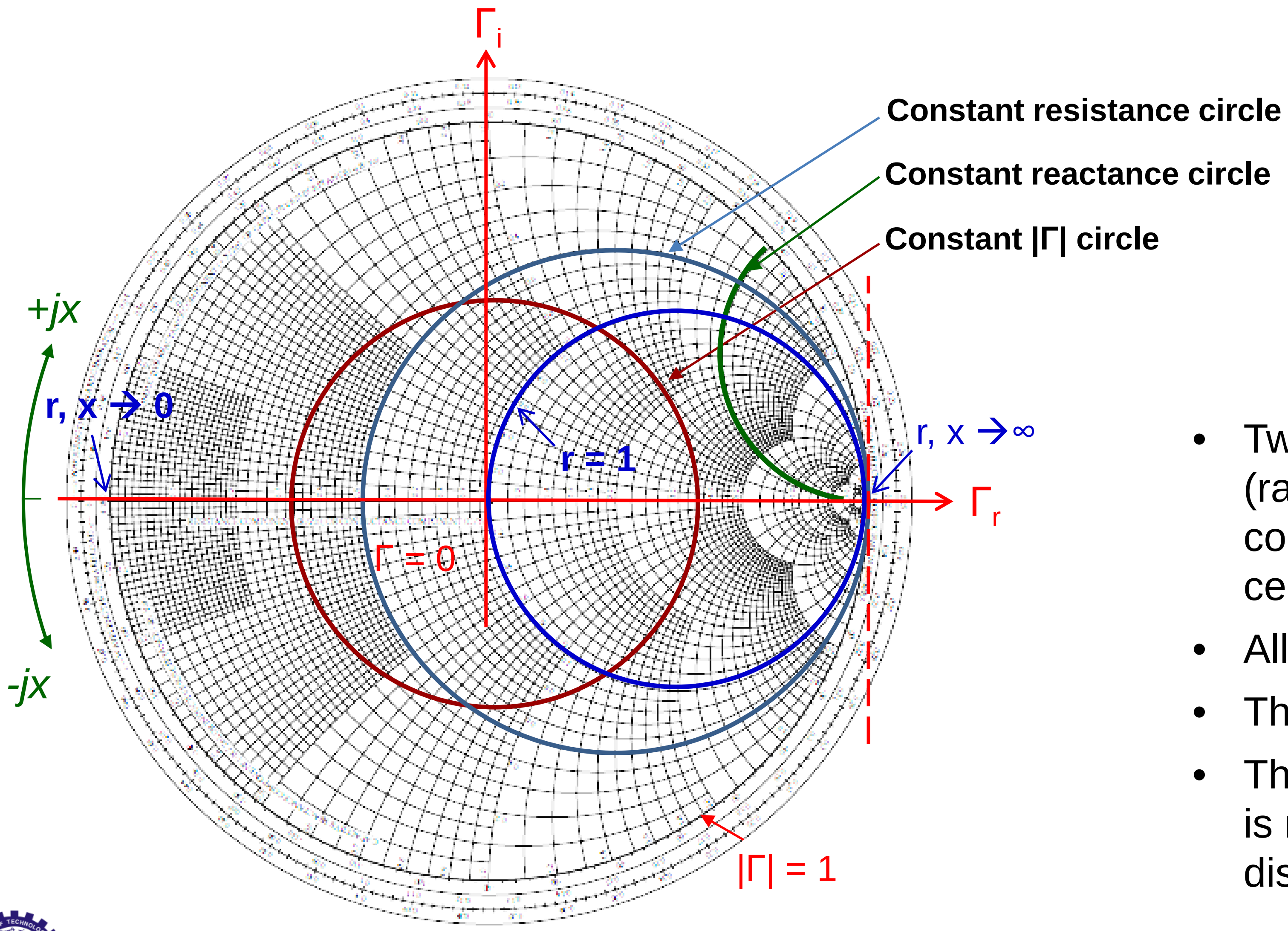
- The complex Γ and $\boldsymbol{z_L}$ are written as $\Gamma = \Gamma_r + j\Gamma_i$ and $z_L = r_L + j\, x_L$.

Therefore, $$r_L + j x_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}.$$

Separating the real and imaginary parts: $$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}, \qquad x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}.$$

Rearranging them: $$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2,$$ and $$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

Smith chart (impedance)

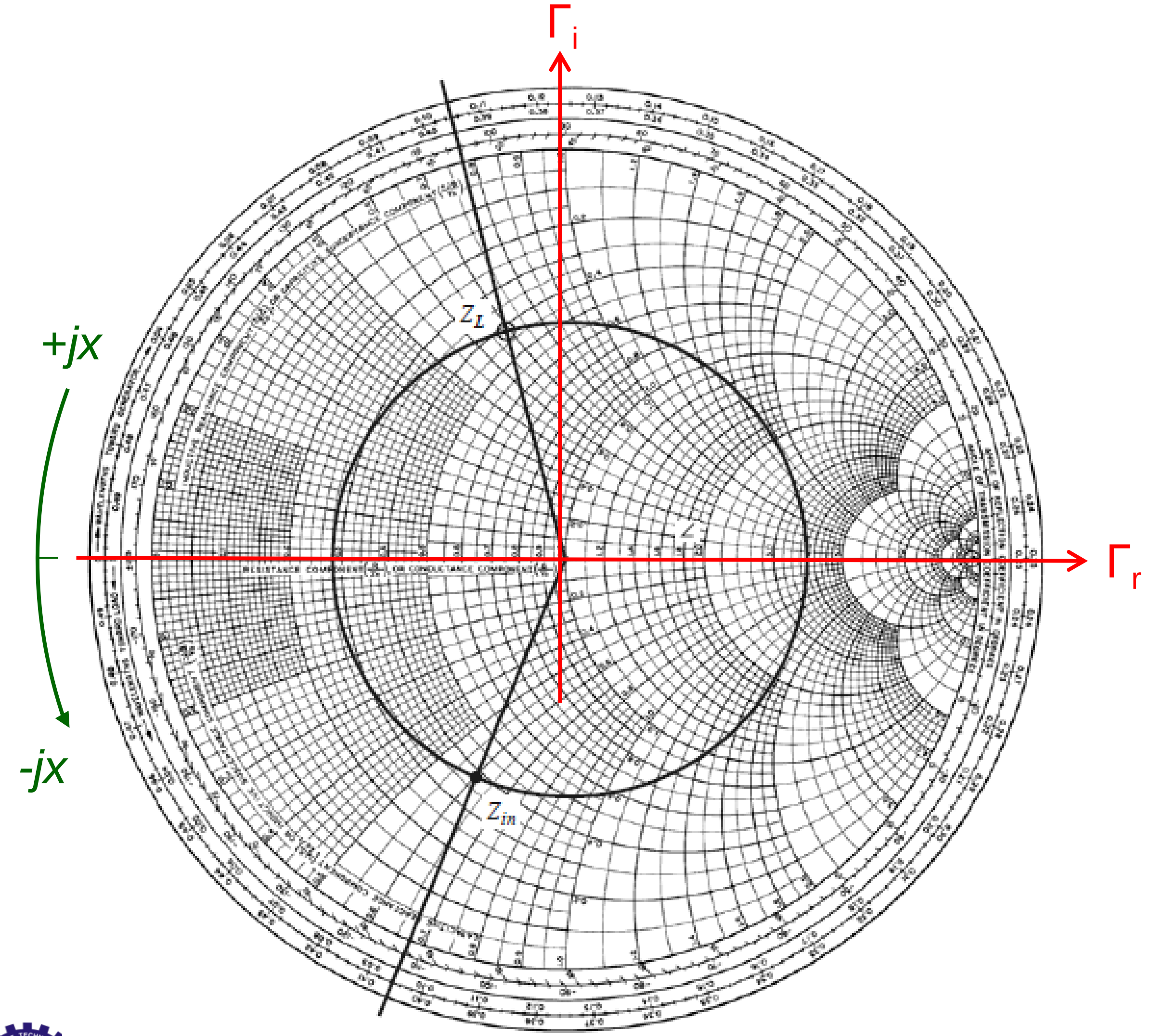


$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2,$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

- Two sets of circles: constant resistance (radius: 1/[1+*r*] and centre: *r*/[1+*r*], 0) and constant reactance (radius: 1/x and centre: 1, 1/x)
- All circles pass through $\Gamma_r = 1$ and $\Gamma_i = 0$.
- The distance around a Smith chart is $\lambda/2$.
- The normalized impedance or admittance is repeated for every half-wavelength of distance.

Smith chart



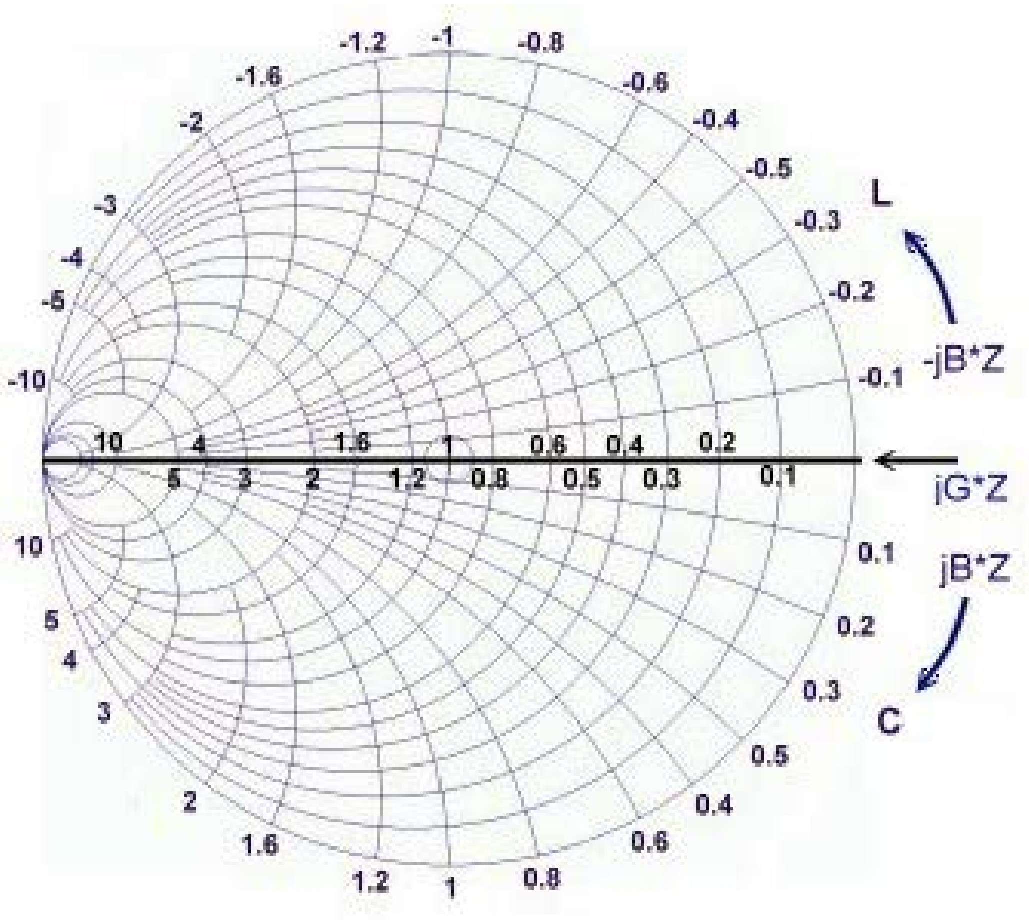
Impedance Smith chart

Problem: A load Z_L = 20 + j 35 Ω is connected to a transmission line of characteristic impedance 50 Ω. A microwave source is connected 0.3λ away. Calculate the Z_{in} seen by the source, the reflection coefficients at the source and load ends.

Steps:

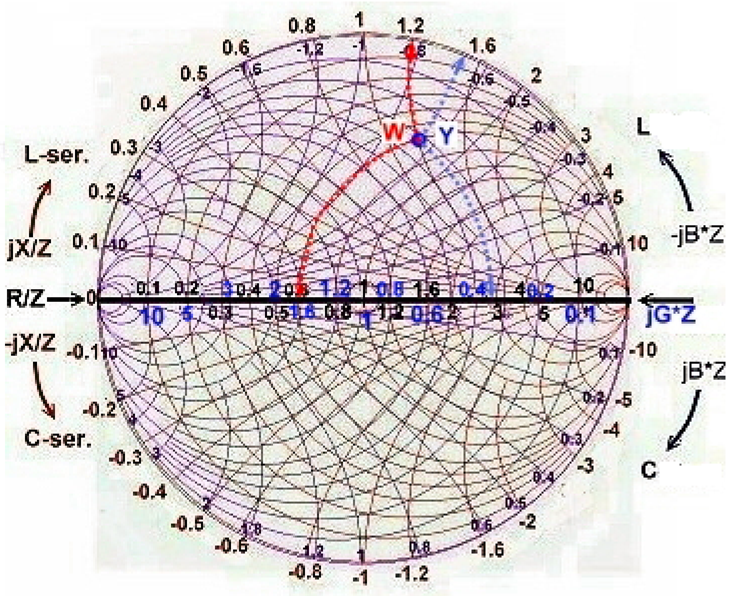
- Normalized imp. z_L = 0.4 + j 0.7 Ω
- Angle of reflection coefficient at load is 104° (at λ = 0.106).
- Using a compass, from the scale (not shown) read off the reflection coefficient magnitude at the load as |Γ| = 0.59.
- Move toward generator by 0.3 λ to 0.406 λ.
- Draw the radius → Γ = 0.59, 104°
- Corresponding z_{in} = 0.365 – j 0.611 Ω.
- → Z_{in} = 18.25 – j 30.55 Ω.

Smith chart



Admittance Smith chart

- Represented in terms of normalized admittance y_{L}.
- Note that if Z_L is connected to a $\lambda/4$ transmission line of characteristic impedance Z_0, the input impedance $z_{in} = 1/\, z_L$.
- All other rules remain similar.
- Combined Smith chart – impedance and admittance on the same Smith chart.



RF and Microwave Engineering (EC 31005)

Guiding Structures (P2)

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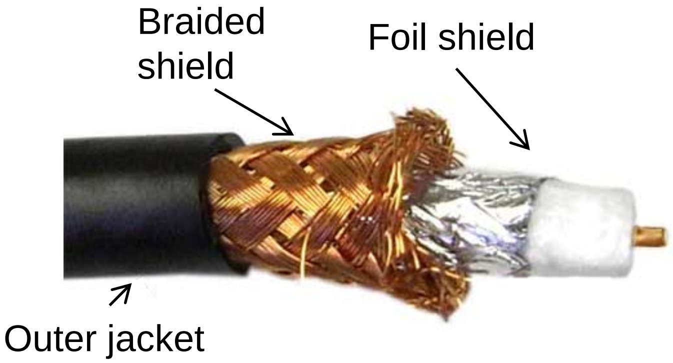
Different guiding structures

Rectangular waveguide
(popular mode - TE)

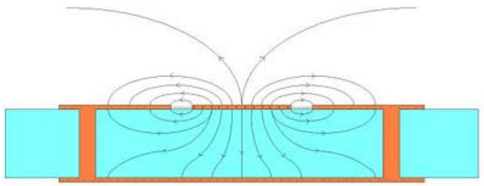
- Low loss, high power handling capacity.
- Bulky, expensive.



Coaxial cable
(popular mode - TEM)

- Flexible.
- Lossy, fabrication problem.

Microstrip line
(popular mode - TEM)

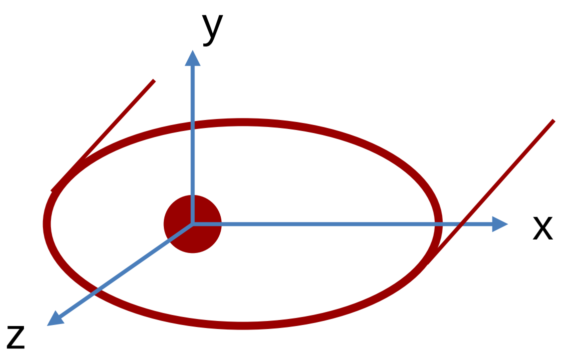


Ground-backed CPW
(popular mode – TEM)

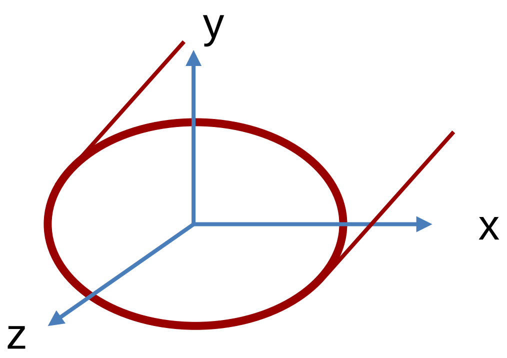
Structures based on PCB:

- Low profile, fabrication simplicity.
- Lossy, low power handling, semi open structures.

General solutions for TEM, TE and TM waves



Two conductor system



Closed waveguide

Assuming time-harmonic fields with $e^{j\omega t}$ dependence and wave propagating along z-axis, the fields are

$$\bar{E}(x, y, z) = [\bar{e}(x, y) + \hat{z}e_z(x, y)]e^{-j\beta z},$$
$$\bar{H}(x, y, z) = [\bar{h}(x, y) + \hat{z}h_z(x, y)]e^{-j\beta z}.$$

$\bar{e}(x, y)$, $\bar{h}(x, y)$ are the transverse components.

Assuming source free region,

$$\nabla \times \bar{E} = -j\omega\mu\bar{H},$$
$$\nabla \times \bar{H} = j\omega\epsilon\bar{E}.$$

General solutions for TEM, TE and TM waves

Then the six field components are

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x, \qquad \frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\epsilon E_x,$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y, \qquad -j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y,$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z, \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z.$$

Notation:

$$E_z(x, y, z) = e_z(x, y)e^{-j\beta z}$$

The transverse components, after rearranging the above equations,

$$E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right), \qquad H_x = \frac{j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right),$$

$$E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right), \qquad H_y = \frac{-j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right),$$

where $k_c^2 = k^2 - \beta^2$ is defined as cutoff wavenumber, $k = \omega\sqrt{\mu\epsilon} = 2\pi/\lambda$

If dielectric loss is present, $\epsilon = \epsilon_o\epsilon_r(1 - j\tan\delta)$

General solutions for TEM wave ($E_z = H_z = 0$)

Eliminate H_x from $\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu H_x, \quad -j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$

→ $\beta^2 E_y = \omega^2 \mu\epsilon E_y$ or $\beta = \omega\sqrt{\mu\epsilon} = k$ → $k_c^2 = 0$, no cutoff frequency.

To find solutions for the fields, start from Helmholtz wave equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) E_x = 0,$$

But considering $e^{-j\beta z}$ dependence, $(\partial^2/\partial z^2)E_x = -\beta^2 E_x = -k^2 E_x$

The above equation reduces to $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) E_x = 0.$ [similar result for the E_y]

Then, for the transverse electric fields, $\nabla_t^2 \bar{e}(x, y) = 0$ where $\nabla_t^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the Laplacian operator.

General solution for TEM wave

Similarly, for the magnetic field, $\nabla_t^2 \bar{h}(x, y) = 0.$

Then the electric field can be represented in terms of a scalar potential,

$\bar{e}(x, y) = -\nabla_t \Phi(x, y)$ where $\nabla_t = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y)$ is the transverse gradient.

For TEM, curl of transverse component must vanish, $\nabla_t \times \bar{e} = -j\omega\mu h_z \hat{z} = 0.$

Now, in charge free region, $\nabla \cdot \bar{D} = \epsilon \nabla_t \cdot \bar{e} = 0$

Therefore, $\nabla_t^2 \Phi(x, y) = 0$ (This equation will be solved under the given boundary condition)

Voltage between two conductors, $V_{12} = \Phi_1 - \Phi_2 = \int_1^2 \bar{E} \cdot d\bar{\ell}$

The current flow on a conductor (Ampere's law), $I = \oint_C \bar{H} \cdot d\bar{\ell}$

General solution for TEM wave

The wave impedance, $Z_{\text{TEM}} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} = \eta$ $\quad Z_{\text{TEM}} = \frac{-E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} = \eta$

The general expression for the transverse magnetic field, $\bar{h}(x, y) = \frac{1}{Z_{\text{TEM}}}\hat{z} \times \bar{e}(x, y).$

Analysis steps:

1. Solve Laplace's equation.
2. Find the unknown constant by applying boundary conditions.
3. Compute the electric and magnetic fields.
4. Compute V and I.
5. Find the characteristics impedance.

General solution for TE wave

Start from the general solutions,

$$E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right), \qquad H_x = \frac{j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right),$$

$$E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right), \qquad H_y = \frac{-j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right),$$

By applying, $E_z = 0$ and $H_z \neq 0$,

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}, \qquad H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x},$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}. \qquad H_y = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial y},$$

where propagation constant, $\beta = \sqrt{k^2 - k_c^2}$

Start from Helmholtz wave equation to find H_z, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) H_z = 0,$

General solution for TE wave

Since $H_z(x, y, z) = h_z(x, y)e^{-j\beta z}$,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z = 0,$$ → two dimensional wave equation with $k_c^2 = k^2 - \beta^2$.

This equation must be solved under the given boundary condition.

The wave impedance, $Z_{\text{TE}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$

The wave impedance is frequency dependent.

Note that a wave to propagate the wave impedance must have a real part. Purely imaginary wave impedance is bounded wave → evanescent.

General solution for TM wave

Start from the general solutions,

$$E_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right), \qquad H_x = \frac{j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right),$$

$$E_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right), \qquad H_y = \frac{-j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right),$$

By applying, $H_z = 0$ and $E_z \neq 0$,

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x}, \qquad H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y},$$

$$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}. \qquad H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x},$$

where propagation constant, $\beta = \sqrt{k^2 - k_c^2}$

Start from Helmholtz wave equation to find E_z, $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\right) E_z = 0,$

General solution for TM wave

Since $E_z(x, y, z) = e_z(x, y)e^{-j\beta z}$

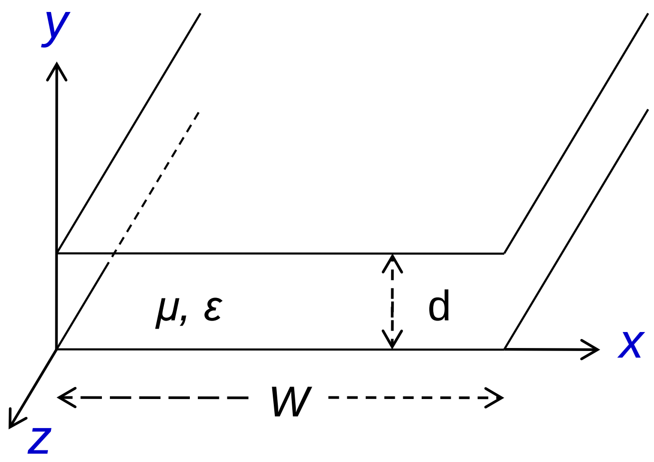
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z = 0$$ → two dimensional wave equation with $k_c^2 = k^2 - \beta^2$

This equation must be solved under the given boundary condition.

The wave impedance, $Z_{\mathrm{TM}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}$

The wave impedance is frequency dependent.

Example: parallel plate waveguide



<u>TEM modes:</u> $E_z = H_z = 0)$

Laplace's equation for electrostatic potential $\Phi(x, y)$ between the two plates:

$$\nabla_t^2 \Phi(x, y) = 0, \quad \text{for } 0 \le x \le W, 0 \le y \le d.$$

Boundary conditions:

$\Phi(x,0) = 0,$ (bottom plate)

$\Phi(x,d) = V_0.$ (top plate)

Example: parallel plate waveguide

No variation with *x* →

$$\Phi(x, y) = A + By$$

Applying BC → $\Phi(x, y) = V_o y/d.$

Electric field: $\bar{E}(x, y, z) = [\bar{e}(x, y) + \hat{z}e_z(x, y)]e^{-j\beta z}$

Then, transverse electric field: $\bar{e}(x, y) = -\nabla_t \Phi(x, y) = -\hat{y}\dfrac{V_o}{d}$

Total electric field: $\bar{E}(x, y, z) = \bar{e}(x, y)e^{-jkz} = -\hat{y}\dfrac{V_o}{d}e^{-jkz}$ where $\boxed{k = \omega\sqrt{\mu\epsilon}}$

Total magnetic field: $\bar{H}(x, y, z) = \bar{h}\,(x, y)\, e^{-jkz} = \dfrac{1}{\eta}\hat{z} \times \bar{E}(x, y, z) = \hat{x}\dfrac{V_o}{\eta d}e^{-jkz}$

where $\eta = \sqrt{\mu/\epsilon}$

The voltage of the top plate with respect to the bottom plate:

$$V = -\int_{y=0}^{d} E_y \, dy = V_o e^{-jkz}$$

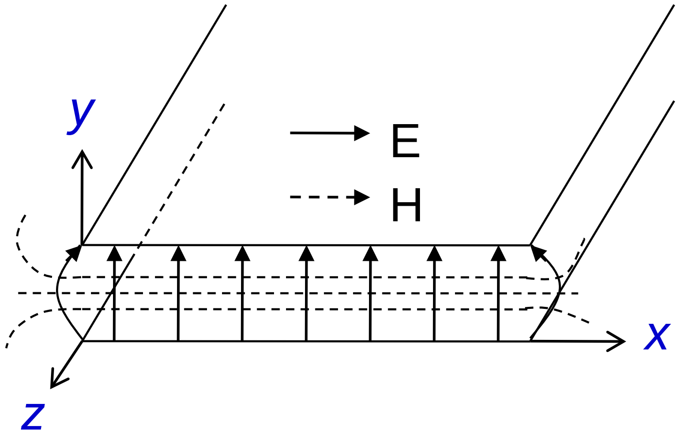
Example: parallel plate waveguide

The total current on the top plate (Ampere's law):

$$I = \int_{x=0}^{W} \bar{J}_s \cdot \hat{z}\, dx = \int_{x=0}^{W} (-\hat{y} \times \bar{H}) \cdot \hat{z}\, dx = \int_{x=0}^{W} H_x dx = \frac{W V_o}{\eta d} e^{-jkz}.$$

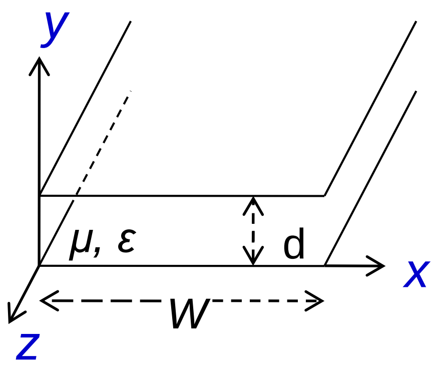
Characteristic impedance: $Z_0 = \frac{V}{I} = \frac{\eta d}{W}$

Phase velocity: $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$



Field distribution.

Example: parallel plate waveguide



TM modes: $H_z = 0$ and for E_z, $\partial/\partial x = 0$

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y}, \quad H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x},$$

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x}, \quad E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}.$$

E_z to be calculated from $\left(\frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x, y) = 0$ where $k_c = \sqrt{k^2 - \beta^2}$

The general solution: $e_z(x, y) = A \sin k_c y + B \cos k_c y$

Boundary condition: $e_z(x, y) = 0, \quad$ at $y = 0,\ d$

→ B = 0 and $k_c = \frac{n\pi}{d}, \quad n = 0,\ 1,\ 2,\ 3, \ldots$

So, propagation constant: $\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (n\pi/d)^2}.$

Therefore, $e_z(x, y) = A_n \sin \frac{n\pi y}{d}$ → $E_z(x, y, z) = A_n \sin \frac{n\pi y}{d} e^{-j\beta z}$

Example: parallel plate waveguide

Solution for other field components:
$$H_x = \frac{j\omega\epsilon}{k_c} A_n \cos\frac{n\pi y}{d} e^{-j\beta z},$$
$$E_y = \frac{-j\beta}{k_c} A_n \cos\frac{n\pi y}{d} e^{-j\beta z},$$
$$E_x = H_y = 0.$$

For $n = 0,\ \beta = k = \omega\sqrt{\mu\epsilon}$

With E_z = 0, E_y and H_x are constant in y → TM_0 mode is identical to TEM mode.

- The cutoff frequency of TM_n mode:

$$f_c = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{n}{2d\sqrt{\mu\epsilon}}.$$

- Cutoff wavelength of TM_n mode: $\lambda_c = \frac{2d}{n}.$

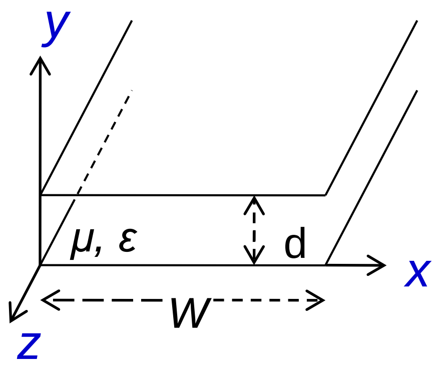
Wave impedance: $Z_{TM} = \frac{-E_y}{H_x} = \frac{\beta}{\omega\epsilon} = \frac{\beta\eta}{k}$

Phase velocity: $v_p = \frac{\omega}{\beta},$

- Guided wavelength: $\lambda_g = \frac{2\pi}{\beta}$

Note that β depends on the mode number.

Example: parallel plate waveguide



TE modes: $E_Z = 0$, $\partial/\partial x = 0$

H_z to be calculated from $\left(\dfrac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x, y) = 0$

where $k_c = \sqrt{k^2 - \beta^2}$ and $H_z(x, y, z) = h_z(x, y)e^{-j\beta z}$

General solution: $h_z(x, y) = A \sin k_c y + B \cos k_c y.$

Therefore, putting it in the general solution,

$$E_x = \frac{-j\omega\mu}{k_c}\left(A \cos k_c y - B \sin k_c y\right) e^{-j\beta z}$$

Now, boundary conditions: $E_x = 0$ at $y = 0, d$.

Applying the BC→ A = 0 and $k_c = \dfrac{n\pi}{d}, \quad n = 1, 2, 3 \ldots$

Therefore, $H_z(x, y) = B_n \cos \dfrac{n\pi y}{d} e^{-j\beta z}.$

Example: parallel plate waveguide

The other field components:

$$H_x = -\frac{j\beta}{k_c^2}\frac{\partial H_z}{\partial x}, \quad H_y = -\frac{j\beta}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_x = -\frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}, \quad E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}.$$

$$\Rightarrow$$

$$E_x = \frac{j\omega\mu}{k_c}B_n \sin\frac{n\pi y}{d}e^{-j\beta z},$$

$$H_y = \frac{j\beta}{k_c}B_n \sin\frac{n\pi y}{d}e^{-j\beta z},$$

$$E_y = H_x = 0.$$

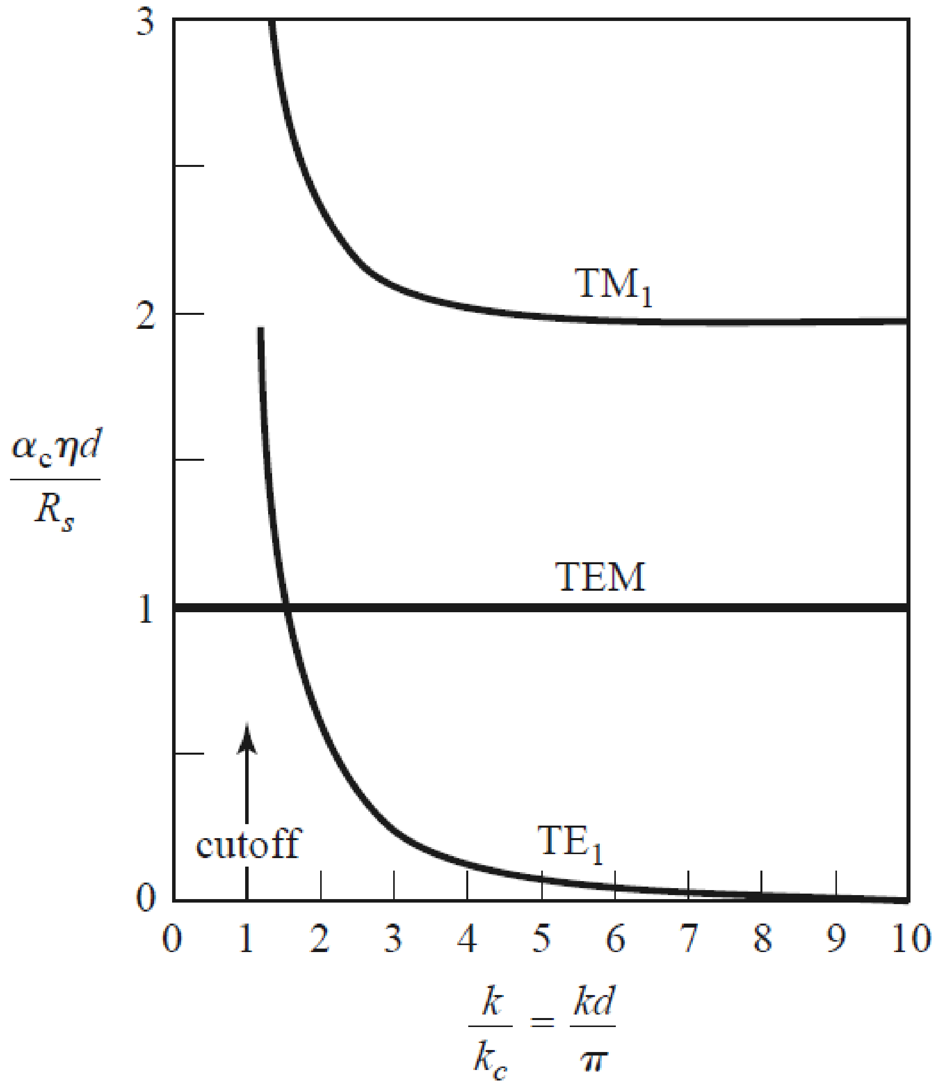
where the propagation constant is $\beta = \sqrt{k^2 - \left(\frac{n\pi}{d}\right)^2}$

The cutoff frequency of TE_n mode: $f_c = \frac{n}{2d\sqrt{\mu\epsilon}}$

The wave impedance: $Z_{\mathrm{TE}} = \frac{E_x}{H_y} = \frac{\omega\mu}{\beta} = \frac{k\eta}{\beta}$

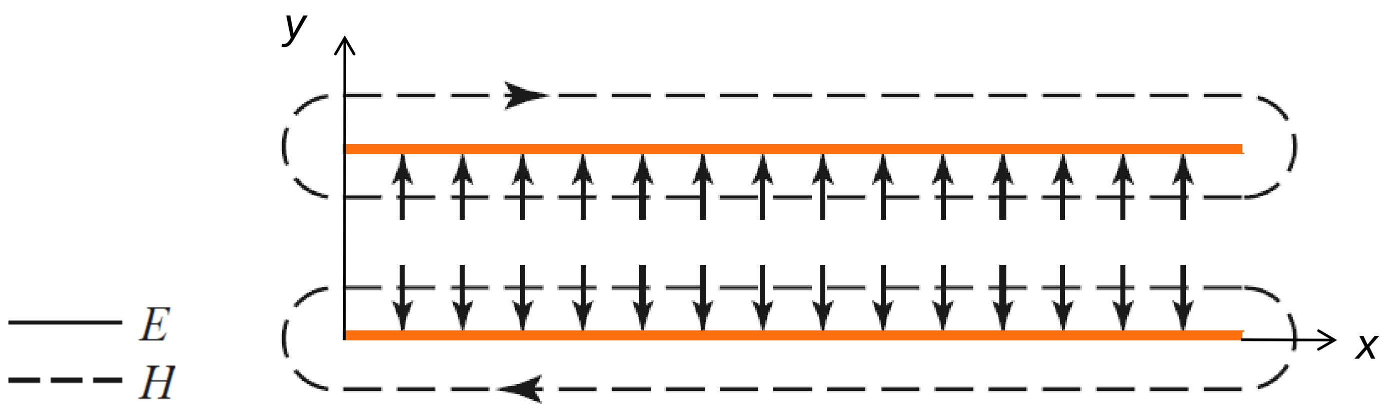
- There is no TE_0 mode.

Example: parallel plate waveguide

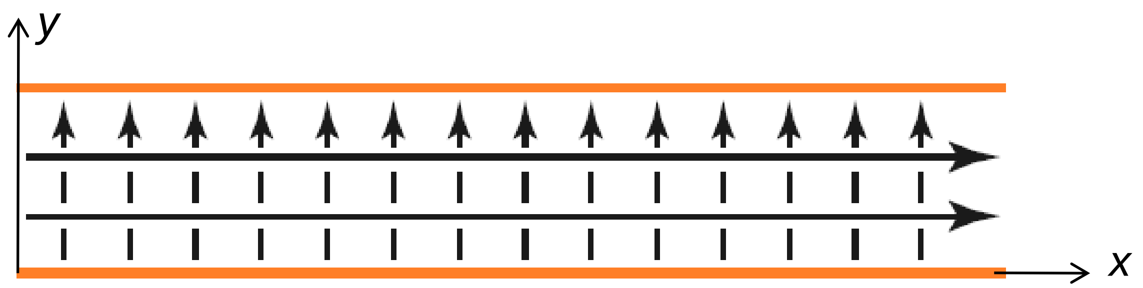


Attenuation due to conductor loss.

Example: parallel plate waveguide

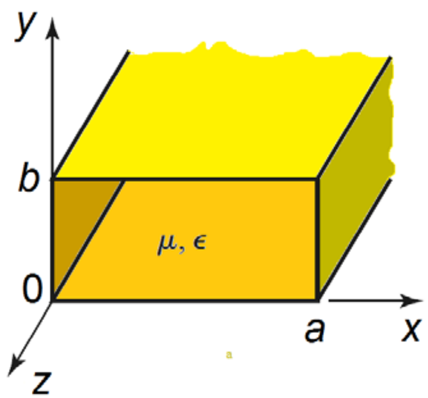


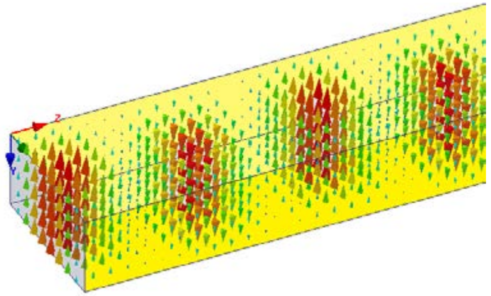
TM_1 mode field distribution.



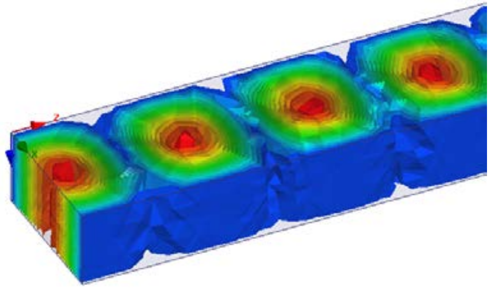
TE_1 mode field distribution.

Rectangular waveguide (TE mode)





Vector Electric field distribution.



Scalar Electric field distribution.

E_z = 0 and h_z must satisfy the reduced wave equation,

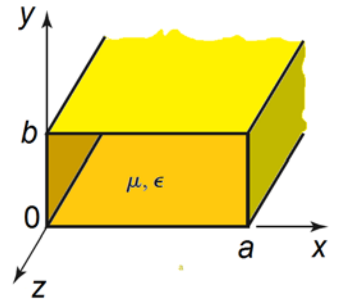
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x, y) = 0, \quad \text{with} \quad H_z(x, y, z) = h_z(x, y)e^{-j\beta z} \quad \text{and} \quad k_c = \sqrt{k^2 - \beta^2}$$

- Separation of variables method:

put $h_z(x, y) = X(x)Y(y)$ in the wave equation → $$\frac{1}{X}\frac{d^2 X}{dx^2} + \frac{1}{Y}\frac{d^2 Y}{dy^2} + k_c^2 = 0.$$

Rectangular waveguide (TE mode)

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + k_c^2 = 0.$$



Then each term must equals a constant → $$\frac{d^2X}{dx^2} + k_x^2 X = 0,$$

$$\frac{d^2Y}{dy^2} + k_y^2 Y = 0,$$ where $$k_x^2 + k_y^2 = k_c^2.$$

Then, the general solution → $$h_z(x, y) = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y).$$

Recall for TE case,

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y},$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}.$$

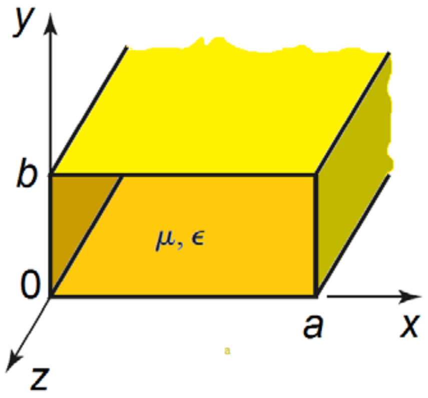
⇨

$$e_y = \frac{j\omega\mu}{k_c^2}k_x(-A\sin k_x x + B\cos k_x x)(C\cos k_y y + D\sin k_y y).$$

$$e_x = \frac{-j\omega\mu}{k_c^2}k_y(A\cos k_x x + B\sin k_x x)(-C\sin k_y y + D\cos k_y y),$$

Rectangular waveguide (TE mode)

Apply the boundary conditions:
$$e_x(x, y) = 0, \qquad \text{at } y = 0, b,$$
$$e_y(x, y) = 0, \qquad \text{at } x = 0, a.$$



Applying the boundary condition:

D = 0, and $k_y = n\pi/b$ for *n* = 0, 1, 2, 3…

B = 0, and $k_x = m\pi/a$ for *m* = 0, 1, 2, 3…

Then the final solution is $H_z(x, y, z) = A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta z},$

Similarly the other field components are

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta z},$$
$$H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta z},$$
$$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-j\beta z},$$
$$H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta z}.$$

The propagation constant is $\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2},$ real for $k>k_c$

Rectangular waveguide (TE mode)

Each mode (*m, n*) has a cutoff frequency, $f_{c_{mn}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}.$

The mode with only $f > f_c$ will propagate.

For $a>b$, the lowest cutoff frequency for ($m = 1$, $n = 0$) is $f_{c_{10}} = \frac{1}{2a\sqrt{\mu\epsilon}}.$

TE_{10} mode is called the dominant TE mode.

There is no TE_{00} mode.

The wave impedance is $Z_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{k\eta}{\beta},$ with $\eta = \sqrt{\mu/\epsilon}$ (intrinsic impedance of the material inside the waveguide)

The guide wavelength $\lambda_g = \frac{2\pi}{\beta} > \frac{2\pi}{k} = \lambda,$

The phase velocity is $v_p = \frac{\omega}{\beta} > \frac{\omega}{k} = 1/\sqrt{\mu\epsilon},$ greater than the velocity of light in that material.

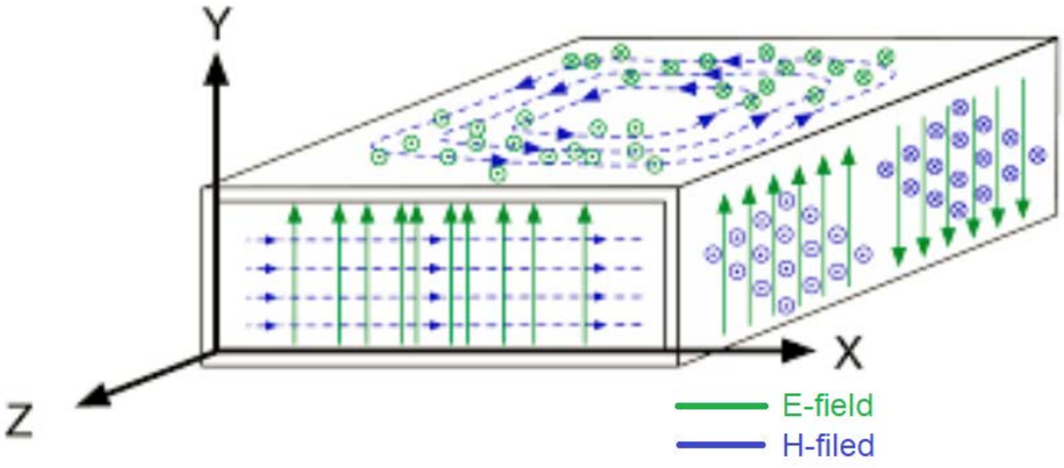
Rectangular waveguide (TE mode)

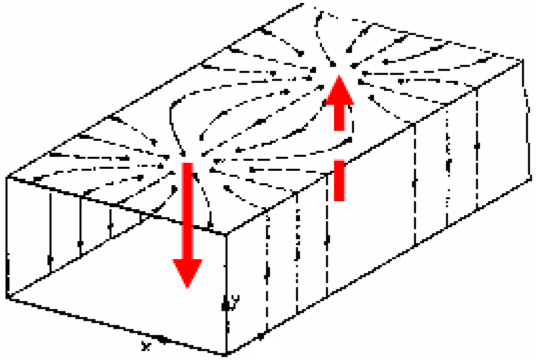
For the TE_{10} mode,

$$E_y = \frac{-j\omega\mu a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z}, \qquad H_z = A_{10}\cos\frac{\pi x}{a} e^{-j\beta z},$$

$$E_x = E_z = H_y = 0. \qquad H_x = \frac{j\beta a}{\pi} A_{10}\sin\frac{\pi x}{a} e^{-j\beta z},$$

and $k_c = \pi/a, \quad \beta = \sqrt{k^2 - (\pi/a)^2}.$





Surface current distribution.

Rectangular waveguide (TE mode)

Power flow down the guide for TE_{10} mode,

$$P_{10} = \frac{1}{2}\mathrm{Re}\int_{x=0}^{a}\int_{y=0}^{b} \bar{E} \times \bar{H}^* \cdot \hat{z}\, dy\, dx \qquad = \frac{1}{2}\mathrm{Re}\int_{x=0}^{a}\int_{y=0}^{b} E_y H_x^*\, dy\, dx$$

$$= \frac{\omega\mu a^2}{2\pi^2}\mathrm{Re}(\beta)|A_{10}|^2 \int_{x=0}^{a}\int_{y=0}^{b} \sin^2\frac{\pi x}{a}\, dy\, dx$$

$$= \frac{\omega\mu a^3 |A_{10}|^2 b}{4\pi^2}\mathrm{Re}(\beta).$$

Power loss due to conductor loss can be calculated from

$$P_\ell = \frac{R_s}{2}\int_C |\bar{J}_s|^2 d\ell,$$

Rectangular waveguide (TM mode)

***H*$_z$ = 0** and e_z must satisfy the reduced wave equation, $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) e_z(x, y) = 0,$$

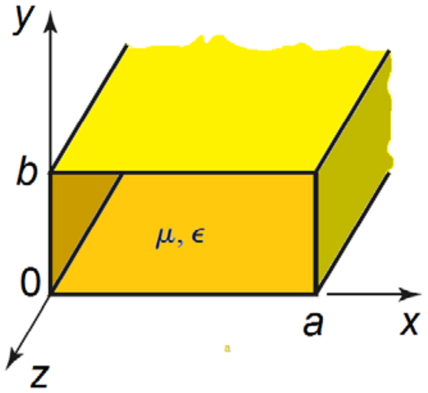
with $E_z(x, y, z) = e_z(x, y)e^{-j\beta z}$ and $k_c^2 = k^2 - \beta^2$

Then the solution is $e_z(x, y) = (A \cos k_x x + B \sin k_x x)(C \cos k_y y + D \sin k_y y).$

Apply the boundary conditions: $e_z(x, y) = 0, \quad \text{at } x = 0, a,$

$$e_z(x, y) = 0, \quad \text{at } y = 0, b.$$

→ $A = 0$ and $k_x = m\pi/a$ and $C = 0$ and $k_y = n\pi/b$



Then, $$E_z(x, y, z) = B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} e^{-j\beta z},$$

Rectangular waveguide (TM mode)

Similarly the other field components are

$$E_x = \frac{-j\beta m\pi}{ak_c^2} B_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}, \qquad H_x = \frac{j\omega\epsilon n\pi}{bk_c^2} B_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z},$$

$$E_y = \frac{-j\beta n\pi}{bk_c^2} B_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}, \qquad H_y = \frac{-j\omega\epsilon m\pi}{ak_c^2} B_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}.$$

The propagation constant is $\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

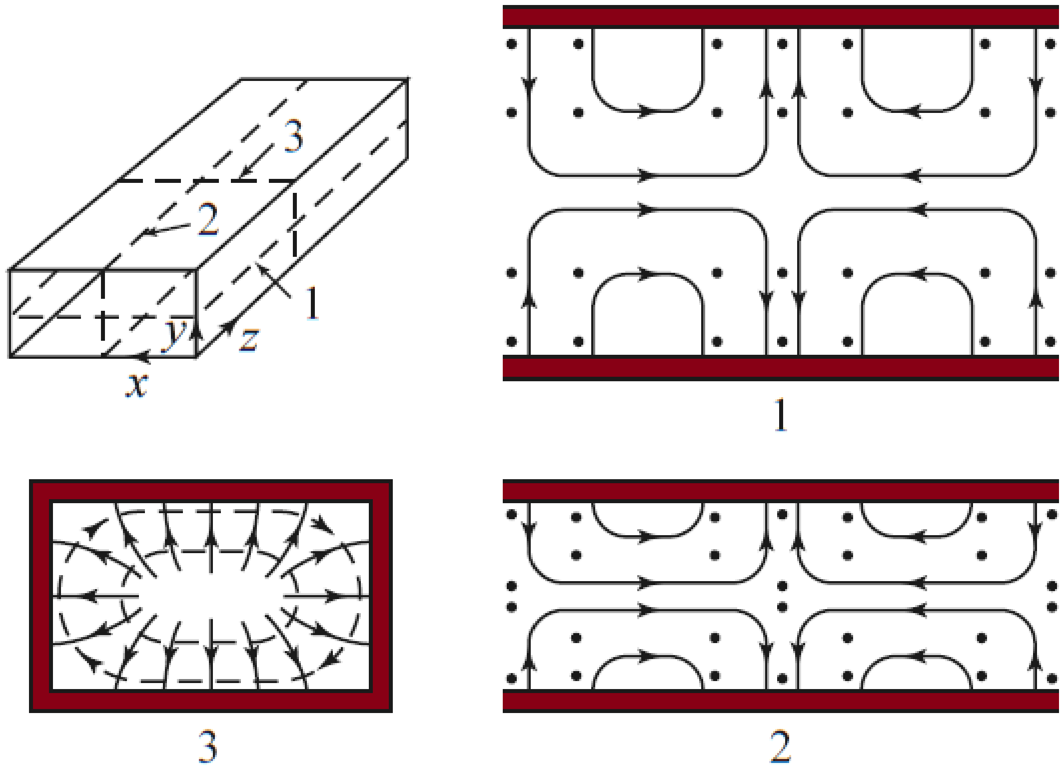
There is no TM_{00} or TM_{10} or TM_{01} mode.

The lowest order TM mode is TM_{11} mode and the corresponding cutoff frequency is

$$f_{c_{11}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2},$$

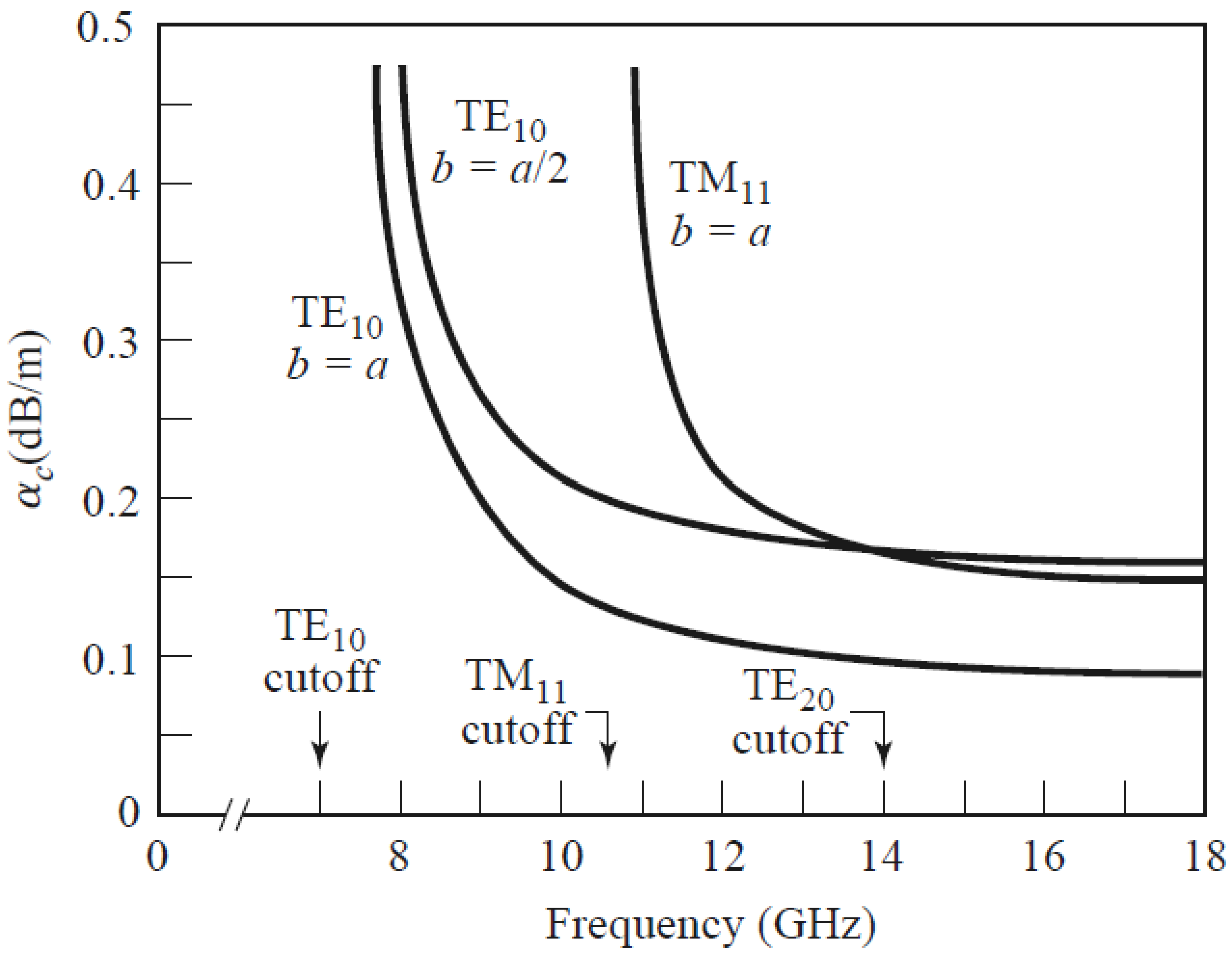
The wave impedance is $Z_{\mathrm{TM}} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{\beta\eta}{k}.$

Rectangular waveguide (TM mode)



Field distribution for TM_{11} mode.

Rectangular waveguide



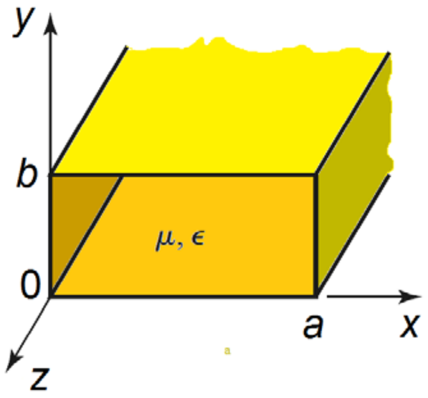
Attenuation of various mode in a rectangular waveguide.

Rectangular waveguide

- Energy transport velocity, group velocity and phase velocity:

$$v_{\text{en}} = \frac{P_T}{W'} = c\sqrt{1 - \frac{\omega_c^2}{\omega^2}} = v_g$$

$$v_g v_p = c^2$$



- Attenuation constant for TE_{10} mode of an air filled waveguide:

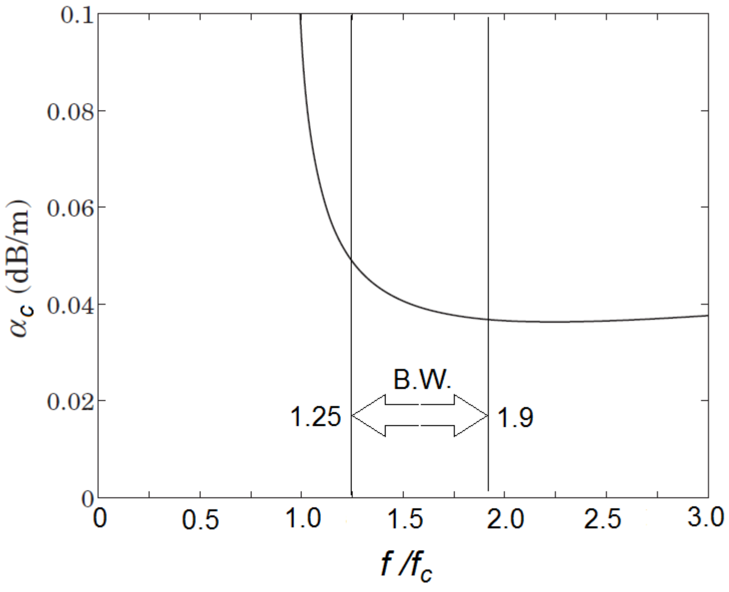
$$\alpha_c = 8.686 \frac{R_s}{\eta b} \frac{\left(1 + \frac{2b}{a}\frac{\omega_c^2}{\omega^2}\right)}{\sqrt{1 - \frac{\omega_c^2}{\omega^2}}} \text{ dB/m}$$

$$\alpha_d = \frac{k^2 \tan\delta}{2\beta} \times 8.686 \text{ dB/m}$$

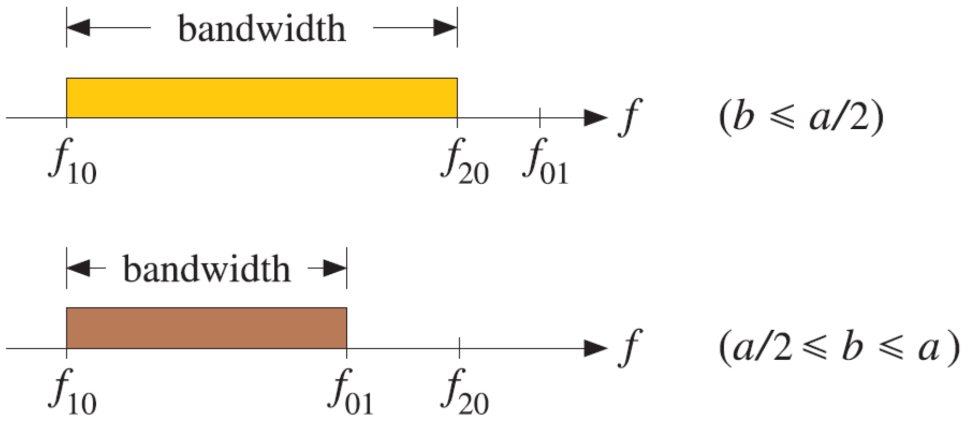
- Surface resistance

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.032\ \Omega \quad \text{for WR-90 @10 GHz}$$

Rectangular waveguide



Variation of α_c.



Operating bandwidth in rectangular waveguides.

- Loss increases with decreasing *b*.
- For a dielectric filled guide, attenuation (conductor + dielectric loss) is minimum approximately at $1.5f_c$ *for* TE_{10} mode excitation.
- Higher order modes offer lower loss.

Standard rectangular waveguides

Bands	U.S.A. (EIA) (JAN)	U.K. WG I.E.C.	Operating Frequency Range (GHz)	Cut-off Frequncy (GHz)	Waveguide Inner Size (Inches)	Cover Flange* MIL-F-3922/ UG	Flange Type
X	WR-90 RG-52/ U	WG-16 R100	8.2 to 12.4	6.56	0.900 x 0.400	53-001 UG-39/ U	Square
	WR-75 RG-346/ U	WG-17 R120	10.0 to 15.0	7.87	0.750 x 0.375	53-007 -	Square
Ku	WR-62 RG-91/ U	WG-18 R140	12.4 to 18.0	9.48	0.622 x 0.311	53-005 UG-419/ U	Square
	WR-51 RG-352/ U	WG-19 R180	15.0 to 22.0	11.57	0.510 x 0.255	70-010 -	Square
K	WR-42 RG-53/ U	WG-20 R220	18.0 to 26.5	14.05	0.420 x 0.170	54-001 UG-595/ U	Square
	WR-34 RG-53/ U	WG-21 R260	22.0 to 33.0	17.33	0.340 x 0.170	- UG-1530/ U	Square
Ka	WR-28 RG-96/ U	WG-22 R320	26.5 to 40.0	21.08	0.280 x 0.140	54-003 UG-599/ U	Square
Q (B)	WR-22 RG-97/ U	WG-23 R400	33.0 to 50.0	26.34	0.224 x 0.112	67B-006 UG-383/ U	Round
U	WR-19 RG-358/ U	WG-24 R500	40.0 to 60.0	31.36	0.188 x 0.094	67B-007 UG-383/ U-M	Round
V	WR-15 RG-98/ U	WG-25 R620	50.0 to 75.0	39.86	0.148 x 0.074	67B-008 UG-385/ U	Round
E	WR-12 RG-99/ U	WG-26 R740	60.0 to 90.0	48.35	0.122 x 0.061	67B-009 UG-387/ U	Round
W	WR-10 RG-359/ U	WG-27 R900	75.0 to 110.0	59.01	0.100 x 0.050	67B-010 UG-387/ U-M	Round
F	WR-8 RG-138/ U	WG-28 R1200	90.0 to 140.0	73.84	0.080 x 0.040	- UG-387/ U-M	Round
D	WR-6 RG-276/ U	WG-29 R1400	110.0 to170.0	90.84	0.065 x 0.0325	- UG-387/ U-M	Round

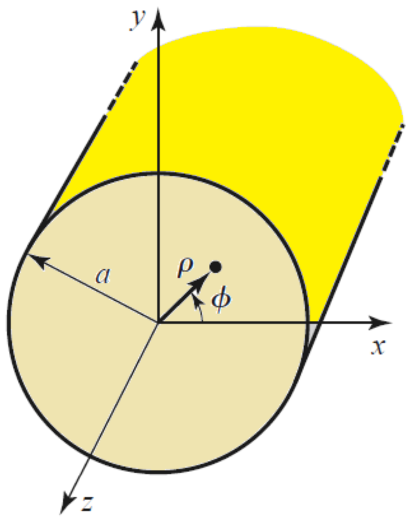
Circular waveguide

The cylindrical components of the transverse fields are

$$E_\rho = \frac{-j}{k_c^2}\left(\beta\frac{\partial E_z}{\partial\rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial\phi}\right), \qquad H_\rho = \frac{j}{k_c^2}\left(\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial\phi} - \beta\frac{\partial H_z}{\partial\rho}\right),$$

$$E_\phi = \frac{-j}{k_c^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial\phi} - \omega\mu\frac{\partial H_z}{\partial\rho}\right), \qquad H_\phi = \frac{-j}{k_c^2}\left(\omega\epsilon\frac{\partial E_z}{\partial\rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial\phi}\right),$$

where $k_c^2 = k^2 - \beta^2$ and propagation direction is z i.e. $e^{-j\beta z}$ is common to all.



Circular waveguide with coordinate system.

TE mode:

For TE modes, $E_z = 0$, and H_z is a solution to the wave equation.

Then start with the Helmholtz wave equation, $\nabla^2 H_z + k^2 H_z = 0.$

and put $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-j\beta z}$

Circular waveguide (TE mode)

Then in the cylindrical coordinate, $$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}+k_c^2\right)h_z(\rho,\phi)=0.$$

Follow the same procedure of separation of variables.

Then, substituting $h_z(\rho,\phi)=R(\rho)P(\phi)$,

from the above equation,

$$\frac{1}{R}\frac{d^2R}{d\rho^2}+\frac{1}{\rho R}\frac{dR}{d\rho}+\frac{1}{\rho^2 P}\frac{d^2P}{d\phi^2}+k_c^2=0,$$

$$\rightarrow \frac{\rho^2}{R}\frac{d^2R}{d\rho^2}+\frac{\rho}{R}\frac{dR}{d\rho}+\rho^2k_c^2=\frac{-1}{P}\frac{d^2P}{d\phi^2}.$$

Left side depends on ρ and right side depends on φ.

Each side equate to a constant, k_φ^2, say.

Circular waveguide (TE mode)

$$\frac{\rho^2}{R}\frac{d^2 R}{d\rho^2} + \frac{\rho}{R}\frac{dR}{d\rho} + \rho^2 k_c^2 = \frac{-1}{P}\frac{d^2 P}{d\phi^2} = k_\phi^2$$

Therefore,

$$\frac{d^2 P}{d\phi^2} + k_\phi^2 P = 0. \quad \text{and} \quad \rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \left(\rho^2 k_c^2 - k_\phi^2\right) R = 0.$$

General solution of the left equation is

$$P(\phi) = A \sin k_\phi \phi + B \cos k_\phi \phi.$$

→ $P(\phi) = A \sin n\phi + B \cos n\phi,$ } Because h_z must be periodic in φ i.e. $h_z(\rho, \phi) = h_z(\rho, \phi \pm 2m\pi)$

Therefore, k_φ must be an integer n.

Similarly, from the right equation,

$$\rho^2 \frac{d^2 R}{d\rho^2} + \rho \frac{dR}{d\rho} + \left(\rho^2 k_c^2 - n^2\right) R = 0,$$ → Bessel's differential equation.

Bessel function

Bessel's differential equation for an arbitrary complex number *α*:

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + \left(x^2 - \alpha^2\right) y = 0$$

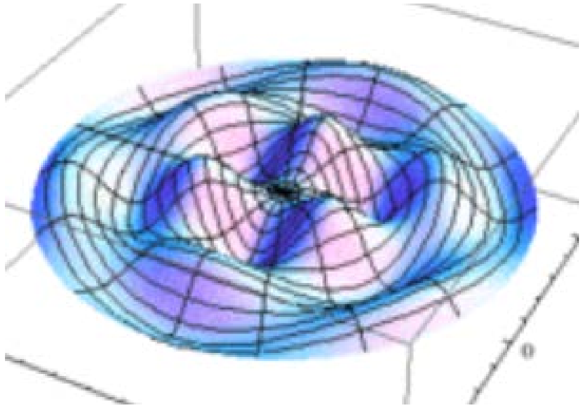
- When *α* is an integer, it is known as cylindrical function or cylindrical harmonics and appears in the solution to Laplace's equation in cylindrical coordinate.
- Spherical Bessel functions with half-integer *α* is obtained when the Helmholtz equation is solved in spherical coordinate.

Bessel functions of the first kind $J_\alpha(x)$:

- Solutions of Bessel's differential equation that are finite at the origin (x = 0) for integer or positive *α.*
- Diverge as x approaches zero for negative non-integer *α*.
- Bessel function, for integer values of *n*, can be represented in integral form as (Bessel's integral)

$$J_n(x) = \frac{1}{\pi}\int_0^\pi \cos(n\tau - x\sin\tau)\,d\tau.$$ or

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^\pi e^{i(n\tau - x\sin\tau)}\,d\tau.$$



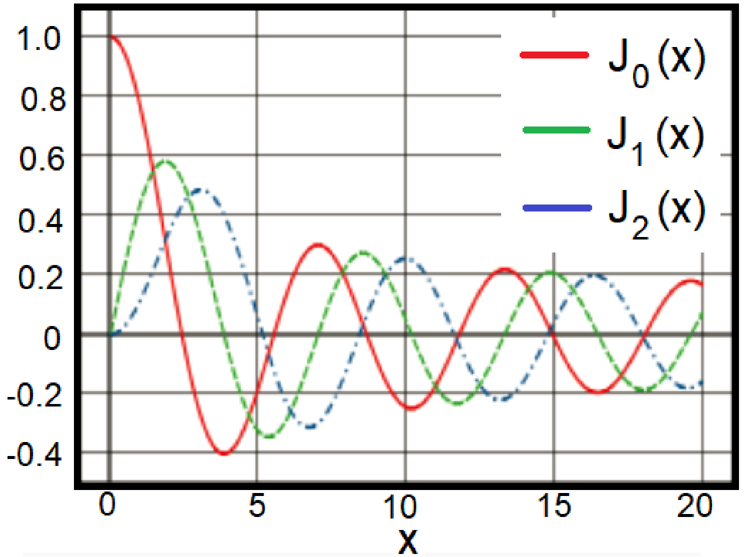
Vibration of a drum.

Bessel function

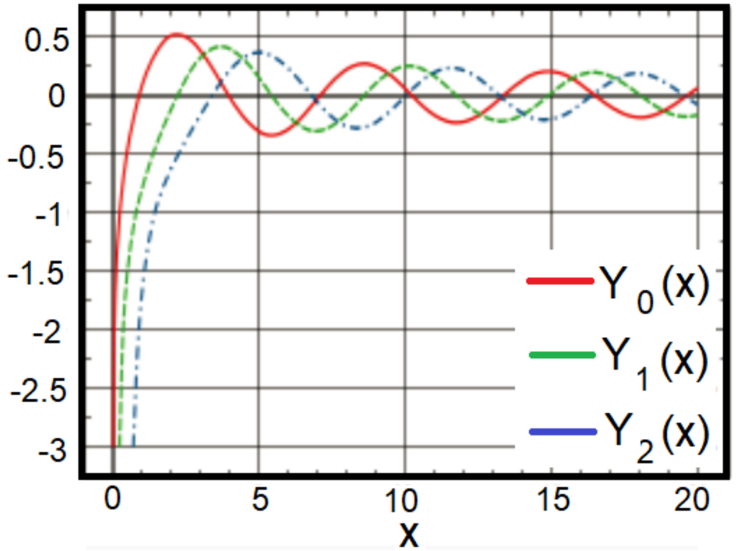
Bessel functions of the second kind Y_α:

- Also denoted by $N_\alpha(x)$, are solutions of the Bessel differential equation that have a singularity at the origin (x = 0) and are multivalued.
- Other names: Weber functions, Neumann functions.
- For integer values of *n*, can be represented in integral form as

$$Y_n(x) = \frac{1}{\pi}\int_0^\pi \sin(x\sin\theta - n\theta)\,d\theta - \frac{1}{\pi}\int_0^\infty \left(e^{nt} + (-1)^n e^{-nt}\right) e^{-x\sinh t}\,dt.$$



Plot of Bessel function of the first kind, $J_\alpha(x)$, for integer orders α = 0, 1, 2



Plot of Bessel function of the second kind, $Y_\alpha(x)$, for integer orders α = 0, 1, 2

Circular waveguide (TE mode)

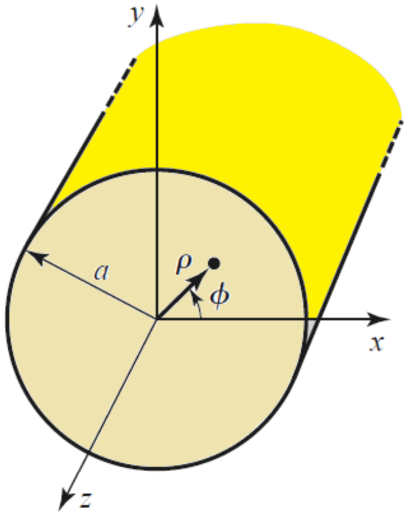
Then, the solution is $R(\rho) = CJ_n(k_c\rho) + DY_n(k_c\rho),$

where $J_n(x)$ and $Y_n(x)$ are the Bessel functions of first and second kinds, respectively.

Since $Y_n(k_c\rho)$ becomes infinite at $\rho = 0$ → physically unacceptable → D = 0.

The solution for h_z simplifies to $h_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c\rho),$

constant *C* is absorbed into the constants *A* and *B.*



Enforce the boundary condition, $E_\phi(\rho, \phi) = 0 \quad \text{at } \rho = a.$

Find E_φ from H_z as, $E_\phi(\rho, \phi, z) = \frac{j\omega\mu}{k_c}(A \sin n\phi + B \cos n\phi) J'_n(k_c\rho) e^{-j\beta z},$

where $J'_n(k_c\rho)$ is the derivative of $J_n(k_c\rho)$

From the boundary condition, $J'_n(k_c a) = 0.$

Let the roots of $J'_n(x)$ are defined as p'_{nm}, so that $J'_n(p'_{nm}) = 0$, where p'_{nm} is the mth root of J'_n, then

$$k_{c_{nm}} = \frac{p'_{nm}}{a}.$$

Circular waveguide (TE mode)

Values of p'_{nm} for TE modes of a circular waveguide

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

n → the number of circumferential (φ) variations
m → the number of radial (ρ) variations ≥1.

- There is no TE_{10} mode, but TE_{01} mode exists.
- The lowest order TE mode is TE_{11} mode.

Then, the propagation constant is $$\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{p'_{nm}}{a}\right)^2},$$

The cutoff frequency is $$f_{c_{nm}} = \frac{k_c}{2\pi\sqrt{\mu\epsilon}} = \frac{p'_{nm}}{2\pi a\sqrt{\mu\epsilon}}.$$

Circular waveguide (TE mode)

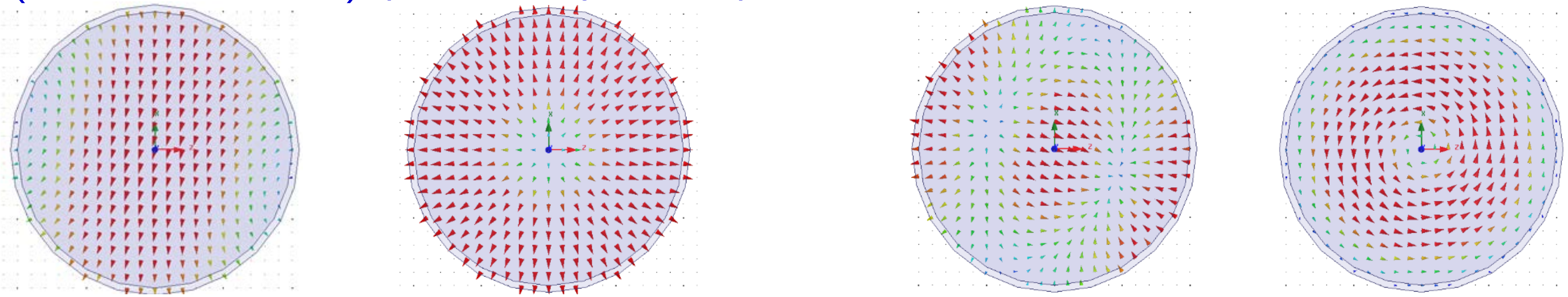
The transverse field components are

$$E_\rho = \frac{-j\omega\mu n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}, \qquad H_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z},$$

$$E_\phi = \frac{j\omega\mu}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}, \qquad H_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z}.$$

The constants *A* and *B* depend on excitation scheme, one of them can be made to zero.

(fundamental mode) (lowest axisymmetric)



TE_{11} TM_{01} (TE_{21}) TM_{11} TE_{01}

E-field distributions for different modes

Circular waveguide (TE mode)

The wave impedance is $$Z_{\mathrm{TE}} = \frac{E_\rho}{H_\phi} = \frac{-E_\phi}{H_\rho} = \frac{\eta k}{\beta}.$$

For the fundamental TE_{11} mode (putting B = 0),

$$H_z = A \sin\phi J_1(k_c\rho)e^{-j\beta z},$$

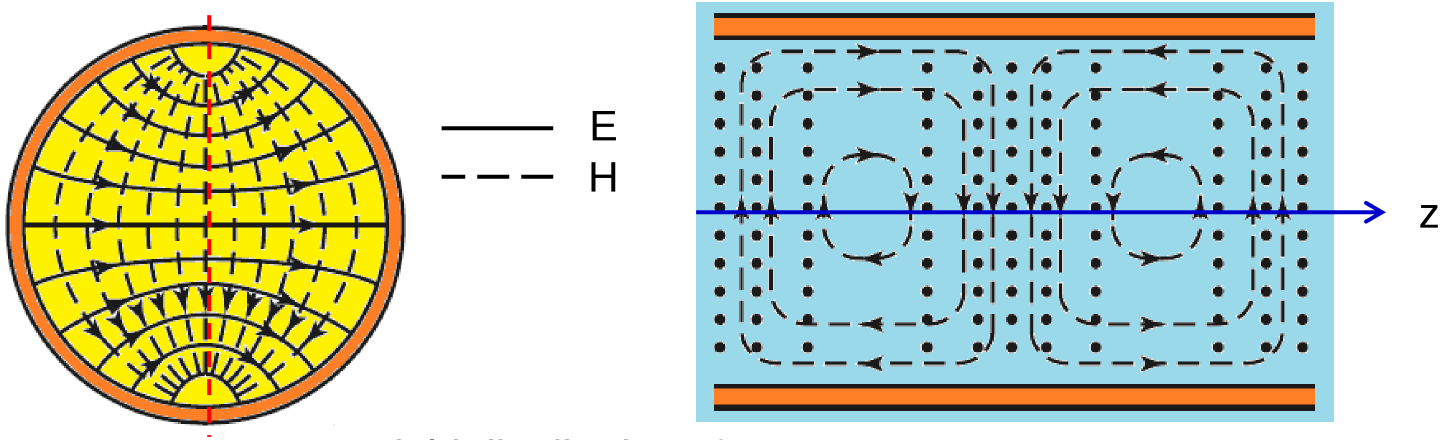
$$E_\rho = \frac{-j\omega\mu}{k_c^2\rho} A \cos\phi J_1(k_c\rho)e^{-j\beta z},$$

$$E_\phi = \frac{j\omega\mu}{k_c} A \sin\phi J_1'(k_c\rho)e^{-j\beta z},$$

$$H_\rho = \frac{-j\beta}{k_c} A \sin\phi J_1'(k_c\rho)e^{-j\beta z},$$

$$H_\phi = \frac{-j\beta}{k_c^2\rho} A \cos\phi J_1(k_c\rho)e^{-j\beta z},$$

$$E_z = 0.$$



Field distributions for TE_{11}

Circular waveguide (TM mode)

For TM modes, $H_z = 0$, and E_z is a solution to the wave equation.

Then, start with the Helmholtz wave equation in cylindrical coordinate,

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + k_c^2 \right) e_z = 0,$$

with $E_z(\rho, \phi, z) = e_z(\rho, \phi) e^{-j\beta z}$ and $k_c^2 = k^2 - \beta^2$

Thus, $e_z(\rho, \phi) = (A \sin n\phi + B \cos n\phi) J_n(k_c \rho).$

Values of p_{nm} for TM modes of a circular waveguide

n	p_{n1}	p_{n2}	p_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

Circular waveguide (TM mode)

Applying the boundary condition, $E_z(\rho, \phi) = 0 \quad \text{at } \rho = a.$

→ $J_n(k_c a) = 0,$

→ $k_c = p_{nm}/a,$

Then, the propagation constant is $\beta_{nm} = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (p_{nm}/a)^2},$

The cutoff frequency is $f_{c_{nm}} = \dfrac{k_c}{2\pi\sqrt{\mu\epsilon}} = \dfrac{p_{nm}}{2\pi a\sqrt{\mu\epsilon}}.$

- There is no TM_{10} mode, but TM_{01} mode exists.
- TE_{11} mode is the fundamental mode.

Circular waveguide (TM mode)

The transverse field components are

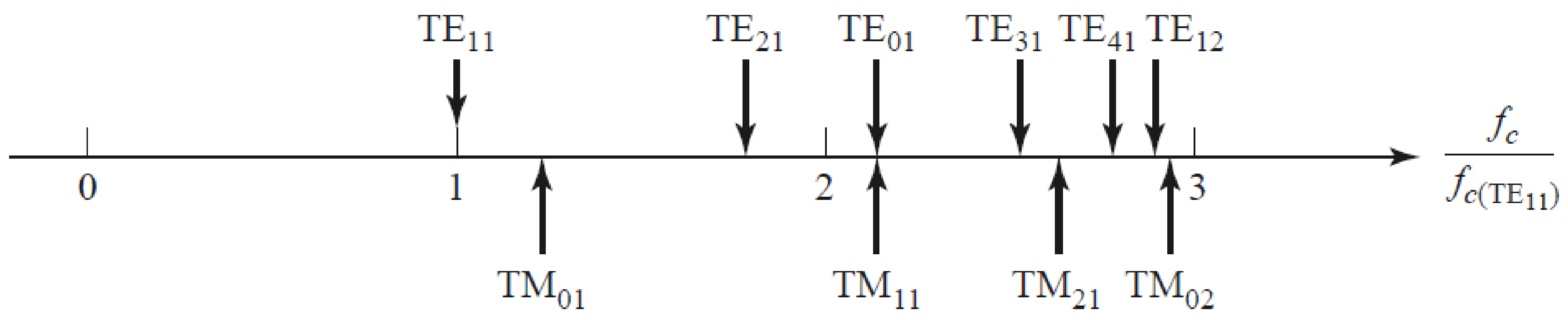
$$E_\rho = \frac{-j\beta}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z},$$

$$E_\phi = \frac{-j\beta n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z},$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2\rho}(A\cos n\phi - B\sin n\phi)J_n(k_c\rho)e^{-j\beta z},$$

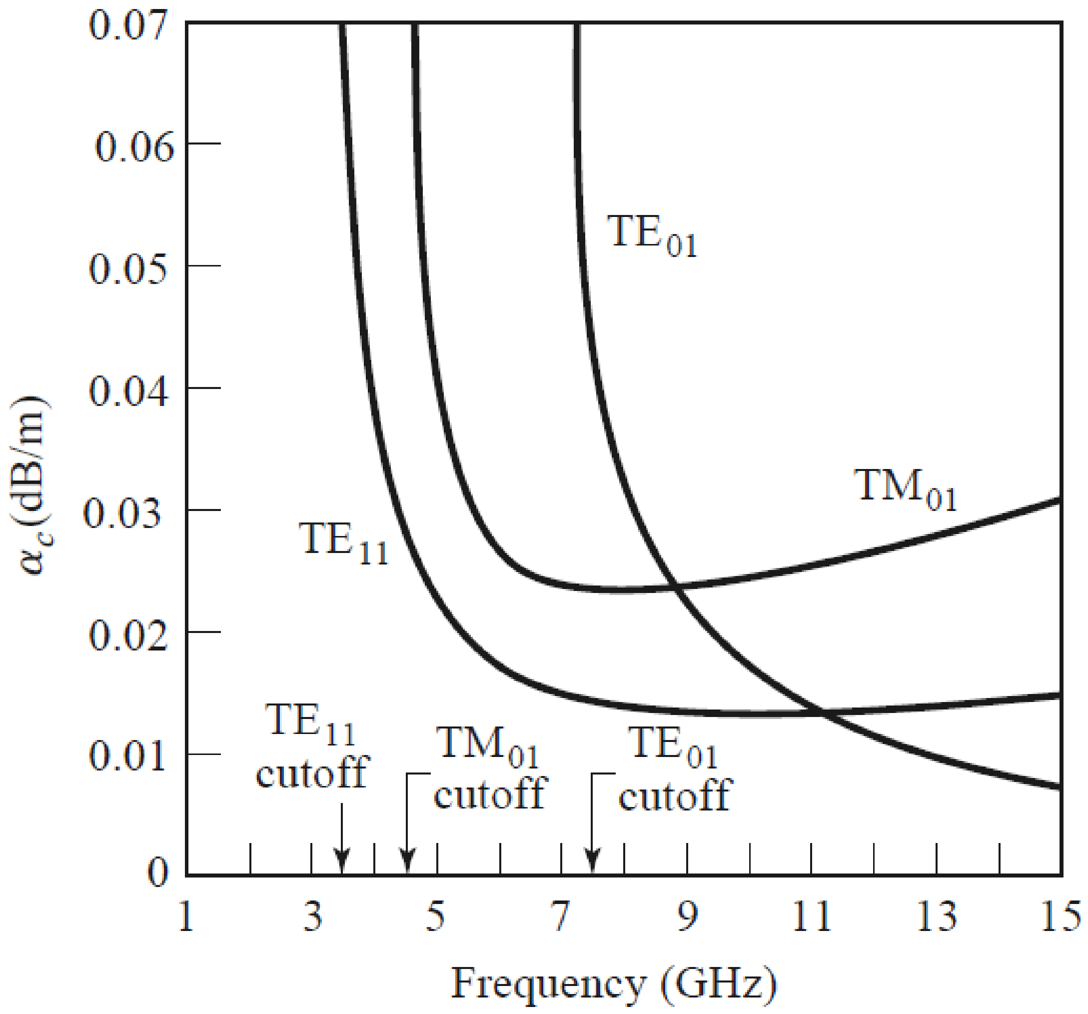
$$H_\phi = \frac{-j\omega\epsilon}{k_c}(A\sin n\phi + B\cos n\phi)J_n'(k_c\rho)e^{-j\beta z}.$$

The wave impedance is $Z_{\mathrm{TM}} = \dfrac{E_\rho}{H_\phi} = \dfrac{-E_\phi}{H_\rho} = \dfrac{\eta\beta}{k}.$



Mode chart: cutoff frequencies.

Circular waveguide (TM mode)



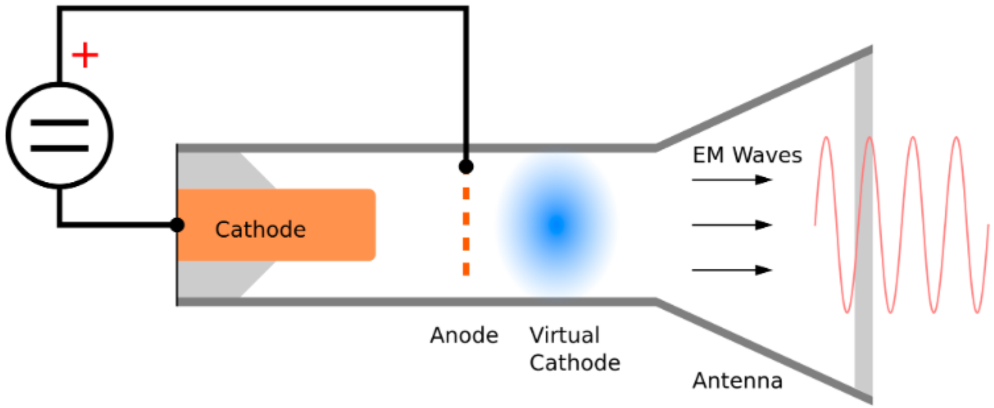
Attenuation of different modes.

Standard circular waveguides

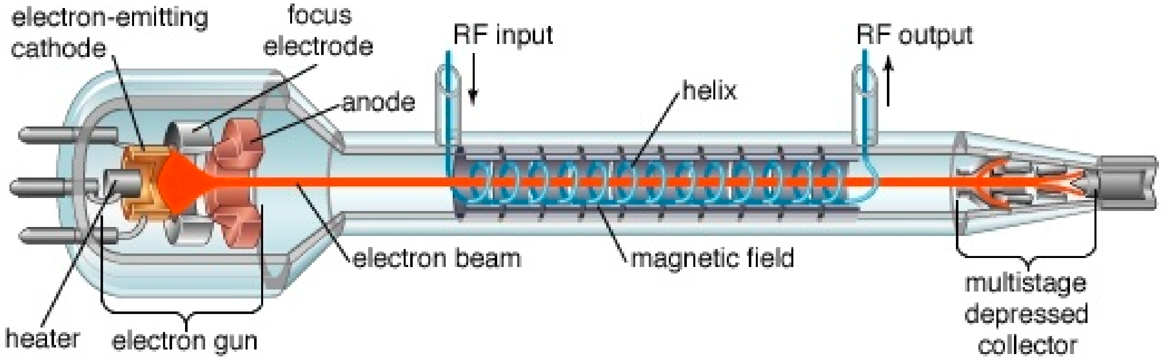
Table Of Internal Diameters for TE_{11} Circular Waveguides

Band Designation	Frequency Range, GHz	Internal Diameter, inch	Band Designation	Frequency Range, GHz	Internal Diameter, inch
Ku-1	12.4-14.6	0.660	E-0	58-68	0.141
Ku-2	14.6-17.5	0.550	E-1	68-77	0.125
K-1	17.5-20.5	0.470	E-2	77-87	0.110
K-2	20.5-24.5	0.396	E-3	87-100	0.094
K-3	24.5-26.5	0.328	W-0	77-87	0.110
Ka-0	26-28.5	0.328	W-1	87-100	0.094
Ka-1	28.5-33	0.281	W-2	100-112	0.082
Ka-2	33-38.5	0.250	F-0	87-100	0.094
Ka-3	38.5-43	0.219	F-1	100-112	0.082
Q-0	33-38.5	0.250	F-2	112-125	0.075
Q-1	38.5-43	0.219	F-3	125-140	0.067
Q-2	43-50	0.188	D-0	100-112	0.082
U-0	38.5-43	0.219	D-1	112-125	0.075
U-1	43-50	0.188	D-2	125-140	0.067
U-2	50-58	0.165	D-3	140-160	0.059
V-0	50-58	0.165	G-0	125-140	0.067
V-1	58-68	0.141	G-1	140-220	0.059
V-2	68-77	0.125			

Circular waveguides



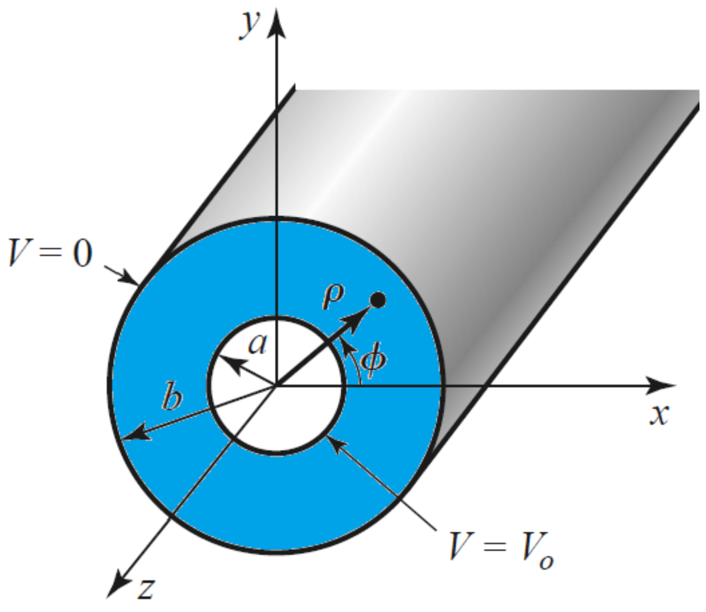
Vircator (VIRtual CAthode oscillator)



Travelling-wave tube.

Coaxial line

A coaxial cable can support TEM, TE and TM modes.



A coaxial cable.

TEM mode:

$E_z = H_z = 0$ → the inner conductor is at potential V_0.

Field can be derived from scalar potential $\Phi(\rho, \phi)$

Start from the Laplace's equation in cylindrical coordinate,

$$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \Phi(\rho,\phi)}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Phi(\rho,\phi)}{\partial \phi^2} = 0.$$

The boundary conditions are $\Phi(a, \phi) = V_o, \quad \Phi(b, \phi) = 0.$

Separation of variables: $\Phi(\rho, \phi) = R(\rho)P(\phi).$

The Laplace's equation can be written as $\displaystyle \frac{\rho}{R}\frac{\partial}{\partial \rho}\left(\rho \frac{dR}{d\rho}\right) + \frac{1}{P}\frac{d^2 P}{d\phi^2} = 0.$

Coaxial line (TEM mode)

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)+\frac{1}{P}\frac{d^2P}{d\phi^2}=0.$$

The two terms must be equal to constants, so that

$$\frac{\rho}{R}\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)=-k_\rho^2, \qquad \frac{1}{P}\frac{d^2P}{d\phi^2}=-k_\phi^2$$

where $k_\rho^2+k_\phi^2=0.$

The general solution is $P(\phi)=A\cos n\phi+B\sin n\phi,$ where $k_\phi=n$ is an integer.

From boundary condition, $\Phi(\rho,\phi)$ should not vary with φ → n must be zero → $k_\rho = 0$.

→ $P(\phi)=A$ and $$\frac{\partial}{\partial\rho}\left(\rho\frac{dR}{d\rho}\right)=0.$$

The solution for R is $R(\rho)=C\ln\rho+D,$

Therefore, $\Phi(\rho,\phi)=C\ln\rho+D.$

Coaxial line (TEM mode)

$$\Phi(\rho, \phi) = C \ln \rho + D.$$

From boundary condition,

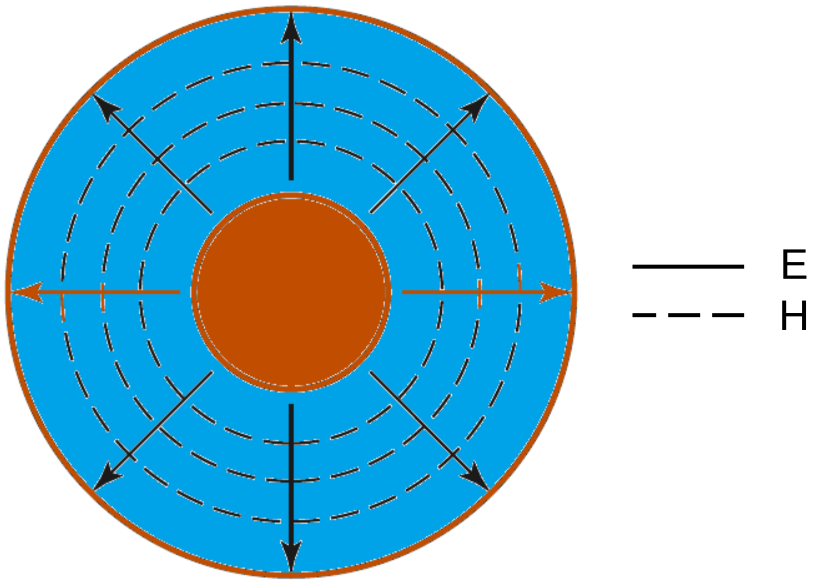
$$\Phi(a, \phi) = V_o = C \ln a + D,$$

$$\Phi(b, \phi) = 0 = C \ln b + D.$$

Therefore, $$\Phi(\rho, \phi) = \frac{V_o \ln b/\rho}{\ln b/a}.$$

Then, the electric and magnetic field can be found from,

$$\bar{e}(x, y) = -\nabla_t \Phi(x, y) \quad \text{and} \quad \bar{h}(x, y) = \frac{1}{Z_{\text{TEM}}} \hat{z} \times \bar{e}(x, y).$$



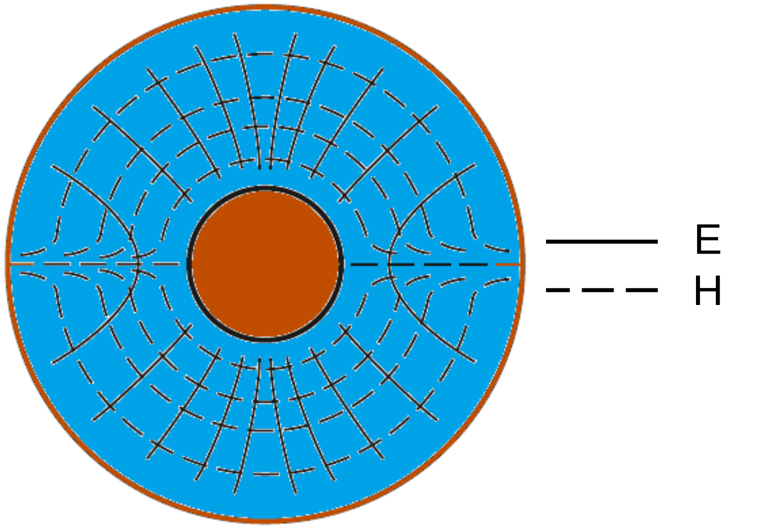
TEM mode in a coaxial cable

Coaxial line (higher order modes)

- Just like the parallel plate waveguide, coaxial line can support both TE and TM modes.
- The dominant mode is TE_{11} mode.

Steps: same as circular waveguide. Start with the wave equation.

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}+k_c^2\right)h_z(\rho,\phi)=0,$$

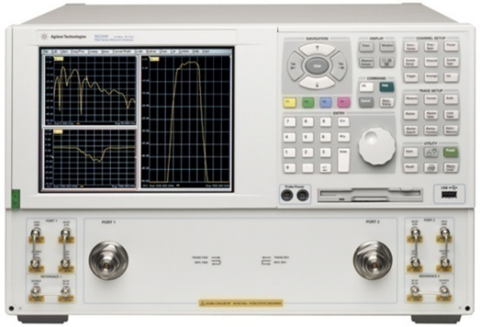


- Cutoff wave number for TE_{11} mode: $k_c = \dfrac{2}{a+b}$. $\rightarrow f_c = \dfrac{c}{\pi(a+b)\sqrt{\varepsilon_r}}$.

TE_{11} mode in a coaxial cable

- Calculate the cutoff frequency for the TE_{11} mode in a SMA connector with 2a = 1 mm and 2*b* = 3.5 mm, dielectric constant = 2.2.

Microwave connectors



VNA

- BNC and TNC are low frequency connectors <1 GHz.

BNC

TNC

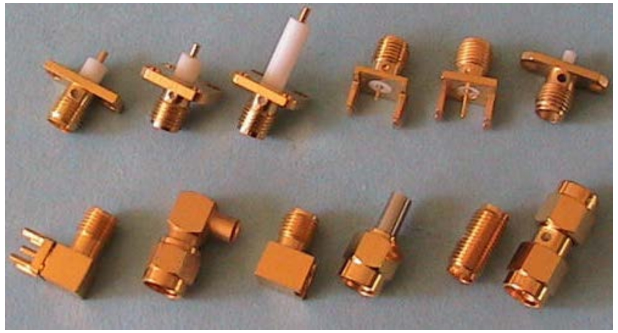
N-type connectors:

- by P. Neill in1940.
- DC-18 GHz
- High power rating.

SMA (sum-miniature version A) connectors:
DC – 18 GHz

Male

female



Different type SMA types: jack, plug, end-launcher etc

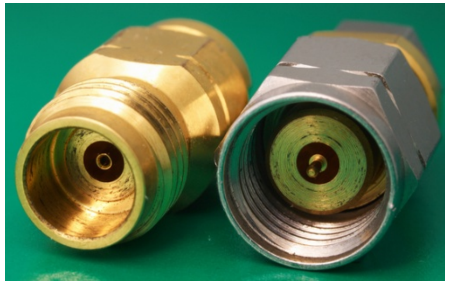
Female Male

Coaxial connectors

Coax Connectors

Connector Type	Frequency Range (GHz)	WiseWave's Designations	
N	DC to 18.0	NF – Female Connector	NM – Male Connector
7mm or APC-7	DC to 18.0	7F– Female Connector	7M – Male Connector
SMA	DC to 18.0	SF – Female Connector	SM – Male Connector
Super SMA	DC to 27.0	SF – Female Connector	SM – Male Connector
3.5 mm	DC to 26.5	3F – Female Connector	3M – Male Connector
2.92 mm or K	DC to 40.0	KF – Female Connector	KM – Male Connector
2.4 mm	DC to 50.0	2F – Female Connector	2M – Male Connector
1.85 mm or V	DC to 65.0	VF – Female Connector	VM – Male Connector
1mm	DC to 110.0	1F – Female Connector	1M – Male Connector

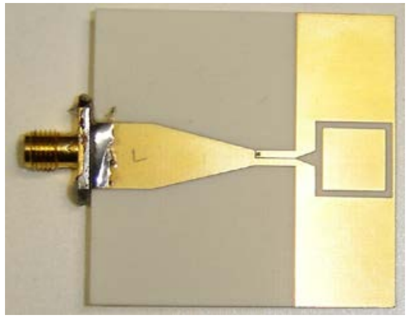
K connectors



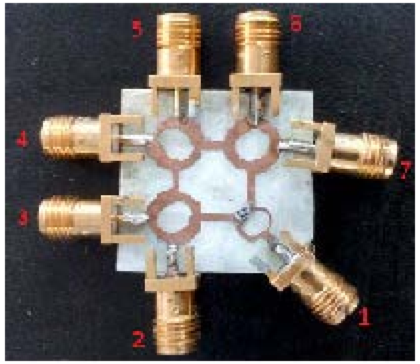
V connectors

Planar guiding structures

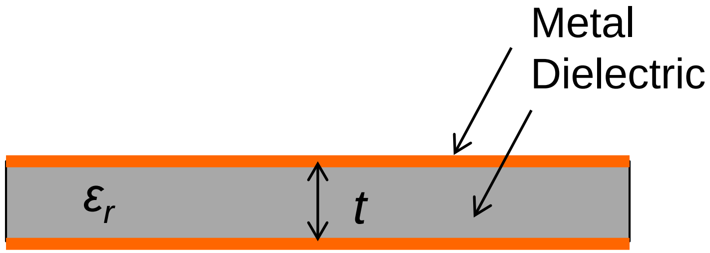
Printed Circuit Boards (PCB)



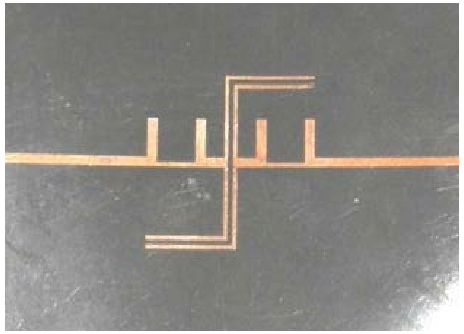
1. A loop antenna with a balun. (RO4003C)



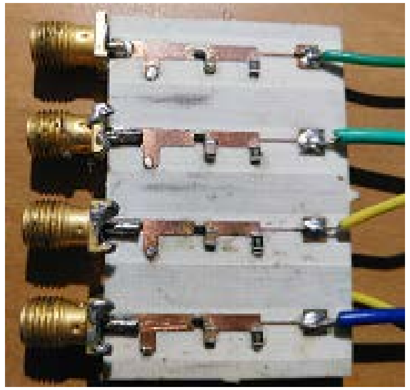
3. Six-port network @10 GHz.



Substrate parameters:
thickness, dielectric constant ε_r, tanδ



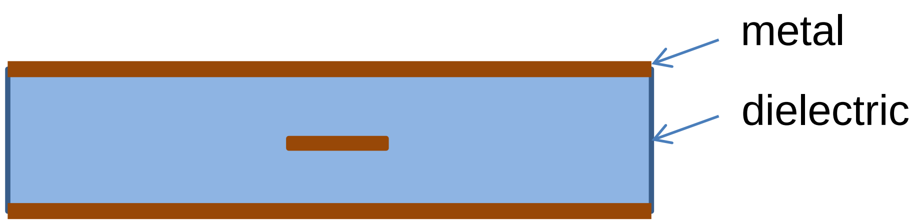
2. An UWB bandpass filter. (Rt/Duroid 5880)



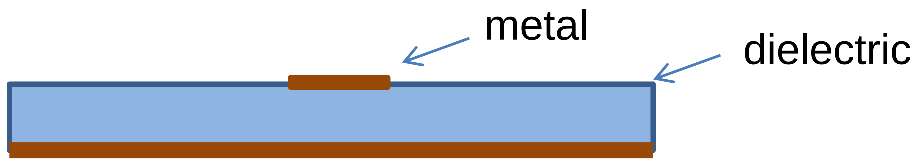
4. Power detectors @10 GHz.

1. M.K. Mandal and Z.H. Chen, Compact dual-band and UWB loop antennas, *IEEE TAP*, Aug. 2011.
2. M.K. Mandal and S. Sanyal, Compact wideband bandpass filter, *IEEE MWCL*, Jan. 2006.

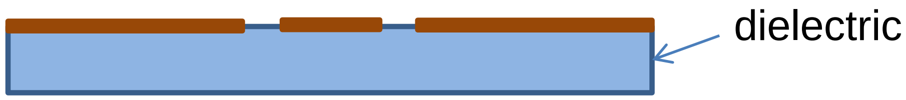
Planar guiding structures



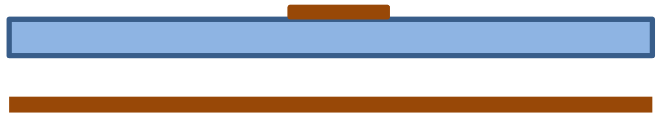
Stripline

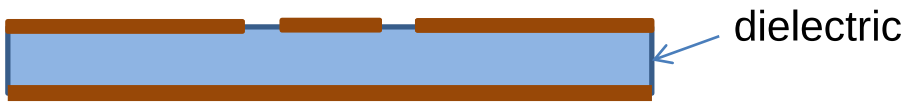


Microstrip line

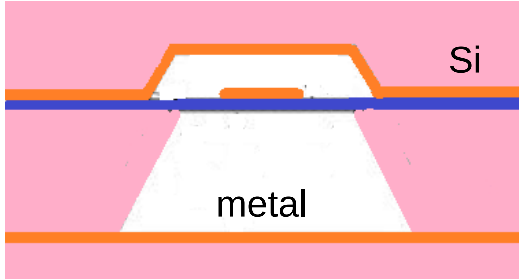


Co-planar waveguide
(CPW)

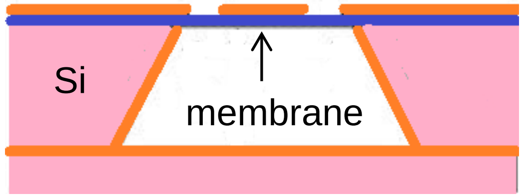

Suspended
microstrip line



Ground-backed CPW



SMM



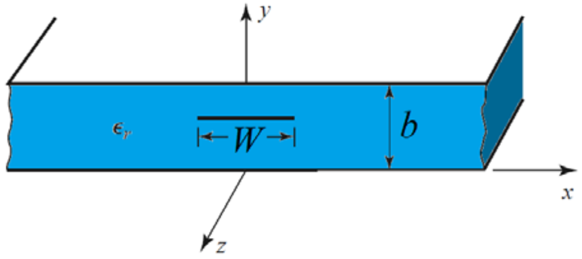
Microshield line

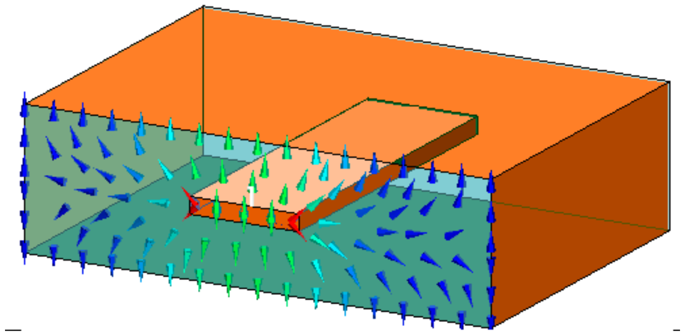
Stripline

- Invented by Robert M. Barrett of the Air Force Cambridge Research Centre in the 1950s.
- Stripline is the earliest form of planar transmission line
- Stripline is always used in TEM mode.
- Can be considered as sliced coaxial line.
- Transmission line equations can be used for initial calculations.
- Electrical length and guided wavelength are, respectively,

$$\theta = \beta l, \quad \lambda_g \approx \frac{\lambda_0}{\sqrt{\varepsilon_r}}.$$

- Phase velocity: $v_p = \dfrac{c}{\sqrt{\varepsilon_r}}.$





E-field distribution.

- Closed form expression for the characteristic impedance

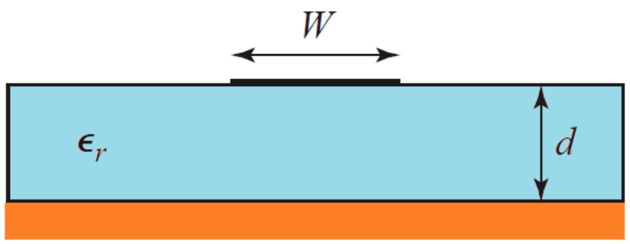
$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b}, \quad \text{where} \quad \frac{W_e}{b} = \frac{W}{b} - \begin{cases} 0 & \text{for } \dfrac{W}{b} > 0.35 \\ (0.35 - W/b)^2 & \text{for } \dfrac{W}{b} < 0.35. \end{cases}$$

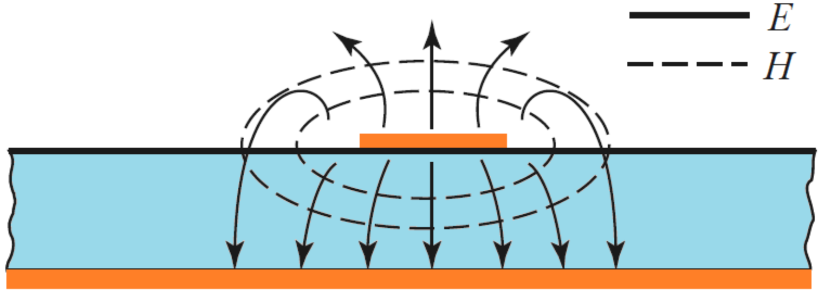
Microstrip line

- One of the most popular planar guiding structure.
- First published by by Grieg and Engelmann in the IRE proceedings, Dec.1952.
- Inhomogeneous medium → modelled by an effective dielectric constant.
- In addition to fields like the TEM mode, a small component of E- and H-fields in the direction of propagation (hybrid mode) – quasi TEM mode.
- Effective dielectric constant:

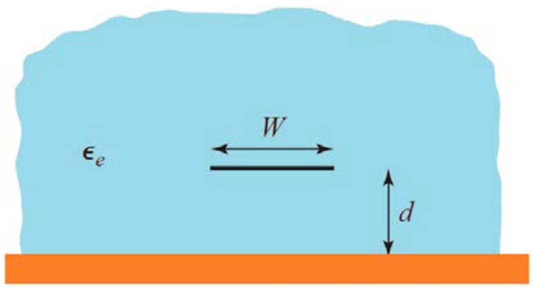
$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \frac{1}{\sqrt{1 + 12d/W}}.$$

- Phase velocity and phase constant: $v_p = \frac{c}{\sqrt{\epsilon_e}}, \ \beta = k_0\sqrt{\epsilon_e}$
- Approximate calculations: $\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{re}}}$ and $\beta = \frac{2\pi}{\lambda_g}$





Field distributions

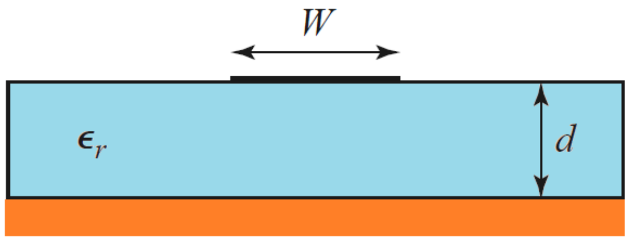


Inside a fictitious dielectric medium.

Microstrip line

Characteristic impedance:

$$Z_0 = \begin{cases} \dfrac{60}{\sqrt{\epsilon_e}} \ln\left(\dfrac{8d}{W} + \dfrac{W}{4d}\right) & \text{for } W/d \leq 1 \\ \dfrac{120\pi}{\sqrt{\epsilon_e}\left[W/d + 1.393 + 0.667 \ln\left(W/d + 1.444\right)\right]} & \text{for } W/d \geq 1. \end{cases}$$

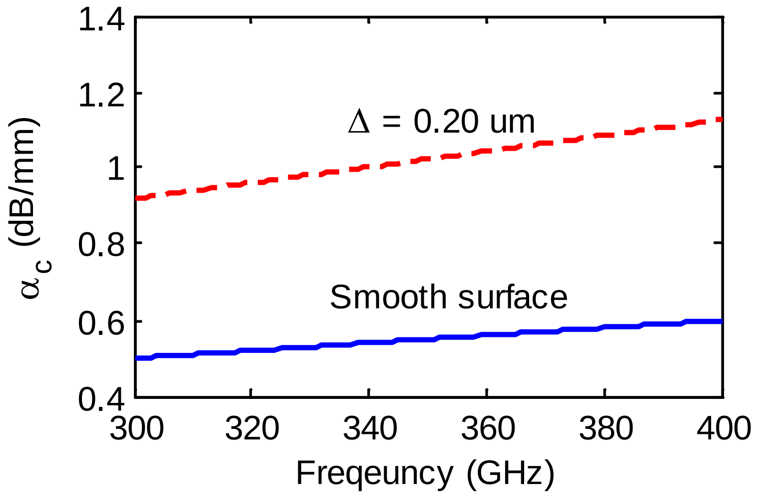


- Attenuation constant due to dielectric loss:

$$\alpha_d = \frac{k_0 \epsilon_r (\epsilon_e - 1) \tan\delta}{2\sqrt{\epsilon_e}(\epsilon_r - 1)} \text{ Np/m}$$

- Attenuation constant due to conductor loss:

$$\alpha_c = \frac{R_s}{Z_0 W} \text{ Np/m,} \quad \text{where} \quad R_s = \sqrt{\omega\mu_0/2\sigma}$$



Plot of attenuation constant for a 50 Ω microstrip line (metal thickness 3 μm on 30 μm BCB).

Microstrip line: frequency dependent parameters

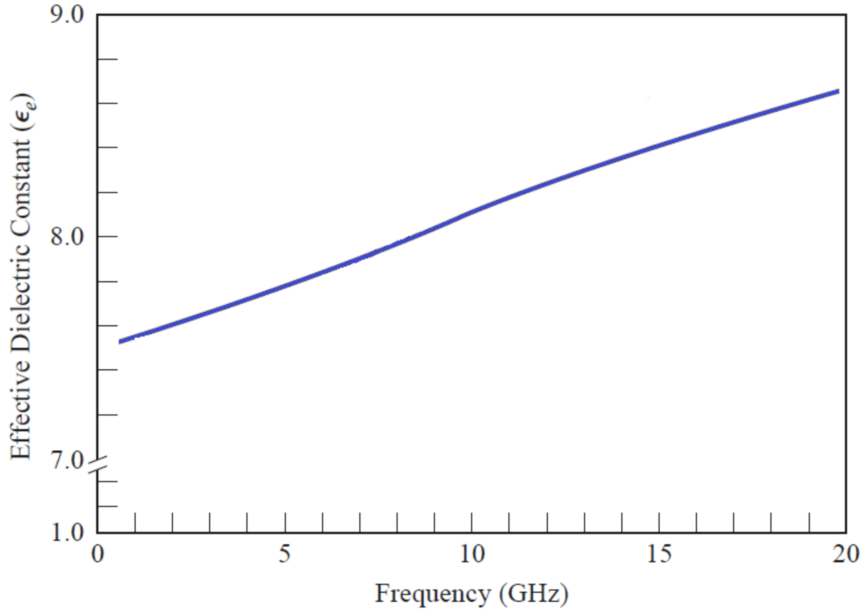
- Frequency dependent ϵ_e :

$$\varepsilon_e(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_e(0)}{1 + (f/f_{50})^m} \qquad \text{where} \quad f_{50} = \frac{f_{T1}}{0.75 + (0.75 - 0.332\varepsilon_r^{-1.73})W/h}$$

- Frequency dependent Z_0 :

$$Z_c(f) = Z_c \frac{\varepsilon_e(f) - 1}{\varepsilon_e(0) - 1} \sqrt{\frac{\varepsilon_e(0)}{\varepsilon_e(f)}}$$

The effect on phase delay is more prominent.



Variation of ϵ_e of a 25 Ω line on a 0.65 mm thick substrate with ϵ_r = 10.

Microstrip line: high frequency limitations

- Threshold frequency of coupling to TM_0 surface wave mode:

$$f_{T1} \simeq \frac{c}{2\pi d}\sqrt{\frac{2}{\epsilon_r - 1}}\tan^{-1}\epsilon_r .$$

$<f_c$ of TM_1 mode.

- Threshold frequency of coupling to TE_1 surface wave mode because of bends, junctions, or even step changes in width :

$$f_{T2} \simeq \frac{c}{4d\sqrt{\epsilon_r - 1}}$$

- Threshold frequency of transverse resonance:

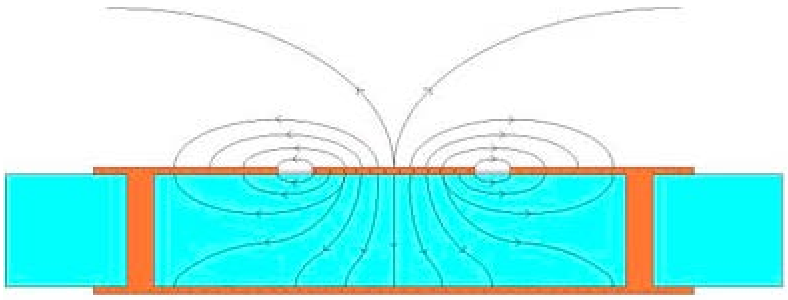
$$f_{T3} \simeq \frac{c}{\sqrt{\epsilon_r}\,(2W + d)}.$$

- Threshold frequency of parallel plate mode:

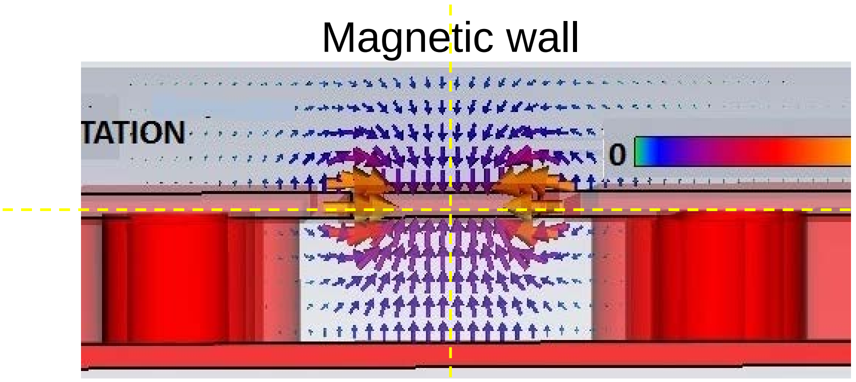
$$f_{T4} \simeq \frac{c}{2d\sqrt{\epsilon_r}}$$

Coplanar waveguide (CPW)

- Coplanar waveguide (CPW) was invented in 1969 by Cheng P. Wen.
- Supports quasi-TEM mode.
- Can be excited in even- and odd-modes.

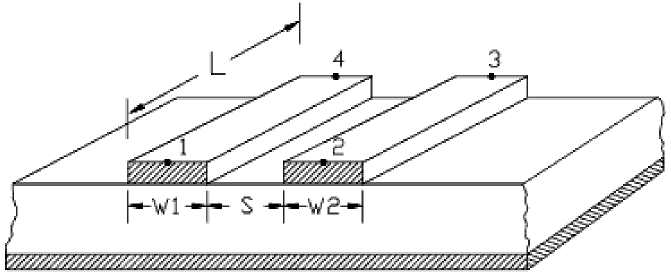


Vector electric field distribution for a CPW (even-mode).



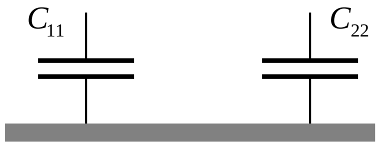
Vector electric field distribution for ground-backed CPW.

Coupled microstrip line



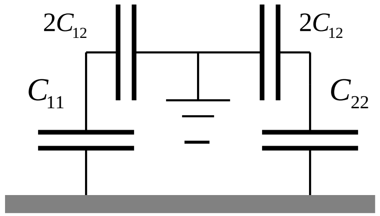
Microstrip coupled lines.

Question: ε_e or ε_o which one is higher?



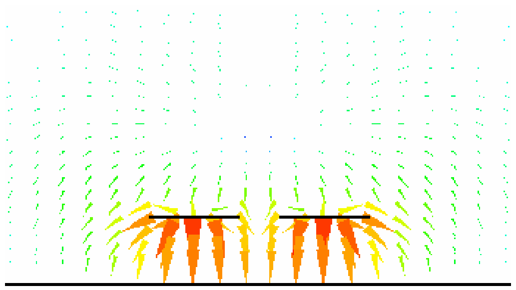
$$C_e = C_{11} = C_{22}$$

$$Z_{0e} = \frac{1}{v_p C_e}.$$

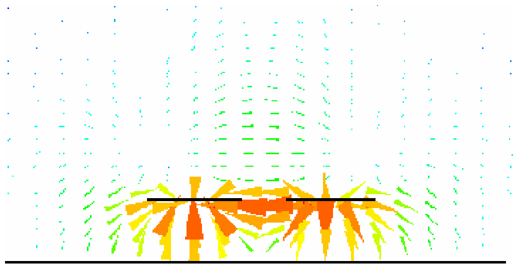


$$C_o = C_{11} + 2C_{12}$$

$$Z_{0o} = \frac{1}{v_p C_o}.$$



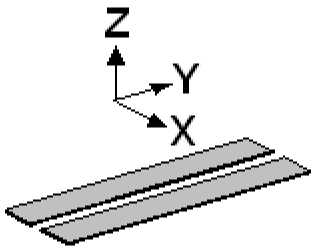
Even-mode

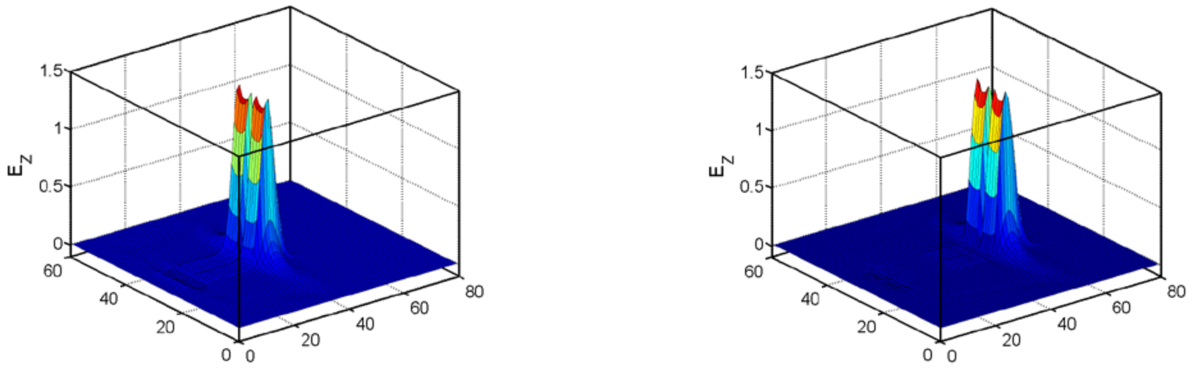


Odd-mode

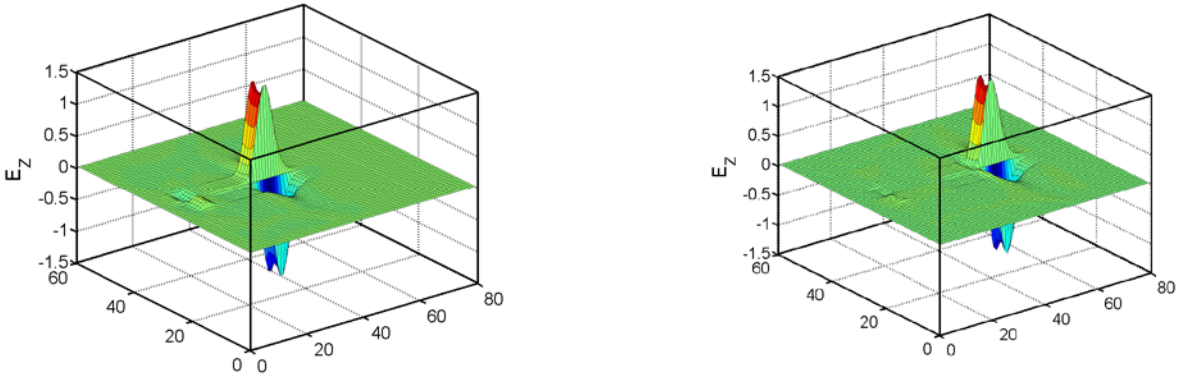
Field distributions in even- and odd-mode, blue lines: electric fields.

Coupled microstrip line





Pulse propagation in even-mode.

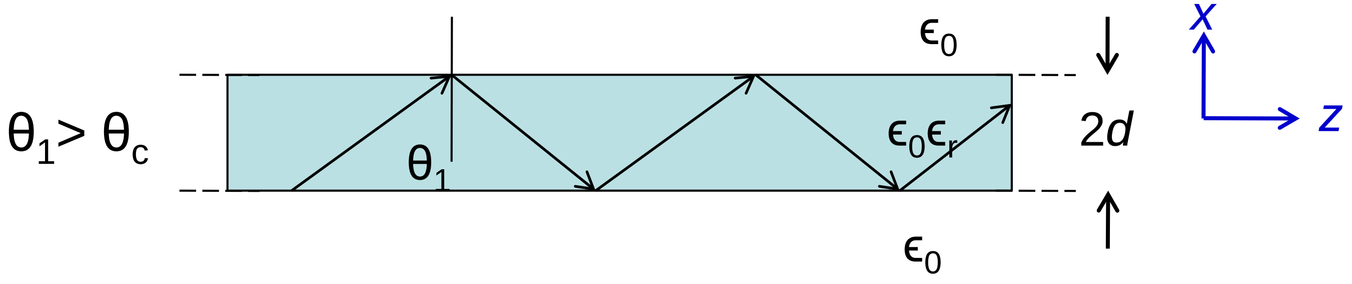


Pulse propagation in odd-mode.

•Different fringing fields→ even-mode electrical length> odd-mode electrical length.

•Solutions→ zigzag inside edge, dielectric overlay, ground plane aperture.

Surface waves



Surface wave modes in a 2d dielectric slab

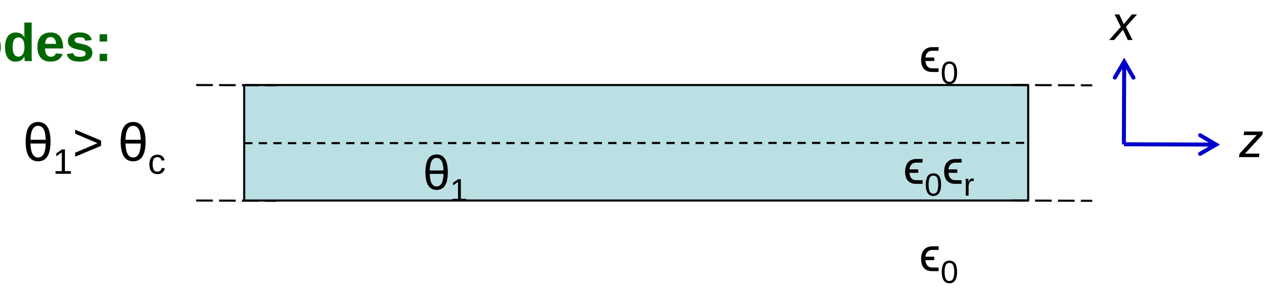
- Both *TM* and *TE* modes are possible.
- *TM* modes (E-modes): **E** is parallel to plane of incidence (*yz*), $E_z \neq 0 \rightarrow H_y, E_x, E_z$.
- *TE* modes (M-modes): **H** is parallel to plane of incidence (*yz*), $H_z \neq 0 \rightarrow E_y, H_x, H_z$.
- The relationship between the propagation constant and attenuation constant:

  $\beta^2 = k_0^2 + \alpha^2$. (+*z* direction: $e^{-j\beta z}$)
- Field decays from the surface as $e^{-\alpha(|y|-d/2)}$

- Field Theory of Guided Waves – R.E. Collin, *McGraw-Hill*.
- Millimeter Wave and Optical Dielectric Integrated Guides and Circuits – Shiban K. Koul, *Wiley*.

Surface waves

TM modes:



Surface wave modes in a 2d dielectric slab

H_y, E_x, E_z

- Two parts: (i) $E_z = 0$, $(\partial H_y/\partial x = 0)$ at $x = 0$ → symmetric mode.

$$H_y = \begin{cases} A\sec(pd)\cos(px), & |x| \le d \\ Ae^{-\alpha(|x|-d)}, & |x| \ge d. \end{cases}$$

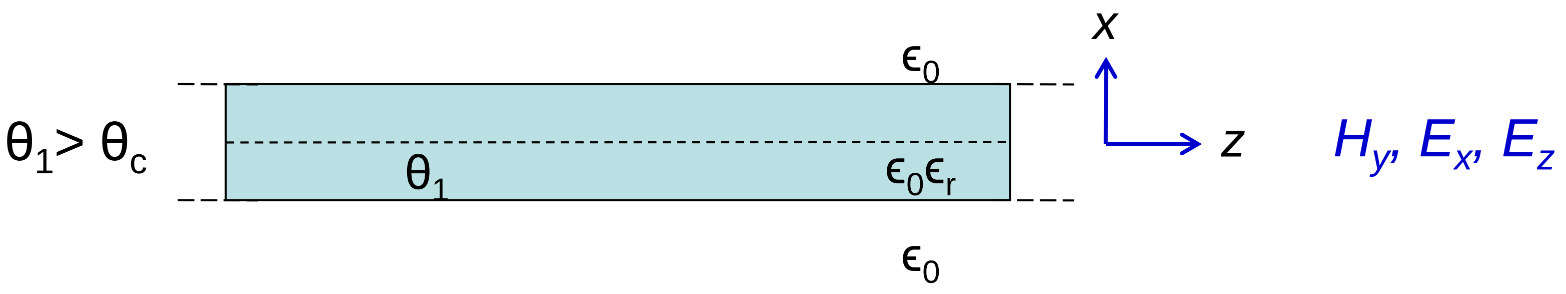
p → x-directed wave number, $v_{ph} = \omega/\beta$.

(ii) $H_y = 0$, $(\partial E_x/\partial z = 0)$ at $x = 0$ → antisymmetric mode.

$$H_y = \begin{cases} Ae^{-\alpha(x-d)}, & |x| \ge d \\ A\,\mathrm{cosec}(pd)\sin(px), & -d \le x \le d \\ -Ae^{-\alpha(x+d)}, & |x| \le -d \end{cases}$$

The eigenvalue equation for each mode is obtained by matching the tangential fields at $x = \pm d$.

Surface waves (TM modes)



Surface wave modes in a 2d dielectric slab

The eigenvalue equations:

$$\varepsilon_r \alpha d = pd \tan(pd), \quad \text{symmetric modes}$$

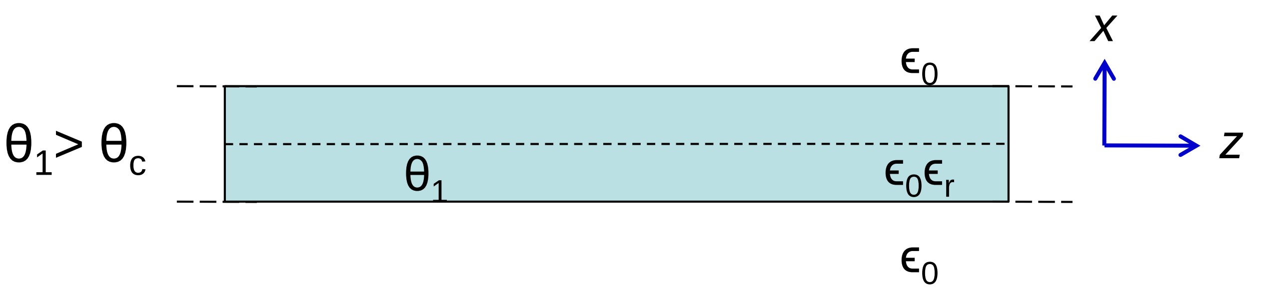
$$\varepsilon_r \alpha d = -pd \cot(pd), \quad \text{antisymmetric modes}$$

where

$$(\alpha d)^2 + (pd)^2 = (\varepsilon_r - 1)(k_0 d)^2$$

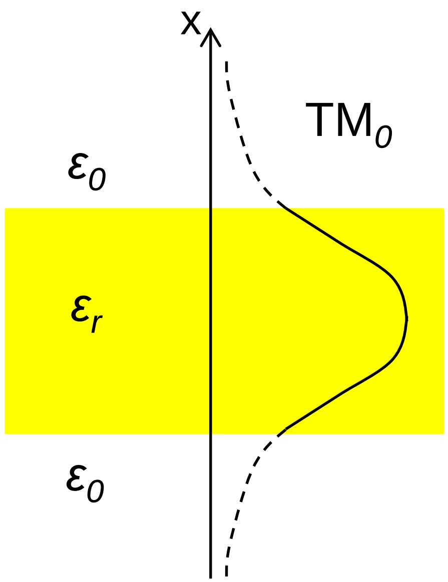
$$\beta^2 = k_0^2 + \alpha^2 = k_0^2 \varepsilon_r - p^2.$$

Surface waves (TM modes)

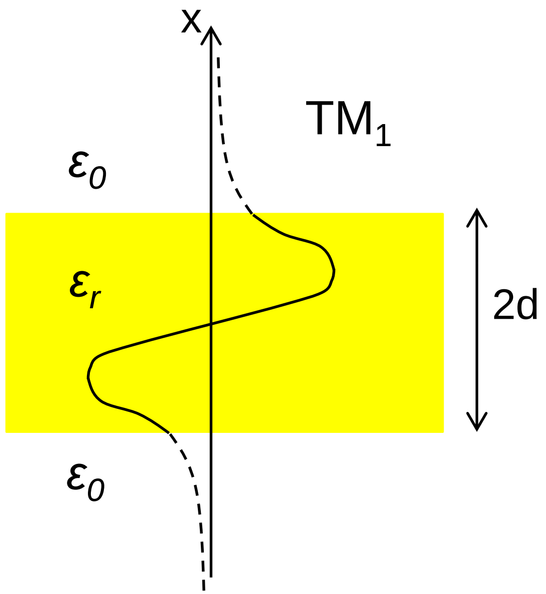


H_y, E_x, E_z

Surface wave modes in a 2d dielectric slab



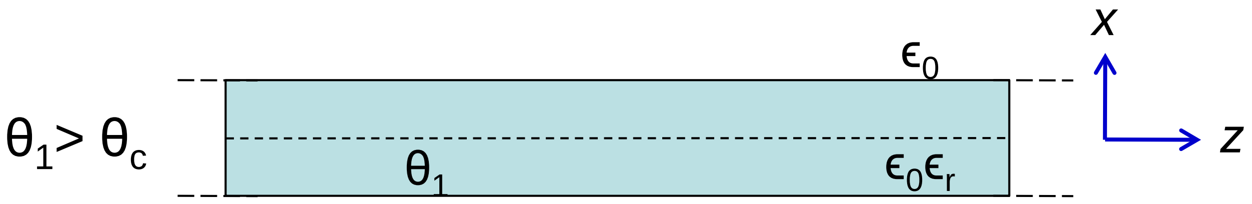
Variation of H_y component.
(symmetric mode)



Variation of H_y component.
(antisymmetric mode)

Surface waves

TE modes:



$E_y, H_x, H_z.$

$$E_y = \begin{cases} A\sec(pd)\cos(px), & |x| \le d \\ Ae^{-\alpha(|x|-d)}, & |x| \ge d. \end{cases}$$

Symmetric mode (H_z = 0 at x = 0)

$$E_y = \begin{cases} Ae^{-\alpha(x-d)}, & |x| \ge d \\ A\,\mathrm{cosec}(pd)\sin(px), & -d \le x \le d \\ -Ae^{-\alpha(x+d)}, & |x| \le -d \end{cases}$$

Antisymmetric mode (E_y = 0 at x = 0)

The eigenvalue equations:

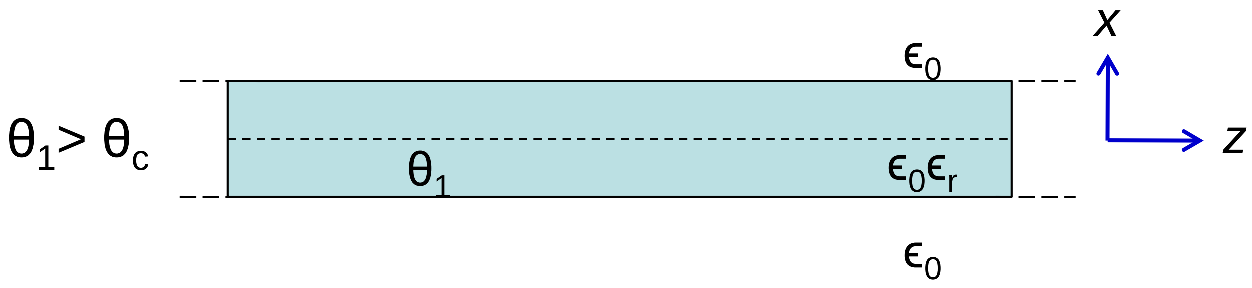
$\alpha d = pd\ \tan(pd)$, symmetric modes

$\alpha d = -pd\ \cot(pd)$, antisymmetric modes

where $(\alpha d)^2 + (pd)^2 = (\varepsilon_r - 1)(k_0 d)^2$

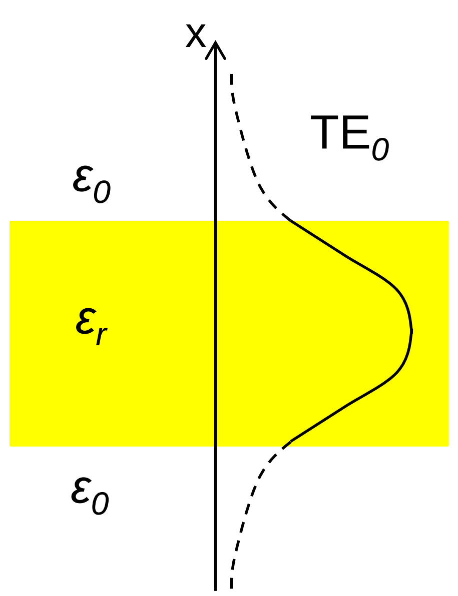
$\beta^2 = k_0^2 + \alpha^2 = k_0^2\varepsilon_r - p^2.$

Surface waves (TE modes)

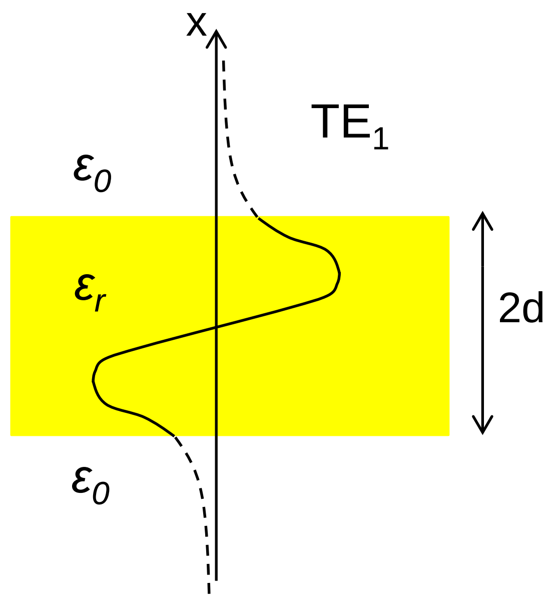


$E_y, H_x, H_z.$

Surface wave modes in a 2d dielectric slab



Variation of E_y component.
(symmetric mode)



Variation of E_y component.
(antisymmetric mode)

Surface waves

Cutoff frequencies:

- TM_0 and TE_0 has no cutoff frequencies.
- For, TM_m and TE_m modes, at cutoff frequencies:

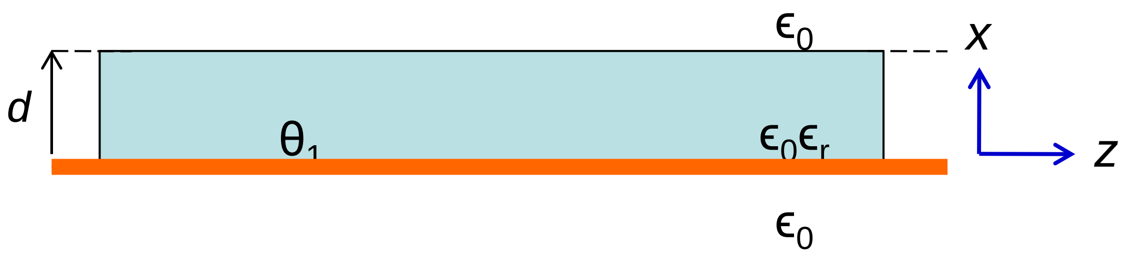
$$\frac{2d}{\lambda_0} = \frac{m}{2\left(\varepsilon_r - 1\right)^{1/2}}, \qquad m = 0, 1, 2...$$

For symmetric mode: m = 0, 2, 4...
for antisymmetric mode: m = 1, 3, 5...

Surface waves: grounded slab

TM modes:

$\theta_1 > \theta_c$



Surface wave modes in a grounded dielectric slab

H_y, E_x, E_z

$$E_z = \begin{cases} A \sin k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$E_x = \begin{cases} \dfrac{-j\beta}{k_c} A \cos k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{-j\beta}{h} A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$H_y = \begin{cases} \dfrac{-j\omega\epsilon_0\epsilon_r}{k_c} A \cos k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{-j\omega\epsilon_0}{h} A \sin k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty. \end{cases}$$

Cutoff wave numbers for the two regions:

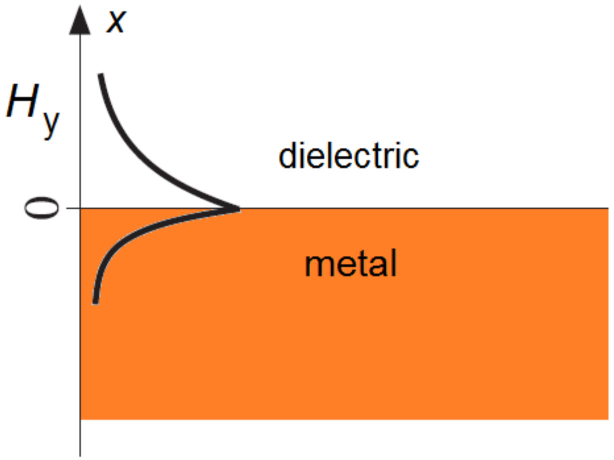
$$k_c^2 = \varepsilon_r k_0^2 - \beta^2,$$
$$h^2 = \beta^2 - k_0^2.$$

- Microwave Engineering – D.M. Pozar, *Wiley*.

Surface waves (TM modes)

where

$$k_c^2 = \epsilon_r k_0^2 - \beta^2, \qquad h^2 = \beta^2 - k_0^2$$



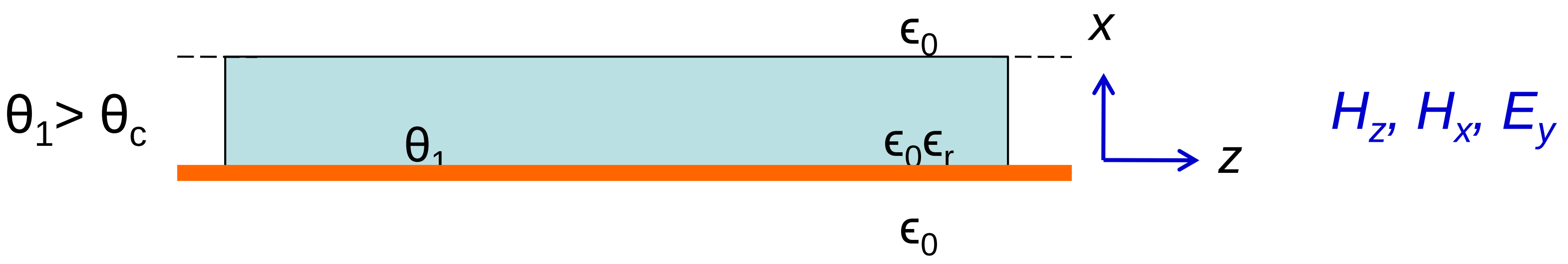
Cutoff frequency for TM_n modes,

$$f_c = \frac{nc}{2d\sqrt{\epsilon_r - 1}}, \qquad n = 0,\ 1,\ 2, \ldots.$$

- TM_0 mode has no cutoff frequency ($k_c = 0$).
- E_z component is zero on ground and maximum at the interface, decays exponentially in air.
- E_x (H_y) is maximum on the ground.
- E_z and E_x are in phase quadrature.

Surface waves

TE modes:



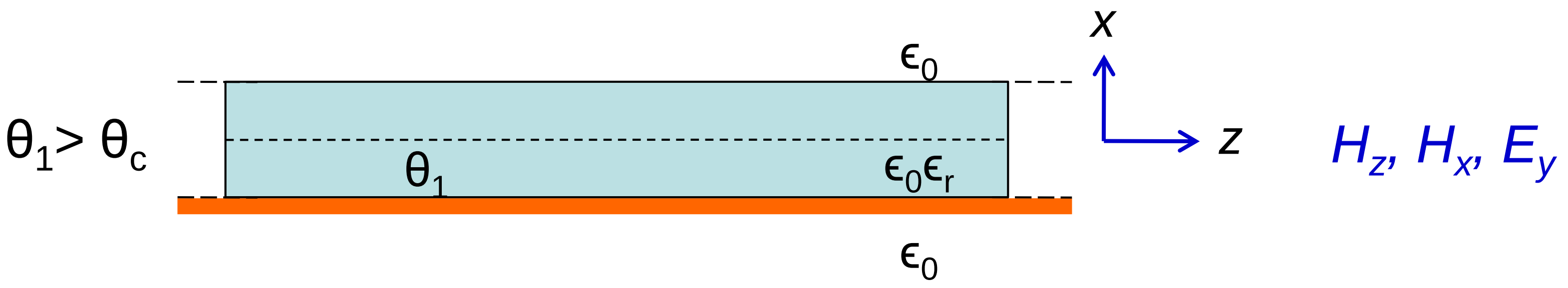
Surface wave modes in a grounded d dielectric slab

$$H_z = \begin{cases} B \cos k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ B \cos k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$H_x = \begin{cases} \dfrac{j\beta}{k_c} B \sin k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{-j\beta}{h} B \cos k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty, \end{cases}$$

$$E_y = \begin{cases} \dfrac{-j\omega\mu_0}{k_c} B \sin k_c x e^{-j\beta z} & \text{for } 0 \le x \le d \\ \dfrac{j\omega\mu_0}{h} B \cos k_c d e^{-h(x-d)} e^{-j\beta z} & \text{for } d \le x < \infty. \end{cases}$$

Surface waves (TE modes)



Surface wave modes in a grounded 2d dielectric slab

Cutoff frequency for TE_n modes,

$$f_c = \frac{(2n-1)c}{4d\sqrt{\epsilon_r - 1}} \qquad \text{for } n = 1,\ 2,\ 3, \ldots .$$

- TE_0 mode has no cutoff frequency ($k_c = 0$).
- H_x component is zero on ground and maximum at the interface, decays exponentially in air.
- H_z is maximum on the ground (half of previous plot).
- H_z and H_x are in phase quadrature.

Sources of loss

1. **Dielectric loss**
2. **Ohmic loss or conductor loss**
3. **Radiation/ surface wave losses.**

- Complex propagation constant: $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\varepsilon}\sqrt{1 - j\dfrac{\sigma}{\omega\varepsilon}}$

- Power flow along a lossy line (without reflection): $P(z) = P_o e^{-2\alpha z}$

Leakage constant:

$$|S_{11}|^2 + |S_{21}|^2 = e^{-2\alpha L}$$

Phase constant by length difference method:

$$\beta = \Delta\theta / \Delta L.$$

- Microstrip lines and slotlines – K.C. Gupta, R. Garg, I. Bahl and P. Bhartia (*Artech House*).
- Microwave Engineering – D.M. Pozar, *Wiley*.

Sources of loss

- Permittivity of a dielectric : $\varepsilon' - j\varepsilon''$

Let the electric field is $\mathrm{E} = \mathrm{E}_0 e^{j\omega t}$

Applying Maxwell's curl equation, $\nabla \times H = j\omega\varepsilon'\mathrm{E} + (\omega\varepsilon'' + \sigma)\mathrm{E}$

Loss due to bound charge (relaxation) + free charge

Electric loss tangent (tanδ) = lossy reaction/ lossless reaction

= (ωϵ'' + σ)/ ωϵ'

≈ ϵ''/ ϵ'

- Permeability: $\mu = \mu' - j\mu''$

Similarly, magnetic loss tangent $\tan\delta_m = \dfrac{\mu''}{\mu'}$

For microstrip and CPW lines, dielectric loss per unit length: $\alpha_d = 2.73 \dfrac{\varepsilon_r}{\sqrt{\varepsilon_{re}}} \dfrac{\varepsilon_{re} - 1}{\varepsilon_r - 1} \dfrac{\tan\delta}{\lambda_o} dB$

Sources of loss

Conductor loss per unit length (assuming smooth surface):

$$\alpha_c = \frac{\beta\delta_s}{4Z_0}\frac{dZ_0}{d\ell} = \frac{R_S}{2Z_0\eta}\frac{dZ_0}{d\ell}$$

where $\frac{dZ_0}{d\ell}$ change in Z_0 if all the metal parts are reduced by d*l*.

$$R_S = \sqrt{\omega\mu_0/2\sigma} = 1/\sigma\delta_S$$

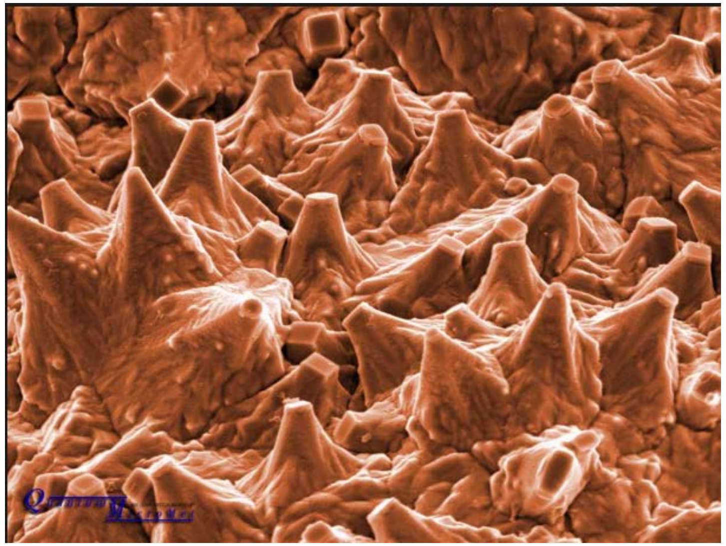
surface resistance.

Conductor loss increases with frequency.

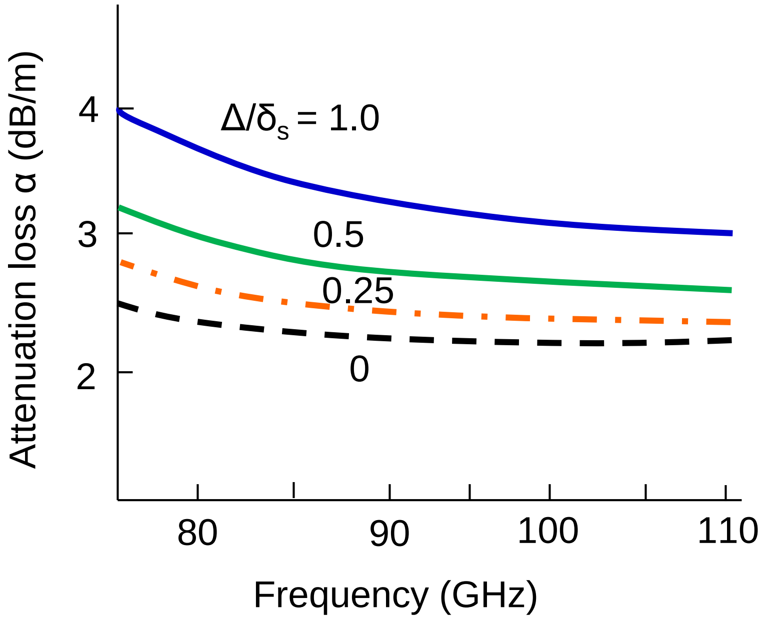
Considering Δ as the *rms* surface roughness, corrected attenuation constant:

$$\alpha_c' = \alpha_c\left[1+\frac{2}{\pi}\tan^{-1}1.4\left(\frac{\Delta}{\delta_s}\right)^2\right]$$

Sources of loss



Electrodeposited copper
(surface roughness ~0.5-0.35 μm)



Variation of calculated attenuation constants of an air-filled waveguide with surface roughness.

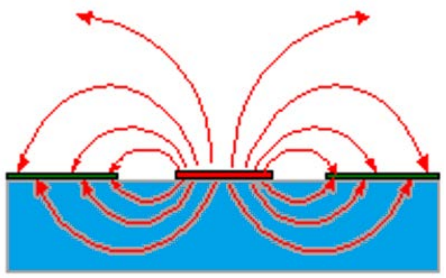
- Effect of surface roughness is more prominent on the attenuation constant than phase constant.

Sources of loss

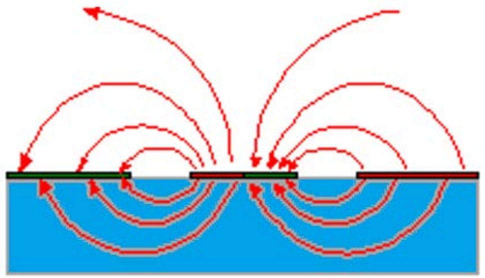
- Radiation from unwanted modes: leakage, substrate parameters, shape of the structure.
- Unwanted modes are created at discontinuities.

Example:

- odd mode in CPW is a radiating mode. Use air bridges at regular interval to avoid.
- Parallel plate modes in conductor backed CPW. Use periodic via to avoid this mode.



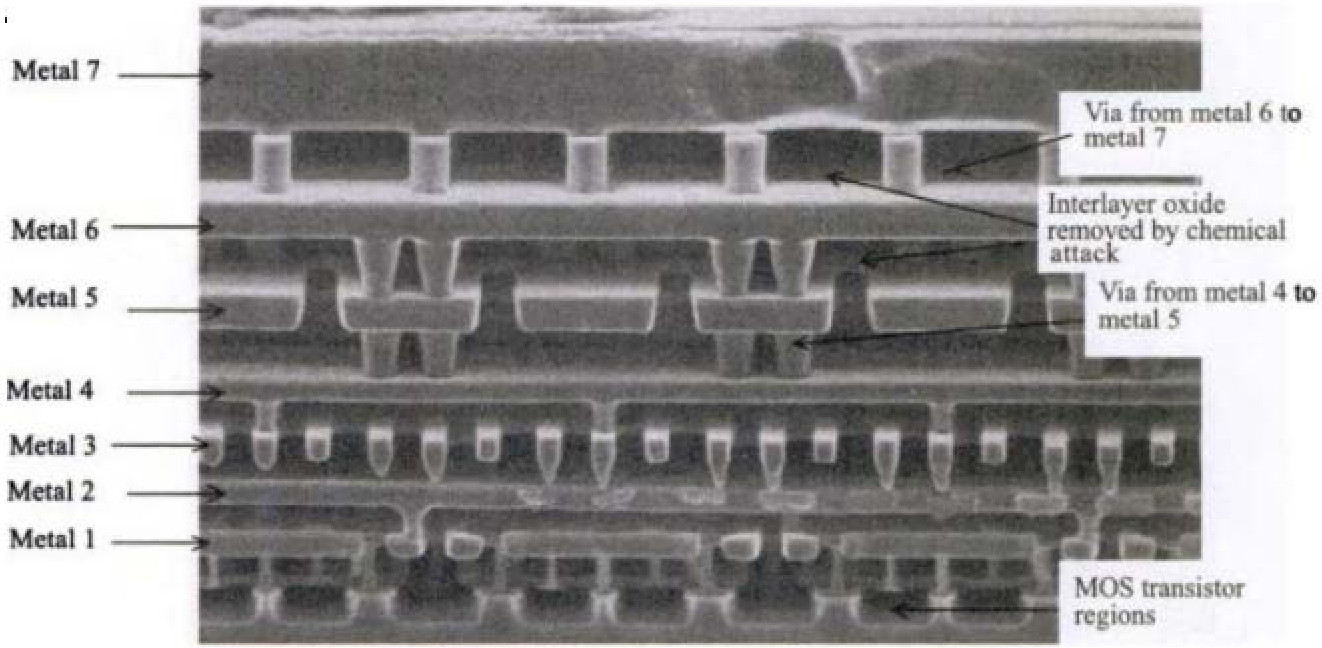
Even mode



Odd mode

- Surface wave or substrate mode give rise to radiation.
- To avoid surface wave $h/\lambda_0 << 1$.

How to design a IC at 94 GHz?



0.12 μm technology: Fujitsu

Attenuation due to dielectric

Three main sources of loss are (*i*) conductor loss, (*ii*) dielectric loss and (*iii*) radiation and surface wave loss.

Attenuation constant $\alpha = \alpha_c + \alpha_d + \alpha_r$.

Considering only the dielectric loss, $\gamma = \alpha_d + j\beta = \sqrt{k_c^2 - k^2}$

$$= \sqrt{k_c^2 - \omega^2 \mu_0 \epsilon_0 \epsilon_r (1 - j \tan\delta)}.$$

For most of the dielectric materials, $(\tan\delta \ll 1)$

and using Taylor expansion, $\sqrt{a^2 + x^2} \simeq a + \frac{1}{2}\left(\frac{x^2}{a}\right), \quad \text{for } x \ll a.$

Complex propagation constant reduces to $\gamma = \sqrt{k_c^2 - k^2 + jk^2 \tan\delta}$

$$\simeq \sqrt{k_c^2 - k^2} + \frac{jk^2 \tan\delta}{2\sqrt{k_c^2 - k^2}}$$

$$= \frac{k^2 \tan\delta}{2\beta} + j\beta,$$

(β is unaffected only for small loss)

Attenuation due to dielectric

The attenuation constant due to dielectric loss is

$$\alpha_d = \frac{k^2 \tan\delta}{2\beta} \text{ Np/m (TE or TM waves).}$$

For TEM wave, $k_c = 0$, $\rightarrow$ $\alpha_d = \dfrac{k \tan\delta}{2} \text{Np/m (TEM waves).}$

1 Np = 10 log e^2 dB = 8.686 dB

- The imaginary component ε″ of permittivity attributed to bound charge and dipole relaxation phenomena, which gives rise to energy loss that is indistinguishable from the loss due to the free charge conduction that is quantified by σ.
- The component ε′ represents lossless permittivity.
- The loss tangent is defined as the ratio (or angle in a complex plane) of the lossy reaction to the electric field **E** in the curl equation to the lossless reaction as

$$\tan\delta = \frac{\omega\varepsilon'' + \sigma}{\omega\varepsilon'}.$$

Thank you

?

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Microwave Network Analysis (P3)

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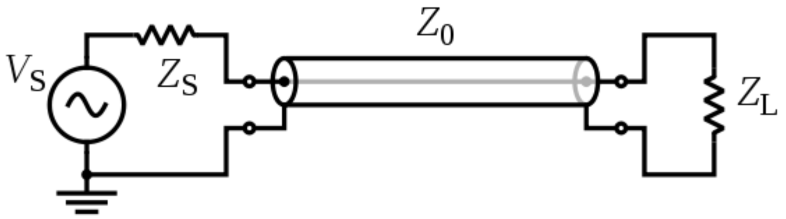
Why network theory..?

- Easier to apply the ideas of circuit analysis to a microwave problem than it is to solve Maxwell's equations for the same problem.
- Field analysis (using Maxwell's equations) gives much more information about a particular problem under consideration than we really need → gives the electric and magnetic fields at all points in space.
- Sometimes terminal solution is enough to design a microwave system →multiple RF chips connected by interconnects.
- The fields is usually assumed to be TEM fields supported by two or more conductors →use transmission line theory and traditional concept of impedance of circuit theory.

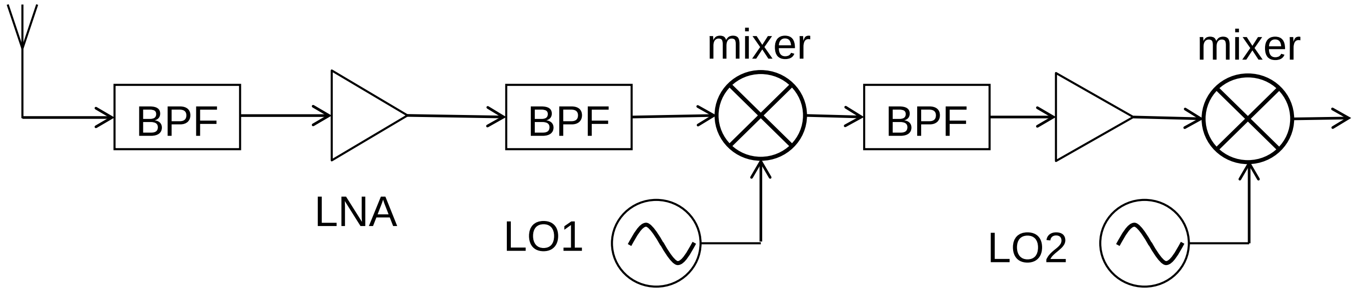


Co-axial cable, a transmission line.

- Input impedance,

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)},$$

$$\text{where} \quad \beta l = \frac{2\pi}{\lambda_g} l = \theta.$$



A superheterodyne receiver.

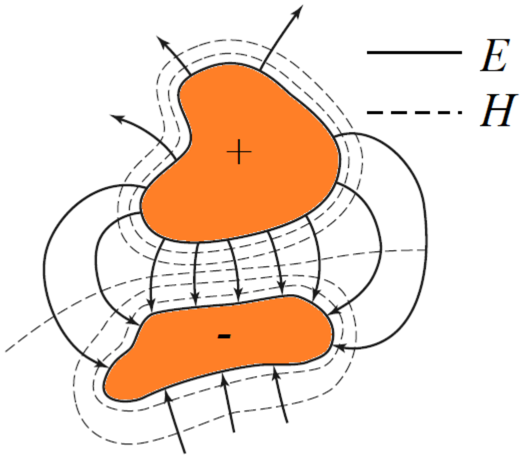
Equivalent voltages and currents

For an arbitrary two-conductor system, the voltages and currents are calculated as (TEM case)

$$V = \int_{+}^{-} \bar{E} \cdot d\bar{\ell}, \quad \text{and} \quad I = \oint_{C+} \bar{H} \cdot d\bar{\ell},$$

Then, characteristic impedance of the line is $Z_0 = \dfrac{V}{I}.$

Examples: coaxial-cable, microstrip line, CPW, parallel plate etc.



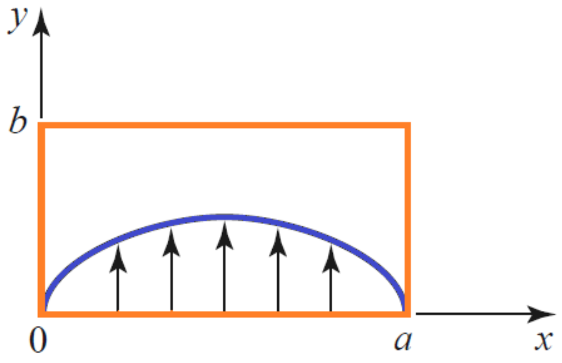
Arbitrary two-conductor line excited in TEM mode.

Waveguides..?

The transverse field components for TE_{10} mode are

$$E_y(x, y, z) = \frac{j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A e_y(x, y) e^{-j\beta z},$$

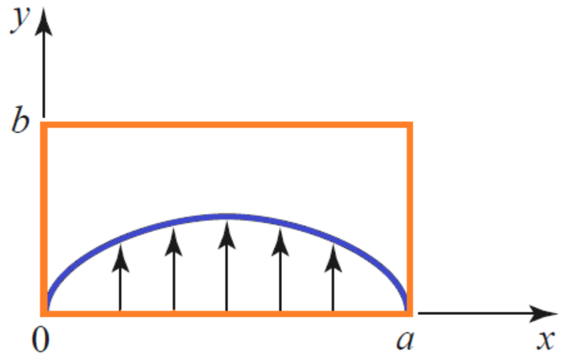
$$H_x(x, y, z) = \frac{j\beta a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A h_x(x, y) e^{-j\beta z}.$$



Electric field lines for TE_{10} mode.

Equivalent voltages and currents

$$E_y(x, y, z) = \frac{j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = A e_y(x, y) e^{-j\beta z},$$



Electric field lines for TE_{10} mode.

Then, like the TEM case (**?**),

$$V = \frac{-j\omega\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} \int_y dy.$$

- No voltage or current can be defined which is unique or pertinent for all applications.
- However, equivalent voltages, currents, and impedances can be obtained for terminal behaviour.
- Voltage and current are defined only for a particular waveguide mode, and are defined so that the voltage is proportional to the transverse electric field and the current is proportional to the transverse magnetic field.
- The product of equivalent voltage and current should gives the power flow of the waveguide mode.
- The ratio of the voltage to the current for a single traveling wave should be equal to the characteristic impedance of the line. This impedance may be chosen arbitrarily, but is usually selected as equal to the wave impedance of the line, or else normalized to unity.

Equivalent voltages and currents

For an arbitrary waveguide mode, the transverse fields are

$$\bar{E}_t(x,y,z) = \bar{e}(x,y)(A^+e^{-j\beta z} + A^-e^{j\beta z}) = \frac{\bar{e}(x,y)}{C_1}\left(V^+e^{-j\beta z} + V^-e^{j\beta z}\right),$$

$$\bar{H}_t(x,y,z) = \bar{h}(x,y)\left(A^+e^{-j\beta z} - A^-e^{j\beta z}\right) = \frac{\bar{h}(x,y)}{C_2}\left(I^+e^{-j\beta z} - I^-e^{j\beta z}\right)$$

The fields are related by the wave impedance Z_w as $\bar{h}(x,y) = \dfrac{\hat{z}\times\bar{e}(x,y)}{Z_w}.$

The above equations can be compared with the equivalent voltage and current waves (TEM case)

$$V(z) = V^+e^{-j\beta z} + V^-e^{j\beta z}, \qquad I(z) = I^+e^{-j\beta z} - I^-e^{j\beta z}, \quad \text{with} \quad V^+/I^+ = V^-/I^- = Z_0.$$

The constants of above equations to be determined are (considering voltage and current are proportional to the corresponding transverse fields)

$$\left.\begin{aligned} C_1 &= V^+/A^+ = V^-/A^- \\ C_2 &= I^+/A^+ = I^-/A^- \end{aligned}\right\} \text{ can be determined from the condition of power and impedance.}$$

Equivalent voltages and currents

The complex power flow of the incident wave is

$$P^+ = \frac{1}{2}|A^+|^2 \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds = \frac{V^+ I^{+*}}{2C_1 C_2^*} \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds.$$ which must be equal to $(1/2)V^+ I^{+*}$

Therefore, $C_1 C_2^* = \int_S \bar{e} \times \bar{h}^* \cdot \hat{z} ds,$ (surface integration over the cross section of the waveguide)

Then, the characteristic impedance is $Z_0 = \frac{V^+}{I^+} = \frac{V^-}{I^-} = \frac{C_1}{C_2},$

If we call the wave impedance as the characteristic impedance, $\frac{C_1}{C_2} = Z_w \; (Z_{\mathrm{TE}} \text{ or } Z_{\mathrm{TM}}).$

Otherwise, $\frac{C_1}{C_2} = 1.$

Similar treatment can be done for any higher order mode.

Comparison of TEM and TE_{10} modes

Waveguide fields	Transmission line model
$E_y = \left(A^+ e^{-j\beta z} + A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$	$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z}$
$H_x = \frac{-1}{Z_{\mathrm{TE}}}\left(A^+ e^{-j\beta z} - A^- e^{j\beta z}\right) \sin\frac{\pi x}{a}$	$I(z) = I^+ e^{-j\beta z} - I^- e^{j\beta z} = \frac{1}{Z_0}\left(V^+ e^{-j\beta z} - V^- e^{j\beta z}\right)$
$P^+ = \frac{-1}{2}\int_S E_y H_x^* dx dy = \frac{ab}{4Z_{\mathrm{TE}}}\lvert A^+\rvert^2$	$P^+ = \frac{1}{2} V^+ I^{+*}$

Therefore, $C_1 = V^+/A^+ = V^-/A^-$ and $C_2 = I^+/A^+ = I^-/A^-$

Equating the incident power, $\frac{ab\left|A^+\right|^2}{4Z_{\mathrm{TE}}} = \frac{1}{2}V^+I^{+*} = \frac{1}{2}\left|A^+\right|^2 C_1 C_2^*.$

Considering, $Z_0 = Z_{\mathrm{TE}}$ $\rightarrow$ $\frac{V^+}{I^+} = \frac{C_1}{C_2} = Z_{\mathrm{TE}}.$ $\rightarrow$ $C_1 = \sqrt{\frac{ab}{2}}, \quad C_2 = \frac{1}{Z_{\mathrm{TE}}}\sqrt{\frac{ab}{2}},$

Concept of impedance

- Impedance is regarded as characteristic of the type of field, as well as of the medium.
- The concept of impedance forms an important link between field theory and transmission line or circuit theory.

Intrinsic impedance: $\eta = \sqrt{\mu/\epsilon}$

This impedance depends only on the material properties of the medium. It is equal to wave impedance for plane waves.

Wave impedance: $Z_w = E_t/H_t = 1/Y_w$

This impedance is a characteristic of the particular type of wave. TEM, TM, and TE waves each have different wave impedances, which may depend on the type of line or guide, the material, and the operating frequency.

Characteristic impedance: $Z_0 = 1/Y_0 = V^+/I^+$

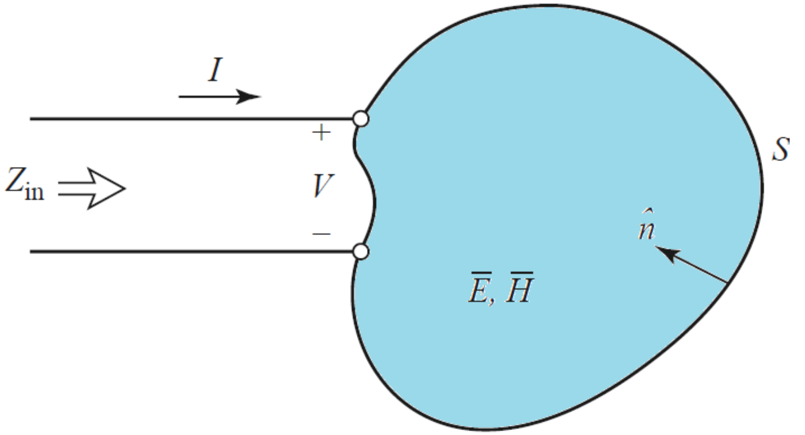
Characteristic impedance is the ratio of voltage to current for a traveling wave on a transmission line (TEM). TE and TM waves do not have a uniquely defined characteristic impedance.

Stored energy and dissipated energy

The complex power delivered to this network is

$$P = \frac{1}{2}\oint_S \bar{E} \times \bar{H}^* \cdot d\bar{s} = P_\ell + 2j\omega(W_m - W_e),$$

= dissipated power + power stored (average)



One port network

Let the fields at the terminal plane of the one port network are

$$\bar{E}_t(x, y, z) = V(z)\bar{e}(x, y)e^{-j\beta z}, \quad \bar{H}_t(x, y, z) = I(z)\bar{h}(x, y)e^{-j\beta z}$$

With the power is normalized as $\int_S \bar{e} \times \bar{h} \cdot d\bar{s} = 1,$

Then, the first equation in terms of voltage and current is $P = \frac{1}{2}\int_S VI^* \bar{e} \times \bar{h} \cdot d\bar{s} = \frac{1}{2}VI^*.$

Then, the input impedance is $Z_{in} = R + jX = \frac{V}{I} = \frac{VI^*}{|I|^2} = \frac{P}{\frac{1}{2}|I|^2} = \frac{P_\ell + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2}.$

If the network is lossless → $P_l = 0$ and the input impedance is purely imaginary → $X = \frac{4\omega(W_m - W_e)}{|I|^2},$

Even and odd properties of Z(ω) and Γ(ω)

Considering the driving point impedance, Z(ω),

→ $V(\omega) = Z(\omega)\, I(\omega)$.

The time-domain voltage is found by taking the inverse Fourier transform of $V(\omega)$ as

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega) e^{j\omega t} d\omega.$$

Since $v(t)$ must be real → $v(t) = v^*(t)$, or

$$\int_{-\infty}^{\infty} V(\omega) e^{j\omega t} d\omega = \int_{-\infty}^{\infty} V^*(\omega) e^{-j\omega t} d\omega = \int_{-\infty}^{\infty} V^*(-\omega) e^{j\omega t} d\omega,$$

This shows that $V(\omega)$ must satisfy the relation $V(-\omega) = V^*(\omega)$,

→Re{$V(\omega)$} is even in ω, while Im{$V(\omega)$} is odd in ω. Similar results hold for $I(\omega)$, and for Z(ω) since $V^*(-\omega) = Z^*(-\omega)I^*(-\omega) = Z^*(-\omega)I(\omega) = V(\omega) = Z(\omega)I(\omega)$.

Even and odd properties of Z(ω) and Γ(ω)

Therefore, if $Z(\omega) = R(\omega) + j\,X(\omega)$, then $R(\omega)$ is even in ω and $X(\omega)$ is odd in ω.

The reflection coefficient at the input port, $\Gamma(\omega) = \dfrac{Z(\omega) - Z_0}{Z(\omega) + Z_0} = \dfrac{R(\omega) - Z_0 + jX(\omega)}{R(\omega) + Z_0 + jX(\omega)}.$

Then, $\Gamma(-\omega) = \dfrac{R(\omega) - Z_0 - jX(\omega)}{R(\omega) + Z_0 - jX(\omega)} = \Gamma^*(\omega)$

→the real and imaginary parts of Γ(ω) are even and odd, respectively.

Then, the magnitude of the reflection coefficient is $|\Gamma(\omega)|^2 = \Gamma(\omega)\Gamma^*(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2,$

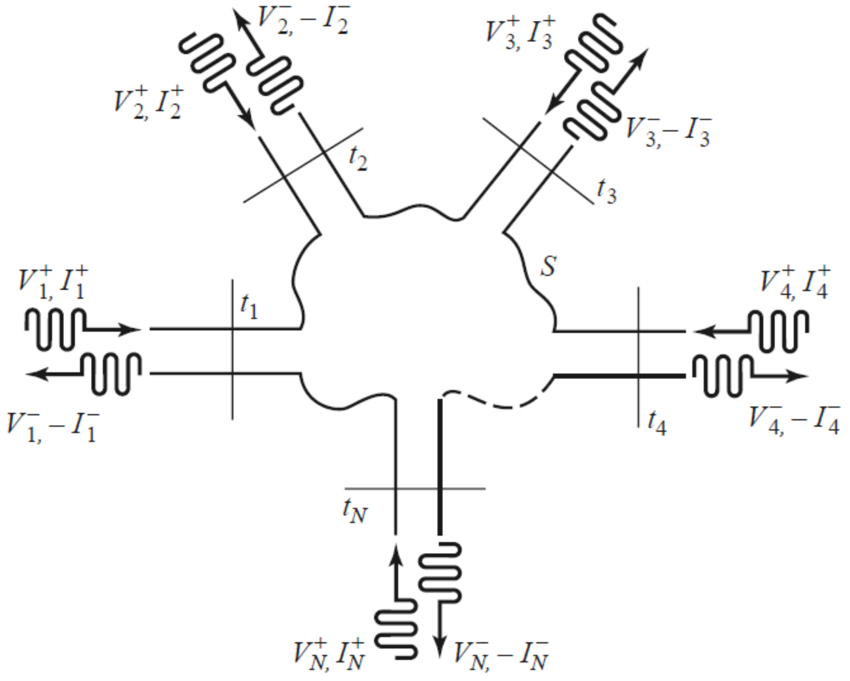
→$\Gamma|(\omega)|^2$ and $\Gamma\ |(\omega)|$ are even functions of ω.

Therefore, only even series of the form $a + b\omega^2 + c\omega^4 + \cdot\cdot\cdot$ can be used to represent $\Gamma|(\omega)|^2$ and $\Gamma\ |(\omega)|$.

Impedance and admittance matrices

- Equivalent voltages and currents are defined at the ports for both TEM and non-TEM waves.
- Only the terminal response is considered.
- The same impedance and/or admittance matrices of circuit theory is applied for matrix description of the network (**terminal behaviour**).
- At a specific point on the nth port, a terminal plane is defined along with equivalent voltages and currents for the incident (V^+_n, I^+_n) and reflected (V^-_n, I^-_n) waves.
- At the nth terminal plane (z = 0), the total voltage and current are

$$V_n = V_n^+ + V_n^-,$$
$$I_n = I_n^+ - I_n^-,$$



N-port network.

Impedance and admittance matrices

The impedance matrix [Z] of the microwave network is

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & & & \vdots \\ \vdots & & & \vdots \\ Z_{N1} & \cdots & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

In matrix form, $[V] = [Z][I].$

The admittance matrix [Y] of the microwave network is

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & & & \vdots \\ \vdots & & & \vdots \\ Y_{N1} & \cdots & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

In matrix form, $[I] = [Y][V].$

Then, $[Y] = [Z]^{-1}.$

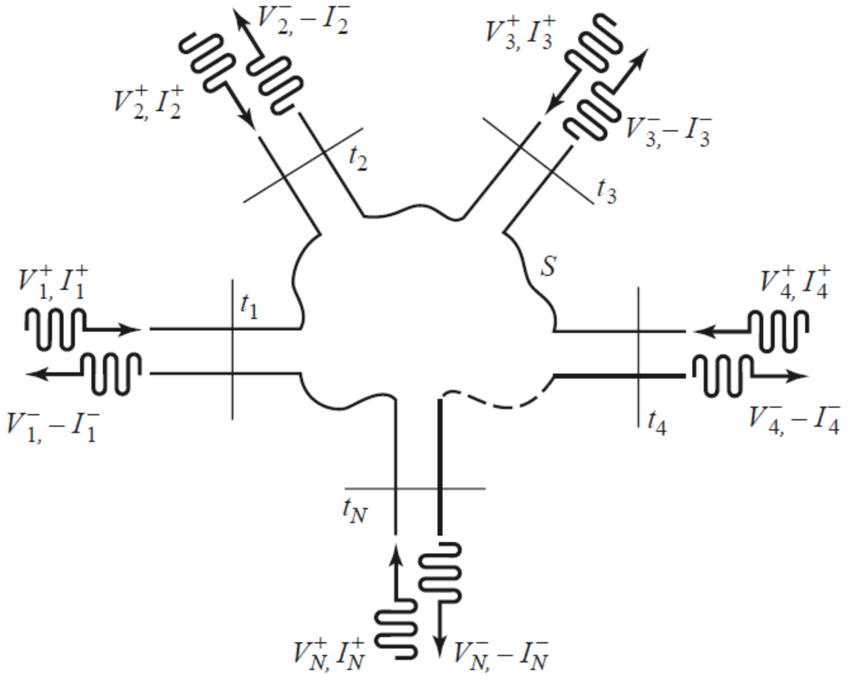
Impedance and admittance matrices

The impedance Z_{ij} can be found by driving port j with the current I_j, and open circuiting all other ports ($I_k = 0$ for $k \neq j$), and measuring the open-circuit voltage at port i,

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{I_k=0 \text{ for } k\neq j}.$$

The admittance Y_{ij} can be found by driving port j with the voltage V_j, and short circuiting all other ports ($V_k = 0$ for $k \neq j$), and measuring the short-circuit voltage at port i,

$$Y_{ij} = \left.\frac{I_i}{V_j}\right|_{V_k=0 \text{ for } k\neq j},$$



N-port network.

- Size of [Z] and [Y] are N × N → $2N^2$ independent quantities.
- However, for reciprocal (not containing any active devices or nonreciprocal media) or lossless (imaginary terms only) or both → the number of independent quantities reduces.

Scattering parameters (S-parameters)

- Let V^+_n is the amplitude of the voltage wave incident on port *n* and V^-_n is the amplitude of the voltage wave reflected from port *n*.
- The scattering matrix ([S] matrix) relates the incident and reflected voltage waves as

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & & & \vdots \\ S_{N1} & \cdots & & S_{NN} \\ \vdots & & & \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

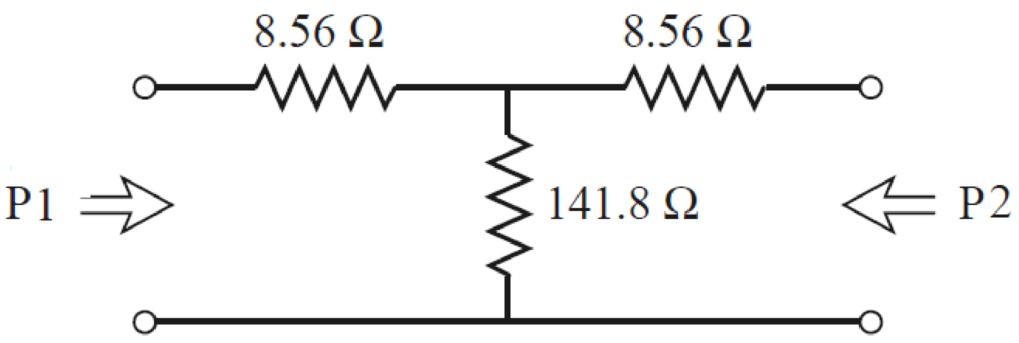
or $[V^-] = [S][V^+].$

A specific element of the scattering matrix is determined as $S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0 \text{ for } k\neq j}.$

(i.e. S_{ij} is found by driving port *j* with an incident wave of voltage V^+_j and measuring the reflected wave amplitude V^-_i coming out of port *i* when all ports are terminated in matched loads to avoid reflections.)

Scattering parameters (S-parameters)

Find the scattering parameters of the circuit and show that it works as a 3 dB attenuator.



Answer:

S_{11} is the reflection coefficient seen at port 1 when port 2 is terminated in a matched load (Z_0 = 50 Ω).

Therefore,

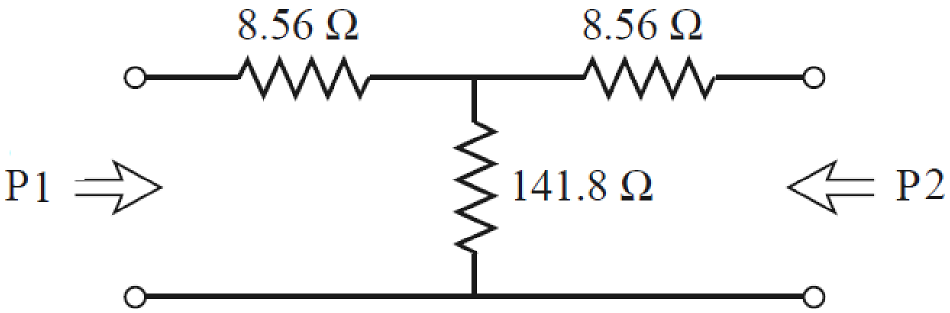
$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \Gamma^{(1)}|_{V_2^+=0} = \left.\frac{Z_{\text{in}}^{(1)} - Z_0}{Z_{\text{in}}^{(1)} + Z_0}\right|_{Z_0 \text{ on port } 2}$$

Now, $Z_{\text{in}}^{(1)} = 8.56 + [141.8(8.56 + 50)]/(141.8 + 8.56 + 50) = 50\ \Omega$ → S_{11} = 0 = S_{22} (from symmetry)

S_{21} is the transmission coefficient seen from port 1 to port → $S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$.

Scattering parameters (S-parameters)

Therefore, voltage at port 2 is



$$V_2^- = V_2 = V_1 \left(\frac{41.44}{41.44 + 8.56} \right) \left(\frac{50}{50 + 8.56} \right) = 0.707 V_1$$

→ $S_{12} = 0.707 = S_{21}$ (from symmetry)

The input power is $|V_1^+|^2/2Z_0$

The output power is $|V_2^-|^2/2Z_0 = |S_{21} V_1^+|^2/2Z_0 = |S_{21}|^2/2Z_0 |V_1^+|^2 = |V_1^+|^2/4Z_0$

Therefore, it works as a 3dB attenuator.

S-parameters from Z- and Y-parameters

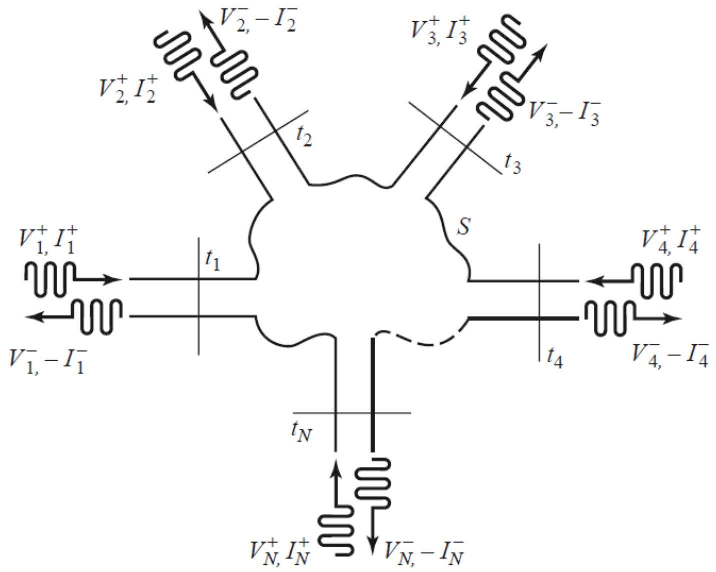
Let characteristic impedances of all the ports are Z_{0n}.

For convenience, assume Z_{0n} = 1.

Then, the total voltage and current at the n^{th} port can be written as

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ - I_n^- = V_n^+ - V_n^-. \quad \text{[Since } Z_{0n} = 1\text{]}$$



N-port network.

Now, using the definition of [Z],

$$[Z][I] = [Z][V^+] - [Z][V^-] = [V] = [V^+] + [V^-]$$

or,

$$([Z] + [U])\,[V^-] = ([Z] - [U])\,[V^+], \qquad \text{where} \quad [U] = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & & \vdots \\ \vdots & & \ddots & \\ 0 & & \cdots & 1 \end{bmatrix}$$

Then, comparing with $[V^-] = [S][V^+]$.

$\rightarrow$ $[S] = ([Z] + [U])^{-1}\,([Z] - [U])$

S-parameters from Z- and Y-parameters

$$[S] = ([Z] + [U])^{-1} ([Z] - [U])$$

For a one port network,

$$S_{11} = \frac{z_{11} - 1}{z_{11} + 1}$$

[Z] in terms of [S]:

From the top most equation,

$$[Z][S] + [U][S] = [Z] - [U],$$

→ $[Z] = ([U] + [S]) ([U] - [S])^{-1}$

Similarly, solve for [Y] matrix.

Reciprocal networks

- Reciprocal network: if the voltage appearing at port 2 due to a current applied at port 1 is the same as the voltage appearing at port 1 when the same current is applied to port 2 (entirely of linear passive components. Exceptions are passive circulators and isolators).
- Impedance, admittance and scattering matrices are symmetric.

The total voltage and current at the n^{th} port can be written as

$$V_n = V_n^+ + V_n^-,$$

$$I_n = I_n^+ - I_n^- = V_n^+ - V_n^-.$$

By adding, $V_n^+ = \frac{1}{2}(V_n + I_n)$ $\rightarrow$ $[V^+] = \frac{1}{2}([Z] + [U])[I].$

By subtracting, $V_n^- = \frac{1}{2}(V_n - I_n)$ $\rightarrow$ $[V^-] = \frac{1}{2}([Z] - [U])[I].$

By eliminating [I] from above two equations, $[V^-] = ([Z] - [U])([Z] + [U])^{-1}[V^+]$

$$\rightarrow [S] = ([Z] - [U])([Z] + [U])^{-1}$$

Reciprocal networks

$$[S] = ([Z] - [U])([Z] + [U])^{-1}$$

Taking the transpose, $[S]^t = \{([Z] + [U])^{-1}\}^t ([Z] - [U])^t$

Now [U] is a diagonal matrix → $[U]^t = [U]$

[Z] is a symmetric matrix → $[Z]^t = [Z]$

Therefore, $[S]^t = ([Z] + [U])^{-1}([Z] - [U])$

→ $[S] = [S]^t$

So, [S] is symmetric.

Lossless network

For a lossless network, the average power delivered to the network

$$P_{\text{avg}} = \frac{1}{2}\text{Re}\{[V]^t[I]^*\} = \frac{1}{2}\text{Re}\{([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*)\}$$

$$= \frac{1}{2}\text{Re}\{[V^+]^t[V^+]^* - [V^+]^t[V^-]^* + [V^-]^t[V^+]^* - [V^-]^t[V^-]^*\}$$

$$= \frac{1}{2}[V^+]^t[V^+]^* - \frac{1}{2}[V^-]^t[V^-]^* = 0,$$

Since the term $-[V^+]^t[V^-]^* + [V^-]^t[V^+]^*$ is of the form A - A*, they are purely imaginary.

$(1/2)[V^+]^t[V^+]^*$ represents total incident power.

$(1/2)[V^-]^t[V^-]^*$ represents total reflected power.

Using the definition, $[V^-] = [S][V^+]$ → $[V^+]^t[V^+]^* = [V^+]^t[S]^t[S]^*[V^+]^*$ (from last equation)

→ $[S]^t[S]^* = [U]$ →(unitary matrix)

→ $[S]^* = \{[S]^t\}^{-1}$

Lossless network

$$[S]^t[S]^* = [U]$$

The above matrix equation can be written in summation form as

$$\sum_{k=1}^{N} S_{ki} S_{kj}^* = \delta_{ij}\text{, for all } i,\ j,$$

where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$, is the Kronecker delta symbol.

Thus, the above equation reduces to

$$\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1 \qquad \text{for } i = j \qquad \text{and} \qquad \sum_{k=1}^{N} S_{ki} S_{kj}^* = 0\text{, for } i \neq j$$

Left relation: the dot product of any column of [S] with the conjugate of that same column gives unity.
Right column: the dot product of any column with the conjugate of a different column gives zero (the columns are orthonormal)

→ $[S][S]^{*t} = [U]$ so the same statements can be made about the rows of the scattering matrix.

Reciprocal and lossless networks

A two-port network has the following scattering matrix: $[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$

Determine if the network is reciprocal and lossless. If port 2 is terminated with a matched load, what is the return loss seen at port 1? If port 2 is terminated with a short circuit, what is the return loss seen at port 1?

Answer:

[S] is not symmetric →the network is not reciprocal.

To be lossless, the scattering parameters must satisfy $\sum_{k=1}^{N} S_{ki} S_{ki}^* = 1$ for $i = j$

Now, taking the first column i.e. $i = 1$,

$$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2 = 0.745 \neq 1$$ →the network is not lossless.

When port 2 is terminated with a matched load, the reflection coefficient seen at port 1 is $\Gamma = S_{11} = 0.15$

→ $\mathrm{RL} = -20 \log |\Gamma| = -20 \log(0.15) = 16.5 \text{ dB}.$

Reciprocal and lossless networks

Port 2 is terminated with a short circuit:

From the definition of the scattering matrix and the fact that $V_2^+ = -V_2^-$ (for a short circuit at port 2),

$$\rightarrow \quad V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ - S_{12}V_2^-,$$

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ - S_{22}V_2^-.$$

From the second equation, $V_2^- = \dfrac{S_{21}}{1+S_{22}}V_1^+$

Dividing the first equation by V_1^+ and using the above relationship,

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12}\frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}$$

$$= 0.15 - \frac{(0.85\angle{-45^\circ})(0.85\angle 45^\circ)}{1+0.2} = -0.452.$$

$$\rightarrow \quad \mathrm{RL} = -20\log|\Gamma| = -20\log(0.452) = 6.9\ \mathrm{dB}$$

Shift in reference plane

Let the [S] represents the scattering matrix for the terminal plane located at $z_n = 0$ for port n.

Let the [S'] is the new matrix when the terminal plane is shifted at $z_n = l_n$ for port n.

Then, in terms of reflected and incident port voltages,

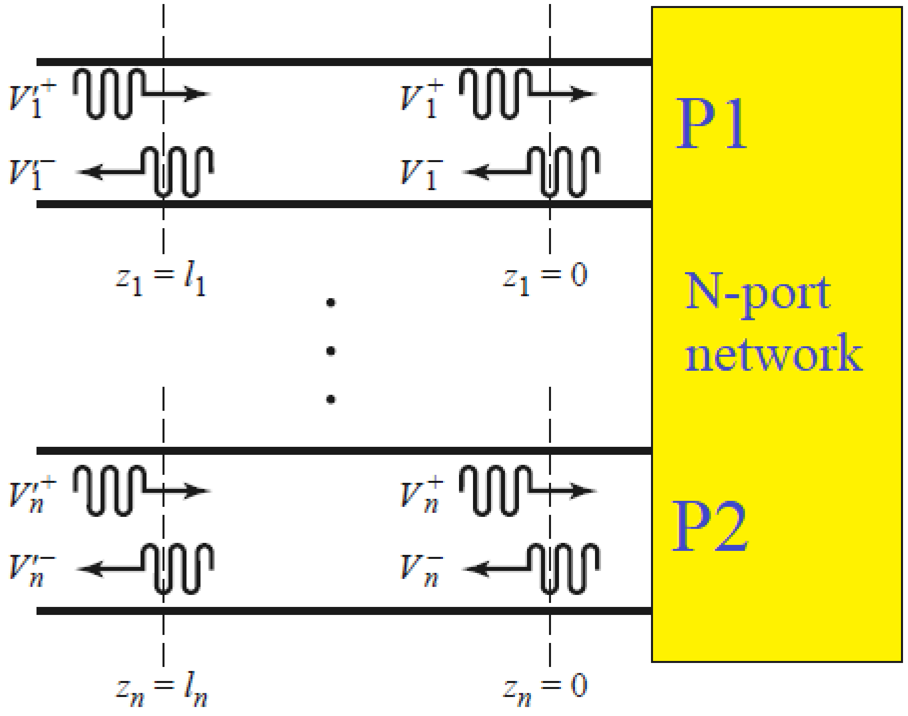
$$[V^-] = [S][V^+], \qquad [V'^-] = [S'][V'^+]$$

For a lossless transmission line,

$$V_n'^+ = V_n^+ e^{j\theta_n}, \qquad V_n'^- = V_n^- e^{-j\theta_n}, \text{ where } \theta_n = \beta_n \ell_n$$

In matrix form,

$$\begin{bmatrix} e^{j\theta_1} & & & 0 \\ & e^{j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{j\theta_N} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & & 0 \\ & e^{-j\theta_2} & & \\ & & \ddots & \\ 0 & & & e^{-j\theta_N} \end{bmatrix} [V'^+].$$



N-port network.

Shift in reference plane

$$\begin{bmatrix} e^{j\theta_1} & & 0 \\ & e^{j\theta_2} & \\ & \ddots & \\ 0 & & e^{j\theta_N} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [V'^+].$$

Multiplying by the inverse of the first matrix on the left gives → $[V'^-] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [V'^+].$

Comparing with $[V'^-] = [S'][V'^+]$ → $[S'] = \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & & 0 \\ & e^{-j\theta_2} & \\ & \ddots & \\ 0 & & e^{-j\theta_N} \end{bmatrix}$

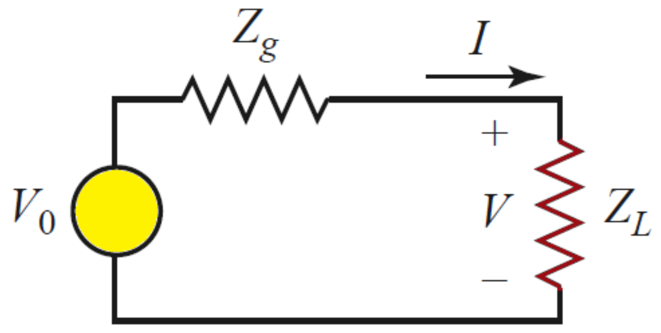
Therefore, $S'_{nn} = e^{-2j\theta_n} S_{nn}$

Generalized scattering parameters

Voltage and current in terms of incident and reflected amplitudes,

$$V = V_0^+ + V_0^-, \qquad I = \frac{1}{Z_0}\left(V_0^+ - V_0^-\right)$$

$$\rightarrow \quad V_0^+ = \frac{V + Z_0 I}{2}, \qquad V_0^- = \frac{V - Z_0 I}{2}.$$



Generator directly connected to load.

Then, the average power delivered to a load,

$$P_L = \frac{1}{2}\mathrm{Re}\left\{V I^*\right\} = \frac{1}{2Z_0}\mathrm{Re}\left\{\left|V_0^+\right|^2 - V_0^+ V_0^{-*} + V_0^{+*} V_0^- - \left|V_0^-\right|^2\right\}$$

$$= \frac{1}{2Z_0}\left(\left|V_0^+\right|^2 - \left|V_0^-\right|^2\right),$$

The result is valid when the characteristic impedance is real..!

(for a lossy line – characteristic impedance is complex)

Generalized scattering parameters

For the power transfer between the source and load, let us call it a power waves.

Let the incident and reflected power wave amplitudes *a* and *b* are defined as the following linear transformations of the total voltage and current:

$$a = \frac{V + Z_R I}{2\sqrt{R_R}}, \qquad b = \frac{V - Z_R^* I}{2\sqrt{R_R}}$$

where Z_R is the reference impedance (arbitrary)

and is $Z_R = R_R + jX_R$

Then, voltage and current in terms of power wave are

$$V = \frac{Z_R^* a + Z_R b}{\sqrt{R_R}}, \qquad I = \frac{a - b}{\sqrt{R_R}}.$$

Then, power delivered to the load is

$$P_L = \frac{1}{2}\mathrm{Re}\{VI^*\} = \frac{1}{2R_R}\mathrm{Re}\left\{Z_R^* |a|^2 - Z_R^* a b^* + Z_R a^* b - Z_R |b|^2\right\}$$

$$= \frac{1}{2}|a|^2 - \frac{1}{2}|b|^2,$$

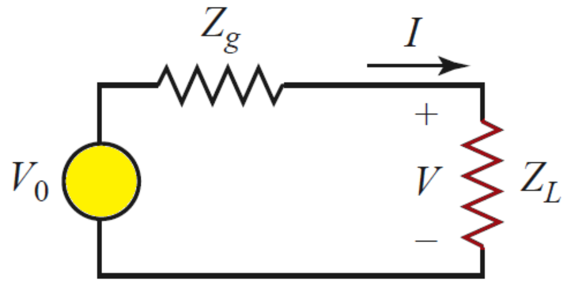
(the term $Z_R a^* b - Z_R^* a b^*$ is purely imaginary)

Generalized scattering parameters

Then, the reflection coefficient is

$$\Gamma_p = \frac{b}{a} = \frac{V - Z_R^* I}{V + Z_R I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}.$$

(putting $V = Z_L I$)



Therefore, to make the reflected power zero → $Z_R = Z^*_L$

Now, from circuit theory,

$$V = V_0 \frac{Z_L}{Z_L + Z_g}, \quad I = \frac{V_0}{Z_L + Z_g}, \quad P_L = \frac{V_0^2}{2} \frac{R_L}{|Z_L + Z_g|^2}$$

Then, the power wave amplitudes with $Z_R = Z^*_L$

$$a = \frac{V + Z_R I}{2\sqrt{R_R}} = V_0 \frac{\dfrac{Z_L}{Z_L + Z_g} + \dfrac{Z_L^*}{Z_L + Z_g}}{2\sqrt{R_R}} = V_0 \frac{\sqrt{R_L}}{Z_L + Z_g}$$

$$b = \frac{V - Z_R^* I}{2\sqrt{R_R}} = V_0 \frac{\dfrac{Z_L}{Z_L + Z_g} - \dfrac{Z_L}{Z_L + Z_g}}{2\sqrt{R_R}} = 0.$$

Generalized scattering parameters

The power delivered to the load is $P_L = \frac{1}{2}|a|^2 = \frac{V_0^2}{2}\frac{R_L}{|Z_L + Z_g|^2},$

→ $P_L = V_0^2/8R_L.$ for $Z_g = Z_L^*$

Next, assume the reference impedance for port i of an N-port network is Z_{Ri}.

Using similar transformation, power wave amplitude vectors are defined as

$$[a] = [F]\,([V] + [Z_R]\,[I]),$$

$$[b] = [F]\left([V] - [Z_R]^*\,[I]\right),$$

where $[F]$ is a diagonal matrix with elements $1/2\sqrt{\mathrm{Re}(Z_{Ri})}$ and $[Z_R]$ is a diagonal matrix with elements Z_{Ri}.

→ $[b] = [F]\left([Z] - [Z_R]^*\right)([Z] + [Z_R])^{-1}\,[F]^{-1}\,[a].$ using [V] = [Z] [I]

→ $\left[S_p\right] = [F]\left([Z] - [Z_R]^*\right)([Z] + [Z_R])^{-1}\,[F]^{-1}$ the diagonal elements can be made to be zero by proper selection of Z_{Ri}

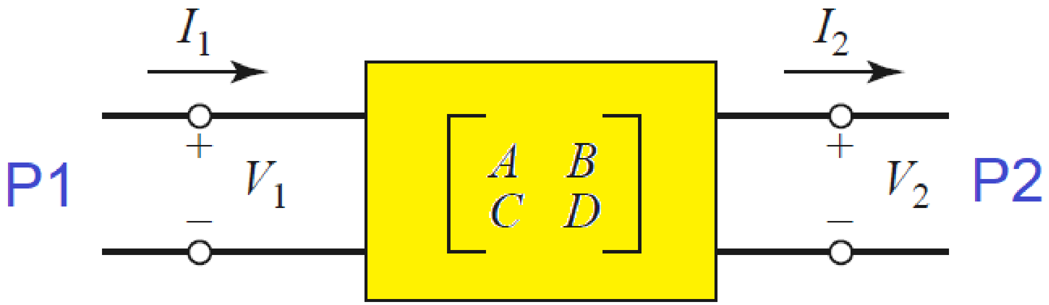
Convert ordinary [S] matrix →to an impedance matrix → use the above relation to obtain the generalized scattering matrix.

Transmission matrix

The ABCD matrix is defined in terms of voltages and currents.

$$V_1 = AV_2 + BI_2,$$
$$I_1 = CV_2 + DI_2,$$

$$\rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}.$$



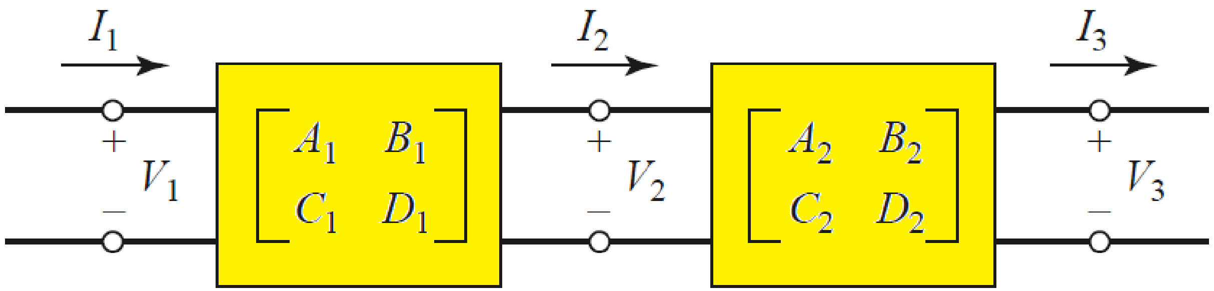
A two port network
(the direction of I_2 is changed to facilitate cascaded network)

For a cascaded network,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix},$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}.$$

$$\rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & C_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$



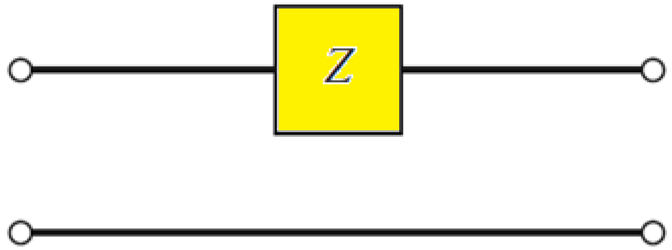
A cascaded network.

Transmission parameters

Find the ABCD parameters of a two-port network consisting of a series impedance Z between ports 1 and 2.

Answer:

Using the definition of ABCD parameters,



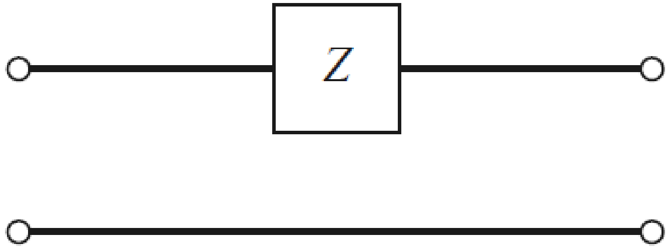
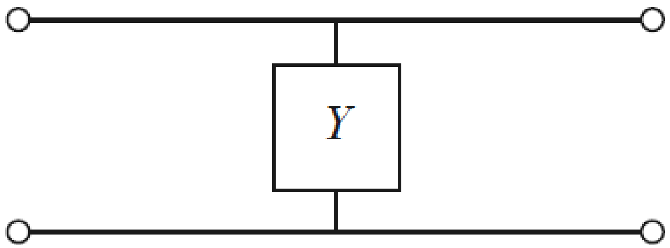
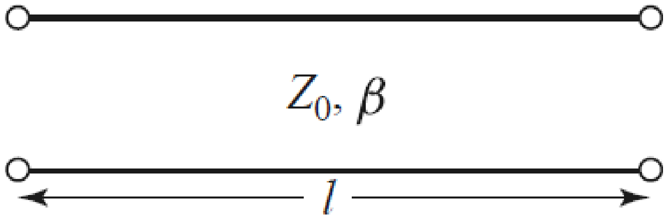
$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}, \qquad = 1$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z,$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0,$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1.$$

Transmission parameters

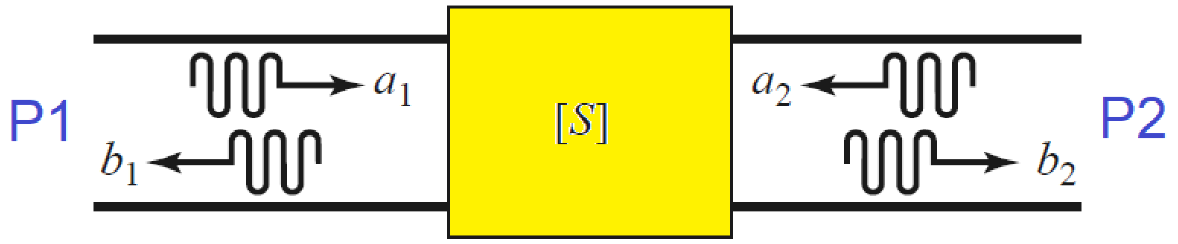
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta \ell$ $C = j Y_0 \sin \beta \ell$	$B = j Z_0 \sin \beta \ell$ $D = \cos \beta \ell$

Transformation

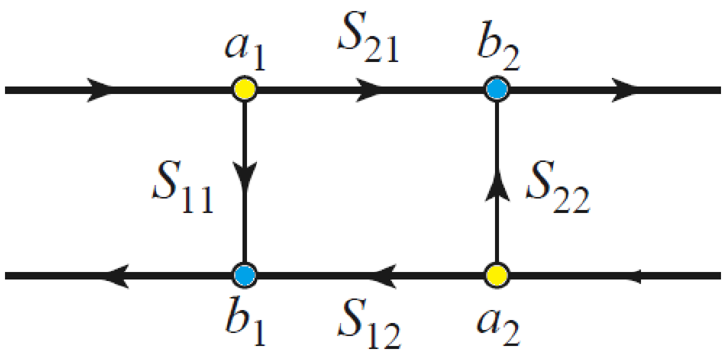
S	$ABCD$
S_{11}	$\dfrac{A+B/Z_0-CZ_0-D}{A+B/Z_0+CZ_0+D}$
S_{12}	$\dfrac{2(AD-BC)}{A+B/Z_0+CZ_0+D}$
S_{21}	$\dfrac{2}{A+B/Z_0+CZ_0+D}$
S_{22}	$\dfrac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D}$

$ABCD$	S
A	$\dfrac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}}$
B	$Z_0\dfrac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$
C	$\dfrac{1}{Z_0}\dfrac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}}$
D	$\dfrac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$

Signal flow graph



A two-port network.

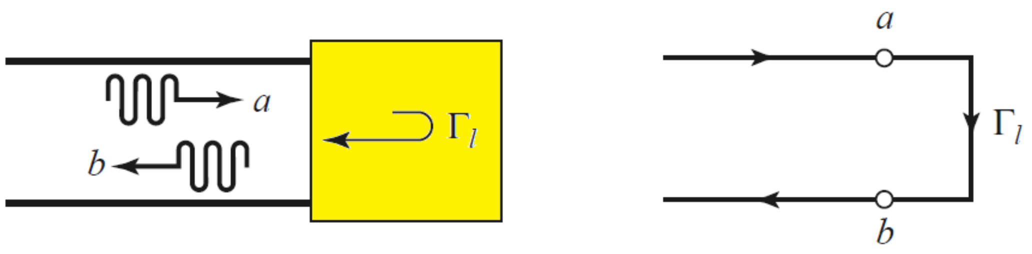


Corresponding signal flow graph.

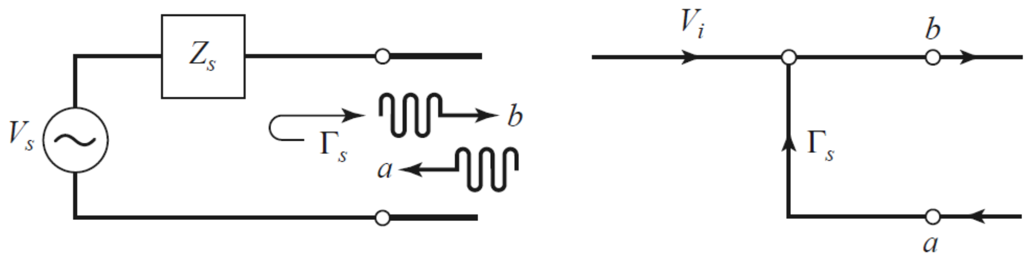
Nodes: Each port *i* has two nodes, a_i (wave entering port *i*) and b_i (a wave reflected from port *i*). The voltage at a node is equal to the sum of all signals entering that node.

Branches: A branch is a directed path between two nodes representing signal flow from one node to another. Every branch has an associated scattering parameter or reflection coefficient.

Signal flow graph makes the calculation simple for the ratio of any combination of wave amplitudes.



Signal flow graph for a load.

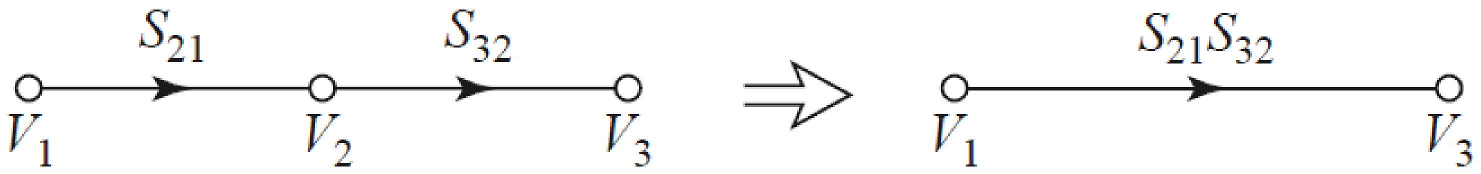


Signal flow graph for a source.

Decomposition of a signal flow graph

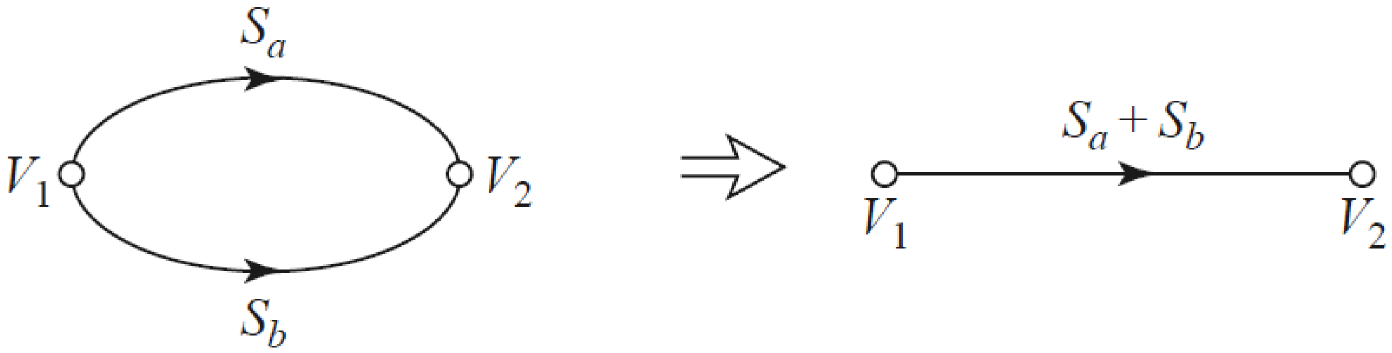
Rule 1 (series rule): Two branches, whose common node has only one incoming and one outgoing wave (branches in series), may be combined to form a single branch whose coefficient is the product of the coefficients of the original branches.

Its derivation follows from the basic relation $V_3 = S_{32} V_2 = S_{32} S_{21} V_1$.



Rule 2 (parallel rule): Two branches from one common node to another common node (branches in parallel) may be combined into a single branch whose coefficient is the sum of the coefficients of the original branches.

Its derivation follows from the basic relation $V_2 = S_a V_1 + S_b V_1 = (S_a + S_b) V_1$.

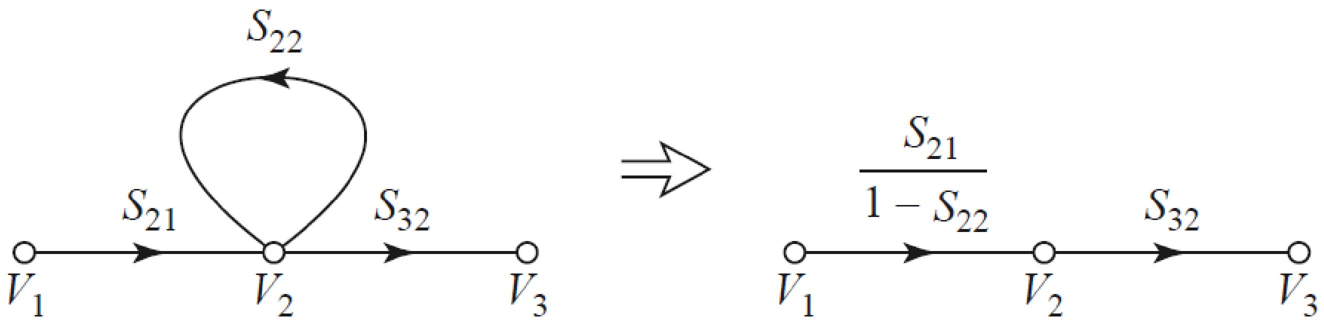


Decomposition of a signal flow graph

Rule 3 (self-loop rule): When a node has a self-loop of coefficient S, the self-loop can be eliminated by multiplying coefficients of the branches feeding that node by 1/(1 − S).

From the original network → $V_2 = S_{21}V_1 + S_{22}V_2, \qquad V_3 = S_{32}V_2.$

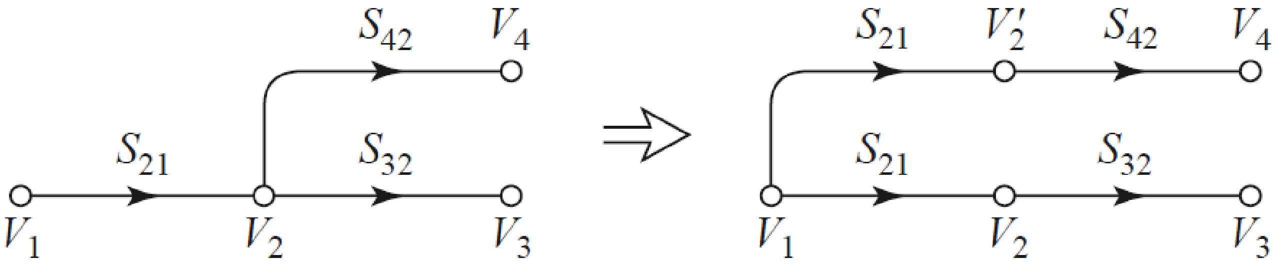
Eliminating V_2 → $V_3 = \dfrac{S_{32}S_{21}}{1 - S_{22}}V_1$



Rule 4 (splitting rule): A node may be split into two separate nodes as long as the resulting flow graph contains, once and only once, each combination of separate (not self-loops) input and output branches that connect to the original node

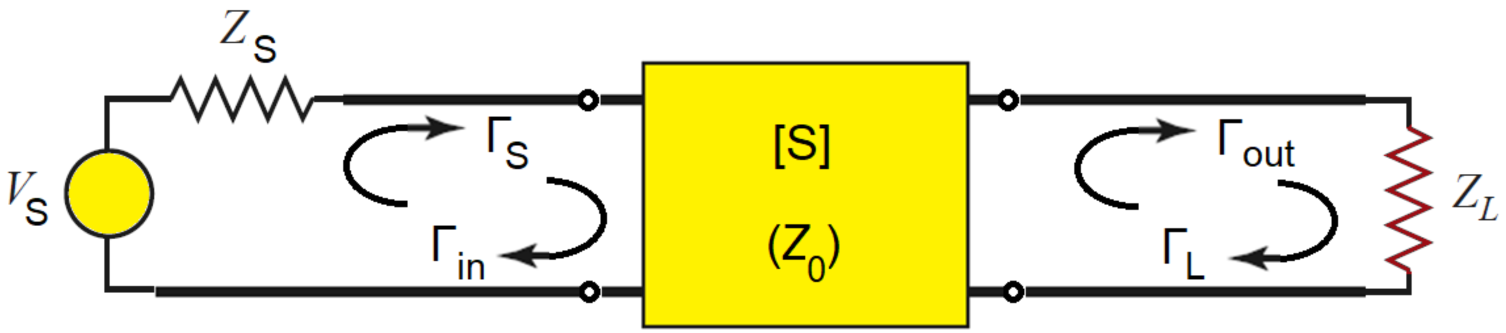
Its derivation follows from the basic relation

$$V_4 = S_{42}V_2 = S_{21}S_{42}V_1$$



Decomposition of a signal flow graph

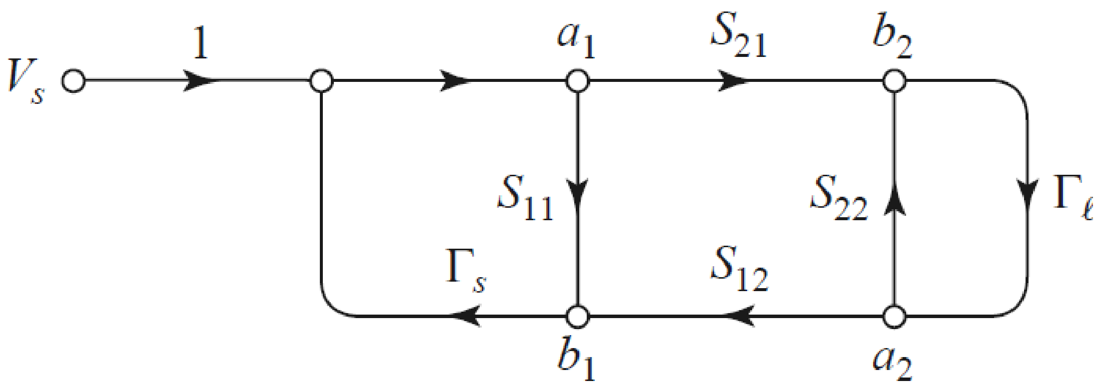
Q. Use signal flow graphs to derive expressions for Γ_{in} and Γ_{out} for the microwave network shown below.



Answer:

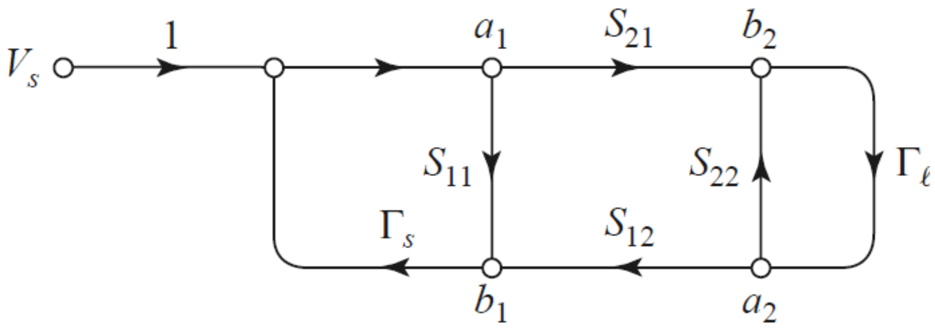
$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell}.$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

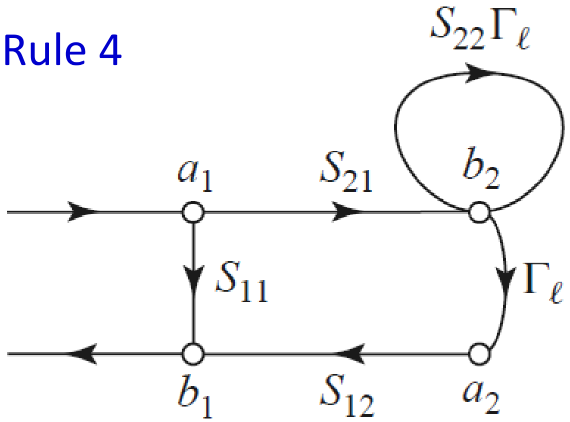


Signal flow graph of the above network.

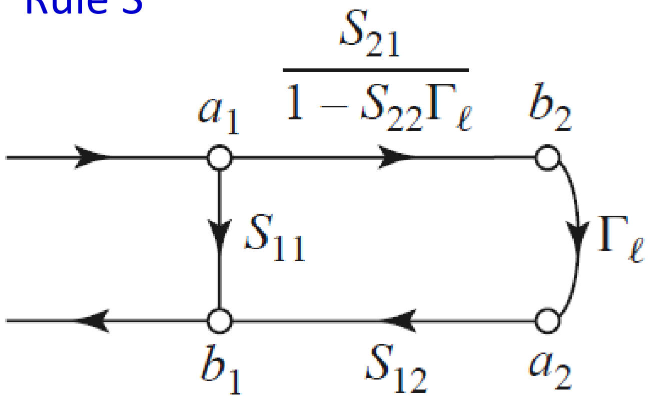
Decomposition of a signal flow graph



Rule 4



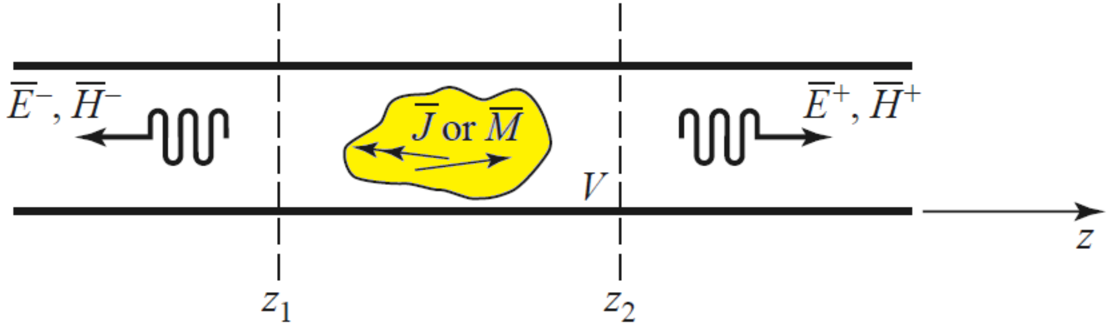
Rule 3



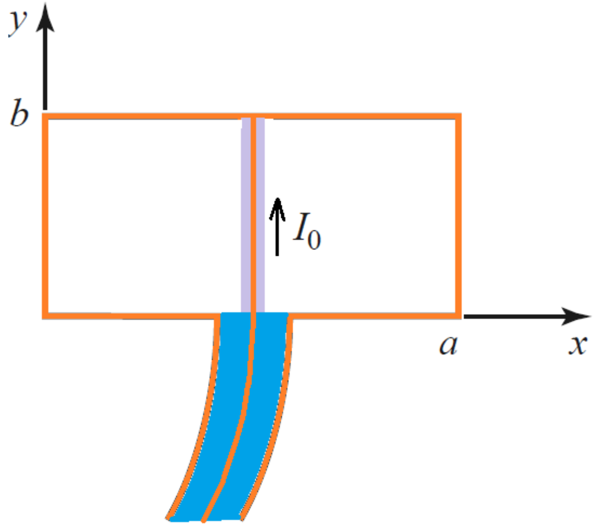
$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1 - S_{22}\Gamma_\ell}.$$

Decomposition.

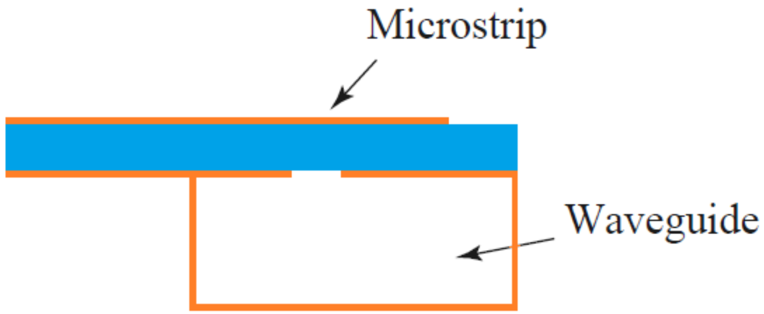
Excitation of waveguides



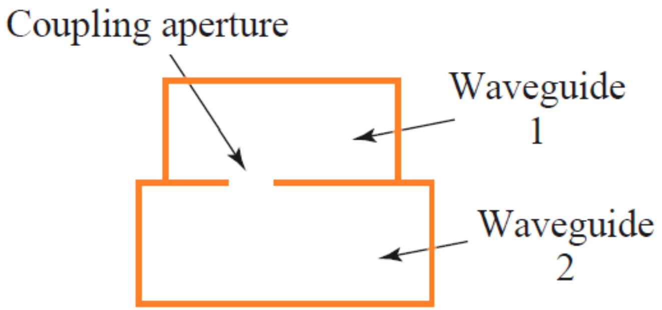
An arbitrary electric or magnetic current source in an infinite waveguide.



Coax-to-waveguide

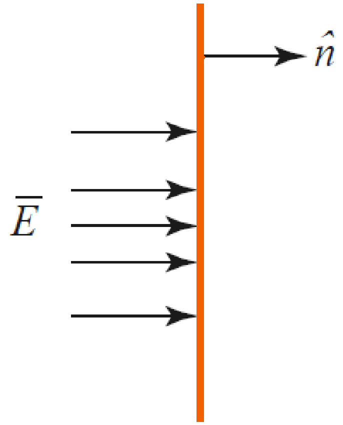


Microstrip-to-waveguide

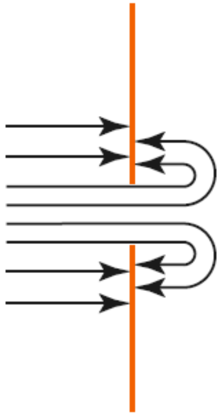


Waveguide-to-waveguide

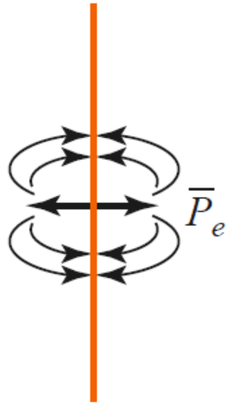
Electric and magnetic field lines



Normal E-field at a conducting wall.

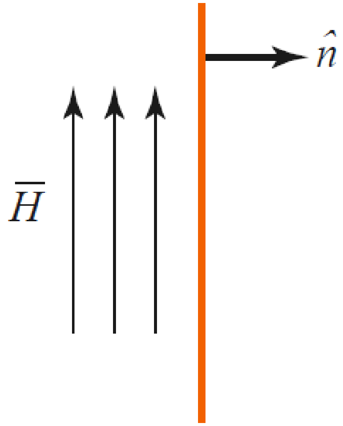


E-field lines around an aperture.

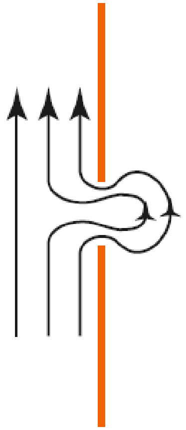


$\boldsymbol{P_e}$ - infinitesimal electric polarization currents normal to a conducting wall

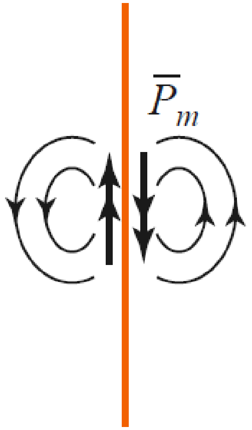
E-field lines around electric polarization currents normal to a conducting wall.



Tangential H-field at a conducting wall.



H-field lines around an aperture.



H-field lines near magnetic polarization currents parallel to a conducting wall.

Electric and magnetic field lines around an aperture

An aperture excited by a normal electric field can be represented by two oppositely directed infinitesimal electric polarization currents, ***P**$_e$*, normal to the closed conducting wall.

The strength of ***P**$_e$* is proportional to the normal electric field.

Thus, $\bar{P}_e = \epsilon_0 \alpha_e \hat{n} E_n \delta(x - x_0)\delta(y - y_0)\delta(z - z_0)$

where α_e is known as the electric polarizability.

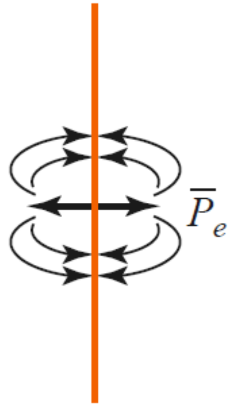
Similarly, for the magnetic polarization current, ***P**$_m$*,

$$\bar{P}_m = -\alpha_m \bar{H}_t \delta(x - x_0)\delta(y - y_0)\delta(z - z_0).$$

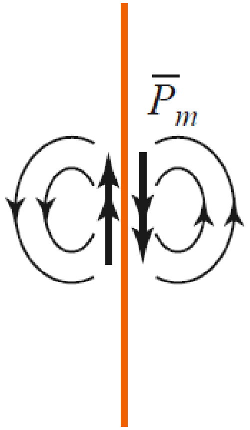
where α_m is the magnetic polarizability.

- From Maxwell's curl equations, $\bar{J} = j\omega \bar{P}_e,$

$$\bar{M} = j\omega\mu_0 \bar{P}_m.$$



E-field lines around electric polarization currents normal to a conducting wall.

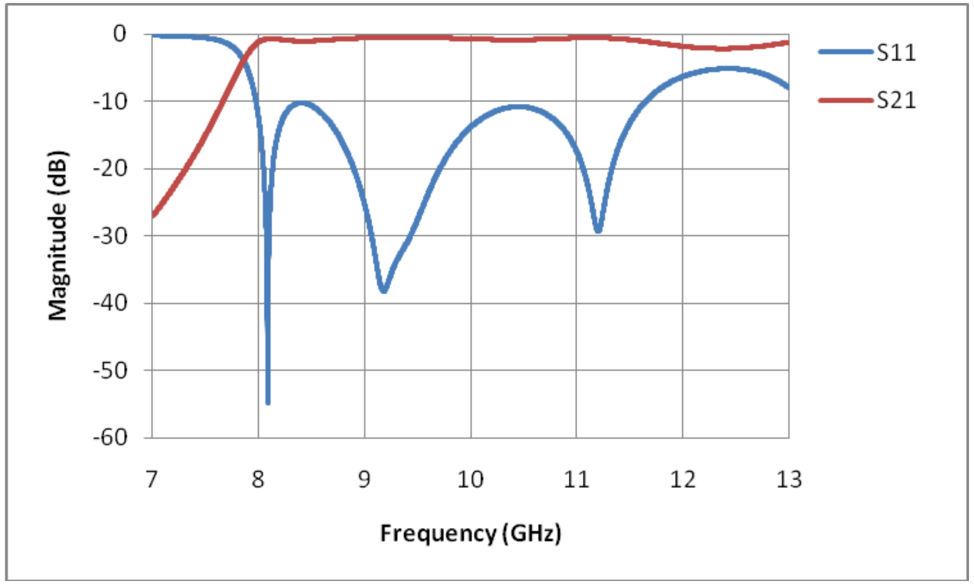


H-field lines near magnetic polarization currents parallel to a conducting wall.

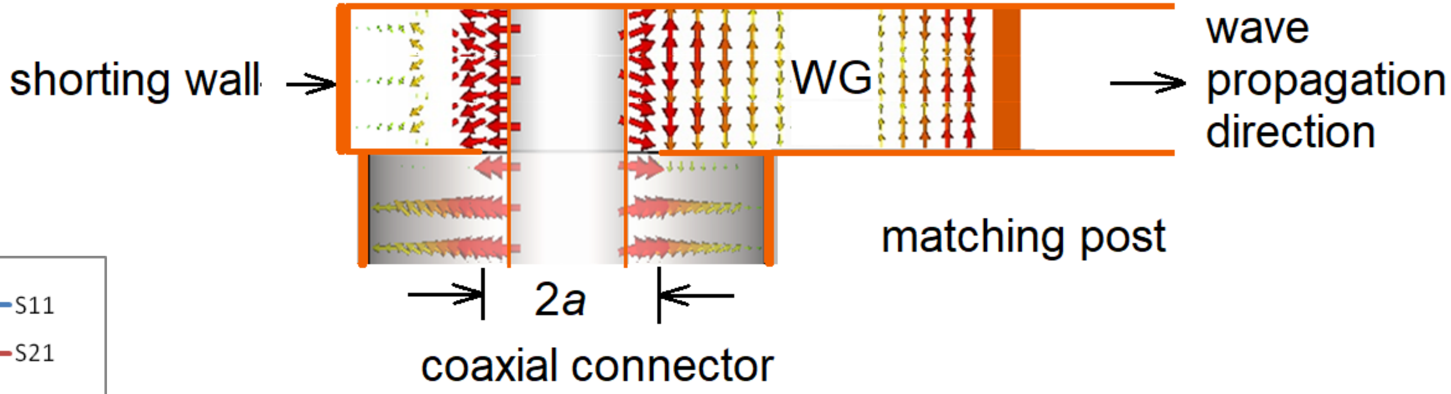
Excitation of waveguide

Design of an adapter:

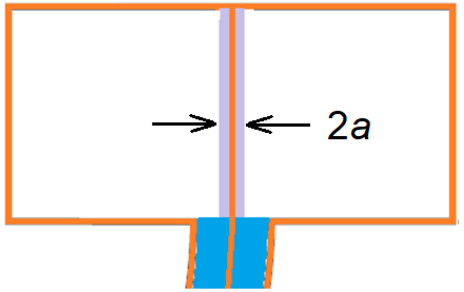
1. Field matching
2. Impedance matching



S-parameters of a back-to-back transition with matching posts.

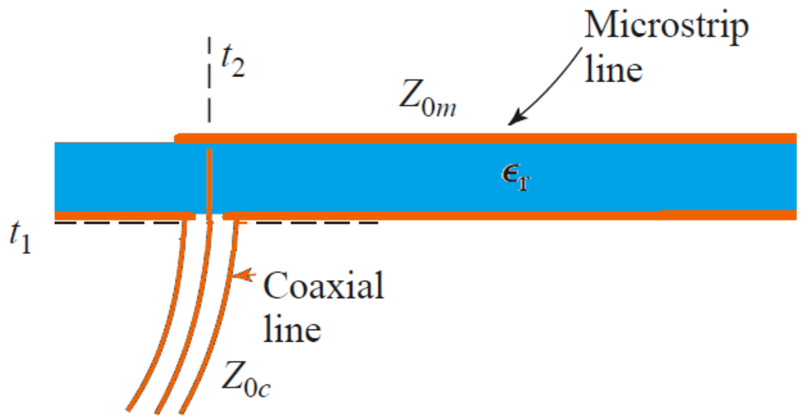


A coaxial connector to waveguide transition (longitudinal view)

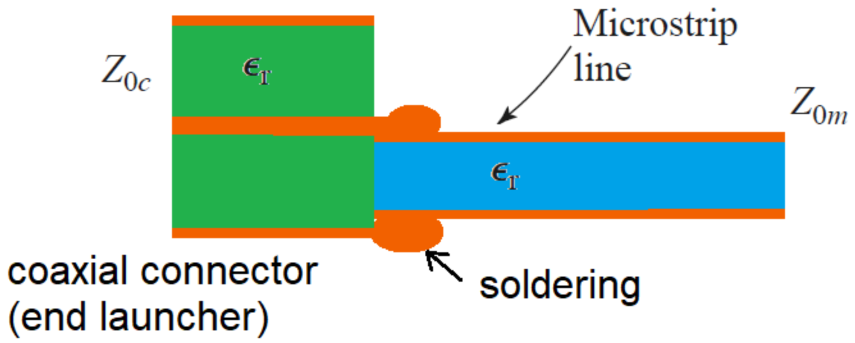


Transition with dielectric post (cross sectional view)

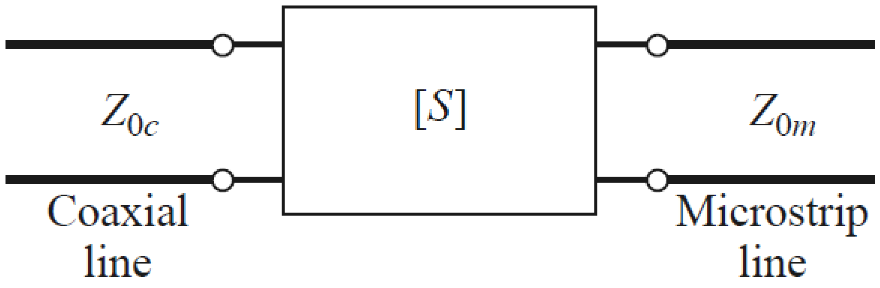
Coaxial to microstrip transition



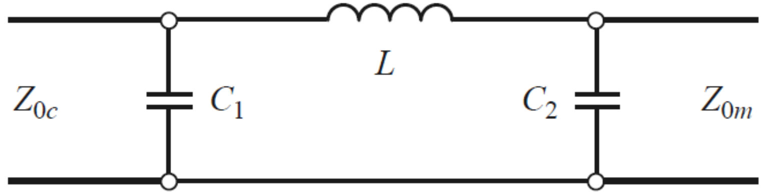
Coaxial to microstrip vertical transition



Coaxial to microstrip horizontal transition

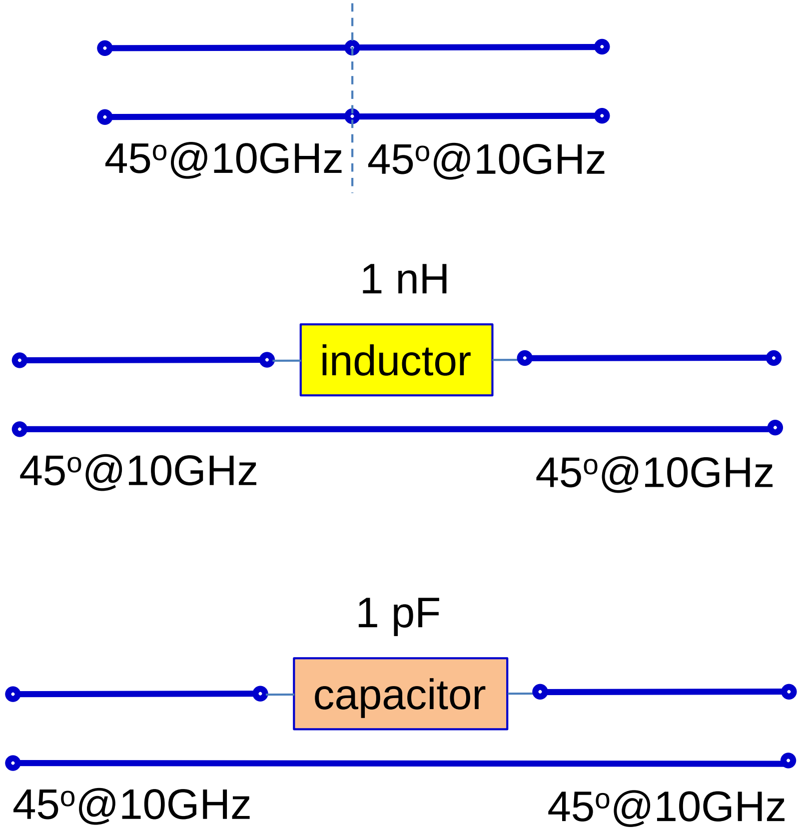


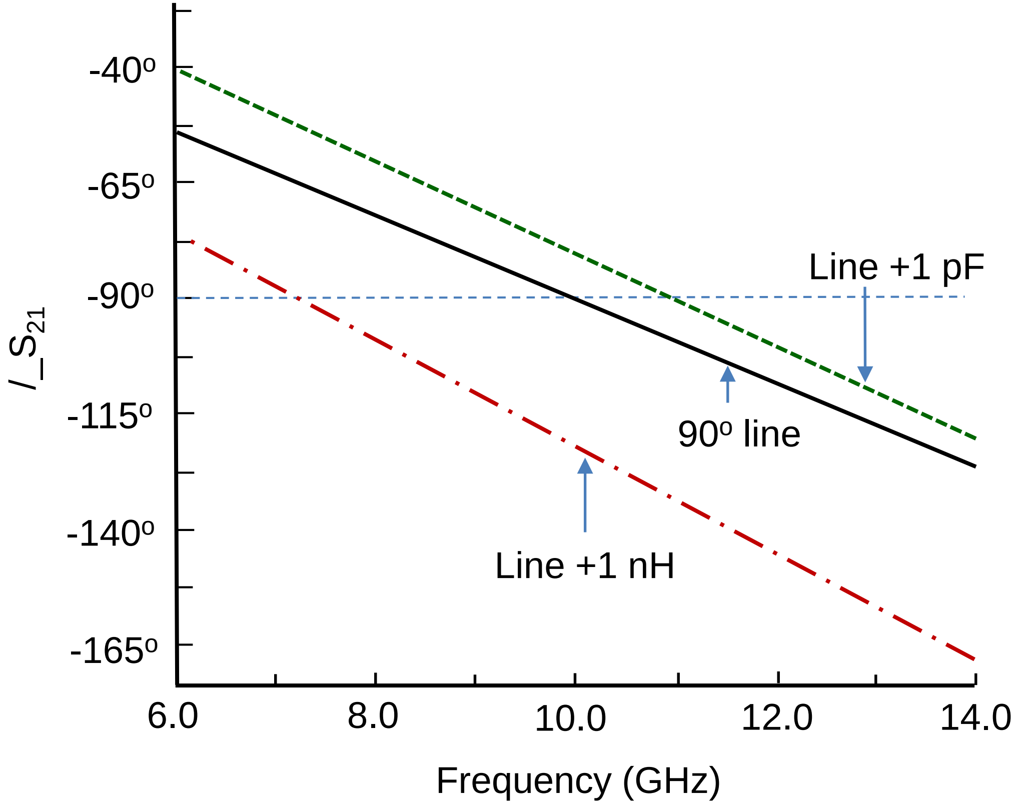
[S] matrix of the transition.



Equivalent circuit of the transition.

Extraction of equivalent model





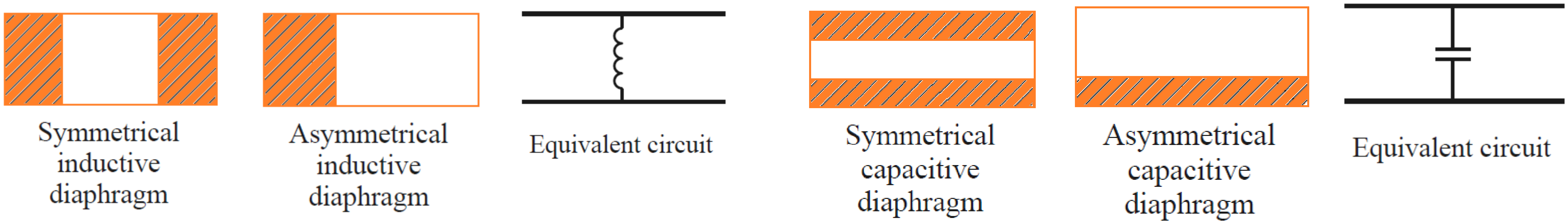
Comparison of phase responses.

Consider only the terminal behaviour (e.g. S-parameters)
Express by equivalent RLC (sometimes intuitively)
In some applications either magnitude or phase response is enough.

Equivalent circuits

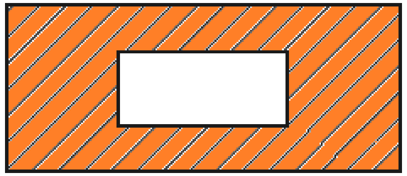
Steps:

- Consider only the terminal behaviour (e.g. ABCD, S-parameters etc.)
- Express by equivalent RLC (sometimes intuitively).
- Compare the ABCD or S-parameters of these two networks to extract the lumped values from full-wave simulation.
- In some applications, magnitude response is enough.

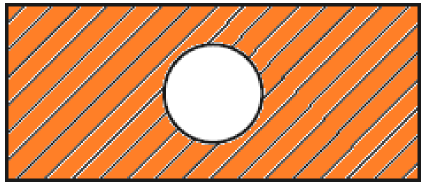


Equivalent circuits of different waveguide discontinuities
(cross sectional view)

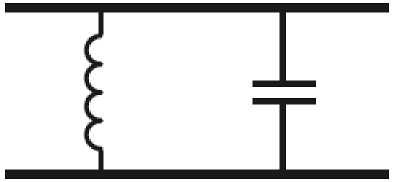
Equivalent circuits



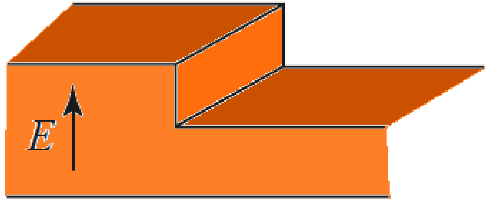
Rectangular resonant iris



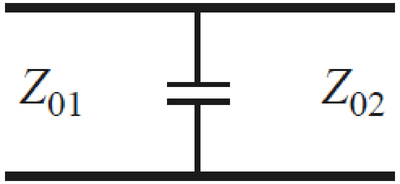
Circular resonant iris



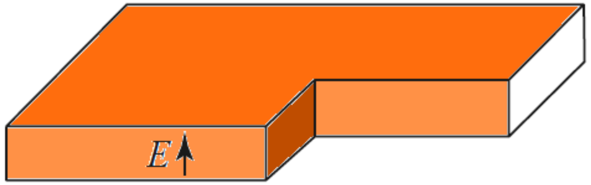
Equivalent circuit



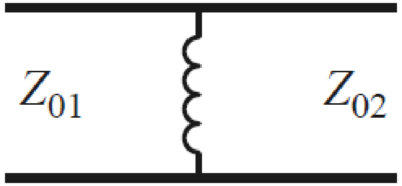
Change in height



Equivalent circuit



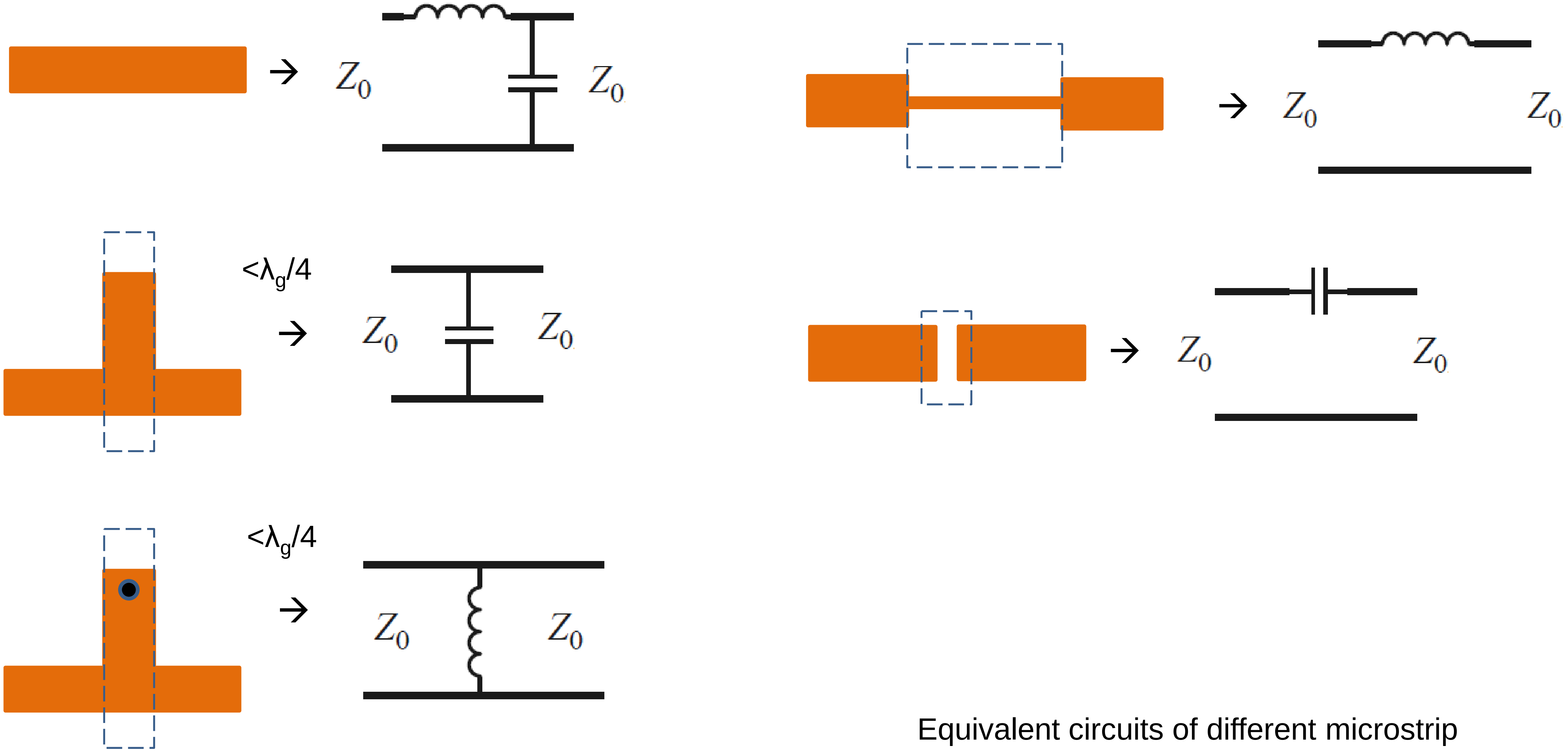
Change in width



Equivalent circuit

Equivalent circuits of different waveguide discontinuities

Equivalent circuits for microstrip discontinuities



Equivalent circuits of different microstrip discontinuities (top views)

RF and Microwave Engineering (EC 31005)

Impedance Matching and Tuning (P4)

Mrinal Kanti Mandal

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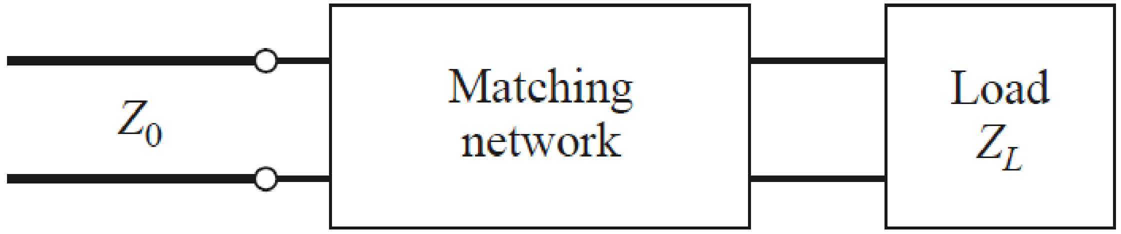
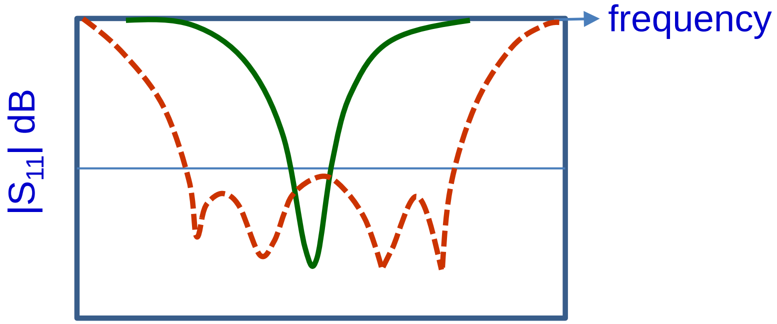
Department of E & ECE

I.I.T. Kharagpur.

Impedance matching circuit





Why impedance matching?

To deliver maximum power
→ improves the signal-to-noise ratio.
→in a power distribution network (e.g. antenna array feed network) may reduce amplitude and phase errors.

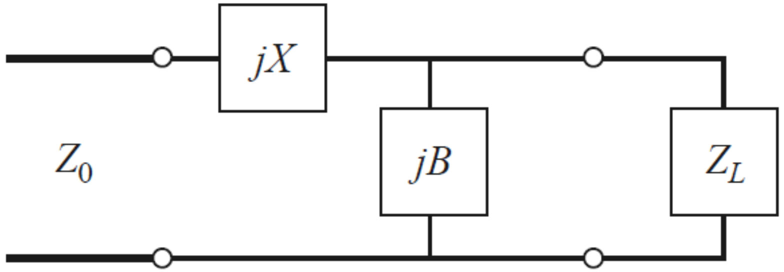
Bandwidth:

- At the design frequency reflection can be made to zero.
- Wideband matching: over the bandwidth, reflection loss is less than a predefined value.

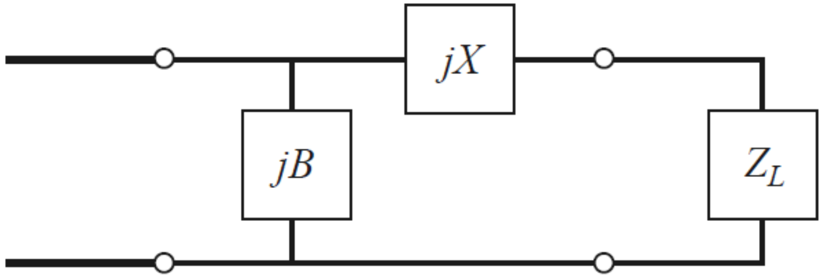
Other issues:

- Complexity of the design, implementation, tunability, small-signal (active devices) etc.

Matching with lumped elements



Network for z_L inside the 1 + $j\,x$ circle



Network for z_L outside the 1 + $j\,x$ circle

- Lumped elements (surface mountable device) at lower frequencies and transmission line sections at higher frequencies are preferred in PCB technology.
- In IC, lumped elements are preferred.
- Two possible scenario: (i) z_L inside the 1 + $j\,x$ circle and (ii) z_L outside the 1 + $j\,x$ circle.
- Eight distinct possibilities for matching circuit.
- Either Smith chart or analytical solution (CAD) can be used.

Analytical solutions

Case I: z_L inside the 1 + $j\,x$ circle → $R_L > Z_0$

At the design frequency, $$Z_0 = jX + \frac{1}{jB + 1/(R_L + jX_L)}.$$

Separating into real and imaginary parts,

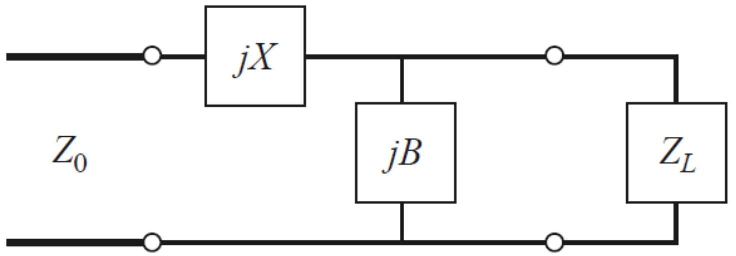
$$B(XR_L - X_L Z_0) = R_L - Z_0,$$
$$X(1 - BX_L) = BZ_0 R_L - X_L.$$

Solving for X and substituting that in 2nd equation,

$$B = \frac{X_L \pm \sqrt{R_L/Z_0}\sqrt{R_L^2 + X_L^2 - Z_0 R_L}}{R_L^2 + X_L^2}.$$

Therefore, $$X = \frac{1}{B} + \frac{X_L Z_0}{R_L} - \frac{Z_0}{BR_L}.$$

- Two solutions possible for X and B, both are feasible (inductor/ capacitor).
- Depending on implementation, bandwidth etc. one of them is chosen.



Network for z_L inside the 1 + $j\,x$ circle

Analytical solutions

Case II: z_L outside the $1 + j\,x$ circle → $R_L < Z_0$

At the design frequency, $$\frac{1}{Z_0} = jB + \frac{1}{R_L + j(X + X_L)}.$$

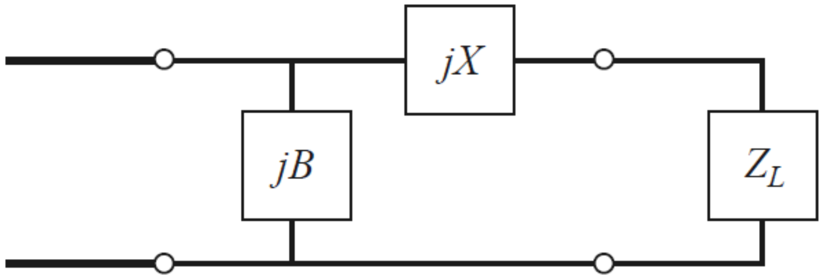
Separating into real and imaginary parts,

$$BZ_0(X + X_L) = Z_0 - R_L,$$

$$(X + X_L) = BZ_0R_L.$$

Solving for X and B,

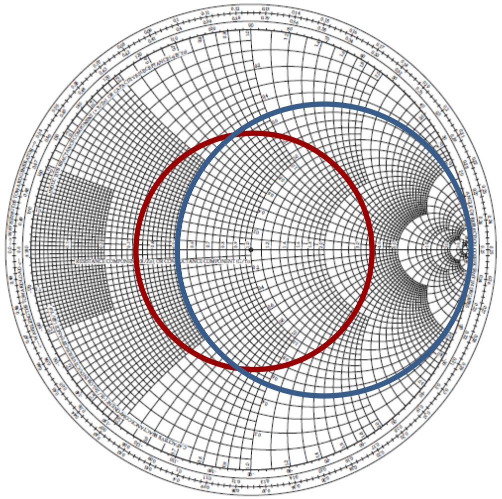
$$\left.\begin{aligned} X &= \pm\sqrt{R_L(Z_0 - R_L)} - X_L, \\ B &= \pm\frac{\sqrt{(Z_0 - R_L)/R_L}}{Z_0}. \end{aligned}\right\}\ \text{Two solutions.}$$



Network for z_L outside the $1 + j\,x$ circle

Smith chart approach

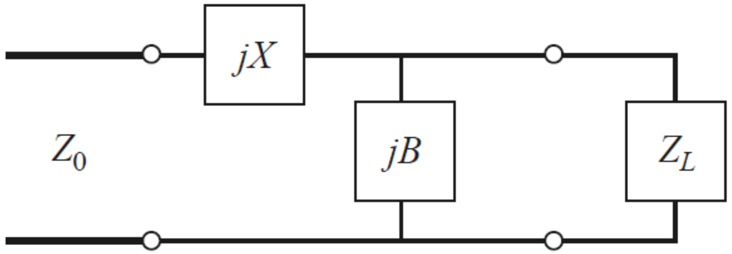
Q. Design an L-section matching network to match a series *RC* load with an impedance Z_L = 200 − j100 Ω to a 100 Ω line at a frequency of 500 MHz.



Answer:

Steps:

- Normalize the load impedance ($z_L = 2 - j$, inside the $1 + j\,x$ circle).
- Convert z_L to y_L (constant SWR circle and straight line through center).
- Use rotated 1 + jx circle (admittance Smith chart)
- Add jB (move on a constant conductance circle).
- Convert admittance to impedance.
- Add jX (impedance Smith chart)



Network for z_L inside the $1 + j\,x$ circle

Solution using Smith chart

- $z_L = 2 - j$
- Convert to $y_L = 0.4 + j\,0.2$
- Add $jB = j\,0.3$
- Convert admittance to impedance: $z = 1 - j\,1.2$ (approximately).
- Add $jX = j\,1.2$ (impedance Smith chart)

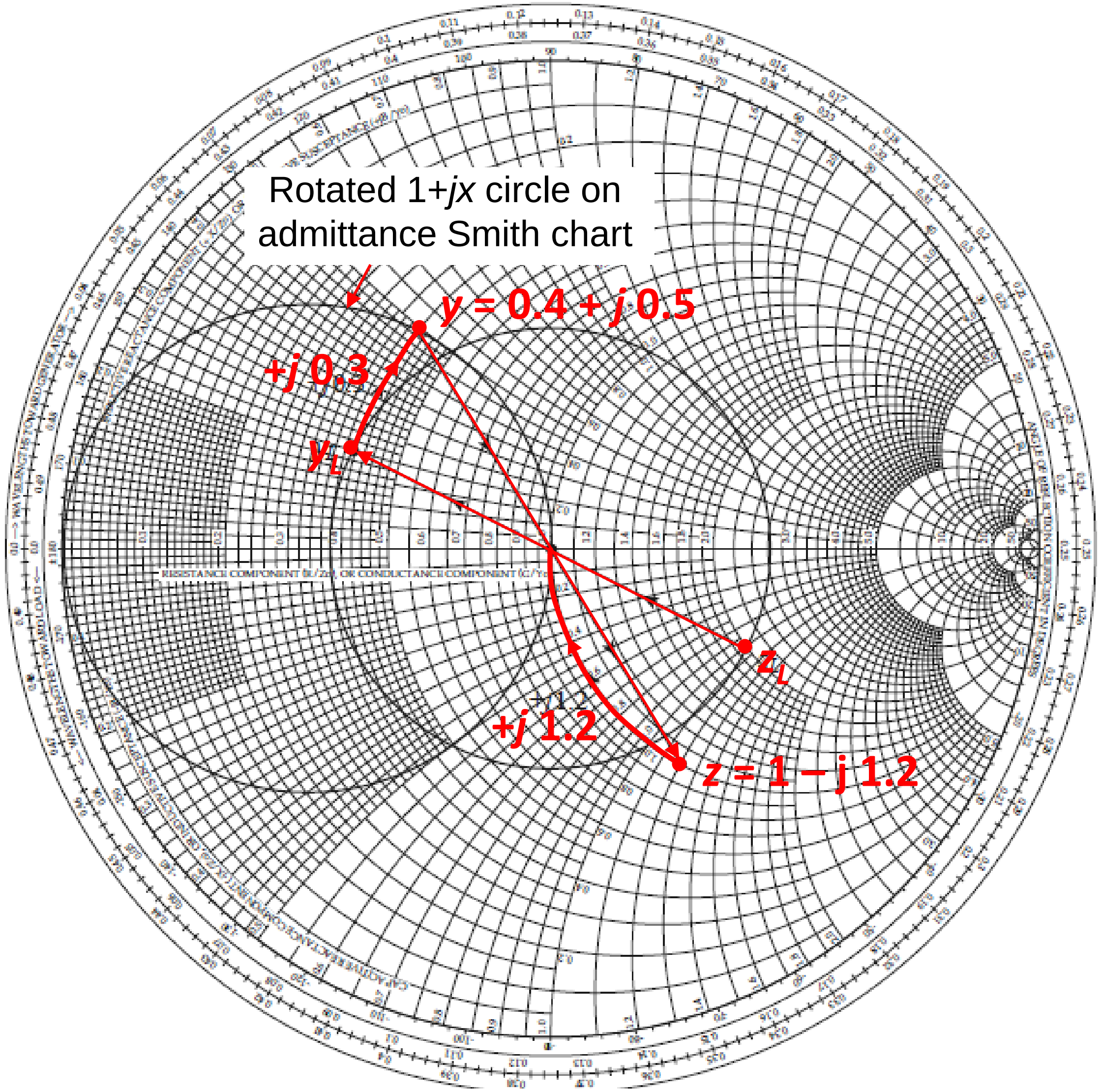
From formula:

$jB = j\,0.29$ and $jX = j\,1.22$.

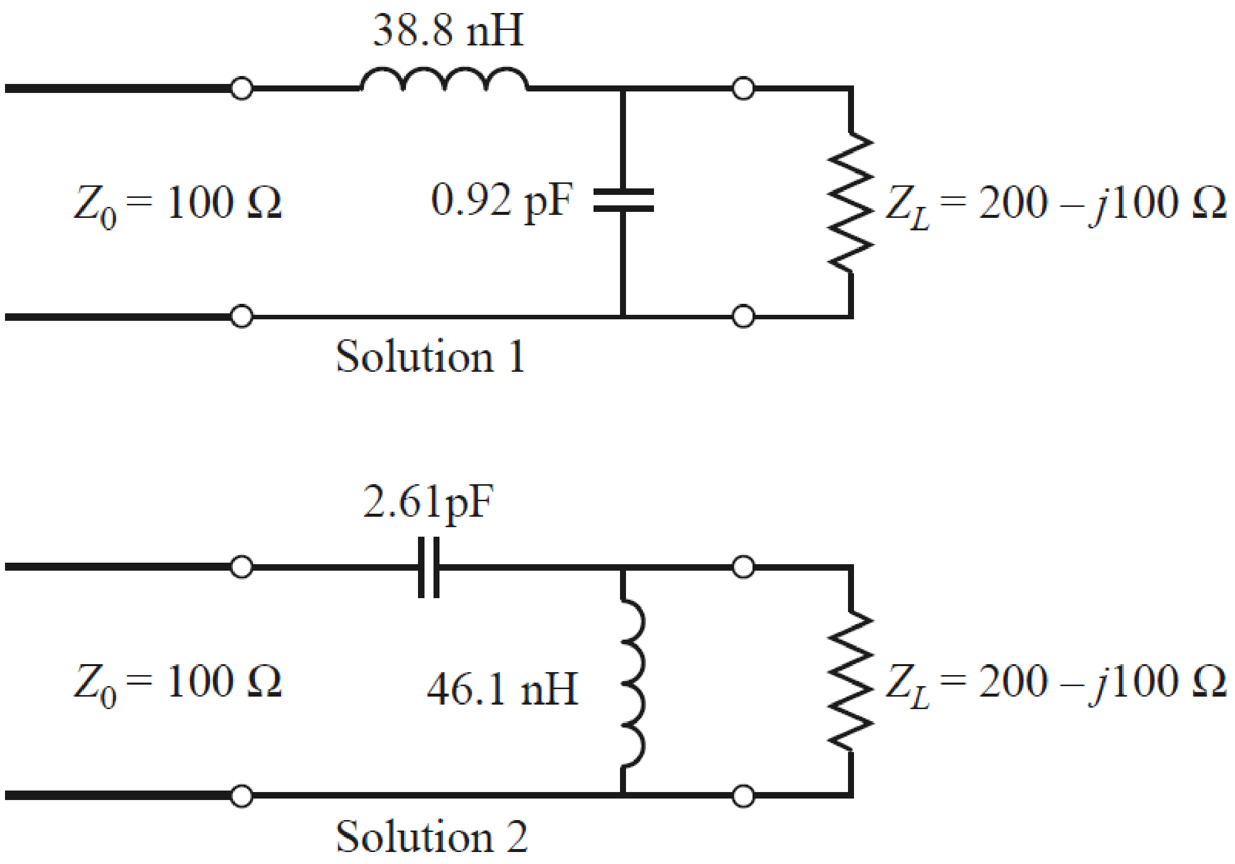
Therefore, at 500 MHz,

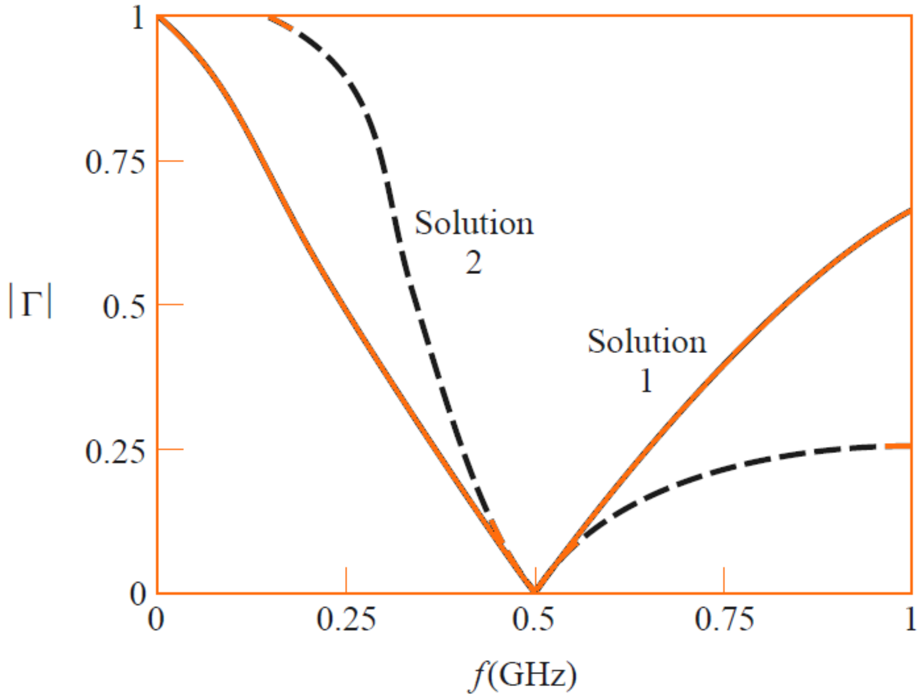
$$C = \frac{b}{2\pi f Z_0} = 0.92\ \text{pF},$$

$$L = \frac{x Z_0}{2\pi f} = 38.8\ \text{nH}.$$



Solution using Smith chart

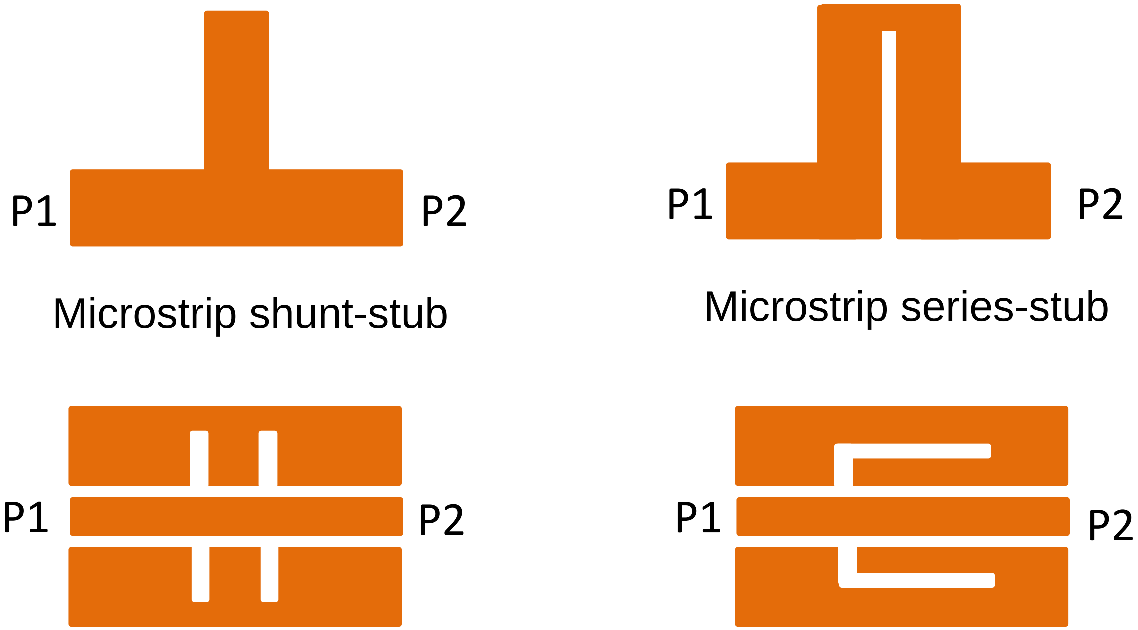




Plot of reflection coefficient for the two circuits.

Single stub matching

- Select an appropriate *d* (minimum) to transform the load admittance (impedance) to $Y_0 + jB$ ($Z_0 + jX$) form.
- Add $-jB$ ($-jX$) to transform to Y_0 (Z_0).
- Calculate *l* for the required susceptance (reactance).
- Either open or short circuited termination can be used.

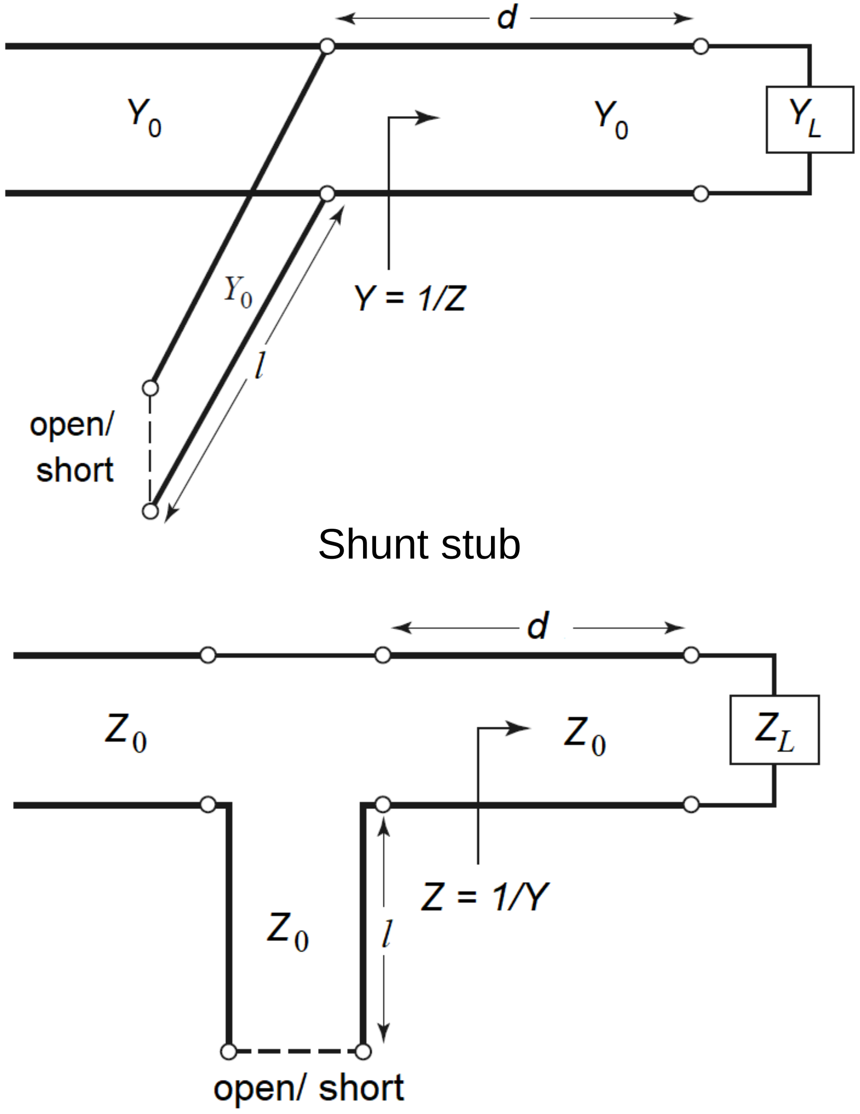


Microstrip shunt-stub

Microstrip series-stub

CPW shunt-stub

CPW series-stub



Shunt stub

Series stub

Single stub matching

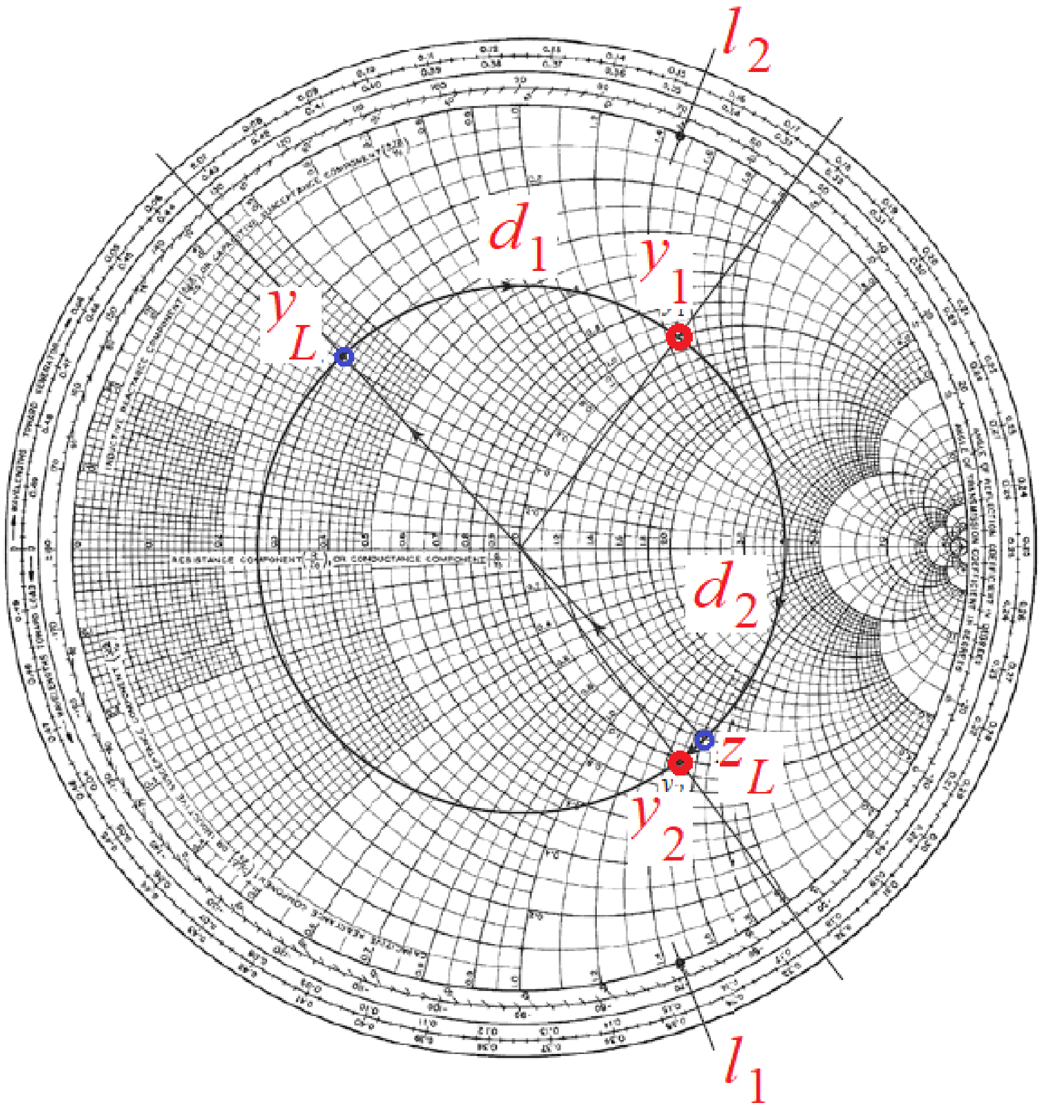
Q. Design a single-stub shunt tuning network for a RC circuit with Z_L = 60 − j80 Ω at 2 GHz. The port impedance is 50 Ω. Provide at least two solutions.

Answer:

- Therefore, $z_L = 1.2 - j\,1.6$.
- Convert z_L to y_L.

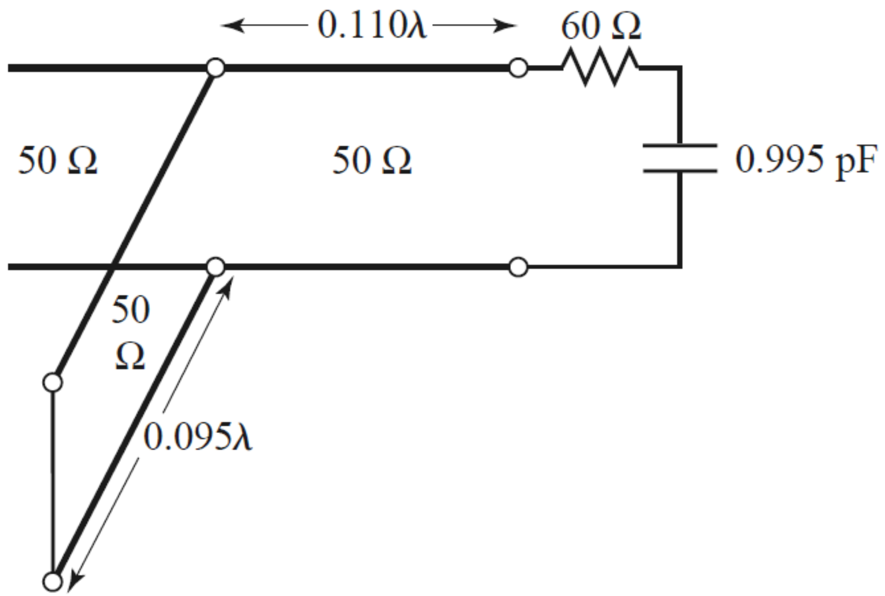
For the remaining steps, consider it as admittance chart:

- The SWR circle intersects the 1 + *jb* circle at two points denoted as y_1 and y_2
- $$d_1 = 0.176 - 0.065 = 0.110\lambda,$$ $$d_2 = 0.325 - 0.065 = 0.260\lambda.$$
- At the two intersection points, $$y_1 = 1.00 + j1.47, \quad y_2 = 1.00 - j1.47.$$
- Stub lengths for short circuited stubs (from y = ∞ towards generator) $$\ell_1 = 0.095\lambda \qquad \ell_2 = 0.405\lambda.$$

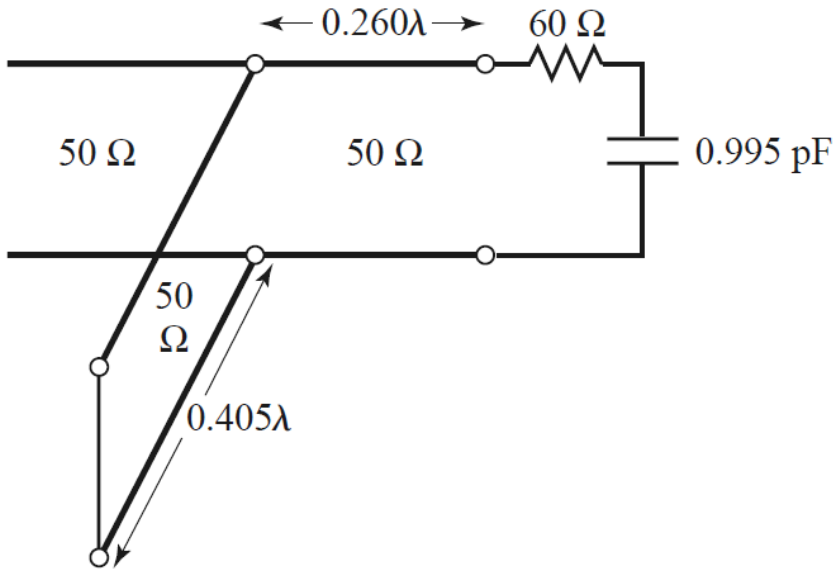


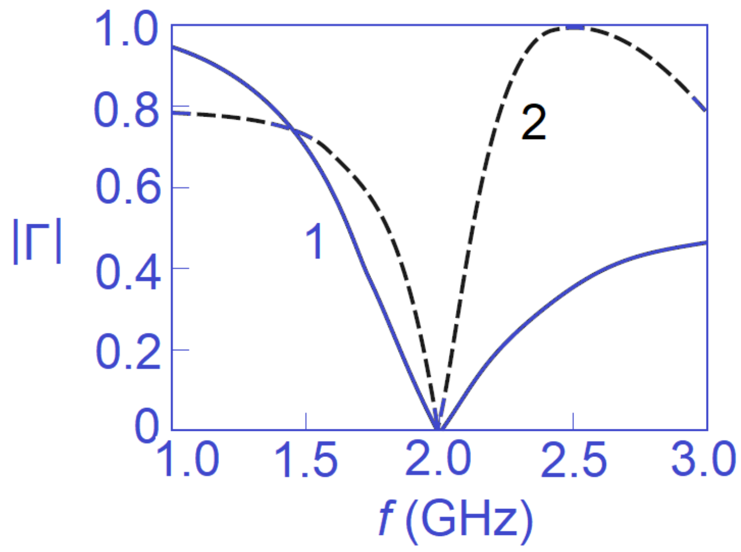
Single stub matching

Solution 1:



Solution 2:

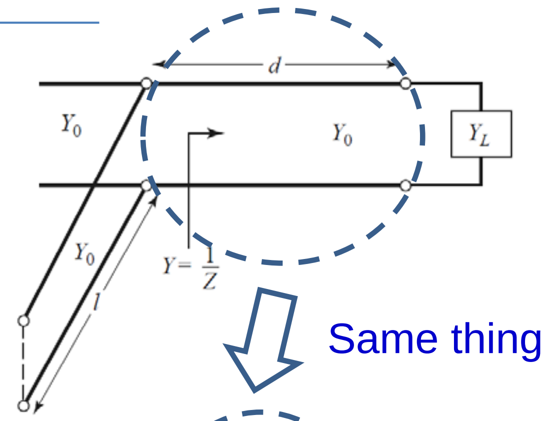


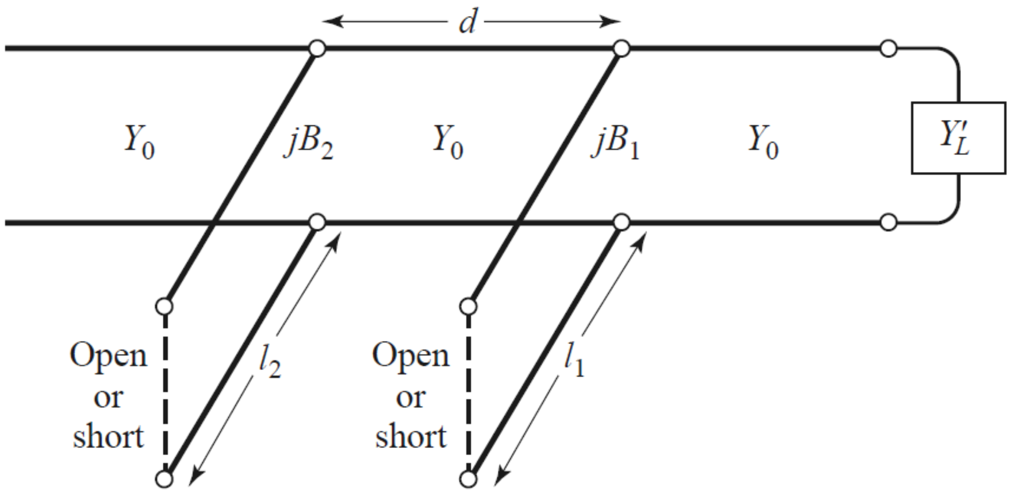


Comparison of the reflection coefficients.

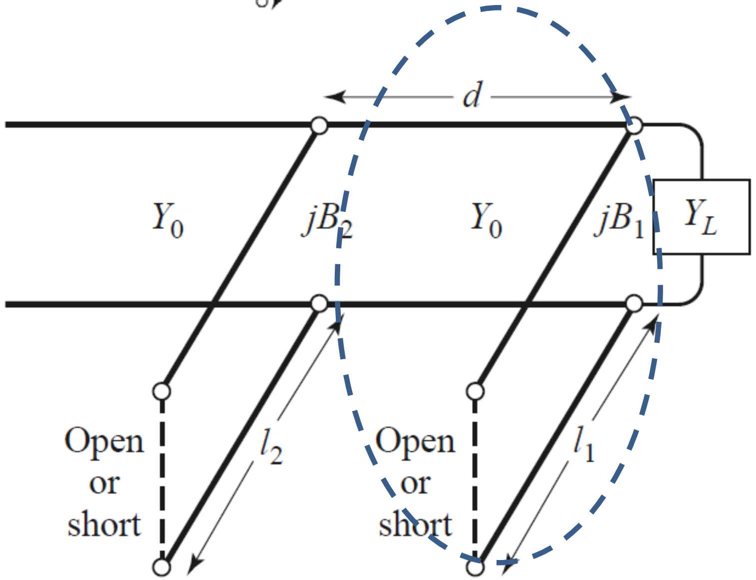
Double stub matching

- Popular for a adjustable tuner.
- Uses two tuning stubs at fixed position (arbitrary separation).
- Limitation: a range of impedance exist that cannot be matched.
- Both series and shunt stub approaches can be used.
- Either open or short termination can be used for the stubs.





Load at an arbitrary distance.



Load transformed to stub plane.

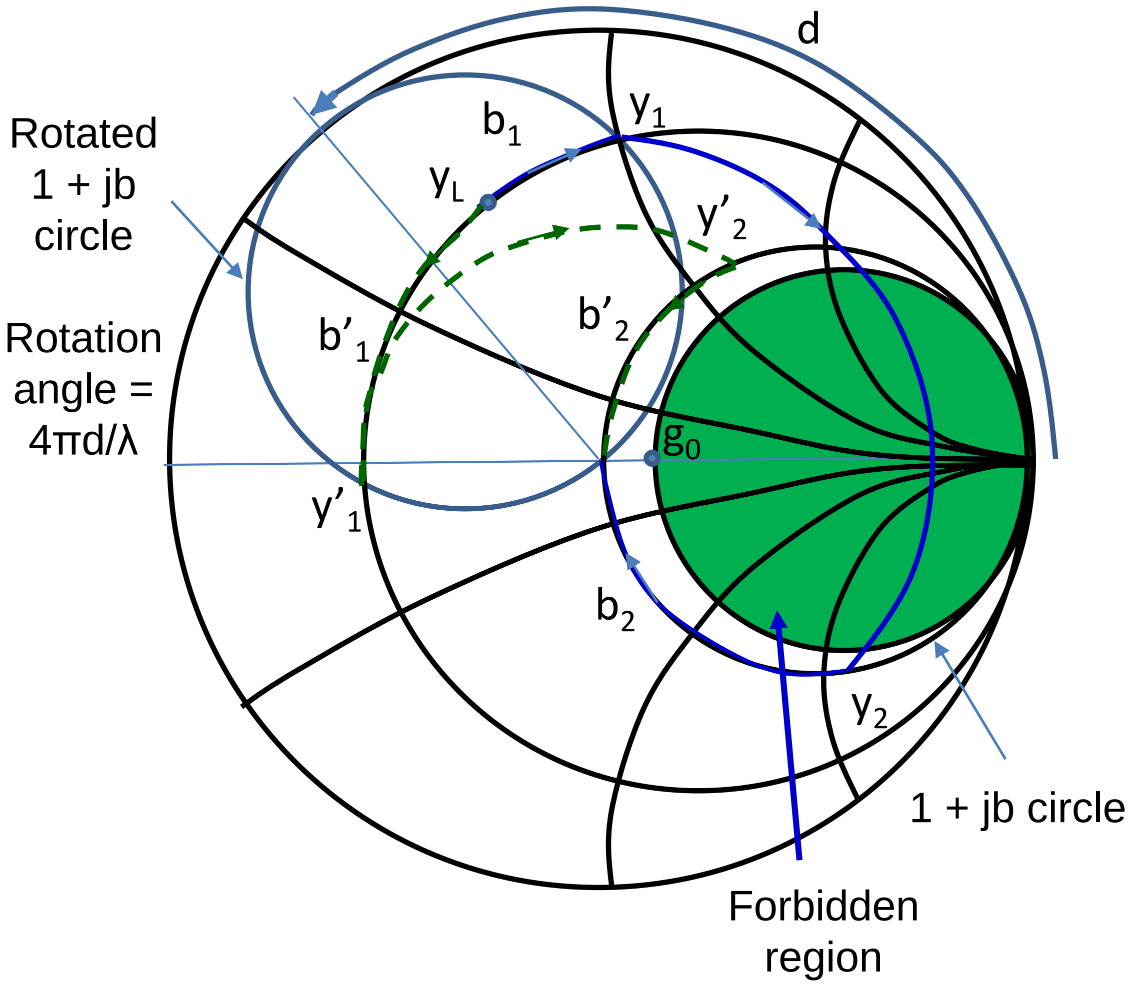
Double stub matching

The length of the first stub is selected so that the admittance at the location of the second stub is y_0.

The length of the second stub is selected to eliminate the imaginary part of the admittance at the location of insertion.

Steps:

- The susceptance of the first stub, b_1 (or b'_1, for the 2nd solution), moves the load admittance to y_1 (y'_1) on rotated 1 + jb circle.
- Transform y_1 by a rotation of *d* (arbitrary = 4πd/λ) wavelength toward load (counter clockwise) to y_2.
- y_2 must be on the 1 + jb circle.
- The second stub adds a susceptance b_2 to bring it to the centre of the Smith chart.

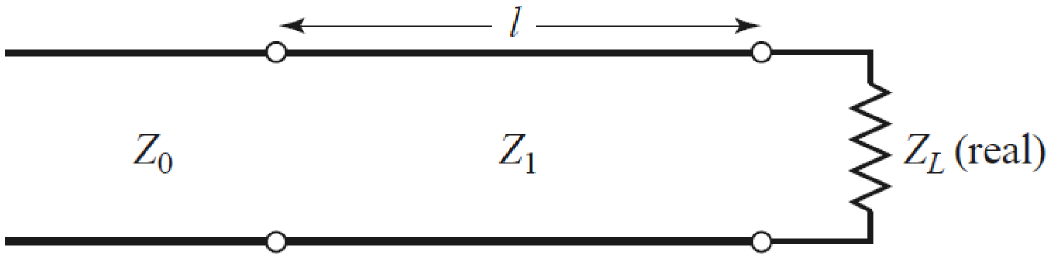


Smith chart for y_2
(rotated Smith chart for y_1)

The quarter wave transformer

- Transform only a real value to the desired Z_0.
- The input impedance looking at the load

$$Z_{\text{in}} = Z_1 \frac{Z_L + jZ_1 t}{Z_1 + jZ_L t}, \quad \text{where } t = \tan\beta\ell = \tan\theta$$



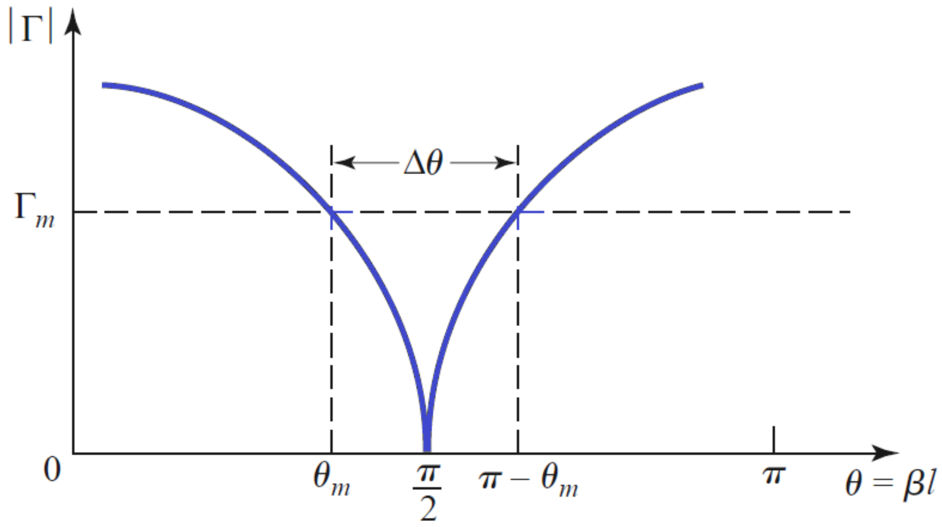
A quarter-wave transformer.

Equating it to Z_0 and for $\theta = 90^{\circ}$,

$$Z_1 = \sqrt{Z_0 Z_L}.$$

The input reflection at any arbitrary frequency,

$$|\Gamma| = \frac{1}{\left\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] \sec^2\theta\right\}^{1/2}}$$



The bandwidth over which reflection is below $|\Gamma_m|$ is

$$\Delta\theta = 2\left(\frac{\pi}{2} - \theta_m\right)$$

Theory of small reflections

Single section transformer:

The partial reflection coefficients are

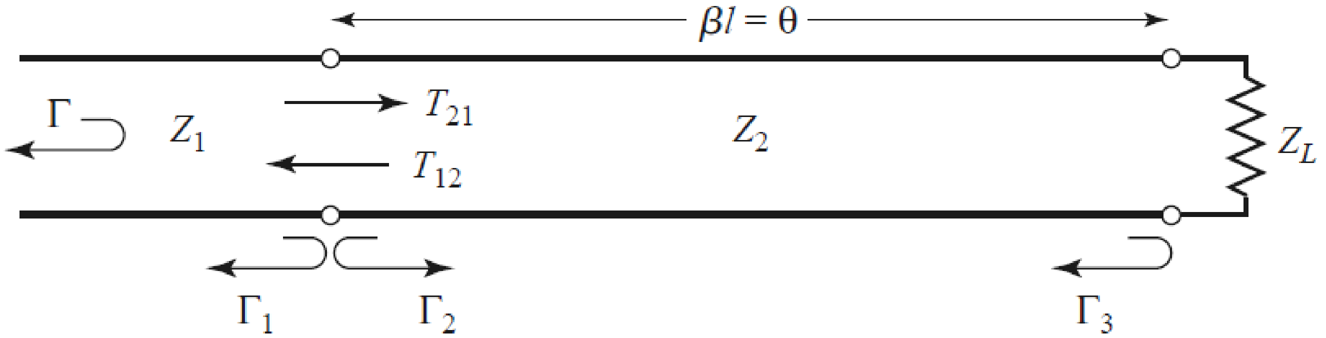
$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}, \quad \Gamma_2 = -\Gamma_1,$$

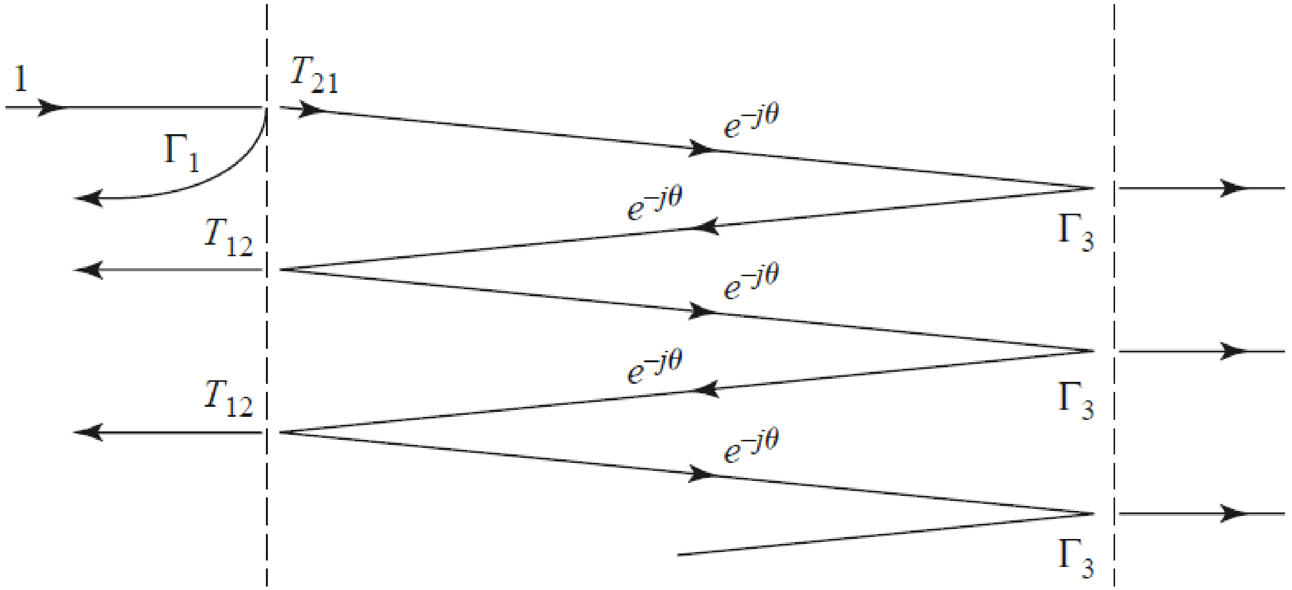
$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2},$$

The partial transmission coefficients are (for lossless network)

$$T_{21} = 1 + \Gamma_1 = \frac{2Z_2}{Z_1 + Z_2},$$

$$T_{12} = 1 + \Gamma_2 = \frac{2Z_1}{Z_1 + Z_2}.$$





Partial reflection and transmission

Theory of small reflection

Then, the total reflection coefficient is

$$\Gamma = \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta} + T_{12}T_{21}\Gamma_3^2\Gamma_2 e^{-4j\theta} + \cdots$$

$$= \Gamma_1 + T_{12}T_{21}\Gamma_3 e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_2^n \Gamma_3^n e^{-2jn\theta}.$$

Therefore,

$$\Gamma = \Gamma_1 + \frac{T_{12}T_{21}\Gamma_3 e^{-2j\theta}}{1 - \Gamma_2\Gamma_3 e^{-2j\theta}}.$$

Since, $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $|x| < 1$

$$= \frac{\Gamma_1 + \Gamma_3 e^{-2j\theta}}{1 + \Gamma_1\Gamma_3 e^{-2j\theta}}.$$

putting $\Gamma_2 = -\Gamma_1$, $T_{21} = 1 + \Gamma_1$, and $T_{12} = 1 - \Gamma_1$

For small discontinuities $|\Gamma_1 \Gamma_3| = 0$,

$\rightarrow \Gamma \simeq \Gamma_1 + \Gamma_3 e^{-2j\theta}.$

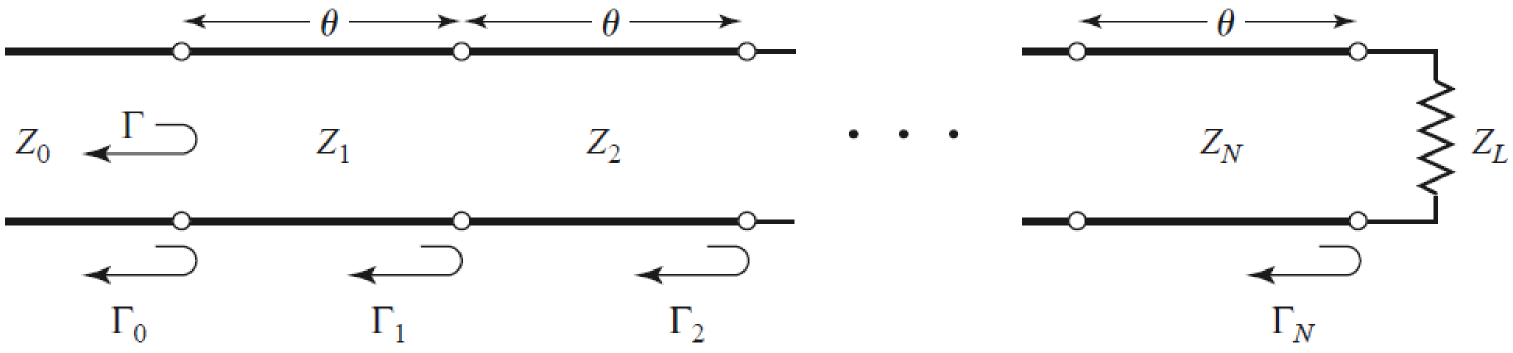
Multisection transformer

The partial reflection coefficients are

$$\Gamma_0 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n},$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}.$$



Multisection transformer

Using the previous relation, approximate reflection coefficient is

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \cdots + \Gamma_N e^{-2jN\theta}.$$

Now, assume that all Z_n increase or decrease monotonically across the transformer and that Z_L is real. Further, the transformer is symmetrical, so that $\Gamma_0 = \Gamma_N$, $\Gamma_1 = \Gamma_{N-1}$, $\Gamma_2 = \Gamma_{N-2}$, and so on. Then,

$$\Gamma(\theta) = e^{-jN\theta}\left\{\Gamma_0[e^{jN\theta} + e^{-jN\theta}] + \Gamma_1[e^{j(N-2)\theta} + e^{-j(N-2)\theta}] + \cdots\right\}.$$

Multisection transformer

When N is odd, the last term in the series is $\Gamma_{(N-1)/2}(e^{j\theta} + e^{-j\theta})$

For N even, the last term in the series is $\Gamma_{N/2}$

The reflection coefficients are

$$\Gamma(\theta) = 2e^{-jN\theta}\left[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta \right.$$
$$\left. + \cdots + \frac{1}{2}\Gamma_{N/2}\right] \quad \text{for } N \text{ even,}$$

$$\Gamma(\theta) = 2e^{-jN\theta}[\Gamma_0 \cos N\theta + \Gamma_1 \cos(N-2)\theta + \cdots + \Gamma_n \cos(N-2n)\theta$$
$$+ \cdots + \Gamma_{(N-1)/2} \cos\theta] \quad \text{for } N \text{ odd.}$$

Binomial multisection transformer

- The passband response is as flat as possible near the design frequency →maximally flat.
- N-section transformer is designed by setting the first (N − 1) derivatives of |Γ(θ)| to zero at the center frequency, f_0.
- Such a response can be obtained with a reflection coefficient of the form $\Gamma(\theta) = A(1 + e^{-2j\theta})^N$.

The magnitude of reflection coefficient is

$$|\Gamma(\theta)| = |A||e^{-j\theta}|^N|e^{j\theta} + e^{-j\theta}|^N \quad = 2^N|A||\cos\theta|^N$$

here $|\Gamma(\theta)| = 0 \text{ for } \theta = \pi/2,$ and $d^n|\Gamma(\theta)|/d\theta^n = 0 \text{ at } \theta = \pi/2 \text{ for } n = 1, 2, \ldots N-1$

($\theta = \pi/2$ corresponds to the center frequency, f_0, for which $l = \lambda/4$ and $\theta = \beta l = \pi/2$.)

The constant A is calculated by letting f → 0. Then $\theta = \beta l = 0$, and the first equation reduces to

$$\Gamma(0) = 2^N A = \frac{Z_L - Z_0}{Z_L + Z_0},$$ (since for $f = 0$ all sections are of zero electrical length)

Therefore, $$A = 2^{-N}\frac{Z_L - Z_0}{Z_L + Z_0}.$$

Binomial multisection transformer

Now, expand the first equation,

$$\Gamma(\theta) = A(1 + e^{-2j\theta})^N = A\sum_{n=0}^{N} C_n^N e^{-2jn\theta},$$ where $C_n^N = \dfrac{N!}{(N-n)!n!}$ (Binomial coefficients)

Here, $C_n^N = C_{N-n}^N,\ C_0^N = 1$, and $C_1^N = N = C_{N-1}^N.$

Compare with the previous relation,

$$\Gamma(\theta) = A\sum_{n=0}^{N} C_n^N e^{-2jn\theta} = \Gamma_0 + \Gamma_1 e^{-2j\theta} + \Gamma_2 e^{-4j\theta} + \cdots + \Gamma_N e^{-2jN\theta}.$$

Therefore, Γ_n should be chosen as $\Gamma_n = AC_n^N.$

For small Γ_n, $\Gamma_n = \dfrac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \simeq \dfrac{1}{2}\ln\dfrac{Z_{n+1}}{Z_n},$ Since, $\ln x \simeq 2(x-1)/(x+1)$ for x close to unity.

$$\ln\frac{Z_{n+1}}{Z_n} \simeq 2\Gamma_n = 2AC_n^N = 2(2^{-N})\frac{Z_L - Z_0}{Z_L + Z_0}C_n^N \simeq 2^{-N}C_n^N \ln\frac{Z_L}{Z_0},$$

Binomial multisection transformer: design table

Z_L/Z_0	N = 2		N = 3			N = 4			
	Z_1/Z_0	Z_2/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110

Z_L/Z_0	N = 5					N = 6					
	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_1/Z_0	Z_2/Z_0	Z_3/Z_0	Z_4/Z_0	Z_5/Z_0	Z_6/Z_0
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

Binomial multisection transformer

Exact design is found by using the transmission line equations for each section and numerically solving for the characteristic impedances .

The bandwidth of the binomial transformer for a specified Γ_m,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{|A|}\right)^{1/N}\right].$$

Q. Design a three-section binomial transformer to match a 50 Ω load to a 100 Ω line and calculate the bandwidth for m = 0.05. Plot the reflection coefficient magnitude versus normalized frequency for the exact designs using 1, 2, 3, 4, and 5 sections.

Answer:

For $N = 3$, $Z_L = 50\ \Omega$, and $Z_0 = 100\ \Omega$ → $A = 2^{-N}\dfrac{Z_L - Z_0}{Z_L + Z_0} \simeq \dfrac{1}{2^{N+1}}\ln\dfrac{Z_L}{Z_0} = -0.0433.$

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{0.05}{0.0433}\right)^{1/3}\right] = 0.70$$

Binomial multisection transformer

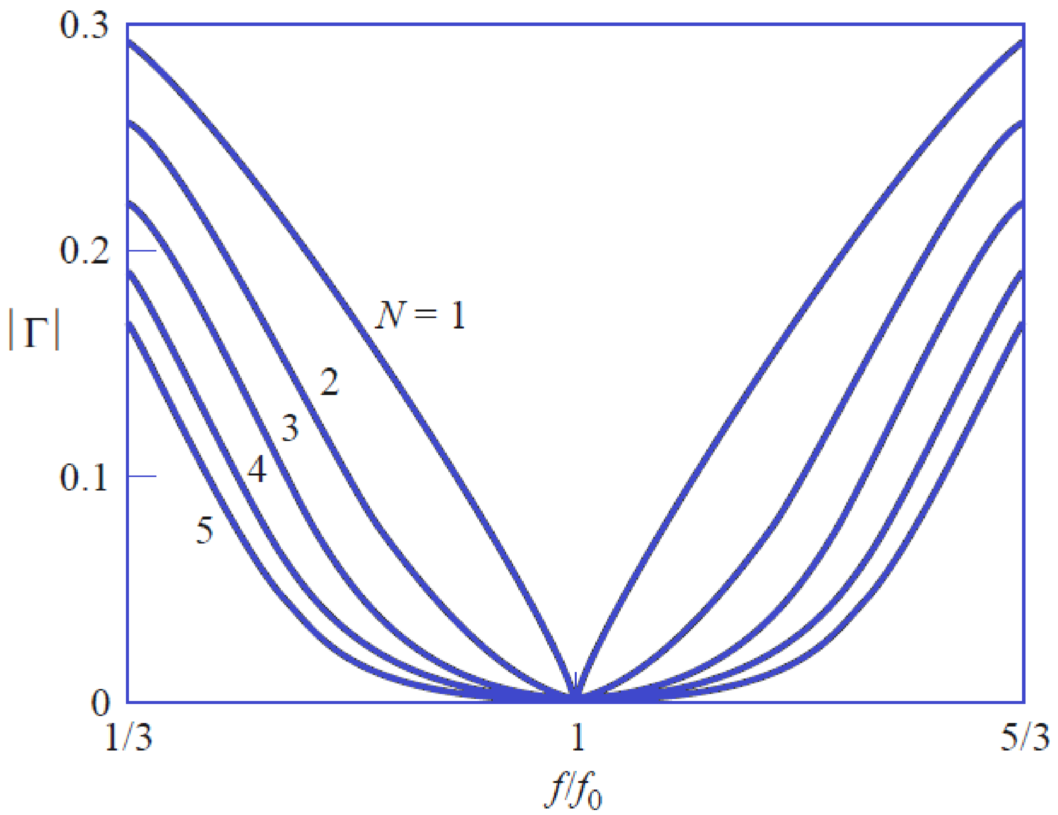
The binomial coefficients are $C_0^3 = \frac{3!}{3!0!} = 1, \quad C_1^3 = \frac{3!}{2!1!} = 3, \quad C_2^3 = \frac{3!}{1!2!} = 3.$

$$n = 0: \quad \ln Z_1 = \ln Z_0 + 2^{-N} C_0^3 \ln \frac{Z_L}{Z_0}$$
$$= \ln 100 + 2^{-3}(1) \ln \frac{50}{100} = 4.518,$$
$$Z_1 = 91.7\ \Omega;$$

$$n = 1: \quad \ln Z_2 = \ln Z_1 + 2^{-N} C_1^3 \ln \frac{Z_L}{Z_0}$$
$$= \ln 91.7 + 2^{-3}(3) \ln \frac{50}{100} = 4.26,$$
$$Z_2 = 70.7\ \Omega;$$

$$n = 2: \quad \ln Z_3 = \ln Z_2 + 2^{-N} C_2^3 \ln \frac{Z_L}{Z_0}$$
$$= \ln 70.7 + 2^{-3}(3) \ln \frac{50}{100} = 4.00,$$
$$Z_3 = 54.5\ \Omega.$$

Binomial multisection transformer



Reflection coefficient magnitude versus frequency for multisection binomial matching transformers (Z_L = 50 Ω and Z_0 = 100 Ω)

Chebyshev multisection transformer

- Compromising on the flatness of the passband response leads to a bandwidth that is substantially better than that of the binomial transformer for a given number of sections.
- The Chebyshev transformer is designed by equating $\Gamma(\theta)$ to a Chebyshev polynomial, which has the optimum characteristics needed for this type of transformer.
- The n^{th}-order Chebyshev polynomial is a polynomial of degree n, denoted by $T_n(x)$ and is given as

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x).$$

First few polynomials are

$$T_1(x) = x,$$

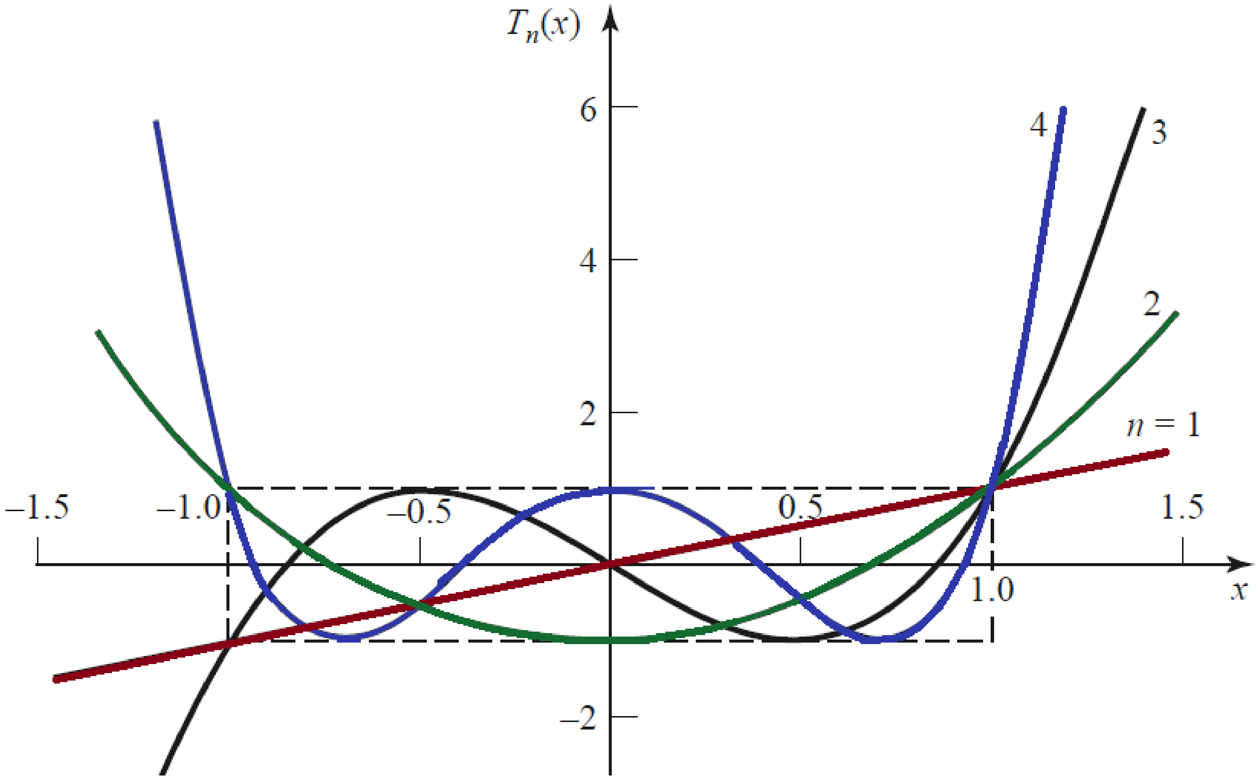
$$T_2(x) = 2x^2 - 1,$$

$$T_3(x) = 4x^3 - 3x$$

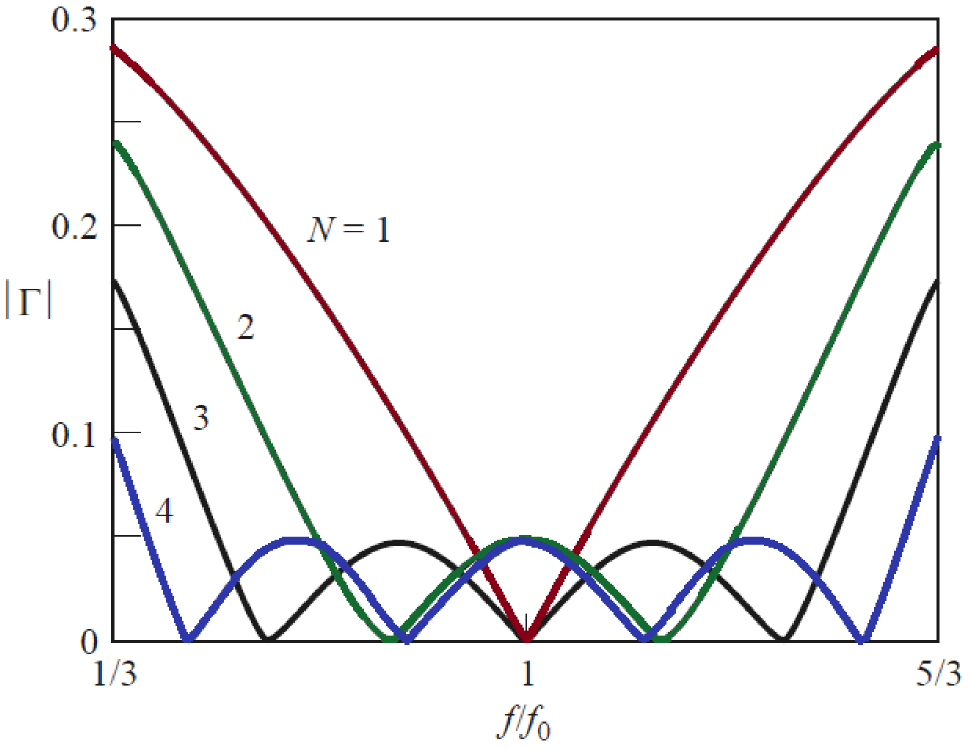
Properties:

- $-1 \le x \le 1$, $|T_n(x)| \le 1$: The Chebyshev polynomials oscillate between ±1. This is the equal-ripple property, and this region will be mapped to the passband of the matching transformer.
- $|x| > 1$, $|T_n(x)| > 1$: This region will map to the frequency range outside the passband.
- $|x| > 1$: The $|T_n(x)|$ increases faster with x as n increases.

Chebyshev multisection transformer

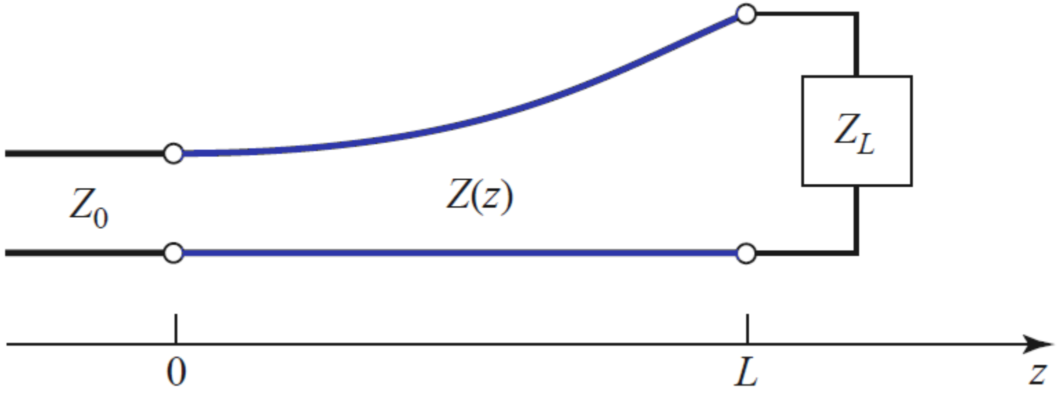


The first four Chebyshev polynomials.

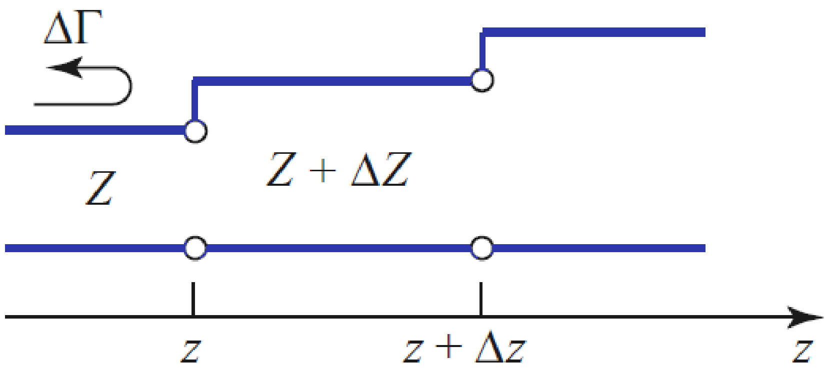


Reflection coefficient magnitude versus frequency for the Chebyshev transformers.

Tapered matching section



Tapered matching section.



Approximation used for analysis.

Reflection from the impedance step, $\Delta\Gamma = \dfrac{(Z+\Delta Z) - Z}{(Z+\Delta Z) + Z} \simeq \dfrac{\Delta Z}{2Z}.$

In the limit ($\Delta z \rightarrow 0$), $d\Gamma = \dfrac{dZ}{2Z} = \dfrac{1}{2}\dfrac{d(\ln Z/Z_0)}{dz}dz$ since, $\dfrac{d(\ln f(z))}{dz} = \dfrac{1}{f}\dfrac{df(z)}{dz}.$

Then, total reflection at z = 0, $\Gamma(\theta) = \dfrac{1}{2}\displaystyle\int_{z=0}^{L} e^{-2j\beta z}\dfrac{d}{dz}\ln\left(\dfrac{Z}{Z_0}\right)dz.$

Tapered matching section

Exponential taper: $Z(z) = Z_0 e^{az} \quad \text{for } 0 < z < L$

Triangular taper: $$Z(z) = \begin{cases} Z_0 e^{2(z/L)^2 \ln Z_L/Z_0} & \text{for } 0 \le z \le L/2 \\ Z_0 e^{(4z/L - 2z^2/L^2 - 1)\ln Z_L/Z_0} & \text{for } L/2 \le z \le L \end{cases}$$

So that the derivative is triangular in form:

$$\frac{d(\ln Z/Z_0)}{dz} = \begin{cases} 4z/L^2 \ln Z_L/Z_0 & \text{for } 0 \le z \le L/2 \\ (4/L - 4z/L^2)\ln Z_L/Z_0 & \text{for } L/2 \le z \le L. \end{cases}$$

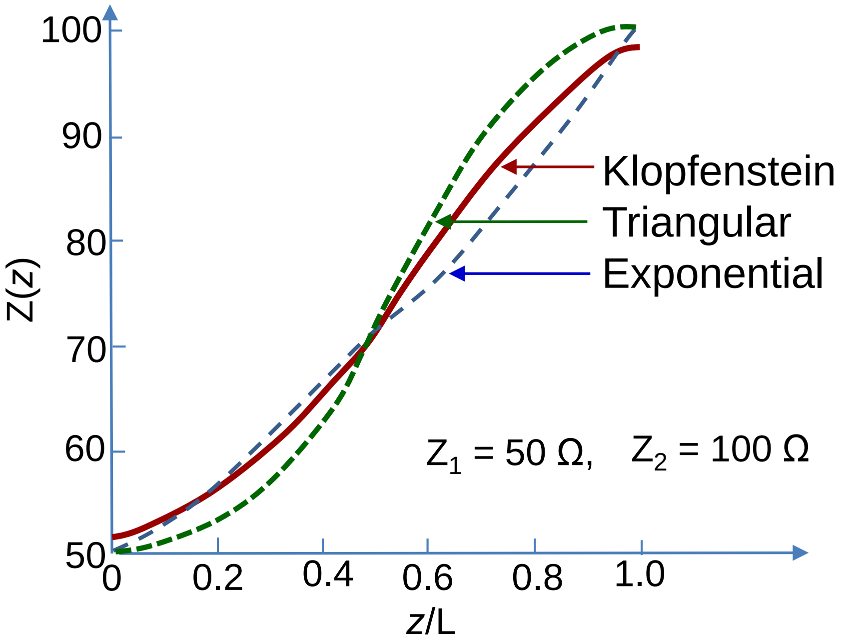
Klopfenstein taper: For a given taper length (greater than some critical value), the Klopfenstein impedance taper provides optimum reflection coefficient over the passband.

Alternatively, for a maximum reflection coefficient specification in the passband, the Klopfenstein taper yields the shortest matching section.

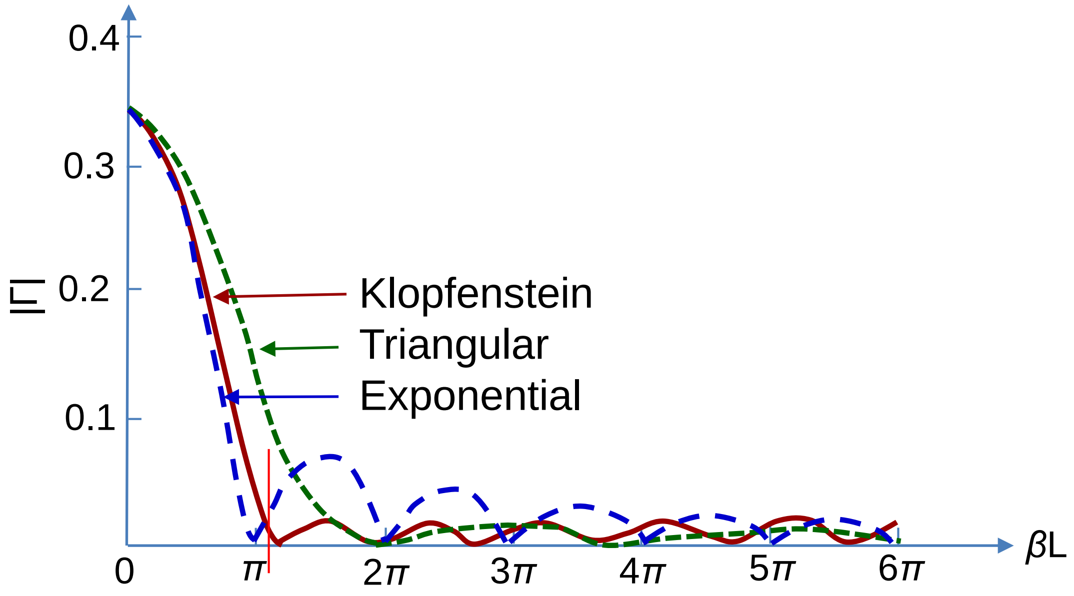
The Klopfenstein taper is derived from a stepped Chebyshev transformer as the number of sections increases to infinity.

Tapered matching section

The partial reflection coefficients are



Impedance variations for the triangular, exponential, and Klopfenstein tapers.



Resulting reflection coefficient magnitude versus frequency for the tapers for $\Gamma_m = 0.02$.

RF and Microwave Engineering (EC 31005)

Resonators (P5)

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I.I.T. Kharagpur.

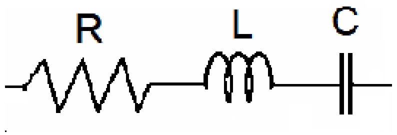
Resonance and resonator

- Resonance is the tendency of a system to oscillate with greater amplitude at some frequencies than at others.

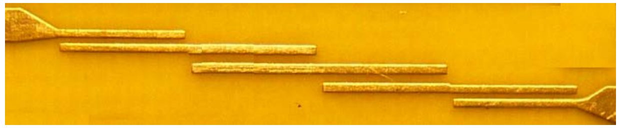
- Examples: quartz crystals, RLC circuit, microwave cavities.

Swing

Quartz osc.



Series resonator.



Microwave resonators.

- Resonator: generate vibrations of a specific frequency (e.g. source), or pick out specific frequencies from a complex vibration containing many frequencies (e.g. filters).

- Microwave resonators: electromagnetic resonators, very similar to RLC resonators. Applications - filters, oscillators, frequency meters, tuned amplifiers, antennas (lossy resonator).

Quality factor

$$Q(\omega) = \omega \times \frac{Maximum\, energy\, stored}{Power\, loss}$$

- At resonance where the stored energy is constant with time,

$$Q = 2\pi \frac{Energy\, stored}{Energy\, dissipated\;\; per\, cycle}$$

$$= 2\pi\, f_r \frac{Energy\, stored}{Power\, loss}$$

- The quality factor of a circuit vary substantially from system to system.

Information from Q-factor:

- How under-damped a resonator is.
- Bandwidth relative to its center frequency.
- Rate of energy loss relative to the stored energy.

Q-factor

•Low Q-factor (Q <0.5): overdamped

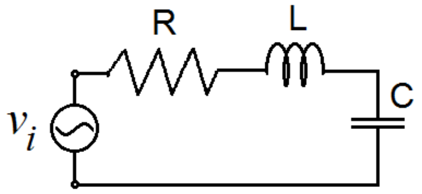
The resonator does not oscillate at all e.g. a second order lowpass RC filter with low Q-factor.

•High Q-factor (Q >0.5): underdamped

The resonator oscillates with a decay of amplitude e.g. a second order Butterworth filter with Q = 1/√2.

•Intermediate Q-factor (Q = 0.5): critically damped

Does not oscillate, fastest response without an overshoot.



Series RLC resonator.

•Q of an inductor: $\dfrac{\omega L}{R_L}$

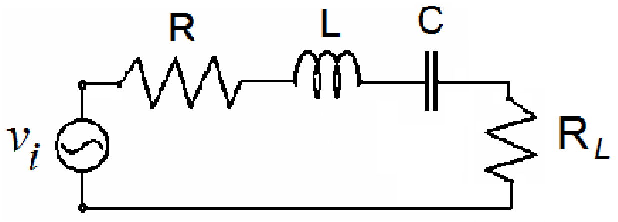
•Q of a capacitor: $\dfrac{1}{\omega C R_C}$

•Q of a parallel RLC circuit: $R\sqrt{\dfrac{C}{L}}$

•Q of a series RLC circuit: $\dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

Loaded, unloaded and external quality factors

- When the resonator is isolated from rest of the world - unloaded quality factor (**Q_{ul}**).
- When a resonator is a part of a circuit, overall loss increases – loaded quality factor (**Q_l**).
- Total loss also includes the loss into the source and load resistances – external quality factor (**Q_e**).



Series resonator with source and loads.

$$Q_{ul} = \omega \frac{\text{average energy stored}}{\text{energy loss / second}}$$

$$= \omega \frac{W_m + W_e}{P_{loss}}$$

$$= \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}.$$

Define external quality factor as

$$Q_e = \frac{\omega_0 L}{R_L} \quad \text{for a series resonator}$$

$$= \frac{R_L}{\omega_0 L} \quad \text{for a parallel resonator.}$$

From energy conservation → $\boxed{\frac{1}{Q_L} = \frac{1}{Q_e} + \frac{1}{Q_{ul}}.}$

Considering the losses of a microwave resonator→ $\boxed{\frac{1}{Q_{uL}} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r}.}$

Some important relationships

•3-dB bandwidth of a resonator is $\Delta f = \dfrac{f_0}{Q_L}$

•The damping ratio is $\varsigma = \dfrac{1}{2Q_L}$

•The attenuation constant is $\alpha = \dfrac{\omega_0}{2Q_L}$

•Insertion loss (dB) is $IL = 20\log_{10}\left(1 - \dfrac{Q_L}{Q_u}\right).$

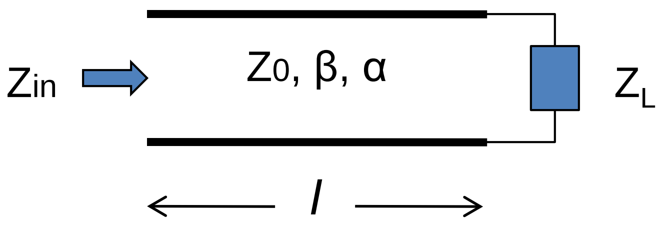
A microstrip line half-wave resonator excited by gap feeding.

•Loaded Q of a component with *Z* = *R* + j*X* is

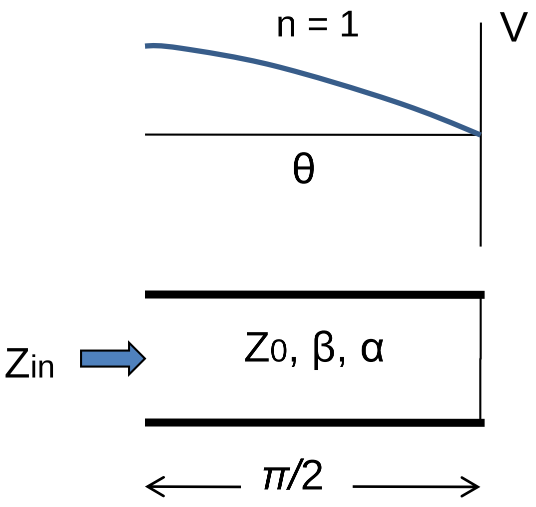
$$Q_L = \frac{\text{susceptance}}{\text{conductance}}$$

- Complexity of the design, implementation, tunability, small-signal (active devices) etc.

Transmission line resonators



Transmission line with a load Z_L.



A shorted λ/4 transmission line.

- Resonance condition: voltage distribution must satisfy the boundary conditions.
- Input impedance of a lossy transmission line

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \quad where,\ \gamma = \alpha + j\beta.$$

- Input impedance of a lossless transmission line

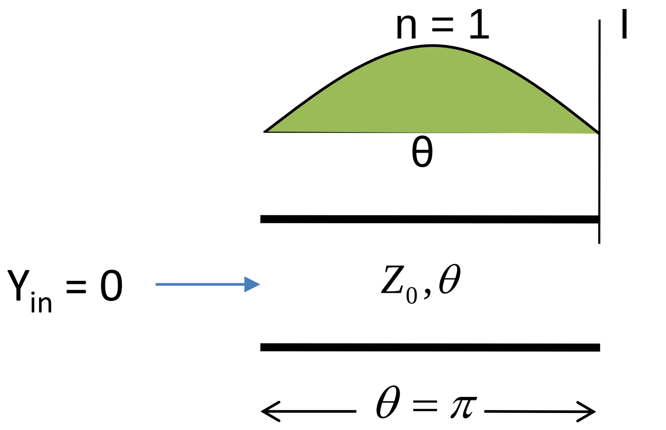
$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \quad for\ \alpha = 0$$

•Resonance condition for shorted load $Z_{in} = 0.$

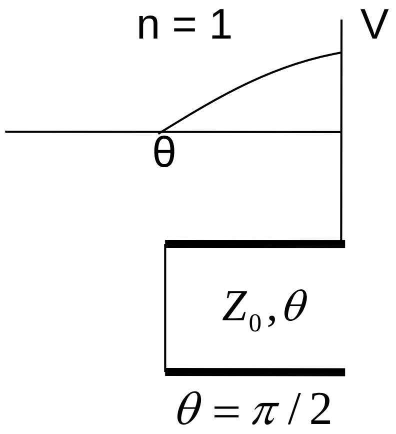
•Resonance condition for open load $Y_{in} = 1/Z_{in} = 0.$

•Quality factor of the resonators: $Q = \dfrac{\beta}{2\alpha}$

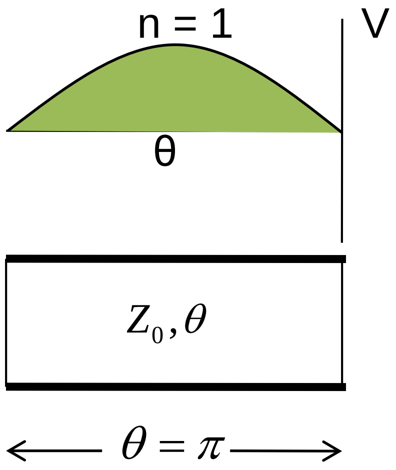
Transmission line resonators



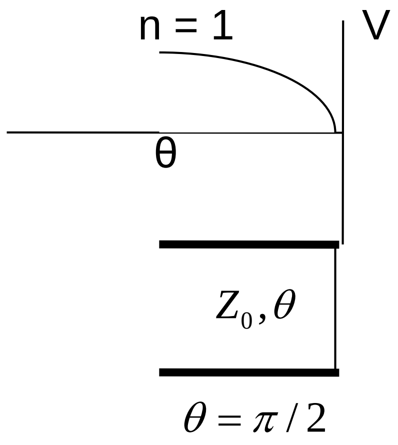
Half-wavelength resonator, both ends open.



Quarter-wavelength resonator, rear end open.

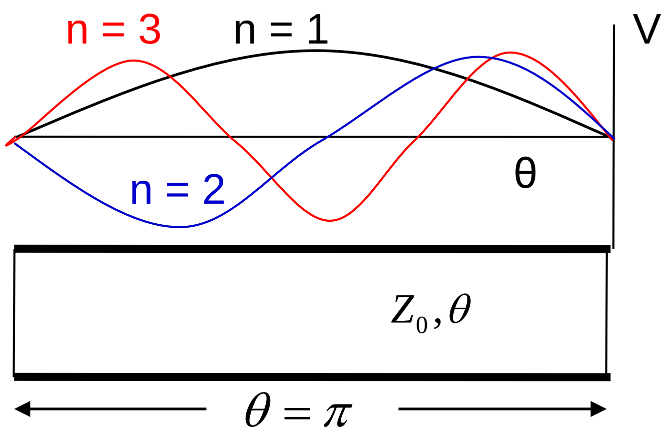


Half-wavelength resonator, both ends shorted.



Quarter-wavelength resonator, rear end shorted.

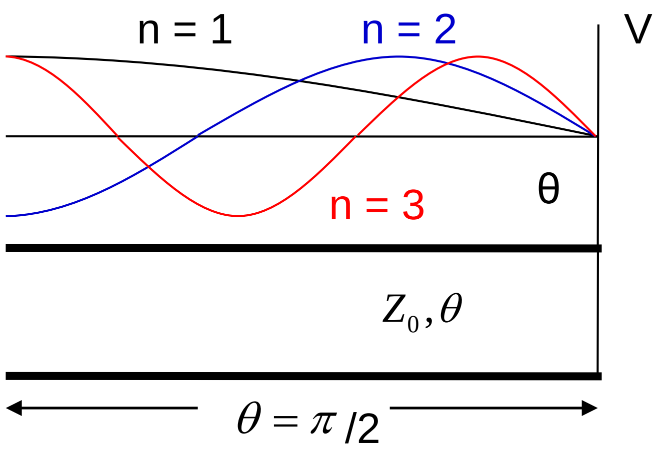
Transmission line resonators: higher order modes



Voltage distributions on a half-wavelength resonator
(both sides shorted).

•Resonant frequencies are

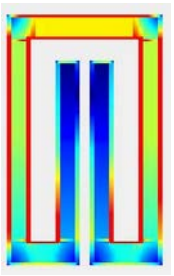
$$f_1,\ 2f_1, 3f_1 \ldots$$



Voltage distributions on a quarter-wavelength resonator.

•Resonant frequencies are

$$f_2,\ 3f_2, 5f_2 \ldots$$



Current distribution on a half-wavelength microstrip resonator, both ends open.

- What is the input impedance at the resonant frequency?
- Parallel or series resonance?

Transmission line resonators

Q. A 50 Ω microstrip line resonator of length $\lambda_g/2$. Compute the physical length and Q at 5 GHz. (Substrate specifications : t = 1.59 mm, ε_r = 2.08, tanδ = 0.0004, copper as metal).

Solution:

The width of a 50Ω microstrip line on this substrate = 5.08 mm and effective permittivity = 1.8.

•Resonant length = $\lambda_g/2$ = 22.4 mm.

•Propagation constant $\beta = \dfrac{\omega}{v_p} = \dfrac{2\pi f\sqrt{\varepsilon_e}}{c} = 151\ rad/m.$

•Attenuation due to conductor loss $\alpha_c = \dfrac{R_S}{Z_0 W} = 0.0724\ Np/m.$ where $R_S = \sqrt{\dfrac{\omega\mu_0}{2\sigma}}.$

•Attenuation due to dielectric loss

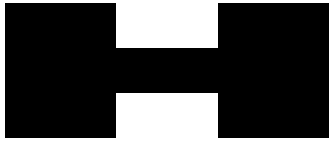
$$\alpha_d = \frac{k_0\varepsilon_r(\varepsilon_e - 1)\tan\delta}{2\sqrt{\varepsilon_e}(\varepsilon_r - 1)} = 0.024\ Np/m. \qquad Q = \frac{\beta}{2\alpha} = \frac{151}{2(0.0724 + 0.024)}$$

$$= 783.$$

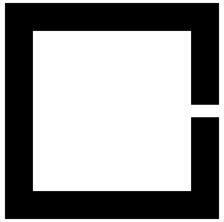
Considering other effects, actual Q is much low..!

Different shapes of microstrip resonators

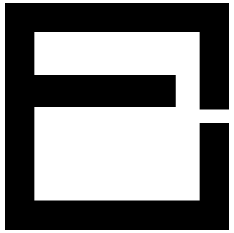
Half-wavelength resonator.



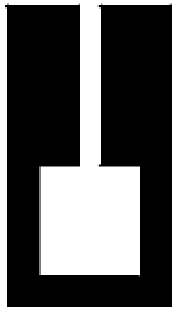
Stepped-imp resonator.



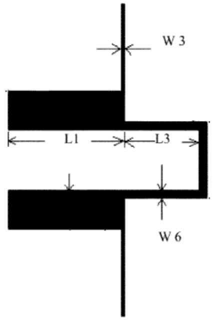
Open-loop resonator (OLR)



Modified OLR



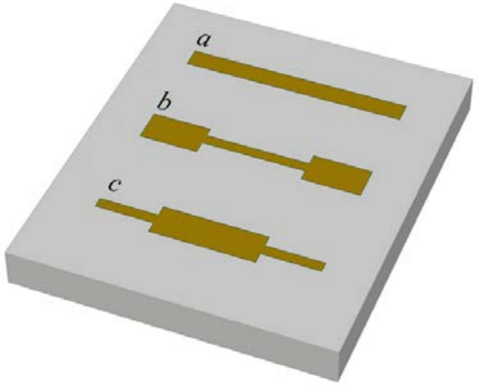
Modified Hair pin resonator



A multi-mode resonator.

- Different shapes are used for
  - Size reduction,
  - Dual-band and multi-band operation,
  - Spurious suppression.

Dual-band resonators

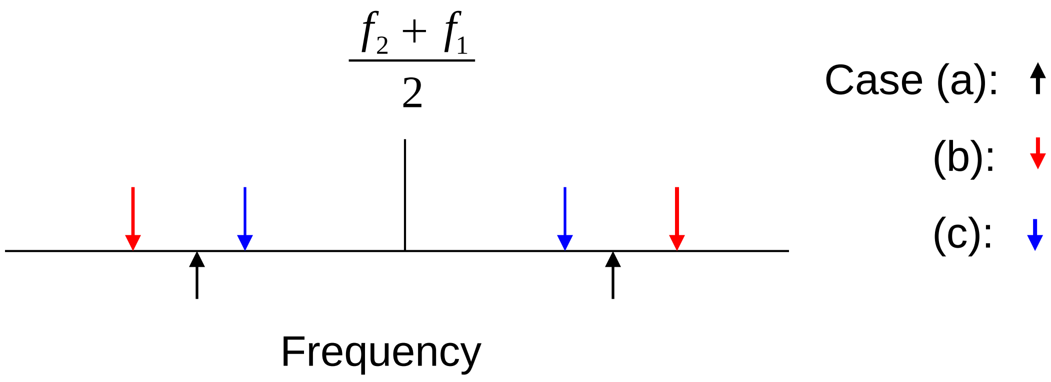


Different microstripline resonators.

$$a)\ \frac{f_2}{f_1}=2,$$

$$b)\ \frac{f_2}{f_1}>2,$$

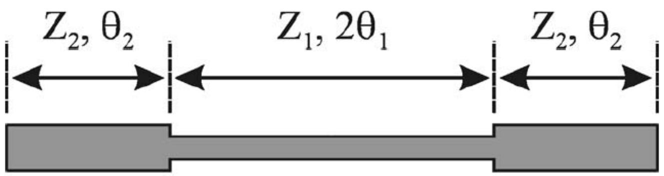
$$c)\ \frac{f_2}{f_1}<2.$$



Properties of Stepped-Impedance Resonator (SIR):

- The fundamental resonant frequency can be reduced.
- Higher order resonant frequencies can be controlled by impedance ratio.

Dual-band resonator



Dual-band SIR

- Resonance condition from Y_{in} = 0,

$$Z_2 \tan\theta_1 \tan\theta_2 / Z_1 = 1.$$

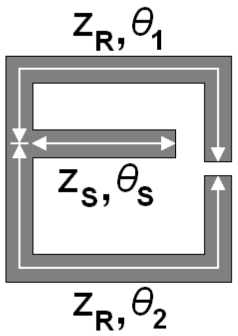
- The frequency ratio for $\theta_1 = \theta_2$,

$$\frac{f_2}{f_1} = \frac{\pi}{2\tan^{-1}\sqrt{Z_2/Z_1}}.$$

- Condition for minimum size of the resonator:

$$\tan\theta_1 = \tan\theta_2 = (Z_1/Z_2)^{1/2}.$$

Dual-band resonator



Dual-Band Open-Loop Resonator.

- An open-loop resonator is similar to a straight $\lambda/2$ line.

Input admittance from one end,

$$Y_{\text{in}} = jY \frac{\tan\theta_1 + \tan\theta_2 + \tan\theta_S}{1 - \tan\theta_1 (\tan\theta_2 + \tan\theta_S)} \qquad \text{where } Y_R = Y_S = Y$$

The resonance conditions

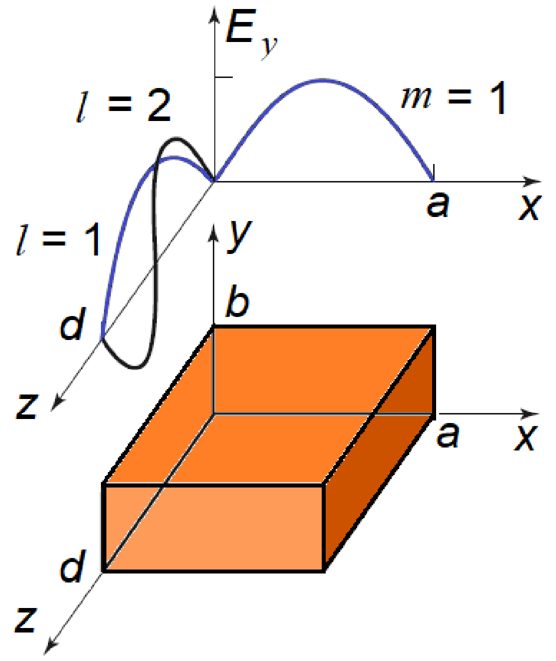
$$\tan(n\theta_1) + \tan(n\theta_2) + \tan(n\theta_S) = 0$$

Ref.: P. Mondal and M. K. Mandal, “Design of dual-band bandpass filters using stub-loaded open-loop resonators”, *IEEE trans on Microw. Theory and Tech.*, Jan. 2008.

Waveguide resonators

- A closed metallic box, or cavity.
- Electric and magnetic energy is stored within the cavity enclosure, and power is dissipated in the metallic walls and in the dielectric material that may fill the cavity.
- Coupling to a cavity resonator may be by a small aperture, or a small probe or loop.
- A rectangular waveguide cavity supports both TE_{mnl} or TM_{mnl} resonant modes, where the indices *m*, *n*, *l* indicate the number of variations in the standing wave pattern in the *x*, *y*, *z* directions, respectively.
- The resonant frequencies are

$$k_{mn\ell} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2}$$



A rectangular waveguide resonator and TE_{101} and TE_{102} modes.

Waveguide cavity resonators

A rectangular waveguide cavity is made from a piece of WR-187 H-band waveguide, with a = 4.755 cm and b = 2.215 cm. The cavity is filled with polyethylene (ε_r = 2.25, tan δ = 0.0004). If resonance is to occur at f_0 = 5 GHz, find the required length, d.

Solution:

The wave number k is $$k = \frac{2\pi f \sqrt{\epsilon_r}}{c} = 157.08 \text{ m}^{-1}.$$

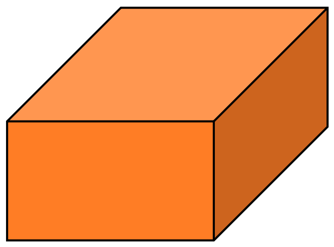
Resonant length for m =1, n = 0, $$d = \frac{\ell\pi}{\sqrt{k^2 - (\pi/a)^2}},$$

Then,

$$\text{for } \ell = 1, \quad d = \frac{\pi}{\sqrt{(157.08)^2 - (\pi/0.04755)^2}} = 2.20 \text{ cm},$$

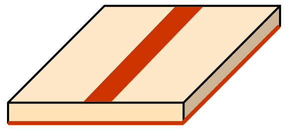
$$\text{for } \ell = 2, \quad d = 2(2.20) = 4.40 \text{ cm}.$$

Improvement of quality factor



TE_{101} ($Q_{ul} > 5000$)

Air filled rectangular waveguide



TE_{101} ($Q_{ul} < 300$)

Microstrip line resonator

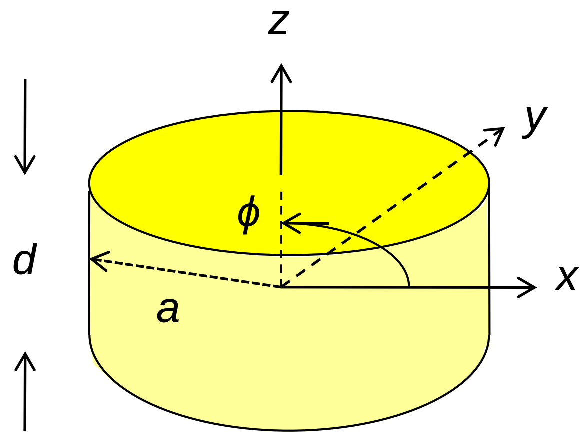
How to improve unloaded quality factor of a resonator?

- Minimize losses (conductor, dielectric, leakage).
- Bigger volume: e.g. conductor loss increases with decreasing *height*, higher order modes offer lower loss, lower dielectric constant (bigger volume) offer lower loss.
- Dielectric with small loss tangent.
- Choose a proper dimension to avoid radiation and surface wave losses (microstrip).
- Polished surface (surface roughness).

A. A. Khan, M. K. Mandal, S. Sanyal, "Unloaded quality factor of a substrate integrated waveguide resonator and its variation with the substrate parameters," in *IEEE Conf. MAP*., 2013.

Dielectric resonators

- Materials: usually ceramic with high ϵ_r (10-100).
- Pros: compact size, lower loss. Cons: sensitive to temperature and mechanical vibration/stress etc.
- Support hybrid modes - HEM_{nml} , and TE/TM to z.
- Hybrid modes are HE_{nml} when E_Z dominates over H_z and EH_{nml}, when H_Z dominates over E_z.
- n – circumferential (ϕ), m – radial (r) and l – axial (z) variations.
- For n = 0, axisymmetric modes → TM_{0ml} and TE_{0ml}.
- l is replaced by δ (0<1< δ) for $d < \lambda_g/2$.
- Most commonly used mode is $TE_{01\delta}$



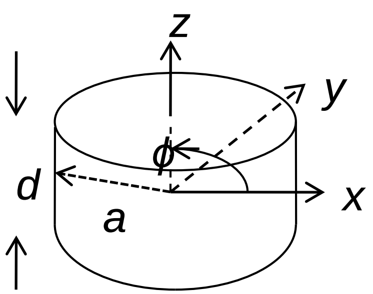
Isolated dielectric resonator

Dielectric resonators

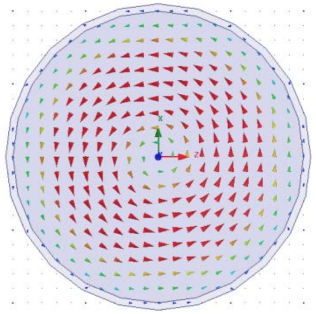
$TE_{01\delta}$ mode:

- $E_z = 0$, have azimuthally symmetric ($\partial/\partial\phi = 0$) and less than a half cycle variation along z.
- Magnetic wall at $r = a$.
- Nonzero field components: E_ϕ, H_r, and H_z.

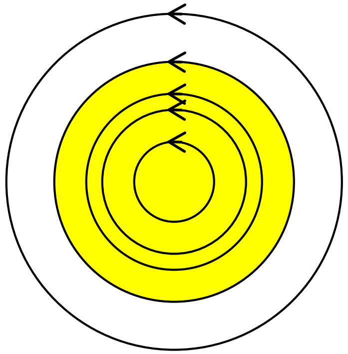
Cylindrical dielectric resonator: $TE_{01\delta}$ mode



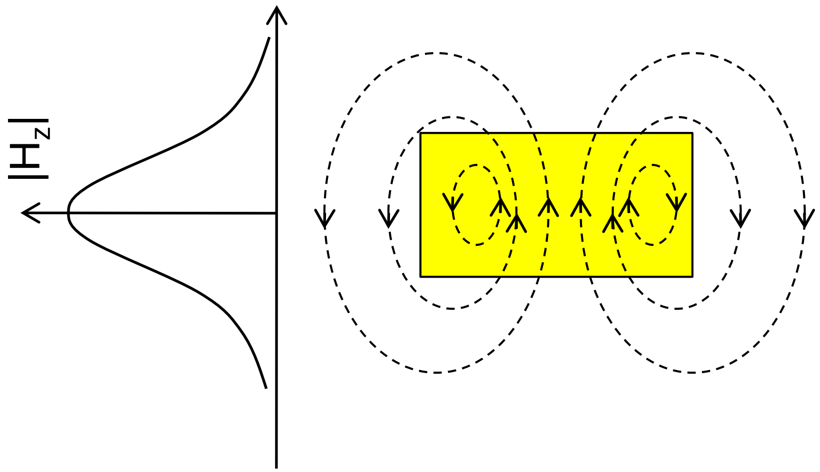
The dielectric resonator.



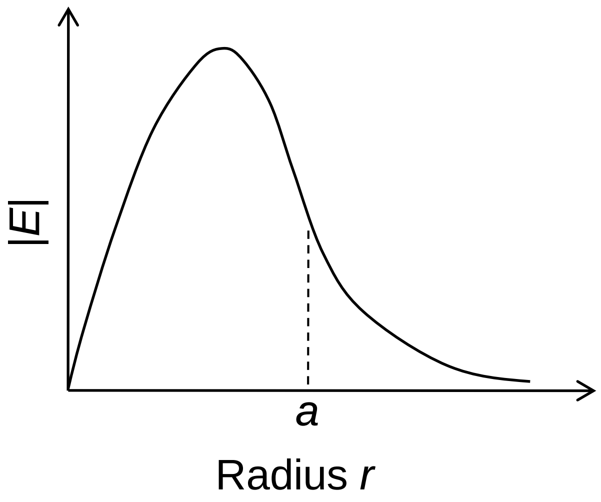
TE_{01} mode in a circular waveguide



E-field lines in z = 0 plane.

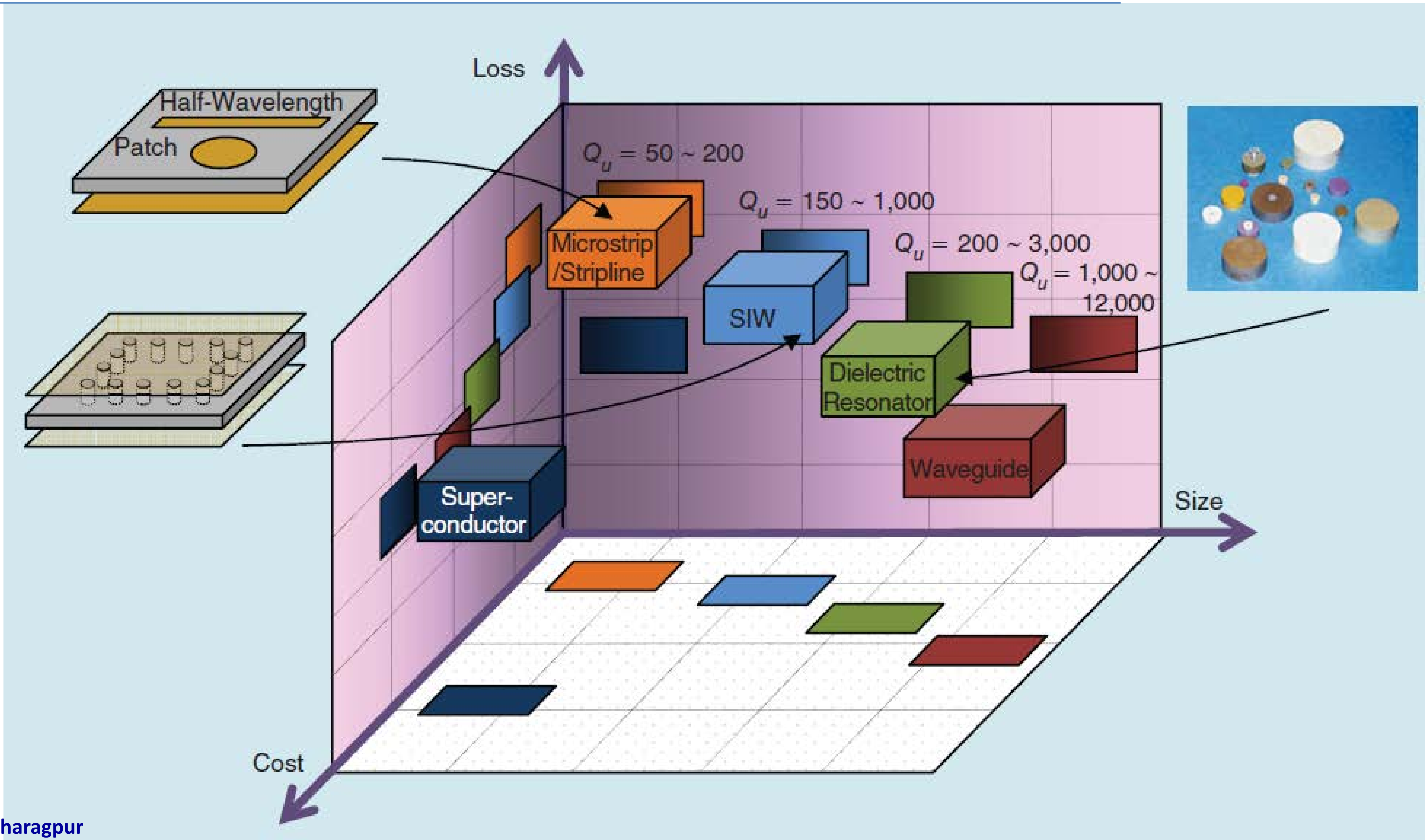


H-field lines in meridian plane.



Electric field intensity variation with r.

Comparison of resonators



Excitation of resonators

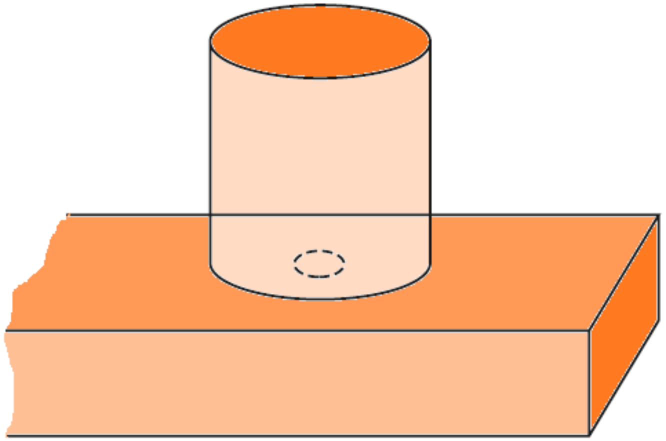
- Uses (low loss): filter (low loss), oscillator, tuned amplifier, frequency meter, antenna (high radiation loss, other loss low)
- Coupling with the source is measured by external quality factor (Q_e).
- External coupling is represented as $g = Q_u / Q_e$.
- Three conditions:

1. g <1: the resonator is under-coupled to the feedline.
2. g =1: the resonator is critically-coupled to the feedline.
3. g >1: the resonator is over-coupled to the feedline.

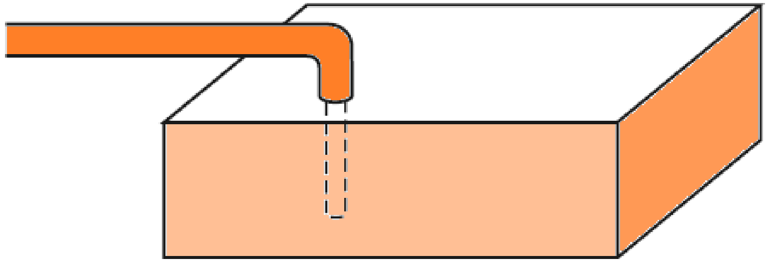
A microstrip line half-wave resonator excited by gap feeding (single port and double port uses).

Excitation of Hairpin resonators:
gap feeding (left) and tapped feeding (right).

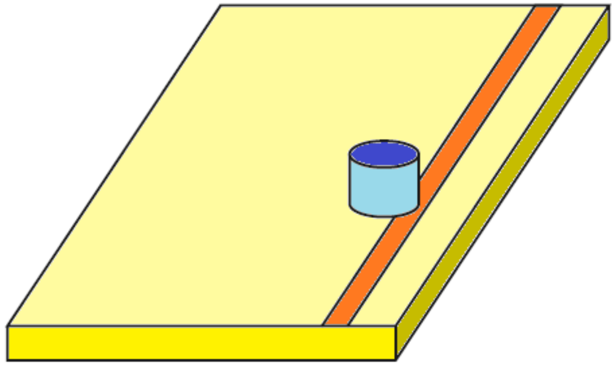
Excitation of resonators



Waveguide excitation of a circular cavity resonator



Coaxial excitation of waveguide resonator



Microstrip excitation of a dielectric resonator

RF and Microwave Engineering (EC 31005)

Power dividers and couplers (P6)

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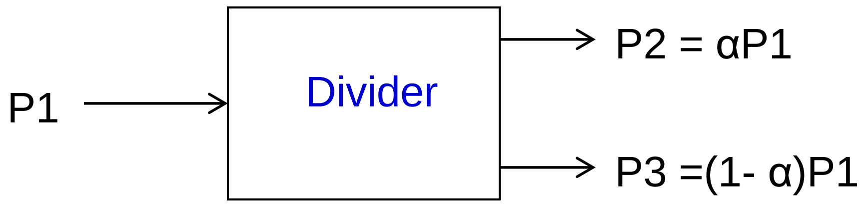
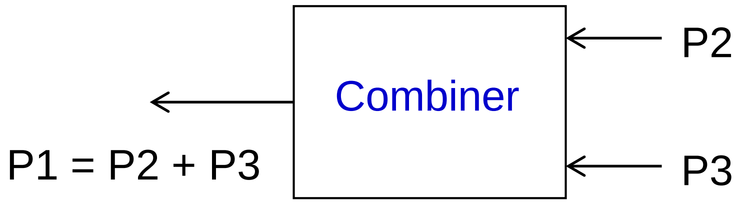
Power dividers and couplers

- They are passive microwave components used for power division or power combining.
- Power is divided into two or more output signals (combiner - accepts two or more input signals and combines them at an output port).
- Power dividers: provide in-phase output signals with an equal (3 dB) or unequal power division ratio.
- Directional couplers: arbitrary power division, either a 90^o or a 180^o phase shift between the output ports.
- Hybrid junctions: usually have equal power division with a 90^o or a 180^o phase shift between the output ports.
- Analysis: considered as lossless devices, even-, odd-mode analysis technique.

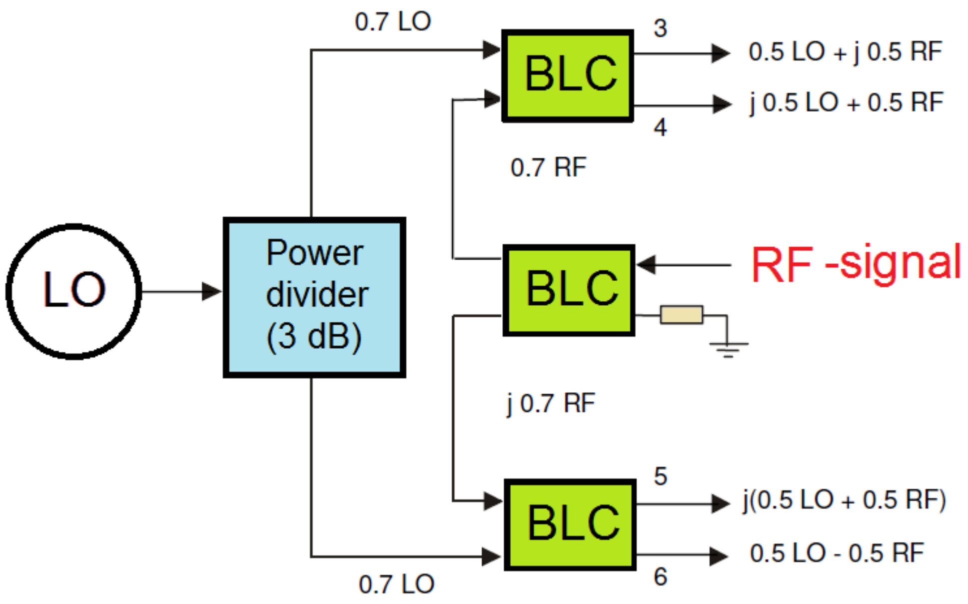


A three-port power divider.



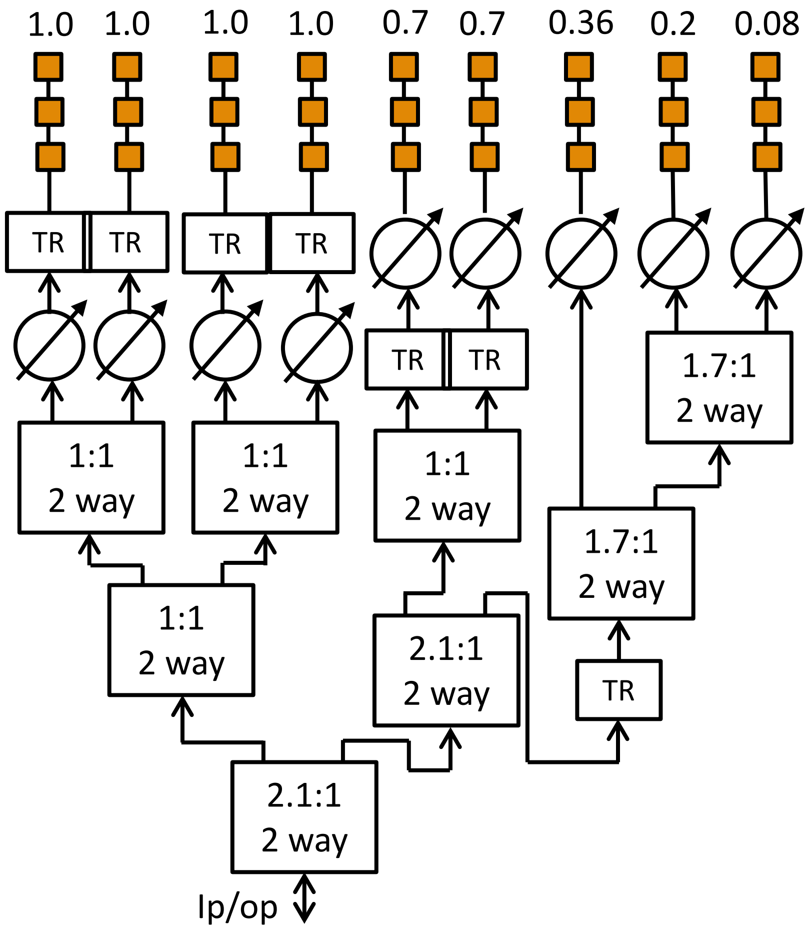
A three-port power combiner.

Applications



A millimeter wave six-port receiver.

- Array antenna feed network.
- Mixer, amplifier.



SIR-C L-band antenna feed system (one-half of symmetrical design) with the active elements for amplitude taper.

Three port network (T-junction)

- Scattering matrix for a three port network: $$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}.$$
- Passive without any anisotropic material →reciprocal and its scattering matrix will be symmetric i.e. $S_{ij} = S_{ji}$.

Desired characteristics: lossless and matched at all ports → $$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}.$$

Impossible to construct such a three-port lossless reciprocal network that is matched at all ports.

For a lossless network, energy conservation requires that the scattering matrix satisfy the unitary properties →

$$\begin{aligned} |S_{12}|^2 + |S_{13}|^2 &= 1, & S_{13}^* S_{23} &= 0, \\ |S_{12}|^2 + |S_{23}|^2 &= 1, & S_{23}^* S_{12} &= 0, \\ |S_{13}|^2 + |S_{23}|^2 &= 1, & S_{12}^* S_{13} &= 0. \end{aligned}$$

The second set of equation shows that at least two of S_{12}, S_{13}, S_{23} must be zero → inconsistent.

Circulator

Any matched lossless three-port network must be nonreciprocal and behaves as a circulator.

The scattering matrix of a matched three-port network has the following form

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}.$$

If the network is lossless, [S] must be unitary, which implies the following conditions

$$|S_{12}|^2 + |S_{13}|^2 = 1, \qquad S_{31}^* S_{32} = 0,$$

$$|S_{21}|^2 + |S_{23}|^2 = 1, \qquad S_{21}^* S_{23} = 0,$$

$$|S_{31}|^2 + |S_{32}|^2 = 1. \qquad S_{12}^* S_{13} = 0,$$

These equations can be satisfied in one of two ways. Either

$$S_{12} = S_{23} = S_{31} = 0, \quad |S_{21}| = |S_{32}| = |S_{13}| = 1, \quad \text{or}$$

$$S_{21} = S_{32} = S_{13} = 0, \quad |S_{12}| = |S_{23}| = |S_{31}| = 1.$$

→ $S_{ij} \neq S_{ji}$ for $i \neq j$

Circulator (nonreciprocal)

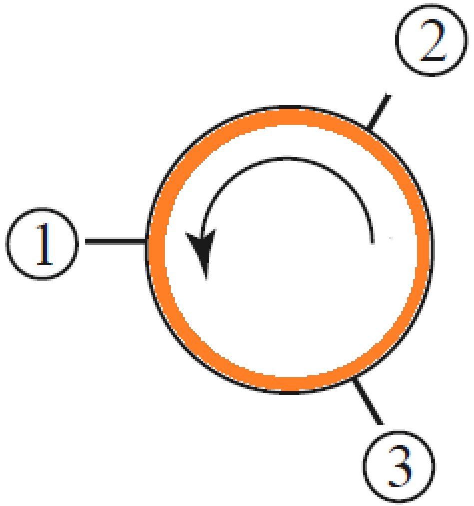
Circulator

Two possible implementation of the two equations:

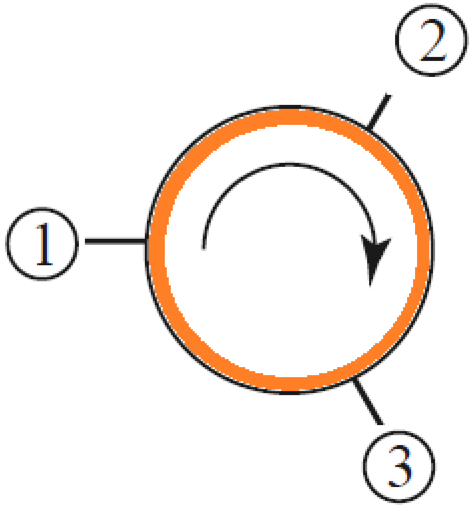
$$S_{12} = S_{23} = S_{31} = 0, \qquad |S_{21}| = |S_{32}| = |S_{13}| = 1,$$

$$S_{21} = S_{32} = S_{13} = 0, \qquad |S_{12}| = |S_{23}| = |S_{31}| = 1.$$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



Properties of three port network

A lossless and reciprocal three-port network can be physically realized if only two of its ports are matched.

Considering ports1and 2 are the matched ports,

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}.$$

To be lossless, the following unitarity conditions must be satisfied

$$S_{13}^* S_{23} = 0,$$
$$S_{12}^* S_{13} + S_{23}^* S_{33} = 0,$$
$$S_{23}^* S_{12} + S_{33}^* S_{13} = 0,$$

$$\left.\begin{aligned} |S_{12}|^2 + |S_{13}|^2 &= 1, \\ |S_{12}|^2 + |S_{23}|^2 &= 1, \end{aligned}\right\} \rightarrow |S_{13}| = |S_{23}|$$

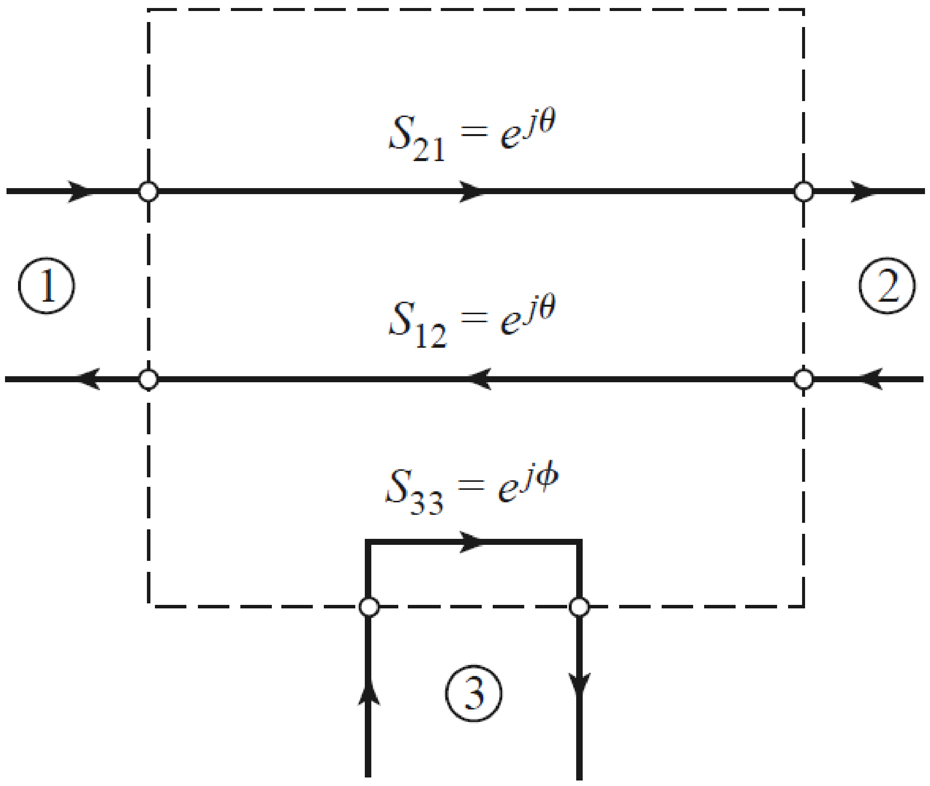
$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 = 1.$$

$\rightarrow S_{13} = S_{23} = 0.$

$\rightarrow |S_{12}| = |S_{33}| = 1$

Three port lossless network

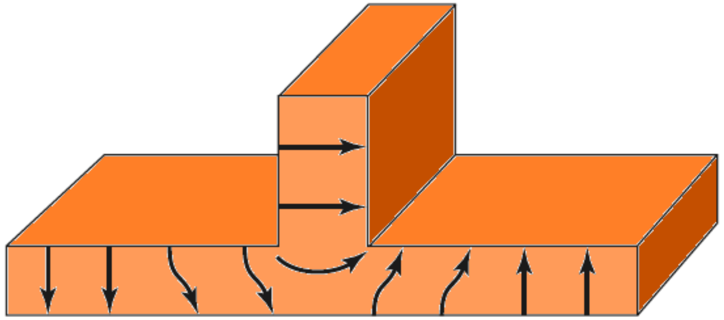
$S_{13} = S_{23} = 0$, $|S_{12}| = |S_{33}| = 1$ provides the following network:



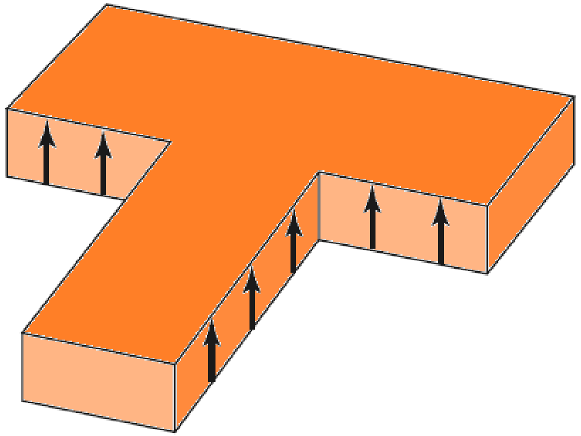
$$[S] = \begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$

However, a lossy three-port network can be made to have isolation between its output ports (e.g., $S_{23} = S_{32} = 0$).

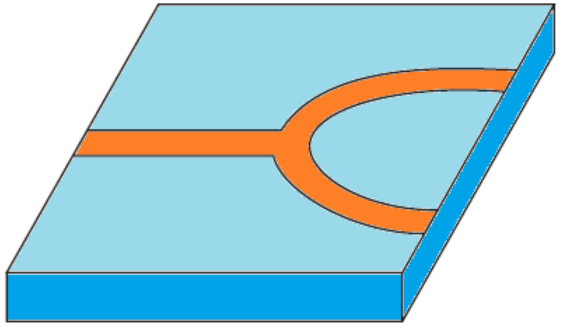
T-junctions



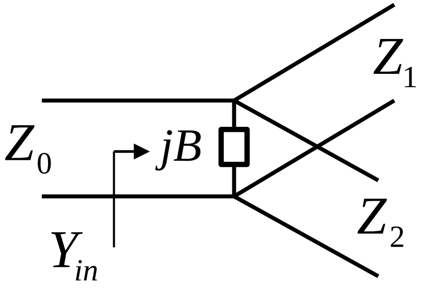
E-plane waveguide T



H-plane waveguide T



Microstrip line T

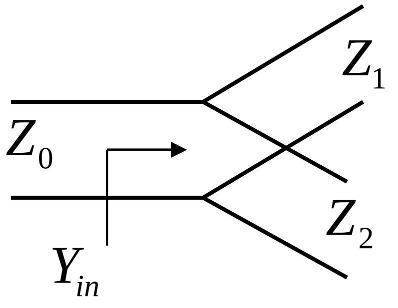


Equivalent circuit.

- Assuming $B = 0$, to match the input impedance: $Y_{in} = \dfrac{1}{Z_0} = \dfrac{1}{Z_1} + \dfrac{1}{Z_2}$.
- For equal power coupling, the output impedances for a 50 Ω input port are 100 Ω.
- The output can be matched to 50 Ω using a quarter wavelength transformer. But, no isolation between the output ports.

T-junction

A lossless T-junction power divider has a source impedance of 50 Ω. Find the output characteristic impedances so that the output powers are in a 2:1 ratio. Compute the reflection coefficients seen looking into the output ports.



Answer:

Considering the voltage at the junction V_0, $P_{\rm in} = \frac{1}{2}\frac{V_0^2}{Z_0}$,

And the output powers are $P_1 = \frac{1}{2}\frac{V_0^2}{Z_1} = \frac{1}{3}P_{\rm in}$, $\quad P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{2}{3}P_{\rm in}$.

Therefore, the characteristic impedances are $Z_1 = 3Z_0 = 150\ \Omega$, $\quad Z_2 = \frac{3Z_0}{2} = 75\ \Omega$.

Then, the input impedance is $Z_{\rm in} = 75||150 = 50\ \Omega$,

The reflection coefficient seen looking at these ports are

$$\Gamma_1 = \frac{30-150}{30+150} = -0.666, \qquad \Gamma_2 = \frac{37.5-75}{37.5+75} = -0.333.$$

Resistive divider

The impedance Z, seen looking into the $Z_0/3$ resistor followed by a terminated output line, is

$$Z = \frac{Z_0}{3} + Z_0 = \frac{4Z_0}{3}.$$

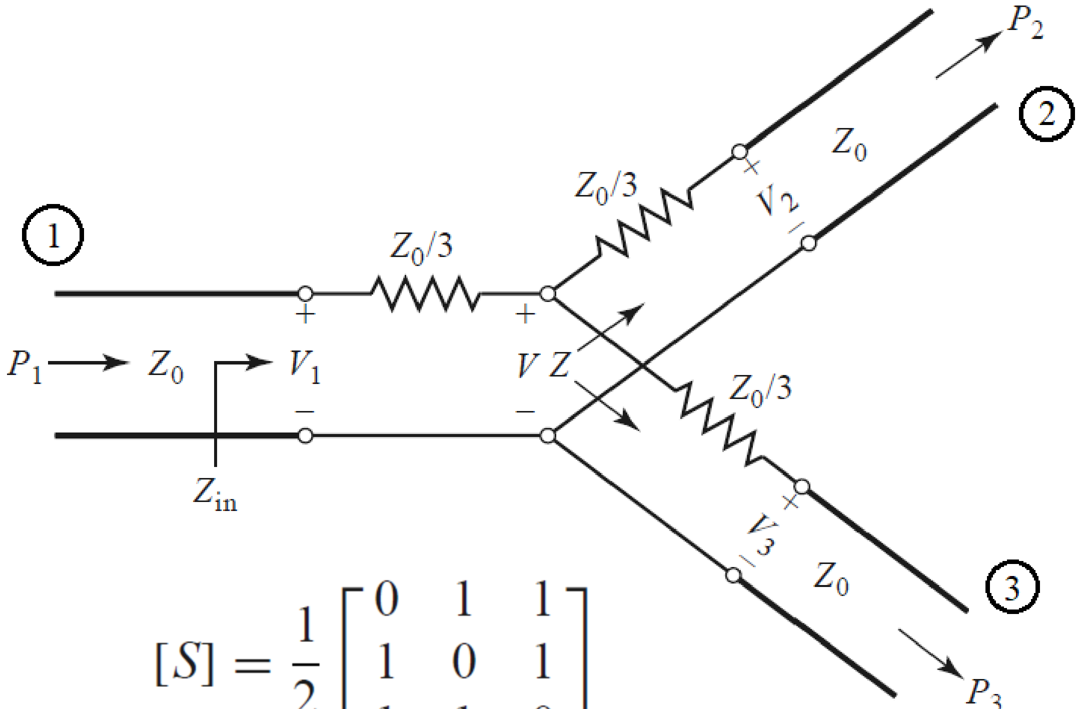
The input impedance of the divider is

$$Z_{\text{in}} = \frac{Z_0}{3} + \frac{2Z_0}{3} = Z_0$$

→ $S_{11} = S_{22} = S_{33} = 0$.

If the voltage at port 1 is V_1, then the voltage at the junction is

$$V = V_1 \frac{2Z_0/3}{Z_0/3 + 2Z_0/3} = \frac{2}{3} V_1,$$



$$[S] = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

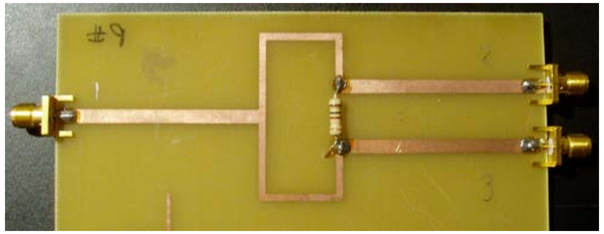
A 3dB resistive power divider

And the output voltages are $V_2 = V_3 = V \dfrac{Z_0}{Z_0 + Z_0/3} = \dfrac{3}{4} V = \dfrac{1}{2} V_1.$

Thus, $S_{21} = S_{31} = S_{23} = 1/2$, so the output powers are 6 dB below the input power level.

Wilkinson power divider

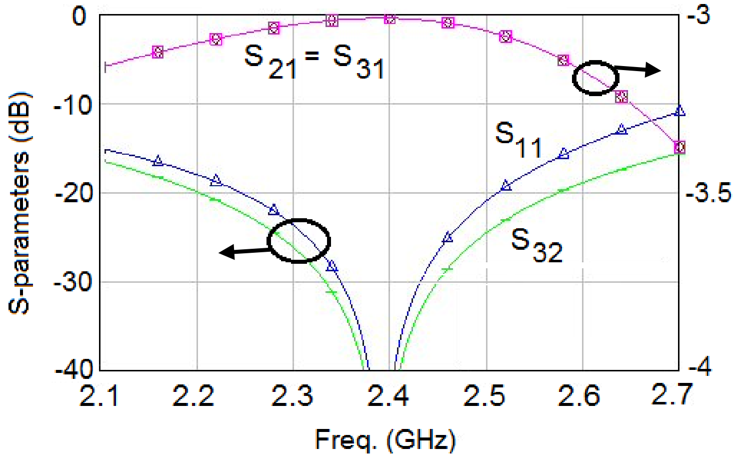
- Invented by Ernest J. Wilkinson in 1960
- The Wilkinson Power Divider achieves isolation between the output ports while maintaining a matched condition on all ports.
- It uses two quarter wave transformers, and a resistor and can be implemented by using any guide which supports TEM mode e.g. coaxial cable, microstrip line, CPW etc. but not the rectangular waveguide or dielectric guide.
- Lumped circuit elements (inductors, capacitors and resistors) are also used at lower frequencies.



Photograph of a Wilkinson power divider (microstrip line).

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

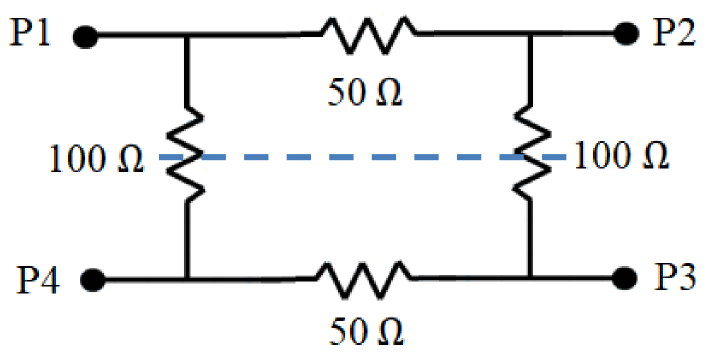
S-matrix at the design frequency



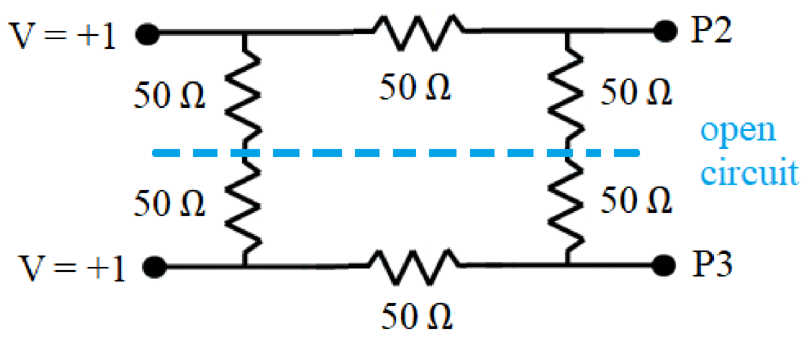
Frequency response.

Even- odd-mode analysis of a symmetrical four port network

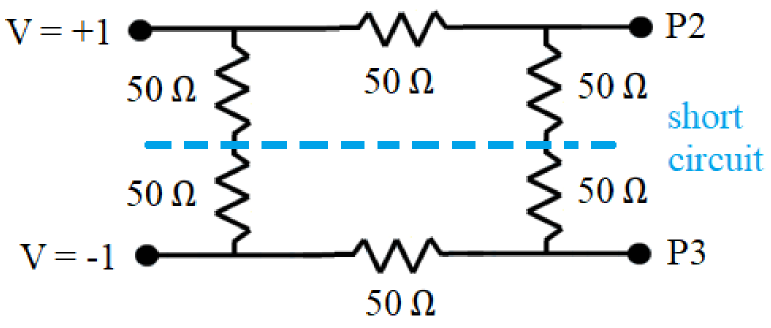
- Any symmetrical (reciprocal), lossless four port network can be decomposed to a two port network excited by the same phase (even-mode) and out-of-phase (odd-mode) components.



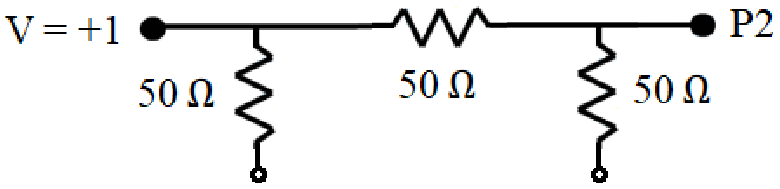
A four port network



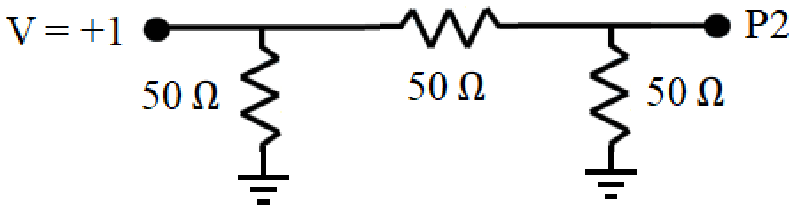
Excitation in even-mode



Excitation in odd-mode



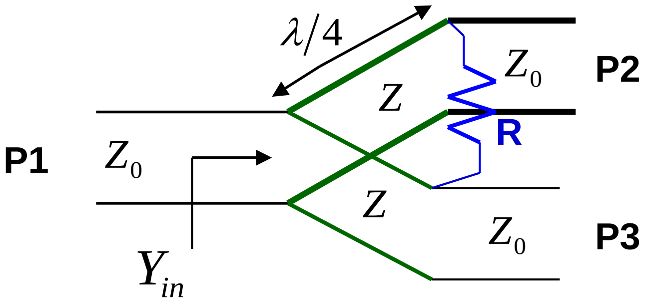
Two-port equivalent circuit in even-mode excitation



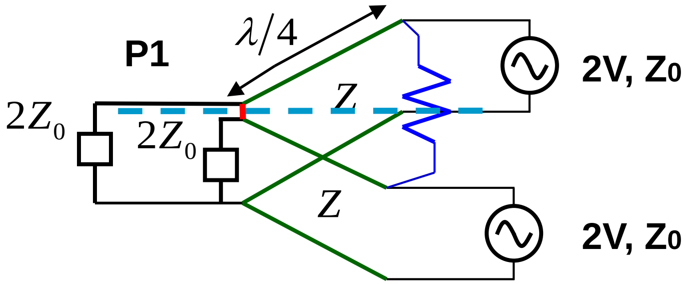
Two-port equivalent circuit in odd-mode excitation

Wilkinson power divider

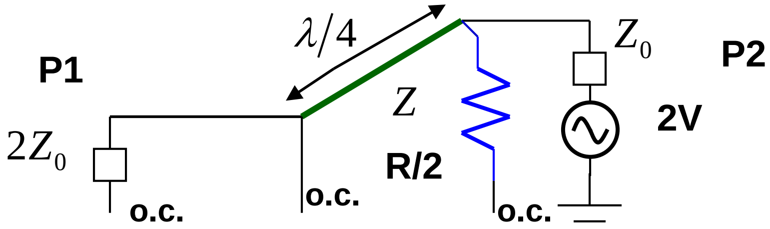
- Connect voltage generators at the output ports.
- This network has been drawn in a form that is symmetric across the midplane.
- For the equal-split power divider, Z =√2 and r = 2.



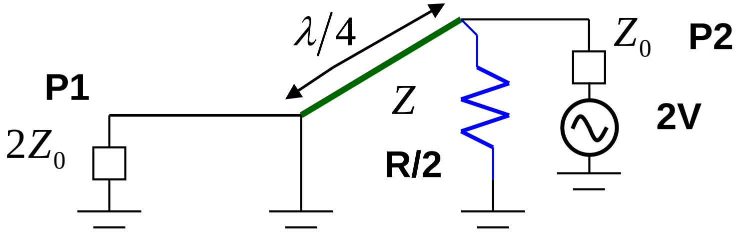
Transmission line circuit.



Even-mode excitation scheme.



Even-mode circuit.



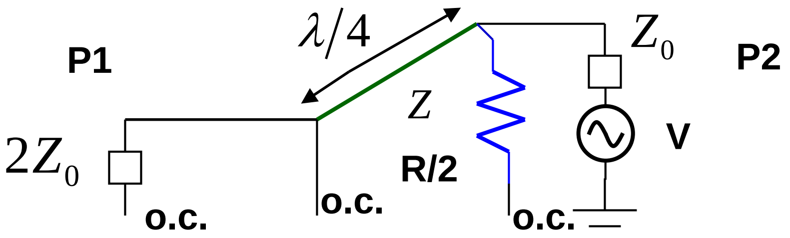
Odd-mode circuit.

Wilkinson power divider

In even-mode excitation:

Looking from port 2, $Z_{in}^{e} = \dfrac{Z^2}{2Z_0}.$

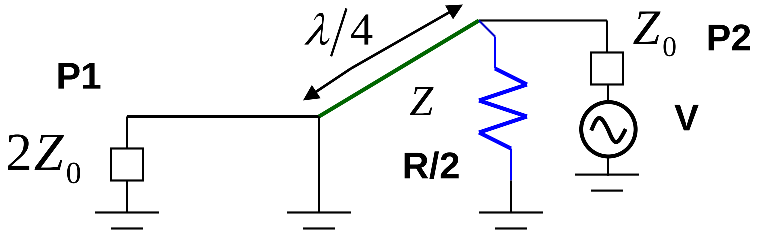
Port 2 will be matched for even-mode, if $Z = Z_0\sqrt{2}.$



Even-mode circuit.

In odd-mode excitation:

- The transmission line segment is seen as open-circuit.
- R should be $2Z_0$ for matching.



Odd-mode circuit.

S-parameters: $S_{11} = S_{22} = S_{33} = 0.$

$$S_{12} = S_{21} = S_{13} = S_{31} = -j/\sqrt{2}.$$

$$S_{23} = S_{32} = 0.$$

Wilkinson power divider

Unequal power division:

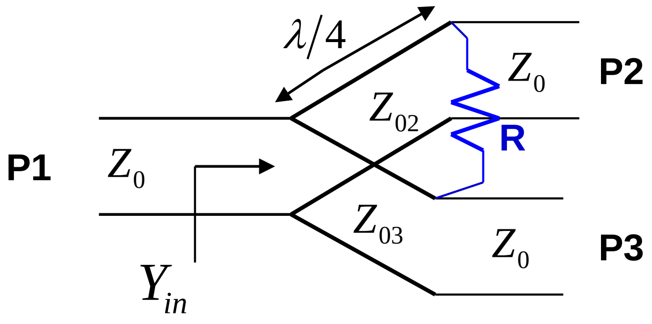
If the power ratio between the ports $K^2 = P_3/P_2$,

Then, the design equations are

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}},$$

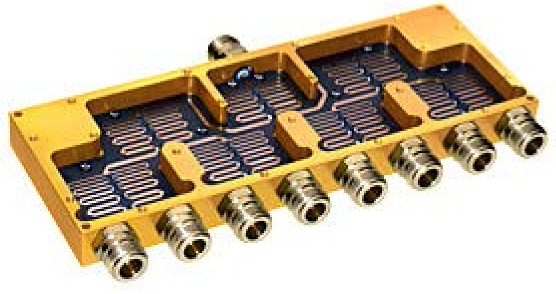
$$Z_{02} = Z_0\sqrt{K\left(1+K^2\right)},$$

$$R = Z_0\left(K+\frac{1}{K}\right).$$



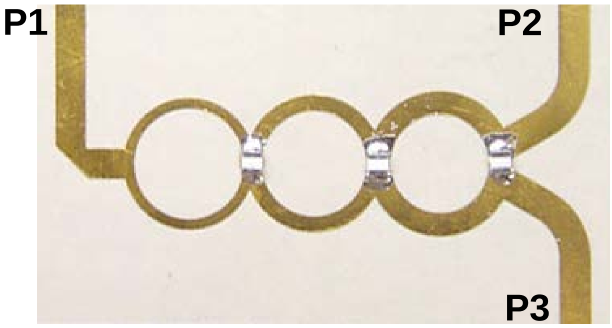
Transmission line circuit.

Multi-port power divider:



A 8-way Wilkinson power divider.

Wideband power divider:



A wideband Wilkinson power divider.

Four port network

The scattering matrix of a reciprocal four-port network matched at all ports has the following form

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

If the network is lossless, 10 equations result from the unitarity condition. Consider the multiplication of row 1 and row 2, and the multiplication of row 4 and row 3 →

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0,$$

$$S_{14}^* S_{13} + S_{24}^* S_{23} = 0.$$

After manipulating the above equations, $S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0.$

Similarly, the multiplication of row 1 and row 3, and the multiplication of row 4 and row 2, gives

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0,$$

$$S_{14}^* S_{12} + S_{34}^* S_{23} = 0.$$

Which after manipulation provides $S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0.$

Four port network

$$S_{14}^*(|S_{13}|^2 - |S_{24}|^2) = 0. \qquad S_{23}(|S_{12}|^2 - |S_{34}|^2) = 0.$$

The above equations would be satisfied if $S_{14} = S_{23} = 0 \rightarrow$ directional coupler.

Then, the self-products of the rows of the unitary scattering matrix yield

$$\left.\begin{aligned} |S_{12}|^2 + |S_{13}|^2 &= 1, \\ |S_{12}|^2 + |S_{24}|^2 &= 1, \\ |S_{13}|^2 + |S_{34}|^2 &= 1, \\ |S_{24}|^2 + |S_{34}|^2 &= 1, \end{aligned}\right\} \rightarrow |S_{13}| = |S_{24}|, \text{ and } |S_{12}| = |S_{34}|$$

Now, choose some reference phases as $S_{12} = S_{34} = \alpha$, $S_{13} = \beta e^{j\theta}$, and $S_{24} = \beta e^{j\varphi}$, where α and β are real, and θ and φ are phase constants to be determined (or one of them you can choose).

Now, the dot product of rows 2 and 3 provides $S_{12}^* S_{13} + S_{24}^* S_{34} = 0,$

The above relation yields a relation between the remaining phase constants as $\theta + \phi = \pi \pm 2n\pi.$

Four port network

$$\theta + \phi = \pi \pm 2n\pi.$$

If we ignore integer multiples of 2π, there are two particular choices that commonly occur in practice:

1. *A Symmetric Coupler:* $\theta = \varphi = \pi/2$. The phases of the terms having amplitude β are chosen equal. Then the scattering matrix has the following form:

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}.$$

2. *An Antisymmetric Coupler*: $\theta = 0$, $\varphi = \pi$. The phases of the terms having amplitude β are chosen to be 180° apart. Then the scattering matrix has the following form:

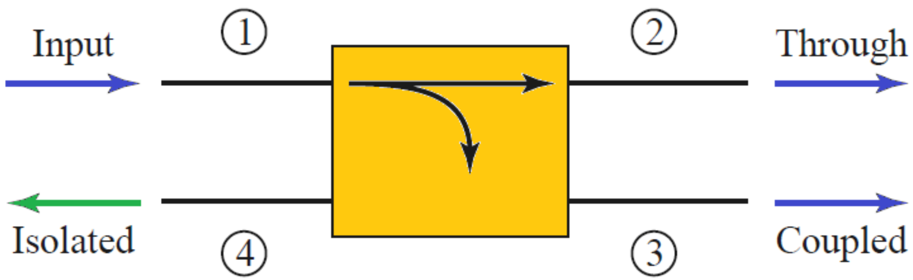
$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}.$$

The amplitude α and β are not independent.
The lossless condition provides,

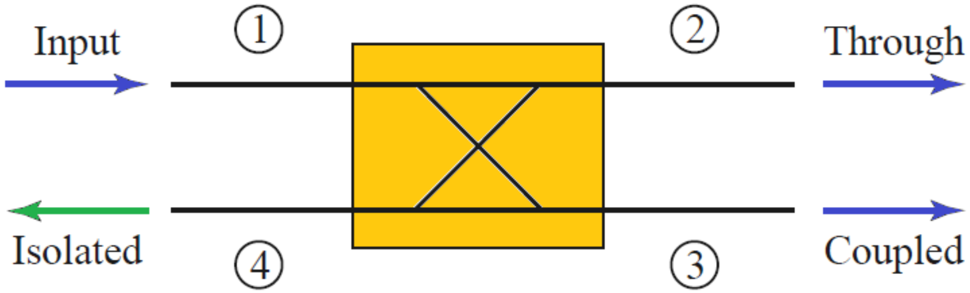
$$\alpha^2 + \beta^2 = 1.$$

Any reciprocal, lossless, matched four-port network is a directional coupler.

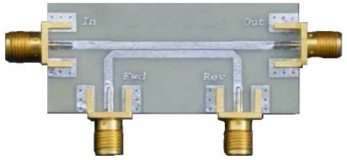
Directional couplers and hybrid junctions



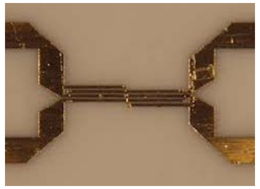
Symbol of a directional coupler



Symbol of a hybrid junction

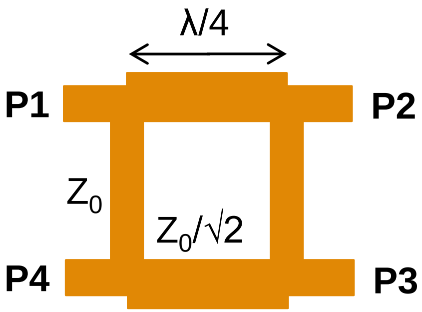


Microstrip line
Directional coupler

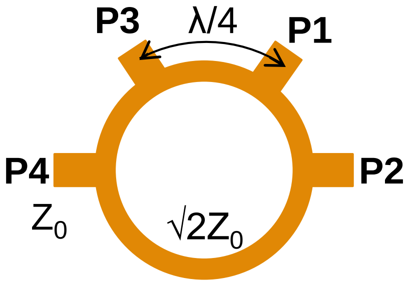


Lange coupler

Directional coupler
in waveguide technology.



A 90º hybrid junction
(branch line coupler) in
microstrip technology.



A 180º hybrid junction
(rat race coupler) in
microstrip technology.

Directional coupler

- The coupling factor $|S_{13}|^2 = \beta^2$.
- Power delivered to through port $|S_{12}|^2 = \alpha^2 = 1 - \beta^2$.

The popular parameters to characterize a directional coupler are

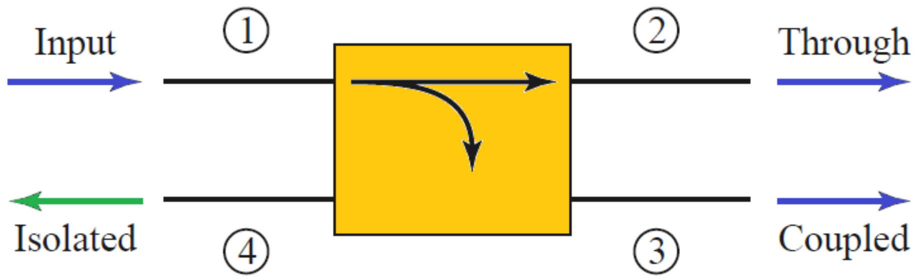
$$\text{Coupling } C = 10\log\frac{P_1}{P_3}\ dB = -20\log\beta \text{ dB},$$

$$\text{Directivity } D = 10\log\frac{P_3}{P_4}\ dB = 20\log\frac{\beta}{|S_{14}|} \text{ dB},$$

$$\text{Isolation } I = 10\log\frac{P_1}{P_4}\ dB = -20\log|S_{14}| \text{ dB},$$

$$\text{Insertion loss } L = 10\log\frac{P_1}{P_2}\ dB = -20\log|S_{12}| \text{ dB},$$

For a loss less coupler, $I = D + C\ dB.$



Directional coupler

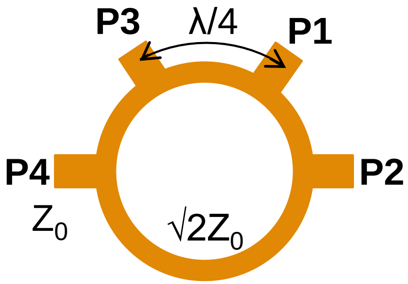
For a 3 dB coupler ($\alpha = \beta = 1/\sqrt{2}$) with 90◦ phase shift between ports 2 and 3 ($\theta = \varphi = \pi/2$), the S-matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}.$$

Four port network

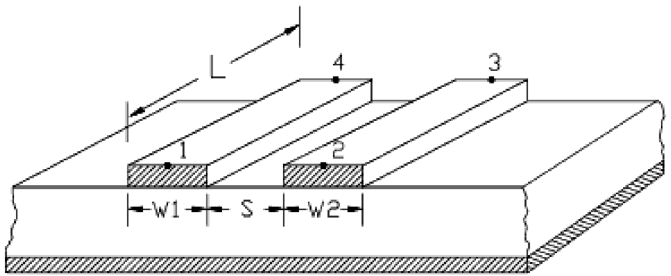
The *magic-T hybrid* and the *rat-race hybrid* have a 180◦ phase difference between ports 2 and 3 when fed at port 4, and are examples of an antisymmetric coupler. Its scattering matrix has the following form:

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}.$$

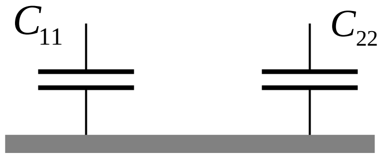


A 180º hybrid junction
(rat race coupler).

Coupled microstrip line

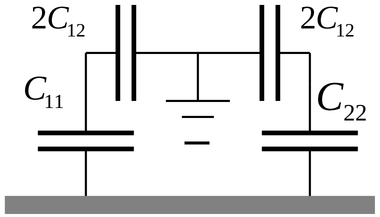


Microstrip coupled lines.



$$C_e = C_{11} = C_{22}$$

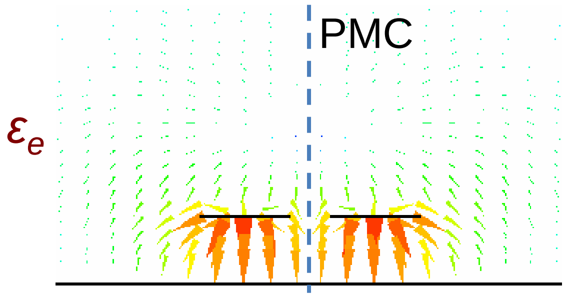
$$Z_{0e} = \frac{1}{v_p C_e}.$$



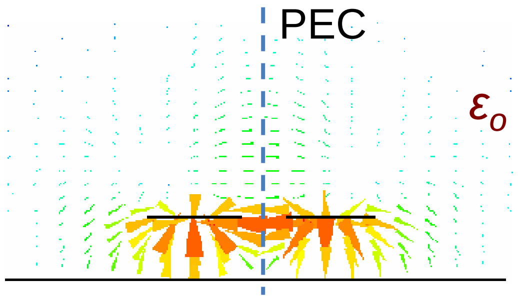
$$C_o = C_{11} + 2C_{12}$$

$$Z_{0o} = \frac{1}{v_p C_o}.$$

Question: ε_e or ε_o which one is higher?



Even-mode

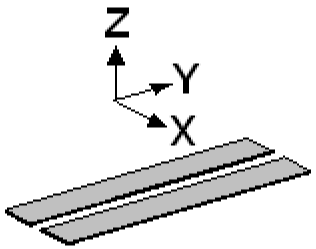


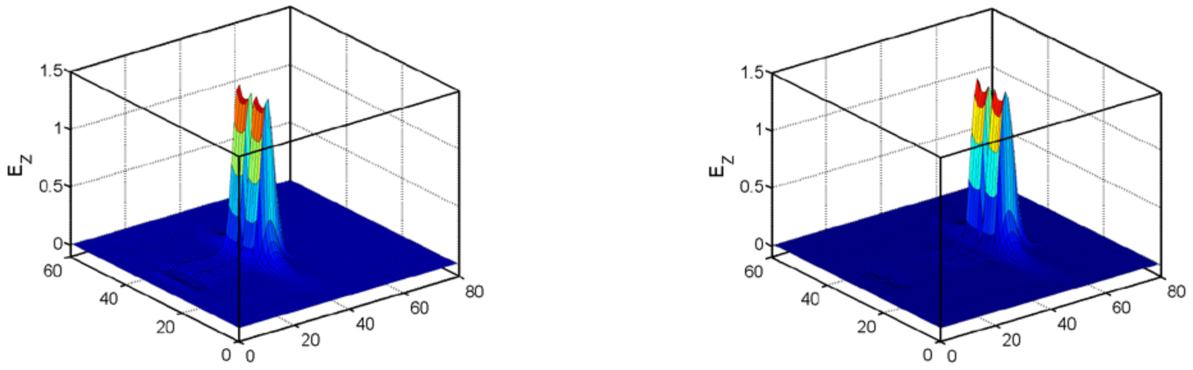
Odd-mode

Electric field distributions in even- and odd-modes.

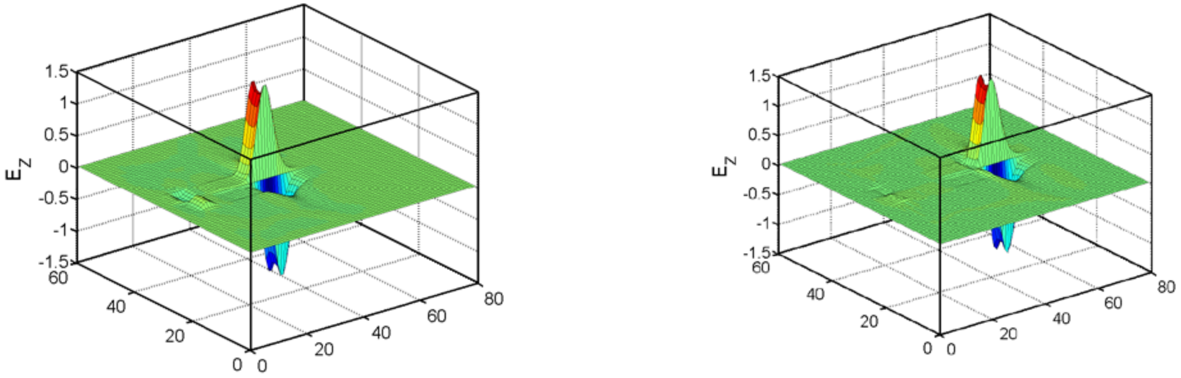
- Even-mode: excited by the same phase signals, odd-mode: excited by opposite phase signals.
- Even-mode: A PMC (open-circuit plane) can be placed between the coupled-lines
- Odd-mode: A PEC (short-circuit plane) can be placed between the coupled-lines.
- Two modes have different phase velocities (non-TEM).

Coupled microstrip line





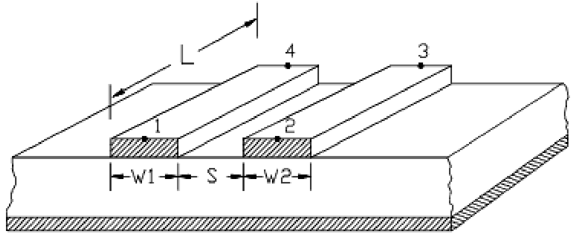
Pulse propagation in even-mode.



Pulse propagation in odd-mode.

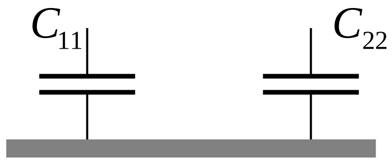
- Different fringing fields→ even-mode electrical length> odd-mode electrical length.
- Solutions→ zigzag inside edge, dielectric overlay, ground plane aperture.

Coupled microstrip line



- Considering the strip conductors are identical in size and location relative to the ground conductor→ $C_{11} = C_{22}$.
- Simplified analysis: assume TEM mode, then, for both of the modes, $\beta = \omega/v_p$ and $v_p = c/\sqrt{\varepsilon_r}$, where ε_r is the relative permittivity of the TEM line.
- Then, the characteristic impedance for the even mode is

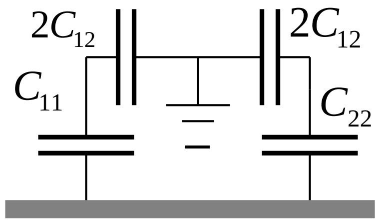
$$Z_{0e} = \sqrt{\frac{L_e}{C_e}} = \frac{\sqrt{L_e C_e}}{C_e} = \frac{1}{v_p C_e},$$ where $v_p = 1/\sqrt{(L_e C_e)} = 1/\sqrt{(L_e C_e)}$.



even-mode.

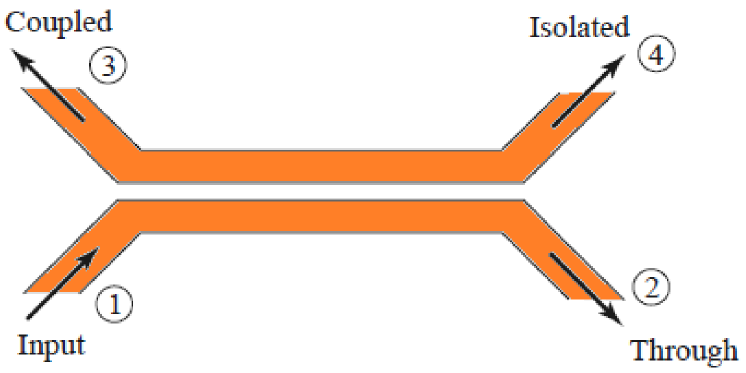
- In odd-mode, the effective capacitance between either strip conductor and ground is

$$Z_{0o} = \sqrt{\frac{L_o}{C_o}} = \frac{\sqrt{L_o C_o}}{C_o} = \frac{1}{v_p C_o}.$$

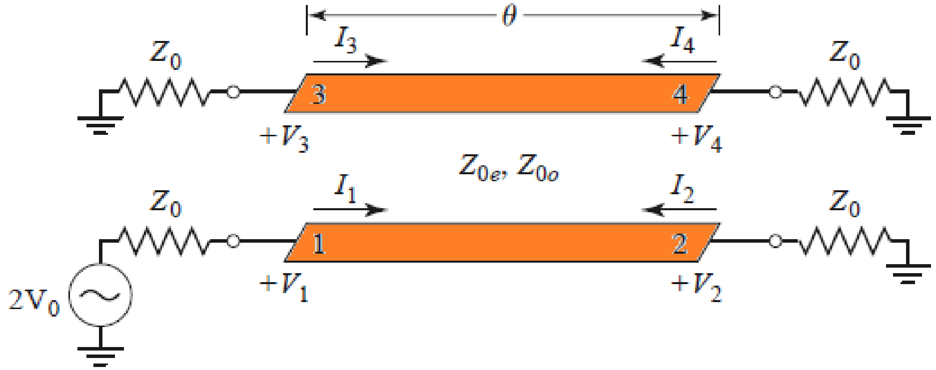


odd-mode.

Coupled-line directional coupler

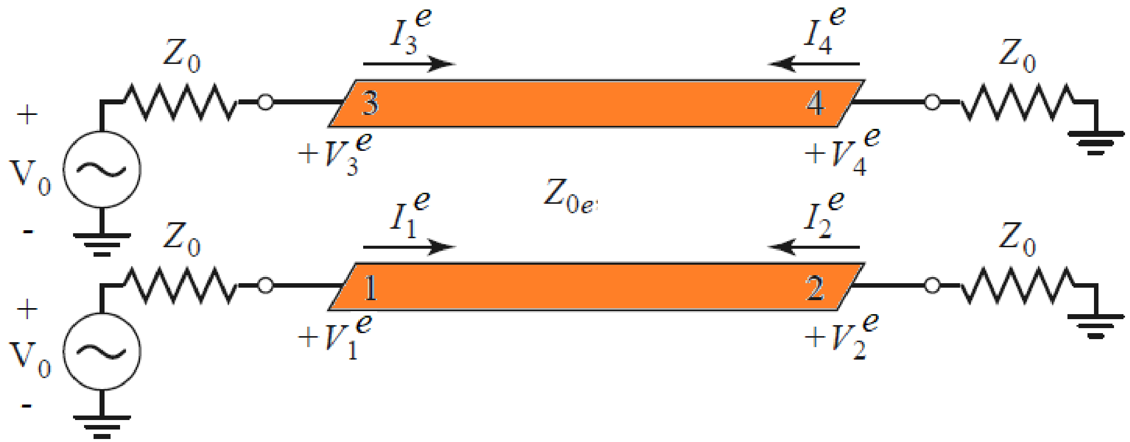


Coupled-line coupler.



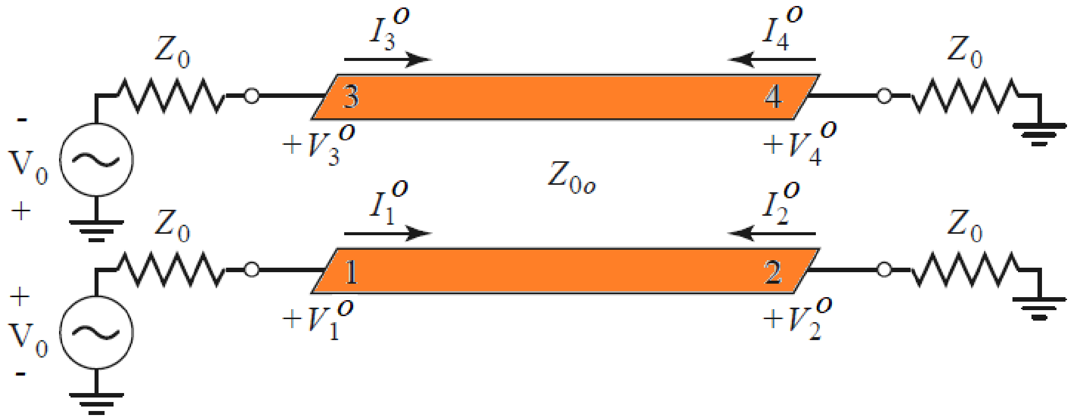
Schematic TEM equivalent circuit.

From symmetry, $I^e_1 = I^e_3$, $I^e_4 = I^e_2$, $V^e_1 = v^e_3$, and $V^e_4 = v^e_2$.



Excitation in even-mode.

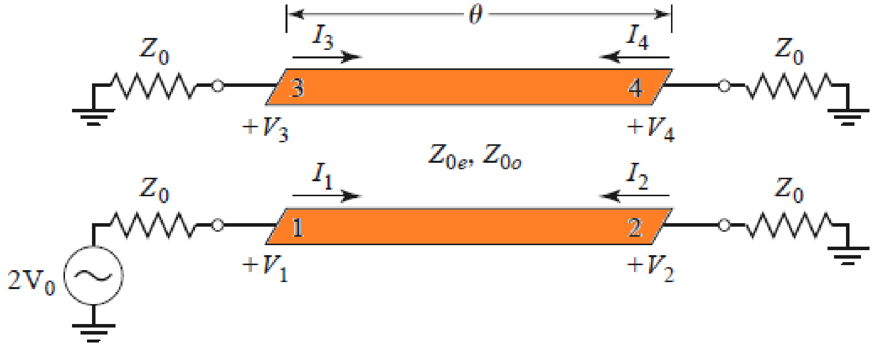
Odd-mode: $I^o_1 = -I^o_3$, $I^o_4 = -I^o_2$, $V^o_1 = -v^o_3$, and $V^o_4 = -v^o_2$.



Excitation in odd-mode.

Coupled-line directional coupler

The input impedance at port 1 of the coupler is



$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{V_1^e + V_1^o}{I_1^e + I_1^o}.$$

Then, in even- and odd-mode (two-port problem),

$$Z_{\text{in}}^e = Z_{0e}\frac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta}, \qquad Z_{\text{in}}^o = Z_{0o}\frac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta},$$

Now, by voltage division, from the even- and odd-mode circuits,

$$V_1^o = V_0\frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0}, \qquad I_1^o = \frac{V_0}{Z_{\text{in}}^o + Z_0},$$

$$V_1^e = V_0\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0}, \qquad I_1^e = \frac{V_0}{Z_{\text{in}}^e + Z_0}.$$

Using these in the first equation… (next slide),

Coupled-line directional coupler

$$Z_{\text{in}} = \frac{Z_{\text{in}}^o(Z_{\text{in}}^e + Z_0) + Z_{\text{in}}^e(Z_{\text{in}}^o + Z_0)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0} = Z_0 + \frac{2(Z_{\text{in}}^o Z_{\text{in}}^e - Z_0^2)}{Z_{\text{in}}^e + Z_{\text{in}}^o + 2Z_0}.$$

Now, putting $Z_0 = \sqrt{Z_{0e} Z_{0o}}$,

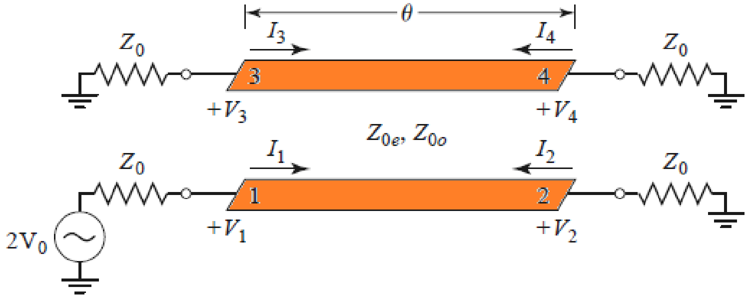
$$Z_{\text{in}}^e = Z_{0e} \frac{Z_0 + jZ_{0e}\tan\theta}{Z_{0e} + jZ_0\tan\theta}, \qquad Z_{\text{in}}^o = Z_{0o} \frac{Z_0 + jZ_{0o}\tan\theta}{Z_{0o} + jZ_0\tan\theta},$$ (from previous slide)

$$\rightarrow Z_{\text{in}}^e = Z_{0e} \frac{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}, \qquad Z_{\text{in}}^o = Z_{0o} \frac{\sqrt{Z_{0e}} + j\sqrt{Z_{0o}}\tan\theta}{\sqrt{Z_{0o}} + j\sqrt{Z_{0e}}\tan\theta}$$

So, $Z_{in} = Z_0$ for $Z_{\text{in}}^e Z_{\text{in}}^o = Z_{0e} Z_{0o} = Z_0^2$ (top most equation)

Coupled-line directional coupler

Now, for $Z_{in} = Z_0$, $V_1 = V_0$. Then, by voltage division at port 3,



$$V_3 = V_3^e + V_3^o = V_1^e - V_1^o = V_0\left[\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} - \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0}\right],$$

Therefore, using the impedance relationships (last slide),

$$\frac{Z_{\text{in}}^e}{Z_{\text{in}}^e + Z_0} = \frac{Z_0 + jZ_{0e}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}, \qquad \frac{Z_{\text{in}}^o}{Z_{\text{in}}^o + Z_0} = \frac{Z_0 + jZ_{0o}\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}$$

So, the above voltage equation reduces to $V_3 = V_0\dfrac{j(Z_{0e} - Z_{0o})\tan\theta}{2Z_0 + j(Z_{0e} + Z_{0o})\tan\theta}$.

Now, define coupling coefficient as $C = \dfrac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}}$, and $\sqrt{1 - C^2} = \dfrac{2Z_0}{Z_{0e} + Z_{0o}}$,

Which provides $V_3 = V_0\dfrac{jC\tan\theta}{\sqrt{1 - C^2} + j\tan\theta}$.

Coupled-line directional coupler

Similarly, for the other port,

$$V_4 = V_4^e + V_4^o = V_2^e - V_2^o = 0,$$

$$V_2 = V_2^e + V_2^o = V_0 \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}.$$

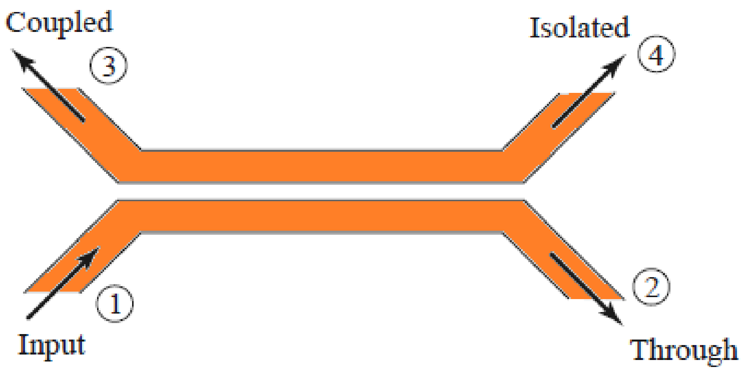
For θ = π/2 i.e. λ/4 line, the voltage equations are $\dfrac{V_3}{V_0} = C, \qquad \dfrac{V_2}{V_0} = -j\sqrt{1-C^2},$

$C < 1$ is called the voltage coupling factor at the design frequency

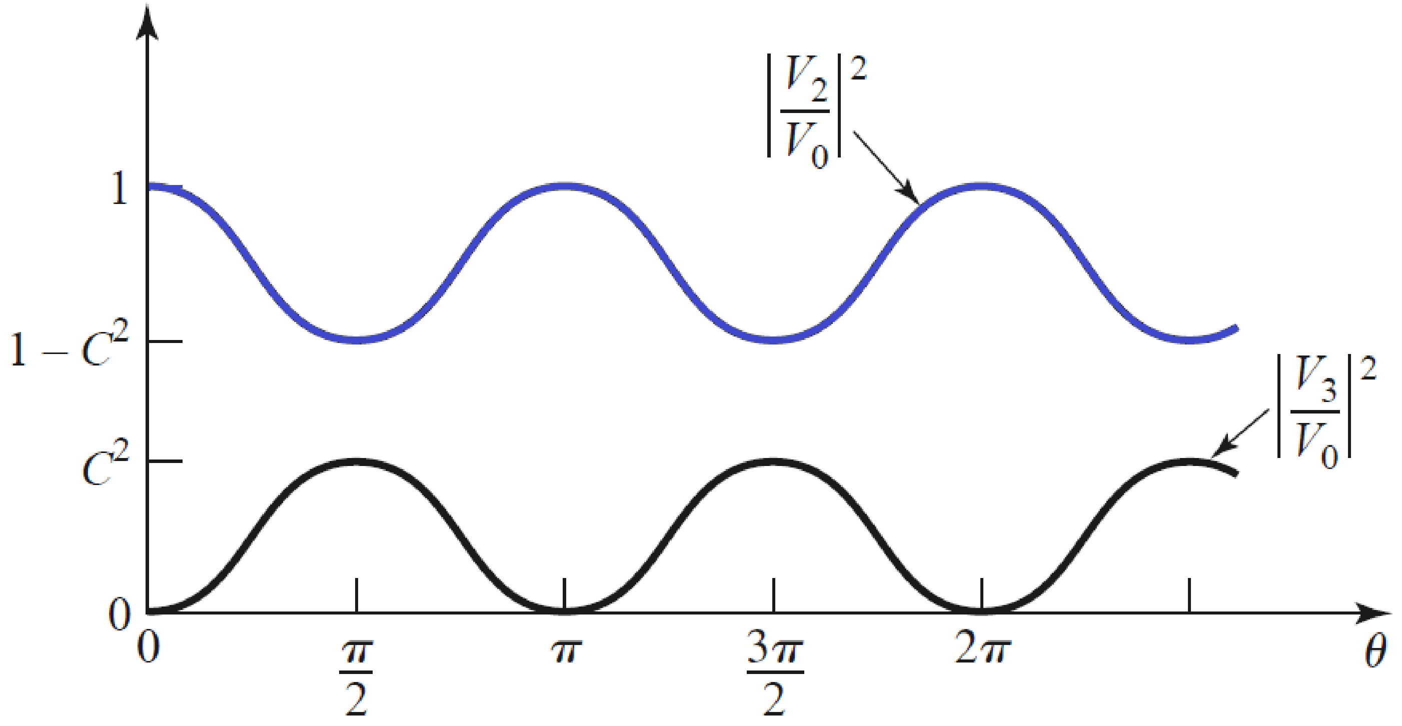
If the characteristic impedance, Z_0, and the voltage coupling coefficient, C, are specified, then the design equations for the required even- and odd-mode characteristic impedances are

$$Z_{0e} = Z_0\sqrt{\frac{1+C}{1-C}}, \qquad Z_{0o} = Z_0\sqrt{\frac{1-C}{1+C}}.$$

Coupled-line directional coupler



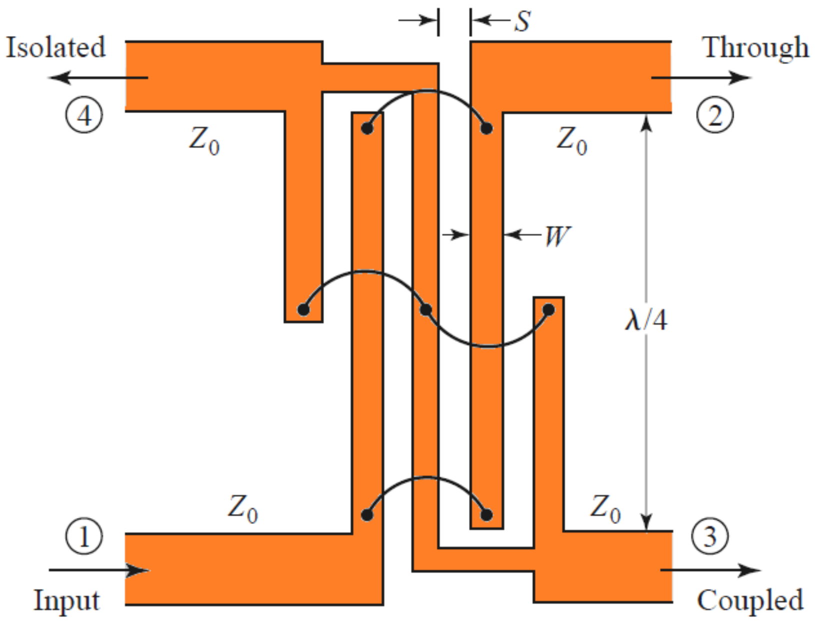
Coupled-line coupler.



Coupled and through port voltages (squared) versus frequency for the coupled line coupler

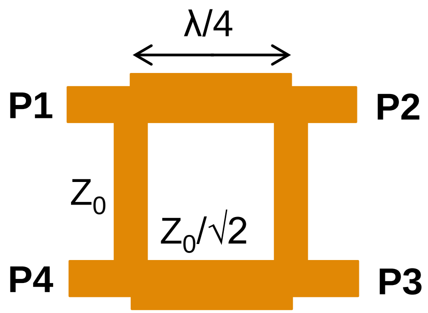
Lange coupler

- Maximum achievable coupling of a conventional direction is limited by the gap between the coupled-lines. Coupling more than 3 dB is hard to achieve.
- One way to increase the coupling between edge-coupled lines is to use several lines parallel to each other, so that the fringing fields at both edges of a line contribute to the coupling → Lange coupler.
- Four parallel coupled lines are used with interconnections to provide tight coupling.
- Lange coupler can easily achieve 3 dB coupling ratios, with an octave or more bandwidth.
- Disadvantage: fabrication complexity. The lines are very narrow and close together, and requires bonding wires across the lines.

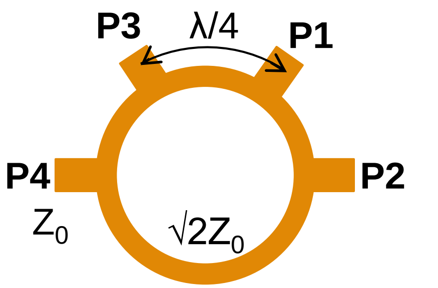


Lange coupler.

Hybrid couplers



Conventional branch-line coupler.

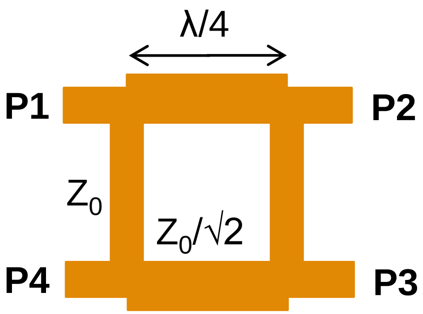


Conventional rat-race coupler.

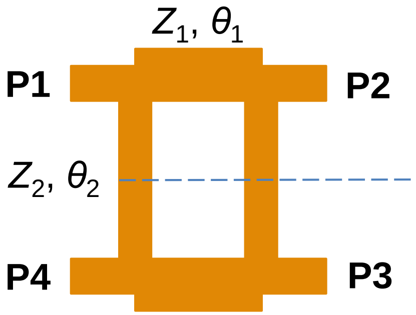
Analysis procedure:

- Use even-odd mode analysis procedure for four port networks.
- Obtain the overall ABCD matrix in even- and odd-mode excitations.
- Obtain the even- and odd-mode reflection and transmission coefficients: $\Gamma_e, \Gamma_o, T_e, T_o$.
- Using formula, convert them to *S*-parameters.
- Apply the conditions (input matching, coupling, isolation and phase relationship) to obtain the impedance and length values.

Hybrid junctions: 90^o hybrid (branch line hybrid)

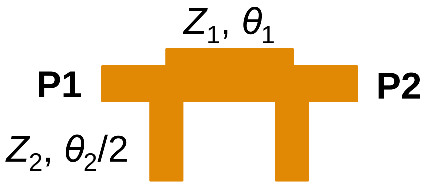


Conventional branch line coupler.

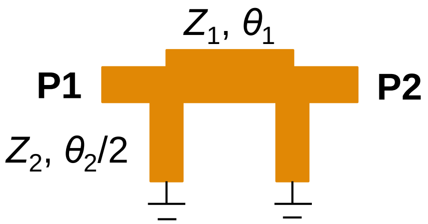


Unequal length branch line coupler.

$$[S] = -\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$



Even-mode eqv. circuit.



Odd-mode eqv. circuit.

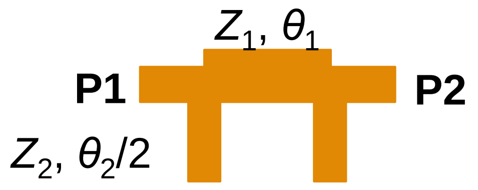
90º hybrid (BLC)

- Even-mode ABCD parameters:

$$A_e = D_e = \cos\theta_1 - \frac{Z_1}{Z_2}\tan\frac{\theta_2}{2}\sin\theta_1$$

$$B_e = jZ_1 \sin\theta_1$$

$$C_e = 2j\frac{1}{Z_2}\tan\frac{\theta_2}{2}\cos\theta_1 + j\frac{1}{Z_2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2}\tan^2\frac{\theta_2}{2}\right)\sin\theta_1.$$



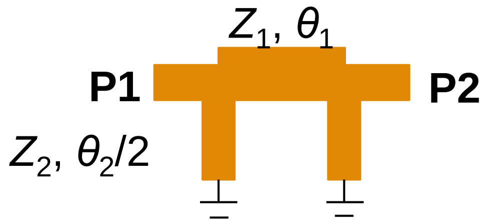
Even-mode eqv. circuit.

- Odd-mode ABCD parameters:

$$A_o = D_o = \cos\theta_1 + \frac{Z_1\sin\theta_1}{Z_2\sin\frac{\theta_2}{2}}$$

$$B_o = jZ_1 \sin\theta_1$$

$$C_o = -2j\frac{1}{Z_2}\frac{\cos\theta_1}{\tan\frac{\theta_2}{2}} + j\frac{1}{Z_2}\left(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2\tan^2\frac{\theta_2}{2}}\right)\sin\theta_1.$$



Odd-mode eqv. circuit.

90º hybrid (BLC)

- Input reflection coefficient in even-mode:

$$\Gamma_e = \frac{B_e/Z_0 - Z_0 C_e}{2A_e + B_e/Z_0 + Z_0 C_e}. \qquad \text{(}S_{11}\text{ in even-mode)}$$

- Transmission coefficient in even-mode:

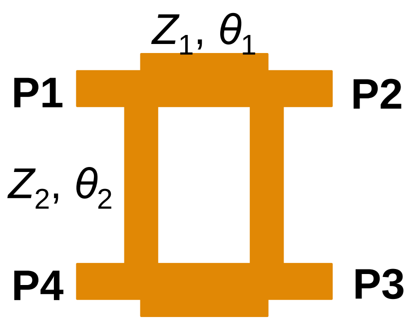
$$T_{21e} = \frac{2}{2A_e + B_e/Z_0 + Z_0 C_e}. \qquad \text{(}S_{21}\text{ in even-mode)}$$

- Similar expressions for odd-mode.

- Design equations by applying, $|S_{11}|=0$, $|S_{41}|=0$, :

$$Z_1 \tan\theta_1 = -Z_2 \tan\theta_2,$$

$$\frac{1}{Z_1^2} - \frac{1}{Z_2^2} = \frac{1}{Z_0^2}.$$

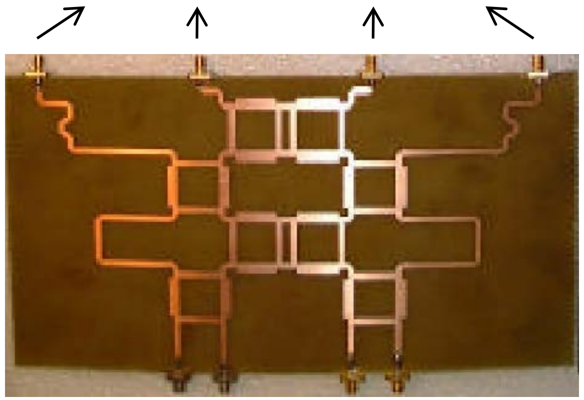


Unequal length branch line coupler.

Dual-band BLC can be designed by introducing open-stubs.

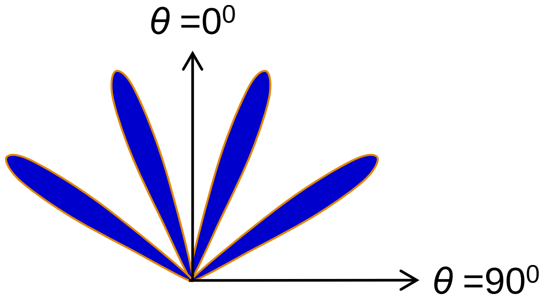
Application examples

To array antenna



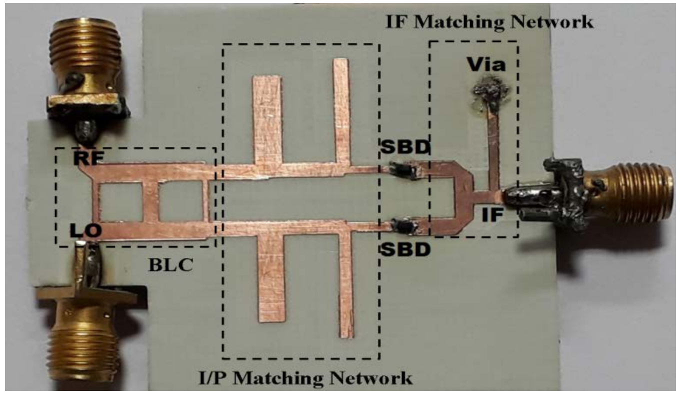
Signal input

A 4x4 Butler matrix



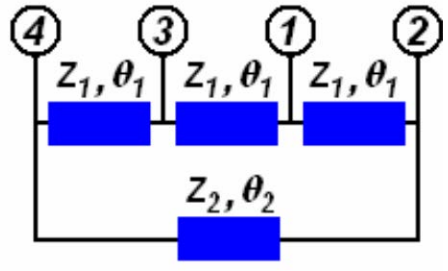
The beam positions.

A 64x64 Butler matrix.



mixer

Hybrid junctions: 180º hybrid (rat race hybrid)



Configuration of a reduced length RRC.

$$[S] = -\frac{j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Characteristics of a RRC:

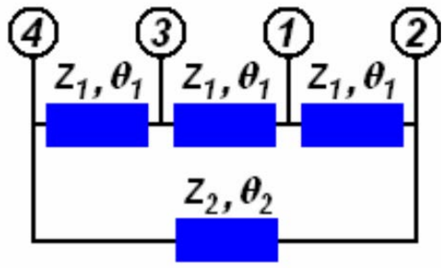
(i). A signal applied to port 1 will be evenly split into two in-phase components at ports 2 and 3, and port 4 will be isolated.

(ii). If the input is applied to port 4, it will be equally split into two components with a 180 deg phase difference at port 2 and 3, and port 1 will be isolated.

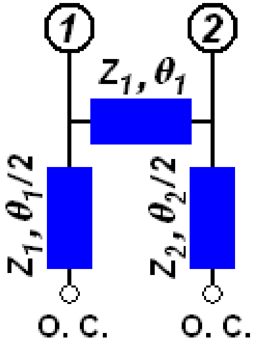
(iii). When operated as a combiner, with input signals applied at ports 2 and 3, the sum of the inputs will be formed at port 1, while the difference will be formed at port 4.

M. K. Mandal and S. Sanyal, Reduced length rat-race coupler, *IEEE T-MTT*, Dec. 2007.

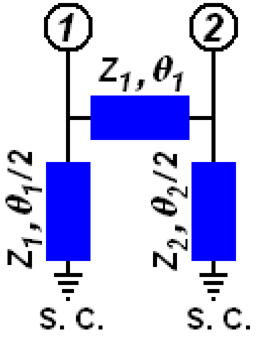
Analysis of rat race coupler



RRC configuration



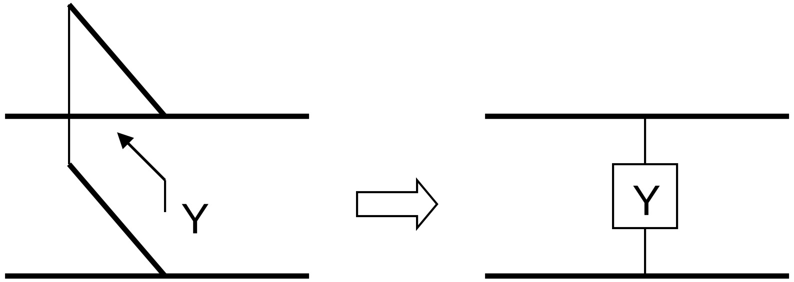
Even-mode circuit.



Odd-mode circuit.

A section of a transmission line

$$\begin{bmatrix} \cos\theta & jZ_0 \sin\theta \\ jY_0 \sin\theta & \cos\theta \end{bmatrix}$$



A stub as shunt admittance.

$$\begin{aligned} Y &= -jY_0 \cot\theta \quad \text{(short circuited stub)} \\ &= jY_0 \tan\theta \quad \text{(open circuited stub).} \end{aligned}$$

Analysis of rat race coupler

- Even-mode, port 1 excitation:

$$\begin{bmatrix} A_e = \cos\theta_1 - \dfrac{Z_1}{Z_2}\dfrac{\sin\theta_1}{\cot\dfrac{\theta_2}{2}}, & B_e = jZ_1\sin\theta_1, \\ C_e = j\dfrac{\cos\theta_1}{Z_1\cot\dfrac{\theta_1}{2}} - j\dfrac{\sin\theta_1}{Z_2\cot\dfrac{\theta_1}{2}\cot\dfrac{\theta_2}{2}} + j\dfrac{\sin\theta_1}{Z_1} + j\dfrac{\cos\theta_1}{Z_2\cot\dfrac{\theta_2}{2}}, & D_e = \cos\theta_1 - \dfrac{\sin\theta_1}{\cot\dfrac{\theta_1}{2}} \end{bmatrix}$$

- Odd-mode, port 1 excitation:

$$\begin{bmatrix} A_o = \cos\theta_1 + \dfrac{Z_1}{Z_2}\dfrac{\sin\theta_1}{\tan\dfrac{\theta_2}{2}}, & B_o = jZ_1\sin\theta_1, \\ C_o = -j\dfrac{\cos\theta_1}{Z_1\tan\dfrac{\theta_1}{2}} - j\dfrac{\sin\theta_1}{Z_2\tan\dfrac{\theta_1}{2}\tan\dfrac{\theta_2}{2}} + j\dfrac{\sin\theta_1}{Z_1} - j\dfrac{\cos\theta_1}{Z_2\tan\dfrac{\theta_2}{2}}, & D_o = \cos\theta_1 + \dfrac{\sin\theta_1}{\tan\dfrac{\theta_1}{2}}. \end{bmatrix}$$

Analysis of rat race coupler

- Even-mode, port 4 excitation:

$$\begin{bmatrix} A_e = \cos\theta_1 - \dfrac{\sin\theta_1}{\cot\dfrac{\theta_1}{2}}, & B_e = jZ_1\sin\theta_1, \\ C_e = j\dfrac{\cos\theta_1}{Z_2\cot\dfrac{\theta_2}{2}} - j\dfrac{\sin\theta_1}{Z_2\cot\dfrac{\theta_1}{2}\cot\dfrac{\theta_2}{2}} + j\dfrac{\sin\theta_1}{Z_1} + j\dfrac{\cos\theta_1}{Z_1\cot\dfrac{\theta_1}{2}}, & D_e = \cos\theta_1 - \dfrac{Z_1}{Z_2}\dfrac{\sin\theta_1}{\cot\dfrac{\theta_2}{2}}. \end{bmatrix}$$

- Odd-mode, port 4 excitation:

$$\begin{bmatrix} A_o = \cos\theta_1 + \dfrac{\sin\theta_1}{\tan\dfrac{\theta_1}{2}}, & B_o = jZ_1\sin\theta_1, \\ C_o = -j\dfrac{\cos\theta_1}{Z_2\tan\dfrac{\theta_2}{2}} - j\dfrac{\sin\theta_1}{Z_2\tan\dfrac{\theta_1}{2}\tan\dfrac{\theta_2}{2}} + j\dfrac{\sin\theta_1}{Z_1} - j\dfrac{\cos\theta_1}{Z_1\tan\dfrac{\theta_1}{2}}, & D_o = \cos\theta_1 + \dfrac{Z_1}{Z_2}\dfrac{\sin\theta_1}{\tan\dfrac{\theta_2}{2}}. \end{bmatrix}$$

Analysis of rat race coupler

$$\Gamma_e = \frac{A_e + B_e/Z_0 - C_e Z_0 - D_e}{A_e + B_e/Z_0 + C_e Z_0 + D_e}. \qquad T_e = \frac{2}{A_e + B_e/Z_0 + C_e Z_0 + D_e}.$$

Assuming, $Z_1 = Z_2 = Z$ and the matching, isolation and phase conditions result

$$\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2} = -1,$$

$$Z = Z_0\sqrt{\left\{3 - \frac{1}{2}\left(\cot^2\frac{\theta_1}{2} - \tan^2\frac{\theta_1}{2}\right)\right\}}.$$

The *S*-parameters are:

$$S_{21} = 1/X$$

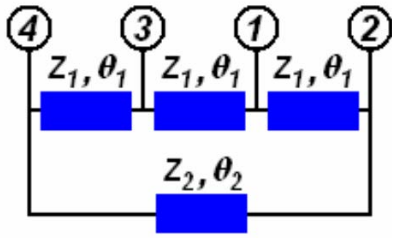
$$S_{31} = 1/X$$

$$S_{24} = -1/X$$

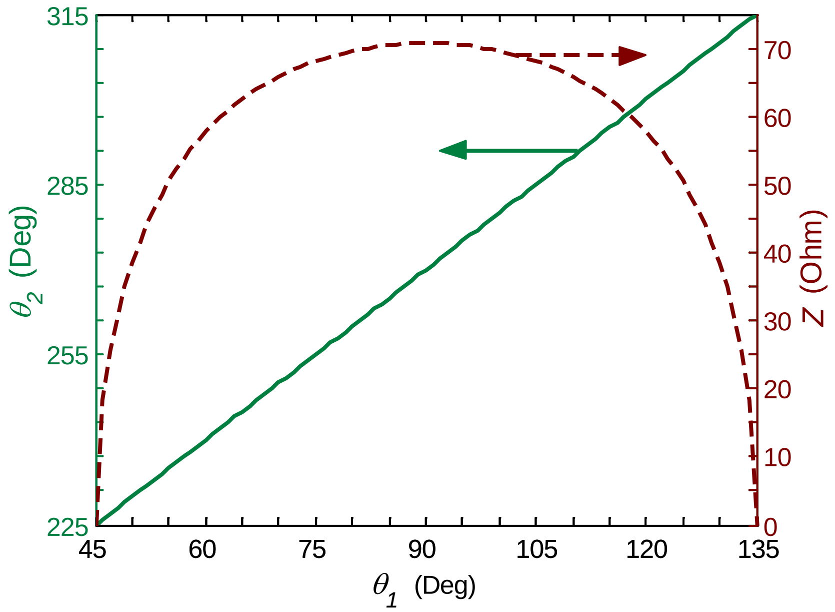
$$S_{34} = 1/X$$

Where, $X = \left[2\cos\theta_1 + j\frac{1}{2}\sin\theta_1\left(\frac{Z}{Z_0} + \frac{2Z_0}{Z}\right) - j\frac{Z_0}{Z}\cos\theta_1\cot\theta_1\right]$

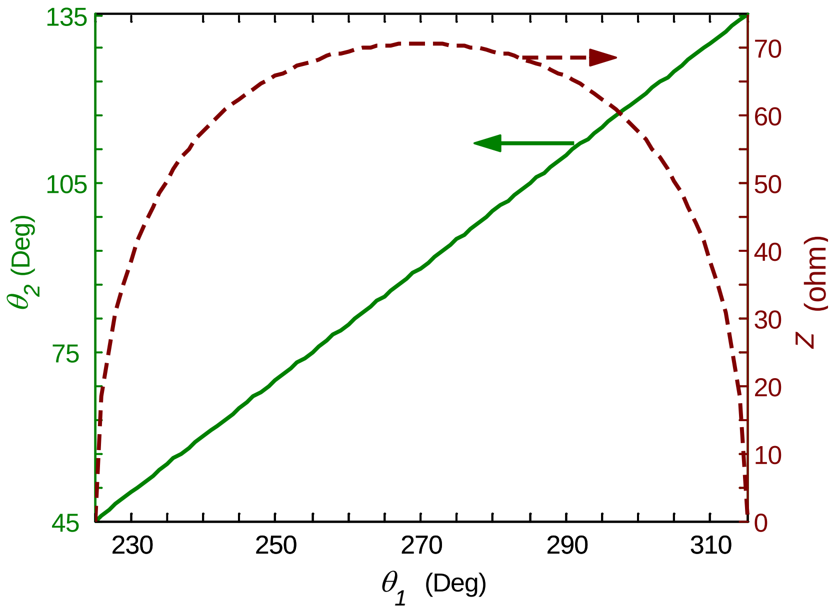
Analysis of rat race coupler



Configuration of the RRC



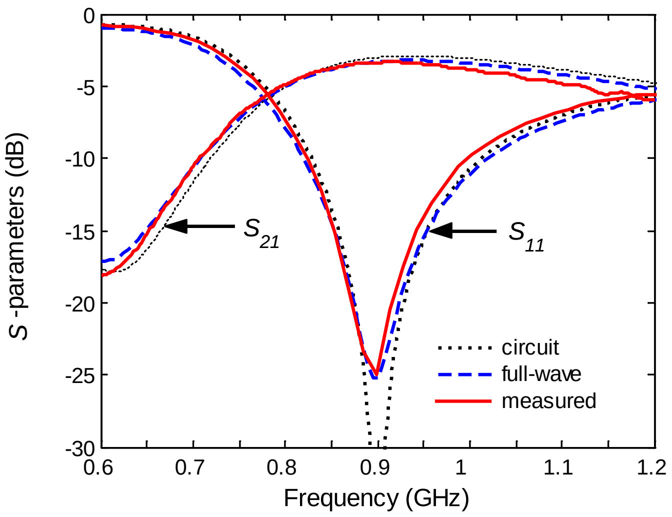
First period



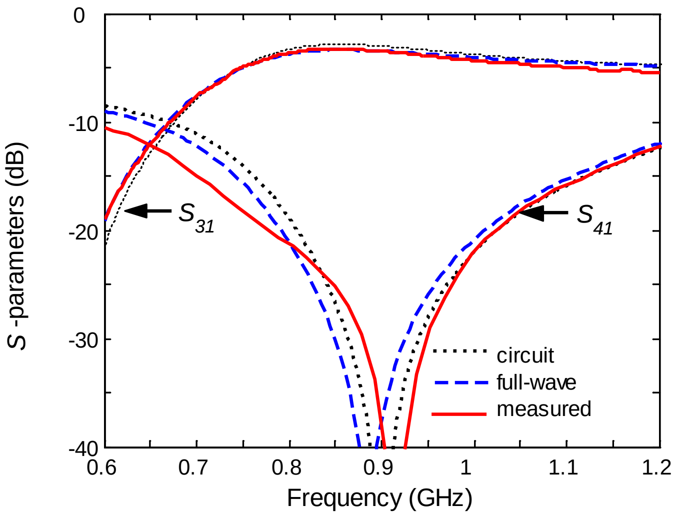
second period

Solutions for Z, θ_1 and θ_2

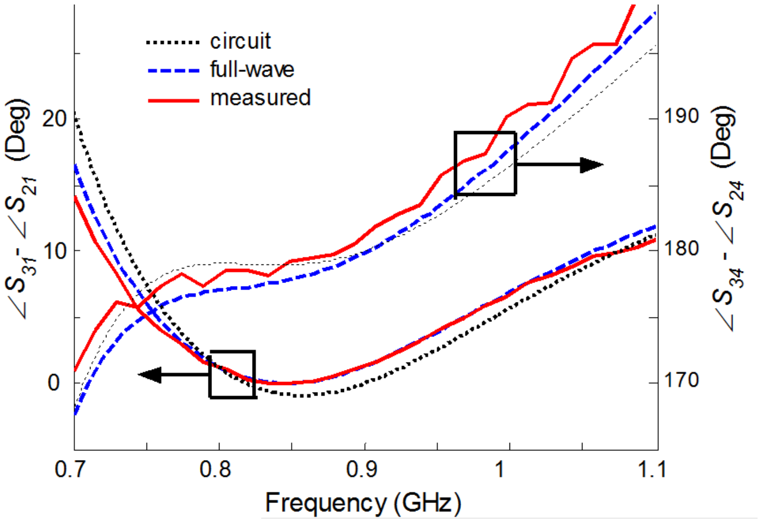
Response of a rat race coupler



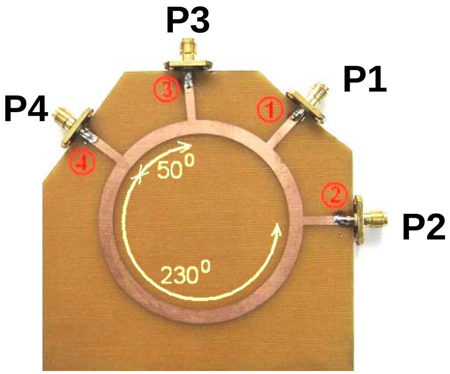
|S|-parameters



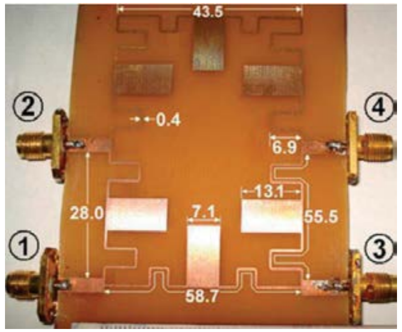
|S|-parameters



Phase imbalance.



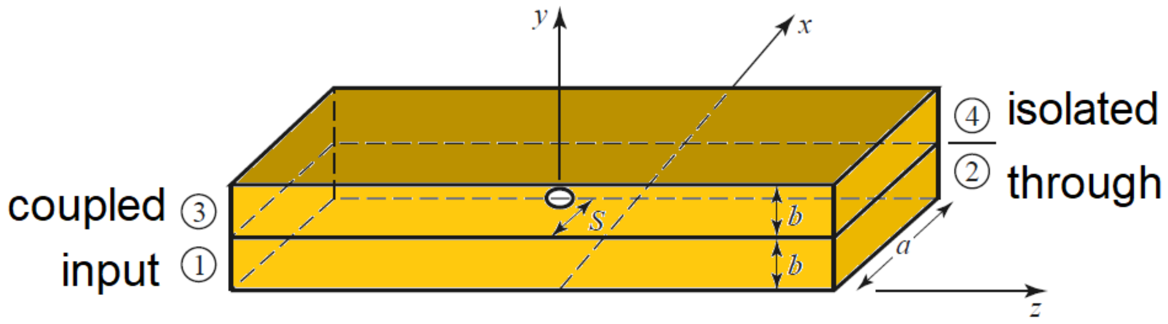
A photograph of the RRC.



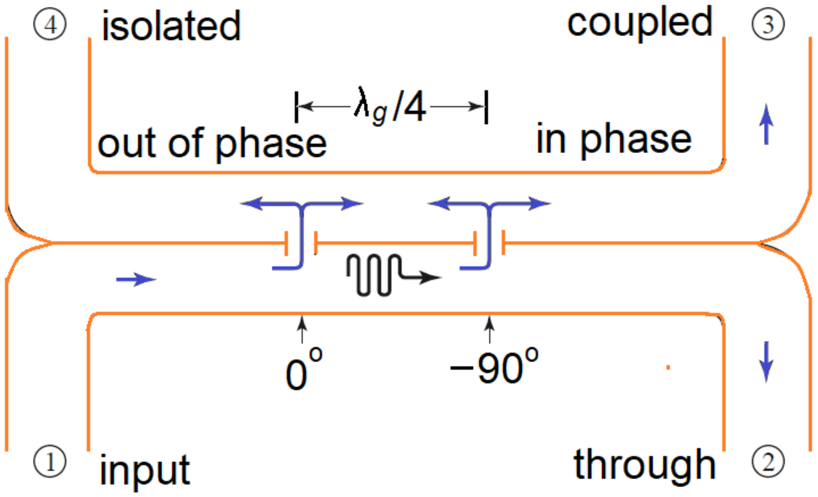
A compact Rat Race coupler.

Bethe hole coupler

- An aperture is modelled by equivalent sources consisting of electric and magnetic dipole moments.
- The normal electric dipole moment and the axial magnetic dipole moment radiate with even symmetry in the coupled guide, while the transverse magnetic dipole moment radiates with odd symmetry.
- By adjusting the relative amplitudes of these two equivalent sources, the radiation in the direction of the isolated port can be cancelled.
- In a multi-hole coupler, the apertures are spaced $\lambda_g/4$ apart and couple the two guides. Each aperture radiates a forward and a backward wave component into the upper guide. In the direction of port 3, both wave are in the same phase.



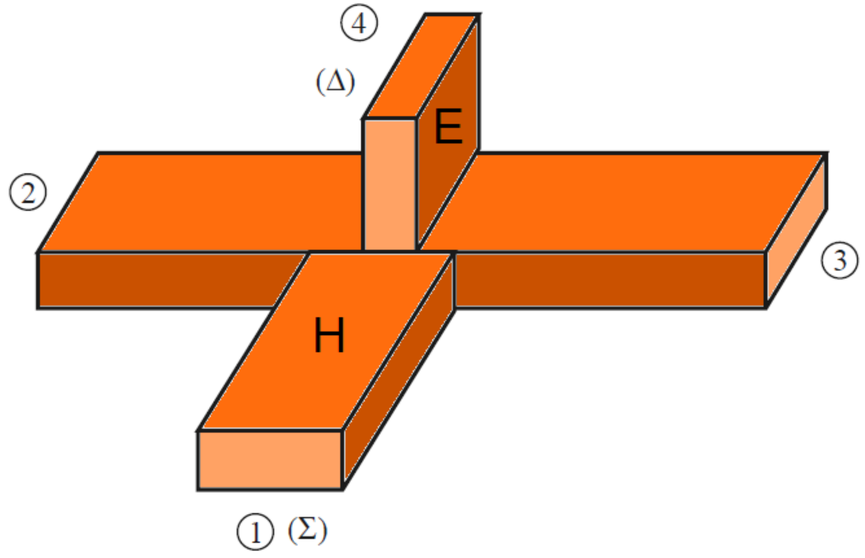
A Bethe hole coupler



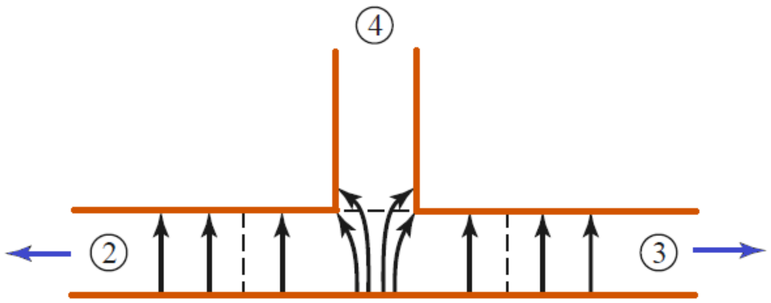
Multi-hole coupler

Magic tee

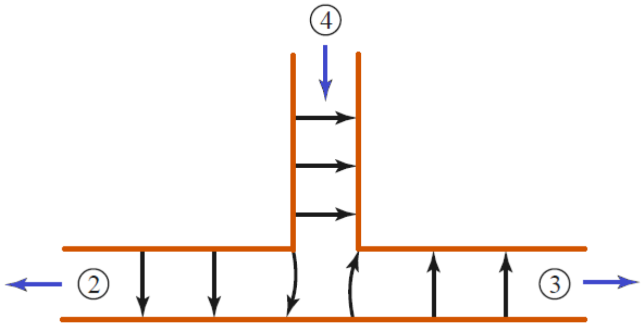
- A magic tee is the waveguide version of a rat-race coupler.
- The magic tee is a combination of E and H plane tees. Arm 1 (Σ port) forms an H-plane tee with arms 2 and 3. Arm 4 (Δ port) forms an E-plane tee with arms 2 and 3.
- Magic: if, by means of a suitable internal structure, the Σ port and Δ port are simultaneously matched, then by symmetry, reciprocity and conservation of energy the two collinear ports are also matched, and are 'magically' isolated from each other.



A waveguide hybrid junction, magic-T



Excitation from port 1



Excitation from port 4

RF and Microwave Engineering (EC 31005)

Microwave Filters (P7)

Mrinal Kanti Mandal

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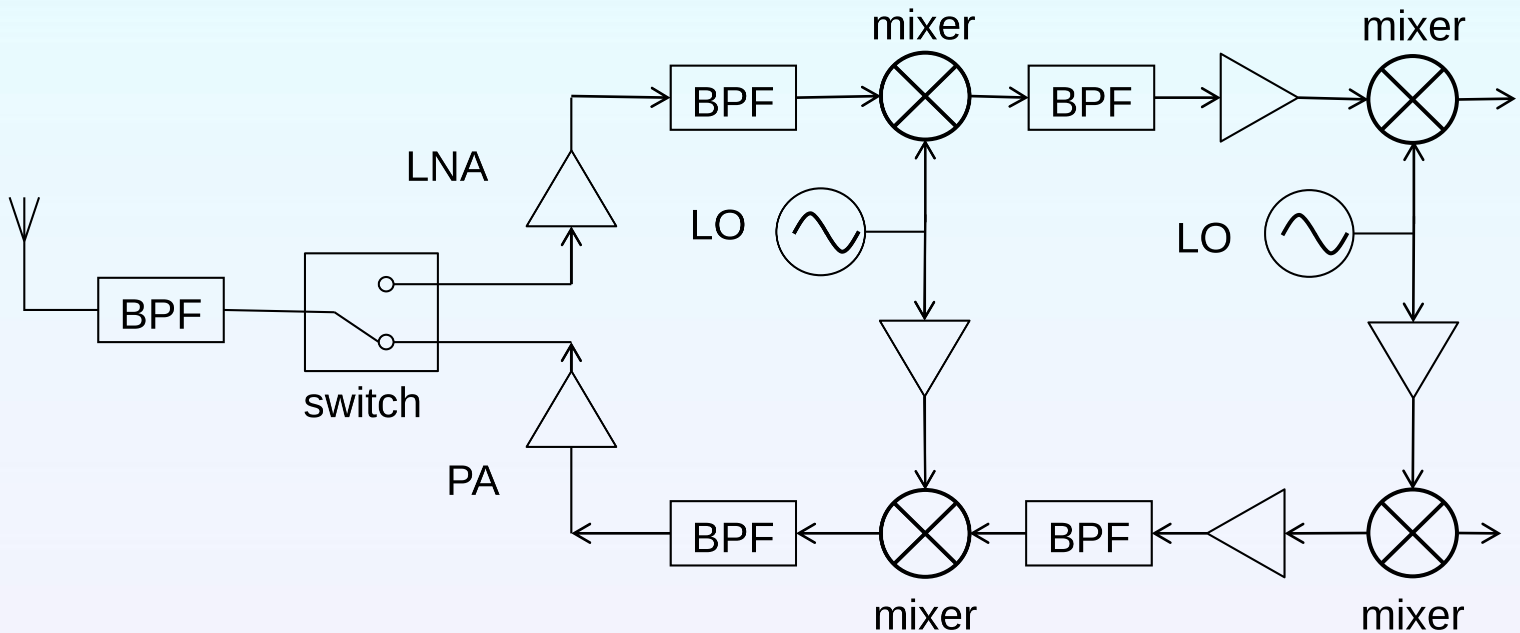
I.I.T. Kharagpur.

Superheterodyne Architecture

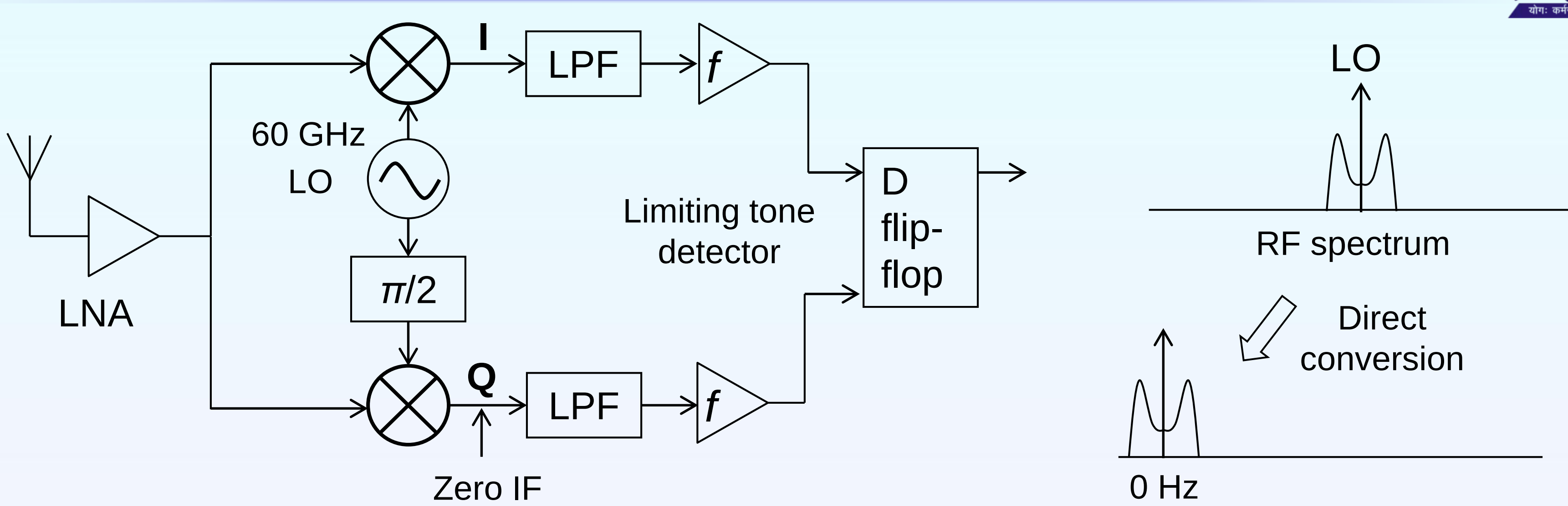


Block diagram of a millimeter wave @60 GHz super heterodyne transceiver.

- The first IF is in the 5 GHz band so that an IEEE 802.11 RF chipset can be used just after that.
- Need many components and high DC power, not preferred for handheld mobile devices.

$$f_{im} = \begin{cases} f + 2f_{IF}, & \text{if} \quad f_{LO} > f \text{ (high side injection)} \\ f - 2f_{IF}, & \text{if} \quad f_{LO} < f \text{ (low side injection)} \end{cases}$$

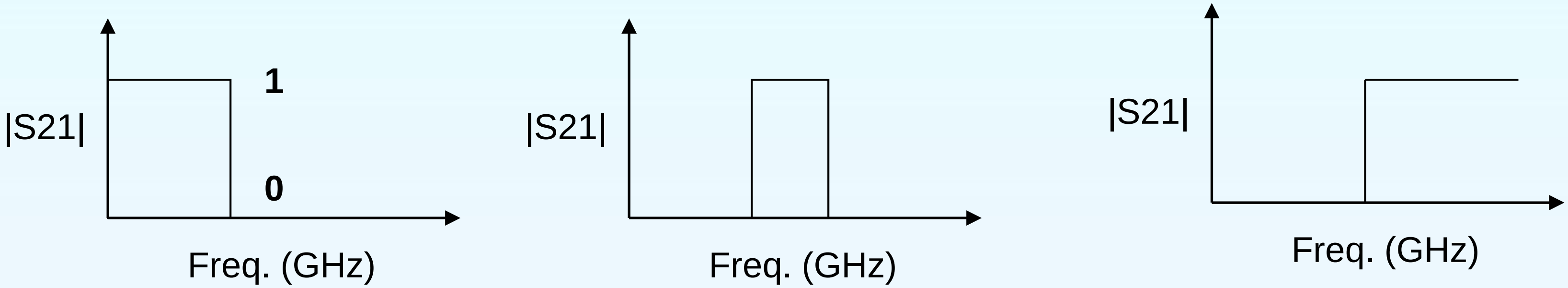
Direct Conversion Architecture



Block diagram of a direct conversion FSK receiver.

- Also known as zero IF.
- Intrinsically simple architecture, well suited for monolithic integration.
- Low power consumption.
- A D flip-flop is used as a detector.
- The LO down convert the signal into two branches, I and Q. It enables the detector to discriminate the signal at positive and negative frequencies (data 1's and 0's)
- Called homodyne receiver when the LO is synchronized in phase with the incoming carrier frequency.

Insertion Loss Method



Ideal lowpass filter   Ideal bandpass filter   Ideal highpass filter

•Perfect filter have zero passband attenuation and infinite attenuation in the stopband.

In insertion loss method: the power loss due to a two port network is represented by a polynomial.

Power loss ratio, $$P_{LR} = \frac{Power\ available\ from\ source}{Power\ delivered\ to\ load} = \frac{P_{inc}}{P_{load}}$$

$$= \frac{1}{1-\left|\Gamma(\omega)\right|^2}.$$

1. J.S. Hong, and M.J. Lancaster, Microstrip filters for RF/Microwave Applications.
2. G. Matthaei , E.M.T. Jones, L. Young, Microwave Filters, Impedance-Matching Networks, and Coupling Structures.
3. David M Pozar, Microwave engineering, Wiley.

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Insertion Loss Method

In general, transfer function is defined as

$$\left|S_{21}(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 F_n^2(\Omega)}$$

where ε is a ripple constant,

$F_n(\Omega)$ represents a filtering function,

Ω is a frequency variable.

•For a linear, time invariant network, the transfer function

$$S_{21}(p) = \frac{N(p)}{D(p)}$$

$N(p)$ and $D(p)$ are polynomials in a complex frequency variable $p=\sigma+j\Omega$.

•The transmission loss response of the filter,

$$L_A(\Omega) = 10\log\frac{1}{\left|S_{21}(j\Omega)\right|^2}\ dB.$$

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Insertion Loss Method

The reflection loss response of the filter,

$$L_R(\Omega) = 10\log\left(1 - \left|S_{21}(j\Omega)\right|^2\right)\ dB.$$

The phase response of the filter,

$$\phi_{21} = Arg\, S_{21}(j\Omega)\ rad.$$

The group delay response of the filter,

$$\tau_d(\Omega) = -\frac{\partial\phi_{21}(\Omega)}{\partial\Omega}\ S.$$

Filter types:

1. Butterworth (maximally flat) response.
2. Chebyshev response.
3. Elliptic function response.
4. Gaussian (flat group-delay) response.

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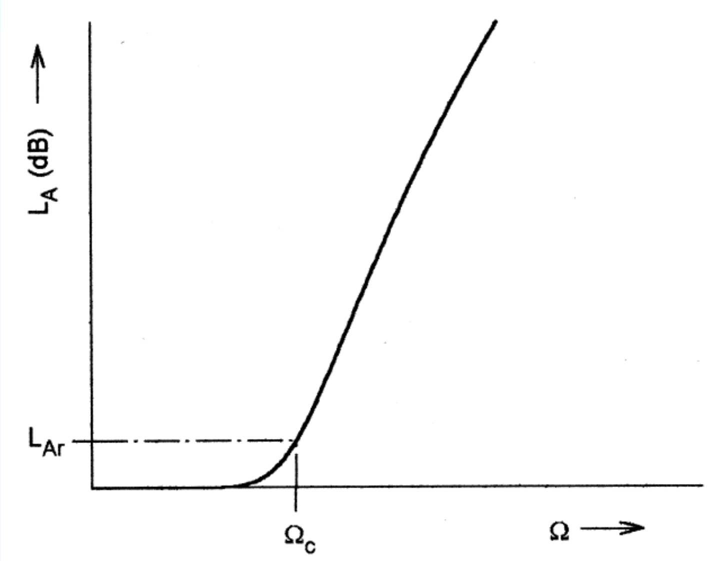
Butterworth (Maximally Flat) Response.

Transfer function is defined as

$$\left|S_{21}\left(j\Omega\right)\right|^2 = \frac{1}{1+\Omega^{2n}}$$



Butterworth lowpass response

$$\text{Maximum number of } \left(2\text{n-}1\right) \text{ zero derivatives at } \Omega = 0$$

$$\text{Passband is best at } \Omega = 0 \text{ but deteriorates as } \Omega \to \Omega_c.$$

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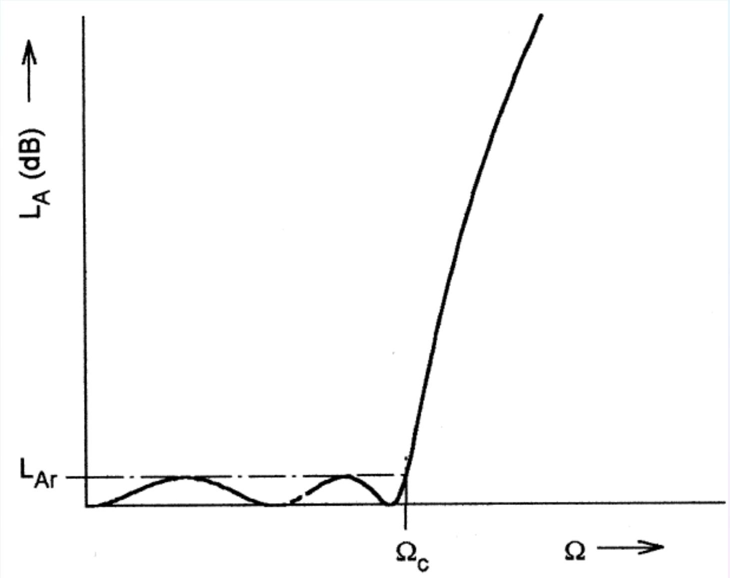
Chebyshev response

Exhibits equal ripple passband and maximally flat stopband.

Transfer function is defined as

$$\left|S_{21}(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 T_n^2(\Omega)} \qquad \text{where ripple constant } \varepsilon = \sqrt{10^{L_{Ar}/10}-1}.$$

$$T_n(\Omega) = \cos\left(n\cos^{-1}\Omega\right) \qquad |\Omega| \le 1,$$
$$= \cosh\left(n\cosh^{-1}\Omega\right) \qquad |\Omega| \ge 1.$$



Chebyshev lowpass response.

Elliptic function response

Transfer function is defined as

$$\left|S_{21}(j\Omega)\right|^2 = \frac{1}{1+\varepsilon^2 F_n^2(\Omega)}$$

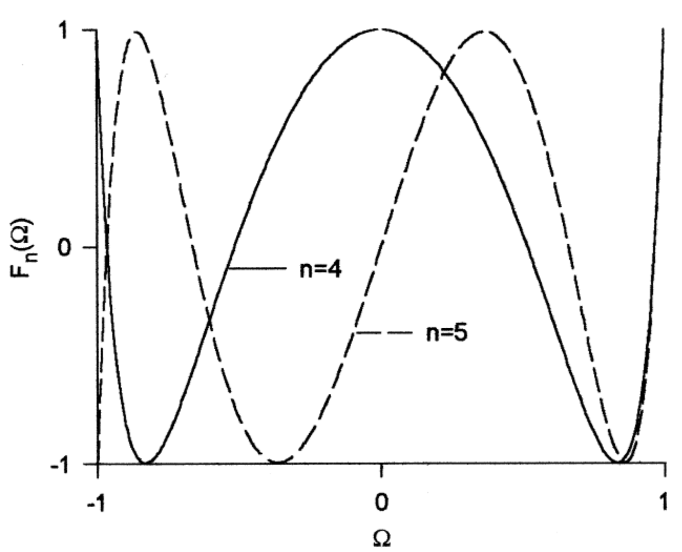
$$F_n(\Omega) = M \frac{\prod_{i=1}^{n/2}\left(\Omega_i^2 - \Omega^2\right)}{\prod_{i=1}^{n/2}\left(\Omega_s^2 - \Omega^2\right)} \quad \text{for } n \text{ even,}$$

$$= N \frac{\Omega \prod_{i=1}^{(n-1)/2}\left(\Omega_i^2 - \Omega^2\right)}{\prod_{i=1}^{(n-1)/2}\left(\Omega_s^2 - \Omega^2\right)} \quad \text{for } n(\geq 3) \text{ odd.}$$

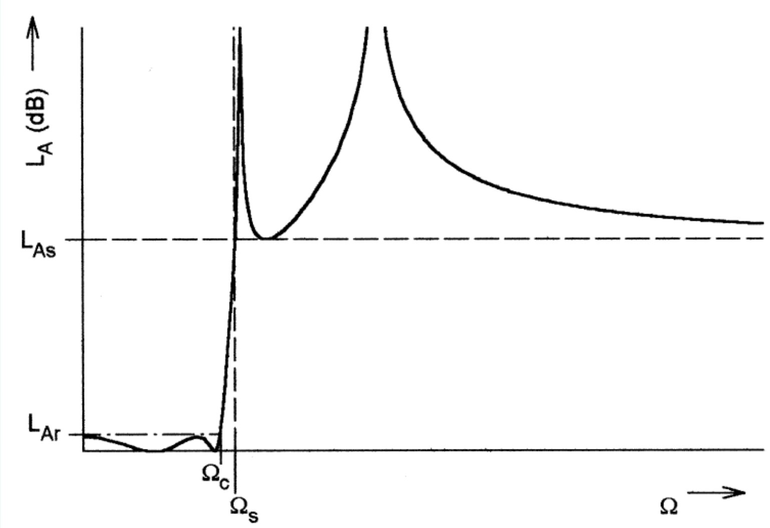
M and N are constants.

$F_n(\Omega)$ oscillate between ± 1 for $|\Omega| \leq 1$, and $\left|F_n(\Omega = \pm 1)\right| = 1$.

Elliptic function response



Plot of elliptic rational function.



Elliptic function lowpass response.

- It also called Cauer filters.
- It has the maximum group delay variation among the four.

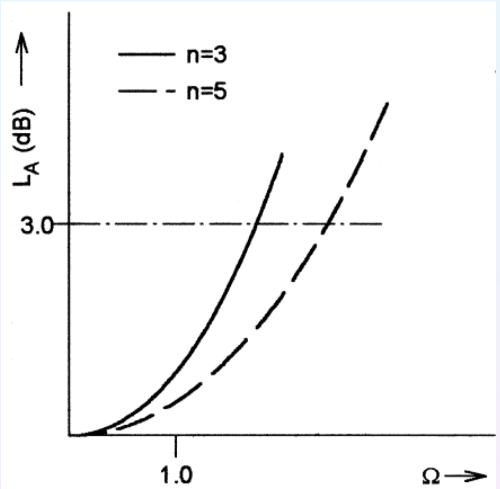
Gaussian (flat group-delay) response

Transfer coefficient is defined as

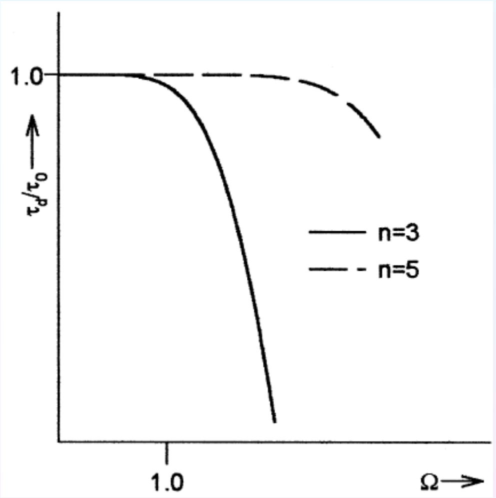
$$\left|S_{21}(p)\right| = \frac{a_0}{\sum_{k=0}^{n} a_k\, p^k}$$

where $p = \sigma + j\Omega$ is the normalized complex frequency variable.

$$a_k = \frac{(2n-k)!}{2^{n-k}\, k!(n-k)!}.$$



Amplitude variation



Group delay variation

Design of a filter

Filter specifications:

•Cutoff frequency f_c (Hz)

•Maximum passband return loss value (dB)

•Stopband minimum attenuation (dB)

•Source/ load impedance (50 Ω)

Filter synthesis:

•Determine the filter type (Butterworth Chebyshev, Equal ripple, elliptic...etc), and order of the filter.

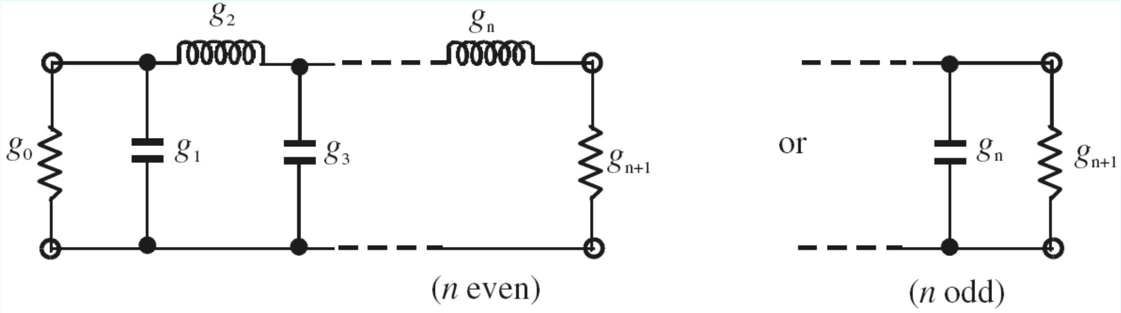
•Obtain $g_0, g_1, \ldots g_n, g_{n+1}$ from the table.

•Calculate the electrical length and characteristic impedance of the stubs.

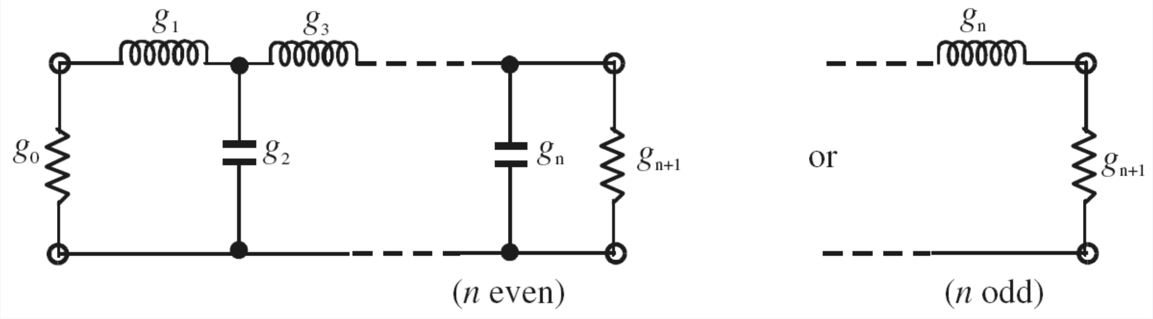
•Optimization using a full wave simulator.

1. Microstrip filters for RF/ Microwave application, *Wiley,* J.S. Hong and M. J. Lancaster.
2. Impedance-matching networks, and coupling structures, *Mc-Graw-Hill,*G. L. Matthaei, L. Young, E. M. T. Jones.

Lowpass prototype filter and elements



Lowpass prototype filters for all-pole filters with a ladder network structure



The dual structure

- g_0, g_{n+1} are the source/ load resiatances.

Butterworth lowpass prototype filters

The elements values may be computed as

considering $L_{Ar} = 3.01dB$ at the cutoff $\Omega_c = 1$,

$$g_0 = 1.0$$

$$g_i = 2\sin\left(\frac{(2i-1)\pi}{2n}\right) \qquad for \;\; i = 1 \;\; to \;\; n$$

$$g_{n+1} = 1.0$$

$$n \geq \frac{\log\left(10^{0.1L_{As}} - 1\right)}{2\log\Omega_s}$$

n is the filter order.

Example:

If $L_{As} = 40\,dB$ and $\Omega_s = 2$ results in $n \geq 6.64 \Rightarrow$ take $n = 7$.

Elements values for a Chebyshev LPF

Passband ripple $\mathrm{L}_{\mathrm{Ar}} = 0.01\ dB\ \left(g_0 = 1,\ \Omega_c = 1\right)$.

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	0.0960	1.0								
2	0.4489	0.4078	1.1008							
3	0.6292	0.9703	0.6292	1.0						
4	0.7129	1.2004	1.3213	0.6476	1.1008					
5	0.7563	1.3049	1.5773	1.3049	0.7563	1.0				
6	0.7814	1.3600	1.6897	1.5350	1.4970	0.7098	1.1008			
7	0.7970	1.3924	1.7481	1.6331	1.7481	1.3924	0.7970	1.0		
8	0.8073	1.4131	1.7825	1.6833	1.8529	1.6193	1.5555	0.7334	1.1008	
9	0.8145	1.4271	1.8044	1.7125	1.9058	1.7125	1.8044	1.4271	0.8145	1.0

- Similar charts are available for different polynomial functions and for different values of ripples.

Practical filter design

Steps:

•Change Ω to practical frequency ω.

•Change terminating impedances to port impedance Z_0.

•It also requires impedance scaling.

Define an impedance scaling factor,

$$\gamma_0 = Z_0 / g_0 \quad \text{when } g_0 \text{ is a resistance}$$
$$= g_0 / Y_0 \quad \text{is a conductance.}$$

The prototype elements are transformed as

$$L \to \gamma_0 L \qquad R \to \gamma_0 R$$
$$C \to C/\gamma_0 \quad \text{and} \quad G \to G/\gamma_0$$

Design of a lowpass and high pass filters

Lowpass transformation:

$$\Omega = \left(\frac{\Omega_c}{\omega_c}\right)\omega$$

$$L = \left(\frac{\Omega_c}{\omega_c}\right)\gamma_0 g \qquad \text{for } g \text{ representing an inductance.}$$

$$C = \left(\frac{\Omega_c}{\omega_c}\right)g \Big/ \gamma_0 \qquad \text{for } g \text{ representing a capacitance.}$$

Highpass transformation:

$$\Omega = -\left(\frac{\Omega_c}{\omega}\right)\omega_c$$

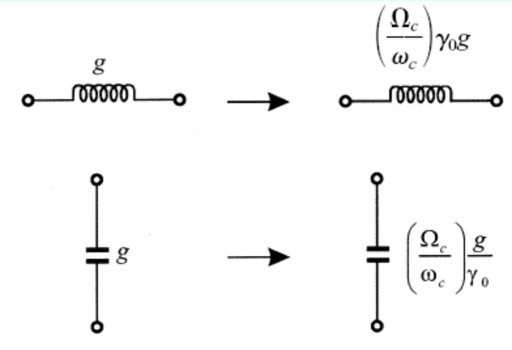
$$C = \left(\frac{1}{\omega_c \Omega_c}\right)\frac{1}{\gamma_0 g} \qquad \text{for } g \text{ representing an inductance.}$$

$$L = \left(\frac{1}{\omega_c \Omega_c}\right)\frac{\gamma_0}{g} \qquad \text{for } g \text{ representing a capacitance.}$$

Example

Design a 3-pole Butteworth LPF with cutoff frequency 2 GHz.

Solution:



Transformation of the prototype element.

From the table,

$$g_0 = g_4 = 1.0\,\text{mho},\ g_1 = g_3 = 1.0\,H,\ g_2 = 2\,F\ \text{for}\ \Omega_c = 1.$$

Given that $\omega_c = 2\pi \times 2 \times 10^9$ rad/S.

Using transformation,

$$\mathrm{L}_1 = L_3 = 3.979\,nH,\ C_2 = 3.183\ pF.$$

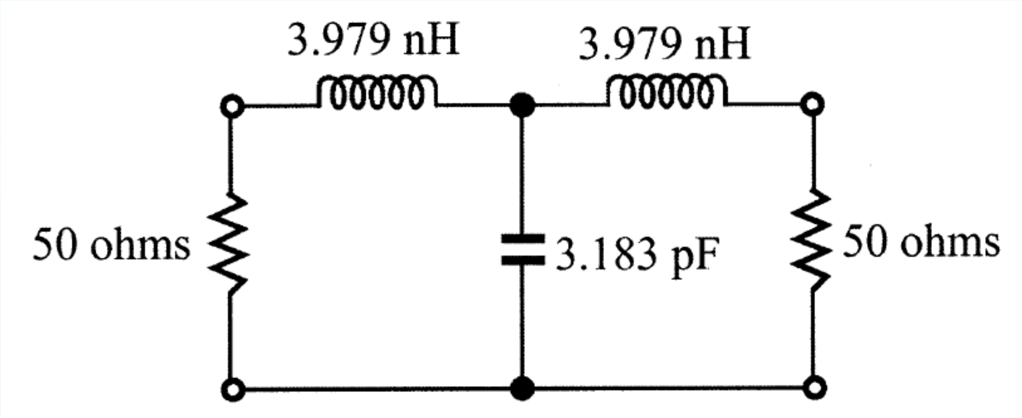
Elements values for a Butterworth LPF

Passband ripple $\mathrm{L}_{\mathrm{Ar}} = 3.01\, dB \left(g_0 = 1, \Omega_c = 1 \right)$.

n	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
1	2.0000	1.0								
2	1.4142	1.4142	1.0							
3	1.0000	2.0000	1.0000	1.0						
4	0.7654	1.8478	1.8478	0.7654	1.0					
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0				
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0			
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0		
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902	1.0	
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0

- Fractional bandwidth (FBW) is defined by the equal ripple bandwidth value.

Example



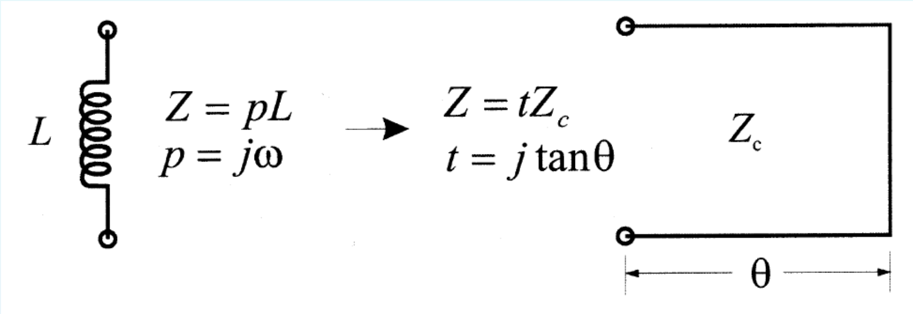
A lowpass filter based on the transformation.

Richard's transformation

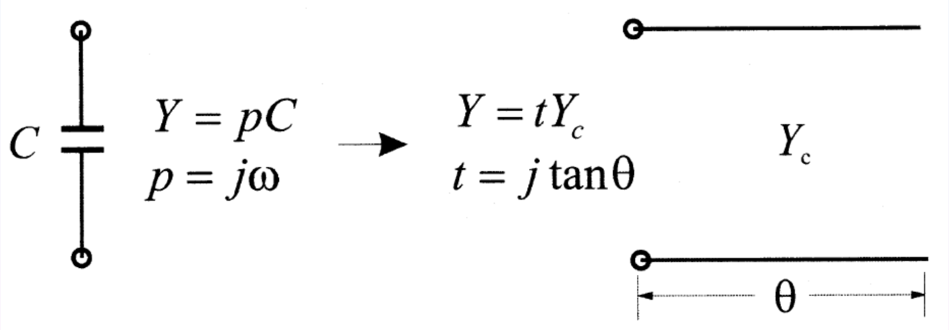
Using the transformation $\Omega = \tan \beta l$,

the reactance of an inductor is $jX_L = j\Omega L = jL \tan \beta l$ and

the susceptance of a capacitor is $jB_C = j\Omega C = jC \tan \beta l$.



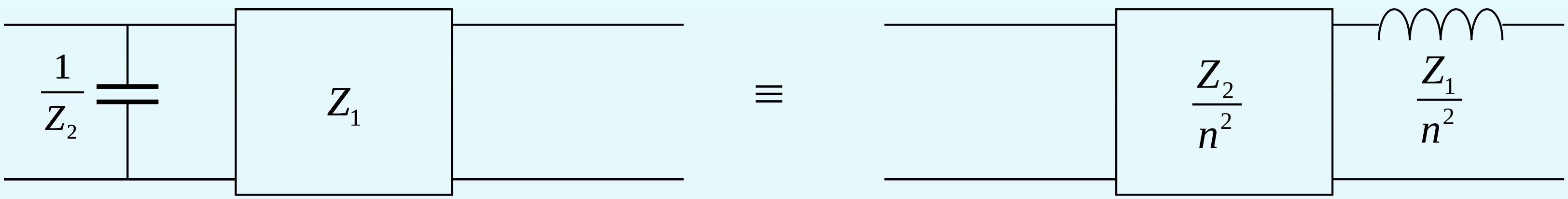
Inductor to a short-circuited stub



Capacitor to an open-circuited stub

Kuroda identities

- Shunt element can be converted to a series element and vice versa.



Shunt to series element

$$n^2 = 1 + Z_2/Z_1$$

Series to shunt element

- Another two identities for transforming a shunt-inductor and series-capacitor.

Design Example: LPF Using High-Low Line

Specifications:

Cutoff frequency = 1 GHz.

Passband ripple = 0.1 dB (corresponding return loss = -16.4 dB).

Port impedance = 50 Ω.

Filter synthesis:

Considering Chebyshev response,

$$g_0 = g_4 = 1.0$$

$$g_1 = g_3 = 1.0316$$

$$g_2 = 1.1474.$$

Using element transformation,

$$L_1 = L_3 = \left(\frac{Z_0}{g_0}\right)\left(\frac{\Omega_c}{2\pi f_c}\right) g_1 = 8.209 \times 10^{-9}\ H$$

$$C_2 = \left(\frac{g_0}{Z_0}\right)\left(\frac{\Omega_c}{2\pi f_c}\right) g_2 = 3.652 \times 10^{-12}\ F.$$

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Example

Let us choose a microstrip line on a substrate with $\varepsilon_r = 10.8,\ t = 1.27\,mm.$
and $Z_{0L} = 93\Omega$ and $Z_{0c} = 24\Omega.$

Physical lengths are

$$l_L = \frac{\lambda_{gL}}{2\pi}\sin^{-1}\left(\frac{\omega_c L}{Z_{0L}}\right) = 11.04\,mm,$$

$$l_c = \frac{\lambda_{gc}}{2\pi}\sin^{-1}\left(\omega_c C\, Z_{0C}\right) = 9.75\,mm.$$

Final results:

Characteristic impedance (ohms)	$Z_{0C} = 24$	$Z_0 = 50$	$Z_{0L} = 93$
Guided wavelengths (mm)	$\lambda_{gC} = 105$	$\lambda_{g0} = 112$	$\lambda_{gL} = 118$
Microstrip line width (mm)	$W_C = 4.0$	$W_0 = 1.1$	$W_L = 0.2$

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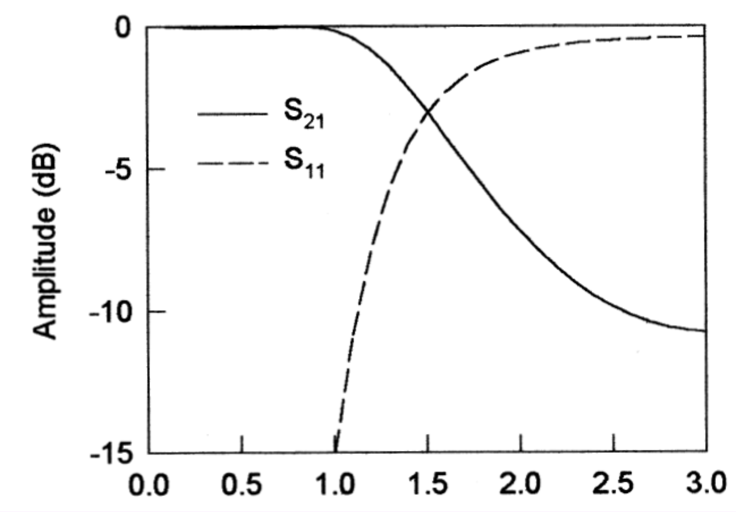
Example

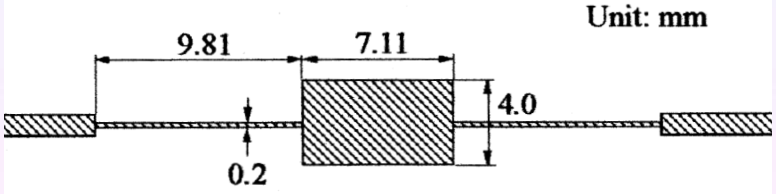
Use these formula for more accurate results:

$$\omega_c L = Z_{0L} \sin\left(\frac{2\pi l_L}{\lambda_{gL}}\right) + Z_{0C} \tan\left(\frac{\pi l_C}{\lambda_{gC}}\right)$$

$$\omega_c C = \frac{1}{Z_{0C}} \sin\left(\frac{2\pi l_C}{\lambda_{gC}}\right) + 2 \times \frac{1}{Z_{0L}} \tan\left(\frac{\pi l_L}{\lambda_{gL}}\right)$$



Simulated results



Layout of the filter with dimensions.

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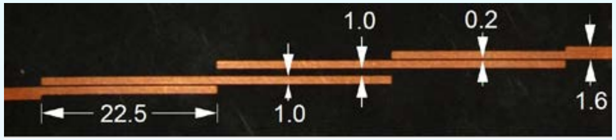
Design of Bandpass Filters

Filter specifications:

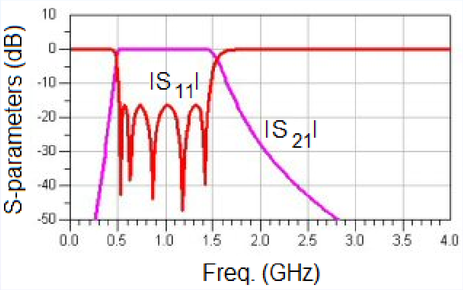
- Cutoff frequencies f_{c1} , f_{c2} (Hz) or bandwidth Δf and the centre frequency f_{0}.
- Maximum passband insertion loss value (dB).
- Stopband minimum attenuation (dB).
- Source/ load impedance (50 Ω).
- Maximum allowed group delay variation.



Coupled-line BPF

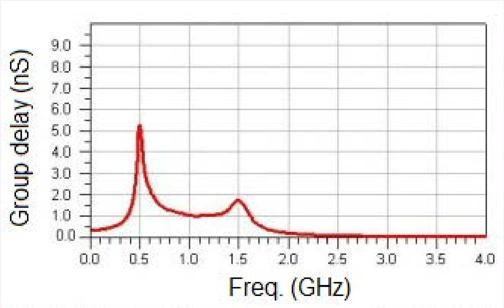
Some important parameters:

- Group delay $= -\partial\angle S_{21}/\partial\omega$
- Unloaded quality factor: Q_{ul}
- External quality factor: Q_{ext}



S-parameters

Group delay results in distorted signals



Group delay

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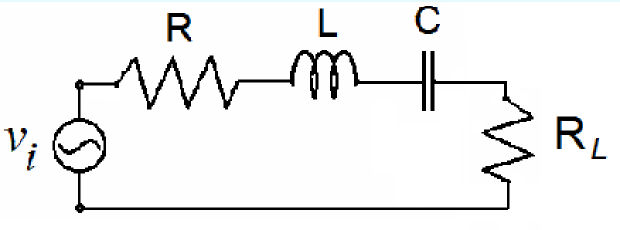
Different Q-factors



Series resonator with source and loads.

- Loaded and unloaded Q-factors:

When a resonator is a part of a circuit, overall loss increases.
Total loss also includes the loss due to the source and load resistances.

Define external quality factor as

$$Q_e = \frac{\omega_0 L}{R_L} \qquad \textit{for a series resonator}$$

$$= \frac{R_L}{\omega_0 L} \qquad \textit{for a parallel resonator.}$$

Then loaded Q is $\dfrac{1}{Q_L} = \dfrac{1}{Q_e} + \dfrac{1}{Q_{ul}}.$

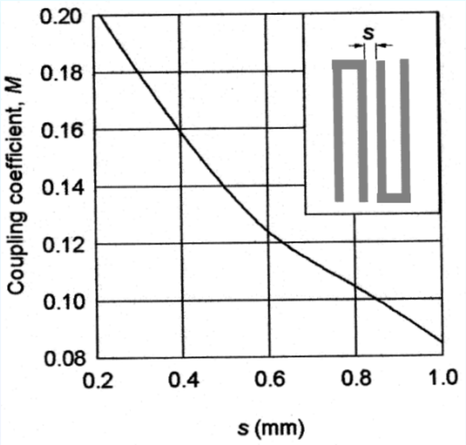
Synthesis of Bandpass Filters

- Determine the filter type (Butterworth Chebyshev, equal ripple, elliptic...etc), and order of the filter.
- Determine the lowpass prototype elements from tabular data: g_0, g_1, ...g_n, g_{n+1}.
- Calculate coupling coefficients:

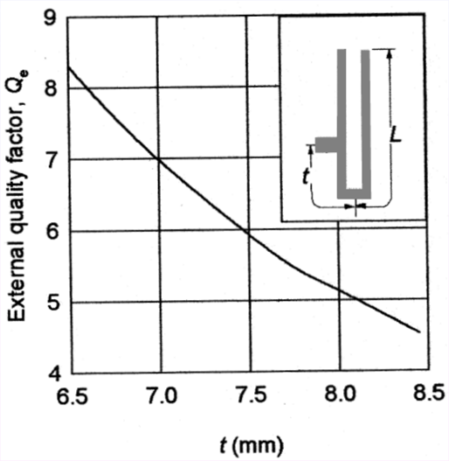
  $M_{i,\,i+1} = FBW/\sqrt{(g_i\, g_{i+1})}$.
- Determine external coupling parameters:

  $Q_{e1} = g_0 g_1/FBW$

  $Q_{en} = g_n g_{n+1}/FBW$.
- Resonator types: hairpin, comb-line, interdigital, open loop, ... etc).
- Determine Filter Dimensions.
- Optimization for final design.



Coupling coefficient



Q_e vs. tapping point

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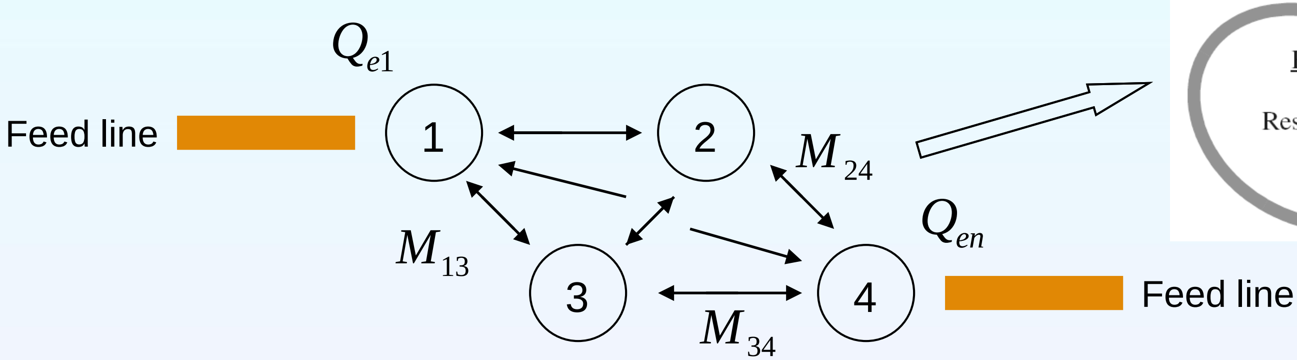
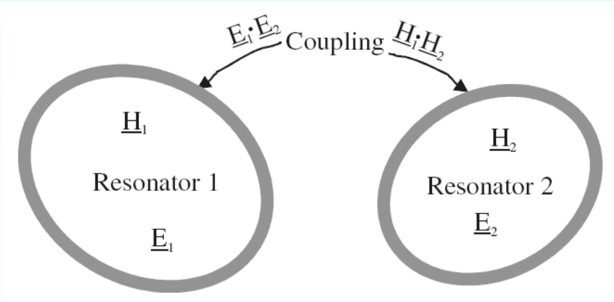
Design of a Bandpass Filter



BPF filter



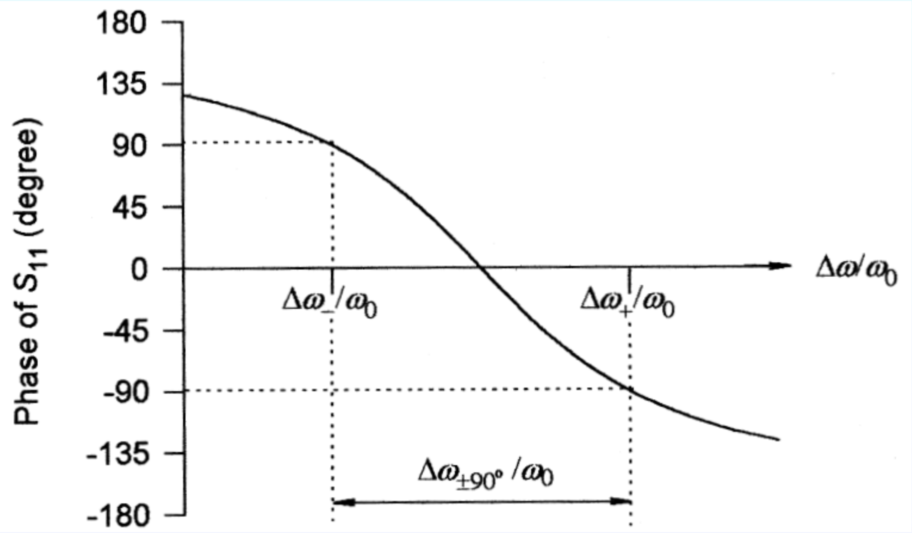
Relation with prototype LPF elements:

$$Q_{e1} = \frac{g_0 g_1}{FBW}, \qquad Q_{en} = \frac{g_n g_{n+1}}{FBW}$$

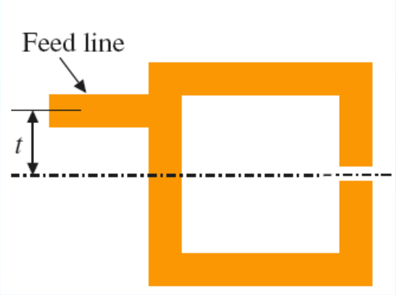
$$M_{i,i+1} = \frac{FBW}{\sqrt{g_i g_{i+1}}} \qquad \text{for } i = 1 \text{ to } n-1$$

FBW is the fractional bandwidth.

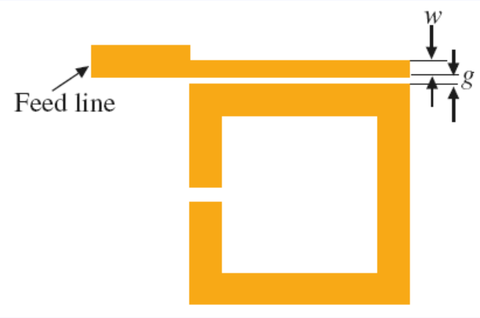
Determination of Qe



Phase of S_{11} for the gap-coupled feed line.



Direct tapped feeding.



Gap coupled feeding.

- External quality factor is given by, $Q_e = \dfrac{\omega_0}{\Delta\omega_{\pm 90^0}}$

- When the susceptance parameter is known,

$$Q_{ei} = R_L \frac{\omega_0}{2} \frac{\partial B}{\partial \omega}\bigg|_{\omega_0}$$

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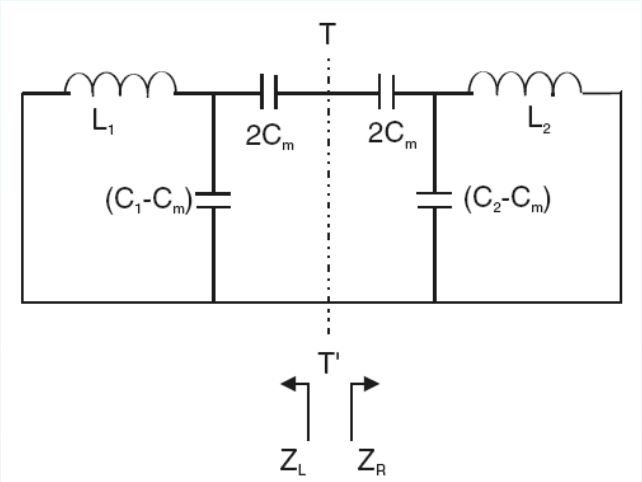
Different Types of Coupling

1. Electric coupling→



Circuit example of electric coupling.

Example of electrically coupled resonators.

A perfect electrical wall can be placed between the coupled resonators.

Coupled-resonators resonate at two frequencies,

$$\omega_{1,2} = \sqrt{\frac{(L_1C_1 + L_2C_2) \pm \sqrt{(L_1C_1 - L_2C_2)^2 + 4L_1L_2C_m^2}}{2(L_1L_2C_1C_2 - L_1L_2\,C_m^2)}}$$

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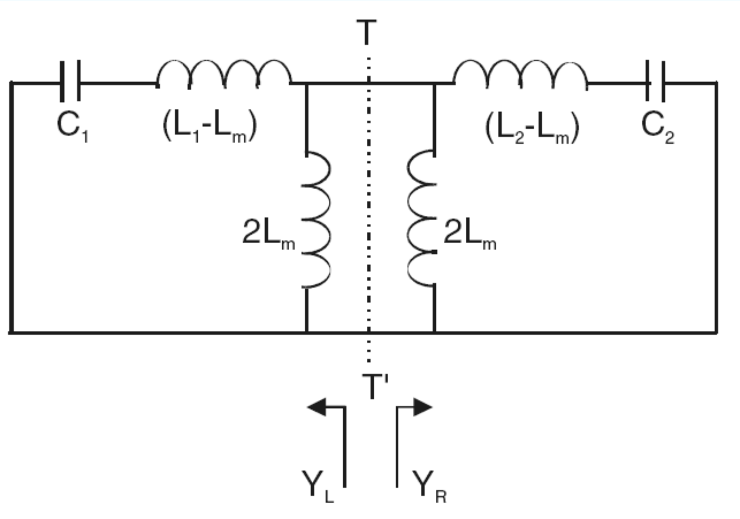
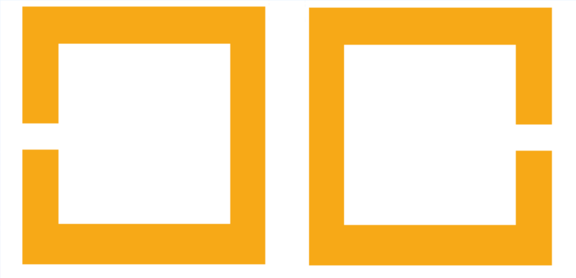
Different Types of Coupling

2. Magnetic coupling→



Circuit example of magnetic coupling.



Example of magnetically coupled resonators.

A perfect magnetic wall can be placed between the coupled resonators.

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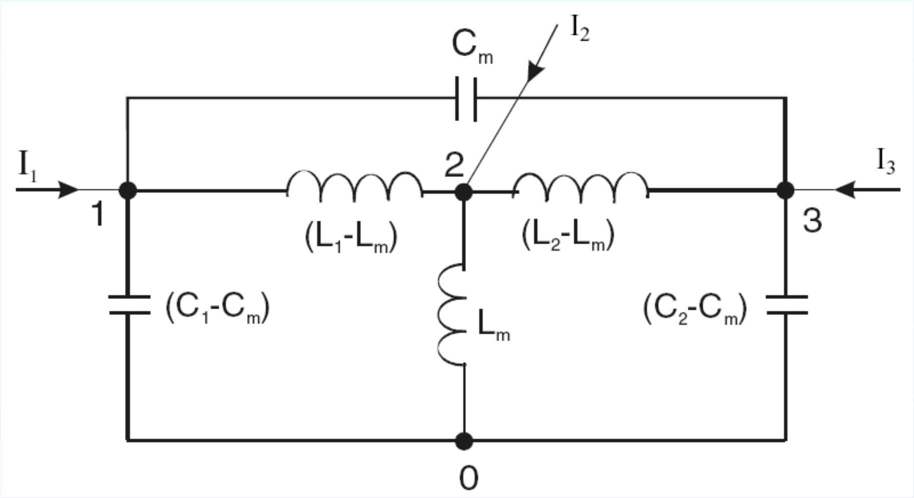
3. Mixed coupling→



Circuit example of mixed coupling.

Example of mixed coupled resonators.

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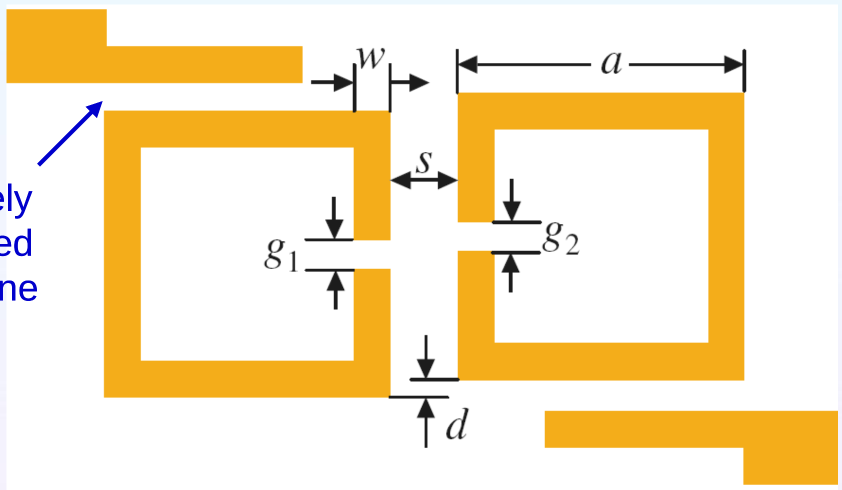
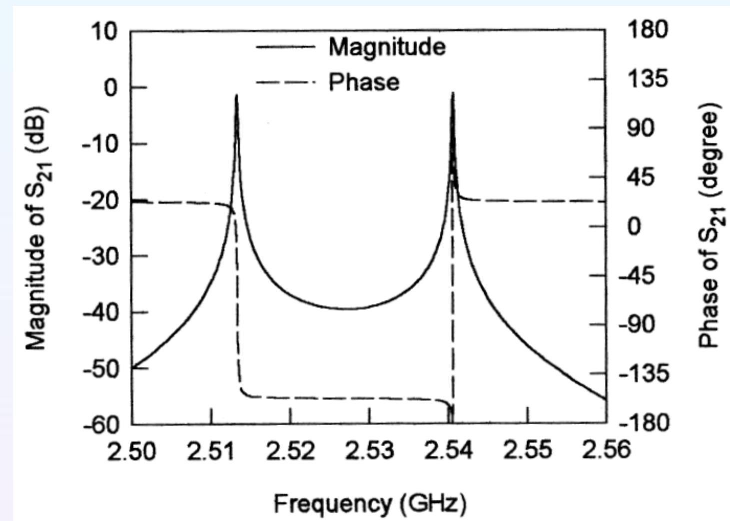
Coupling Co-efficient

Coupling coefficient is $k = \dfrac{\omega_2^2 - \omega_1^2}{\omega_2^2 + \omega_1^2} = \dfrac{f_2^{\,2} - f_1^{\,2}}{f_2^{\,2} + f_1^{\,2}}$.

Loosely coupled feed line



Determination of k_E.



Corresponding S_{21} plot.

$$k > 1/Q_e + 1/Q_u \quad \text{overcoupled}$$
$$k < 1/Q_e + 1/Q_u \quad \text{undercoupled.}$$

•External feed lines should be loosely coupled to avoid any loading effects on the resonators.

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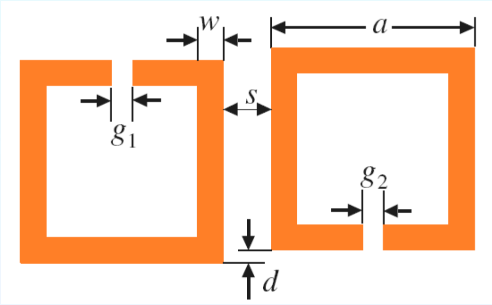
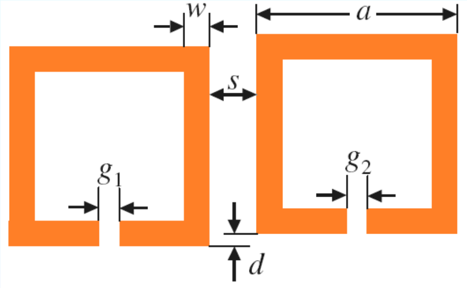
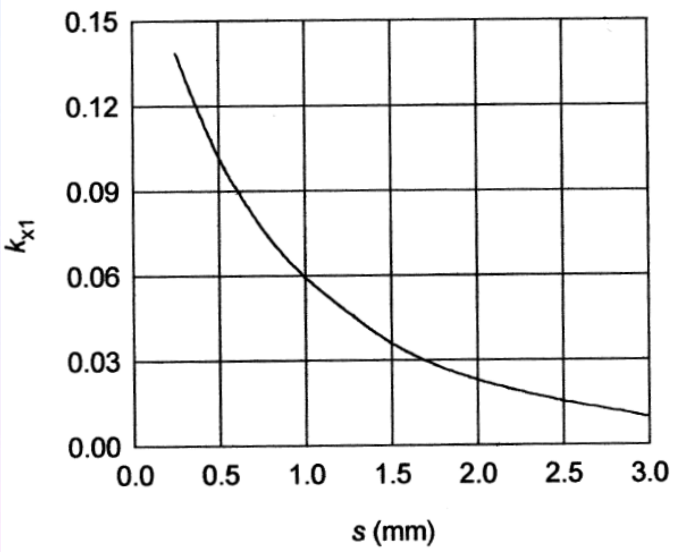
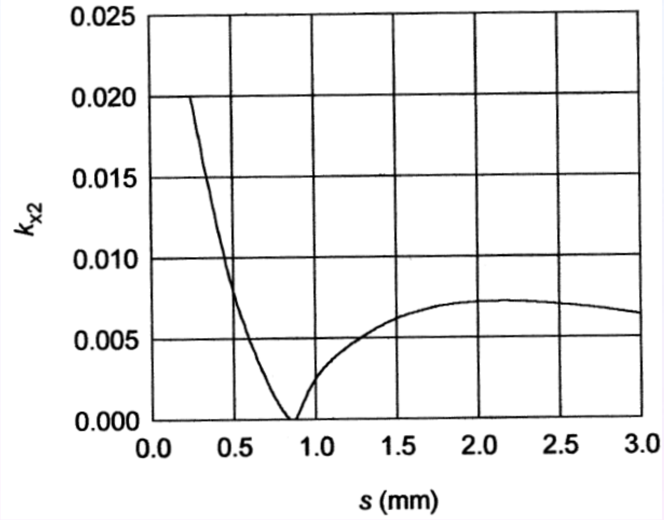
Example







Extracted *k* for the above coupling scheme.



Extracted *k* for the above coupling scheme.

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BPF Example

Design a five-pole bandpass filter at a mid-band frequency of 2 GHz with a fractional bandwidth of 20%.

Solutions:

Let us choose a Chebyshev lowpass prototype with a passband ripple of 0.1 dB.

The lowpass prototype parameters are

$$g_0 = g_6 = 1.0,\ g_1 = g_5 = 1.1468,\ g_2 = g_4 = 1.3712,\ g_3 = 1.9750.$$

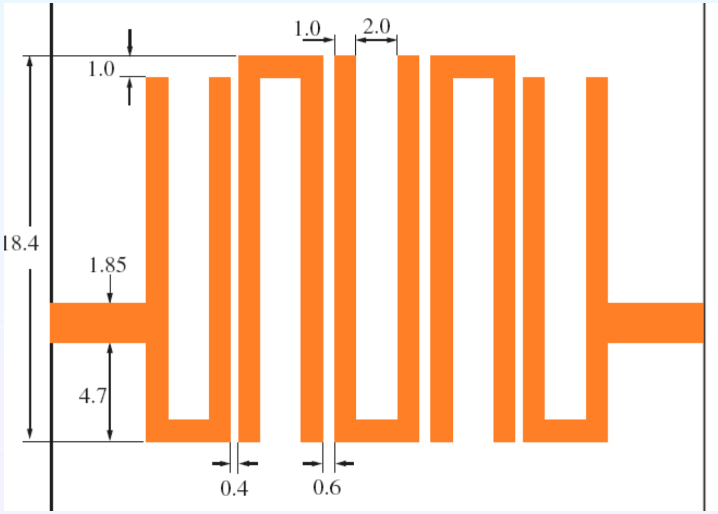
Using the relationships for Qe and coupling-coefficient,

$$Q_{e1} = Q_{e5} = 5.734$$

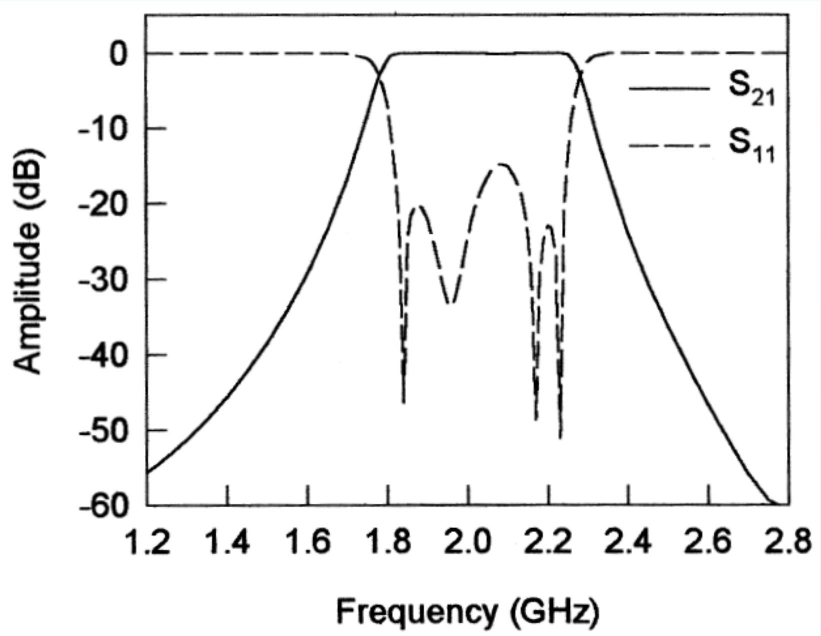
$$M_{1,2} = M_{4,5} = 0.160$$

$$M_{2,3} = M_{3,4} = 0.122$$

Filter Response



Layout of the filter.



Simulated filter response.

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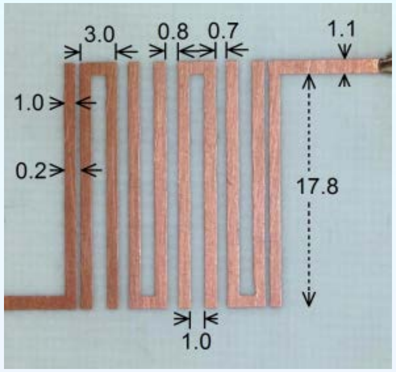
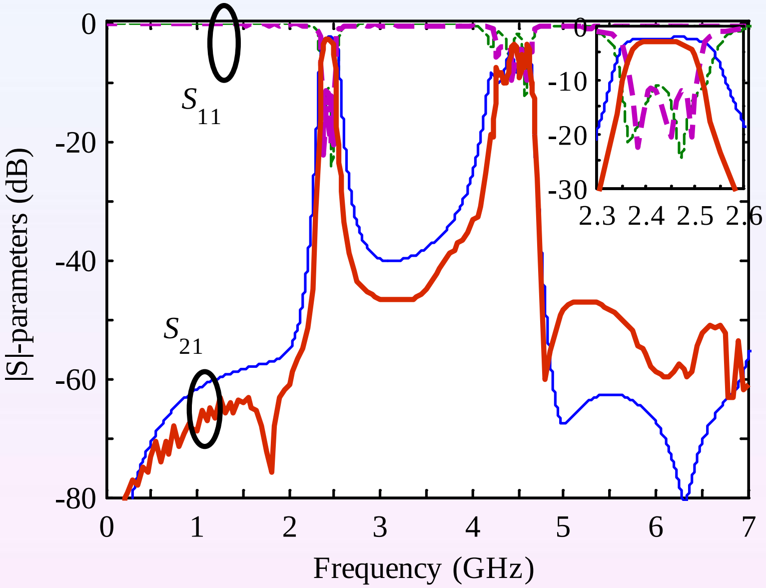
Practical Design Considerations

- Proper choice of resonators: implementation area, power consumption, required bandwidth (Q_{ul})
- Asymmetrical skirt selectivity.
- Higher harmonics.
- Fabrication tolerance.
- Packaging issues.





Measured and simulated response.

Different shapes of resonator.

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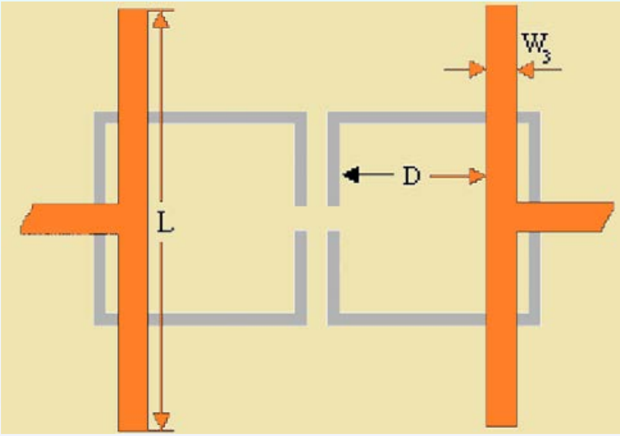
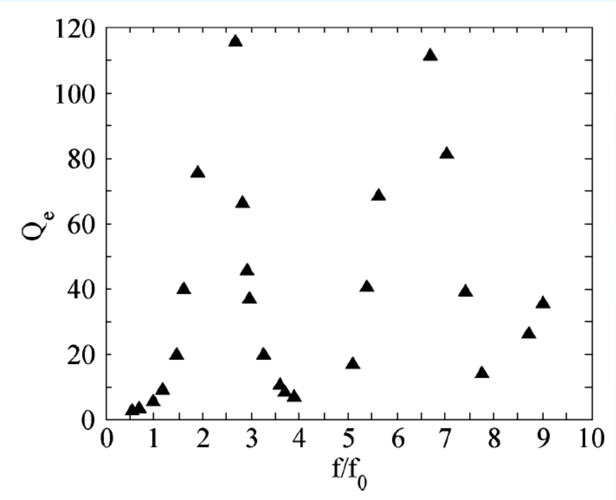
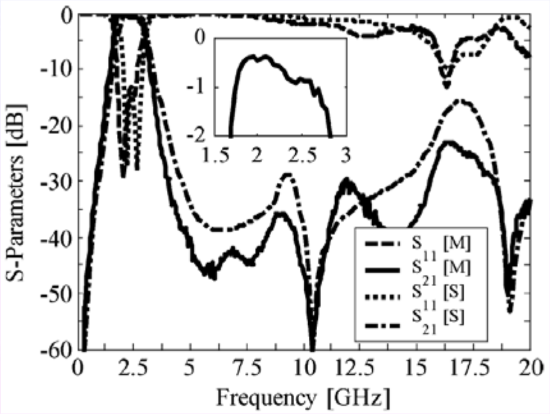
Frequency Dependent Qe



Configuration of the filter.



Tuning of Q_e



Filter response.

P. Mondal et. Al., IEEE MWCL, 2007.

Dual-Band BPF

Steps:

1. Determine the resonator dimensions so that the frequency ratio condition is satisfied.
2. Follow the conventional synthesis procedure.
3. To avoid the problem with matching Q_e at two frequencies, choose a proper coupling scheme.
4. An external dual-band matching network can be used to match the Q_e at two frequencies.

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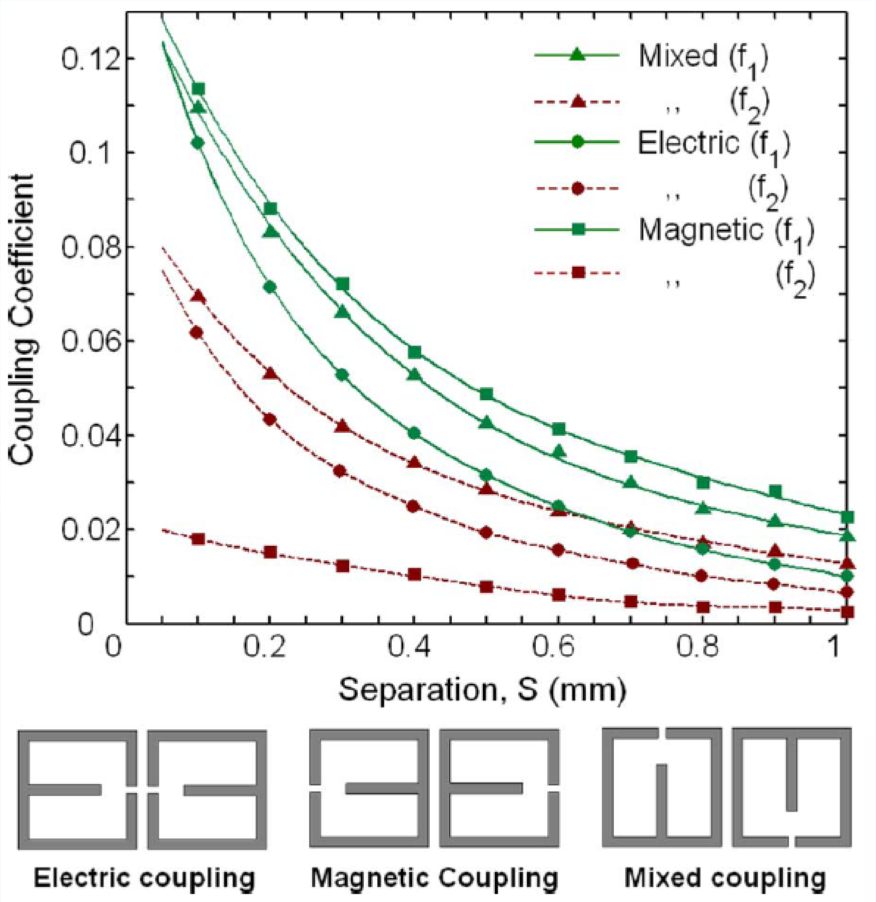
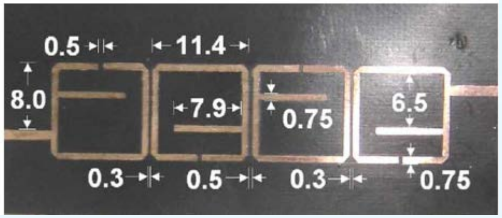
Dual-Band BPF



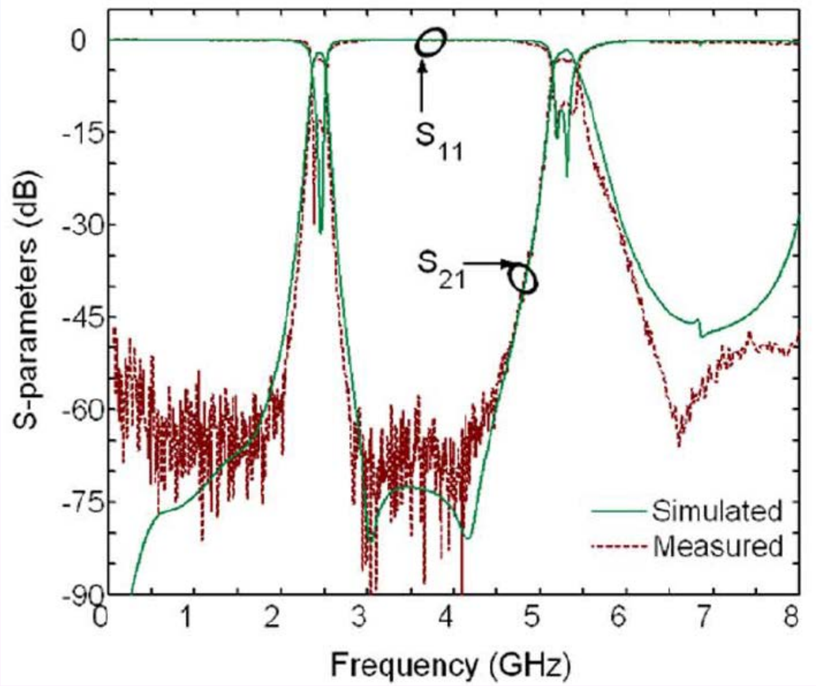
Extracted coupling coefficient plot, f_1 = 2.45 GHz, f_2 = 5.25 GHz.

P. Mondal and M. K. Mandal, IEEE T-MTT, Jan. 2008.

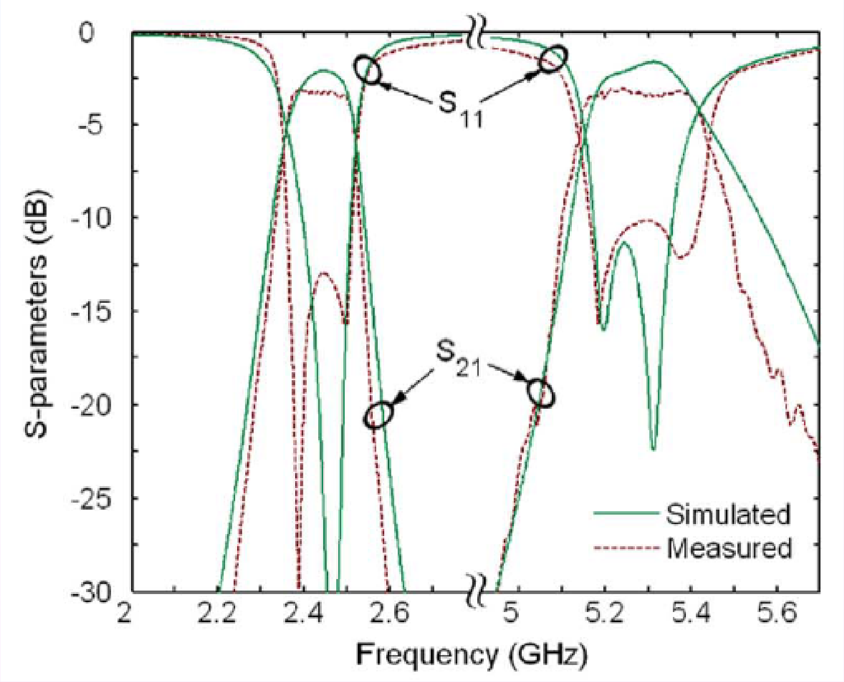
Dual-Band BPF



Photograph of the filter.



Wideband filter response



Narrowband filter response

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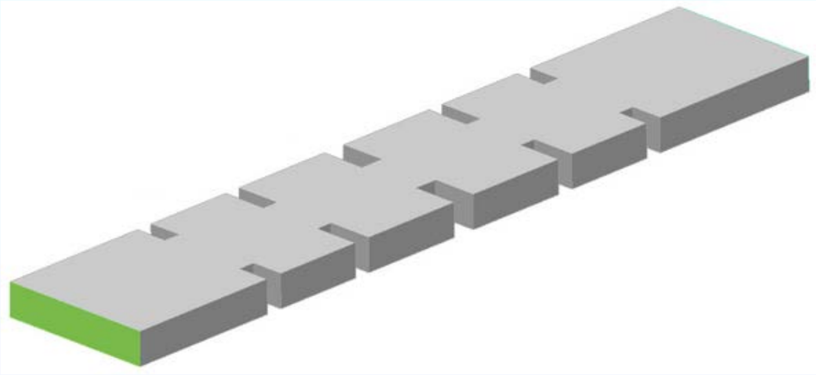
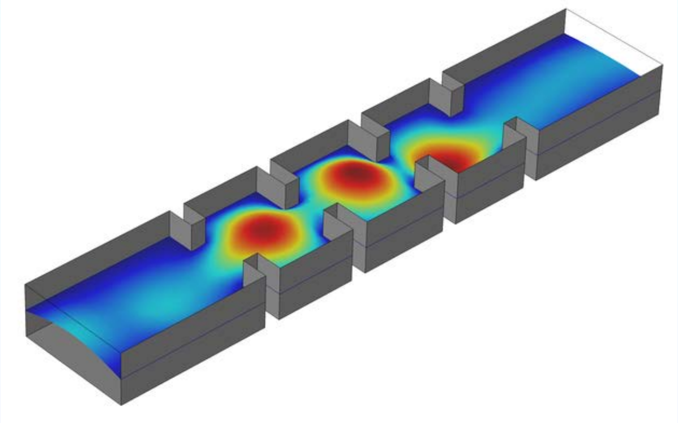
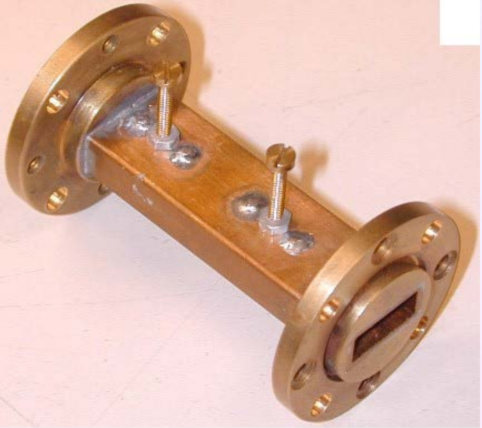
Waveguide Filters



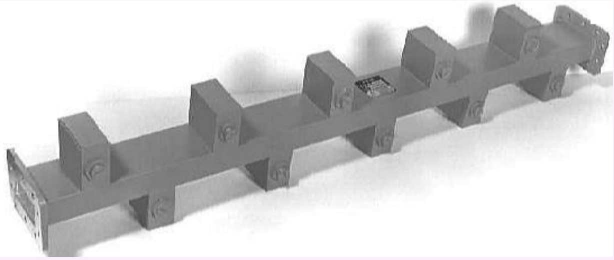
A waveguide bandpass filter



Time averaged electric field distributions.

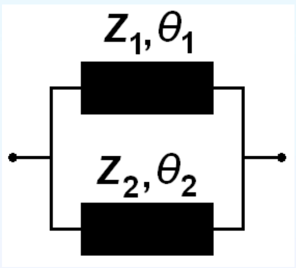


Bandpass filter with tuning adjustment.

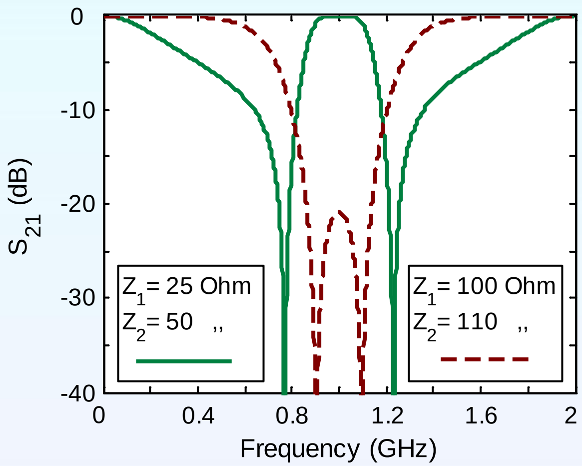


A bandstop filter.

Signal Interference Technique



$$\theta_1 = f\left(\frac{\theta_{10}}{f_0}\right), \quad \theta_2 = f\left(\frac{\theta_{20}}{f_0}\right)$$



Basic structure

Calculated S-parameters

$$S_{21} = \frac{-j2Z_0\left(Z_2 \csc\theta_1 + Z_1 \csc\theta_2\right)}{Z_1 Z_2 + Z_0{}^2\left\{\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1} + 2\left(\csc\theta_1 \csc\theta_2 - \cot\theta_1 \cot\theta_2\right)\right\} - j2Z_0\left(Z_2 \cot\theta_1 + Z_1 \cot\theta_2\right)} \quad (1)$$

•Condition of zeros:

$$\frac{Z_2}{Z_1} = -\frac{\sin(\theta_1)}{\sin(\theta_2)} \quad (2)$$

M. K. Mandal and S. Sanyal, IET Electronics Lett., Jan. 2007.

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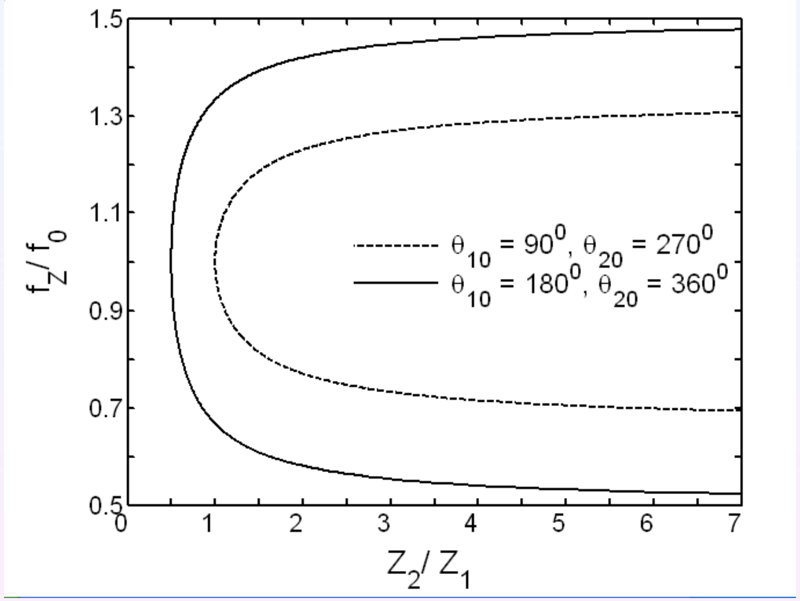
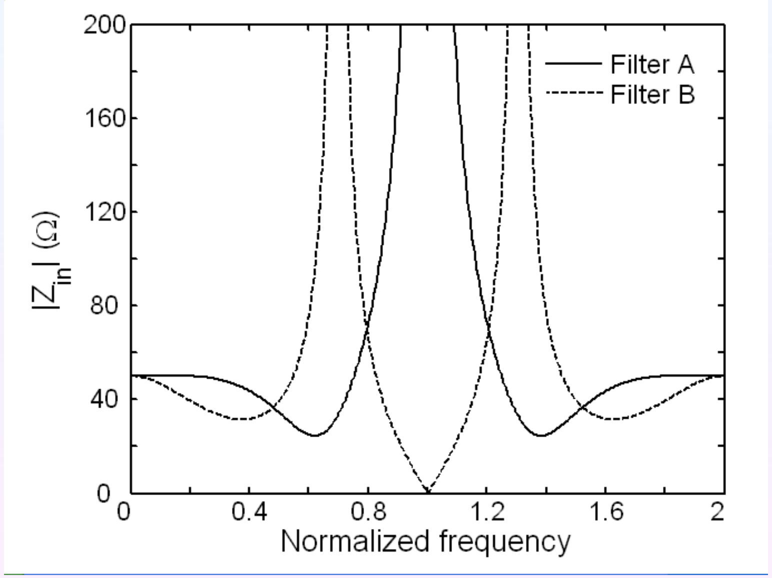
Signal Interference Technique

Transmission zero conditions:

(i) θ_1, θ_2 are $(2m - 1)\pi/2$ and $(2n + 1)\pi/2$, respectively, provided $\theta_1 \neq \theta_2$;
(ii) θ_1, θ_2 are $m\pi$ and $n\pi$ respectively provided $\theta_1 \neq \theta_2$.
Where m, n are any positive integer.



Dependence of the tx. zeros.



Input impedance plot.

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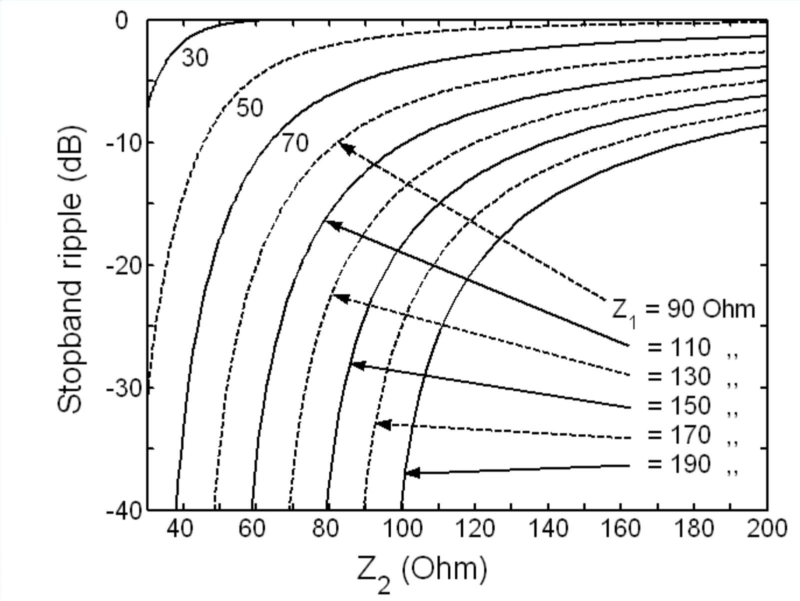
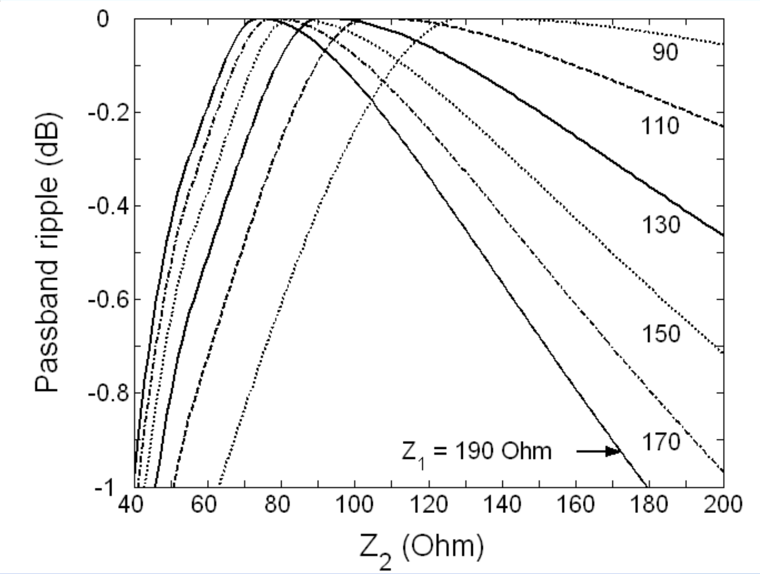
Signal Interference Technique: BSF

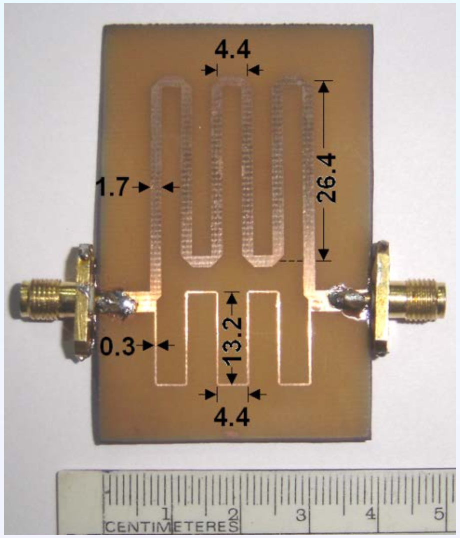


Dependence of the stopband ripple on the impedance values.

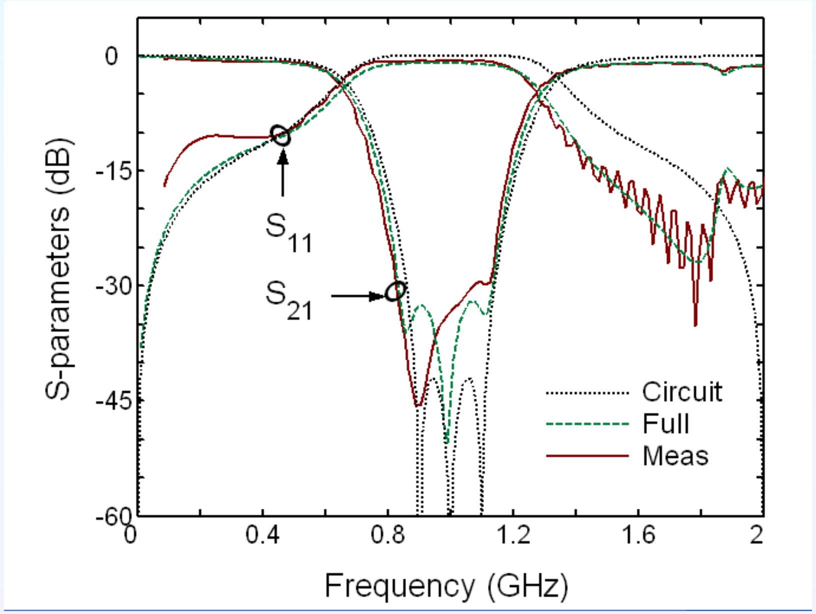


Dependence of the passband ripple on the impedance values.

BSF Example



Photograph of the filter.



Comparison of the S-parameters.

Filter specifications:
At least 30 dB attenuation over a FBW of 30% at 1 GHz.

Z_1 = 130 Ω, Z_2 =70 Ω, electrical lengths are θ_{10} = 180 deg, θ_{20} = 360 deg.

M. K. Mandal et. Al., IET MAP., Jun. 2008.

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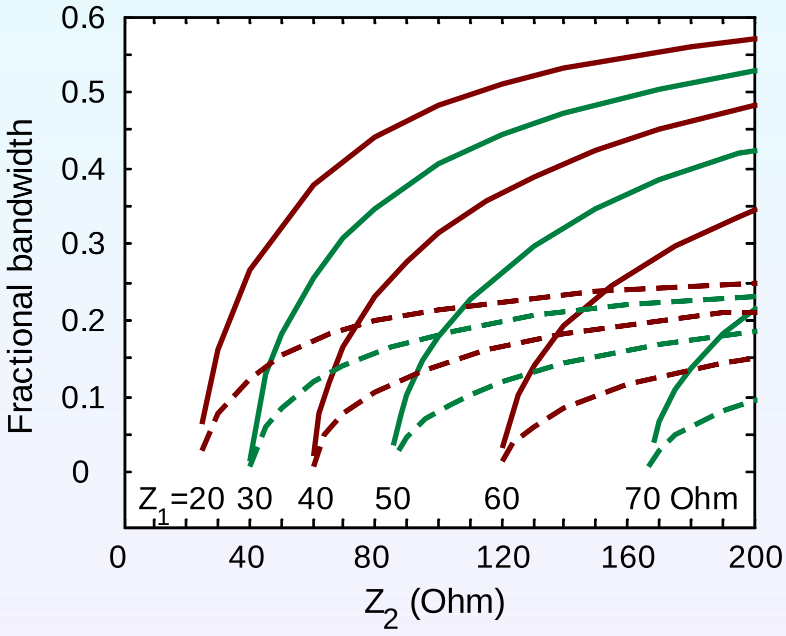
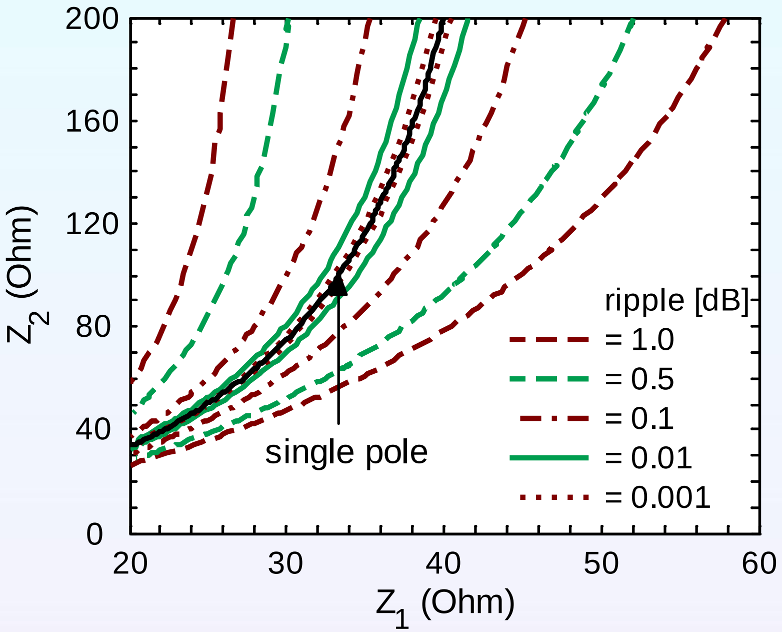
Signal Interference Technique: BPF

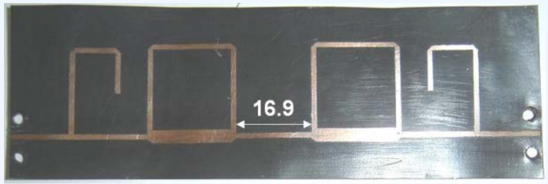


FBW variation with impedance. Solid lines: m = 1, n = 1 and dashed lines: m = 1, n = 3.

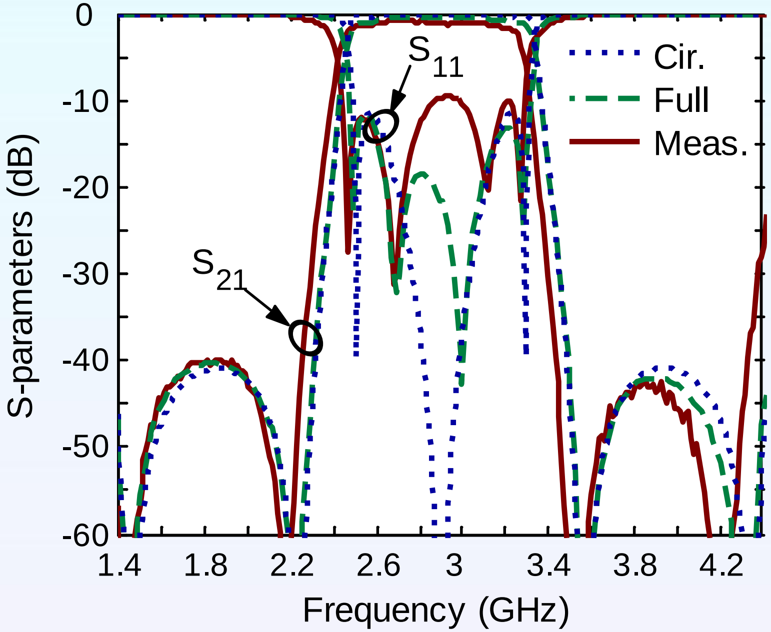


Impedance solutions for passband ripples, m = 1, n = 1.

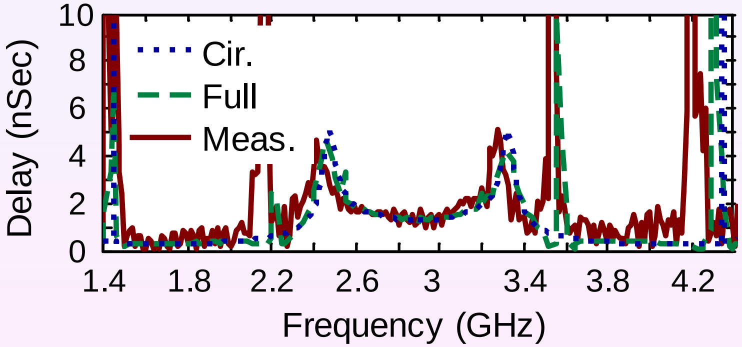
BPF Example



Photograph of the filter.



(*a*) S-parameters.



(*b*) group delay

M. K. Mandal et. Al., IET MAP., Jan. 2010.

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Some Other Components

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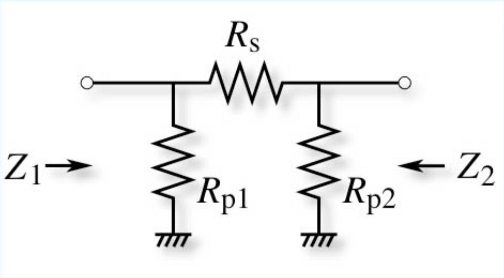
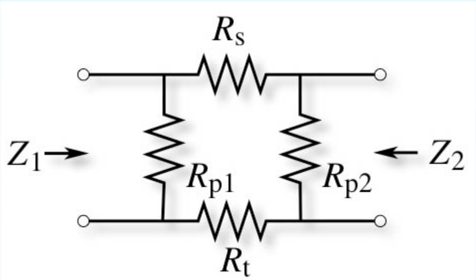
Attenuators

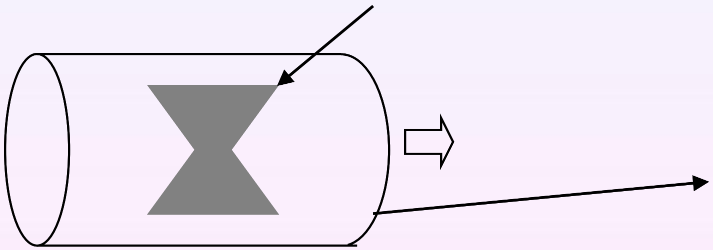


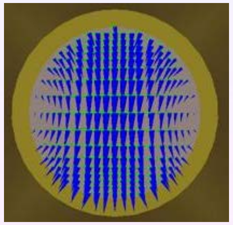
Unbalanced attenuator



Balanced attenuator

- Attenuates the signal without distorting the waveform and without reflection.
- Attenuation: $10\log\frac{P_{out}}{P_{in}}\ dB.$





TE_{11}

A rotary attenuator.

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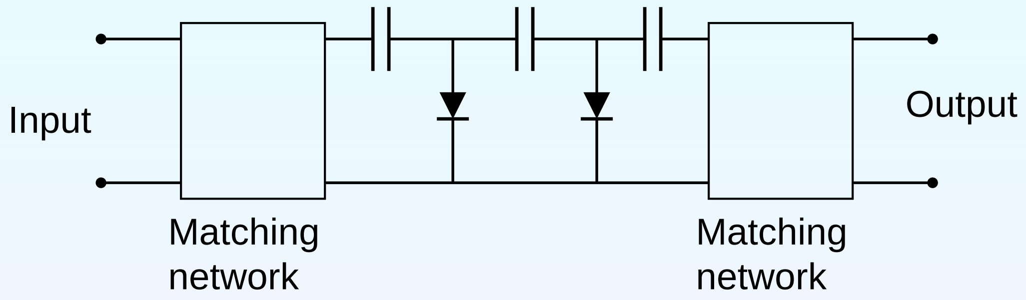
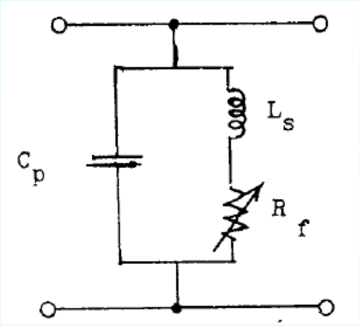
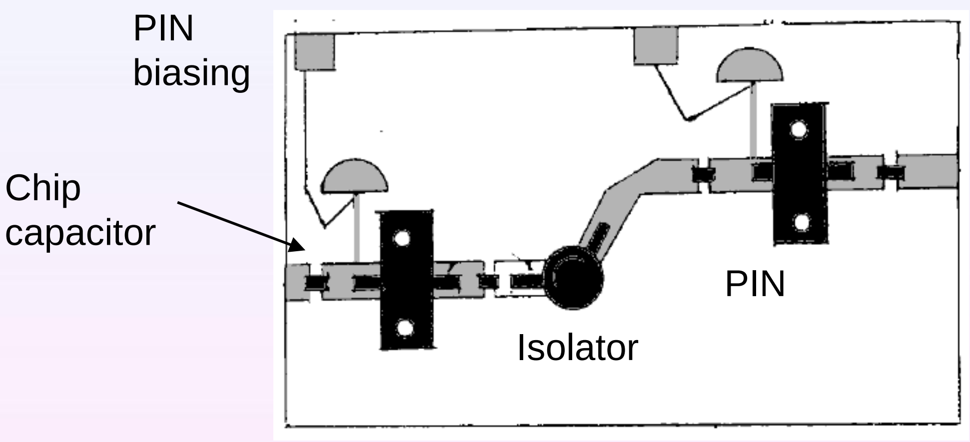
Attenuators



Block diagram of a microstrip attenuator.



Equivalent circuit of a PIN.



Microstrip layout.

- Isolator to reduce backward coupling between the coupling.

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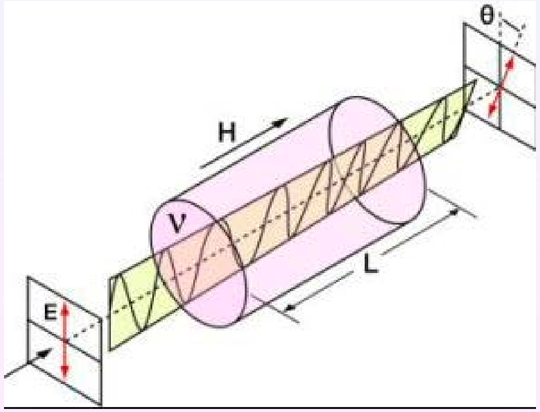
Isolators

A resonance absorption isolator.

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

- An isolator transmits wave in one direction.
- Three types: a) resonance absorption, b) field displacement, and c) using a circulator.



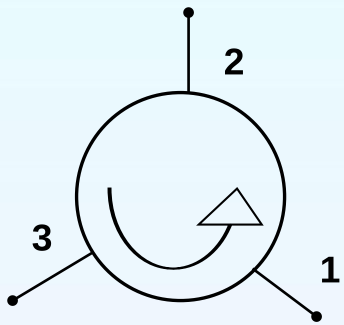
Effect of magnetic field.

- Angle of rotation:

$$\theta = \nu B d \qquad \nu \text{ is the Verdet constant.}$$

$$+\nu \rightarrow \text{anticlockwise.}$$

Circulators



Circulator block diagram.

$$S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

S-parameters.

A high power waveguide circulator.

- Circulator circulates signal using Farday rotation.

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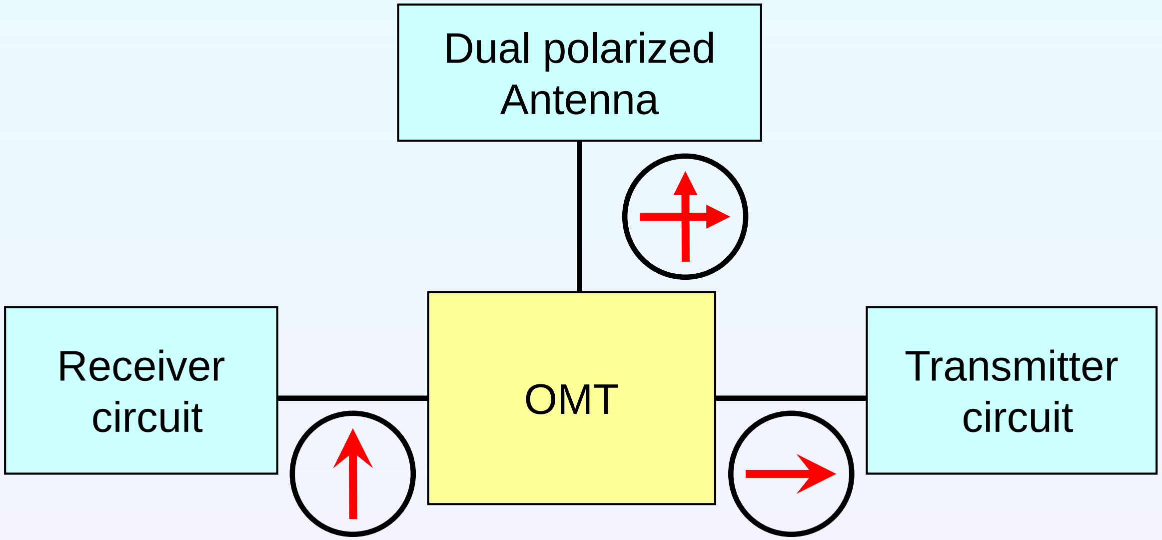
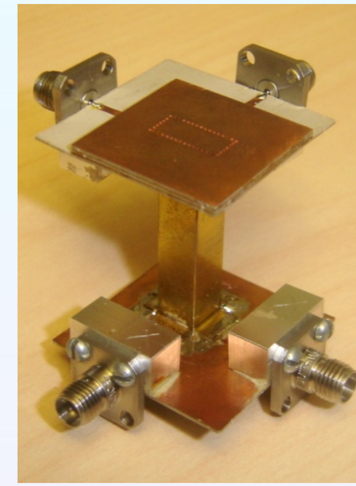
Ortho-Mode Transducer (OMT)





OMT

- Orthomode transducer (OMT) is a polarization duplexer.
- Applications in VSAT, radar antennas, radiometers, and communications links.

M.K. Mandal, K. Wu and D. Deslandes, A Compact Planar Orthomode Transducer, *IEEE MTTS – 2011*, Baltimore, USA.

RF and Microwave Engineering (EC 31005)

Noise and Non-linearities (P8)

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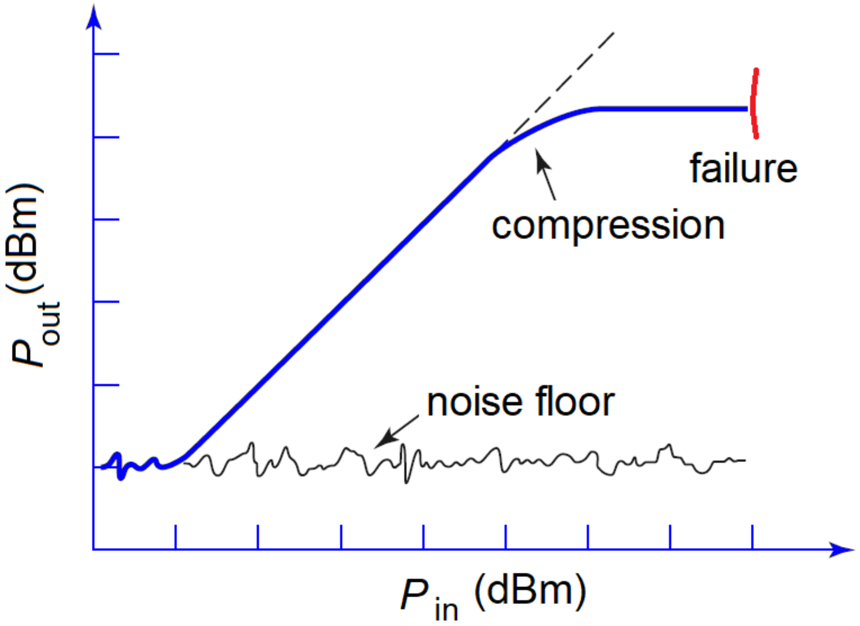
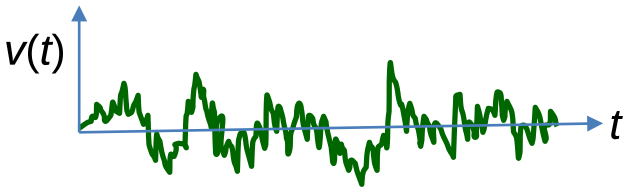
Effect of noise

- Noise power is a result of random processes such as the flow of charges or holes in a solid-state device, the thermal vibrations in any component at a temperature above absolute zero.
- Noise power cannot be completely eliminated. An amplifier amplifies both the signal and noise, in addition, will generate some noise.
- Noise level of a system sets the lower limit on the strength of a signal that can be detected.



Dynamic range of a realistic amplifier



Noise voltage generated by a resistor.

Different sources of noise

Internal noises:

- **Thermal noise** is the most basic type of noise, being caused by thermal vibration of bound charges. Other names Johnson, Nyquist noise.
- **Shot noise** is due to random fluctuations of charge carriers in an electron tube or solid-state device.
- **Flicker noise** occurs in solid-state components and vacuum tubes. Flicker noise power varies inversely with frequency, and so is often called 1/ f -noise.
- **Plasma noise** is caused by random motion of charges in an ionized gas, such as a plasma, the ionosphere, or sparking electrical contacts.
- **Quantum noise** results from the quantized nature of charge carriers and photons; it is often insignificant relative to other noise sources.

External noises:

- Thermal noise from the ground
- Cosmic background noise from the sky
- Noise from stars (including the sun)
- Lightning
- Gas discharge lamps
- Radio, TV, and cellular stations
- Wireless devices
- Microwave ovens
- Deliberate jamming devices

Noise power and equivalent noise temperature

- Kinetic energy of the electrons in a resistor at a physical temperature T (K) are proportional to T.
- These random motions produce small, random voltage fluctuations at the resistor terminals.
- The voltage has a zero average value but a nonzero *rms* value.

Noise power (Planck's blackbody radiation law),

$$N(f) = 4kTR\frac{hf}{kT}\Big/\left[\exp\left(\frac{hf}{kT}\right)-1\right].$$

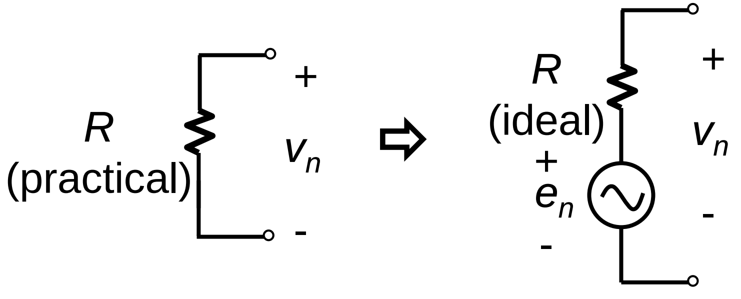
Considering, $\frac{hf}{kT} << 1.$ (Rayleigh-Jeans approximation)

$$N(f) = 4kTR \qquad \text{W/Hz.}$$

Noise voltage (rms), $v_n(f) = \sqrt{4kTR} \qquad \text{V/Hz.}$

Power delivered to a match load, $N_a(f) = kT \qquad \text{W/Hz.}$

where
h = 6.626 × 10^{-34} J-sec is Planck's constant.
k = 1.380 × 10^{-23} J/K is Boltzmann's constant.
T = the temperature in degrees kelvin (K).
B = the bandwidth of the system in Hz.
f = the center frequency of the bandwidth in Hz.
R = the resistance in Ohm.



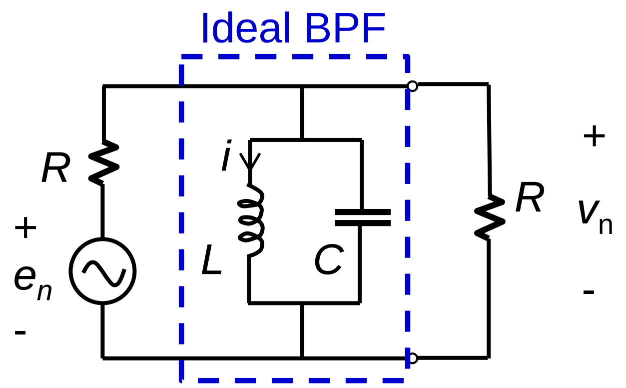
Representation of a resistor.

Noise power delivered to a practical system

- For a practical system, a load resistor *R* results in maximum power transfer from the noisy resistor.
- The maximum power delivered to the load in a bandwidth *B* is

$$P_n = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R} = kTB$$

Maximum available noise power is independent of resistor.



A noisy resistor transferring maximum noise power to a load.

- Systems with smaller bandwidths collect less noise power.
- Cooler devices and components generate less noise power.
- As $B \rightarrow \infty$, $P_n \rightarrow \infty$. This is the so-called ultraviolet catastrophe, which does not occur in reality because above equation is not valid when $f \rightarrow \infty$.

Considering ideal scenario, theoretical thermal noise floor at room temperature (290K) is

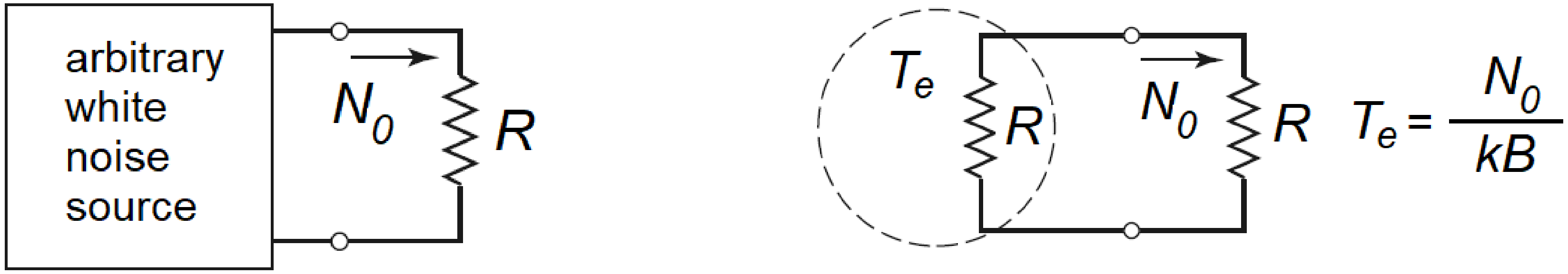
$$k T\overline{B} = 4.005 \times 10^{-21}\ \mathrm{W} \quad = -174\,\mathrm{dBm/\ Hz}$$

Noise temperature

- If an arbitrary source of noise (thermal or nonthermal) is “white,” the noise power is not a strong function of frequency.
- It can be modelled as an equivalent thermal noise source, and characterized with an equivalent noise temperature T_e.
- Thus, a noisy source (white noise) can be replaced by a noisy resistor of value R at temperature T_e, where

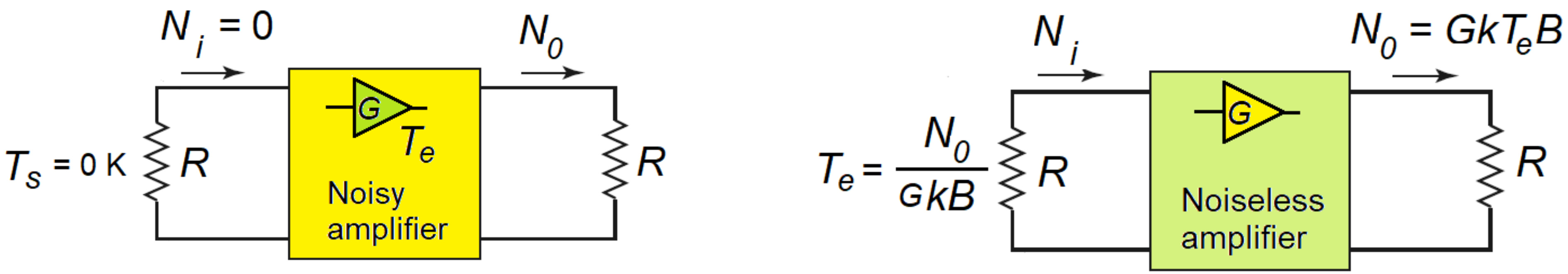
$$T_e = N_0(f)/kB.$$

N_0 is the maximum power delivered to load R.



Equivalent noise temperature for an arbitrary white noise source.

Noisy amplifier



Representation of a noisy amplifier.

- Let a noisy amplifier with a bandwidth B and gain G be matched to noiseless source and load resistors, at a temperature of T_s = 0 K.
- Then, the output noise power N_o will be only due to the noise generated by the amplifier itself.
- It is modelled by an equivalent load noise power by driving an ideal noiseless amplifier with a resistor at the temperature, $T_e = N_o/GkB$ so that the output power in both cases is $N_o = GkT_eB$.
- Then, T_e is the equivalent noise temperature of the amplifier.

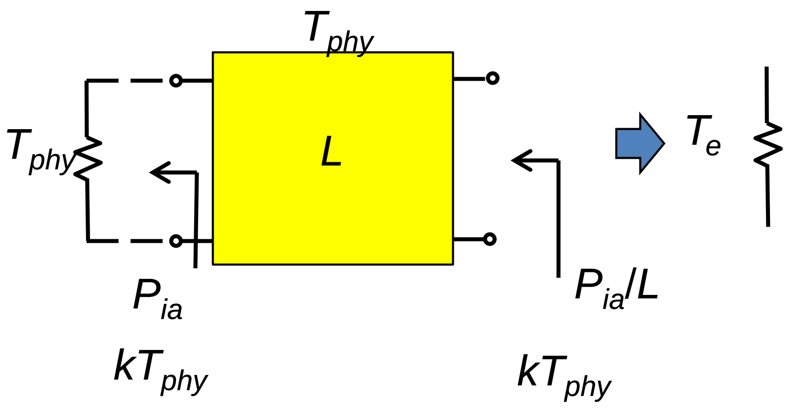
Passive devices

Circuit generated internal noise,

$$N_{int} = kT_{phy}\left(1 - 1/L\right), \quad L \text{ is the loss (ratio>1).}$$

Let this noise power is generated by a resistor connected at the source. Then the equivalent noise temperature,

$$\boxed{T_e = LN_{\text{int}}/k = T_{phy}\left(L-1\right).} \quad \text{per Hz}$$



Available power for a noisy system
(T_{phy} is the physical temp. and not equivalent noise temp.).

Example:

Let a 3dB (L = 1.995) matched attenuator at 290 K is connected to a RF source.

Then, T_e of the attenuator is 288.5 K.

Calibrated noise source

- A resistor is placed at a constant known temperature (oven or cryogenic flask).
- Active noise sources may use a diode, transistor, or tube to provide a calibrated noise power output.
- Noise power from such generator is defined as the excess noise ratio (ENR),

$$\text{ENR (dB)} = 10\log\frac{N_g - N_o}{N_o} = 10\log\frac{T_g - T_0}{T_0}$$

where N_g and T_g are the noise power and equivalent noise temperature of the generator, and N_o and T_0 are the noise power and temperature associated with a matched load at room-temperature (T_0 = 290 K).

Y-factor method:

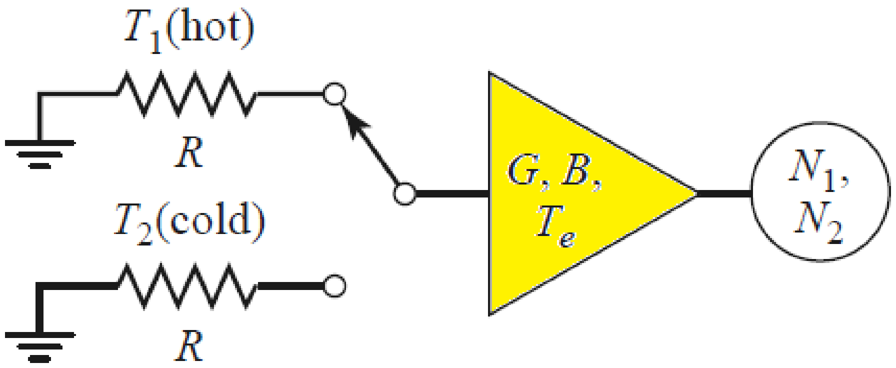
- The noise powers at the output are

$$N_1 = GkT_1B + GkT_eB,$$

$$N_2 = GkT_2B + GkT_eB,$$

- Define the Y-factor as $Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1,$

Then, $T_e = \frac{T_1 - YT_2}{Y - 1}$



Measurement set up.

Noise temperature

Q1. An X-band amplifier has a gain of 20 dB and a BW of 1 GHz. The measured noise powers are N_1 = −62.0 dBm for T_1 = 290 K and N_2 = −64.7 dBm for T_1 = 77 K. Find its equivalent noise temperature by the Y -factor method.

Answer: Y = (N_1 − N_2) dB = 2.7 dB or Y = 1.86

Therefore, T_e = 170 K.

Q2. If a source with an equivalent noise temperature of T_s = 450 K drives the above amplifier, calculate the total noise power out of the amplifier.

Answer: The noise power into the amplifier is $kT_s B$.

Then, the total noise power out of the amplifier is

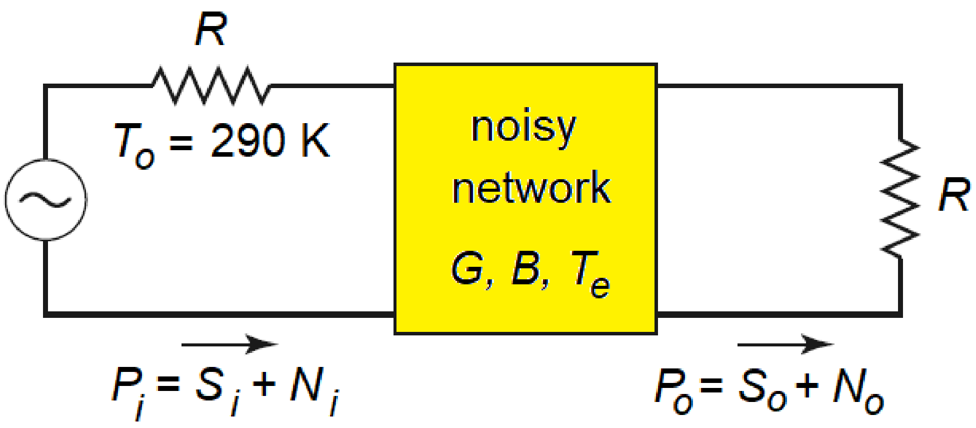
$$N_o = GkT_sB + GkT_eB = 100(1.38 \times 10^{-23})(10^9)(450 + 170)$$
$$= 8.56 \times 10^{-10}\ \text{W} = -60.7\ \text{dBm}.$$

Noise factor and noise figure

- Noise figure of a component is a measure of the degradation in the signal-to-noise ratio (SNR) between the input and output of the component.
- The SNR is dependent on the signal power.
- Noise factor is defined as

$$F = \frac{SNR_i}{SNR_o} \geq 1$$

Noise figure, NF = 10 $\log_{10}$ (F) dB, a positive number.



Determination of noise figure

- By definition, the input noise power is assumed to be the noise power resulting from a matched resistor at T_0 = 290 K; i.e. $N_i = kT_0B = 4.005 \times 10^{-21} \times B$ W.
- Considering the above figure,

$$N_i = kT_0B \quad \text{and} \quad N_o = kGB(T_0 + T_e)\,.$$

$$\therefore F = \frac{S_i}{kT_0B}\frac{kGB(T_0 + T_e)}{GS_i} = 1 + \frac{T_e}{T_0}.$$

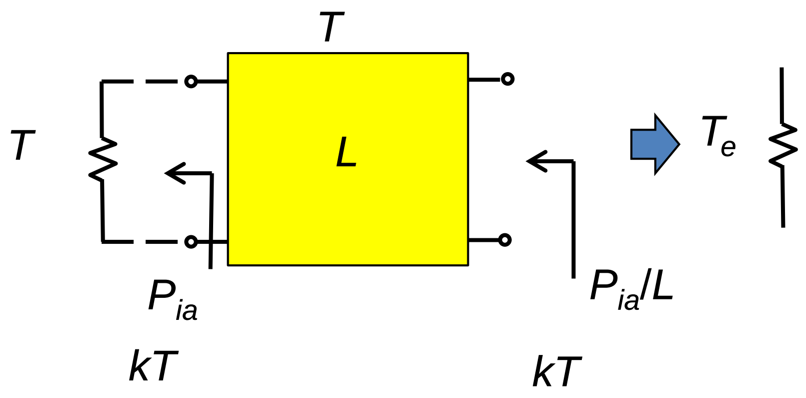
Noise figure and equivalent noise temperatures are interchangeable characterizations of the noise properties of a component.

Noise figure of a lossy network

- The power gain, G, of a lossy network is less than unity.

  → the loss factor, $L = 1/G > 1$.
- Considering the entire system is in thermal equilibrium at T, and has a driving point impedance of R,

  → $N_o = kTB$.
- The output noise power is from the source resistor (attenuated by the lossy line), and from the noise generated by the network itself. Thus,



A lossy network.

$$N_o = kTB = GkTB + GN_{\text{added}},$$ and

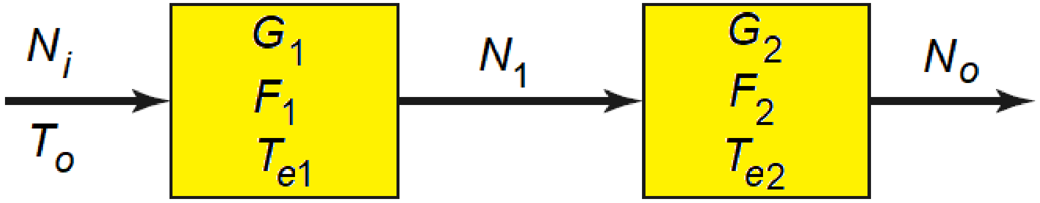
$$N_{\text{added}} = \frac{1-G}{G}kTB = (L-1)kTB.$$

Now, from previous slide, $T_e = \dfrac{1-G}{G}T = T(L-1).$

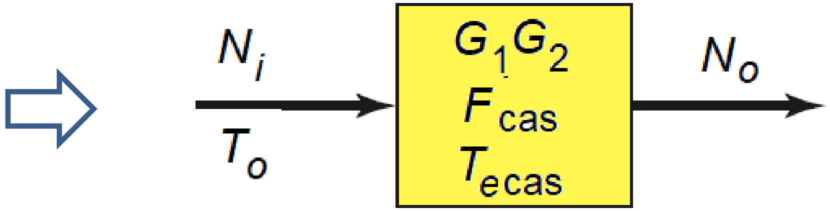
Therefore, $F = 1 + (L-1)\dfrac{T}{T_0}.$

If the network is at temperature T_0, then $F = L$. Thus, a 6 dB attenuator at room temperature has a noise figure of NF = 6 dB.

Noise figure of a cascaded system



Two cascade networks



Equivalent network.

The noise power at the output of the first stage is $N_1 = G_1 k T_0 B + G_1 k T_{e1} B$

The noise power at the output of the second stage is $N_o = G_2 N_1 + G_2 k T_{e2} B$

$$= G_1 G_2 k B \left(T_0 + T_{e1} + \frac{1}{G_1} T_{e2} \right).$$

For the equivalent system, $N_o = G_1 G_2 k B (T_{\text{cas}} + T_0)$

Therefore, comparing with the previous equation, noise temperature of the cascade system is

$$T_{\text{cas}} = T_{e1} + \frac{1}{G_1} T_{e2}.$$

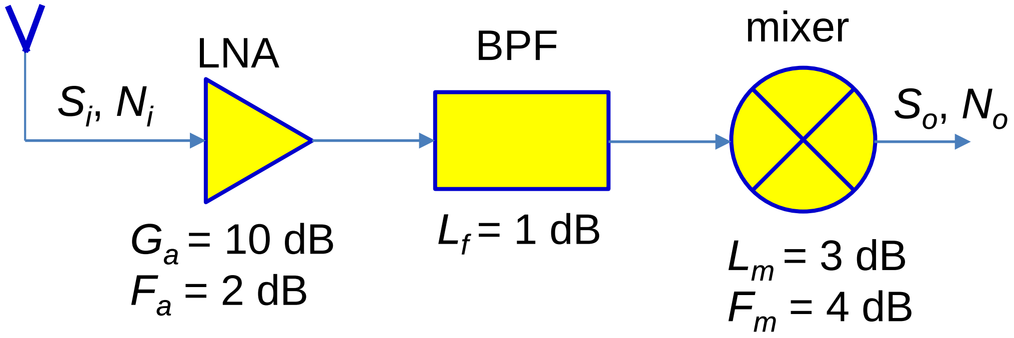
Therefore, NF of the cascaded system is $F_{\text{cas}} = F_1 + \dfrac{1}{G_1}(F_2 - 1).$

Noise figure of a cascaded system

- For arbitrary number of stages,

$$F_{\text{cas}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots. \quad \text{and} \quad T_{\text{cas}} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots,$$

Q. Compute the output noise power of this subsystem. Given that the antenna is at a temperature of T_A = 150 K, system temperature is T_0 and IF bandwidth is 10 MHz. If we require a minimum signal-to-noise ratio (SNR) of 20 dB at the output of the receiver, what is the minimum signal voltage that should be applied at the receiver input? Assume the system is matched to 50 Ω.



A receiver front-end.

Answer:

$$G_a = 10 \text{ dB} = 10 \qquad G_f = -1.0 \text{ dB} = 0.79 \qquad G_m = -3.0 \text{ dB} = 0.5$$ (All G's are power gain)

$$F_a = 2 \text{ dB} = 1.58 \qquad F_f = 1 \text{ dB} = 1.26 \qquad F_m = 4 \text{ dB} = 2.51$$

$$\rightarrow NF = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f} = 1.80 = 2.55 \text{ dB}.$$ (overall noise figure of the system)

Noise calculation

Then, equivalent noise temperature of the system is

$$T_e = (F - 1)T_0 = (1.80 - 1)(290) = 232 \text{ K}.$$

Overall gain of the system is $G = (10)(0.79)(0.5) = 3.95.$

Then, the output noise power is

$$\begin{aligned} N_o &= k(T_A + T_e)BG = (1.38 \times 10^{-23})(150 + 232)(10 \times 10^6)(3.95) \\ &= 2.08 \times 10^{-13} \text{ W} = -96.8 \text{ dBm}. \end{aligned}$$

For an output SNR of 20 dB = 100, the input signal power must be

$$S_i = \frac{S_o}{G} = \frac{S_o}{N_o}\frac{N_o}{G} = 100\frac{2.08 \times 10^{-13}}{3.95} = 5.27 \times 10^{-12} \text{ W} = -82.8 \text{ dBm}.$$

For a 50 Ω system impedance, this corresponds to an input signal voltage of

$$V_i = \sqrt{Z_o S_i} = \sqrt{(50)(5.27 \times 10^{-12})} = 1.62 \times 10^{-5} \text{ V} = 16.2 \ \mu\text{V (rms)}.$$

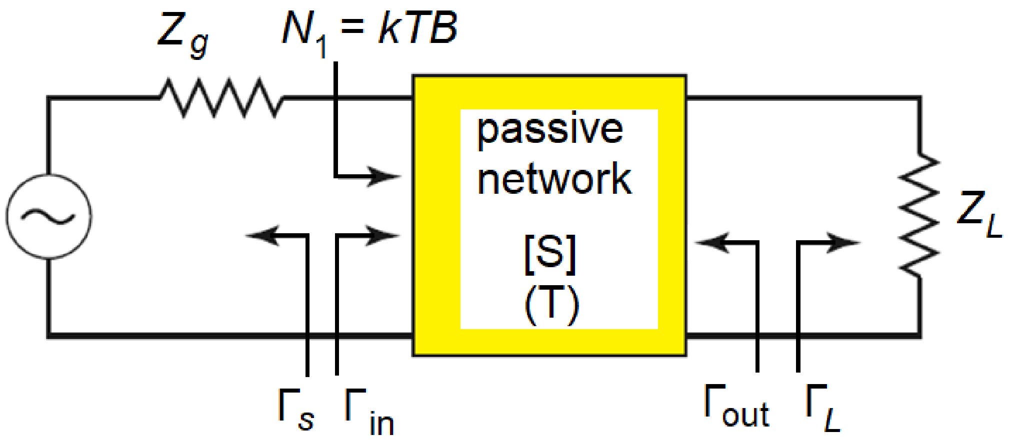
Noise figure of passive two port network

- Consider a case when the network is not matched.
- Let the available noise power at the input is $N_1 = kTB$.

Then, available noise power at the output port is

$$N_2 = G_{21}kTB + G_{21}N_{\text{added}},$$

where N_{added} is added by the network (referenced to port 1), and G_{21} is the available power gain of the network from port 1 to port 2.



A passive two port network with impedance mismatch.

The available power gain of the network from port 1 to port 2 in terms of the S-parameter is

$$G_{21} = \frac{\text{power available from network}}{\text{power available from source}} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)}{|1 - S_{11}\Gamma_S|^2(1 - |\Gamma_{\text{out}}|^2)}.$$

where the output port mismatch (derived in the network analysis) is $\Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}.$

The available gain of the network does not depend on Γ_L. Because available gain is defined in terms of the maximum power that is available from the network (conjugate matching).

Noise figure of passive two port network

- Since the system is in thermal equilibrium, both the input and output noise powers should be *kTB*.

$$N_2 = G_{21}kTB + G_{21}N_{\text{added}}, \quad \rightarrow \quad N_{\text{added}} = \frac{1 - G_{21}}{G_{21}}kTB.$$

- Then, the equivalent noise temperature of the network is $T_e = \frac{N_{\text{added}}}{kB} = \frac{1 - G_{21}}{G_{21}}T,$

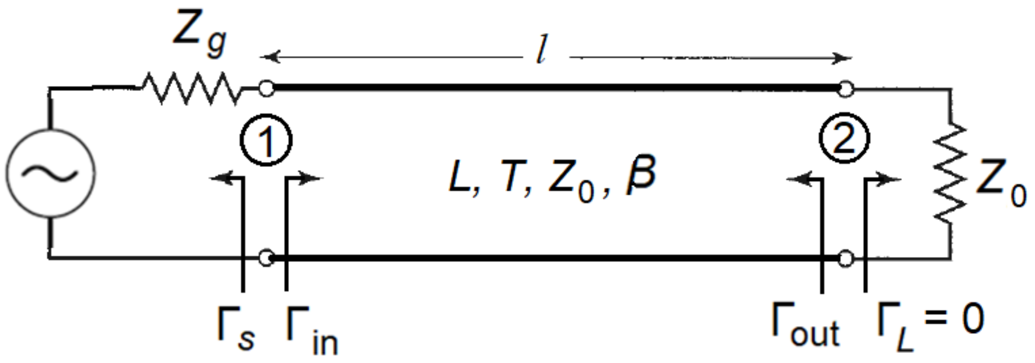
and the noise figure of the network is $F = 1 + \frac{T_e}{T_0} = 1 + \frac{1 - G_{21}}{G_{21}}\frac{T}{T_0}.$

Noise figure of a mismatched loosy line:

Reflection coefficient looking toward the generator is

$$\Gamma_s = \frac{Z_g - Z_0}{Z_g + Z_0} \neq 0.$$

The scattering matrix of the lossy line is $[S] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{e^{-j\beta\ell}}{\sqrt{L}},$

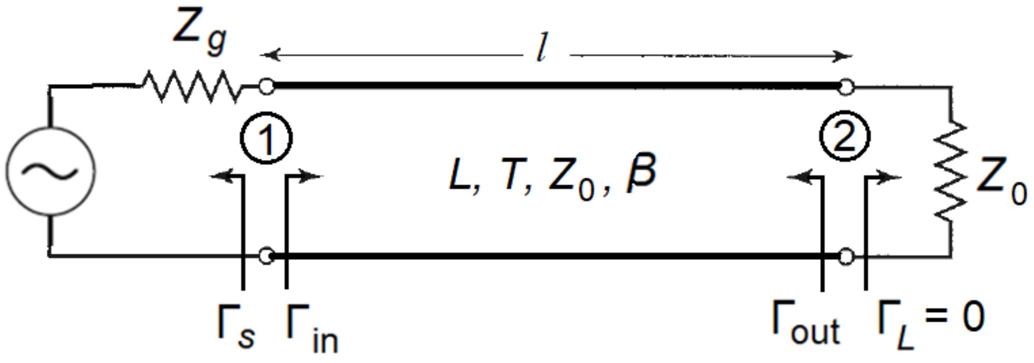


A mismatched lossy transmission line at temperature T.

Noise figure of a mismatched loosy line

Then, using S-parameters

$$\rightarrow \Gamma_{\text{out}} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = \frac{\Gamma_s}{L}e^{-2j\beta\ell}.$$



A lossy transmission line matched only at its output.

Therefore, the available gain is

$$G_{21} = \frac{\frac{1}{L}(1 - |\Gamma_s|^2)}{1 - |\Gamma_{\text{out}}|^2} = \frac{L(1 - |\Gamma_s|^2)}{L^2 - |\Gamma_s|^2}.$$

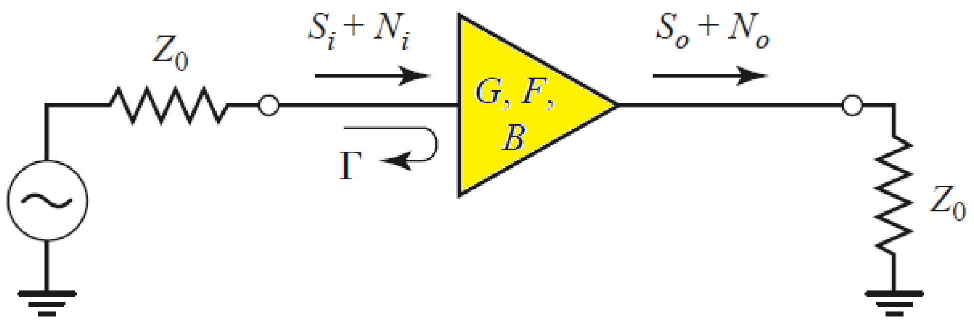
Therefore, the equivalent noise temperature is $T_e = \dfrac{1 - G_{21}}{G_{21}}T = \dfrac{(L-1)(L + |\Gamma_s|^2)}{L(1 - |\Gamma_s|^2)}T.$

Q. Find the noise figure of a Wilkinson power divider.

Answer: $$F = 1 + \frac{T_e}{T_0} = 1 + (2L - 1)\frac{T}{T_0}.$$

Noise figure of a mismatched amplifier

- Consider an amplifier is matched only at the output port.
- The relationship for passive network cannot be directly applied here.
- Better to use noise temperature approach.



A mismatched amplifier.

Let the input noise power to the amplifier be $N_i = kT_0B$. Then the output noise power from the amplifier (referenced to the input) is

$$N_o = kT_0GB\left(1 - |\Gamma|^2\right) + kT_0\left(F - 1\right)GB$$

Now, for an input signal power S_i, the output signal power is $S_o = G\left(1 - |\Gamma|^2\right)S_i$.

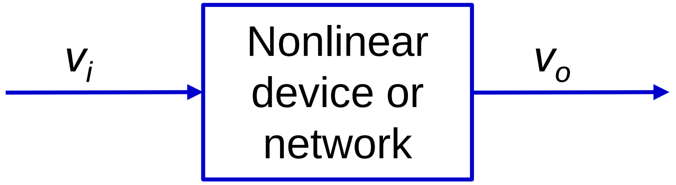
Then, the overall noise figure is $$F_m = \frac{S_i N_o}{S_o N_i} = 1 + \frac{F - 1}{1 - |\Gamma|^2}.$$

The minimum noise figure that can be achieved is in matched condition and is F.

Nonlinear distortion

The effect of nonlinear distortion are

- Harmonic generation (multiples of a fundamental signal)
- Saturation (gain reduction in an amplifier)
- Intermodulation distortion (products of a two-tone input signal)
- Cross-modulation (modulation transfer from one signal to another)
- AM-PM conversion (amplitude variation causes phase shift)
- Spectral regrowth (intermodulation with many closely spaced signals)



A nonlinear device or network.

The output signal is represented by a Taylor series as $v_o = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots,$

where

$a_0 = v_o(0)$ (DC output) →rectification

$a_1 = \left.\dfrac{dv_o}{dv_i}\right|_{v_i=0}$ (linear output) →attenuation or amplification

$a_2 = \left.\dfrac{d^2v_o}{dv_i^2}\right|_{v_i=0}$ (squared output) →mixing or frequency translation

Gain compression

Let the input is a continuous wave as $v_i = V_0 \cos \omega_0 t.$

Then, the output of a nonlinear device is

$$v_o = a_0 + a_1 V_0 \cos \omega_0 t + a_2 V_0^2 \cos^2 \omega_0 t + a_3 V_0^3 \cos^3 \omega_0 t + \cdots$$
$$= \left(a_0 + \frac{1}{2} a_2 V_0^2\right) + \left(a_1 V_0 + \frac{3}{4} a_3 V_0^3\right) \cos \omega_0 t + \frac{1}{2} a_2 V_0^2 \cos 2\omega_0 t + \frac{1}{4} a_3 V_0^3 \cos 3\omega_0 t + \cdots .$$

Voltage gain at a frequency ω_0 is

$$G_v = \frac{v_o^{(\omega_0)}}{v_i^{(\omega_0)}} = \frac{a_1 V_0 + \frac{3}{4} a_3 V_0^3}{V_0} = a_1 + \frac{3}{4} a_3 V_0^2,$$ (considering till third term)

In most of the amplifier, a_1 and a_3 have opposite sign →gain decreases with large voltage.

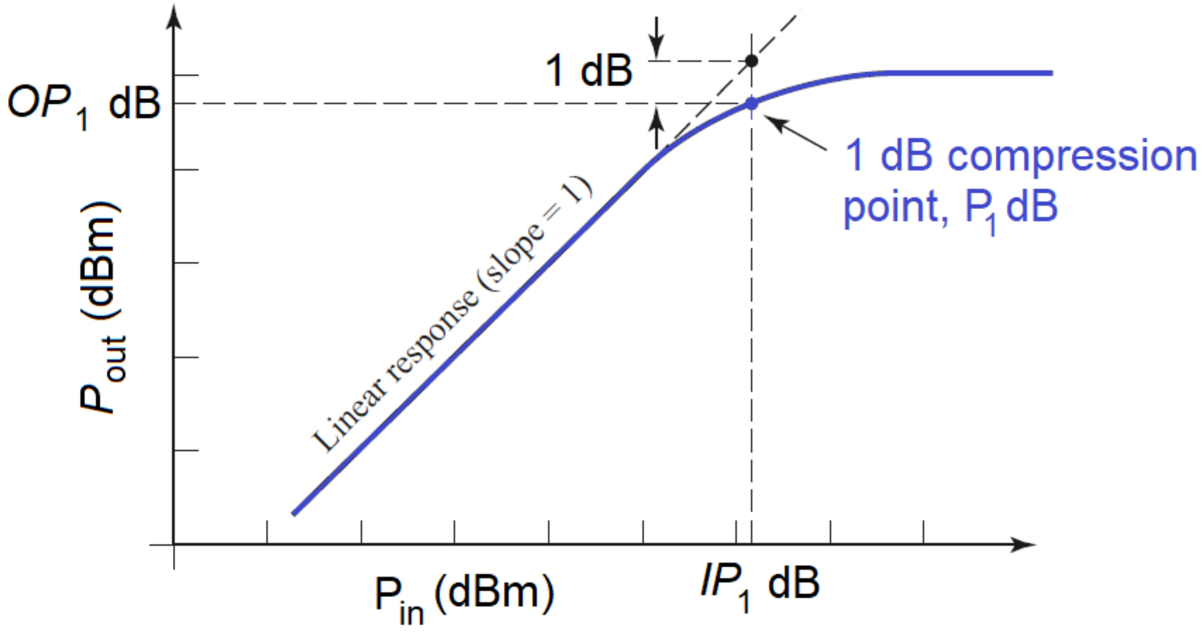
→ Gain compression or saturation.

→ Physically, it is limited by the bias voltage and nonlinearity of the active device.

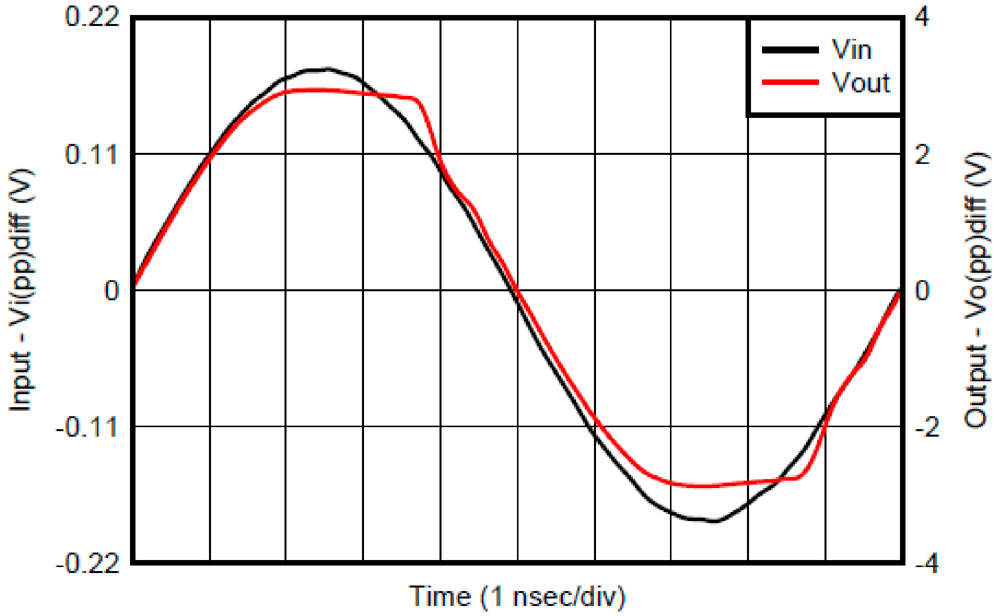
Gain compression

- The upper limit is defined by the 1 dB compression point.
- Higher one between IP_1, OP_1 and is usually specified for a device.
- The relation between a compression point referenced at the input versus the output is given as.

$$OP_{1\,\mathrm{dB}} = IP_{1\,\mathrm{dB}} + G - 1\ \mathrm{dB}$$



Response of a practical amplifier



Measured output voltage of a RF amplifier at the 1 dB compression point.

Harmonic and intermodulation distortion

- For a single input tone, ω_0, the output consists of harmonics of the input frequency, $n\omega_0$, for n = 0, 1, 2, ...
- The situation is complicated when the input signal consists of two closely spaced frequencies.
- Consider a two-tone input voltage, consisting of two closely spaced frequencies ω_1 and ω_2,

$$v_i = V_0(\cos\,\omega_1 t + \cos\,\omega_2 t).$$

Then, the output voltage is

$$\begin{aligned}
v_o &= a_0 + a_1 V_0(\cos\omega_1 t + \cos\omega_2 t) + a_2 V_0^2(\cos\omega_1 t + \cos\omega_2 t)^2 + a_3 V_0^3(\cos\omega_1 t + \cos\omega_2 t)^3 + \cdots \\
&= a_0 + a_1 V_0 \cos\omega_1 t + a_1 V_0 \cos\omega_2 t + \frac{1}{2}a_2 V_0^2(1 + \cos 2\omega_1 t) + \frac{1}{2}a_2 V_0^2(1 + \cos 2\omega_2 t) + a_2 V_0^2 \cos(\omega_1 - \omega_2)t \\
&\quad + a_2 V_0^2 \cos(\omega_1 + \omega_2)t + a_3 V_0^3\left(\frac{3}{4}\cos\omega_1 t + \frac{1}{4}\cos 3\omega_1 t\right) + a_3 V_0^3\left(\frac{3}{4}\cos\omega_2 t + \frac{1}{4}\cos 3\omega_2 t\right) \\
&\quad + a_3 V_0^3\left[\frac{3}{2}\cos\omega_2 t + \frac{3}{4}\cos(2\omega_1 - \omega_2)t + \frac{3}{4}\cos(2\omega_1 + \omega_2)t\right] \\
&\quad + a_3 V_0^3\left[\frac{3}{2}\cos\omega_1 t + \frac{3}{4}\cos(2\omega_2 - \omega_1)t + \frac{3}{4}\cos(2\omega_2 + \omega_1)t\right] + \cdots
\end{aligned}$$

Harmonic and intermodulation distortion

The output contains term in the form of $m\omega_1 + n\omega_2$ with $m, n = 0, \pm 1, \pm 2, \pm 3, \ldots .$

The above terms are called the intermodulation products and the order of a product is |*m*|+|*n*|.

For example, the previous expression contains the following four intermodulation products of second order:

$$\begin{array}{llll}
2\omega_1 & \text{(second harmonic of } \omega_1) & m = 2 \;\; n = 0 & \text{order} = 2, \\
2\omega_2 & \text{(second harmonic of } \omega_2) & m = 0 \;\; n = 2 & \text{order} = 2, \\
\omega_1 - \omega_2 & \text{(difference frequency)} & m = 1 \;\; n = -1 & \text{order} = 2, \\
\omega_1 + \omega_2 & \text{(sum frequency)} & m = 1 \;\; n = 1 & \text{order} = 2.
\end{array}$$

- The second-order intermodulation products ($\omega_1 - \omega_2$) or ($\omega_1 + \omega_2$) are used in mixer.
- The ratio of the amplitude of the second-order intermodulation product ($\omega_1 - \omega_2$) or ($\omega_1 + \omega_2$) to the amplitude of a second harmonic $2\omega_1$ or $2\omega_2$ is 2.0, so the second-order harmonic power is 6 dB less than the power in the second-order sum or difference terms.
- Similarly, it contains six third-order intermodulation products: $3\omega_1$, $3\omega_2$, $2\omega_1 + \omega_2$, $2\omega_2 + \omega_1$, **$2\omega_1 - \omega_2$, $2\omega_2 - \omega_1$**.
  →This effect is called third-order intermodulation distortion.
- The output power of third-order products increases as the cube of the input power.

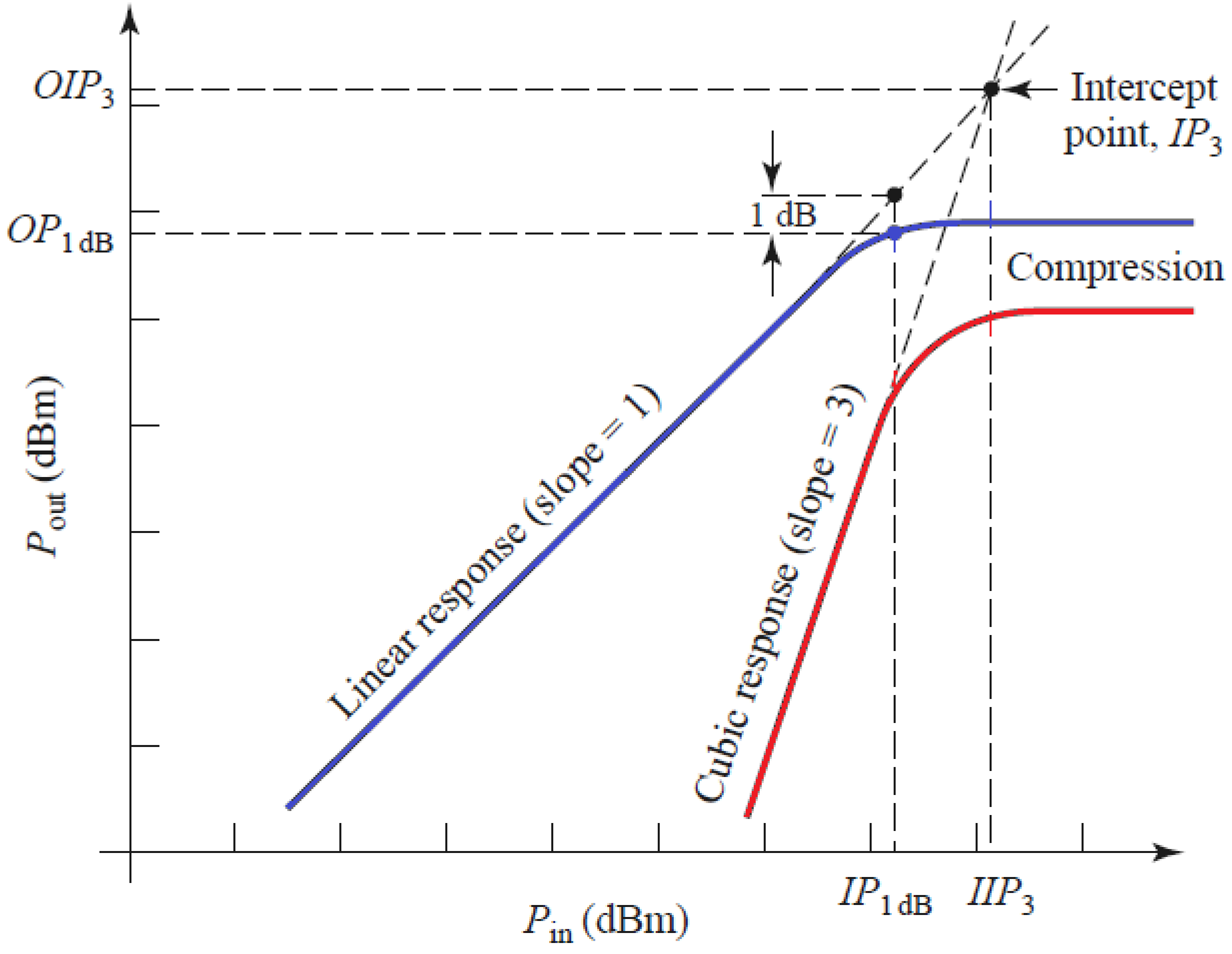
Third order intermodulation distortion

- The hypothetical intersection point where the first-order and third-order powers would be equal is called the third-order intercept point, denoted as IP_3.
- For most of the practical components IP_3 is 10–15 dB greater than P_1dB.
- The output power of the desired signal at frequency ω_1 is

$$P_{\omega_1} = \frac{1}{2} a_1^2 V_0^2.$$

- Similarly, the output power of the intermodulation product of frequency $2\omega_1 - \omega_2$

$$P_{2\omega_1-\omega_2} = \frac{1}{2}\left(\frac{3}{4} a_3 V_0^3\right)^2 = \frac{9}{32} a_3^2 V_0^6.$$



Third order intercept diagram of a nonlinear device.

Third order intermodulation distortion

- Now, the two powers are equal at the intercept point. Define the input signal voltage as V_{IP} at the intercept point. Then,

$$\frac{1}{2}a_1^2 V_{IP}^2 = \frac{9}{32}a_3^2 V_{IP}^6.$$

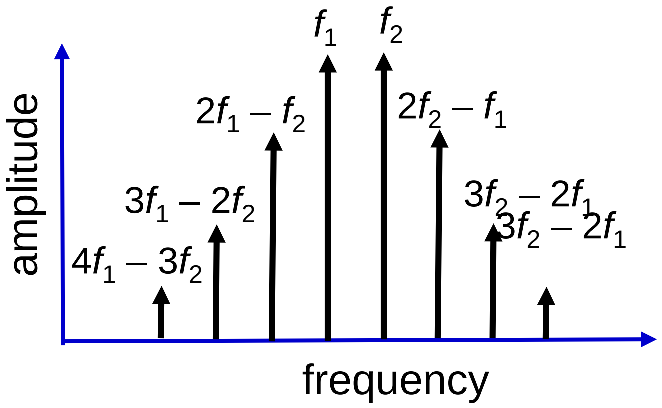
- Solving for V_{IP}, $V_{IP} = \sqrt{\frac{4a_1}{3a_3}}.$

- Since OIP_3 is equal to the linear response of $P_{\omega 1}$ at the intercept point, from the above equations,

$$OIP_3 = P_{\omega_1}\big|_{V_0 = V_{IP}} = \frac{1}{2}a_1^2 V_{IP}^2 = \frac{2a_1^3}{3a_3},$$

Passive intermodulation

- Passive intermodulation (PIM): intermodulation products generated by passive nonlinear effects in connectors, cables, antennas, or almost any component where there is a metal-to-metal contact.
- Caused by poor mechanical contact, oxidation of junctions between ferrous-based metals, contamination of conducting surfaces at RF junctions, or the use of nonlinear materials such as carbon fiber composites or ferromagnetic materials.
- It is similar to the intermodulation effects in amplifiers and mixers, it occurs when signals at two or more closely spaced frequencies mix to produce spurious products.
- Difficult to predict PIM levels. Measurement techniques (two tone) must usually be used.
- PIM is usually only significant when input signal powers are relatively large (e.g. satellite transmitter, cellular telephone base station etc.).



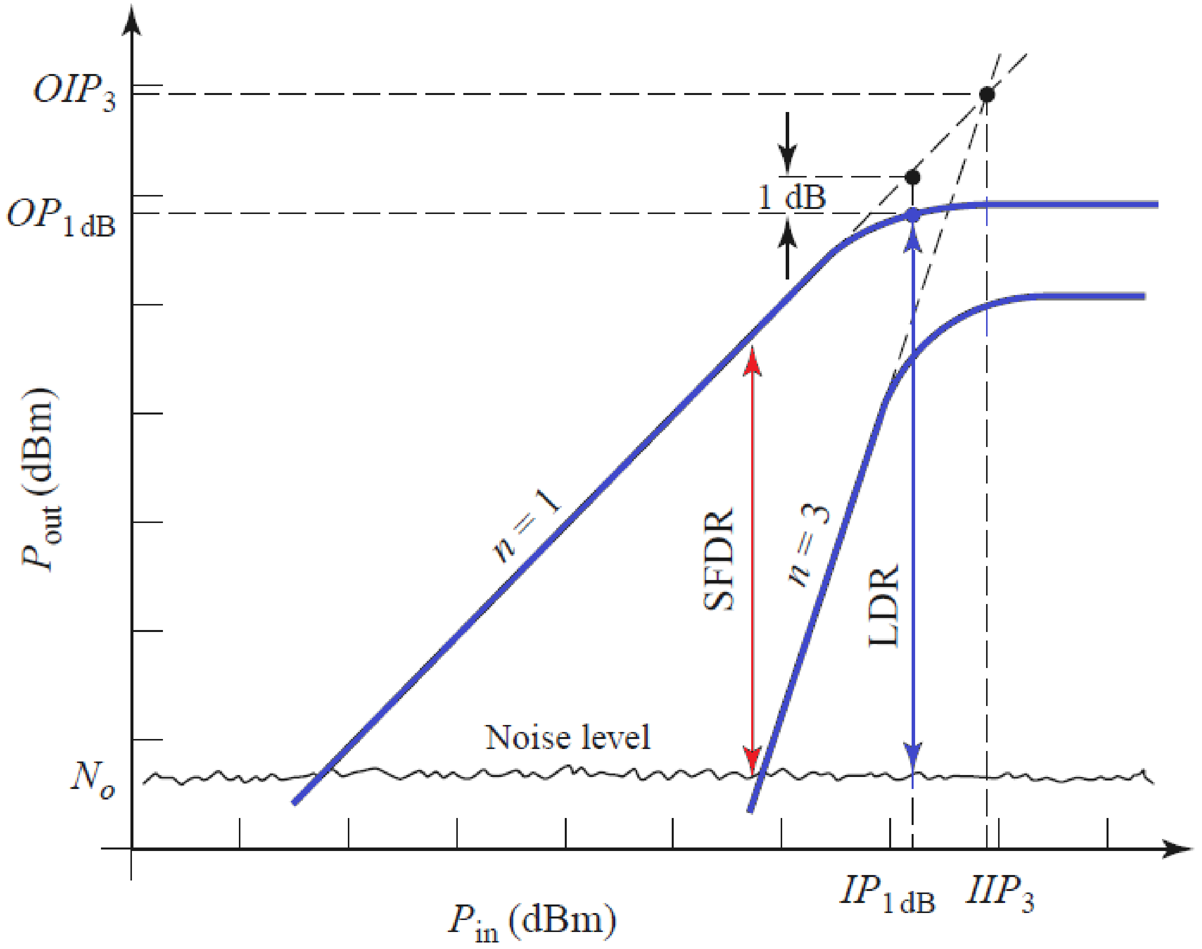
Sidebands due to PIM.

Dynamic range

- Definition of dynamic range varies with components.
- Power amplifier: noise level to 1 dB gain compression point →linear dynamic range (LDR).
- LNA and mixer: noise level to unacceptable intermodulation distortion → spurious free dynamic range (SFDR).

Then, $$\text{LDR (dB)} = OP_{1\,\text{dB}} - N_o$$

SFDR: It is defined as the maximum output signal power for which the power of the third-order intermodulation product is equal to the noise level of the component, divided by the output noise level. If $P_{\omega 1}$ is the output power of the desired signal at frequency ω_1, and $P_{2\omega 1-\omega 2}$ is the output power of the third-order intermodulation product, then



Dynamic range of a realistic amplifier

$$\text{SFDR} = \frac{P_{\omega_1}}{P_{2\omega_1-\omega_2}}$$ with $P_{2\omega 1-\omega 2}$ is taken as the noise level of the component.

Dynamic range

Now, $P_{2\omega1-\omega2}$ in terms of OIP_3 and $P_{\omega1}$ is $P_{2\omega_1-\omega_2} = \dfrac{(P_{\omega_1})^3}{(OIP_3)^2}$.

Therefore, from the above two equations, $$\text{SFDR} = \left.\frac{P_{\omega_1}}{P_{2\omega_1-\omega_2}}\right|_{P_{2\omega_1-\omega_2}=N_o} = \left(\frac{OIP_3}{N_o}\right)^{2/3}.$$

In decibel scale, $$\text{SFDR (dB)} = \frac{2}{3}(OIP_3 - N_o)$$ (the same result for 2ω2−ω1)

- For a receiver, considering a minimum SNR is required for detection,

$$\text{SFDR (dB)} = \frac{2}{3}(OIP_3 - N_o) - \text{SNR}.$$

Q. A receiver has a NF of 7 dB, a 1 dB compression point of 25 dBm (referenced to output), a gain of 40 dB, and a third-order intercept point of 35 dBm (referenced to output). If the receiver is fed with an antenna having a noise temperature of T_A = 150 K, and the desired output SNR is 10 dB, find the linear and spurious free dynamic ranges. Assume a receiver bandwidth of 100 MHz.

Dynamic range

Answer:

The noise power at the receiver output is

$$N_o = GkB[T_A + (F-1)T_0] = 10^4(1.38\times10^{-23})(10^8)[150 + (4.01)(290)]$$
$$= 1.8\times10^{-8}\text{ W} = -47.4\text{ dBm}.$$

Then, the linear dynamic range is

$$\text{LDR} = OP_{1\text{dB}} - N_o = 25\text{ dBm} + 47.4\text{ dBm} = 72.4\text{ dB}.$$

Then, spurious free dynamic range is

$$\text{SFDR} = \frac{2}{3}(OIP_3 - N_o) - \text{SNR} = \frac{2}{3}(35 + 47.4) - 10 = 44.9\text{ dB}.$$

RF and Microwave Engineering (EC 31005)

Microwave Mixers and Multipliers (P9)

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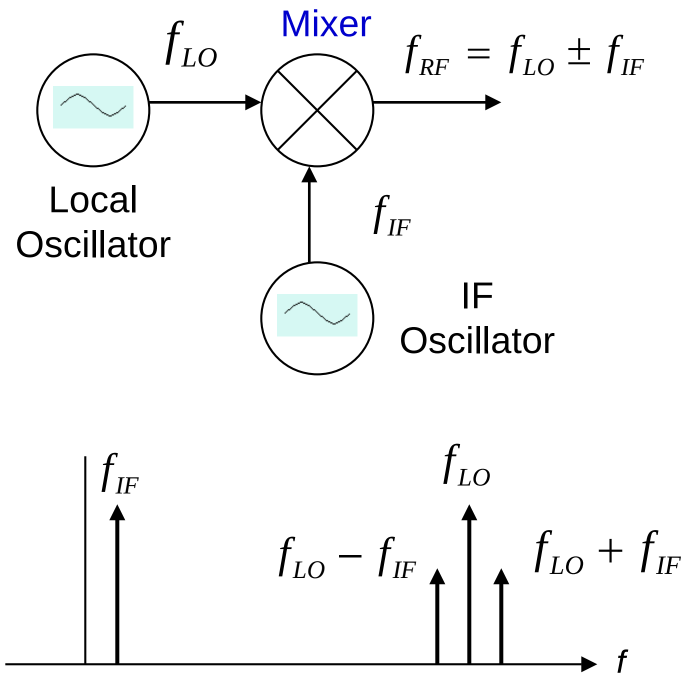
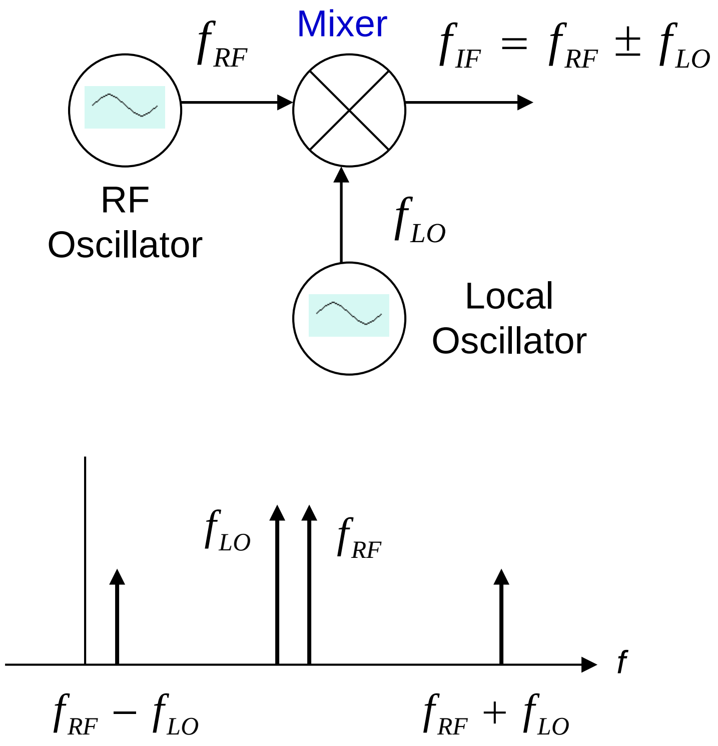
Mixers

- A mixer is a three-port device that uses a nonlinear or time-varying element to achieve frequency conversion.
- Mixer generates the sum and difference frequencies of its two input signals.
- The nonlinearity of a diode or transistor is used for this purpose.
- A nonlinear component can generate a wide variety of harmonics and other products of input frequencies, so filter must be used.



Up-conversion.



Down-conversion.

- Impedance matching: each mixer port would be matched at its particular frequency (RF, LO, or IF) → nonlinear analysis.
- Undesired frequency products are absorbed with resistive loads, or blocked with reactive terminations.

Mixer

Up-conversion in a transmitter:

Input to mixer: $v_{LO}(t) = \cos 2\pi f_{LO} t, \qquad v_{IF}(t) = \cos 2\pi f_{IF} t.$

Output of the mixer (only the 2nd term):

$$v_{RF}(t) = K \cos 2\pi f_{LO} t \, \cos 2\pi f_{IF} t = \frac{K}{2}\left[\cos 2\pi (f_{LO} - f_{IF}) t + \cos 2\pi (f_{LO} + f_{IF}) t\right].$$

(K – voltage conversion loss).

Desired output after band pass filtering: $f_{IF} = f_{RF} + f_{LO}.$

Microwave mixer.

Down-conversion in a receiver:

Input to mixer: $v_{LO}(t) = \cos 2\pi f_{LO} t, \qquad v_{RF}(t) = \cos 2\pi f_{RF} t.$

Output of the mixer:

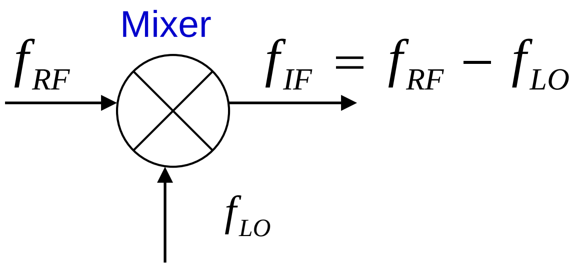
$$v_{RF}(t) = K \cos 2\pi f_{LO} t \, \cos 2\pi f_{RF} t = \frac{K}{2}\left[\cos 2\pi (f_{RF} - f_{LO}) t + \cos 2\pi (f_{RF} + f_{LO}) t\right].$$

Desired output after lowpass filtering: $f_{IF} = f_{RF} - f_{LO}.$

Mixer parameters

- **Image frequency in a receiver:**

$$f_{RF} = f_{LO} + f_{IF} \;\Rightarrow f_{IF},$$
$$f_{RF(IM)} = f_{LO} - f_{IF} \;\Rightarrow -f_{IF}.$$



Down-conversion.

Image response: $f_{IM} = f_{LO} - f_{IF}$

Example: for f_{RF} = 20 GHz and f_{LO} = 18 GHz, image frequency is f_{IM} = 16 GHz.

- **Conversion loss in a receiver:**

Accounts for all losses → resistive, conversion, higher harmonic losses etc.

Conversion loss, $L_c = 10\log\dfrac{\text{available RF input power}}{\text{available IF output power}} \geq 0\ dB.$

Diode mixers (passive): typical conversion loss is 4 to 10 dB in the 1–100 GHz range.

Transistor mixers have lower conversion loss, and they may even have conversion gain of a few dB.

Conversion loss depends on the LO power level → accurate characterization requires nonlinear analysis.

Mixer parameters: noise figure

$$\text{Noise figure} = 10\log\frac{\text{Input SNR}}{\text{Output SNR}}\,dB.$$

Input noise power $N_i = kT_0B \quad (k = 1.380638\times10^{-23}\ J/K, \quad T = 290\ K, \quad B = \text{IF bandwidth})$

Output noise power = input noise + N_{added} → $N_0 = \dfrac{\left(kT_0B + N_{added}\right)}{L_c}$

Noise figure relationship for DSB signal:

Consider a DSB input signal as $v_{\rm DSB}(t) = A[\cos(\omega_{\rm LO} - \omega_{\rm IF})t + \cos(\omega_{\rm LO} + \omega_{\rm IF})t].$

Down converted IF signal after mixing and lowpass filtering,

$$v_{\rm IF}(t) = \frac{AK}{2}\cos(\omega_{\rm IF}t) + \frac{AK}{2}\cos(-\omega_{\rm IF}t) = AK\cos\omega_{\rm IF}t,$$

(constant K accounts for conversion loss for each side band)

The average power of DSB input signal is $S_i = \dfrac{A^2}{2} + \dfrac{A^2}{2} = A^2$

Mixer parameters: noise figure

The average power of the output IF signal is $S_o = \dfrac{A^2 K^2}{2}.$

Then, using the definition of noise figure, $F_{\text{DSB}} = \dfrac{S_i N_o}{S_o N_i} = \dfrac{2}{K^2 L_c}\left(1 + \dfrac{N_{\text{added}}}{kT_0 B}\right).$

Noise figure relationship for SSB signal:

Consider a SSB input signal as $v_{\text{SSB}}(t) = A\cos(\omega_{\text{LO}} - \omega_{\text{IF}})t.$

Down converted IF signal after mixing and lowpass filtering, $v_{\text{IF}}(t) = \dfrac{AK}{2}\cos(\omega_{\text{IF}} t).$

The average power of SSB input signal, $S_i = \dfrac{A^2}{2}$

The average power of the output IF signal is $S_o = \dfrac{A^2 K^2}{8}.$

Then, $F_{\text{SSB}} = \dfrac{S_i N_o}{S_o N_i} = \dfrac{4}{K^2 L_c}\left(1 + \dfrac{N_{\text{added}}}{kT_0 B}\right).$

Therefore, $F_{\text{SSB}} = 2F_{\text{DSB}}.$

Other mixer parameters

IIP3: nonlinearity→ produce intermodulation products. Typical values of *IIP*3 for mixers range from 15 to 30 dBm.

Isolation: since LO power is high, most important is LO-RF isolation. Otherwise LO power radiates through antenna.

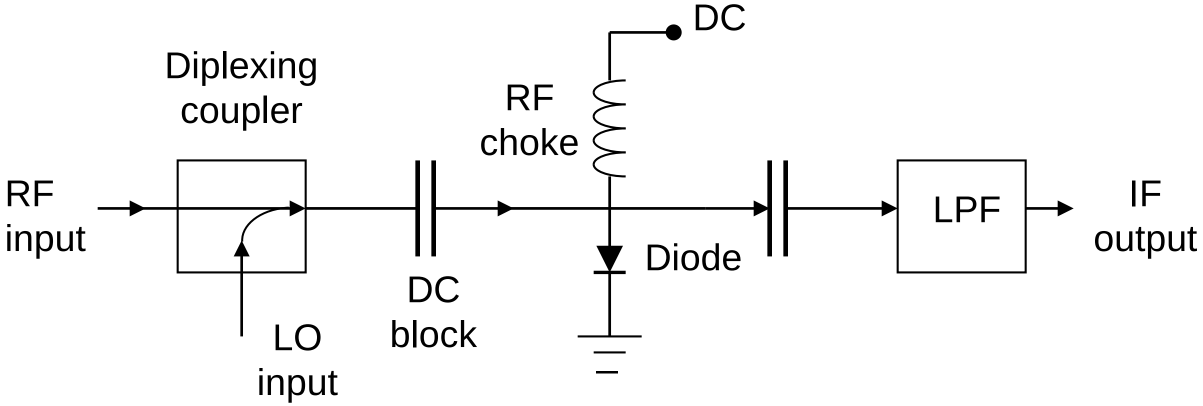
Remedy – use a BPF and/or tuned amplifier before mixer.

- Single balanced mixer: provides isolation between LO and RF port. Sometimes called just balanced mixer.
- Double balanced mixer: provides isolation among all the ports. Better spurious suppression, higher IP3 point. Disadvantage→ complex design and requires higher LO drive power.

Single ended diode mixer

Input RF signal: $v_{RF}(t) = V_{RF}\cos\omega_{RF}t.$

LO signal: $v_{LO}(t) = V_{LO}\cos\omega_{LO}t.$



Single-ended diode mixer circuit.

Diode current is

$$i_D(t) = I_0 + G_d[v_{RF} + v_{LO}] + \frac{G'_d}{2}[v_{RF} + v_{LO}]^2 + \ldots$$

Using only the third term:

$$i(t) = \frac{G'_d}{2}[V_{RF}\cos\omega_{RF}t + V_{LO}\cos\omega_{LO}t]^2.$$

$$= \frac{G'_d}{2}\left(V_{\mathrm{RF}}^2\cos^2\omega_{\mathrm{RF}}t + 2V_{\mathrm{RF}}V_{\mathrm{LO}}\cos\omega_{\mathrm{RF}}t\cos\omega_{\mathrm{LO}}t + V_{\mathrm{LO}}^2\cos^2\omega_{\mathrm{LO}}t\right)$$

$$= \frac{G'_d}{4}\Big[V_{\mathrm{RF}}^2(1+\cos 2\omega_{\mathrm{RF}}t) + V_{\mathrm{LO}}^2(1+\cos 2\omega_{\mathrm{LO}}t) + 2V_{\mathrm{RF}}V_{\mathrm{LO}}\cos(\omega_{\mathrm{RF}}-\omega_{\mathrm{LO}})t$$

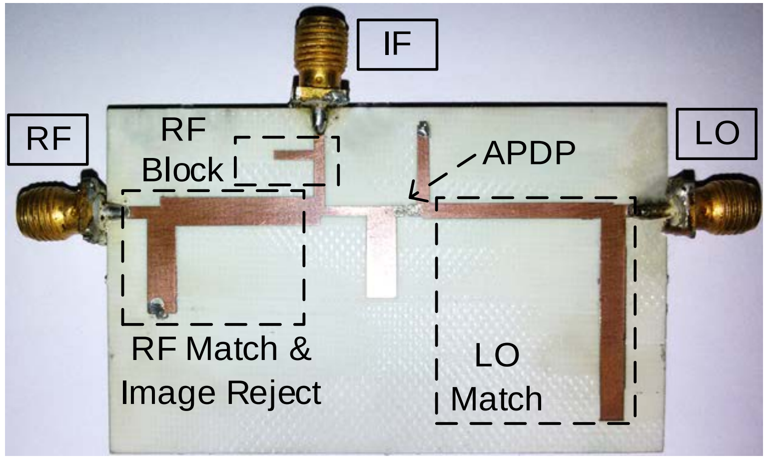
$$+ 2V_{\mathrm{RF}}V_{\mathrm{LO}}\cos(\omega_{\mathrm{RF}}+\omega_{\mathrm{LO}})t\Big].$$

Single ended diode mixer

After lowpass filtering and dc blocking,

$$i_{IF}(t) = \frac{G'_d}{2}\left[V_{RF}V_{LO}\cos\left(\omega_{RF} - \omega_{LO}\right)t\right]$$
$$= \frac{G'_d}{2}\left[V_{RF}V_{LO}\cos\omega_{IF}t\right].$$

Problem: high conversion loss.



A sub-harmonic mixer using anti-parallel diode pair.

Sub-harmonic mixer:

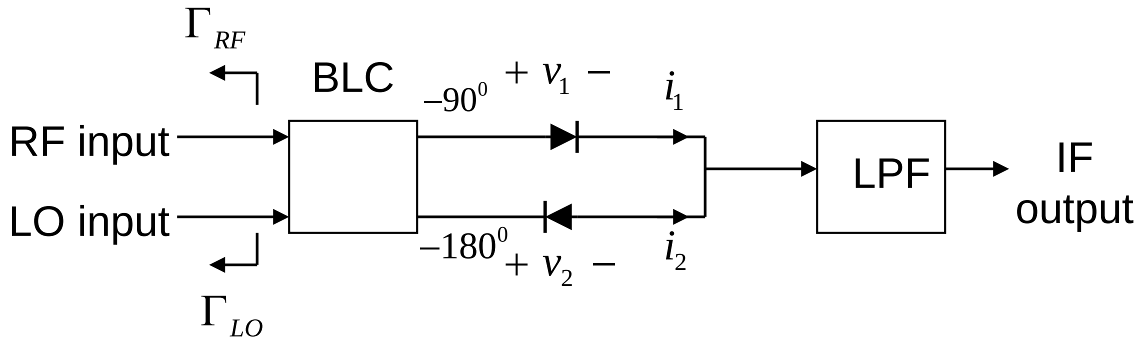
Anti-parallel diodes and higher order harmonics of the LO is used for mixing.

Balanced mixer

- RF input matching and RF-LO isolation is improved.

Input RF signal: $v_{RF}(t) = V_{RF} \cos \omega_{RF} t.$

LO signal: $v_{LO}(t) = V_{LO} \cos \omega_{LO} t.$



Balanced mixer circuit using a 90° hybrid.

Total voltage applied to diode 1,

$$v_1(t) = \frac{1}{\sqrt{2}}\left[V_{RF}\cos\left(\omega_{RF}t - 90^0\right) + V_{LO}\cos\left(\omega_{LO}t - 180^0\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left[V_{RF}\sin\omega_{RF}t - V_{LO}\cos\omega_{LO}t\right].$$

Total voltage applied to diode 2,

$$v_2(t) = \frac{1}{\sqrt{2}}\left[V_{RF}\cos\left(\omega_{RF}t - 180^0\right) + V_{LO}\cos\left(\omega_{LO}t - 90^0\right)\right]$$

$$= \frac{1}{\sqrt{2}}\left[-V_{RF}\cos\omega_{RF}t + V_{LO}\sin\omega_{LO}t\right].$$

S-matrix of a quadrature hybrid is

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

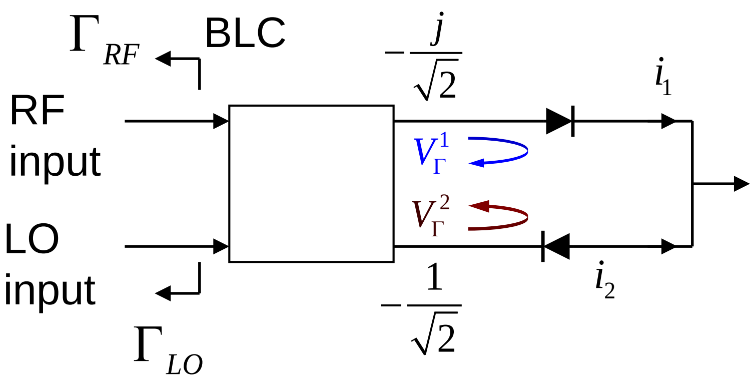
Balanced mixer

The third term of the diode currents in Diode 1 and 2, respectively, are

$$i_1(t)=\frac{K}{2}\left[V_{RF}^2\sin^2\omega_{RF}t+V_{LO}^2\cos^2\omega_{LO}t-2V_{RF}V_{LO}\sin\omega_{RF}t\cos\omega_{LO}t\right]$$

$$i_2(t)=-\frac{K}{2}\left[V_{RF}^2\cos^2\omega_{RF}t+V_{LO}^2\sin^2\omega_{LO}t-2V_{RF}V_{LO}\cos\omega_{RF}t\sin\omega_{LO}t\right].$$

$$\therefore\ i_1(t)+i_2(t)=-\frac{K}{2}\left[V_{RF}^2\cos 2\omega_{RF}t-V_{LO}^2\cos 2\omega_{LO}t+2V_{RF}V_{LO}\sin\omega_{IF}t\right]$$



Single balanced mixer circuit using 90° hybrid.

After filtering, the IF output is

$$i_{IF}(t)=-KV_{RF}V_{LO}\sin\omega_{IF}t,\qquad \text{where}\quad \omega_{IF}=\omega_{RF}-\omega_{LO}.$$

A 90° hybrid provides perfect RF match and a 180° hybrid provides perfect RF-LO isolation.

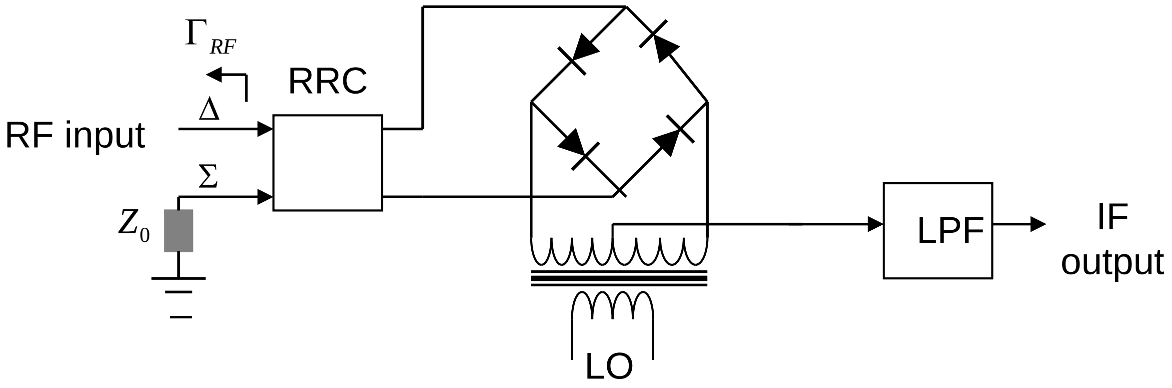
Reflected RF voltages at the diodes are $V_{\Gamma1}=-\frac{j\Gamma V_{RF}}{\sqrt{2}},\quad V_{\Gamma2}=-\frac{\Gamma V_{RF}}{\sqrt{2}}.$

They combine at the RF and LO ports as $V_\Gamma^{RF}=-j\frac{V_{\Gamma1}}{\sqrt{2}}-\frac{V_{\Gamma2}}{\sqrt{2}}=-\frac{1}{2}\Gamma V_{RF}+\frac{1}{2}\Gamma V_{RF}=0,$ →perfect match at the RF port.

$$V_\Gamma^{LO}=-\frac{V_{\Gamma2}}{\sqrt{2}}-j\frac{V_{\Gamma1}}{\sqrt{2}}=\frac{1}{2}j\Gamma V_{RF}+\frac{1}{2}j\Gamma V_{RF}=j\Gamma V_{RF}.$$

Double balanced mixer

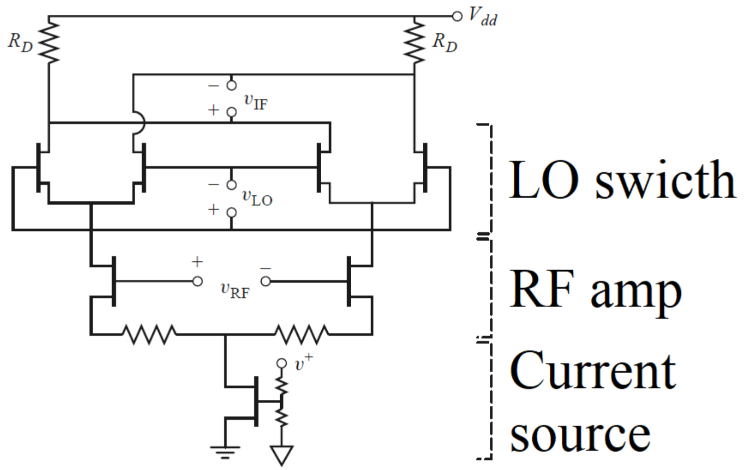
- The double balanced mixer provides good isolation among all ports, reject all even harmonics but consumes much higher power.



Double balanced mixer circuit using rat race coupler.

Other mixers:

- **Image reject filter:** LSB ($\omega_{LO} - \omega_{IF}$) and the USB ($\omega_{LO} + \omega_{IF}$) are separated as two signals and they are available at two isolated output ports.
- **Differential FET mixer**: a differential FET pair is used for switching→ popular in CMOS technology.
- **Gilbert cell mixer:** uses two singly balanced FET mixers to form a double-balanced mixer → popular in CMOS technology.



Gilbert cell mixer.

Frequency multipliers

- Use the non-linear properties of a device →non-linear reactance, non-linear-resistance.
- Non-linear reactive diode multiplier:

  Use the non-linear capacitance variation of a junction. Theoretically 100% efficiency can be achieved by terminating all other undesired harmonics into reactive elements.
- Non-linear resistive diode multiplier:

  Use the non-linear V-I characteristics. Typically Schottky junctions are used. All other undesired harmonics are terminated into reactive loads. Efficiency is poor drops as the square of the multiplication factor. But easy to design.
- Transistor multiplier:

  Compared to diode frequency multipliers, offer better bandwidth and the possibility of conversion efficiencies greater than 100%. FET multipliers require less input and DC power than diode multipliers.

RF and Microwave Engineering (EC 31005)

Microwave Amplifiers (P10)

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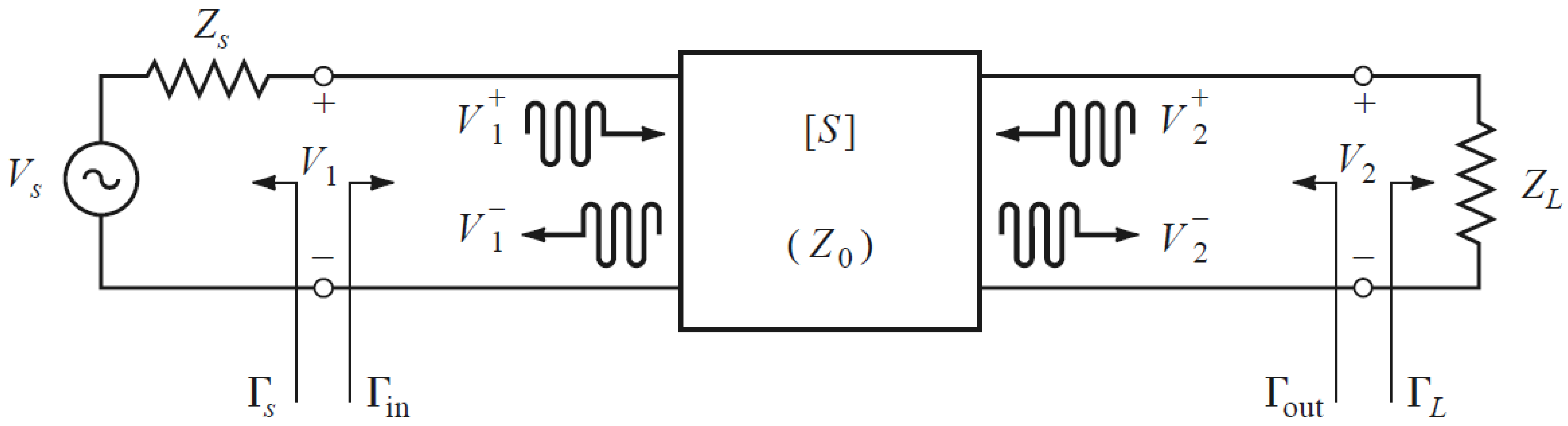
Two port power gain



A two-port network with arbitrary source and load.

- **Power gain ($G = P_L/P_{in}$):** the ratio of power dissipated in the load Z_L to the power delivered to the input of the two-port network. This gain is independent of Z_S, although the characteristics of some active devices may be dependent on Z_S.
- **Available power gain ($G_A = P_{avn}/P_{avs}$):** the ratio of the power available from the two-port network to the power available from the source. This assumes conjugate matching of both the source and the load, and depends on Z_S, but not Z_L.
- **Transducer power gain ($G_T = P_L/P_{avs}$):** the ratio of the power delivered to the load to the power available from the source. This depends on both Z_S and Z_L.

Two port power gain

Reflection coefficient seen looking toward load is $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Reflection coefficient seen looking toward source is $\Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0}$

Now, from the definition of scattering parameters,

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+ = S_{11} V_1^+ + S_{12} \Gamma_L V_2^-,$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+ = S_{21} V_1^+ + S_{22} \Gamma_L V_2^-.$$

Eliminating V^-_2 from the first equation and solving for V^-_1 / V^+_1 gives,

$$\Gamma_{\text{in}} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} = \frac{Z_{\text{in}} - Z_0}{Z_{\text{in}} + Z_0}$$

Similarly, looking into port 2, $\Gamma_{\text{out}} = \dfrac{V_2^-}{V_2^+} = S_{22} + \dfrac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S}.$

Two port power gain

Now, by voltage division $$V_1 = V_S \frac{Z_{\text{in}}}{Z_S + Z_{\text{in}}} = V_1^+ + V_1^- = V_1^+ (1 + \Gamma_{\text{in}}).$$

From the previous relation for Γ_{in}, $$Z_{\text{in}} = Z_0 \frac{1 + \Gamma_{\text{in}}}{1 - \Gamma_{\text{in}}}$$

Solving for V^+_1 in terms of V_S gives, $$V_1^+ = \frac{V_S}{2} \frac{(1 - \Gamma_S)}{(1 - \Gamma_S \Gamma_{\text{in}})}.$$ [put the value of Z_{in} and $Z_S = Z_0 \frac{1 + \Gamma_S}{1 - \Gamma_S}$]

If peak values are assumed for all voltages, the average power delivered to the network is,

$$P_{\text{in}} = \frac{1}{2Z_0} \left|V_1^+\right|^2 \left(1 - |\Gamma_{\text{in}}|^2\right) = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{|1 - \Gamma_S \Gamma_{\text{in}}|^2} \left(1 - |\Gamma_{\text{in}}|^2\right),$$ (using the previous relation)

The power delivered to the load is $$P_L = \frac{\left|V_2^-\right|^2}{2Z_0} \left(1 - |\Gamma_L|^2\right).$$

Two port power gain

Solving for V_2^- from the scattering parameter relation and using the previous relation,

$$P_L = \frac{|V_1^+|^2}{2Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{|1 - S_{22}\Gamma_L|^2} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right) |1 - \Gamma_S|^2}{|1 - S_{22}\Gamma_L|^2 \, |1 - \Gamma_S \Gamma_{in}|^2}.$$

The power gain is $$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 \left(1 - |\Gamma_L|^2\right)}{\left(1 - |\Gamma_{in}|^2\right) |1 - S_{22}\Gamma_L|^2}.$$

Now, the power available from the source, P_{avs}, is the maximum power that can be delivered to the network. Thus, considering input impedance of the terminated network is conjugately matched to the source impedance,

$$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*} = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_S|^2}{\left(1 - |\Gamma_S|^2\right)}.$$

Similarly, the power available from the network, P_{avn}, is the maximum power that can be delivered to the load. Thus,

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 \left(1 - |\Gamma_{out}|^2\right) |1 - \Gamma_S|^2}{|1 - S_{22}\Gamma_{out}^*|^2 \, |1 - \Gamma_S \Gamma_{in}|^2}\Bigg|_{\Gamma_L=\Gamma_{out}^*}$$

(Γ_{in} must be evaluated for $\Gamma_L = \Gamma^*_{out}$)

Two port power gain

Now, putting the values, $$\left|1-\Gamma_S\Gamma_{\text{in}}\right|^2\Big|_{\Gamma_L=\Gamma_{\text{out}}^*} = \frac{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{\text{out}}|^2\right)^2}{\left|1-S_{22}\Gamma_{\text{out}}^*\right|^2},$$

Then, $$P_{\text{avn}} = \frac{|V_S|^2}{8Z_0}\frac{|S_{21}|^2\,|1-\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{\text{out}}|^2\right)}.$$

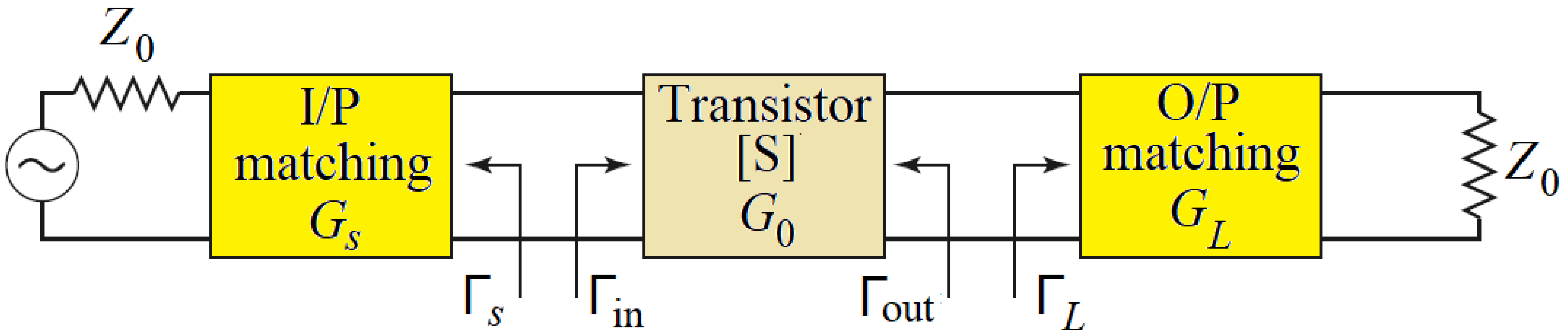
Therefore, available power gain, $$G_A = \frac{P_{\text{avn}}}{P_{\text{avs}}} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-S_{11}\Gamma_S|^2\left(1-|\Gamma_{\text{out}}|^2\right)}.$$

Transducer power gain, $$G_T = \frac{P_L}{P_{\text{avs}}} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-\Gamma_S\Gamma_{\text{in}}|^2\,|1-S_{22}\Gamma_L|^2}.$$ (For $\Gamma_L = \Gamma_S = 0$, $G_T = |S_{21}|^2$)

Now, $\Gamma_{in} = S_{11}$ for $S_{12} = 0$ → unilateral power gain, $$G_{TU} = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)\left(1-|\Gamma_L|^2\right)}{|1-S_{11}\Gamma_S|^2\,|1-S_{22}\Gamma_L|^2}.$$

Amplifier with matching networks

The most useful gain definition for amplifier design is the transducer power gain, which accounts for both source and load mismatch.



An amplifier with matching networks.

The overall transducer gain is then $G_T = G_S G_0 G_L$, where

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}, \quad G_0 = |S_{21}|^2, \quad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$

For unilateral transistor ($S_{12} = 0$),

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}, \quad G_0 = |S_{21}|^2, \quad G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$

Stability

- Oscillation is possible if either the input or output port impedance has a negative real part → $|\Gamma_{in}| > 1$ or $|\Gamma_{out}| > 1$.
- Since Γ_{in} and Γ_{out} depend on the source and load matching networks, the stability of the amplifier depends on Γ_S and Γ_L as presented by the matching networks.

Unconditional stability: The network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances (i.e., $|\Gamma_S| < 1$ and $|\Gamma_L| < 1$).

Conditional stability: The network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for a certain range of passive source and load impedances. This case is also referred to as potentially unstable.

Stability circle:

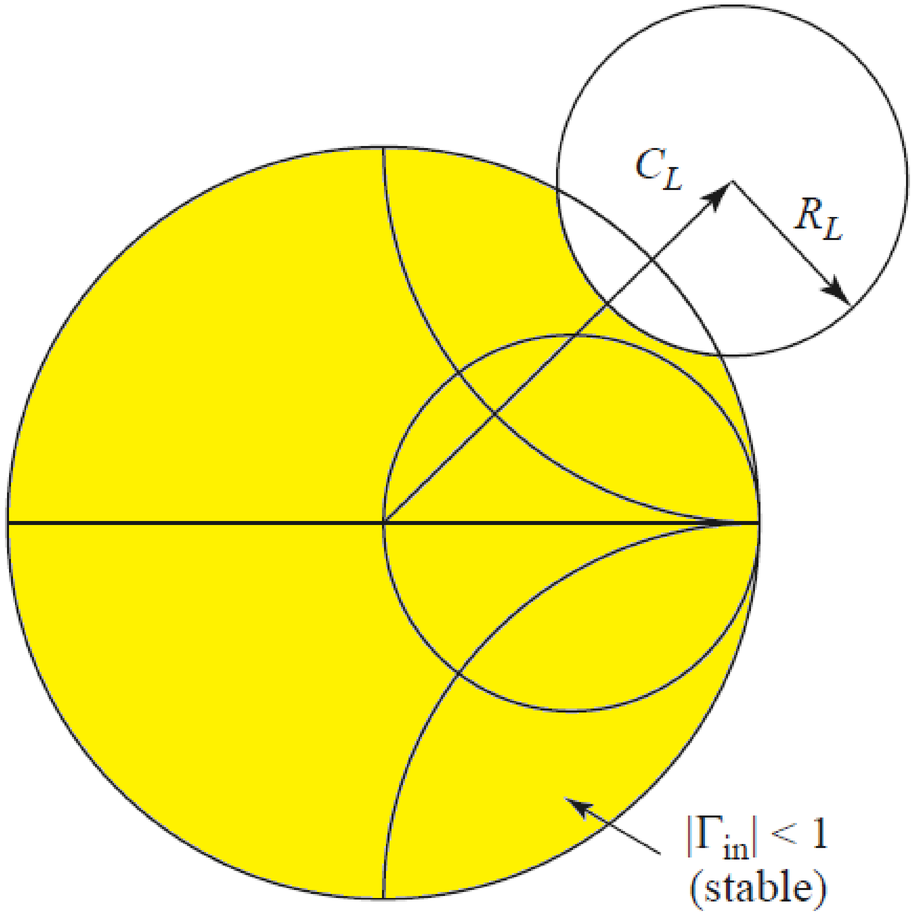
The unconditionally stable criteria provides two condition as

$$|\Gamma_{\rm in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1, \quad |\Gamma_{\rm out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1.$$

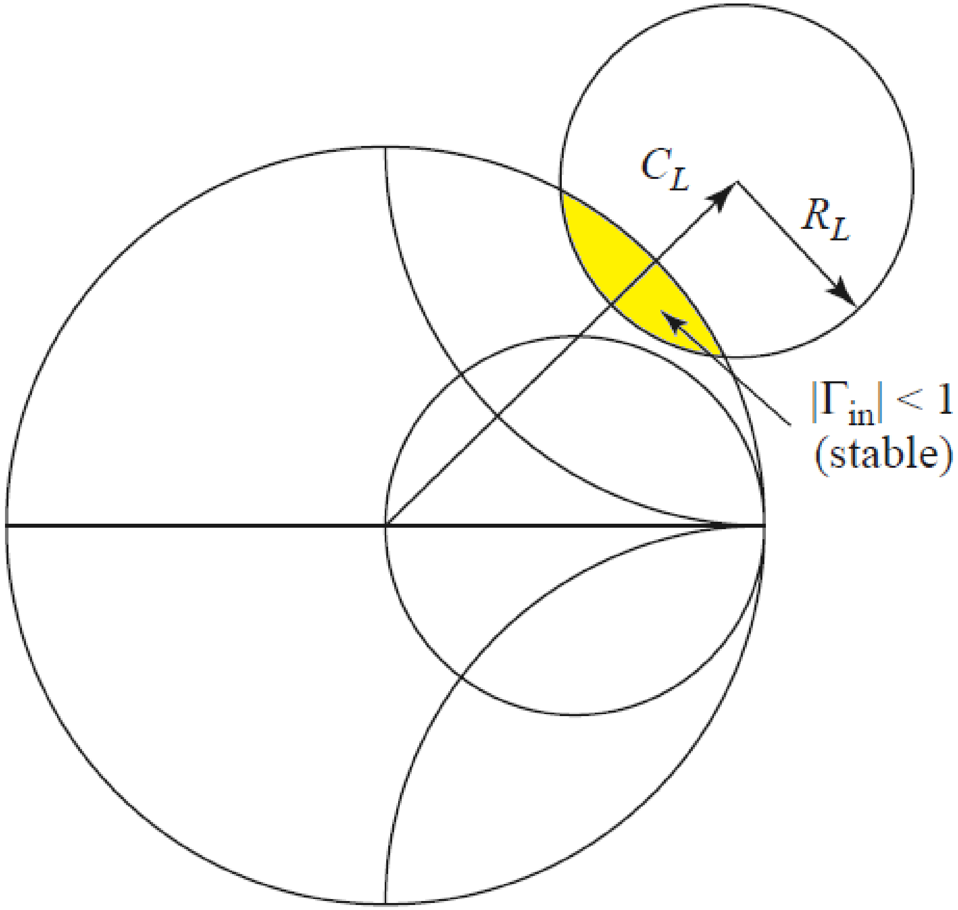
For unilateral transistor, the above condition reduces to $|S_{11}| < 1$ and $|S_{22}| < 1$.

The stability circles are defined as the loci in the Γ_S and Γ_L planes for which $|\Gamma_{in}| = 1$ and $|\Gamma_{out}| = 1$, respectively.

Stability circles



(a)



(b)

Output stability circles for a conditionally stable device (a) $|S_{11}| < 1$ and (b) $|S_{11}| > 1$.

Test for unconditional stability

A simple method to determine unconditional stability is $K-\Delta$ test:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1, \quad \text{(Rollet's condition)}$$

$$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1 \quad \text{(auxiliary condition)}$$

- For unconditional stability, both the above conditions must be satisfied. These two conditions are necessary and sufficient for unconditional stability, and are easily evaluated.
- If the device scattering parameters do not satisfy the $K-\Delta$ test, the device is not unconditionally stable, and stability circles must be used to determine if there are values of Γ_S and Γ_L for which the device will be conditionally stable with $|S_{11}| < 1$ and $|S_{22}| < 1$.

Relative stability:

$$\mu = \frac{1 - |S_{11}|^2}{\left|S_{22} - \Delta S_{11}^*\right| + |S_{12}S_{21}|} > 1.$$

- If $\mu > 1$, the device is unconditionally stable (only one parameter). In addition, it can be said that larger values of μ imply greater stability.

Design of amplifiers

Design for Maximum Gain (Conjugate Matching):

- Overall transducer gain of the amplifier will be controlled by the gains, G_S and G_L, of the matching sections.
- Maximum gain will be realized when these sections provide a conjugate match between the amplifier source or load impedance and the transistor.
- Narrowband design.
- Maximum gain is $$G_{T_{\max}} = \frac{1}{1-|\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}.$$

Broadband transistor amplifier:

- Microwave transistors typically are not well matched to 50 Ω, and large impedance mismatches are governed by the Bode–Fano gain–bandwidth criterion.
- $|S_{21}|$ decreases with frequency at the rate of 6 dB/octave.
- Some of the common approaches are *Compensated matching networks, Resistive matching networks, Negative feedback, Balanced amplifiers, Distributed amplifiers, Differential amplifiers.*

Low noise amplifier

- Generally it is not possible to obtain both minimum noise figure and maximum gain for an amplifier.
- Use constant-gain circles and circles of constant noise figure to select a usable trade-off between noise figure and gain.
- Noise figure of a two-port amplifier is $F = F_{\min} + \frac{R_N}{G_S}|Y_S - Y_{\text{opt}}|^2,$

  where

$$
\begin{aligned}
Y_S &= G_S + jB_S = \text{source admittance presented to transistor.} \\
Y_{\text{opt}} &= \text{optimum source admittance that results in minimum noise figure.} \\
F_{\min} &= \text{minimum noise figure of transistor, attained when } Y_S = Y_{\text{opt}}. \\
R_N &= \text{equivalent noise resistance of transistor.} \\
G_S &= \text{real part of source admittance.}
\end{aligned}
$$

- MOSFET LNA: They have relatively low AC input resistance, making them difficult to impedance match.
- An external series resistance can be added to the gate, but this approach increases noise power and degrades efficiency.
- A series inductor at the source of a MOSFET create a resistive input impedance without adding noisy resistors → **inductive source degeneration**.

Power amplifiers

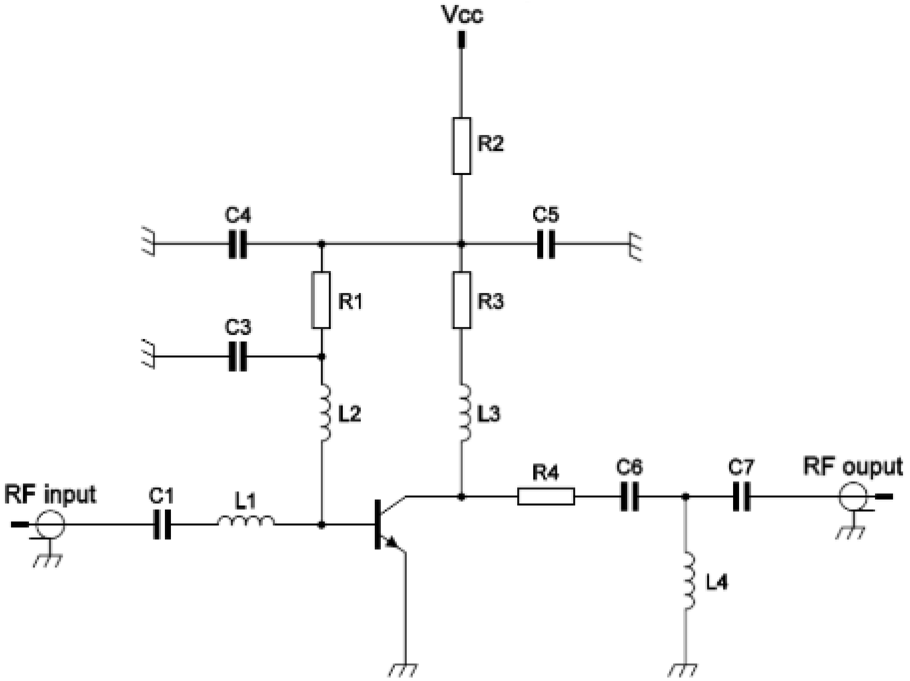
- Large signal characterization: involves non-linear analysis.
- Load-pull contours: plot contours of constant power output on a Smith chart as a function of the load reflection coefficient, with the transistor conjugately matched at its input.
- Efficiencies: Class A – 50%, class B – 78%, class C – 100% (only for constant envelope). Class D, E, F, and S, use the transistor as a switch to pump a highly resonant tank circuit, and may achieve very high efficiencies at microwave frequencies.
- **Amplifier efficiency** is the ratio of RF output power to DC input power: $\eta = P_{out}/P_{DC}$ (also called drain efficiency or collector efficiency).
- **Power added efficiency:**

  Consider RF power delivered at the input to the amplifier. It is given by

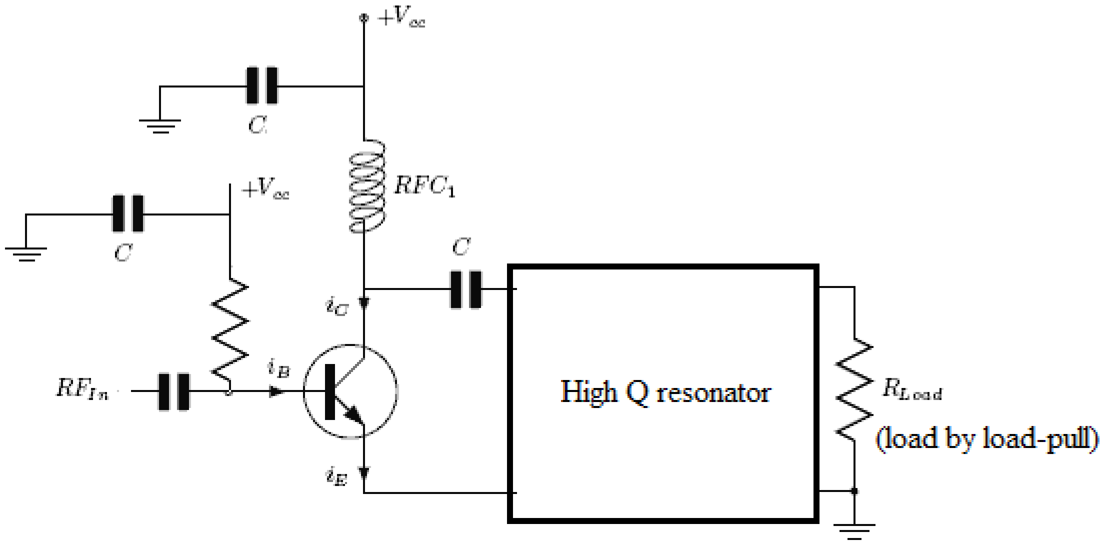
$$\eta_{PAE} = \text{PAE} = \frac{P_{\text{out}} - P_{\text{in}}}{P_{\text{DC}}} = \left(1 - \frac{1}{G}\right)\frac{P_{\text{out}}}{P_{\text{DC}}} = \left(1 - \frac{1}{G}\right)\eta$$

- **Compressed gain:** defined as the gain of the amplifier at the 1 dB compression point.

Amplifiers



A low noise amplifier



A power amplifier

RF and Microwave Engineering (EC 31005)

Microwave systems (P11)

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System aspects of antennas

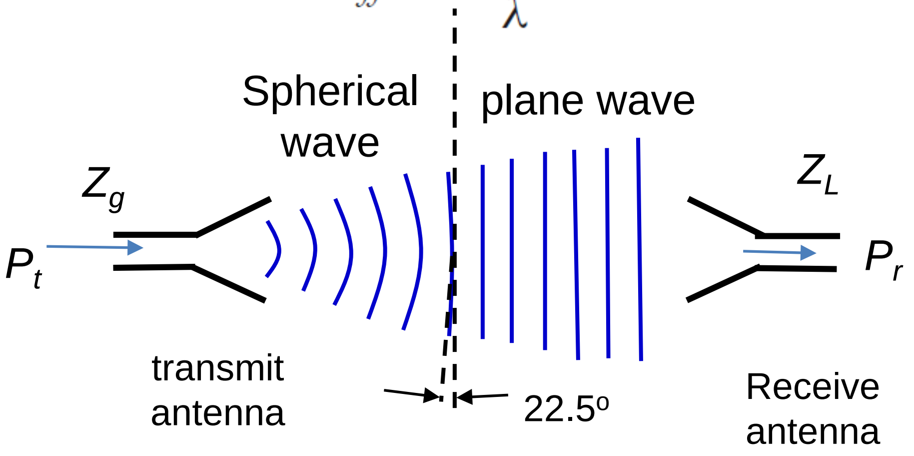
- A transmitting antenna converts a guided electromagnetic wave into a plane wave propagating in free space.
- The opposite is for the receiving antenna.
- Antennas are reciprocal devices.
- A wide varieties of antennas are used for different specifications: wire antennas, printed antennas, aperture antennas, reflector antennas, array antennas etc.
- The electric field in the far-field (TEM wave) is

$$\bar{E}(r,\theta,\phi) = \left[\hat{\theta} F_\theta(\theta,\phi) + \hat{\phi} F_\phi(\theta,\phi)\right] \frac{e^{-jk_0 r}}{r} \text{ V/m}$$

- The magnetic fields associated with this electric field are $H_\phi = \dfrac{E_\theta}{\eta_0}, \quad H_\theta = \dfrac{-E_\phi}{\eta_0}$
- The Poynting vector is $\bar{S} = \bar{E} \times \bar{H}^* \text{ W/m}^2$ and is time average value is

$$\bar{S}_{\text{avg}} = \frac{1}{2}\text{Re}\{\bar{S}\} = \frac{1}{2}\text{Re}\{\bar{E} \times \bar{H}^*\} \text{ W/m}^2.$$

$$R_{ff} = \frac{2D^2}{\lambda} \text{m.}$$

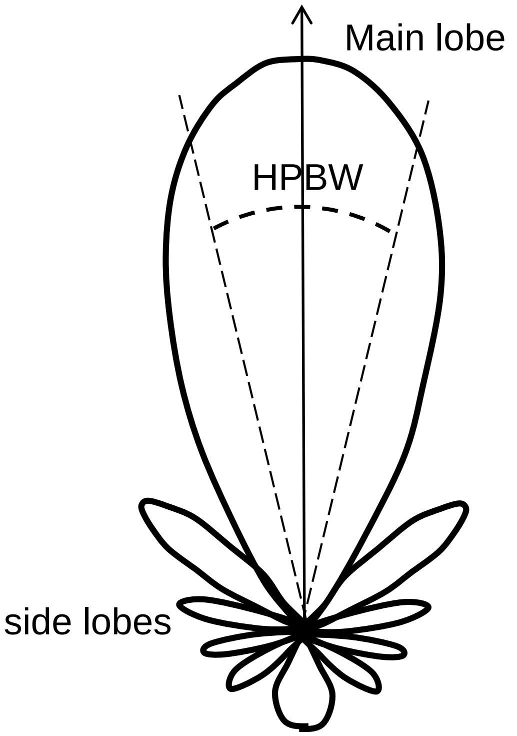


Antennas in a wireless system.

Antenna pattern characteristics

Learn from antenna book…!

- **Far field radiation pattern:**
  - Beamwidth: half-power beamwidth and first null beamwidth.
  - Radiation Intensity: power radiated from an antenna per unit solid angle in a given direction
  - Directivity: The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions (ratio of its radiation intensity in a given direction over that of an isotropic source).
  - Gain = 4π×radiation intensity/ total input accepted power
  - Polarization: direction of electric field.
- **Impedance bandwidth**



Radiation pattern of an antenna

Aperture efficiency and effective aperture

- For aperture antennas (e.g. reflector antennas, horn antennas, lens antennas, array antennas etc.), maximum directivity that can be obtained from an electrically large aperture of area A is

$$D_{\max} = \frac{4\pi A}{\lambda^2}.$$

- Practically, achieved directivity is much lower for nonideal amplitude or phase characteristics of the aperture field, aperture blockage, etc.
- Define an aperture efficiency as the ratio of the actual directivity of an aperture antenna to the maximum directivity. Then, directivity of an aperture antenna is

$$D = \eta_{ap} \frac{4\pi A}{\lambda^2}.$$

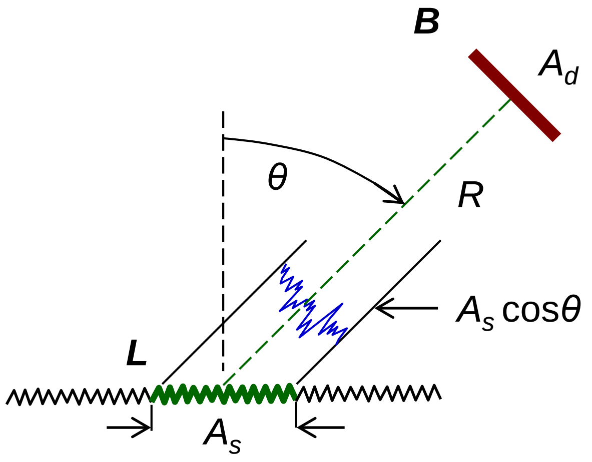
- For a receiving antenna, considering the effective aperture A_e, the received power is $P_r = A_e S_{\text{avg}}$

- Then, maximum effective aperture area of an antenna is $A_e = \dfrac{D\lambda^2}{4\pi}$

Background and brightness temperature

External source of noise:

- Background radiation from sky.
- Reflected sun rays.
- Ground radiation.
- Man made noise – other users, radio/TV stations, ignition, discharge lamp etc.



Brightness B W/m^2 sr Hz at a collector due to surface of radiance L.

Represented by an equivalent resistor producing an available output noise power $N_o = kTB$.

External noise from nature can be estimated and represented by background noise temperature, T_B.

- Sky (toward zenith) 3–5 K
- Sky (toward horizon) 50–100 K
- Ground 290–300 K

The effective brightness temperature seen by the antenna can be found by weighting the spatial distribution of background temperature by the pattern function of the antenna and Planck's law of radiation.

Friis transmission equation

- Power density radiated by an isotropic antenna (D = 1 = 0 dB), $S_{avg} = \frac{P_t}{4\pi r^2}\ W/m^2,\ \ P_t$ is transmitted power.
- Power density radiated by an antenna with gain G_t, $S_{avg} = \frac{G_t P_t}{4\pi r^2}\ W/m^2$.
- Power received by an antenna with effective area A_e, $P_r = A_e S_{avg} = \frac{G_t P_t A_e}{4\pi r^2}\ W$.
- Considering losses in the receiver antenna, $P_r = A_e S_{avg} = \frac{G_t G_r \lambda^2}{(4\pi r)^2} P_t\ \ \mathrm{W}$.

Effective isotropic radiated power = $P_t\, G_t$

- Considering ideal scenario, $\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2$, P_r, P_t *are antenna powers*, $G_t,\ G_r$ are antenna gains,

  λ freespace wavelength, r is the distance.

$$P_r = P_t + G_t + G_r + 20\log_{10}\left(\frac{\lambda}{4\pi r}\right)\ \mathrm{dB}.$$

Friis transmission equation

Ideal scenario:

- $r >> \lambda$
- The antennas are unobstructed, no multipath, no atmospheric scattering or attenuation effect,
- Antennas are ideal (no feed line loss) and are in right polarization.
- Bandwidth is narrow enough for single frequency consideration (no pulse).
- The above conditions can be achieved only in anechoic chamber or in free space.

Considering the effect of antennas, $$\frac{P_r}{P_t} = G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r) \left(\frac{\lambda}{4\pi r}\right)^2 \left(1 - |\Gamma_t|^2\right)\left(1 - |\Gamma_r|^2\right) |a_t . a_r^*|^2 e^{-\alpha r}$$

where, Γ_t, Γ_r are reflection coefficients of the transmit and receive antennas

a_t, a_r are the polarization vectors taken in appropriate directions.

α is the absorption coefficient in the medium.

- The above model does not include the multipath effects.
- Sometimes empirical adjustment is made to the Friis equation, $\frac{P_r}{P_t} \propto G_t G_r \left(\frac{\lambda}{r}\right)^n$

Link budget and link margin

Link budget: signal power plan under the given condition.

Free space path loss: $20\log_{10}\left(\dfrac{4\pi r}{\lambda}\right)$ dB.

Transmit power	P_t
Transmit antenna line loss	(-) L_t
Transmit antenna gain	G_t
Free space path loss	(-) L_o
Atmospheric attenuation	(-)L_A
Receive antenna gain	G_r
Receive antenna line loss	(-) L_r
Receive power	P_r

In decibel scale, $P_r = P_t + G_t + G_r - PL(r) - (L_t + L_r)$ dB

In practical communications systems, the received power level should be greater than the threshold level specified by a minimum carrier-to-noise ratio (CNR), or minimum SNR.

A design allowance for received power is referred to as the *link margin*, and is expressed as

Link margin = $P_r - P_{r(min)}$.

Link margin example

A geostationary satellite link operates over 35-36 GHz band with a transmit carrier power of 120 W, transmit antenna gain = 34 dB, IF bandwidth = 20 MHz, maximum distance = 39,000 km. The receive antenna gain is 30 dB and has an effective temperature T_e = 100 K. The required minimum SNR is 10 dB. Check for link margin (suggested minimum value is 3 dB).

Answer:

Therefore, minimum SNR required at the output of the receive antenna is 13 dB.

$$\text{Path loss} = 20\log\left(\frac{4\pi r}{\lambda}\right) = 20\log\left(\frac{4\pi\times 39\times 10^{9}}{0.00845}\right) = 215.3\text{ dB}$$

$$P_t = 120\ W = 50.8\text{ dBm}$$

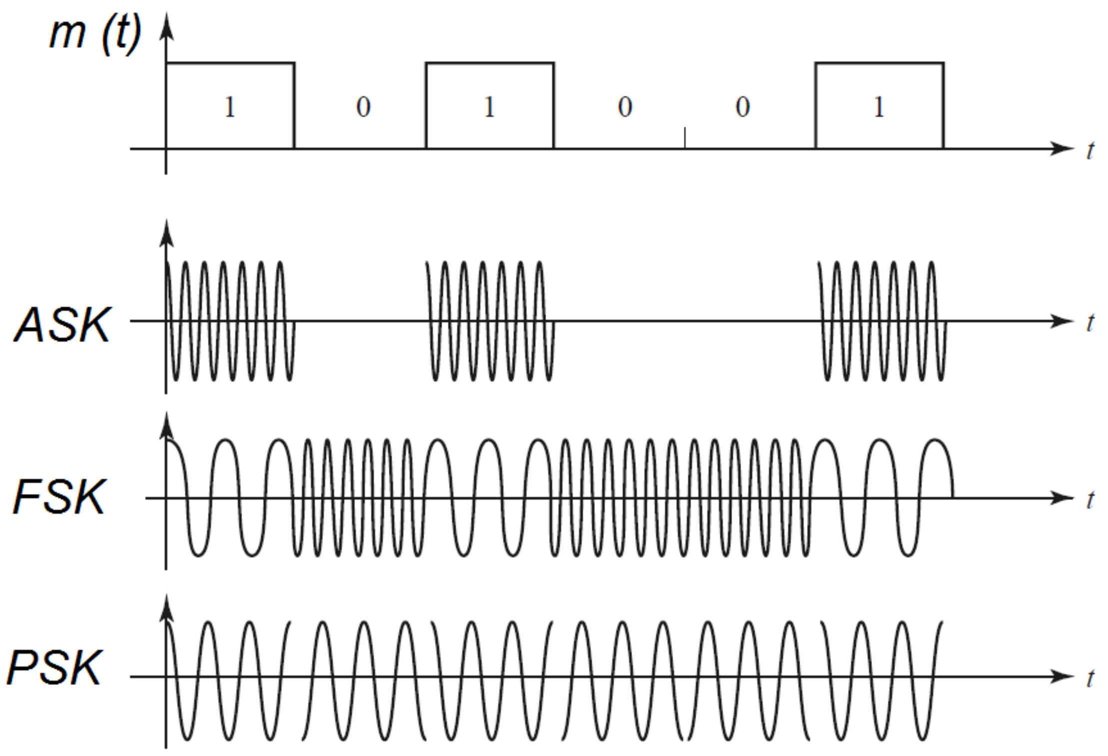
$$\therefore \mathrm{P}_r = 50.8 + 34 - 215.3 + 30 = -100.5\text{ dBm} = 8.91\times 10^{-14}\ \mathrm{W}.$$

SNR at the output: $$SNR = \frac{P_r}{kT_eB} = \frac{8.91\times 10^{-14}}{1.38\times 10^{-23}\times 100\times 20\times 10^{6}} = 3.23 = 5.09\text{ dB}.$$

Digital modulation

Advantages over analog:

- Superior performance in presence of noise and fading
- Low power requirement
- Error correction and encryption.



Different modulation schemes.

Digital modulation and bit error rate

Bit error probability (P_b) depends on the ratio of received bit energy (E_b) to noise power density in the channel (n_o) .

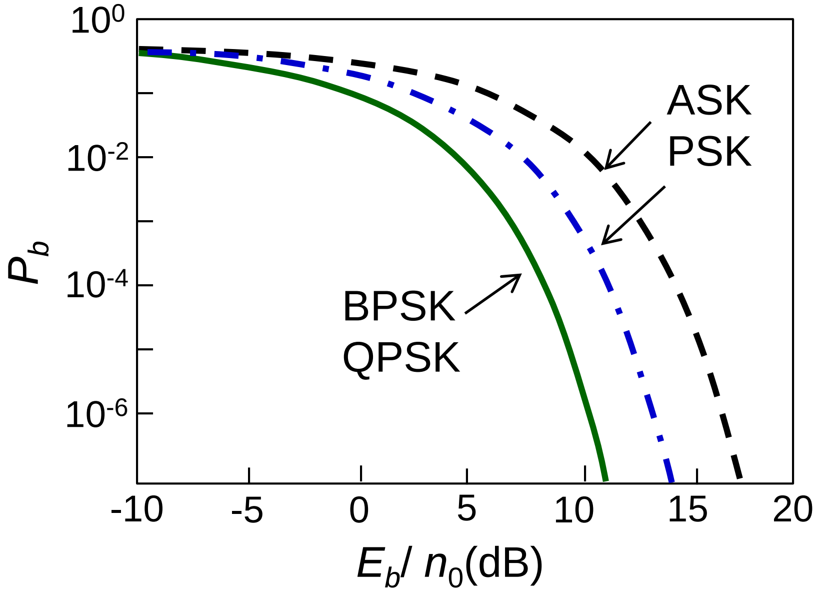
$$E_b = ST_b = S/R_b \quad W-\text{Sec.},$$

S – received signal power (carrier),

T_b-bit period, R_b – bit rate.

$$\therefore \frac{E_b}{n_o} = \frac{ST_b}{n_o} = \frac{S}{n_o R_b} = \frac{S}{N} BT_b,$$

N – noise power, B – bw of the receiver.



Comparison of bit error rate considering coherent demodulation.

Digital modulation and bit error rate

Modulation Type	E_b/n_0(dB) for $P_b = 10^{-5}$	Bandwidth Efficiency
Binary ASK	15.6	1
Binary FSK	12.6	1
Binary PSK	9.6	1
QPSK	9.6	2
8-PSK	13.0	3
16-PSK	18.7	4
16-QAM	13.4	4
64-QAM	17.8	6

Comparison of required E_b/ n_0(dB) for $P_b= 10^{-5}$ for different modulation schemes.

Minimum SNR

An UAV uses QPSK to communicate with its base (r = 100 km). What is the maximum data rate for a bit error probability of 10^{-5} (i.e. required E_b/n_0 = 9.6 dB = 9.12)? Given that transmit power is 1 W, carrier frequency = 77 GHz, transmit antenna gain = 10 dB, receive antenna gain 30 dB, system temperature = 750 K, atmospheric loss = 2 dB, required link margin = 10 dB.

Answer: $$\text{Path loss} = 20\log\left(\frac{4\pi r}{\lambda}\right) = 20\log\left(\frac{4\pi\times100\times10^{3}}{0.0039}\right) = 170.2\ \text{dB}$$

$$\therefore \mathrm{P}_r = P_t + G_t - PL - L_a + G_r = 30 + 10 - 170.2 - 2 + 30 = -102.2\ \text{dBm}.$$

Considering LM = 10 dB,

$$\mathrm{S}_{\min} = P_r - LM = -102.2 - 10 = -112.2\ dBm = 6.03\times10^{-15}\ W.$$

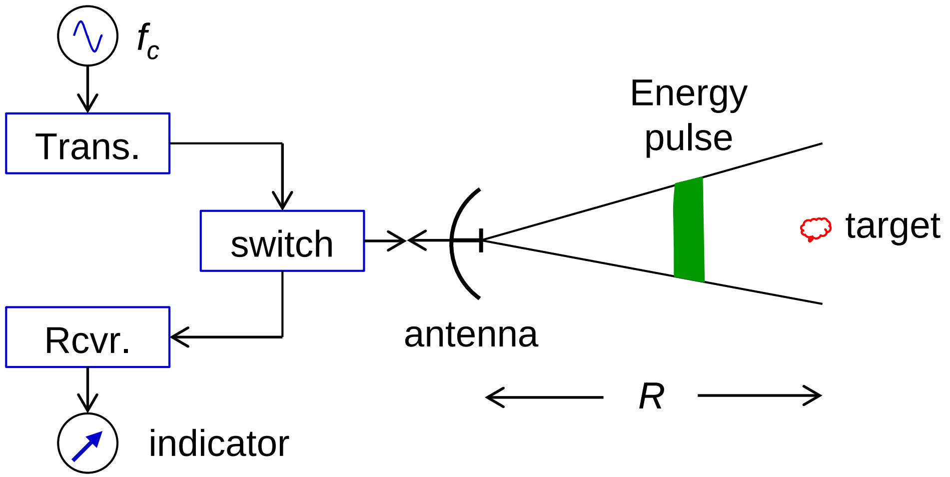
$$\therefore R_b = \left(\frac{S_{\min}}{n_0}\right)\Big/\left(\frac{E_b}{n_0}\right) = \left(\frac{S_{\min}}{kT_{sys}}\right)\Big/\left(\frac{E_b}{n_0}\right) = \frac{1}{9.12}\times\frac{6.03\times10^{-15}}{1.38\times10^{-23}\times750} = 63.9\ kbps.$$

Radar equation

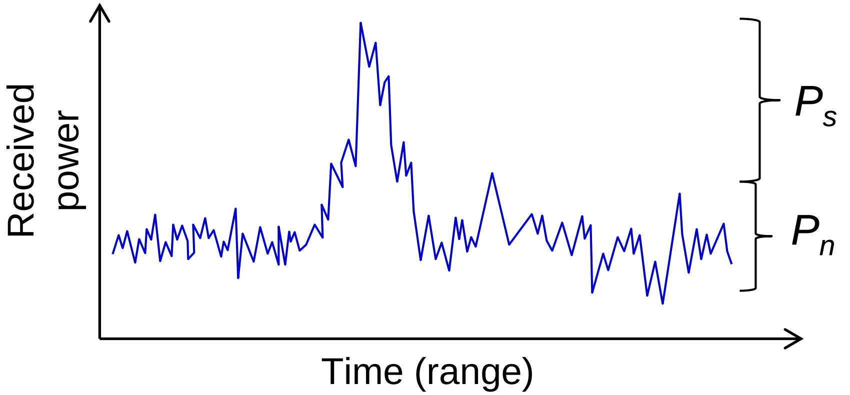
SNR at the input of a receiver:

$$SNR_i = P_t G^t \sigma A_e \Big/ \left[\left(4\pi R^2\right)^2 FkT_A B \right]$$

- Assuming a point target and omnidirectional scattering.
- P_t – average transmitted power during transmission.
- A_e – receiving aperture.
- σ – effective radar cross section of the target →depends on the scattered power in the direction of receiving antenna (aspect angle of the target, material, wavelength, presence of resonance phenomena etc). If the target is not a point target, define backscatter coefficient (direction sensitive).
- $P_t G_t$ – Effective isotropic radiated power W/m^2.
- F – noise factor of the receiver.



A radar system.



Received pulse.

Radar equation

Different type of radars: pulse radar, Doppler radar.

- The scattered power density at a range *R*,

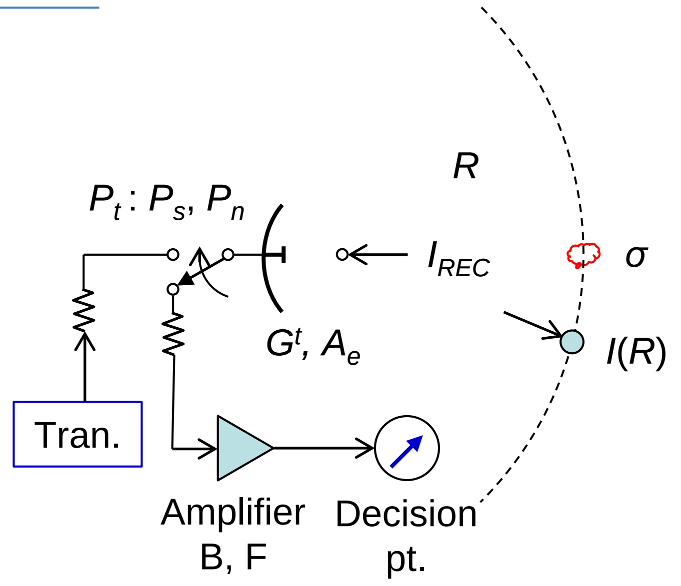
$$I(R) = P_t G^t \sigma \Big/ \left(4\pi R^2\right)^2 \qquad \text{W/m}^2.$$

- The receiver input noise,

$$kT_A B \quad \text{W}, \qquad k = 1.38\times 10^{-23}\ J/K, \quad T_A = \text{antenna temp.}, \quad B = B.W.$$

- Received power,

$$P_r = I(R) A_r, \quad A_r \text{ receiving antenna aperture.}$$



Relation of radar equation parameters to system elements.

object	Radar cross section	object	Radar cross section
Sphere	πr^2 (max value)	bicycle	1-2 m^2
Square plane	$4\pi a^4/\lambda^2$(max value)	truck	100-200 m^2
Human	1 m^2	Small plane	1-2 m^2
bird	0.01 m	Airliner	50-150 m^2

Radiometer

- Radiometer is a calibrated passive receiver, which develops information about a target solely from the blackbody radiation (noise) that it either emits directly or reflects from surrounding bodies.
- A radiometer is a sensitive receiver specially designed to measure this noise power.

Environmental applications

- Measurement of soil moisture
- Flood mapping
- Snow cover/ice cover mapping
- Ocean surface wind speed
- Atmospheric temperature profile
- Atmospheric humidity profile

Military applications

- Target detection
- Target recognition
- Surveillance
- Mapping

Astronomy applications

- Planetary mapping
- Solar emission mapping
- Mapping of galactic objects
- Measurement of cosmological background radiation

Brightness temperature of an object

- The radiated power kTB is true for a perfect blackbody.
- For a practical surface define emissivity as

$$e = \frac{P}{kTB}$$

P is the radiated power by a nonideal body.

- Define a brightness temperature as $T_B = eT$.

Effective emissivity of common materials at various frequencies.

	Effective Emissivity		
Surface	44 GHz	94 GHz	140 GHz
Bare metal	0.01	0.04	0.06
Painted metal	0.03	0.10	0.12
Painted metal under canvas	0.18	0.24	0.30
Painted metal under camouflage	0.22	0.39	0.46
Dry gravel	0.88	0.92	0.96
Dry asphalt	0.89	0.91	0.94
Dry concrete	0.86	0.91	0.95
Smooth water	0.47	0.59	0.66
Rough or hard-packed dirt	1.00	1.00	1.00

Dicke radiometer