

## 一. 选择题

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1. Suppose that  $A, B$  are two events, and  $P(AB) = 0$ , then
  - A.  $AB$  is a null event
  - B.  $A$  and  $B$  are mutually exclusive
  - C.  $P(A)=0$  or  $P(B)=0$
  - D.  $AB$  may not be a null event
2.  $X_1, X_2$  are two random variables with distribution functions  $F_1(x), F_2(x)$  respectively. If  $aF_1(x) + bF_2(x)$  is also a distribution for a random variable. Then  $a, b$  could be
  - A.  $a = 3/5, b = -1/2$
  - B.  $a = -1/2, b = 3/2$
  - C.  $a = 1, b = -1$
  - D.  $a = 1/2, b = 2/3$
3.  $Z = \max[X, Y]$ . Then  $F_z(Z)$  equals?
  - A.  $\max[F_X, F_Y]$
  - B.  $F_x$
  - C.  $F_X + F_Y$
  - D.  $F_X F_Y$
4.  $\text{Var}[X] = 4, \text{Var}[Y] = 9$ . The correlation coefficient is 0.5. find the  $\text{Var}[2X - Y]$
5.  $E[XY] = E[X]E[Y]$ , then

## 二. 填空题

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1. 2 red balls, 5 blue balls in the urn. The probability of the second ball is blue is ?
2.  $P(x) = ax + b, x \in [1, 3]$  And  $P[2 < x < 3] = 2P[1 < x < 2]$  find  $a$  and  $b$
3.  $X, Y$  are independent Poisson random variables  $E[x] = 2, D[y] = 4$ . Then  $P[X + Y = 3]$  ?
4.  $X \sim N(0, 4)$   $Y = -2X + 1$  Then  $Y \sim ?$
5.  $K$  uniformly distribute on  $[0, 4]$ . find Probability that equation  $4x^2 + 4Kx + K + 2$  has real root.
6. Let  $X$  and  $Y$  's joint probability density function is  $f(x, y) = 8xy, 0 < x < y < 1$  then  $P[X + Y > 1] = ?$

## 三. 计算题

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1. Jack Mike Tom want to eat the last piece of cake. The probability that Jack want is 0.4 , Mike want is 0.35, Tom want is 0.25. Find the probability that only one of them want to eat the cake.
2.  $X$  is uniformly distributed on  $[-1, 1]$ 
  - (a) find  $E[2X]$  and  $D[2X]$
  - (b) find the probability density function of  $Y = |X|$
3. Tossing  $n$  fair coins  $X$  is the number of head and  $Y$  is the number of tail
  - (a) find the probability mass function of  $X$

(b) Prove the correlation coefficient of  $X + Y$  and  $X - Y$  is 0

4.  $f(x, y) = Ae^{-3x-2y}, x > 0, y > 0$

(a) find  $A$

(b) find  $P[X + Y < 3]$

(c) find the probability density function of  $X$

(d) Does  $X$  and  $Y$  independent. Why or Why not

5.

(X,Y)	(0,1)	(0,2)	(1,1)	(1,2)
P	1/8	3/8	1/4	1/4

(a) find the probability mass function of  $Z = X + Y$

(b) find  $E[X]$ .