Tasha Asyiqin, Sophia Godfrey, Mikael Pineda, and Diego Rufino

ECGR 2254

10/17/2022

Robert Cox

Project 1

Problem 1

1aa. Determining the differential equation requires to do KVL. This resulted in:

$$-V_s + V_L + V_R$$

When converting each voltage to its respected conversion, L was then divided on all sides, resulting in the following steps:

$$V_s = L\left(\frac{di}{dt}\right) + IR$$

$$\frac{V_s}{L} = \frac{di}{dt} + \frac{IR}{L}$$

$$\frac{480\sqrt{2}\cos{(120\pi)}}{0.045} = \frac{di}{dt} + \frac{0.1}{0.0045} * I$$

1ab & 1ac. The following steps show how the steady-state solution of the differential equation was solved:

a.b) $y_{ss}(t) = |B|\cos(2\pi 60t + 0^\circ) = \text{Real}\{|B|e^{j0^\circ}e^{j2\pi 60t}\}$

$\frac{di}{dt} + 22.2\, i = \frac{480\sqrt{2}}{0.0045}\cos(120\pi t) \Rightarrow \frac{d}{dt}(Be^{j2\pi 60}) + 22.2(Be^{j2\pi 60}) = \frac{480\sqrt{2}}{0.0045}e^{j0}e^{j120\pi}$

$\frac{j2\pi 60 + 22.2(Be^{j2\pi 60})}{j2\pi 60 + 22.2} = \frac{\frac{480\sqrt{2}}{0.0045}e^{j2\pi 60}e^{j0}}{j2\pi 60 + 22.2}$ $\quad \frac{480\sqrt{2}}{0.0045} = 150{,}849.45$

$|B| = \frac{150{,}849.45}{\sqrt{(22.2)^2+(j120\pi)^2}} = 399.45 \quad \angle B = 0^\circ - \tan^{-1}\left(\frac{120\pi}{22.2}\right) = 0 - 86.63 = -86.63^\circ$

$\boxed{399.45\cos(2\pi 60t - 86.63^\circ)}$ a.c) $\boxed{399.45}$

1ba. When transforming into the form of transient solution and steady-state solution, it results in:

$$V_c(t) = Ce^{-\frac{t}{\tau}} + |B|\cos(2\pi 60t + B^\circ)$$

Next was to calculate delta_t, delta_t / 100, and phi.

1b.a) $V_c(t) = Ce^{-t/\tau} + |B|\cos(2\pi 60t + \angle B)$

$\Delta t \ll 5\tau = (5)(0.045) = 0.225$ $\quad \Delta t \ll T = \frac{2\pi}{2\pi 60} = 0.017$ secs

$\Delta t < \frac{0.225}{100} = 0.00225$ secs $\quad \Delta t < \frac{0.017}{100} = \boxed{0.00017 \text{ secs}}$

$\tau = \frac{L}{R} = 0.045$

1bb. Plug in both choices of the transient response and the steady-state response for finding t_end:

1b.b) $\boxed{\text{Choice 1: } t_{end} = 7\tau = 7(0.045) = 0.315 \text{ secs}}$ Choice 2: $t_{end} = 2T = 0.034$ secs

t = [0 : delta_t : 7·tau]

The larger result must be picked in order to see the characteristics of the solution.

1bc. Using what the approximation is for dy/dt, the following steps were taken to find the discretized equation:

1b,c) $\frac{di_L}{dt} \approx \frac{i_L(t+\Delta t)-i_L(t)}{\Delta t}$ $\quad V_s(t)=V_c(t)+\tau\left[\frac{i_L(t+\Delta t)-i_L(t)}{\Delta t}\right]$

$V_c(t+\Delta t)=f(V_c(t),V_s(t))$ $\quad V_s(t)\left(\frac{\Delta t}{\tau}\right)=V_c(t)\left(\frac{\Delta t}{\tau}\right)+V_c(t+\Delta t)-V_c(t)$

$V_c(t+\Delta t)=V_s(t)\underbrace{\left[\frac{\Delta t}{\tau}\right]}_{a}+V_c(t)\underbrace{\left[1-\frac{\Delta t}{\tau}\right]}_{b}$ $\quad t=n\Delta t$ $\quad V_c(n\Delta t+\Delta t)=V_c((n+1)\Delta t)$

$V_c(n+1)=V_s(n)a+V_c(n)b$

Entering into MATLAB, the current after t = 0 and plotting the graph was made:

```
% PROJECT 1
% part A and B
clear all
% values for circuit
Ls = 0.0045;
R1s = 0.1;
phi = deg2rad(0);

% finding time variables
omega = 2*pi*60;
T = (2*pi)/omega;
tau = Ls/R1s;

delta_t = tau/100;
t = [0:delta_t:7*tau];

% defining x & y
v_s = (480*sqrt(2)/.1)*cos(omega*t - phi);
v_c = [];
v_c_0 = 0;

a = delta_t/tau;
b = (1-(a));

for n = 1:1:length(t)-1
    if n == 1
        v_c = v_c_0;
    end
        v_c(n+1) = a*v_s(n)+b*v_c(n);
end

figure(1);
plot(t,v_c);
title('Current over time');
xlabel('Time: sec');
ylabel('Current (amps)');
```

1bd. 0 degrees = 400, 45 degrees = 640, 90 degrees = 733.5, 135 degrees = 631, & 180 degrees = 410. I chose 90 degrees.

1ca. This was the finding differential equation.

$$1c\,a)\quad V_s(t) = v_L(t) + (R_1 + R_2)\,L\frac{di}{dt}$$

1cb. These were the steps to find the steady-state solution, using the finding value of phi.

$$1c.b)\quad 480\sqrt{2}\cos(2\pi 60t - 90^\circ) = V_{ss}(t) + \tau\frac{di}{dt}$$

$$V_{ss}(t) = |B|\cos(2\pi 60t + \angle B) = Re\{Be^{j2\pi 60t}\} \quad B = |B|e^{j\angle B}$$

$$480\sqrt{2}e^{-j90^\circ} \cdot e^{j2\pi 60t} = Be^{j2\pi 60t} + \tau\frac{d}{dt}\left(Be^{j2\pi 60t}\right)$$

$$e^{j2\pi 60t}\,480\sqrt{2}e^{-j90^\circ} = Be^{j2\pi 60t}(1 + j2\pi 60\tau)$$

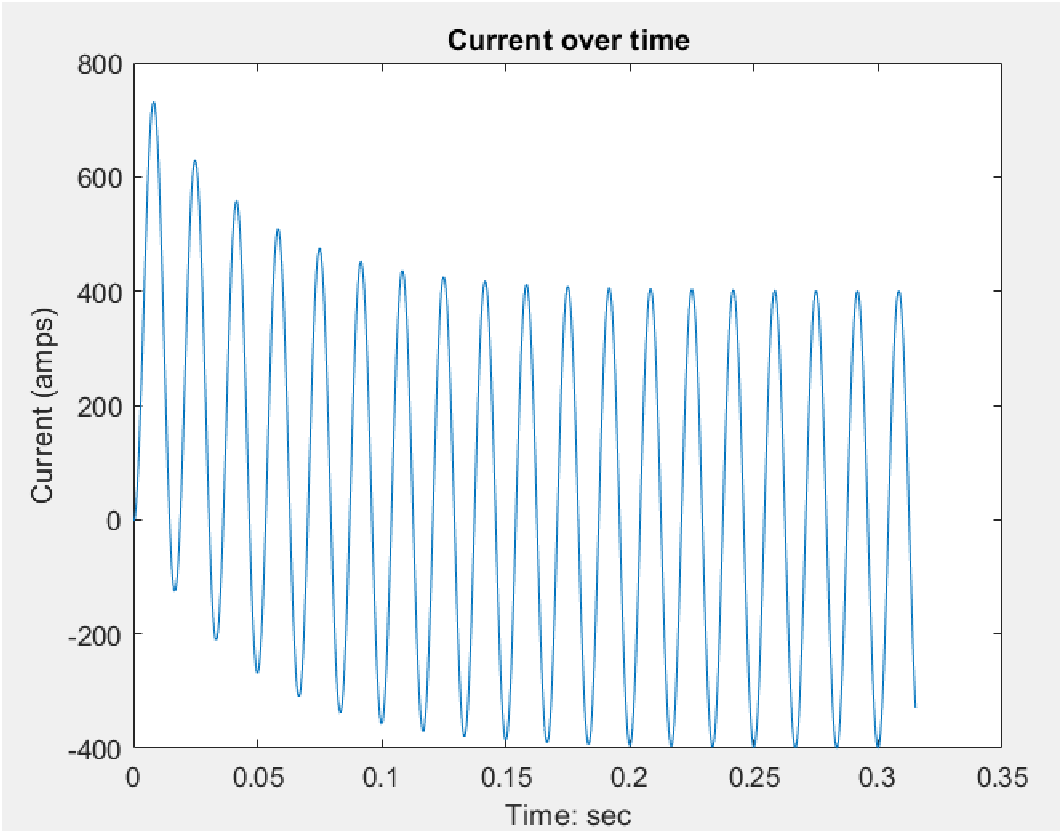
$$\frac{V_s}{R} = I \quad \frac{480\sqrt{2}e^{(-j90^\circ)}}{0.1 + 23.04} = 29.335A$$

$$29.335\cos(120\pi t - 92^\circ)$$

1cc. Entering 0 into t, this was what was found.

```
% PROJECT 1
% part C
clear all
% values for circuit
Ls = 0.0045;
R1s = 0.1;
R2 = 23.04;
phi = deg2rad(90);


% finding time variables
omega = 2*pi*60;
delta_t = 2*pi/(100*omega);
tau = Ls/(R1s);

t = [0:delta_t:7*tau];

B = (480*sqrt(2))*exp(j*(-phi))/(j*omega*(Ls)+R1s);
B2 = (480*sqrt(2))*exp(j*(-phi))/(j*omega*(Ls)+R1s+R2);

% defining x & y
v_s = (1/(R1s))*480*sqrt(2)*cos(omega*t - phi);
v_c = [];
v_c_0 = abs(B2)*cos(angle(B2));

a = delta_t/tau;
b = (1-(a));

for n = 1:1:length(t)-1
    if n == 1
        v_c = v_c_0;
    end
        v_c(n+1) = a*v_s(n)+b*v_c(n);
end

figure(2);
plot(t,v_c);
title('Current over time');
xlabel('Time: sec');
ylabel('Current (amps)');
```

1cd. Graph:



Problem 2 – Audio Filtering

2a & 2b. Write KVL for the loop. Afterwards, write approximation for discretization dx/dt. The discretization for dx/dt is the following:

$$\frac{dx}{dt} \approx \frac{x(n+1) - x(n)}{\Delta t}$$

2a. $-x(t) + V_c + y(t) \Rightarrow x(t) = \frac{1}{C}\left(\int_0^t I\,dt\right) + Ry(t) \quad \frac{dx}{dt} = y(t) + \tau\frac{dy}{dt} \Rightarrow \frac{dx}{dt} - \frac{y}{\tau} + \frac{dy}{dt}$

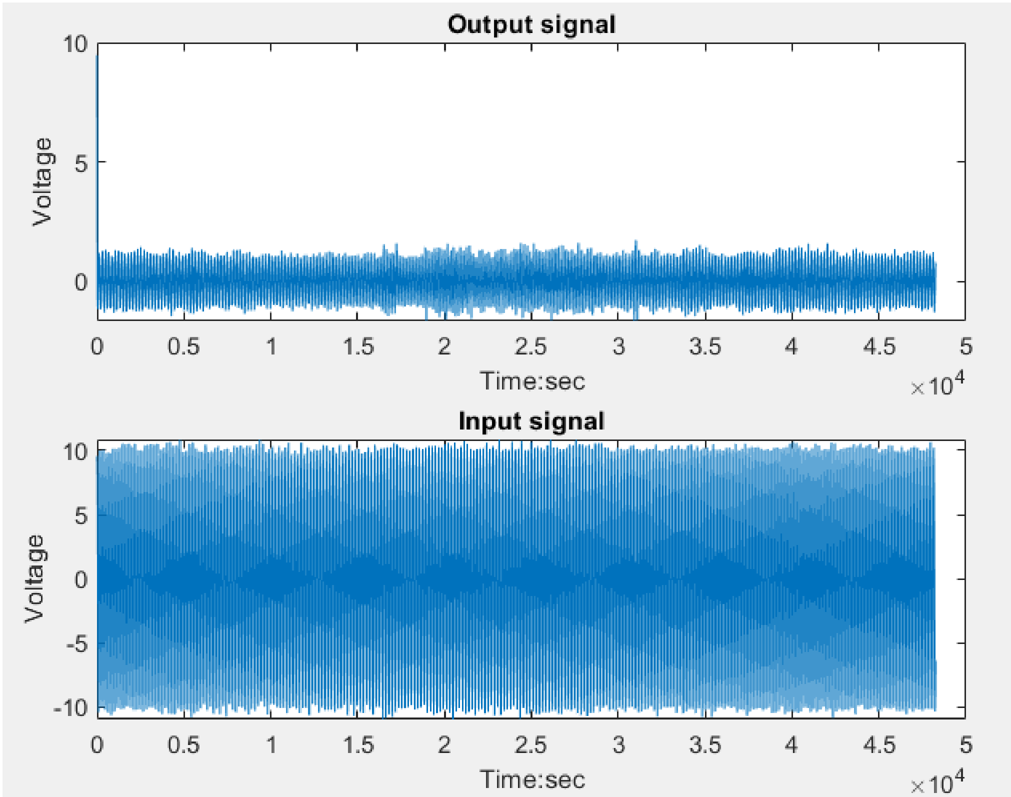
2b. $y(t+\Delta t) = \frac{\Delta t}{\tau}x(t) + \left(1 - \frac{\Delta t}{\tau}\right)y(t) \Rightarrow y((n+1)\Delta t) = \frac{\Delta t}{\tau}x(n\Delta t) + \left(1 - \frac{\Delta t}{\tau}\right)y(n\Delta t)$

$y((n+1)) = \frac{\Delta t}{\tau}x(n) + \left(1 - \frac{\Delta t}{\tau}\right)y(n)$

```
%PROJECT 1
%problem 2
clear all
load('sampleaudio.mat');

%implement given #'s
R = 1;
C = 256.26*10^-6;

tau = R*C;
delta_t = 1/Fs;
A = tau/(tau+delta_t);

t = [0:1:length(x)-1];
y = zeros(size(x));

%plot
for n = 1:1:length(x)-1
    if n == 1
        y(1) = x(1);
    else
        y(n+1) = A * y(n) + A*(x(n+1)-x(n));
    end
end

sound(y,Fs);
subplot(2,1,1)
plot(t, y);
title('Output signal');
xlabel('Time:sec');
ylabel('Voltage');

subplot(2,1,2)
plot(t,x);
title('Input signal');
xlabel('Time:sec');
ylabel('Voltage');
```

2c.

2d. The sentence heard was “If only you knew the power of the dark side.”

2e. 

Problem 3 – Traditional Communications

3a. Using Euler's identity relationships to figure out how to implement the demodulator, the following steps were done:

3a. $x(t) = [x_{audio}(t)\cos(2\pi ft)][\cos(2\pi ft)] \Rightarrow$

$$\left(\frac{x_{audio}(t)}{2}e^{j2\pi ft} + \frac{x_{audio}(t)}{2}e^{-j2\pi ft}\right)\cdot\left(\frac{1}{2}e^{j2\pi ft} + \frac{1}{2}e^{-j2\pi ft}\right)$$

$$\left(\frac{x_{audio}(t)}{4}e^{j4\pi ft} + \frac{x_{audio}(t)}{4}e^{-j4\pi ft} + \frac{x_{audio}(t)}{2}\right)$$

$$\frac{x_{audio}}{2}\left(\frac{1}{2}e^{j4\pi ft} + \frac{1}{2}e^{-j4\pi ft} + 1\right) = \frac{x_{audio}(t)}{2}\left(\cos(4\pi ft) + 1\right)$$

$$\Rightarrow \frac{1}{2}x_{audio}(t) + \frac{1}{2}x_{audio}(t)\cos(4\pi ft)$$

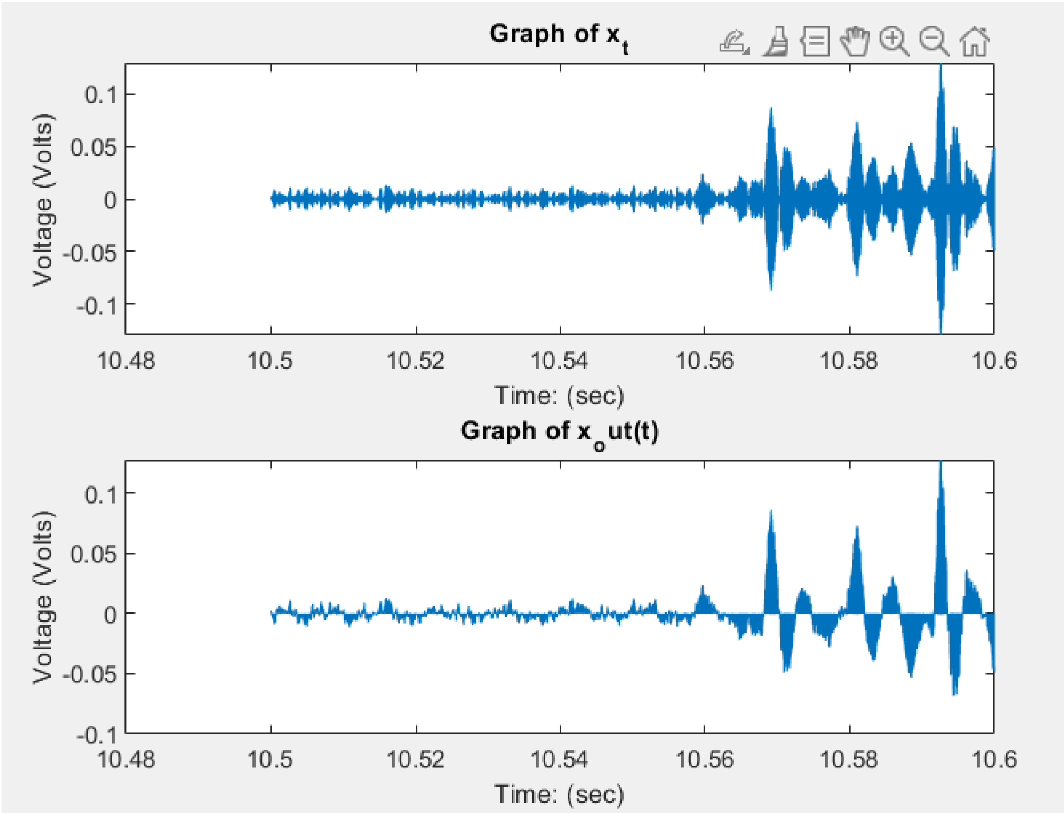
3ba. Setup Code:

```
%PROJECT 1
%Problem 3
clear all

fileID = fopen('problem3.bin', 'r');
x=fread(fileID,'single');
fclose(fileID);
```

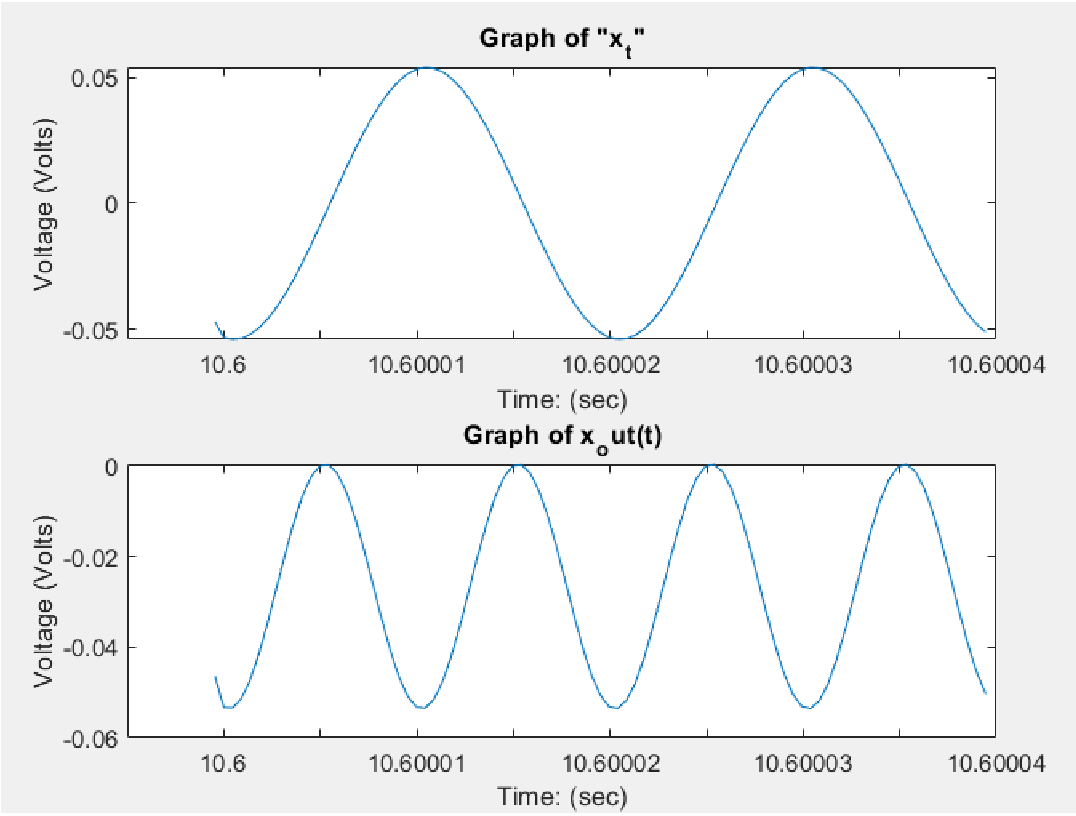
3bb. Implementation Code: (delta_t1 and delta_t2 is n = t/delta_t)

```
f = 50*10^3;
fsamp = 2.205 * 10^6;
delta_t = 1/fsamp;
x = x';

delta_t1 = 10.5/delta_t;
delta_t2 = 10.6/delta_t;

t = (0:delta_t:(length(x)-1)*delta_t);
vector = cos(2*pi*f.*t);
x_out = x.*vector;
time = t(delta_t2:88+delta_t2);

figure(1)
subplot(2,1,1);
plot( t(delta_t1:delta_t2) , x(delta_t1:delta_t2) );
title('Graph of x_t');
xlabel('Time: (sec)');
ylabel('Voltage (Volts)');

subplot(2,1,2);
plot( t(delta_t1:delta_t2) , x_out(delta_t1:delta_t2) );
title('Graph of x_out(t)');
xlabel('Time: (sec)');
ylabel('Voltage (Volts)');

figure(2)
subplot(2,1,1)
plot( time, x(delta_t2:88+delta_t2) );
title('Graph of "x_t"');
xlabel('Time: (sec)');
ylabel('Voltage (Volts)');

subplot(2,1,2);
plot( time, x_out(delta_t2:88+delta_t2) );
title('Graph of x_out(t)');
xlabel('Time: (sec)');
ylabel('Voltage (Volts)');
```

3bc. Graph 1:



Graph of x_t
Voltage (Volts)
Time: (sec)
Graph of $x_o$ut(t)
Voltage (Volts)
Time: (sec)



Graph of "x_t"
Voltage (Volts)
Time: (sec)
Graph of $x_o$ut(t)
Voltage (Volts)
Time: (sec)

3bd.

3ca. The amplitude from the signal varies because of amplitude modulation.

3cb. When working Xout(t), what was noticed was that the frequency of Xout(t) should be twice of the input x(t). This is due to the coefficient of pi*f*t was changed input -> output.

3da & 3db. Making the differential equation and discrete-time equation:

3d. $i = i_R + i_C \ldots i_R = \frac{y(t)}{R}$ & $i_C = C\frac{dy(t)}{dt} \Rightarrow \frac{di}{dt} = \frac{1}{R}\frac{dy(t)}{dt} + C\frac{d^2y(t)}{dt^2}$

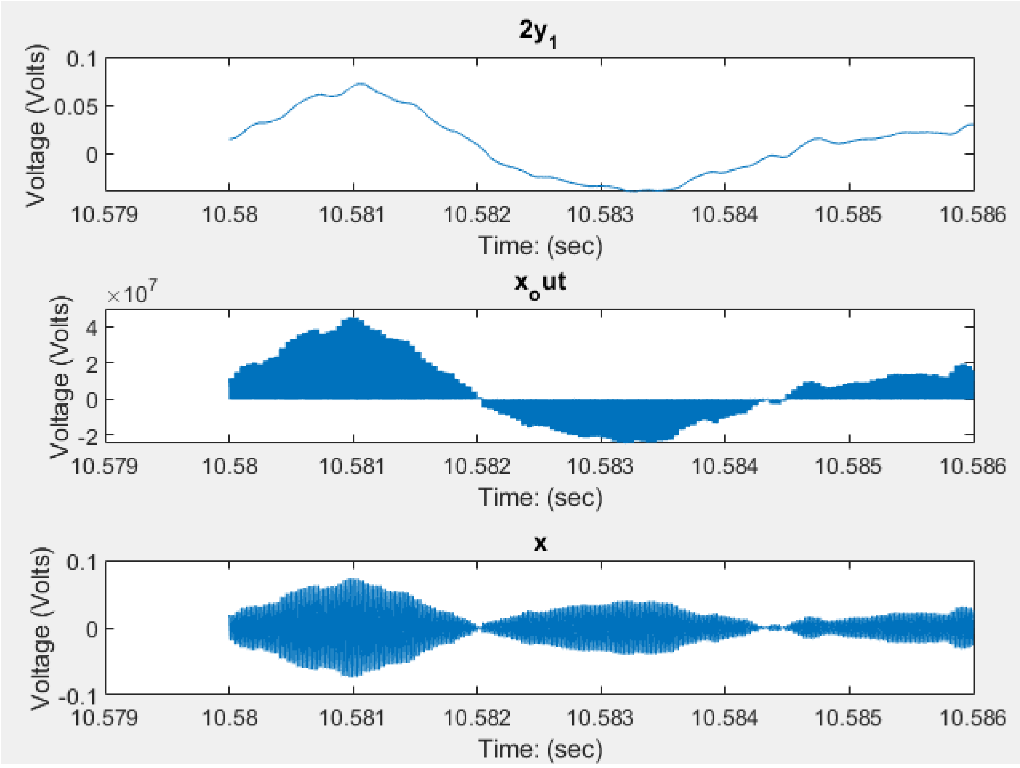
Then loop equation is made... $X_{out}(t) = L\frac{di}{dt} + y(t)$

$X_{out}(t) = L\left(\frac{1}{R}\frac{dy(t)}{dt} + C\frac{d^2y(t)}{dt^2}\right) + y(t) \Rightarrow \frac{d^2y(t)}{dt} + \frac{1}{RC}\frac{dy(t)}{dt} + \frac{1}{LC}y(t) = \frac{1}{LC}x_{out}(t)$

3dc. Code: (nstart and nend is (seconds*fsamp))

```
%constants for network
R = 28.135;
L = 1.61*10^-3;
C = 1*10^-6;
y0 = 0;
dy0 = 0;
c_1 = 1/(L*C);
c_2 = 1/(R*C);

x_out = x_out/(L*C);
y_1 = zeros(size(t));
y_2 = zeros(size(t));

for n = 1:1:length(t)-1
    if n == 1
        y_1(1) = y0;
        y_2(1) = dy0;
    end
        y_1(n+1) = y_1(n)+delta_t*y_2(n);
        y_2(n+1) = delta_t*(x_out(n)-c_2*y_2(n)-c_1*y_1(n))+y_2(n);
end

nstart = 10.58*fsamp;
nend = 10.586*fsamp;

newplot = t(nstart:nend); %%10.58*fsamp <-> 10.586*fsamp
figure(3);
subplot(3,1,1);
plot(newplot,2*y_1(nstart:nend))
title('2y_1');

subplot(3,1,2);
plot(newplot,x_out(nstart:nend))
title('x_out');
subplot(3,1,3);
plot(newplot,x(nstart:nend))
title('x');
```

3dd. Graphs:



3ea. What happened to the cos(4*pi*f*t) by passing xout(t) through the filter was that it was multiplied by cos(4*pi*f*t).

3eb. 2y(t) appears to relate to x(t) since x(t) is the input that contains the original audio going through the filter that has the carrier cosine wave. Y(t) is what has passed through the filter, where it is the original audio signal with no carrier wave. Y(t) is the amplitude of the input wave.

3f. What plays is a sample recording of the Gettysburg address. Xout is noisier than the result because the volume of the output signal was cut in half.

Problem 4 – Complex Communications

4a. x_3(t) would be equal to e^(-j*2*pi*f*t) because it eliminates e^(j*2*pi*f*t) in x(t).

4b.

```
%PROJECT 1
%Problem 4
clear all

fileID = fopen('problem4.bin', 'r');
x_in = fread(fileID, 'single');
fclose(fileID);

for n = 2:2:length(x_in)
    x(n/2) = x_in(n-1) + j*x_in(n);
end

%done
f = 50*10^3;
fsamp = 2.205 * 10^6;
delta_t = 1/fsamp;

t = (0:delta_t:(length(x)-1)*delta_t);

x3 = exp(-2*j*pi*f.*t);

xout = x3 .* x;

x1 = real(xout);
x2 = imag(xout);
sound(downsample(x1,100),fsamp/100); %light
sound(downsample(x2,100),fsamp/100); %dark
```

4c. The light part is the Star Wars main title theme and the dark part is the Imperial March.

Problem 5 – DC Motor Control

5a. Do KVL.

5a. $-V_a(t)+R_a+L_a+K\Omega(t) \Rightarrow V_a(t)=RI+L\frac{di}{dt}+K\Omega(t) \Rightarrow \frac{di}{dt}+\frac{R}{L}i=\frac{V_a-K\Omega(t)}{L}$

5b. Using approximation of di/dt and d(ohm)/dt, electrical and mechanical discrete time equations turn out to be:

5b. $\frac{di}{dt} \approx \frac{i(t+\Delta t) - i(t)}{\Delta t}$ $\frac{d\Lambda}{dt} \approx \frac{\Lambda(t+\Delta t) - \Lambda(t)}{\Delta t}$

$$\frac{i(t+\Delta t) - i(t)}{\Delta t} + \frac{R}{L} i = \frac{V_a - K\Lambda(t)}{L} \rightarrow \left[\frac{i(t+\Delta t) - i(t)}{\Delta t} = \frac{V_a - K\Lambda(t)}{L} - \frac{R}{L} i\right] \cdot \Delta t$$

electrical: $i(t+\Delta t) - i(t) = \Delta t\left[\frac{V_a - K\Lambda(t)}{L} - \frac{R}{L} i\right] \Rightarrow i(t+\Delta t) = \Delta t\left[\frac{V_a - K\Lambda(t)}{L} - \frac{R}{L} i\right] + i(t)$

$$\frac{d\Lambda}{dt} = \frac{K i_a - B\Lambda}{J} \rightarrow \frac{\Lambda(t+\Delta t) - \Lambda(t)}{\Delta t} = \frac{K i_a - B\Lambda}{J}$$

after algebra...

mechanical: $\Lambda(t+\Delta t) = \Delta t\left[\frac{K}{J} I_a(t) - \frac{B}{J}\Lambda(t)\right] + \Lambda(t)$

5c. Finding tau and delta_t, the following was calculated for electrical and mechanical:

5c. electrical: $\tau = \frac{L}{R} = \frac{0.01}{3.38} = 2.959 \cdot 10^{-3}$, $\boxed{\Delta t = \frac{5\tau}{100} = 1.4795 \cdot 10^{-4}}$

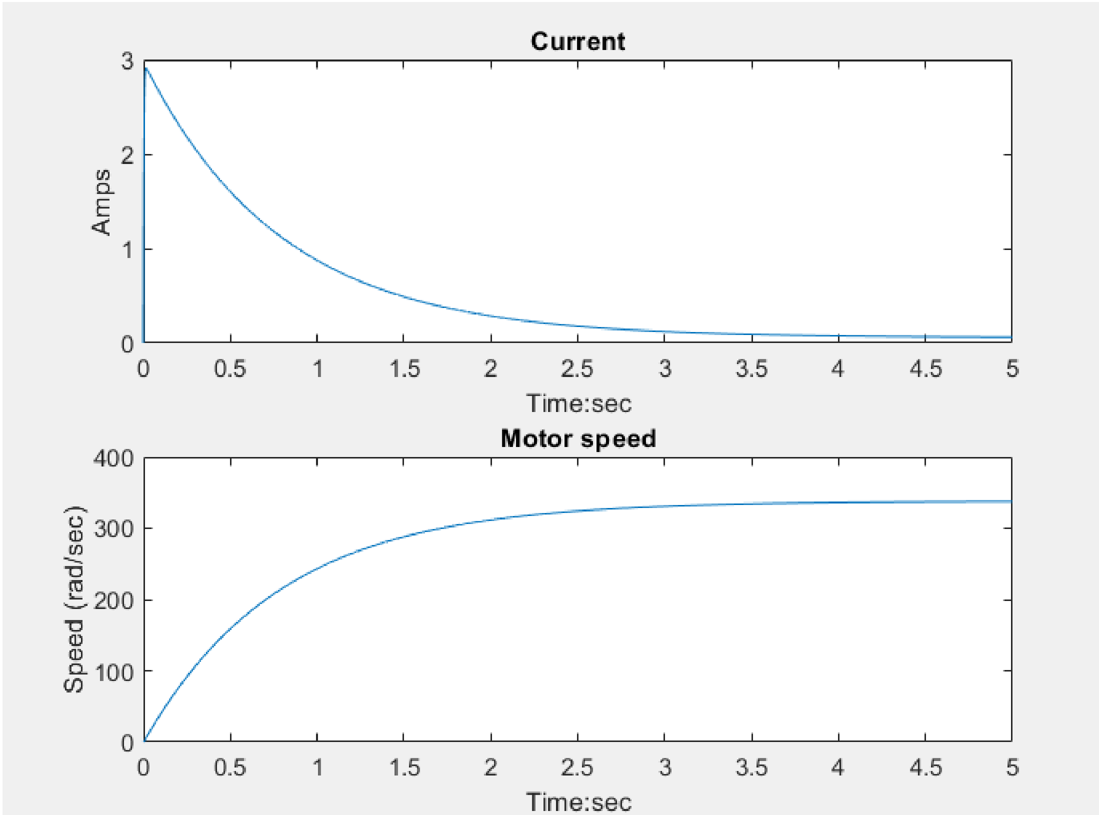
mechanical: $\tau = \frac{J}{B} = \frac{2 \cdot 10^{-4}}{5 \cdot 10^{-6}} = 40$, $\Delta t = \frac{5\tau}{100} = 2$

To choose which delta_t to use, the smaller value for delta_t was chosen in order to see the full characteristics in the system.

5d. Code:

```
%PROJECT 1
%Problem 5 part d
clear all
L = 0.01;
R = 3.38;
K = 0.029; % motor constant
J = 2*10^-4; % moment of inertia
B = 0.5*10^-5; % damping ratio

tau = L/R;
tauint = 5;
dt = 5*tau/100;
t = 0:dt:tauint;

va = 10;

ia = zeros(size(t));
om = zeros(size(t));

for n = 1:1:length(t)-1
    if n == 1
        ia(1) = 0;
        om(1) = 0;
        th(1) = 0;
    end
    ia(n+1) = (dt/L)*(va-R*ia(n)-K*om(n))+ia(n);
    om(n+1) = dt*((K/J)*ia(n)-(B/J)*om(n))+om(n);
end

subplot(2,1,1)
plot(t, ia);
title('Current');
xlabel('Time:sec');
ylabel('Amps');

subplot(2,1,2)
plot(t,om);
title('Motor speed');
xlabel('Time:sec');
ylabel('Speed (rad/sec)');
```

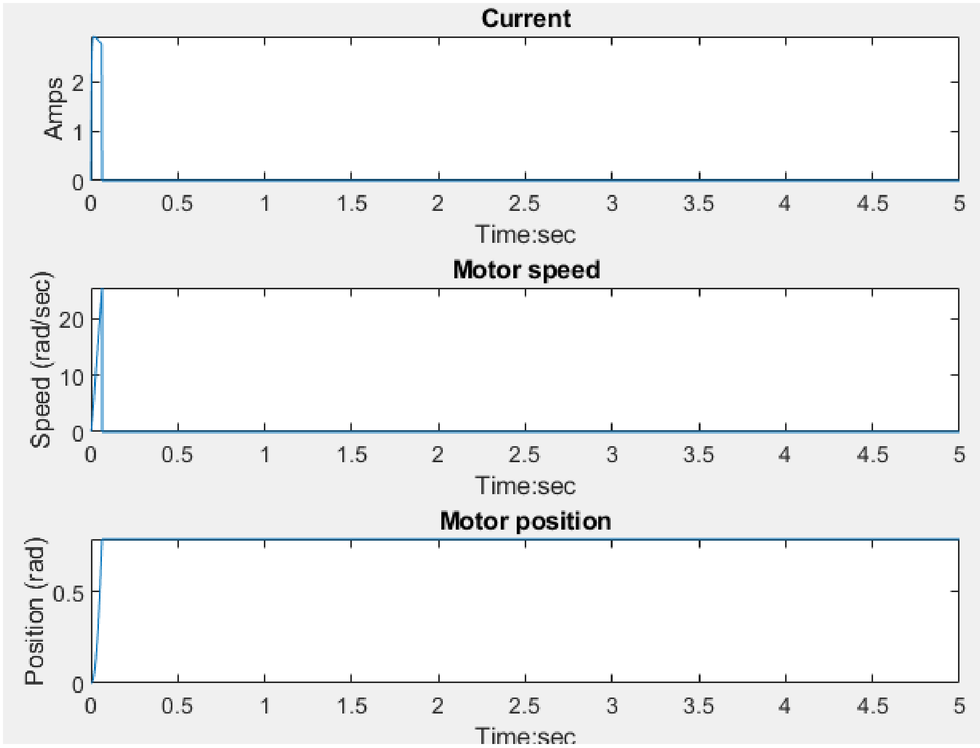
Results:



5e. Code: (another for loop was added)

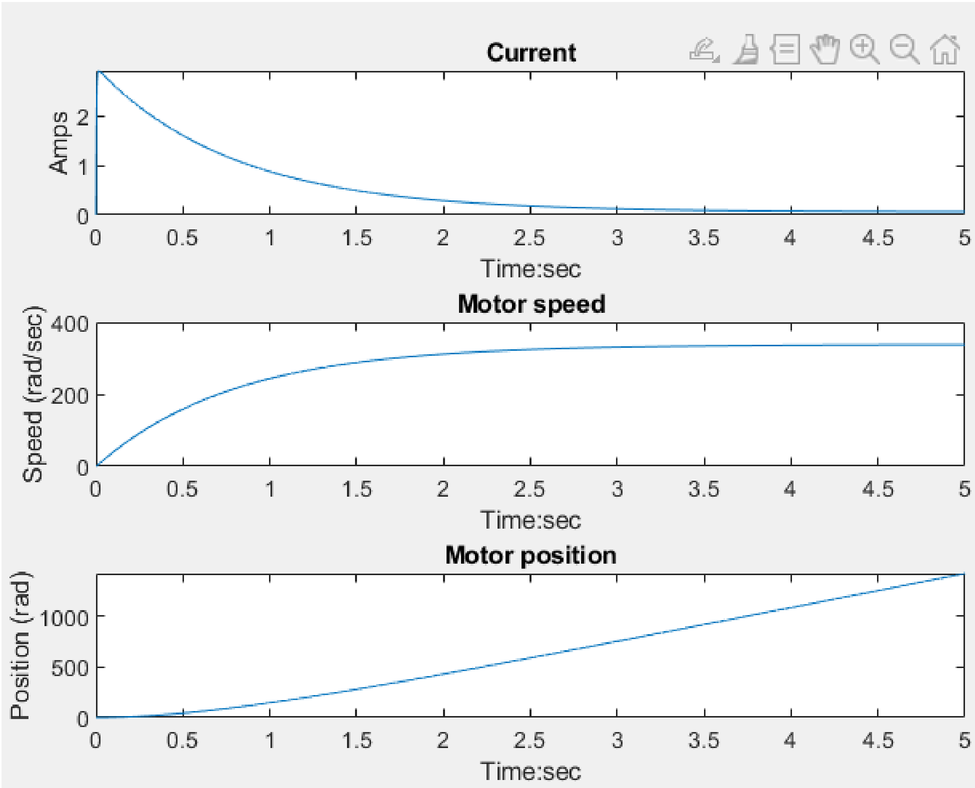
```
for n = 1:1:length(t)-1
    if n == 1
        ia(1) = 0; % current
        om(1) = 0; % motor speed
        th(1) = 0; % motor position
    end
    ia(n+1) = (dt/L)*(va-R*ia(n)-K*om(n))+ia(n); % current
    om(n+1) = dt*((K/J)*ia(n)-(B/J)*om(n))+om(n); % motor speed
    th(n+1) = dt*om(n) + th(n); % motor position
end

for n = 1:1:length(t)-1
    if th(n) >= pi/4
        ia(n+1) = 0;
        om(n+1) = 0;
        th(n+1) = th(n);
    end
end
```

Graphs:



Current
Amps
0
1
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec
Motor speed
Speed (rad/sec)
0
10
20
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec
Motor position
Position (rad)
0
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec

```
%PROJECT 1
%Problem 5 part d
clear all
L = 0.01;
R = 3.38;
K = 0.029; % motor constant
J = 2*10^-4; % moment of inertia
B = 0.5*10^-5; % damping ratio

tau = L/R;
tauint = 5;
dt = 5*tau/100;
t = 0:dt:tauint;

va = 10;

ia = zeros(size(t)); % current
om = zeros(size(t)); % motor speed
th = zeros(size(t)); % motor position

for n = 1:1:length(t)-1
    if n == 1
        ia(1) = 0; % current
        om(1) = 0; % motor speed
        th(1) = 0; % motor position
    end
    ia(n+1) = (dt/L)*(va-R*ia(n)-K*om(n))+ia(n); % current
    om(n+1) = dt*((K/J)*ia(n)-(B/J)*om(n))+om(n); % motor speed
    th(n+1) = dt*om(n) + th(n); % motor position
end

subplot(3,1,1)
plot(t, ia);
title('Current');
xlabel('Time:sec');
ylabel('Amps');

subplot(3,1,2)
plot(t,om);
title('Motor speed');
xlabel('Time:sec');
ylabel('Speed (rad/sec)');

subplot(3,1,3)
plot(t,th);
title('Motor position');
xlabel('Time:sec');
ylabel('Position (rad)');
```


Current
Amps
0
1
2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec
Motor speed
Speed (rad/sec)
0
200
400
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec
Motor position
Position (rad)
0
500
1000
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Time:sec