Chapter 2: Lending — Risk, Capital, and the Time Value of Money

1 Lending as a Financial Contract

Lending is the process by which capital is transferred from a lender to a borrower under a contractual agreement specifying repayment amount, schedule, and conditions. Unlike payments, lending is inherently time-dependent and risk-bearing. The lender exchanges present capital for uncertain future cash flows, making lending fundamentally a stochastic optimization problem.

Formally, a loan can be represented as a sequence of cash flows:

$$\mathcal{L} = \{-P, C_1, C_2, \ldots, C_n\}$$

where P is the principal disbursed at time $t = 0$, and C_t represents the repayment cash flow at time t.

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2 Time Value of Money

The core principle underlying lending is that money today is worth more than the same amount in the future. This is captured through discounting.

The present value of a future cash flow C_t received at time t with discount rate r is:

$$PV(C_t) = \frac{C_t}{(1+r)^t}$$

The net present value (NPV) of a loan is:

$$NPV = -P + \sum_{t=1}^{n} \frac{C_t}{(1+r)^t}$$

A loan is economically viable only if $NPV \geq 0$.

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3 Loan Amortization and EMI Derivation

Most retail loans are repaid through equated periodic installments. Let:

- P = Principal
- r = periodic interest rate
- n = total number of periods

The installment amount EMI is derived by equating the present value of repayments to the principal:

$$P = \sum_{t=1}^{n} \frac{EMI}{(1+r)^t}$$

Solving the geometric series yields:

$$EMI = \frac{P \cdot r(1+r)^n}{(1+r)^n - 1}$$

Each installment consists of an interest component and a principal component, whose proportions evolve over time.

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4 Default as a Stochastic Event

Borrower default is an uncertain event influenced by income stability, leverage, behavioral factors, and macroeconomic conditions. Default is modeled probabilistically.

Define a binary random variable:

$$D = \begin{cases} 1, & \text{if default occurs} \\ 0, & \text{otherwise} \end{cases}$$

The probability of default (PD) is:

$$PD = P(D = 1)$$

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5 Expected Loss Framework

Credit risk is quantified using the Expected Loss (EL) framework:

$$EL = PD \times LGD \times EAD$$

where:

- PD = Probability of Default
- LGD = Loss Given Default
- EAD = Exposure at Default

This formulation converts uncertain borrower behavior into an expected monetary loss.

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6 Risk-Adjusted Loan Pricing

The interest rate charged on a loan must compensate for expected loss and operational costs.

Let:

- C = operational cost
- EL = expected loss

The minimum sustainable interest rate r_{min} satisfies:

$$r_{min} \cdot P = EL + C$$

Thus:

$$r_{min} = \frac{EL + C}{P}$$

Charging below this rate guarantees negative expected returns.

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7 Portfolio-Level Risk Aggregation

A lending institution holds a portfolio of loans $\{\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_N\}$. The total expected loss is:

$$EL_{portfolio} = \sum_{i=1}^{N} PD_i \cdot LGD_i \cdot EAD_i$$

If defaults are correlated, portfolio variance increases:

$$\sigma^2 = \sum_i \sigma_i^2 + \sum_{i \neq j} \rho_{ij} \sigma_i \sigma_j$$

where ρ_{ij} denotes default correlation.

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8 Delinquency and Early Warning Signals

Loan repayment behavior is monitored using delinquency metrics, commonly expressed in days past due (DPD). The probability of eventual default increases sharply with early missed payments.

Let $P(D|DPD = k)$ denote conditional default probability. Empirically:

$$\frac{\partial P(D)}{\partial DPD} > 0$$

This justifies intervention and restructuring strategies.

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9 Liquidity and Maturity Mismatch

Lending institutions often fund long-term loans with short-term capital, creating maturity mismatch risk.

If:

- T_L = loan maturity
- T_F = funding maturity

Then instability arises when:

$$T_L \gg T_F$$

This mismatch exposes the system to refinancing risk and liquidity shocks.

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10 Summary

Lending transforms present capital into uncertain future cash flows under time, risk, and liquidity constraints. Its mathematical foundation rests on discounting, probabilistic default modeling, and portfolio risk aggregation. Sustainable lending requires precise alignment between pricing, expected loss, and capital structure.