SDM5013:
Deep Learning and Reinforcement Learning

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System Design and
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Lecture: Regression and Linear Regression

- Regression Problems
- Linear Regression

Problems for Today

- What should I watch this Friday?

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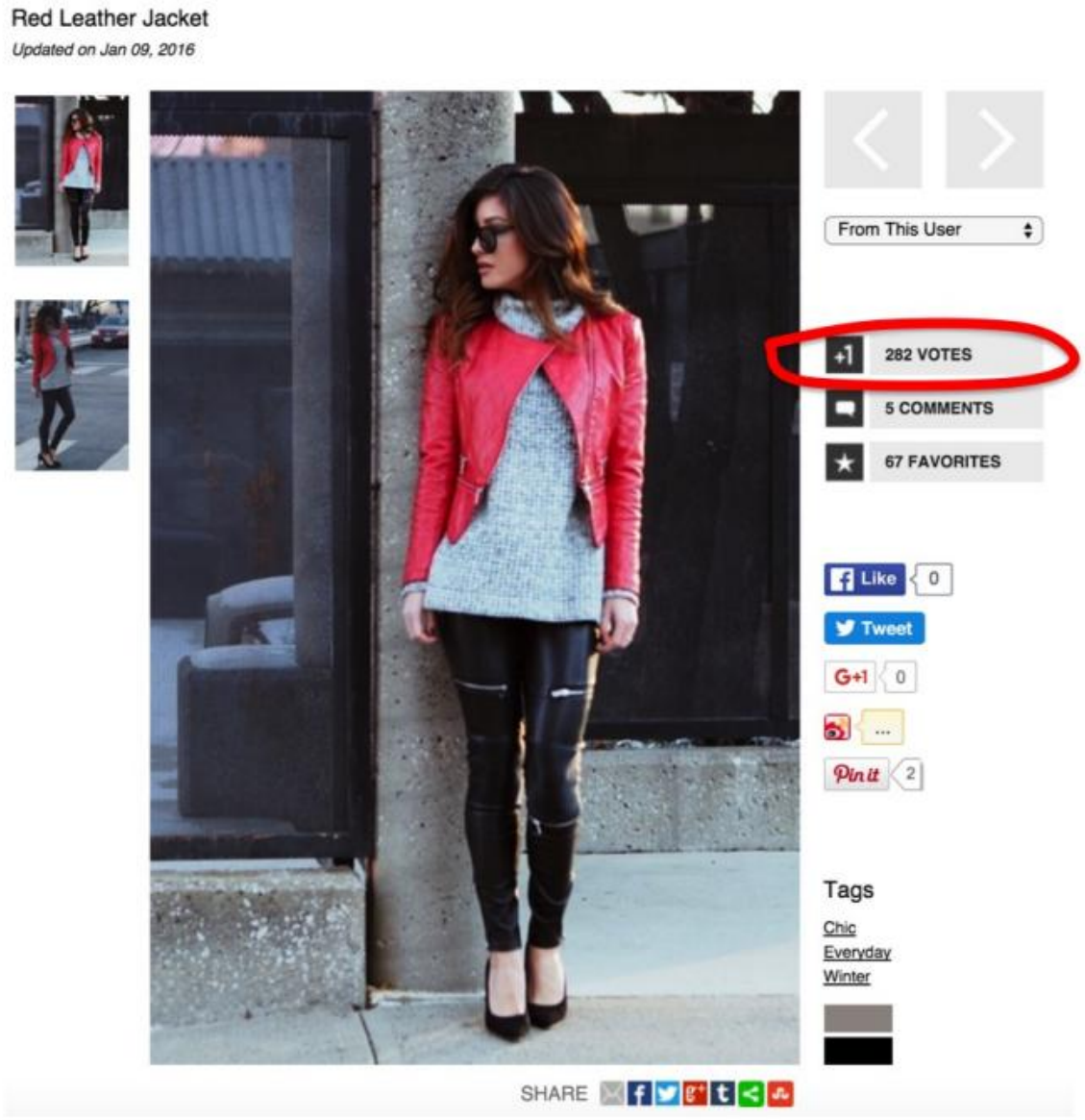
- **Goal**: Predict movie rating automatically!

Factors:
-Year of release of the film in cinemas
-Length of the film in minutes
-Budget for the films production
-Number of positive votes received by viewers
-Genre of the film including Action, Animation, Comedy, Documentary, Drama, Romance and Short

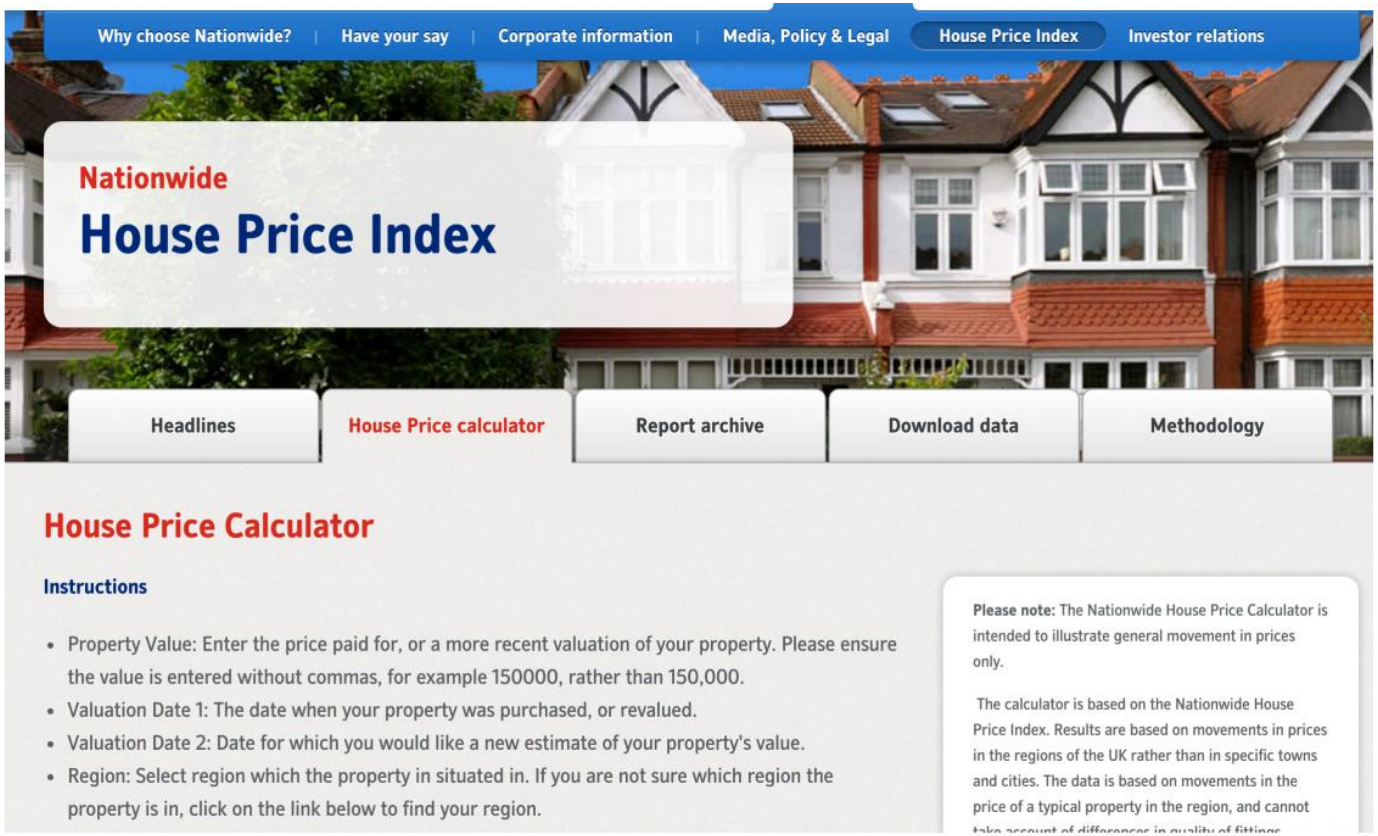
Problems for Today

- **Goal:** How many followers will I get?



Problems for Today

- **Goal:** Predict the price of the house



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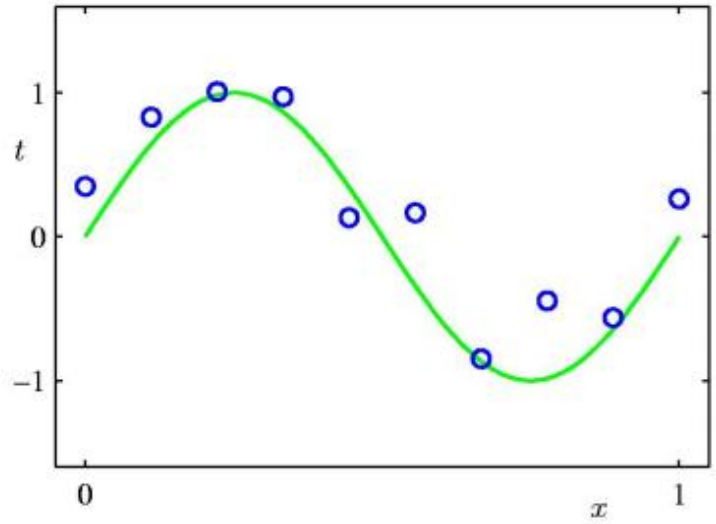

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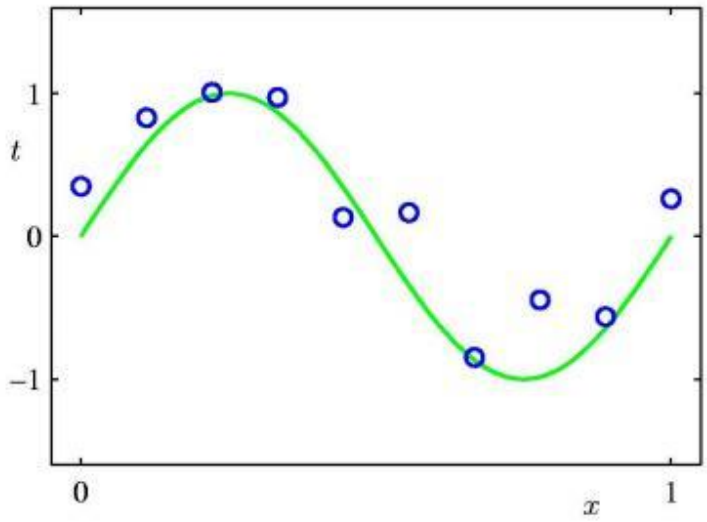

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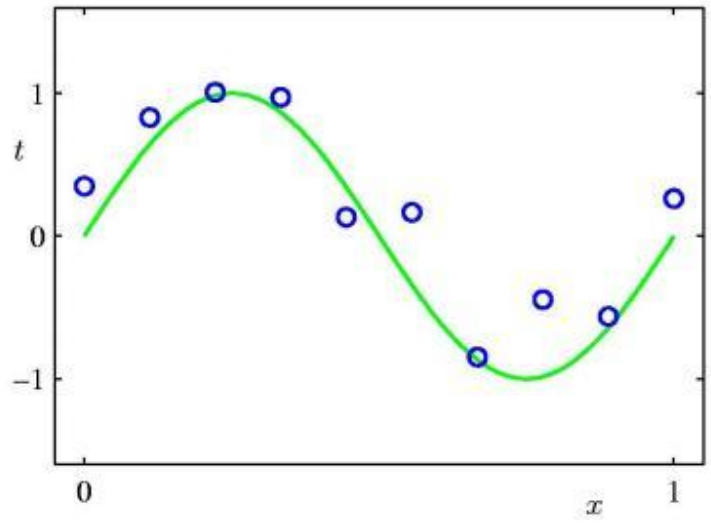

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- In green is the "true" curve that we don't know
- Goal: We want to fit a curve to these points

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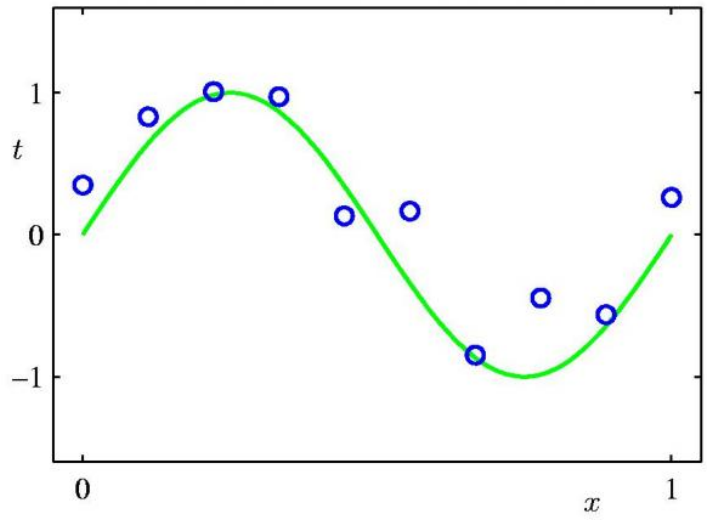

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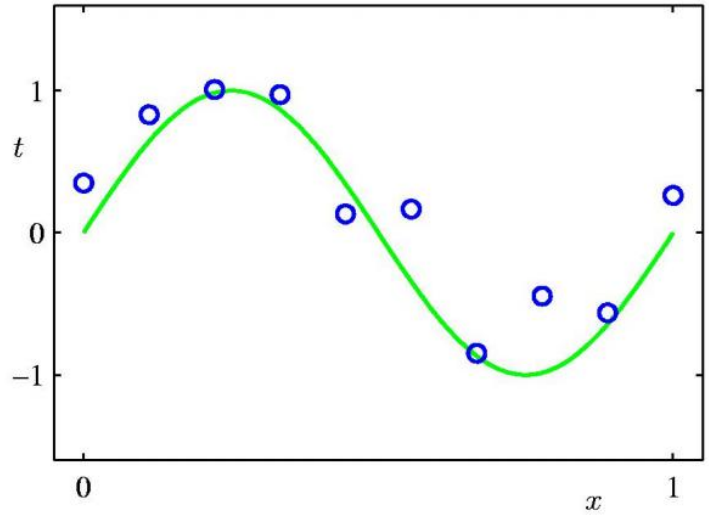

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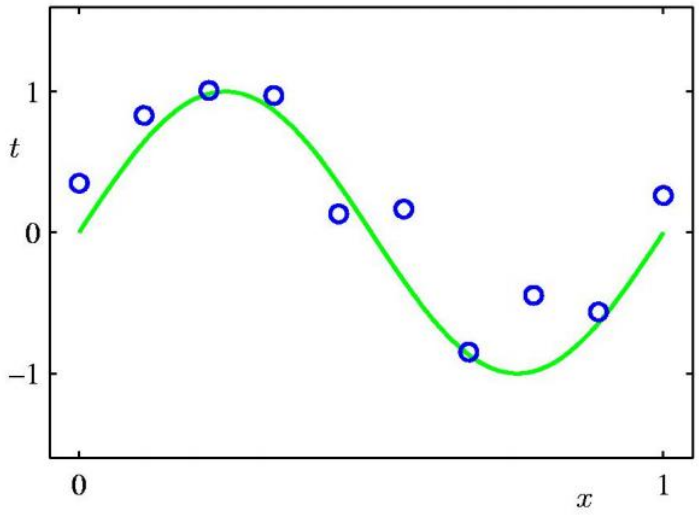

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  - How do we optimize fit to unseen test data (generalization)?

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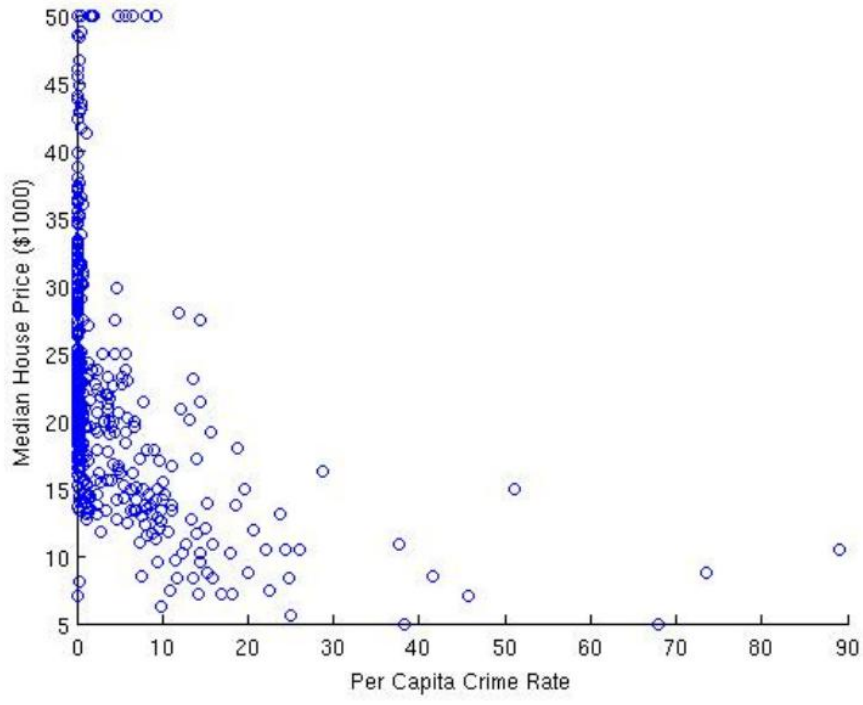

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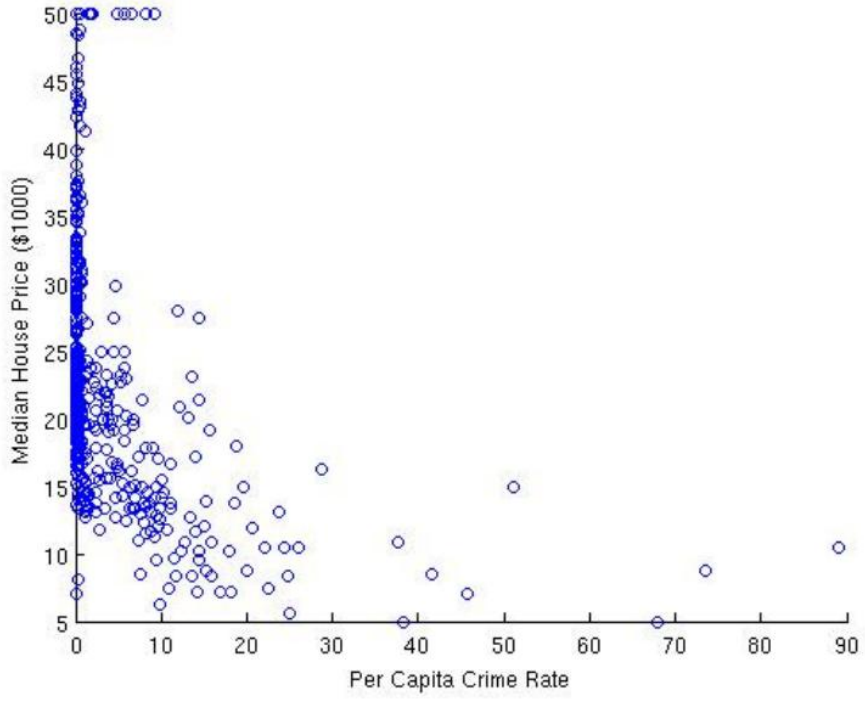

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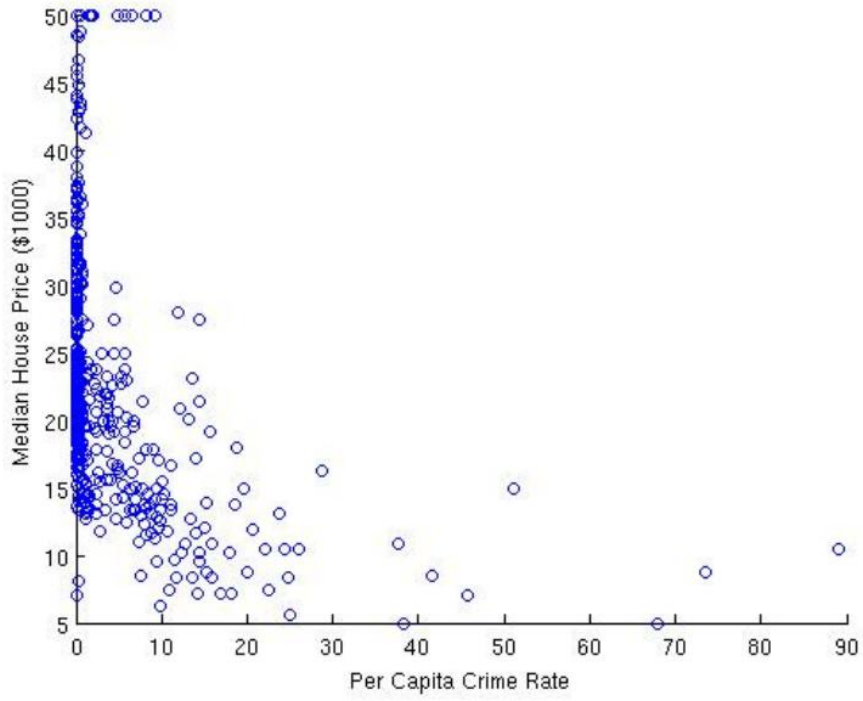

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- Is this a good input (attribute) to predict house prices?

Represent the Data

- Data is described as pairs $\mathcal{D} = \{(x^{(1)}, t^{(1)}), \cdots, (x^{(N)}, t^{(N)})\}$
  - $x \in \mathbb{R}$ is the input feature (per capita crime rate)
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  - Evaluate hypothesis on test set

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  - Model may be too simple to account for data targets

Summary of Regression

- Regression: to predict continuous outputs *t*
  - Consider proper features (inputs): *x* (or **x** if vectors)
  - Training examples, many $x(\mathrm{i})$ for which *t(i)* is known (labeled)
  - A model, a function that represents the relationship between *x* and t

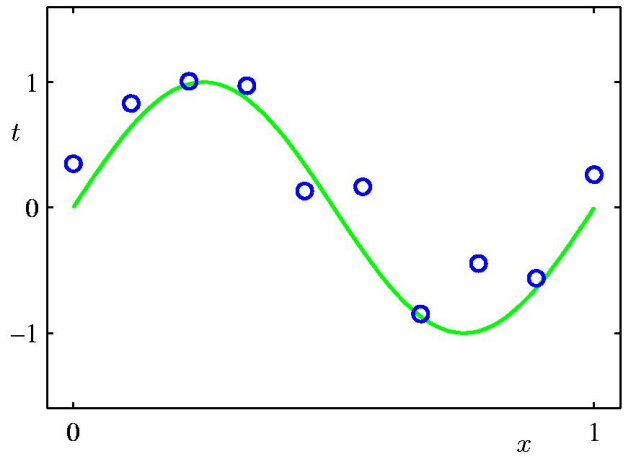
$$y = f(x, w)$$

  - A loss or a cost or an objective function, which tells us how well our model approximates the training examples
  - Optimization, a way of finding the parameters w of our model that minimizes the loss function

Today: Linear Regression

- Linear regression
  - continuous outputs
  - simple model (linear)
- Introduce key concepts:
  - loss functions
  - generalization
  - optimization
  - model complexity
  - regularization

Squared Error (SSE/MSE) Loss



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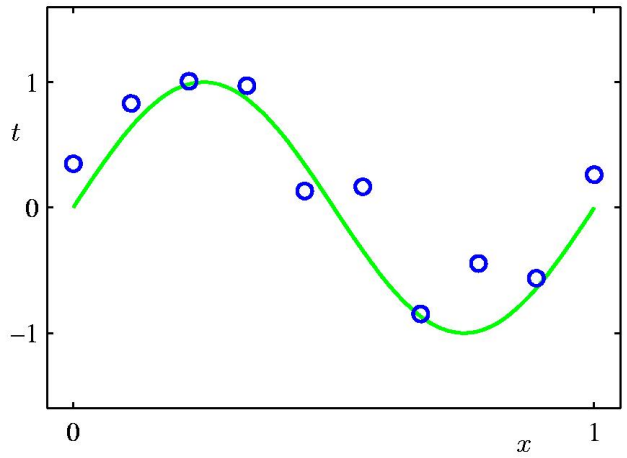

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$$y(x) = \text{function}(x, \mathbf{w})$$

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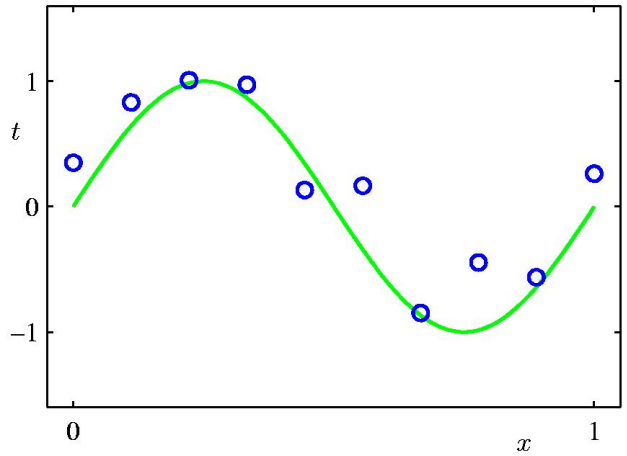

- Define a model

$$\text{Linear:} \qquad y(x) = w_0 + w_1 x$$

- Standard loss/cost/objective function measures the squared error between y and the true value t

$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - y(x^{(n)})]^2$$

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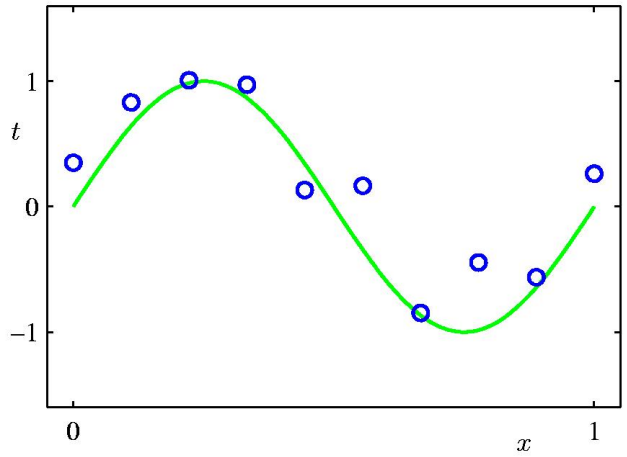

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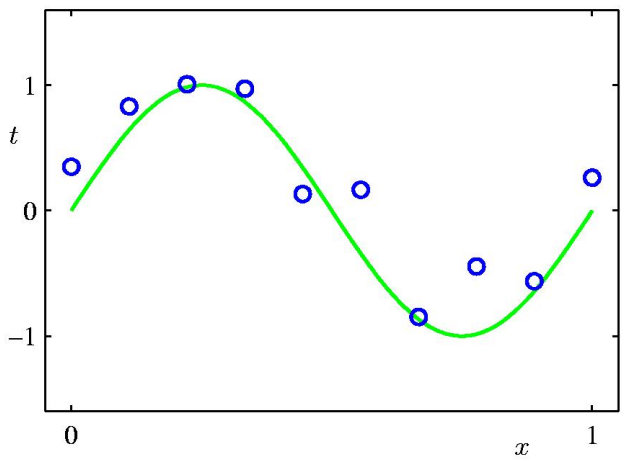

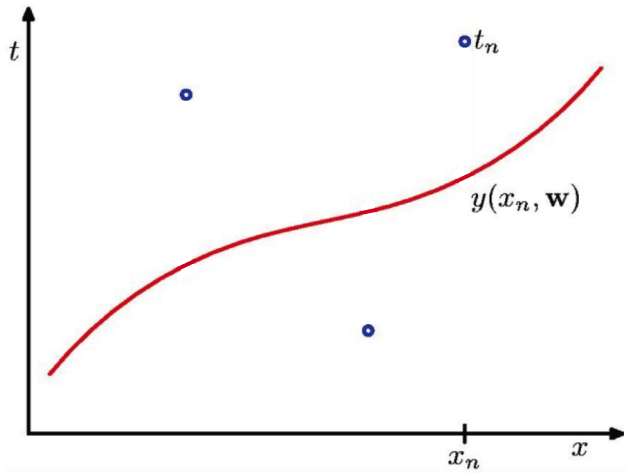

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- For a particular hypothesis ($y(x)$ defined by a choice of $\mathbf{w}$, drawn in red), what does the loss represent geometrically?

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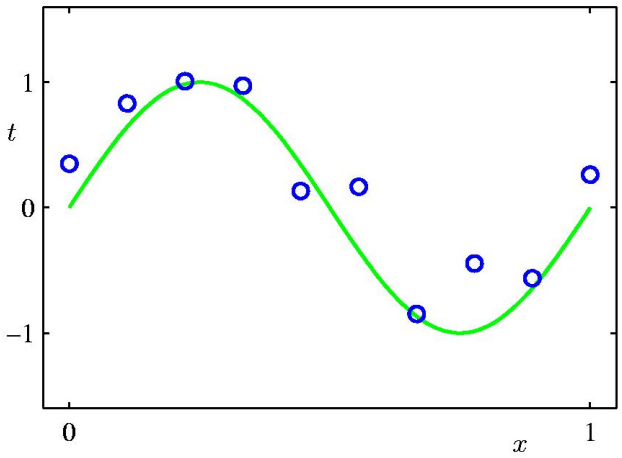

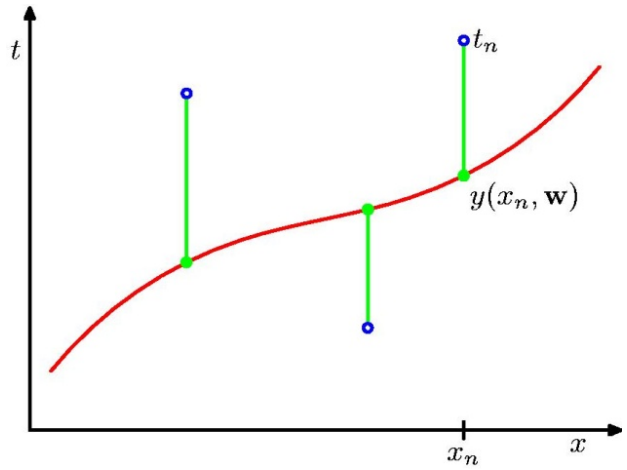

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- The loss for the red hypothesis is the **sum of the squared vertical errors** (squared lengths of green vertical lines)

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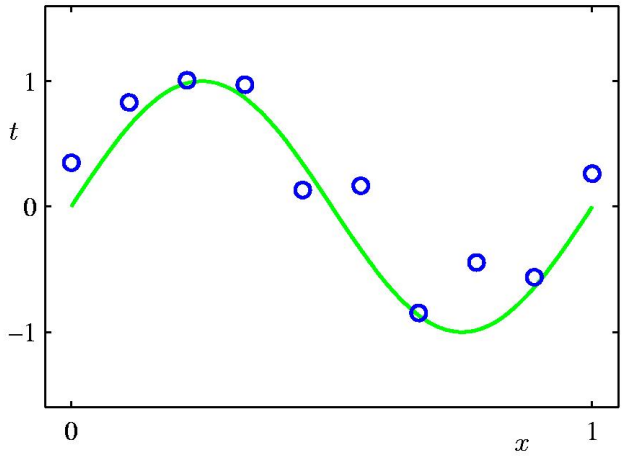

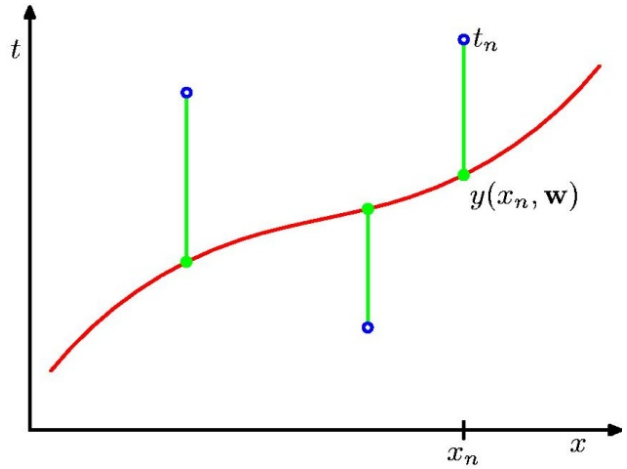

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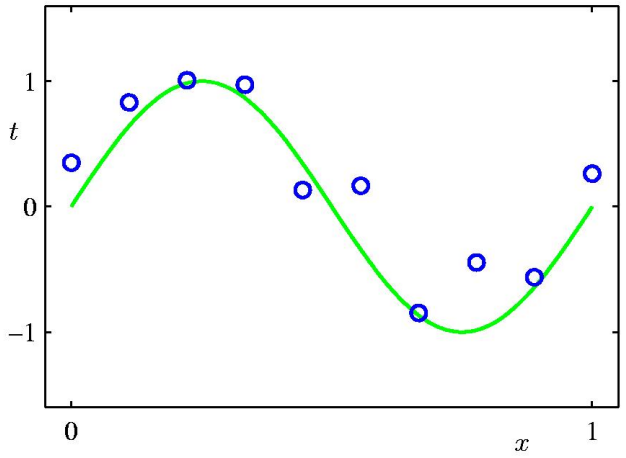

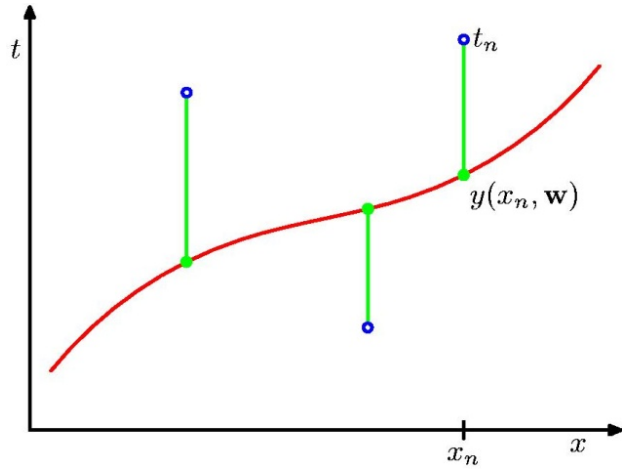

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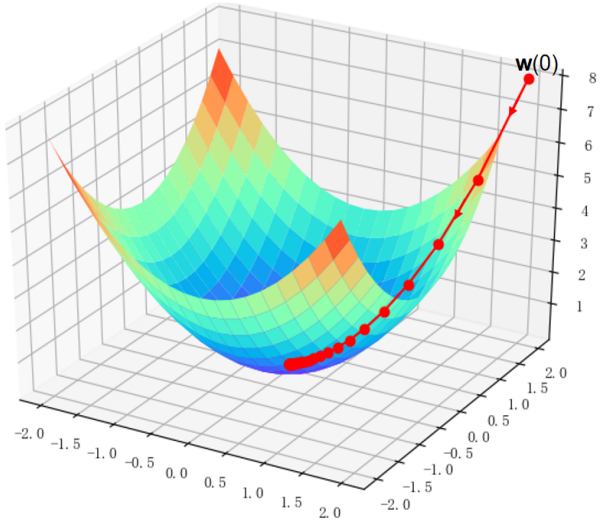

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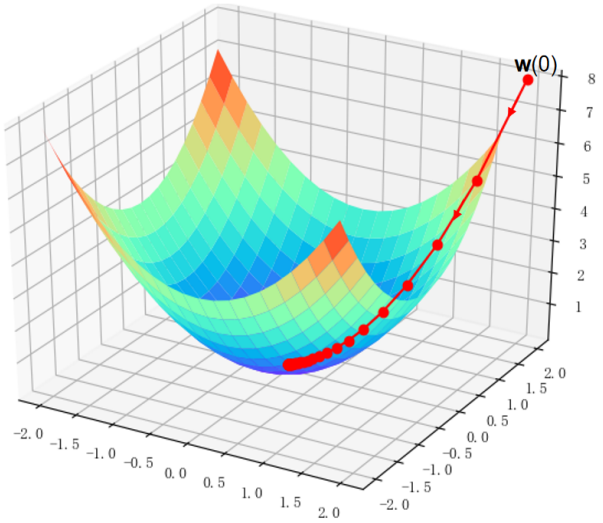

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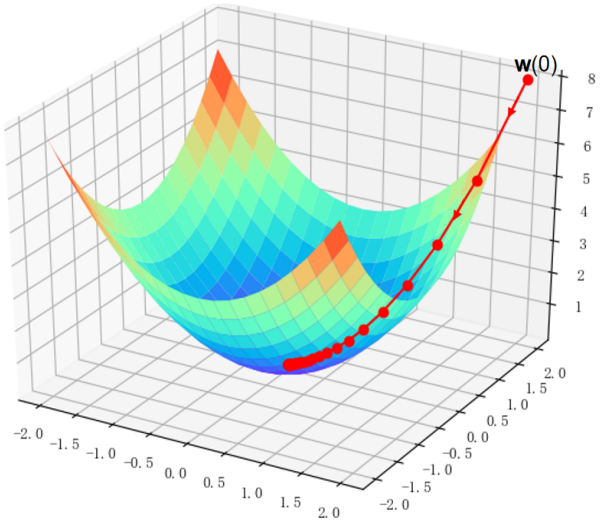

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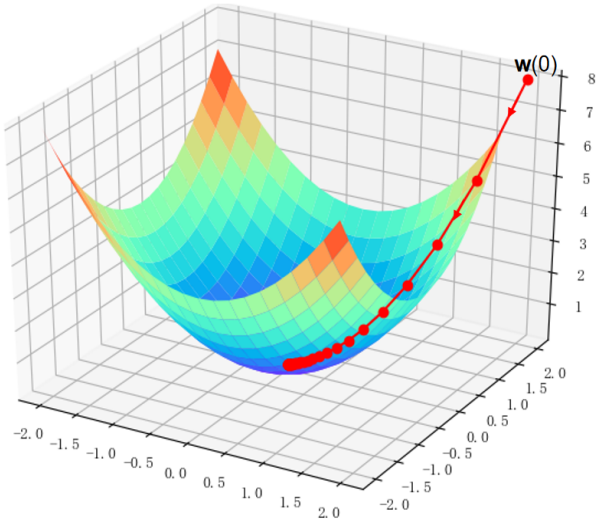

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$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda \underbrace{(t^{(n)} - y(x^{(n)}))}_{\text{error}} \begin{bmatrix} 1 \\ x^{(n)} \end{bmatrix}$$

- Note: As error approaches zero, so does the update ($\mathbf{w}$ stops changing)

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Algorithm 1 Stochastic gradient descent

1: Randomly shuffle examples in the training set
2: **for** $i = 1$ to N **do**
3: Update:

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda (t^{(i)} - y(x^{(i)})) \begin{bmatrix} 1 \\ x^{(n)} \end{bmatrix} \text{ (update for a linear model)}$$

4: **end for**

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  - Underlying assumption: sample is independent and identically distributed (i.i.d.)

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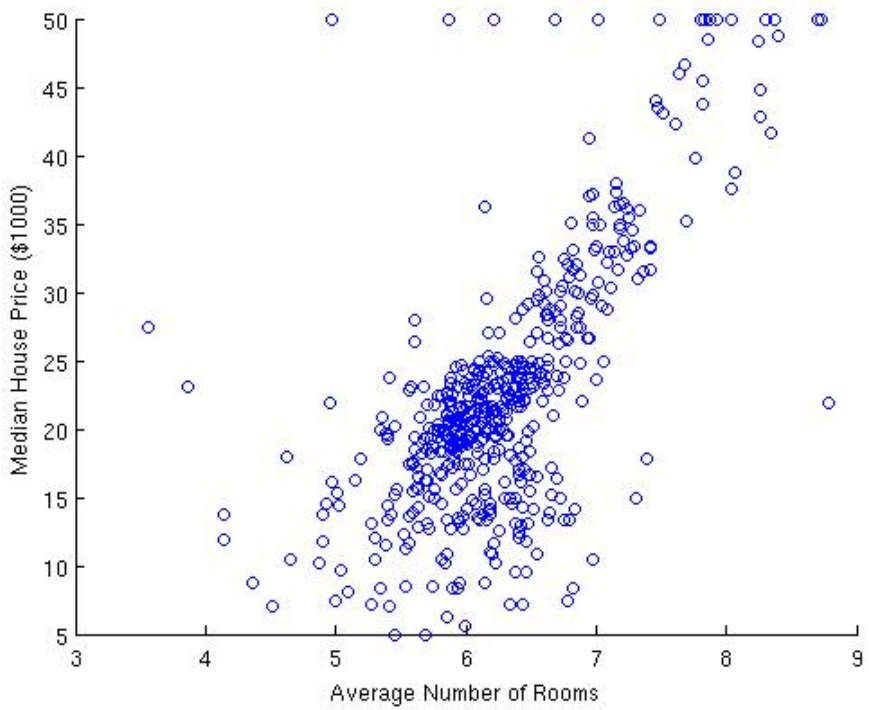
- In the Boston housing example, we can look at the number of rooms



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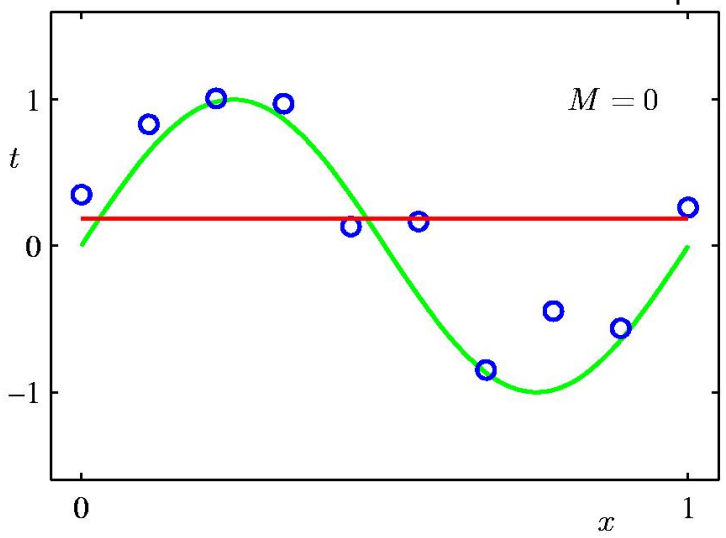
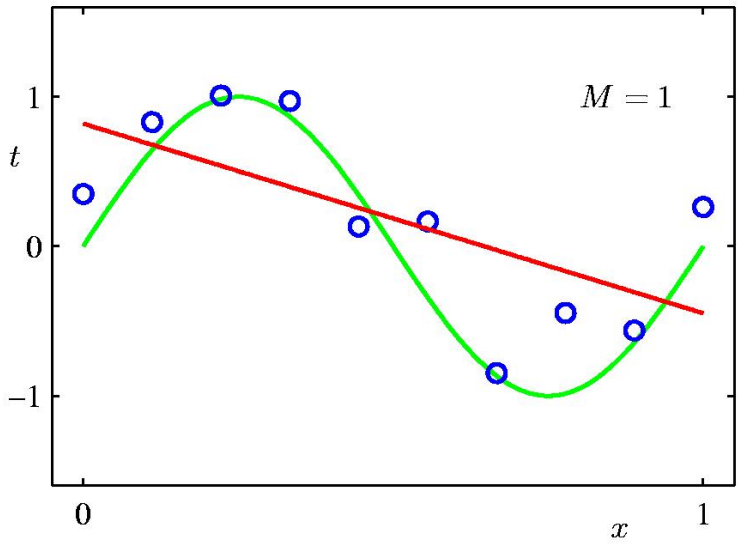
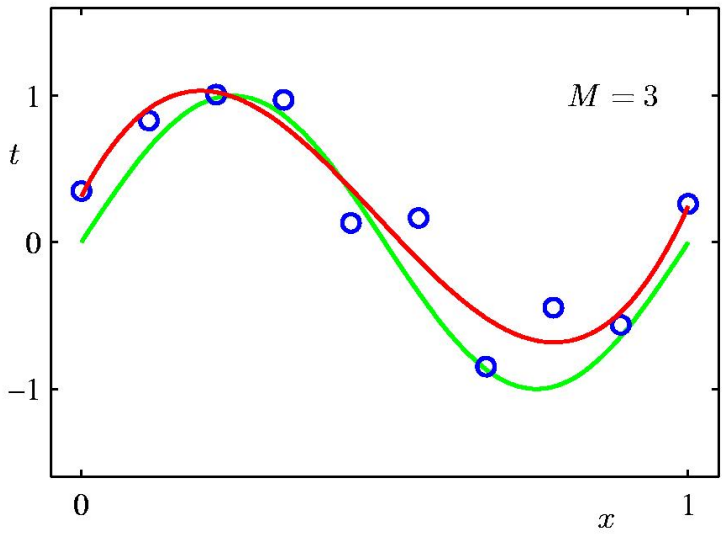
- We can use gradient descent to solve for each coefficient, or compute $\mathbf{w}$ analytically (how does the solution change?)

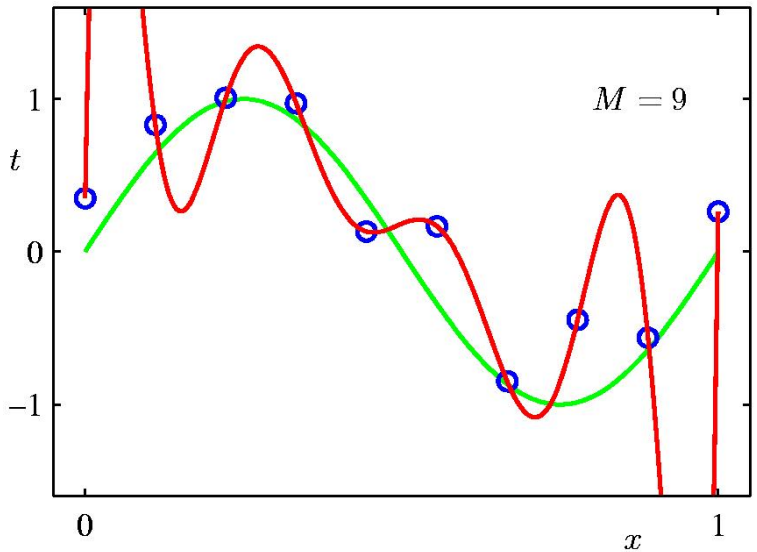
Which Fit is Best?

from Bishop









Generalization

- Generalization = model's ability to predict the held out data
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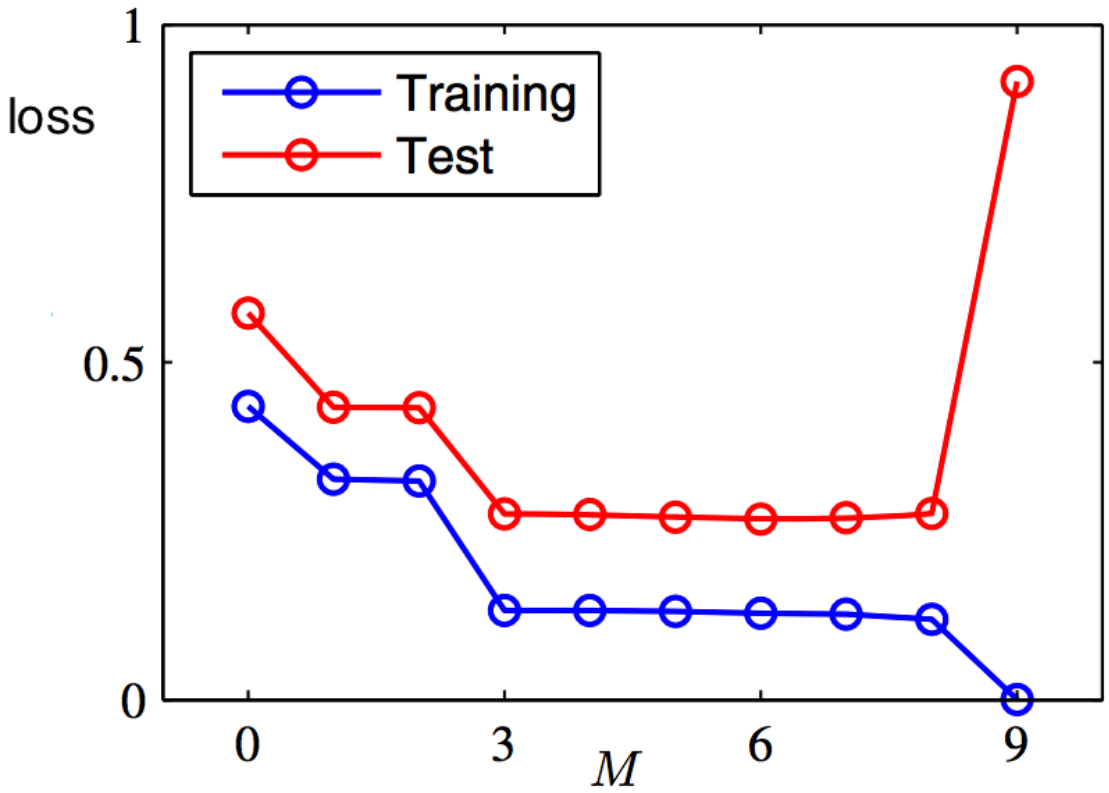

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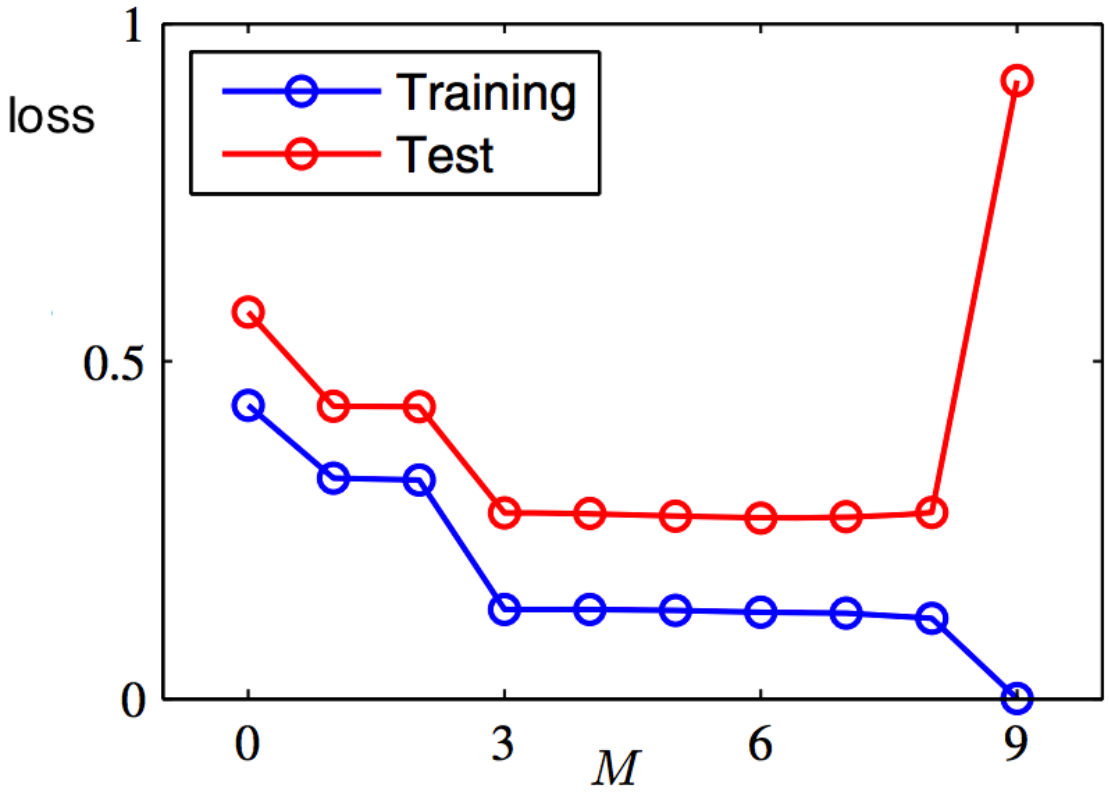

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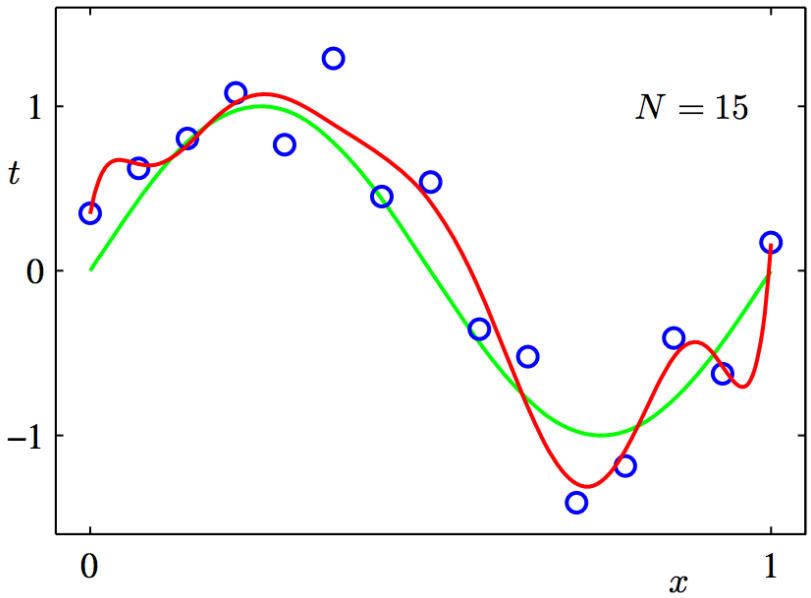

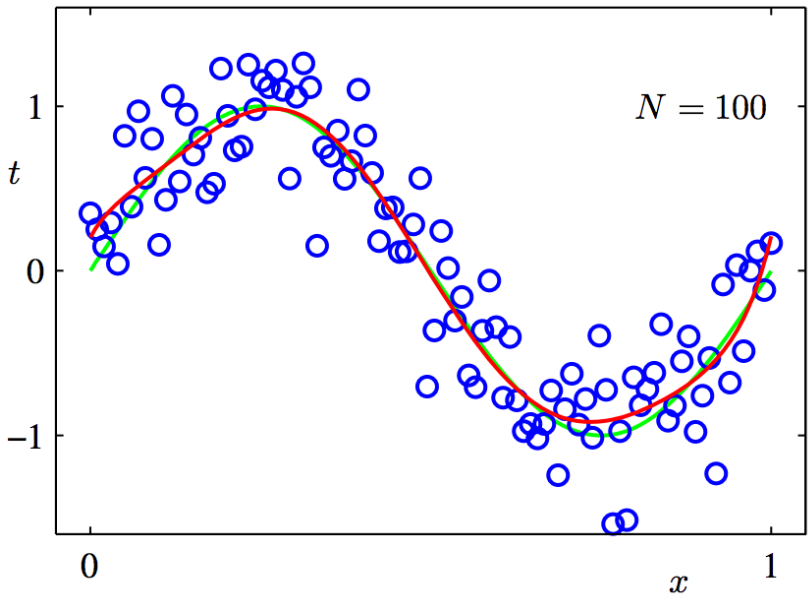

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	$M = 0$	$M = 1$	$M = 6$	$M = 9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
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- The weights are becoming huge to compensate for the noise
- One way of dealing with this is to encourage the weights to be small (this way no input will have too much influence on prediction). This is called regularization

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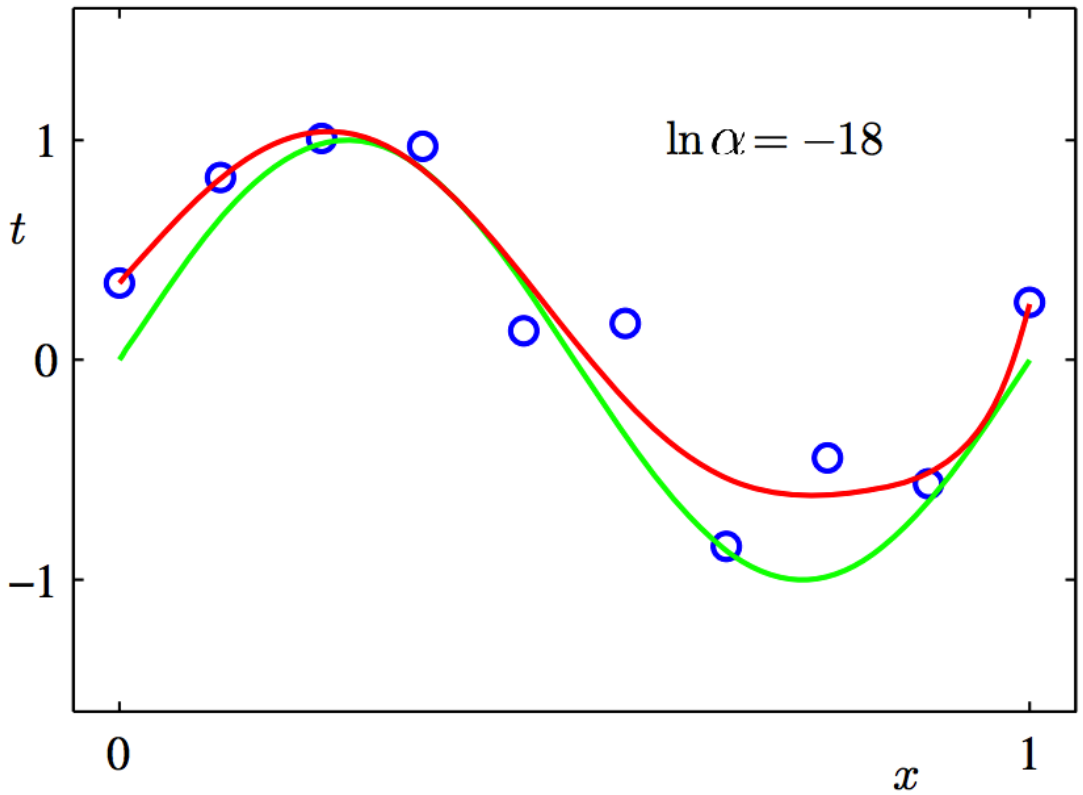
Regularized least squares

- Better generalization
- Choose α carefully



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Training set vs. Test set

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- One method of assessing fit: test generalization = model's ability to predict the held out data
- Optimization is essential: stochastic and batch iterative approaches; analytic when available