$$\overrightarrow{F}_{1,2} = \frac{q_1 q_2 K}{r^2}\hat{r}_{1,2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{1,2}$$

$$\overrightarrow{E} = \frac{\overrightarrow{F}}{q} = \frac{QK}{r^2}\hat{r} \qquad E = \frac{\sigma}{2\epsilon_0}$$ （电场Electric Field / g / F/q）

$$d\phi = \overrightarrow{E} \cdot \overrightarrow{dA} \qquad \phi = \frac{\sum Q_{inside}}{\epsilon_0}$$ （电通量Electric Flux）

$$U = \int_R^\infty \overrightarrow{F}_{el} \cdot \overrightarrow{dr} = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$ （静电势能Electrostatic Potential Energy / W）

$$V = \int_r^\infty \frac{\overrightarrow{F}_{el}}{q} \cdot \overrightarrow{dr} = \int_r^\infty \overrightarrow{E} \cdot \overrightarrow{d}\,r = \frac{Q}{4\pi\epsilon_0 R}$$ （电势Electric Potential，电势差/电压 / W/q）

$$\frac{dV}{dr}\vec{r} = \frac{d}{dr}\frac{Q}{4\pi\epsilon_0 r}\vec{r} = -\frac{Q}{4\pi\epsilon_0 r^2}\vec{r} = -\overrightarrow{E} \qquad \overrightarrow{E} = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

$$C = \frac{Q}{V} = 4\pi\epsilon_0 R$$ （电容Capacitance）

<u>L1. WHAT HOLDS OUR WORLD TOGETHER? ELECTRIC CHARGES (HISTORICAL), POLARIZATION, ELECTRIC FORCE, COULOMB'S LAW</u>

$\overrightarrow{F}_{1,2} = \frac{q_1 q_2 K}{r^2}\hat{r}_{1,2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{1,2}$ (K: Coulomb's Constant 库伦常数, $K = 9 \times 10^9 \frac{Nm^2}{C^2}$)

$q_{p^+} = q_{e^-} \approx 1.6 \times 10^{-19}C$ (C 库伦)

Nuclear force → Electric force → Gravitational force

<u>L2. ELECTRIC FIELD, FIELD LINES, SUPERPOSITION, INDUCTIVE CHARGING, DIPOLES, INDUCED DIPOLES</u>

-Electric Field 电场

$\overrightarrow{E} = \frac{\overrightarrow{F}}{q} = \frac{QK}{r^2}\hat{r}$ (unit: $\frac{N}{C}$)

$$\overrightarrow{E} = \sum_i \overrightarrow{E}_i$$

$\overrightarrow{F} = q\overrightarrow{E}$

-Graphical

-Electrical Field Lines (F: tangential direction)

-Dipole

-Demo

van de Graaff - Balloon - Prof. W.L.

(van de Graaff: 范德格拉夫起电机)

L3. ELECTRIC FLUX, GAUSS'S LAW, EXAMPLES

-Electric Flux 电通量

$$d\phi = \overrightarrow{E} \cdot \hat{n}\, dA = \overrightarrow{E} \cdot \overrightarrow{dA} = E\ dA \cos\theta$$ (unit: $\frac{N}{C}m^2$)

-on Closed Surface

$$\phi = \oint \overrightarrow{E} \cdot \overrightarrow{dA}$$

-on Sphere

$$\phi = 4\pi R^2 E \qquad \overrightarrow{E}_R = \frac{Q}{4\pi\epsilon_0 R^2}\hat{r} \qquad (k = \frac{1}{4\pi\epsilon_0})$$

$$\phi = \frac{Q}{\epsilon_0}$$

-Gaussian Law (Maxwell's 1st Equation)

$$\phi = \oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{\sum Q_{inside}}{\epsilon_0}$$

-Symmetry

-Sphere (with charge Q uniformly distributed, radius R)

$$for\ \ r < R, \qquad 4\pi r^2 E = \frac{Q_{inside}}{\epsilon_0}, \qquad E = 0$$ (内电场为0)

$$for\ \ r > R, \qquad 4\pi r^2 E = \frac{Q}{\epsilon_0}, \qquad E = \frac{Q}{4\pi r^2 \epsilon_0}$$ (外电场同电荷在圆心的电场)

-Infinite Plane (with charge Q uniformly distributed, $\sigma = \dfrac{Q}{A}$ $[\dfrac{C}{m^2}]$

$$E = \frac{\sigma}{2\epsilon_0}$$

L4. Electrostatic Potential, Electric Energy, eV, Conservative Field, Equipotential Surfaces

-Electrostatic Potential Energy U

$$W = \int_{\infty}^{R} \overrightarrow{F}_{push} \cdot \overrightarrow{dr} = \int_{R}^{\infty} \overrightarrow{F}_{el} \cdot \overrightarrow{dr} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_{R}^{\infty} \frac{dr}{r^2}$$

$U = \dfrac{q_1 q_2}{4\pi\epsilon_0 R}$ (unit: J) ***(U: Energy to bring all charges to place)***

-Electric Potential V

Work per unit charge to go from infinity to this location

$V_p = \dfrac{Q}{4\pi\epsilon_0 R}$ (unit: $J/C = Volts$)

$$V = \int_{r}^{\infty} \frac{\overrightarrow{F}_{el}}{q} \cdot \overrightarrow{dr} = \int_{r}^{\infty} \overrightarrow{E} \cdot \overrightarrow{d}r$$

$$W = \Delta V q$$

-Hollow Sphere

Potential inside hollow metal sphere is constant = same as surface

-Equipotential lines

always Perpendicular to Field Lines

Different … lines/surfaces can Never Intersect

Every metal … will be an equipotential…

-Potential Difference

$$V_A = \int_{A}^{\infty} \overrightarrow{E} \cdot \overrightarrow{dr}$$

$$V_B = \int_B^\infty \overrightarrow{E} \cdot \overrightarrow{dr}$$

$$V_A - V_B = \int_A^B \overrightarrow{E} \cdot \overrightarrow{dr}$$

-Demo: light tube close to a van de Graaff

L5. E = -grad V, More on Equipotential Surfaces, Conductors, Electrostatic Shielding (Faraday Cage)

-Recap

$$\oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{\sum Q_{inside}}{\epsilon_0}$$

$$V_P = \int_P^\infty \overrightarrow{E} \cdot \overrightarrow{dl}$$

$$V_A - V_B = \int_A^B \overrightarrow{E} \cdot \overrightarrow{dl}$$

$$\oint \overrightarrow{E} \cdot \overrightarrow{dl} = 0$$

-$E - V$ (1)

$$\frac{dV}{dr}\vec{r} = \frac{d}{dr}\frac{Q}{4\pi\epsilon_0 r}\vec{r} = -\frac{Q}{4\pi\epsilon_0 r^2}\vec{r} = -E$$

-$E - V$ (2)

$$|E_x| = |\frac{\Delta V}{\Delta x}|\Big|_{yz}$$

$$\overrightarrow{E} = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

$$\overrightarrow{E} = -gradV$$

-Electrostatic shielding - Faraday cage

Charge in conductors re-arranges itself until potential is equal

Goes to outer surface

-Demo: Prof. W.L. in a Faraday cage (a cage made of metal net)

L6. High-Voltage Breakdown, Lightning, Sparks - St Elmo's Fire

-Irregularly shaped object has higher charge density at more pointy parts

Sphere A of radius R_A connected with wire to Sphere B of radius R_B

Charge Q_A on A, Q_B on B

$$V_A = \frac{Q_A}{4\pi\epsilon_0 R_A} = V_B = \frac{Q_B}{4\pi\epsilon_0 R_B}$$

$$\frac{Q_B}{Q_A} = \frac{R_B}{R_A} \qquad + \qquad \sigma = \frac{Q}{4\pi R^2} \qquad + \qquad E = \frac{\sigma}{\epsilon_0} \qquad \rightarrow$$

$$\frac{E_B}{E_A} = \frac{\sigma_B}{\sigma_A} = \frac{R_A}{R_B} \qquad \rightarrow \qquad E \propto \sigma \propto \frac{1}{R}$$

-smaller radius → larger charge density → stronger electric field

-Electric Breakdown

Dry air, room temperature

1 micron between collisions

To ionize oxygen = 12.5 eV

To ionize nitrogen = 15 eV

about 10^7 V/m **(observed 3 x 10^6 V/m)**

-Van de Graff Generator

$$V = ER \ (radius); \quad E < 3 \times 10^6$$

L7. Capacitance, Field Energy

-Field Energy

e.g. Top plate $+Q = +\sigma A$ Bottom Plate $-Q = -\sigma A$

$$E = \frac{\sigma}{\epsilon_0}$$

To move top plate from h to $h + x$, and ***creating Electric Field***

$$F_{W.L.} = \frac{1}{2}QE$$ (Electric Force is the sum of two electric field on two sides of the charged layer with $E = \frac{\sigma}{\epsilon_0}$ and $E = 0$)

$$W = \frac{1}{2}QEx = \frac{1}{2}\epsilon_0 E^2 Ax = \frac{1}{2}\epsilon_0 E^2 (Volume)$$

(work to do to create electric field in certain volumes)

-Field Energy Density

$$\frac{W}{(Volume)} = \frac{1}{2}\epsilon_0 E^2 \left[\ \mathrm{J/m\hat{\ }3}\right]$$ (Field energy density)

$$U = \int_{all\ space} \frac{1}{2}\epsilon_0 E^2 d(Volume)$$ ***(U: integrate all electric field in space)***

-Potential Energy between Two Plates

$$U = \int \frac{1}{2}\epsilon_0 E^2 d(Volume) = \frac{1}{2}\epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2 Ah = \frac{1}{2}\sigma A \frac{\sigma}{\epsilon_0} h = \frac{1}{2}QEh = \frac{1}{2}QV$$

-Capacitance

the capability of holding charge for a given electric potential

$$C = \frac{Q}{V}$$ [unit: F - Farad] = charge / potential difference

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2$$

-Sphere: $V_p = \frac{Q}{4\pi\epsilon_0 R}$

$$C = 4\pi\epsilon_0 R$$

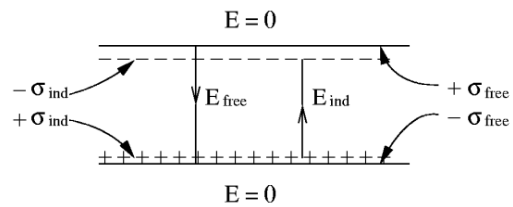
-Two Plates:

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{A\epsilon_0}{d}$$

L8. Polarization, Dielectrics, The Van de Graaff, More on Capacitors

-Dielectrics (Dielectric Constant)

$$E_{free} = \frac{\sigma_{free}}{\varepsilon_0} \quad E_{\text{ind}} = \frac{\sigma_{\text{ind}}}{\varepsilon_0}$$



$$\vec{E} = \vec{E}_{free} + \vec{E}_{ind}$$

$$E = E_{free} - E_{ind}$$

Assume that $E_{ind} \propto E_{free}$, thus $E_{ind} = bE_{free}$ $\quad \rightarrow \vec{E} = (1-b)\vec{E}_{free}$

Let $(1-b) = \frac{1}{\kappa}$ ***(κ: Dielectric Constant, depends on the material)***

$$\vec{E} = \frac{\vec{E}_{free}}{\kappa}$$

-Capacitance with Dielectrics

$$V \downarrow = E \downarrow d$$

$$C \uparrow = \frac{Q_{free}}{V \downarrow}$$

-Van de Graaf

Charge inside the sphere

L9. Currents, Resistivity, Ohm's Law

$Cu \qquad T = 300K \qquad \langle v_e \rangle = 10^6 \text{ m/sec}$

time between collision (due to thermal motion) $\tau = 3 \times 10^{-14}\text{sec}$

number of free electrons per cubic meter $n = 10^{29}$

$F = eE \qquad a = \frac{F}{m_e} \qquad v_d = a\tau = \frac{eE}{m_e}\tau$ (Drift Velocity)

$I = v_d Ane = \frac{e^2 n\tau}{m_e} AE$ **(Current, unit: A)**

$\sigma = \frac{e^2 n\tau}{m_e}$ **(Conductivity)**

$$= \frac{\sigma AV}{l}$$

$$V = \frac{l}{\sigma A} I$$

$$\rho = \frac{1}{\sigma}$$ **(Resistivity)**

$$R = \frac{l}{\sigma A} = \frac{l\rho}{A}$$ **(Resistance, unit: Ω)**

$$V = IR$$ **(Ohm's law)**

L10. Batteries, EMF, Energy Conservation, Power, Kirchhoff's Rules, Circuits, Kelvin Water Dropper

-Power Supply

-Copper-Zinc battery: Cu+ , Zn-

SO4- ion flows against electric field because when it reacts with Zn+, more energy is released than it costs to flow

-Electromotive Force

$$\varepsilon = I\left(R + r_i\right)$$ (EMF - Electromotive Force)

$$V_b = IR = \varepsilon - Ir_i$$

-Power

$$dW = dq\left(V_A - V_B\right) \quad \rightarrow \quad \frac{dW}{dt} = \frac{dq}{dt}\left(V_A - V_B\right) \quad \rightarrow$$

$$P = IV = I^2R = \frac{V^2}{R}$$ (Power, unit: $J/sec = W(att)$)

-Battery Power

$$P = I\varepsilon = I^2\left(R + r_i\right) = \frac{\varepsilon^2}{R + r_i}$$

-Kirchhoff's Rules

1. $\oint \vec{E} \cdot d\vec{l} = 0$

2. Charge Conservation

3. $\int_1^2 \vec{E} \cdot d\vec{l} = V_1 - V_2$

-Demo: Water Tank Battery