L11. Magnetic Field, Lorentz Force, Torques, Electric Motors (DC), Oscilloscope

-Magnetic Field

$\overrightarrow{B}$ (Electric Wire: direction of right hand rule)

$\hat{F} = \hat{I} \times \hat{B}$

-Lorentz Force

$\overrightarrow{F}_b = q(\overrightarrow{v} \times \overrightarrow{B})$ ($\overrightarrow{B}$ unit: $T(esla) = \dfrac{N \cdot sec}{C \cdot m}$)

$$\overrightarrow{F}_{tot} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

-Force on wires

$$d\overrightarrow{F}_B = dq\left(\overrightarrow{v_d} \times \overrightarrow{B}\right)$$

$$I = \frac{dq}{dt}$$

$$d\overrightarrow{F}_B = Idt\left(\overrightarrow{v}_d \times \overrightarrow{B}\right) = I(d\vec{l} \times \overrightarrow{B})$$

-Building a Electric Motor

-Demo: Prof. W.L. as a Commutator

L13. Moving Charges in B-fields, Cyclotron, Synchrotron, Mass Spectrometer, Cloud Chamber

$$d\overrightarrow{F}_B = dq\left(\overrightarrow{v_d} \times \overrightarrow{B}\right)$$

-for $\overrightarrow{B} \perp \overrightarrow{v}$ and $\overrightarrow{B}$ constant

$qvB = \dfrac{mv^2}{R} \quad \Rightarrow \quad R = \dfrac{mv}{qB} = \sqrt{\dfrac{2mV}{qB^2}}$ ($qV = \dfrac{1}{2}mv^2$)

-Mass Spectrometer

-Cyclotron

-Cloud Chamber (Bubble Chamber)

-Demo: Cloud Chamber (to see nuclear particles)

L14. Biot-Savart Law, Gauss' Law for Magnetic Fields, Revisit the "Leyden Jar", High-Voltage Power Lines

$B \propto \frac{I}{R}$ (Experimental Fact - straight wire)

-Analogy: Electric field of a charged pole is proportional to $\frac{1}{r^2}$

a charged wire is proportional to $\frac{1}{r}$

-Biot-Savart Law

$$\overrightarrow{dB} = \frac{CI}{r^2}\overrightarrow{dl} \times \hat{r}$$ ($C = 10^{-7} = \frac{\mu_0}{4\pi}$, μ_0: permeability of free space)

-Straight Wire

$$B = \frac{\mu_0 I}{2\pi R}$$

-Wire Loop

$$B = \frac{\mu_0 I}{2R}$$

-Maxwell's 2nd Equation

$$\oint \vec{B} \cdot d\vec{A} = 0$$ (since there's no magnetic monopole as an experimental fact)

-High-Voltage Power Lines

$V_a - V_b = IR$ $V_b = V_a - IR$ $V_b I = V_a I - I^2 R$

$V_b I =$ Power consumed $V_b I =$ Power consumed $I^2 R =$ Power lost

-Leyden Jar

Corona Discharge

-Demo: Leyden Jar with glass also discharged

<u>L15. Ampere's Law, Solenoids, Revisit the Kelvin Water Dropper, Midterm Evaluation</u>

-Ampere's Law (Part of Maxwell's 3rd Equation)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed\ (penetration)}$$

(enclosed: any closed-loop surface penetrated by current-carrying wire)

-Magnetic Field outside/inside Wire

-outside: $B = \dfrac{\mu_0 I}{2\pi r}$

-inside: $B = \dfrac{\mu_0 I r}{2\pi R^2}$

-Solenoids

$$B = \frac{\mu_0 I N}{L} \quad (\, L \gg R \,)$$

-Demo: Kelvin Water Dropper Revisit

<u>L16. Electromagnetic Induction, Faraday's Law, Lenz Law, Complete Breakdown of Intuition, Non-Conservative Fields</u>

-Electromagnetic Induction

<u>Faraday: Changing magnetic field causes current</u>

profound discovery: contributed largely to the technological revolution of the late nineteenth and early twenty century

-Lenz's Law

generated current resist the change of magnetic field

$$\varepsilon_{ind} = I_{ind} R$$

-Faraday's Law

Experiment: Two parallel Wire Loops

current through Loop1 generating changing magnetic field

changing magnetic field induced current in Loop2

Observation: $\varepsilon_2 \propto \frac{dB_1}{dt}$ $\varepsilon_2 \propto area_2$

⇒ ***<u>EMF is the result of the change of the magnetic flux</u>***

-Magnetic Flux

$$\phi_B = \int \overrightarrow{B} d\overrightarrow{A}$$

$$\varepsilon = -\frac{d\phi_b}{dt} = -\frac{d}{dt}\int_{open\ surface} \overrightarrow{B} \cdot \overrightarrow{dA} = \oint \overrightarrow{E} \cdot \overrightarrow{dl}$$

$$\oint \overrightarrow{E} . \overrightarrow{dl} = -\frac{d}{dt}\int \overrightarrow{B} \cdot \overrightarrow{dA}$$ **(Maxwell's 4th Equation)**

<u>Electric field caused by magnetic flux change is non-conservative</u>

Kirchhoff's law is a special case of Faraday's when there is no change in magnetic flux

<u>-Demo: 2 voltmeters can be connected to the same place in the circuit and read different values</u>

<u>L17. Motional EMF, Dynamos, Eddy Currents, Magnetic Braking</u>

-2D Loop

$$\oint \overrightarrow{E} . \overrightarrow{dl} = -\frac{d}{dt}\int \overrightarrow{B} \cdot \overrightarrow{dA} = \frac{d}{dt}\int BA\cos\theta$$

change in B, A, θ will cause EMF

-Spinning coil in magnetic field

$$\omega = \frac{2\pi}{\text{Period}} \qquad \theta = \omega t \qquad \phi_b = AB\cos\omega t$$

$$\varepsilon(t) = -\frac{d\phi}{dt} = AB\omega\sin\omega t$$

$$I(t) = \frac{\varepsilon(t)}{R}$$ (AC: Alternating Current)

-Dynamo

conducting windings that are being moved forcefully through magnetic fields

US: $60Hz, 110\sqrt{2}V$ Europe: $50Hz, 220\sqrt{2}V$

-Story

Faraday was once interviewed by reporters when he came up with this law, and they said to him, "So what?

So fine, so you moved a winding through a magnetic field and so you get a little bit of electricity?

So what?" And his answer was, some day you will tax it.

-Changing area by sliding one side of loop

-Eddy Current (magnetic braking/damping)

Demo: Two pendulums oscillating between magnetic poles, one with teeth

current will be induced -> heat

force will be applied opposing motion (magnetic braking)

-Demo: conducting windings with light bulb moved through magnet (so quick to break the bulb)

-Demo: metal ring free falling through strong magnetic field slowly (magnetic damping)

L18. Displacement Current (Difficult Concept), Synchronous Motors, Induction Motors, Secret Top, How does it work?

-Ampere's Law doesn't work with capacitor $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enclosed\ (penetration)}$

-Displacement Current (what's in a name — Shakespeare)

a changing magnetic flux gives rise to an electric field

maybe a changing electric flux gives rise to a magnetic field

$$d\phi = \vec{E} \cdot \overrightarrow{dA}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \kappa \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{dA} \right)$$

$\epsilon_0 \kappa \frac{d}{dt} \int \vec{E} \cdot \overrightarrow{dA}$: displacement current

-Induction Motor / Synchronous Motor (Eddy Current, Lorentz Force $\vec{F}_b = q(\vec{v} \times \vec{B})$)

Three phase current

-Demo: rotating metal egg

-Demo: Rotation can (Two phase current)

L19. Vacation Special: How do magicians levitate women? (with demo); Electric Shock Treatment (no demo); Electrocardiogram (with demo) - Pacemakers; Superconductivity (with demo)- Levitating Bullet Trains; Aurora Borealis

-Aurora Borealis

Plasma (solar wind)

-Superconductivity

no electric field ($R = 0$)

$\varepsilon = 0 = I_{induction} R$

$I^2 R = 0$ — high current — high magnetic field — cyclotron

When approached by a magnet, EMF must remain 0, so eddy currents will flow to never allow magnetic flux

$$P = \frac{B^2}{2\mu_0} \left[\mathrm{N/m^2} \right] \quad \text{(Magnetic Pressure)}$$

-Magnetic Levitation (Maglev)

1. superconductivity (no resistance, eddy force undamped)
2. high speed (high speed magnetic flux change)
3. AC

L20. Inductance, RL Circuits, Magnetic Field Energy

-Self-Inductance L (unit: Henry (H) = $V \cdot sec/A$)

$\phi_B = LI \quad \Rightarrow$

$$\varepsilon_{ind} = \frac{-d\phi}{dt} = L\frac{dI}{dt}$$

-Solenoid

$$B = \frac{\mu_0 I N}{l} \quad \Rightarrow$$

$$\phi_b = \pi r^2 N B = \pi r^2 N^2 \frac{\mu_0 I}{l} \quad \Rightarrow$$

$$L = \pi r^2 \frac{N^2}{l} \mu_0$$

*Every circuit has a finite self-inductance

-RL Circuits

$$\oint E dl = \frac{-d\phi}{dt} = -L \frac{dI}{dt} \quad \Rightarrow$$

$$\oint E dl = 0 + IR - V = -L \frac{dI}{dt}$$ (differential equation) $\Rightarrow$

$$I = I_{\max}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\boldsymbol{I}_{\max} = \frac{V}{R}$$

-Discharge

$$\oint E dl = 0 + IR - 0 = -L \frac{dI}{dt}$$ (differential equation) $\Rightarrow$

$$I = I_{max} e^{-\frac{R}{L}t}$$

<u>heat produced by IR^2 is stored in the self-inductor during charging</u>

-Magnetic Field Energy Density

$$\int_0^\infty I^2 R dt = I_{max}^2 R \int_0^\infty e^{-\frac{2R}{L}t} dt = \frac{1}{2} L I_{max}^2$$

-Substitute L, I: $\quad B = \frac{\mu_0 I N}{l}; \quad L = \pi r^2 \frac{N^2}{l} \mu_0$

$$\frac{1}{2} L I^2 = \frac{B^2}{2\mu_0} \pi r^2 l$$

where $\pi r^2 l$ is the volume of the self-inductor

$$\frac{B^2}{2\mu_0} = \text{ **Magnetic field energy density** } \left[\mathrm{J/m^3}\right]$$

work to get a current going in a pure self-inductor

-AC Current

$V = V_0 \cos \omega t$

$I = \dfrac{V_0}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t - \phi)$, where $I_{max} = \dfrac{V_0}{\sqrt{R^2 + (\omega L)^2}}$

$\tan \phi = \dfrac{\omega l}{R}$ (Phase Lag)

-Phase lag for magnetic levitation (coil on top of a metal plate)

AC Current and the ε_{ind} are synchronized, but the ***Eddy Current*** is delayed

$\tan \phi = \dfrac{\omega l}{R}$: Superconductor $R \to 0 \quad \Rightarrow \quad \phi \to 90^\circ \quad \Rightarrow$

The Coil and the Metal Plate always repel each other

-Demo: metal ring above coil with AC Current

metal ring with a slot: $R \to \infty \quad \Rightarrow \quad \phi \to 0 \quad \Rightarrow$

half time repel, half time attract, no levitation