"Heavy-Flavor Transport in QCD Matter" ECT*-Workshop pre-homework

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1a) *Current best estimate of $D_s(2\pi T)$ as function of T over available T-range (both charm and bottom, if available).*

We have made two different extractions of heavy quark momentum diffusion coefficient $\kappa/T^3 = 2/(D_s T)$ from lattice:

- We have recently measured [1] heavy quark momentum diffusion coefficient from lattice in temperature range $T = 1.1 - 10^4 T_\mathrm{c}$ in pure gauge $T_\mathrm{c} = 1.24\Lambda_{\overline{\mathrm{MS}}}^{N_\mathrm{f}=0}$. We got the following values for κ:

$$\begin{aligned}
1.91 <& \frac{\kappa}{T^3} < 5.4 && \text{and} \quad 0.37 < D_\mathrm{s}T < 1.05 && \text{for} \quad T = 1.1T_\mathrm{c}\,,\\
1.31 <& \frac{\kappa}{T^3} < 3.64 && \text{and} \quad 0.55 < D_\mathrm{s}T < 1.52 && \text{for} \quad T = 1.5T_\mathrm{c}\,,\\
0.63 <& \frac{\kappa}{T^3} < 2.20 && \text{and} \quad 0.91 < D_\mathrm{s}T < 3.17 && \text{for} \quad T = 3T_\mathrm{c}\,,\\
0.43 <& \frac{\kappa}{T^3} < 1.05 && \text{and} \quad 1.9 < D_\mathrm{s}T < 4.65 && \text{for} \quad T = 6T_\mathrm{c}\,,\\
0 <& \frac{\kappa}{T^3} < 0.72 && \text{and} \quad 2.78 < D_\mathrm{s}T && \text{for} \quad T = 10T_\mathrm{c}\,,\\
0 <& \frac{\kappa}{T^3} < 0.10 && \text{and} \quad 20 < D_\mathrm{s}T && \text{for} \quad T = 10^4T_\mathrm{c}\,.
\end{aligned}$$

  These results are plotted in figure 1, where on left we plot all our results on κ compared to perturbation theory and the fit explained at point 1c, and on the right we show the D_s compared to the existing lattice results.

- Our group has also recently extracted κ of quarkonium from the pre-existing un-quenched lattice measurements of quarkonium in medium width Γ [2]. For example for 1S Coulombic quarkonium state we have relation $\kappa = \Gamma/(3a_0)$ with a_0 being the Bohr radius $a_0 = 2/(MC_\mathrm{F}\alpha_\mathrm{s})$ [3, 4]. Using the un-quenched lattice results from [5–7] we get the results shown in figure 2. The left side of the figure shows the most accurate determinations we have for $\Upsilon(1S)$ at two different temperatures compared to existing results of κ. The $T = 407, 440$MeV point is constructed from two separate lattice studies [5, 6] used as upper and lower boundaries and for $T = 334$MeV we take the data from [7]. The Ref. [7] has more available temperatures, but since our hierarchy of scales assumes $\pi T \gg E$ and using the Coulombic binding energy we find $E = -1/(ma_0^2)$ which for bottomonium is -460 MeV. This is fulfilled only very marginally at the lower temperatures, which is why we are confident only on the two points listed above. For consistency, we do however, list

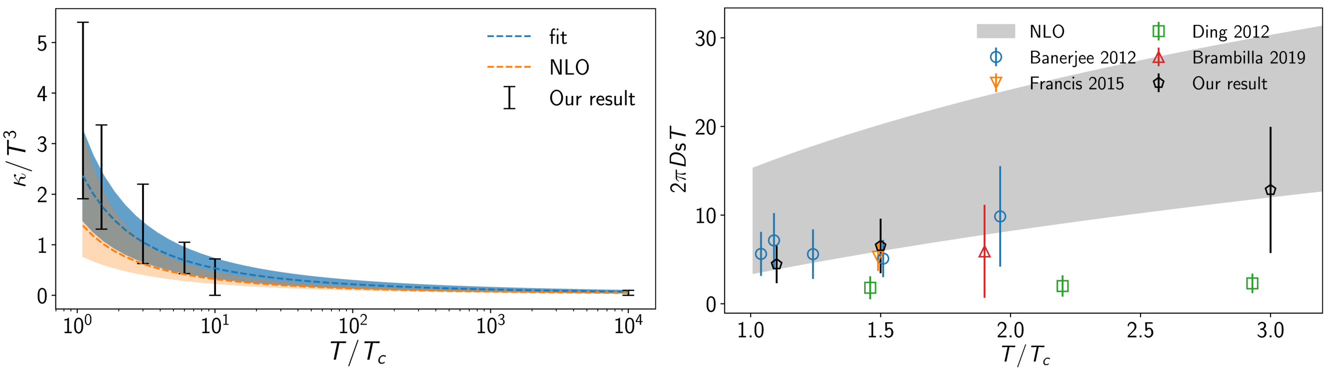

Figure 1: Quenched lattice measurement of the heavy quark momentum diffusion coefficient. On left: κ and a simple fit for temperature dependence. On right: Our D_s compared to existing lattice measurements.

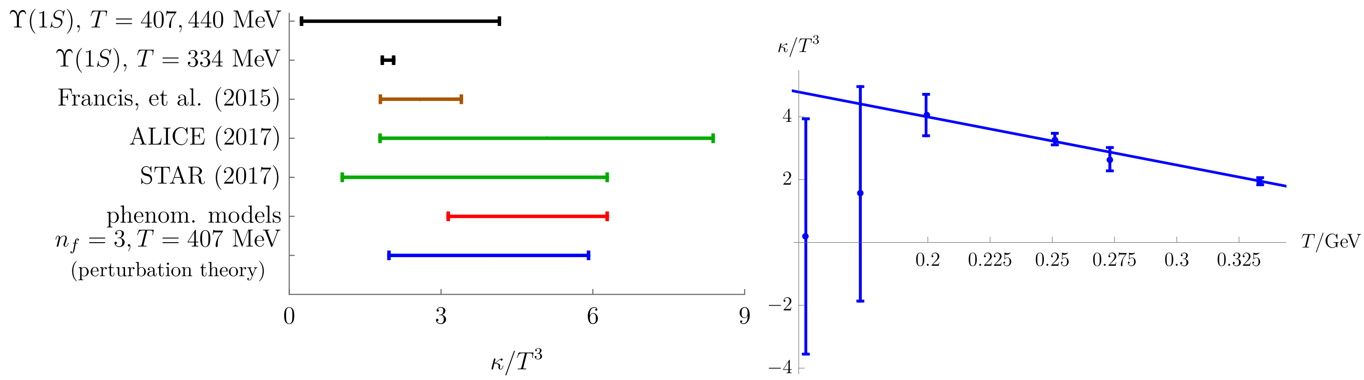

Figure 2: Left: The κ/T^3 extracted in [2] from [5–7] compared to some existing results. Right: Estimate of the temperature dependence of κ/T^3.

κ extractions for all the points available in [7] below:

$$
\begin{array}{rlll}
0.24 < \kappa/T^3 < 4.2 & \text{and} \quad 0.48 < D_\mathrm{s}T < 8.3 & \text{for} \quad T = 407,\, 440\,\mathrm{MeV}\,, \\
1.84 < \kappa/T^3 < 2.06 & \text{and} \quad 0.97 < D_\mathrm{s}T < 1.09 & \text{for} \quad T = 334\,\mathrm{MeV}\,, \\
2.28 < \kappa/T^3 < 3.02 & \text{and} \quad 0.66 < D_\mathrm{s}T < 0.89 & \text{for} \quad T = 273\,\mathrm{MeV}\,, \\
3.10 < \kappa/T^3 < 3.47 & \text{and} \quad 0.58 < D_\mathrm{s}T < 0.65 & \text{for} \quad T = 251\,\mathrm{MeV}\,, \\
3.40 < \kappa/T^3 < 4.72 & \text{and} \quad 0.42 < D_\mathrm{s}T < 0.59 & \text{for} \quad T = 199\,\mathrm{MeV}\,, \\
\kappa/T^3 < 4.96 & \text{and} \quad 0.40 < D_\mathrm{s}T & \text{for} \quad T = 173\,\mathrm{MeV}\,, \\
\kappa/T^3 < 3.94 & \text{and} \quad 0.51 < D_\mathrm{s}T & \text{for} \quad T = 151\,\mathrm{MeV}\,,
\end{array}
$$

Further, we would like to remind that in the literature the heavy quark momentum diffusion coefficient defined with fundamental links is expected to be equivalent to the heavy quarkonium momentum diffusion coefficient defined with adjoint link between electric fields. While this picture has recently been questioned, we observe nice agreement between our heavy quark and quarkonium results. Nevertheless, the κ in our models is same for bottom and charm, and likewise shared between charmonium and bottomonium. Any corrections to this static limit picture would be suppressed by at least $1/M_{\mathrm{HQ}}$.

1b) *Normalized momentum dependence of friction coefficient, $A(p;T)/A(p=0;T)$, for current best estimate.*

The drag coefficient is related to κ with the usual fluctuation dissipation relation $\eta_{\mathrm{D}} = \kappa/(2M_{\mathrm{HQ}}T)$. We do not have momentum dependence in our measurements of κ due to the fact that $1/M_{\mathrm{HQ}}$ effects are not included.

1c) *Table of current best estimates of charm friction and momentum-diffusion coefficients for $p = 0-40\,\mathrm{GeV}$ (in steps of $\mathrm{d}p = 0.2\,\mathrm{GeV}$) and $T = 0.16-0.6\,\mathrm{GeV}$ (steps $\mathrm{d}T = 0.02\,\mathrm{GeV}$) for $\mu_B = 0$. The idea is to run them through a Langevin simulation in a common hydrodynamic medium evolution.*

As stated in point 1c we do not have momentum dependence in our measurements of κ due to the fact that $1/M_{\mathrm{HQ}}$ effects are not included. We can, however, extract the temperature dependence of κ/T^3 at $p = 0$ for both of our models. Instead of a table we provide a functional form for the temperature dependence.

- For the quenched lattice results we can do a very simple fit inspired by NLO form of spectral function:
$$\frac{\kappa}{T^3} = \frac{g(\mu)^4 C_{\mathrm{F}} N_{\mathrm{c}}}{18\pi}\left[\left(\ln\frac{2T}{m_{\mathrm{E}}} + \xi\right) + \frac{m_{\mathrm{E}}}{T}C\right]$$
where $m_{\mathrm{E}} = g(\mu)T$, $\xi \simeq -0.64718$, we measure the running coupling using 5-loop running, and set the scale μ such that the NLO correction to the gauge coupling in EQCD vanishes [8]:
$$\ln(\mu_\omega) = \ln(4\pi T) - \gamma_{\mathrm{E}} - \frac{N_{\mathrm{c}} - 8N_{\mathrm{f}}\ln(2)}{2(11N_{\mathrm{c}} - 2N_{\mathrm{f}})}.$$
Using this fit we get $C = 3.81(1.33)$, which can be compared to the perturbative NLO result of $C = 2.3302$ [9]. The errors include a variation of scale μ by a factor of two. The resulting curve is shown in figure 1.
- For the unquenched study, we show on the right hand side of figure 2 the estimation of temperature dependence with this method. Here we go with simple linear regression to get a rough estimate of temperature dependence within the available temperature range $T \approx 1-2.2T_{\mathrm{c}} = 150-334\,\mathrm{MeV}$. The function has a form $\kappa/T^3 = 7.2(3) - 15(1)\frac{T}{\mathrm{GeV}}$, which translates to $D_{\mathrm{s}}T = 0.8(4) + 6(1)\frac{T}{\mathrm{GeV}}$.

Our group has nothing to contribute to questions 2 and 3 at the moment.

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