GU TECH, AL GHAZALI UNIVERSITY

Fall-2024 Department of Computer Science
Final Exam
SOLUTION PAPER
6^{th} February 2025, 10:00am – 01:00pm

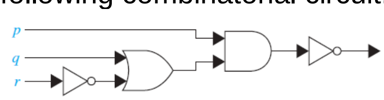
Course Code: CS103	**Course Name:** Discrete Structures
Instructor Name: Dr. M. Shahzad	
Student ID:	**Name:**

Instructions:

- Calculators are **NOT ALLOWED**.
- Answer all questions on the question paper. Do all ROUGH work on the plane sides of the pages.
- Left column contains questions. Right column contains the space for ANSWERS.
- In case of any ambiguity, you may make assumption. However, your assumption should not any statement in the question paper.
- Write **Student ID** and **Name** first page of the question paper.

Total Time: 3 Hours **Max Points**: 50

Q1 [CLO-1]: Choose the best answer: **[10 points, 30 mins]**

1. Which of the following is a tautology? a) p∧¬p b) p∨¬p c) p→p d) p↔¬p	2. The negation of ∀x(P(x)→Q(x)) is: a) ∀x(P(x)∧¬Q(x)) b) ∃x(P(x)∧Q(x)) c) ∃x(P(x)∧¬Q(x)) d) ∀x(P(x)∨Q(x))
3. The inverse of the function f(x)=2x+3 is: a) $f^{-1}(x) = \frac{x-3}{2}$ b) $f^{-1}(x) = 2x - 3$ c) $f^{-1}(x) = \frac{x+3}{2}$ d) $f^{-1}(x) = x - 3$ Sol. (a)	4. To prove that the sum of two even numbers is even, we assume two numbers a and b, and write them as: a) a=2m+1, b=2n+1 b) a=2m, b=2n c) a=m/2, b=n/2 d) a=2m+1, b=2n
5. What is the negation of "$n \leq 5$" a) n is less then OR equal to 5 b) n is less then AND equal to 5 c) n is neither less then nor equal to 5 d) None of these	6. The contrapositive statement for $(\sim p \wedge q) \rightarrow \sim q$ is: a) $\sim(\sim p \wedge q) \rightarrow q$ b) $\sim q \rightarrow (\sim p \wedge q)$ c) $\sim q \rightarrow \sim(p \vee \sim q)$ d) $q \rightarrow \sim(\sim p \wedge q)$
7. Which of the following is expression(s) true? a) p→ q ≡ t b) (p→ q) → p ≡ t c) ((p→ q) → p) → p ≡ t d) None of these	8. A relation R on set A is called antisymmetric if: a) $(a,a) \in R \rightarrow (b,a) \in R$ b) $(a,a) \in R$ and $(b,a) \in R$ imply $a = b$ c) $(a,a) \in R\ \ \forall a \in A$ d) $(a,b) \notin R\ \ \forall a \neq b$
9. Suppose a universal set U contains 50 elements, and two subsets A and B satisfy |A|=30, |B|=25, and |A∩B|=10. What is $\|A^c \cup B^c\|$? $A^c \cup B^c = U - (A \cap B)$ $\|A^c \cup B^c\| = 50 - 15 = 35$ a) 25 b) 30	10. What is correct Boolean expression for the following combinatorial circuit:  a) $\overline{p.(q+\overline{r})}$

c) 35 d) 40	b) $p.(q+\overline{r})$ c) $\overline{(p.q)+\overline{r}}$ d) $(p+q).r$

Q2 [CLO-1]: SETS and FUNCTIONS **[10 points, 50 mins]**

a) List the members of $\{x \mid x$ is a real number such that $x^2 = 1\}$	**{-1,1}**
b) List the members of $\{x \mid x$ is an integer such that $x^2 = 2\}$	**{} or Ø**
c) If {∅, {*a*}, {∅, *a*}} is the power set of a set S, write S.	**The given set is not a power set of any set**
d) If {∅, {*a*}, {*b*}, {*a*, *b*}} is the power set of a set S, write S.	**S = {a,b}**
e) What can you say about the sets *A* and *B* if $A \oplus B = A$?	**Either A is null set or B is null set**
f) Let A_i be the set of all nonempty bit strings of length not exceeding *i*. Find A_1, A_2 and A_1UA_2	**A_1 = {0,1}** **A_2 = {0,1,00,01,10, 11}** **$A_1UA_2 = A_2$**
g) Determine whether the function $f: Z \to Z$ given $f(x) = x^2$ by is a) injective, b) surjective	**a) Not injective because f(−1)=1 and f(1)=1,** **b) Not surjective because negative integers are not in the range.**
h) Let $f: Z \to Z$ be $f(x) = 2x - 1$ What is the range of *f*? Is *f* onto?	**Range: All odd numbers** **Not onto because even integers have no preimage**
i) Let $f: Z \to Z$ be $f(x) = 2x - 1$ Is *f* one-to-one?	**Yes *f* is one-to-one**
j) Let $f(x) = x + 3$ and let $g(x): 2x$ find $(f\ o\ g)(x)$	$(f \circ g)(x) = f(g(x))$ • First, apply $g(x)$: $g(x) = 2x$ • Now, apply $f(x)$ to $g(x)$: $f(g(x)) = f(2x) = 2x + 3$ • So, $(f \circ g)(x) = 2x + 3$

Q3 [CLO-1]: RELATIONS **[10 points, 50 mins]**

For the relations in parts (a-e), determine the properties of the relations.			
Note: Relations in parts (d-f) are defined on Z. Relations in parts (a-c) are defined on set {1,2,3,4}	Reflexive/Irreflexive/None	Symmetric/Asymmetric/Anti-symmetric/None	Transitive/Not Transitive
a) R: {(1, 2), (2, 3), (3, 4)}	**Irreflexive**	**Asymmetric**	**Not Transitive**
b) R: {(1, 1), (2, 2), (3, 3), (4, 4)}	**Reflexive**	**Symmetric, Anit-sym**	**Transitive**
c) R: {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}	**Irreflexive**	**None**	**Not Transitive**
d) R: $x = y^2$	**None**	**Anti-symmetric**	**Not Transitive**
e) R: $x \geq y^2$	**None**	**Anti-symmetric**	**Transitive**
f) R: $x \neq y$	**Irreflexive**	**Symmetric**	**Transitive**
g) Let A={1, 2, 3, 4}. Consider the following two relation on *A*. R_1={(1,1), (1,2), (2,2), (2,1), (3,3)}	$\boldsymbol{R_2^2\ o\ R_1}$ = **{(1,2),(1,1),(3,1),(3,2)}**		

R_2= {(1,3), (3,2), (2,1)} **Find $R_2^2\ o\ R_1$**	
h) Is (Z,=) a poset? How?	• **The relation "=" is reflexive because a=a.** • **It is antisymmetric because if a=b and b=a, then a=b.** • **It is transitive because if a=b and b=c, then a=c.**
i) Determine is it a poset? $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	• **Reflexive** ✅ • **Antisymmetric** ❌ **(fails because M_{12}=1 and M_{21}=1)** • **Transitive** ✅ **Since it fails antisymmetric, the relation is not a poset.**
j) Draw Hasse diagram for the poset (S,<=) S={1,2,4}	**4** **|** **2** **|** **1**

Q4 [CLO-1]: LOGICS [10 points, 20 mins]

a) Let p, q, and r be the propositions p :You have the flu. q :You miss the final examination. r :You pass the course. Express the proposition (p → ¬ r) ∨ (q → ¬ r) as an English sentence.	**If you have the flu then you don't pass the course or if you miss the final examination then you don't pass the course.**
b) State the converse, contrapositive, and inverse of the following conditional statements. "I come to class whenever there is going to be a quiz☺"	Converse: "If I come to class, then there will be a quiz." Contrapositive: "If I do not come to class, then there will not be a quiz." Inverse: "If there is not going to be a quiz, then I don't come to class."
Translate (c-e) of these statements into logical expressions using predicates, quantifiers, and logical connectives.	
c) Something is not in the correct place.	P(x): x is in the correct place. Q(x):x is in excellent condition. The domain of x consists of all objects under consideration. ∃x¬P(x)

d) All tools are in the correct place and are in excellent condition.	$\forall x(T(x)\rightarrow(P(x)\wedge Q(x)))$
e) Nothing is in the correct place and is in excellent condition.	$\neg\exists x(P(x)\wedge Q(x))$ or $\forall x\neg(P(x)\wedge Q(x))$

*********End of The Exam*********