

# Chapter 6: KYC / AML — Surveillance, Anomaly Detection, and Regulatory Compliance

## 1 KYC and AML as Financial Control Systems

Know-Your-Customer (KYC) and Anti-Money-Laundering (AML) systems are designed to prevent misuse of financial infrastructure for illicit purposes. Unlike fraud detection, which focuses on immediate transactional abuse, AML systems focus on long-term behavioral patterns and structural risk.

These systems operate under regulatory mandates and therefore impose non-negotiable constraints on onboarding, monitoring, and reporting. Mathematically, KYC/AML systems can be viewed as continuous surveillance and anomaly detection mechanisms over customer and transaction state spaces.

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## 2 Customer Identity as a State Variable

Each customer is associated with an identity state vector:

$$\mathbf{u} = (u_1, u_2, \dots, u_k)$$

where components may represent identity attributes, verification outcomes, jurisdictional risk, and historical behavior.

KYC aims to estimate the probability that a customer identity is legitimate:

$$P(I = 1 \mid \mathbf{u})$$

where  $I$  is a binary variable indicating identity validity.

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### 3 Risk-Based Approach to Compliance

Modern compliance systems operate on a risk-based framework. Each customer is assigned a composite risk score:

$$R = \sum_{i=1}^k w_i \cdot u_i$$

where weights  $w_i$  reflect regulatory sensitivity and empirical risk contribution. Higher-risk customers are subjected to enhanced due diligence and stricter monitoring.

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### 4 Customer Lifecycle Monitoring

Compliance does not end at onboarding. Let  $R_t$  denote customer risk at time  $t$ . Risk evolves as:

$$R_{t+1} = R_t + \Delta R_t$$

where  $\Delta R_t$  captures behavioral changes, transaction anomalies, and external signals. Sudden increases in  $R_t$  trigger reviews or restrictions.

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### 5 Transaction Monitoring as a Time Series Problem

Let  $\{X_t\}$  denote a time series of transactions for a customer. AML systems analyze patterns such as volume, frequency, counterparties, and geographic dispersion.

The expected transaction behavior is modeled as:

$$E[X_t \mid \mathcal{H}_{t-1}]$$

where  $\mathcal{H}_{t-1}$  is historical behavior up to time  $t - 1$ .

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### 6 Anomaly Detection Using Statistical Distance

Anomalies are identified as deviations from historical norms. For a transaction feature  $x$  with mean  $\mu$  and standard deviation  $\sigma$ :

$$Z = \frac{x - \mu}{\sigma}$$

Large absolute values of  $Z$  indicate statistically unusual behavior.

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## 7 Multivariate Anomaly Detection

When monitoring multiple features simultaneously, the Mahalanobis distance is used:

$$D^2 = (\mathbf{x} - \boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x} - \boldsymbol{\mu})$$

where  $\Sigma$  is the covariance matrix of features. This captures correlated anomalies across dimensions.

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## 8 Rule-Based AML Systems

Many AML frameworks incorporate deterministic rules such as threshold breaches:

$$\sum_{t=1}^n Amount_t > L$$

within a defined time window. While simple, rule-based systems are transparent and regulator-friendly but prone to high false positive rates.

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## 9 Probabilistic Suspicious Activity Modeling

Let  $S$  denote the event that activity is suspicious. The objective is to estimate:

$$P(S = 1 \mid \mathbf{x}_{1:t})$$

where  $\mathbf{x}_{1:t}$  represents the sequence of transactions. This probability guides alert generation.

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## 10 Alert Generation and Review Capacity

Alerts are generated when risk exceeds a threshold  $\tau$ :

$$\text{Alert if } R_t \geq \tau$$

Operational capacity constraints require that alert volume remains bounded:

$$Alerts_{daily} \leq Capacity_{review}$$

This introduces an optimization constraint absent in pure detection problems.

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## 11 False Positives and Compliance Cost

Let:

- $FP$  = legitimate customers flagged
- $TP$  = true suspicious cases detected

Compliance cost is modeled as:

$$Cost = FP \cdot C_{review} + FN \cdot C_{regulatory}$$

where false negatives may incur severe penalties.

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## 12 Reporting Obligations

Confirmed suspicious activity must be reported to authorities within defined time limits.

Let  $T_{report}$  denote reporting latency:

$$T_{report} \leq T_{max}$$

Violations introduce legal and systemic risk.

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## 13 Data Retention and Auditability

Compliance systems require immutable audit trails. Let  $\mathcal{D}$  represent stored records. Regulatory requirements impose:

$$\mathcal{D}_{retention} \geq N \text{ years}$$

Auditability constrains data deletion and system design.

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## 14 Cross-Border and Jurisdictional Risk

Customer risk increases with exposure to high-risk jurisdictions. Let  $J$  denote jurisdictional risk weight:

$$R_{geo} = \sum_j J_j \cdot Exposure_j$$

Geographic dispersion amplifies compliance complexity.

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## 15 Concept Drift in AML Systems

Behavioral norms evolve over time. Let  $P_t(X)$  denote transaction distributions. Concept drift occurs when:

$$P_t(X) \neq P_{t+k}(X)$$

Static thresholds become ineffective, requiring recalibration.

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## 16 Systemic Risk Perspective

AML systems contribute to macro-level financial stability by preventing accumulation of illicit flows. Failure modes propagate beyond individual institutions, making AML a systemic control layer rather than a local optimization problem.

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## 17 Summary

KYC and AML systems operate as continuous surveillance and anomaly detection mechanisms under strict regulatory constraints. Their mathematical foundation combines risk scoring, time-series analysis, statistical distance measures, and constrained optimization. Effectiveness depends on balancing detection accuracy, operational capacity, and regulatory compliance over time.