- **三角积分类型：**

$$\int \frac{\sin x}{x^n}dx，\int \frac{\cos x}{x^n}dx, \int \frac{\tan x}{x^n}dx. \text{ 这里} n \in \mathbb{N}.$$

- **高斯积分类型：**

$$\int e^{ax^2}dx，\int x^{2n}e^{ax^2}dx, n \in \mathbb{N}, a \neq 0$$

- **菲涅尔积分类型：**

$$\int \sin x^2 dx, \int \cos x^2 dx, \int \tan x^2 dx$$

- **指数积分类型：**

$$\int \frac{e^x}{x}dx, \int \frac{e^{-x}}{x}dx, \int \frac{e^x}{1+x}dx, \int \frac{e^x}{1+x^2}dx, \int \frac{e^x}{x(1+x)}dx$$

- **对数积分类型：**

$$\int \frac{dx}{\ln x}, \int \frac{\ln x}{1+x}dx, \int \frac{\ln x}{1+x^2}dx, \int \ln(\sin x)dx, \int \ln(\cos x)dx, \int \ln(\tan x)dx,$$
$$\int \ln(1+\sin x)dx, \int \ln(1+\cos x)dx, \int \ln(1+\tan x)dx, \int \frac{\ln x}{x(1+x)}dx$$

- **椭圆积分类型：**

$$\int \sqrt{1-k^2\sin^2 x}dx, \int \sqrt{1-k^2\cos^2 x}dx, \int \frac{dx}{\sqrt{1-k^2\sin^2 x}}, \int \frac{dx}{\sqrt{1-k^2\cos^2 x}},$$

$$\int \frac{dx}{(1-n\sin^2 x)\sqrt{1-k^2\sin^2 x}}dx, \int \frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}}dx, \int \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}dx,$$

$$\int \sqrt{1-x^3}dx, \int \sqrt{1-x^4}dx, \int \frac{dx}{\sqrt{1-x^3}}, \int \frac{dx}{\sqrt{1-x^4}}; \text{ 这里} 0 < k^2 < 1.$$

$$\int \sqrt{1+\sin^2 x}dx, \int \sqrt{1+\cos^2 x}dx.$$

- **贝塞尔积分类型：**

  $$\int \cos(nx - a\sin x)dx,\ n \in \mathbb{N}, a \neq 0$$

- **超几何函数类型：**

  $$\int \sqrt{1-x^n}dx, \int \frac{dx}{\sqrt{1-x^n}}dx,\ \text{这里} n \geq 5$$

- **伯努利函数/多重对数函数类型：**

  $$\int \frac{x}{e^x-1}dx,\ \int \frac{x^2}{e^x-1}dx, \int \frac{x^3}{e^x-1}dx$$

- **其他一些：**

  $$\int e^{\sqrt{x}}\sin x dx,\ \int x^x dx,\ \int \ln(\ln x)dx,\ \int e^{e^x}dx,$$

  $$\int \sin(\cos x), \int \cos(\sin x)dx,\ \int \sinh(\cosh x)dx,\ \int \cosh(\sinh x)dx$$

  $$\int e^{a\cos x}dx,\ \int e^{a\sin x}dx\ \ a \neq 0$$

有的函数没有初等原函数，但是在特定区间上的定积分可以计算，比如：

- $\displaystyle\int_0^{+\infty} \frac{\sin x}{x}dx = \frac{\pi}{2}$
- $\displaystyle\int_0^{+\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{4a}},\ a > 0$
- $\displaystyle\int_0^{+\infty} x^{2n}e^{-ax^2}dx = \frac{(2n-1)!!}{2^{n+1}}\sqrt{\frac{\pi}{a^{n+1}}},\ n \in \mathbb{N}, a > 0$
- $\displaystyle\int_0^{+\infty} \frac{x}{e^x-1}dx = \frac{\pi^2}{6}$

- $\displaystyle\int_0^{+\infty} \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$
- $\displaystyle\int_0^{+\infty} \frac{\ln x}{1 + x^2} dx = 0$
- $\displaystyle\int_0^{\frac{\pi}{4}} \ln(1 + \tan x) dx = \frac{\pi}{8} \ln 2$