动量冲量角动量

答案

1.证: 米从空中掉落时间为$t, h = \frac{1}{2}gt^2$，得$t = \sqrt{2h/g}$，空中米的质量$m = ct$，c是常数，重力$G = mg = c\sqrt{2gh}$.由动量定理$Fdt = dm \cdot v$,得

$$F = \frac{dp}{dt} = cv = c\sqrt{2gh}$$

所以所作合理

2.证：由角动量定理

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} = -m\omega^2\hat{r},$$

$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times (-m\omega^2\hat{r}) = 0$$

$$\vec{M} = \frac{d\vec{L}}{dt} = 0,\vec{L}\text{是恒量。}$$

方法二：$\vec{p} = m\vec{v} = m\frac{d\vec{r}}{dt} = m(-a\omega\sin\omega t\vec{i} + b\omega\cos\omega t\vec{j})$

$$\vec{L} = \vec{r} \times m\vec{v} = abm\omega\vec{k}, \vec{L}\text{是衡量}$$

计算题

1.水平方向动量守恒

$(m + M)v_0\cos\theta = Mv + m(v - u)$,得

$$v = \frac{(m+M)v_0\cos\theta=mu}{M+m}$$

$$\Delta v = v = v_0\cos\theta = \frac{mu}{m+M}$$

$$\Delta t = \frac{v_0\sin\theta}{g}, \Delta x = \Delta v\Delta t = \frac{muv_0\sin\theta}{(M+m)g}$$

2.解：链子下落s距离时，平台上链子的重量为ρsg,链子下端接触台面处取ds段，重量为$dm = \rho ds$,

$$F = \frac{dmv}{dt} = \frac{ds\rho v}{dt} = \rho v^2$$

根据$\frac{1}{2}mv^2 = mgs$,得$F = 2\rho gs$,所以

$$N = \rho gs + F = 3\rho gs.$$

3.解：根据受力分析，$mg - T = ma$，$T = ma$,得$a = g/2$,有运动公式

$$\frac{1}{2}a\Delta t^2 = 0.4, \quad \Delta t = 0.4s\ (g = 10),$$

$$v_A = v_B = \frac{g}{2}\Delta t = 0.2g = 2m \cdot s^{-1},$$

$$2mv_A = (3)mv_C,$$

$$v_c = \frac{4}{3}m \cdot s^{-1}$$

4.解:根据动量守恒,$mv = (m + M)v_1$,由能量守恒

$$\frac{1}{2}mv^2 = \frac{1}{2}(m + M)v_1^2 + \frac{1}{2}k\Delta x^2$$

$$\Delta x = \sqrt{\frac{mM}{(m+M)k}}v$$

6.解：$\Delta m = ct = 40t$

竖直方向：$F_v = \frac{dp}{dt} = \frac{dm}{dt}v = c\sqrt{2gh} = 160N$

水平方向：$F_p = \frac{dp}{dt} = cv = 120N$

$$F = \sqrt{F_v^2 + g_p^2} = 200N$$

$$\vec{F} = F_p\vec{i} + F_v\vec{j} = 120\vec{i} + 160\vec{j}$$