L21. Magnetic Materials, Dia-, Para-, and Ferromagnetism, Prize Ceremony of Motor Contest

-Magnetic Dipole Moment

$$\overrightarrow{\mu} = I\overrightarrow{A} \quad (A : area)$$

-Diamagnetism

dipole moment is induced to oppose magnetic field

internal field is smaller than external

repelled from field

-Paramagnetism

randomized magnetic dipoles align with magnetic field

dipoles can be modeled as flowing current - Lorentz force component towards magnet

causes material to be attracted to magnet

returns to chaos when field is removed

normally weaker than weight (exception is liquid oxygen)

-Demo: hanging liquid oxygen (paramagnetic) from a strong electric magnet

-Ferromagnetism

-Magnetic Domain

domains where dipoles are aligned

some domains may remain oriented when field is removed

-Demo: Barkhausen effect

forces may be stronger than weight

-Common material

Cobalt, Nickel, Iron

Gadolinium (ferromagnetic in the winter, Curie point: $16C^{\circ}$)

-Demo: 15kg ferromagnetic bar into solenoids

-Relative Permeability κ_M

$$B = \kappa_M B_{vacuum}$$

(magnetic field with magnetic material is proportional to the vacuum one)

$\kappa_M = 1 + X_M$

-Diamagnetic $\quad X_M \to -0 \quad \Rightarrow \quad \kappa_M < 1$

-Paramagnetic $\quad X_M \to +0 \quad \Rightarrow \quad \kappa_M > 1$

-Ferromagnetic $\quad X_M \simeq \kappa_M \simeq 10^2 \to 10^5$

-Curie Point

the temperature when ferromagnetic material loose its structure of magnetic domain

-Demo: heat up an iron nut to loose its magnetic domain to become paramagnetic

L22. Hysteresis, Electromagnets, Bohr Magneton; Maxwell's Equations; 600 Daffodils

-Hysteresis Curve

-Maxwell's Equations

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{free}}{\epsilon_0 \kappa}$$

$$\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0$$

$$\oint \overrightarrow{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$ (Faraday's Law)

$$\oint \overrightarrow{B} \cdot d\vec{l} = \kappa_M \mu_0 (I_{penetration} + \epsilon_0 \kappa \frac{d\phi_B}{dt})$$

L24. Transformers, Car Coils, RC Circuits

-RC Circuits

$$+V_C + IR - V_0 = 0 \quad \Rightarrow$$

$$\frac{Q}{C} + R\frac{dQ}{dt} - V_0 = 0 \quad \Rightarrow$$

$$Q = V_0 C \left(1 - e^{-t/RC}\right) \quad \Rightarrow$$

$$I = \frac{dQ}{dt} = \frac{V_0}{R} e^{-t/RC} \qquad V_c = \frac{Q}{C} = V_0\left(1 - e^{-t/RC}\right)$$

-Discharge

$$\frac{Q}{C} + R\frac{dQ}{dt} = 0 \quad \Rightarrow$$

$$I = \frac{dQ}{dt} = -\frac{V_0}{R} e^{-t/RC}$$

-Demo: RC circuit under a switching voltage

$$f = 8msec$$

$$RC = 6k\Omega \times 0.1\mu F = 0.6msec \ll \frac{1}{2}f$$

-Transformers

-primary and secondary coils with magnetic flux coupling

$$V_1 = -L_1\frac{dI_1}{dt} = \varepsilon_1 = -N_1\frac{d\phi_B}{dt}$$

$$V_2 = -L_2\frac{dI_2}{dt} = \varepsilon_1 = -N_2\frac{d\phi_B}{dt}$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

-Assuming

$$R \ll \omega L$$

No energy loss through eddy currents in core

Perfect flux coupling

$$\Rightarrow \quad I_1V_1 = I_2V_2 \quad \Rightarrow \quad \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

-Demo: melting an iron nail $\left(\frac{I_2}{I_1} = \frac{N_1}{N_2}\right)$

-Spark Plugs - Car coils (The Ruhmkorff) $\left(\frac{V_2}{V_1} = \frac{N_2}{N_1}\right)$

High voltage from 12V car battery

Run DC through a transformer where $N_2 \gg N1$

When circuit is broken ⇒ high current difference ⇒ high magnetic flux difference

⇒ high voltage pulse in secondary coil

L25. Driven LRC Circuits, Resonance, Metal Detectors (Beach/Airport)

-LRC Circuits

$$V_c + 0 + IR - V_0 \cos \omega t = -L\frac{dI}{dt} \quad \Rightarrow$$

$$I = \frac{dQ}{dt} \qquad V_c = \frac{Q}{C}$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \cos \omega t \quad \Rightarrow$$

$$I = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}} \cos(\omega t - \phi) \qquad \left(\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} \right)$$

-Reactance: $\mathrm{X} = \omega L - \frac{1}{\omega C}$

-Impedance: $Z = \sqrt{R^2 + \mathrm{X}^2}$ (unit: Ω)

$$\tan \phi = \frac{\mathrm{X}}{R}$$

$$\overline{P} = \overline{VI} = \frac{V_0^2}{Z}\overline{\cos \omega t \cos(\omega t - \phi)} = \frac{V_0^2}{2Z}\cos \phi$$

-Resonance

$$\mathrm{X} = 0 \quad \Rightarrow \quad \omega L = \frac{1}{\omega C} \quad \Rightarrow \quad \int$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

-Quality

In the graph of I to ω:

$\Delta\omega = \frac{R}{L} =$ width of region greater than $70\,\%$ of maximum current $=$ half power

$Q = \frac{\omega_0}{\Delta\omega} = \frac{1}{R}\sqrt{\frac{L}{C}}$ (Q: narrowness of curve)

-Metal detectors

L26. Traveling Waves, Standing Waves; Musical Instruments

-Traveling Wave

$$y = y_0 \sin(kx - \omega t)$$

y_0: amplitude

λ: wave length

k: wave number $k = \frac{2\pi}{\lambda}$

ω: angular frequency

v: traveling velocity $v = \frac{\omega}{k}$

-Standing Wave

$$y_1 = y_0 \sin(kx - \omega t) \qquad y_2 = y_0 \sin(kx + \omega t)$$

$$y = y_1 + y_2 = 2y_0 \sin kx \cos \omega t$$

-Musical Instrument

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L}$$

$$v_{string} \propto \sqrt{\frac{\text{Tension}}{\frac{\text{mass}}{\text{length}}}}$$

$$v_{gas} \propto \sqrt{\frac{\text{temp}}{\text{molecular weight}}}$$

L27. Resonance, Destructive Resonance, Electromagnetic Waves, Speed of Light; Radio, TV, Distance Determinations using Radar and Lasers

-Electromagnetic Waves

-Plane Waves

$$\vec{E} = E_0\hat{x}\cos(kz - \omega t)$$

$$\vec{B} = B_0\hat{y}\cos(kz - \omega t)$$

-Satisfies Maxwell's Equations if:

$$B_0 = \frac{E_0}{c}$$

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

-Proof

Ampere's Law $\Rightarrow B_0 = \epsilon_0\mu_0 E_0 c$

Faraday's Law $\Rightarrow B_0 = \dfrac{E_0}{c}$

L28. Index of Refraction, Poynting Vector; Oscillating Charges, Radiation Pressure, Comet Tails, Polarization (Linear, Elliptical, and Circular)

-Electromagnetic Energy Density

Electric: $U_E = \dfrac{1}{2}\varepsilon_0 E^2$

Magnetic: $U_B = \dfrac{1}{2\mu_0}B^2$

$$= \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{1}{2}\varepsilon_0 E^2$$

$$U_{total} = \epsilon_0 E^2 = \epsilon_0 EBc \qquad \left(B_0 = \frac{E_0}{c} \right)$$

-Poynting Vector (Energy Flux)

$$Volume = speed \times area = c \ [m^3]$$

$$U_{total}c = \varepsilon_0 EBc^2 = \frac{EB}{\mu_0} \ [J/m^2sec]$$

$$\overrightarrow{S} = \frac{E \times B}{\mu_0} \ [W/m^2]$$ (Poynting Vector)

$$< S > = \frac{1}{2}\frac{E_0B_0}{\mu_0} = \frac{1}{2}\frac{E_0^2}{\mu_0 c}$$ (Time Average)

-Photon

Momentum of Photon: $p = \dfrac{\text{energy of one photon}}{c}$

$$\frac{S}{c} = \frac{1}{c}\frac{\text{energy}}{m^2sec} = \frac{\text{energy}}{c}\frac{1}{m^2sec} = \frac{p}{sec}\frac{1}{m^2} = \frac{F}{m^2}$$

$\dfrac{\bar{S}}{c}\alpha =$ Pressure

$\alpha = 0$ Full transparency

$\alpha = 1$ Full absorption

$\alpha = 2$ Full reflection

<u>L29. Snell's Law, Refraction, Total Reflection, Dispersion, Prisms, Huygen's Principle; The Illusion of Color, The Weird Benham Top, Land's Famous Demo</u>

$$\theta_0 = \theta_{refl}$$

-Snell’s Law

$$\frac{\sin\theta_0}{\sin\theta_{refr}} = \frac{n_{refr}}{n_0}$$ (n: Index of Refraction)

$n_{water} = 1.3$ $\qquad n_{glass} = 1.5$

-Total Reflection (e.g. light from water to air)

$$\sin\theta_{critical} = \frac{n_{refr}}{n_0}\left(n_0 > n_{refr}\right)$$

Application: Fibre Optics

-Speed of light

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0 \kappa \kappa_M}} = \frac{c}{\sqrt{\kappa \kappa_M}} = \frac{c}{n}$$

-dielectric constants depend on frequency- at high frequency, the dipoles can't follow fast enough

n_{water} for radio waves $= 8.9$

-Dispersion

Index of Refraction for light in different color is also different

$n_{red} = 1.331$ $n_{blue} = 1.343$ (both in water)

-3 Primary Color for Light: Red, Green, Violet

-Color TV

-Fail of the theory

-Bentham Top

-Edwin Land Slides