Tutorial - 2

RF & Microwave Theory

(EC31005)

1. A two-port network is driven at both ports such that the port voltage and currents have the following values (Z_0= 50 Ω)

$$V_1 = 10\angle 90^0, \quad I_1 = 0.2\angle 90^0$$

$$V_2 = 8\angle 0^0, \quad I_2 = 0.16\angle -90^0.$$

Determine the input impedance seen at each port, and find the incident and reflected voltage at each port.

Given that $V_1 = 10\angle 90^0$, $I_1 = 0.2\angle 90^0$ and $V_2 = 8\angle 0^0$, $I_2 = 0.16\angle -90^0$.

The input impedance seen at port 1,

$$Z_{in(1)} = \frac{V_1}{I_1} = \frac{10\angle 90^0}{0.2\angle 90^0} = 50\ \Omega.$$

And input impedance seen at port 2,

$$Z_{in(2)} = \frac{V_2}{I_2} = \frac{8\angle 0^0}{0.16\angle -90^0} = 50\angle 90^0 = (0+j50)\ \Omega.$$

As the incident and reflected voltage at port (n) is given by,

$$V_n^{+} = \frac{(Vn+Z_0 I_n)}{2} \text{ and } V_n^{-} = \frac{(V_n - Z_0 I_n)}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$$

contd.

So incident voltage at port 1,

$$V_1^{\ +} = \frac{(10\angle 90^{0} + 50 * 0.2\angle 90^{0})}{2} = \frac{j10 + j10}{2} = \mathrm{j}10 = 10\angle 90^{0}$$

And the reflected Voltage At Port 1,

$$V_1^{\ -} = \frac{(10\angle 90^{0} - 50 * 0.2\angle 90^{0})}{2} = 0$$

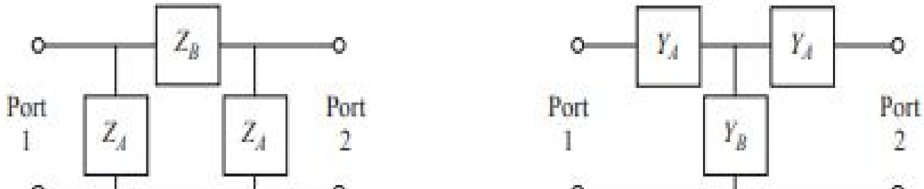
Now The Incident Voltage At Port 2,

$$V_2^{\ +} = \frac{(8\angle 0^{0} + 50 * 0.16\angle -90^{0})}{2} = \frac{8 - j8}{2} = 5.65\angle -45^{0}$$

And hence the reflected voltage at port 2,

$$V_2^{\ -} = \frac{(8\angle 0^{0} - 50 * 0.16\angle -90^{0})}{2} = \frac{8 + j8}{2} = 5.65\angle 45^{0}$$

2. Derive the [Z] and [Y] matrices for the two-port networks shown in the figure below



(a) (b)

If we have voltage and current at port 1 and port 2 is (V_1,I_1) and (V_2,I_2) respectively, then from the two port network, applying KVL and KCL,

$$Z_{11} = \frac{V_1}{I_1}(given\ I_2 = 0) = \frac{V_1}{V_1(\frac{2ZA+Z_B}{Z_A(ZA+ZB)})} = \frac{Z_A(ZA+ZB)}{2ZA+ZB} = Z_{22}\ \ (By\ Symmetry)$$

$$Z_{21} = \frac{V_2}{I_1}\ (\text{given } I_2 = 0) = \frac{I_1 Z_{11}(\frac{Z_A}{Z_A+ZB})}{I_1} = \frac{Z^2{}_A}{2ZA+ZB} = Z_{12}\ \ (By\ Symmetry)$$

$$Y_{11} = \frac{I_1}{V_1}\ \ (given\ V_2 = 0) = \frac{I_1}{I_1(\frac{Z_A Z_B}{Z_A+ZB})} = \frac{Z_A+ZB}{Z_A Z_B} = Y_{22}\ \ \ (By\ Symmetry)$$

Contd.

$$\mathrm{Y}_{21} = \frac{I_2}{V1} \ \ (given\ V_2 = 0) = \frac{-V_1}{Z_B V_1} = \frac{-1}{Z_B} = Y_{22} \quad (By\ Symmetry)$$

(b) If we have voltage and current at port 1 and port 2 are (V_1,I_1) and (V_2,I_2) respectively, then from the two port network, applying KVL and KCL,

$$\mathrm{Z}_{11} = \frac{V_1}{I_1} (given\ I_2 = 0) = \frac{(Y_A+Y_B)}{Y_A Y_B} = Z_{22}\ \ (By\ Symmetry)$$

$$\mathrm{Z}_{21} = \frac{V_2}{I_1}\ (\text{given } \mathrm{I}_2 = 0) = \frac{-1}{Y_B} = Z_{12}\ \ (By\ Symmetry)$$

$$\mathrm{Y}_{11} = \frac{I_1}{V_1}\ \ (given\ V_2 = 0) = \frac{Y_A(YA+YB)}{2Y_A+Y_B} = Y_{22} \quad (By\ Symmetry)$$

$$\mathrm{Y}_{21} = \frac{I_2}{V_1}\ \ (given\ V_2 = 0) = \frac{Y^2{}_A}{2Y_A+Y_B} = Y_{22} \quad (By\ Symmetry)$$

3. Consider a lossless air-filled rectangular waveguide with dimensions a = 22.86 mm and b = 10.16 mm. Calculate the value of the propagation constant (per meter) of the corresponding propagation mode at 10 GHz operating frequency.

Ans: a = 22.86 mm, b = 10.16mm, frequency = 10 GHz
Assume dominant mode of propagation in the waveguide i.e. TE10 mode.
m = 1 and n = 0

Cut off frequency for TE10 mode is given by

$$f_c = \frac{c}{2}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2} = \frac{c}{2a} = \frac{3*10^{10}}{2*2.286} = 6.56 \text{ GHz}$$

Since cut off frequency is 6.56 GHz, the frequency of 10 GHz will propagate in the waveguide.
Propagation constant $\boldsymbol{\gamma}=\alpha+j\beta$
Since the waveguide is lossless, $\alpha=0$, β is phase constant

$$\beta=\frac{2\pi}{\lambda_g} = 2\pi \Big/ \lambda_0/\sqrt{1-(\frac{\lambda_0}{\lambda_c})^2} = \frac{2\pi f}{c}\sqrt{1-(\frac{f_c}{f})^2}$$

$$= \frac{2\pi.\ 10*10^9}{3*10^8}\sqrt{1-(\frac{6.56}{10})^2} \qquad = \mathbf{158}\ \boldsymbol{rad.m^{-1}}$$

4. An air – filled rectangular waveguide of cross sectional dimension a $\times b$ ($a > b$) has a cut off frequency of 6 GHz for the dominant TE10 mode. If the cutoff frequency of the TM11 mode is 15 GHz, calculate the cut off frequency of the TE01 mode

Ans: Cut off frequency f_c = 6GHz for dominant mode

For dominant mode cut off wavelength $\lambda_c = 2$a

λ_{c10}=$\frac{c}{f_{c10}}$ = $\frac{3*10^{10}}{6*10^{9}}$ = 5cm= 2a

a=2.5cm

f_c for TM11 mode is 15GHz

$f_c = \frac{c}{2}\sqrt{(\frac{1}{a})^2+(\frac{1}{b})^2}$ = $15^{x}10^{9}$

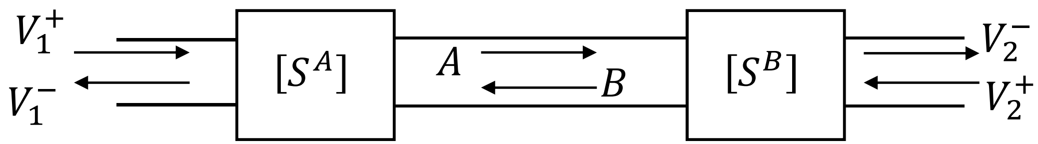
b=$\frac{2.5}{\sqrt{5.25}}$

f_c for TE01 mode = $\frac{c}{2b}$ = $\frac{3*10^{10}}{2.5}$ $\sqrt{5.25}$ =**13.75GHz**

5. Consider two two-port networks with individual scattering matrices $[S^A]$ and $[S^B]$. Show that the overall S_{21} parameter of the cascade of these networks is given by:

$$\boldsymbol{S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}}$$

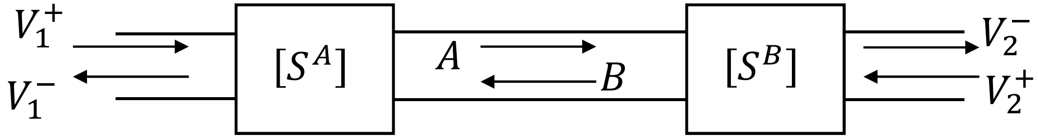
Considering wave amplitude as shown,



Then,

$$\begin{bmatrix} V_1^- \\ A \end{bmatrix} = [S^A] \begin{bmatrix} V_1^+ \\ B \end{bmatrix} \qquad \begin{bmatrix} B \\ V_2^- \end{bmatrix} = [S^B] \begin{bmatrix} A \\ V_2^+ \end{bmatrix} \qquad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = [S] \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix} \qquad S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0}$$

So, we have $B = S_{11}^B A$, and $V_2^- = S_{21}^B A$



$$A = S_{21}^A V_1^+ + S_{22}^A B = S_{21}^A V_1^+ + S_{22}^A S_{11}^B A$$

Putting the value of A, we get,

$$\frac{V_2^-}{S_{21}^B} = S_{21}^A V_1^+ + S_{22}^A S_{11}^B \frac{V_2^-}{S_{21}^B}$$

So,

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B}$$

6. A four-port network has the scattering matrix shown as follows.
(a) Is this network lossless?
(b) Is this network reciprocal?
(c) What is the return loss at port 1 when all other ports are terminated with matched loads?
(d) What is the insertion loss and phase delay between ports 2 and 4 when all other ports are terminated with matched loads?
(e) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are terminated with matched loads?

$$[S] = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix}.$$

(a) To be lossless, [S] must be unitary. From 1st row:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 0.552 \neq 1$$ => So, NOT lossless.

(b) The [S] matrix is symmetric, so it is reciprocal.

(c) When ports 2, 3, 4 are matched,

So, $RL = -20log|\Gamma| = -20\log(0.178) = 15\ dB$

(d) For ports 1 and 3 terminated with Z_0, we have

$V_1^+ = 0$, $V_3^+ = 0$, So, $V_4^- = S_{42}V_2^+$

$IL = -20log|S_{42}| = -20\log(0.3) = 10.5\ dB$

phase delay = +45^0

(e) For a short at port 3, Z_0 on the other ports, we have

$$V_2^+ = V_4^+ = 0$$
$$V_3^+ = -V_3^-$$
$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+ = S_{11}V_1^+ - S_{13}V_3^-$$
$$V_3^- = S_{31}V_1^+$$

Then,

$$\Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{13}S_{31} = 0.178j - (0.4\angle 45^0)(0.4\angle 45^0) = 0.178j - 0.16j = 0.018j =$$
$$0.018\angle 90^0$$

7. A transmission line resonator is fabricated from a λ/4 length of open-circuited line. Find the unloaded Q of this resonator if the complex propagation constant of the line is α + jβ.

Ans. Unloaded Q as Q_0 , Input Impedence as Z_{in}

$$l = \lambda / 4 = \frac{\pi v_p}{2\omega_0}$$

at $\omega = \omega_0$

$$\beta l = \omega_0 l \big/ v_p + \Delta\omega l \big/ v_p = \frac{\pi}{2}(1 + \frac{\Delta\omega}{\omega_0})$$

$$\tan\beta l = -\cot(\frac{\Delta\omega\pi}{2\omega_0}) = -\frac{2\omega_0}{\Delta\omega\pi}$$

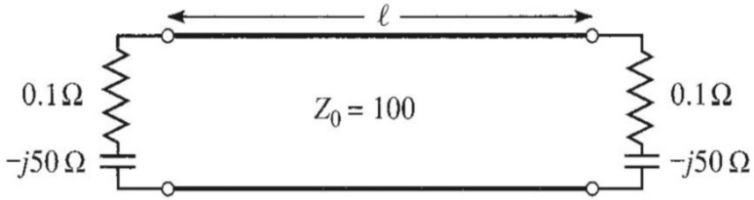
$$\tanh\alpha l = \alpha l$$

$$Z_{in} = Z_0 \frac{1 + j\tan\beta l \tanh\alpha l}{\tanh\alpha l + j\tan\beta l} = Z_0 \frac{1 - j\dfrac{2\omega_0}{\Delta\omega\pi}\alpha l}{\alpha l - j\dfrac{2\omega_0}{\Delta\omega\pi}} = Z_0(\alpha l + j\frac{\Delta\omega\pi}{2\omega_0}) = R + 2jL\Delta\omega$$

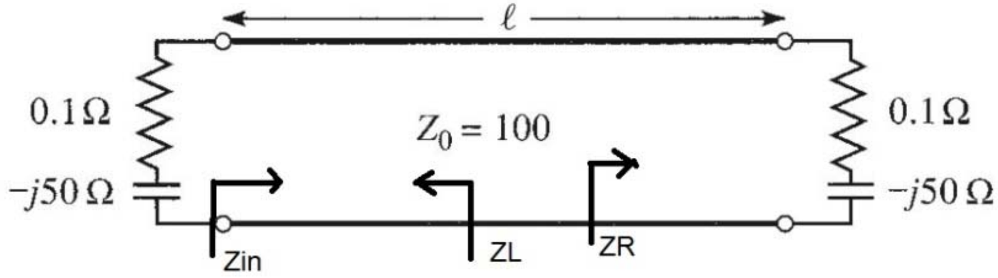
$$R = Z_0 \alpha l, L = \frac{\pi Z_0}{4\omega_0}$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{\pi}{4\alpha l} = \frac{\beta}{2\alpha}$$

8. A transmission line resonator is made from a length *l* of lossless transmission line of characteristic impedance Z_0 = 100 Ω . If the line is terminated at both ends as shown below, find l/λ for the first resonance, and the unloaded Q of this resonator.



Ans.



Since the resonator is symmetrical, at midpoint of line we must have , $Z_L = Z_R^* = Z_R$
Let t = tan βl/2 and $Z_L = R_L + jX_L$, (R_L=0.1, X_L=-50)

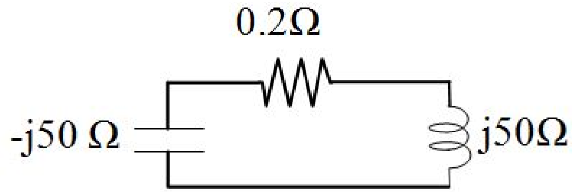
$$Z_R = Z_O \frac{Z_L + jZ_O t}{Z_O + jZ_L t} = Z_O \frac{R_L + j(Z_L\,\mathrm{t} + X_L)}{(Z_O - X_L\,\mathrm{t}) + jR_L\,\mathrm{t}}$$

$$\mathrm{Im}\left\{Z_R\right\} = 0 \Rightarrow (X_L + Z_O t)(Z_o + X_L t) - R_L{}^2 t = 0$$

$$t = -0.75 \pm 1.25$$

$$\beta l = 53.1^o$$

So, $$l = \frac{53.1^o}{360^o}\lambda = 0.1475\lambda$$



$$\therefore l/\lambda = 0.1475$$

$$\tan\beta l = 1.332$$

$$Z_{in} = Z_O \frac{Z_L + jZ_O \tan\beta l}{Z_O + jZ_L \tan\beta l} = 100\frac{(.1 - j50) + j133.2}{100 + j(0.1 - j50)1.332} = 0.1 + j50\Omega$$

$$Q_0 = \frac{\omega_0 L}{R} = \frac{X_L}{R} = 50/0.2 = 250$$