UNIT1. ELECTROSTATICS

-Electric Force F

$$\overrightarrow{F}_{1,2} = \frac{q_1 q_2 K}{r^2}\hat{r}_{1,2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{r}_{1,2}$$

$$K = 9 \times 10^9 \frac{Nm^2}{C^2} = \frac{1}{4\pi\epsilon_0}$$ = Coulomb Constant

ϵ_0**: Vacuum Permittivity**

C: Coulomb $\quad 1C = 1A \times 1sec$

$q_{p^+} = q_{e^-} \approx 1.6 \times 10^{-19}C$

Electricity is times stronger than gravity.

But large objects are typically neutral, so gravity dominates at large scales.

-Electric Field E

$$\overrightarrow{E} = \frac{\overrightarrow{F}}{q} = \frac{QK}{r^2}\hat{r} \qquad [\frac{N}{C}]$$

$$\overrightarrow{E} = \sum_i \overrightarrow{E}_i$$

-Electric Flux ϕ

$$d\phi = \overrightarrow{E} \cdot \overrightarrow{dA}$$

-Electrostatic Potential Energy U

$$Work = \int_R^\infty \overrightarrow{F}_{el} \cdot \overrightarrow{dr} = \frac{q_1 q_2}{4\pi\epsilon_0}\int_R^\infty \frac{dr}{r^2}$$ (*Energy to bring all charges to place)*

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 R} \qquad [J]$$

-Electric Potential V

$$V = \int_r^\infty \frac{\overrightarrow{F}_{el}}{q} \cdot \overrightarrow{dr} = \int_r^\infty \overrightarrow{E} \cdot \overrightarrow{d}r$$ *(Work per unit charge from infinity)*

$$V_p = \frac{Q}{4\pi\epsilon_0 R} \qquad [J/C = Volts]$$

-Equipotential lines

Every metal … will be an equipotential…

-Potential Difference

$$V_A - V_B = \int_A^B \overrightarrow{E} \cdot \overrightarrow{dr}$$

-Electric Field Energy Density

$$\frac{W}{(Volume)} = \frac{1}{2}\epsilon_0 E^2 \qquad [J/m^3]$$

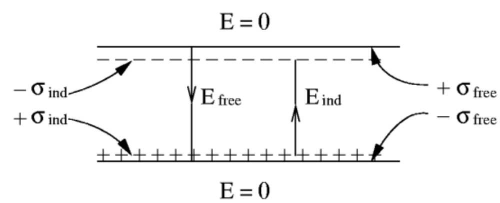
-Capacitance

$$C = \frac{Q}{V} \qquad [C/V = Farad]$$

(the capability of holding charge for a given electric potential)

-Dielectric Constant κ

$$\overrightarrow{E} = \overrightarrow{E}_{free} + \overrightarrow{E}_{ind} = \frac{\overrightarrow{E}_{free}}{\kappa}$$



κ: Dielectric Constant, depends on material

-Ohm's Law

-Notations

τ = Time between electron collisions (due to thermal motion)

n = Number of free electrons per cubic meter

$v_d = a\tau = \frac{eE}{m_e}\tau$ = Drift Velocity

$\sigma = \frac{e^2 n\tau}{m_e}$ = Conductivity

$\rho = \frac{1}{\sigma}$ = Resistivity

$$I = v_d A n e = \frac{e^2 n \tau}{m_e} A E = \sigma A E$$

$$V = El = \frac{l}{\sigma A} I$$

$$R = \frac{l}{\sigma A} = \frac{l\rho}{A}$$

$V = IR$ (only holds when current doesn't cause temperature change)

-Power

$$dW = dq\left(V_A - V_B\right) \qquad \rightarrow \qquad \frac{dW}{dt} = \frac{dq}{dt}\left(V_A - V_B\right) \qquad \rightarrow$$

$$P = IV = I^2R = \frac{V^2}{R} \qquad [J/sec = Watt]$$

$$\overrightarrow{E} = \frac{\overrightarrow{F}}{q} \quad \Rightarrow \quad V = \int_r^\infty \overrightarrow{E} \cdot \overrightarrow{dr} \quad + \quad \overrightarrow{E} = -gradV \quad \Rightarrow \quad \Delta U = \Delta Vq$$

-Gaussian Law

$$\phi = \oint \overrightarrow{E} \cdot \overrightarrow{dA} = \frac{\sum Q_{inside}}{\epsilon_0}$$

σ: Charge Density [$\frac{C}{m^2}$]

-Sphere (with charge Q uniformly distributed, radius R)

-Electric Field

$$for\ r < R, \quad 4\pi r^2 E = \frac{Q_{inside}}{\epsilon_0}, \quad E = 0$$ (内电场为0)

$$for\ r > R, \quad 4\pi r^2 E = \frac{Q}{\epsilon_0}, \quad E = \frac{Q}{4\pi r^2 \epsilon_0} = \frac{\sigma}{\epsilon_0}$$(外电场同电荷在圆心的电场)

-Conducting Sphere with Charge $+q$ Somewhere Inside

Uniform outside charge, independent of position of charge inside

-Capacitance

$$V_p = \frac{Q}{4\pi\epsilon_0 R} \quad \Rightarrow \quad C = 4\pi\epsilon_0 R$$

-Two Infinite Plane ($\sigma = \frac{Q}{A}$)

-Electric Field

$$E = \frac{\sigma}{\epsilon_0}$$

-Field Energy

$$W = \frac{1}{2}QEx = \frac{1}{2}\epsilon_0 E^2 Ax = \frac{1}{2}\epsilon_0 E^2 V \text{ (Volume)}$$

-Potential Energy in-between

$$U = \int \frac{1}{2}\epsilon_0 E^2 dV = \frac{1}{2}\epsilon_0 \left(\frac{\sigma}{\epsilon_0}\right)^2 Ah = \frac{1}{2}\sigma A \frac{\sigma}{\epsilon_0} h = \frac{1}{2}QEh = \frac{1}{2}QV$$

as capacitor: $U = \frac{1}{2}QV = \frac{1}{2}CV^2$

-Capacitance

$$C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \frac{A\epsilon_0}{d}$$

-Electric Breakdown

about $10^7 V/m$ (observed $3 \times 10^6 V/m$)

-Kirchhoff's Rules

$\oint \vec{E} \cdot d\vec{l} = 0$ (only holds when there is no inductor)

UNIT2. MAGNETIC FIELDS AND FORCES

-Magnetic Field B

$\overrightarrow{B}$ $[\frac{N \cdot sec}{C \cdot m} = Tesla]$ or $[Gauss = 10^{-4}T]$

(Around Electric Wire: direction of right hand rule)

(Electric field can do work on a charge, but a magnetic field cannot

- force is always perpendicular to motion)

-Lorentz Force

$$\overrightarrow{F}_B = q(\overrightarrow{v} \times \overrightarrow{B})$$

-Biot-Savart Law

$$\overrightarrow{dB} = \frac{CI}{r^2}\overrightarrow{dl} \times \hat{r} = \frac{\mu_0 I}{4\pi r^2}\overrightarrow{dl} \times \hat{r}$$

$$Constant = 10^{-7} = \frac{4\pi}{\mu_0}$$

μ_0**: Vacuum Permeability** $[H/m]$

-Ampere's Law

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 I_{penetration}$$

-Electromagnetic Induction

Faraday: Changing magnetic field causes current

-Lenz's Law

Induced current resist the change of magnetic field

-Faraday's Law

-Magnetic Flux

$$\phi_B = \int \overrightarrow{B} \cdot \overrightarrow{dA}$$

$$\oint \overrightarrow{E} \cdot \overrightarrow{dl} = \varepsilon = -\frac{d\phi_b}{dt} = -\frac{d}{dt}\int_{open\ surface} \overrightarrow{B} \cdot \overrightarrow{dA}$$

$$\oint \overrightarrow{E} . \overrightarrow{dl} = -\frac{d}{dt}\int \overrightarrow{B} \cdot \overrightarrow{dA}$$

Electric field caused by magnetic flux change is non-conservative

-Eddy Current

-Amended Ampere's Law

<u>a changing electric flux gives rise to a magnetic field</u>

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \left(I + \epsilon_0 \kappa \frac{d}{dt} \int \overrightarrow{E} \cdot \overrightarrow{dA} \right)$$

$\epsilon_0 \kappa \dfrac{d}{dt} \displaystyle\int \overrightarrow{E} \cdot \overrightarrow{dA}$**: Displacement Current**

-Induction Motor, Multi-phase current

-Self-Inductance L

Let $\phi_B = LI \quad \Rightarrow$

$$\varepsilon_{ind} = -\frac{d\phi_B}{dt} = -L\frac{dI}{dt} \qquad [Henry = \frac{V \cdot sec}{A}]$$

-Relative Permeability κ_M

$$B = \kappa_M B_{vacuum}$$

$$\kappa_M = 1 + X_M$$

-Diamagnetic	$X_M \rightarrow -0$	$\Rightarrow$	$\kappa_M < 1$
-Paramagnetic	$X_M \rightarrow +0$	$\Rightarrow$	$\kappa_M > 1$
-Ferromagnetic	$X_M \simeq \kappa_M \simeq 10^2 \rightarrow 10^5$		

-Ampere-Maxwell Equation

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \kappa_M \mu_0 (I_{penetration} + \epsilon_0 \kappa \frac{d\phi_B}{dt})$$

-Maxwell's 2nd Equation

$$\oint \overrightarrow{B} \cdot \overrightarrow{dA} = 0$$

(since there's no magnetic monopole as an experimental fact)

-Superconductivity

-Magnetic Levitation (Maglev)

-Curie Point

the temperature when ferromagnetic material loose its structure of magnetic domain

-Moving Charges in B-fields

-for $\overrightarrow{B} \perp \overrightarrow{v}$ and $\overrightarrow{B}$ constant

$$qvB = \frac{mv^2}{R} \quad \Rightarrow \quad R = \frac{mv}{qB} = \sqrt{\frac{2mV}{qB^2}} \qquad (qV = \frac{1}{2}mv^2)$$

-Cyclotron

-Straight Wire

-Biot-Savart Law

$$B = \frac{\mu_0 I}{2\pi R}$$

-Ampere's Law

-outside: $B = \dfrac{\mu_0 I}{2\pi r}$

-inside: $B = \dfrac{\mu_0 I r}{2\pi R^2}$

-Solenoids

-Biot-Savart Law (single loop)

$$B = \frac{\mu_0 I}{2R}$$

-Ampere's Law

$$B = \frac{\mu_0 I N}{L} \quad (L \gg R)$$

-Inductance

$$B = \frac{\mu_0 I N}{l} \quad \Rightarrow \quad \phi_b = \pi r^2 N B = \pi r^2 N^2 \frac{\mu_0 I}{l} \quad \Rightarrow$$

$$L = \pi r^2 \frac{N^2}{l} \mu_0$$

-Dynamo

$$\oint \overrightarrow{E} . \overrightarrow{dl} = -\frac{d}{dt} \int \overrightarrow{B} \cdot \overrightarrow{dA} = \frac{d}{dt} \int BA \cos\theta$$

change in B, A, θ will cause EMF

-Spinning coil in magnetic field

$$\omega = \frac{2\pi}{\text{Period}} \qquad \theta = \omega t \qquad \phi_b = AB \cos \omega t$$

$$\varepsilon(t) = -\frac{d\phi}{dt} = AB\omega \sin \omega t$$

-Transformers

$$V_1 = -L_1\frac{dI_1}{dt} = \varepsilon_1 = -N_1\frac{d\phi_B}{dt}$$

$$V_2 = -L_2\frac{dI_2}{dt} = \varepsilon_1 = -N_2\frac{d\phi_B}{dt}$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

-Spark Plugs - Car coils (The Ruhmkorff)

UNIT3. CIRCUITS

-RL Circuits

$$\oint Edl = 0 + IR - V = -L\frac{dI}{dt}$$

$$I_{charge} = I_{\max}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$I_{discharge} = I_{max}e^{-\frac{R}{L}t}$$

-Magnetic Field Energy Density

$$U = \int_0^\infty I^2 R dt = I_{max}^2 R \int_0^\infty e^{-\frac{2R}{L}t} dt = \frac{1}{2}LI_{max}^2$$
$$= \frac{B^2}{2\mu_0}\pi r^2 l = \frac{B^2}{2\mu_0}V \text{ (Volume)}$$

$\frac{B^2}{2\mu_0} =$ **Magnetic field energy density** $[\mathrm{J/m^3}]$

-AC Current

$$V = V_0 \cos \omega t$$

$$I = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}} \cos(\omega t - \phi), \quad \text{where } I_{max} = \frac{V_0}{\sqrt{R^2 + (\omega L)^2}}$$

$$\tan \phi = \frac{\omega l}{R} \quad \text{(Phase Lag)}$$

-RC Circuits

$$+V_C + IR - V_0 = 0 \quad \Rightarrow$$

$$\frac{Q}{C} + R\frac{dQ}{dt} - V_0 = 0 \quad \Rightarrow$$

$$Q = V_0 C\left(1 - e^{-t/RC}\right) \quad \Rightarrow$$

$$I_{charge} = \frac{dQ}{dt} = \frac{V_0}{R}e^{-t/RC} \qquad V_C = \frac{Q}{C} = V_0\left(1 - e^{-t/RC}\right)$$

$$I_{discharge} = -\frac{V_0}{R}e^{-t/RC}$$

-LRC Circuits

$$V_c + 0 + IR - V_0 \cos \omega t = -L\frac{dI}{dt} \quad \Rightarrow$$

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = V_0 \cos\omega t \quad \Rightarrow$$

$$I = \frac{V_0}{Z}\cos(\omega t - \phi)$$

-Reactance: $\mathrm{X} = \omega L - \dfrac{1}{\omega C}$

-Impedance: $Z = \sqrt{R^2 + \mathrm{X}^2}$ (unit: Ω)

$$\tan\phi = \frac{\mathrm{X}}{R}$$

$$\overline{P} = \overline{VI} = \frac{V_0^2}{Z}\overline{\cos\omega t\,\cos(\omega t - \phi)} = \frac{V_0^2}{2Z}\cos\phi$$

-Resonance

$$\mathrm{X} = 0 \quad \Rightarrow \quad \omega L = \frac{1}{\omega C} \quad \Rightarrow \quad \int$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

-Quality

In the graph of I to ω:

$\Delta\omega = \dfrac{R}{L} =$ width of region greater than $70\,\%$ of maximum current

$Q = \dfrac{\omega_0}{\Delta\omega} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$ (Q: narrowness of curve)

UNIT4. MAXWELL'S EQUATIONS AND ELECTROMAGNETISM

-Maxwell's Equations

$$\oint \overrightarrow{E} \cdot d\overrightarrow{A} = \frac{Q_{free}}{\epsilon_0 \kappa}$$ (Gaussian Law)

$$\oint \overrightarrow{B} \cdot d\overrightarrow{A} = 0$$

$$\oint \overrightarrow{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$ (Faraday's Law)∫

$$\oint \overrightarrow{B} \cdot d\vec{l} = \kappa_M \mu_0 (I_{penetration} + \epsilon_0 \kappa \frac{d\phi_B}{dt})$$ (Ampere's Law)

-Electromagnetic Waves

-Plane Waves

$$\overrightarrow{E} = E_0 \hat{x} \cos(kz - \omega t)$$

$$\overrightarrow{B} = B_0 \hat{y} \cos(kz - \omega t)$$

-Satisfies Maxwell's Equations if:

$$B_0 = \frac{E_0}{c}$$

$$\frac{\omega}{k} = c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

-Electromagnetic Energy Density

Electric: $U_E = \frac{1}{2} \varepsilon_0 E^2$

Magnetic: $U_B = \frac{1}{2\mu_0} B^2$

$$= \frac{1}{2\mu_0} \frac{E^2}{c^2} = \frac{1}{2} \varepsilon_0 E^2$$

$$U_{total} = \epsilon_0 E^2 = \epsilon_0 EBc \qquad (B_0 = \frac{E_0}{c})$$

-Poynting Vector (Energy Flux)

$$Volume = speed \times area = c \ [m^3]$$

$$U_{total} c = \varepsilon_0 EBc^2 = \frac{EB}{\mu_0} \ [J/m^2 sec]$$

$$\overrightarrow{S} = \frac{E \times B}{\mu_0} \ [W/m^2] \qquad \text{(Poynting Vector)}$$

$$< S > = \frac{1}{2}\frac{E_0 B_0}{\mu_0} = \frac{1}{2}\frac{E_0^2}{\mu_0 c} \qquad \text{(Time Average)}$$

-Snell's Law

$$\frac{\sin\theta_0}{\sin\theta_{refr}} = \frac{n_{refr}}{n_0} \qquad \text{(} n\text{: Index of Refraction)}$$

$$n_{water} = 1.3 \qquad n_{glass} = 1.5$$

-Speed of light

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0 \kappa \kappa_M}} = \frac{c}{\sqrt{\kappa \kappa_M}} = \frac{c}{n}$$

-Traveling Wave

$$y = y_0 \sin(kx - \omega t)$$

y_0: amplitude

λ: wave length

k: wave number $\qquad k = \frac{2\pi}{\lambda}$

ω: angular frequency

v: traveling velocity $\qquad v = \frac{\omega}{k}$

-Standing Wave

$$y_1 = y_0 \sin(kx - \omega t) \qquad y_2 = y_0 \sin(kx + \omega t)$$

$$y = y_1 + y_2 = 2y_0 \sin kx \cos \omega t$$

-Total Reflection (e.g. light from water to air)

$$\sin\theta_{critical} = \frac{n_{refr}}{n_0} \left(n_0 > n_{refr}\right)$$

Application: Fibre Optics

-Dispersion

<u>Index of Refraction for light in different color is also different</u>

$n_{red} = 1.331 \qquad n_{blue} = 1.343$ (both in water)

-Linearly Polarization

-Interference

-Diffraction (Single-slit Interference)

-Doppler Shift Equation

$$f' = f\,\frac{v_{sound} - v_{receiver}}{v_{sound} - v_{transmitter}}$$

-Musical Instrument

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L}$$

$$v_{string} \propto \sqrt{\frac{\text{Tension}}{\frac{\text{mass}}{\text{length}}}}$$

$$v_{gas} \propto \sqrt{\frac{\text{temp}}{\text{molecular weight}}}$$