Math 3808 Final Project

Bridge: Probability Distribution of a Random Hand's Shape

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Introduction

The shape of a bridge hand is the set of lengths of the suits in the hand, i.e., an unordered 4-tuple (w,x,y,z) where w+x+y+z=13, and each w, x, y, z describes the length of a suit in the hand**.** For this project, we wrote a program that calculates the probability distribution of the shape of a random hand. The program is for Python 3.8 or above and uses the matplotlib and tkinter libraries. The program calculates all the probabilities of all hand shapes, plots the distribution, and includes a function to calculate probability of incomplete bridge shapes (e.g., (6,4,X,X)). To run the program, simply open BridgeShapeTool.exe. The start-up/runtime is about 10 seconds. The code is at https://github.com/startrekdude/BridgeShapeTool.

Description of Calculations

Bridge Hand Probability Calculations:

To calculate the probability of a given shape the program does the following:

1. Calculates the number of "floating suits", i.e., the number of suit lengths that are equal in the shape of a hand. For example, (4,3,3,3) has 3 floating suits and (5,4,3,1) has 0 floating suits. Any permutation of the suits among the floating suits does not change the shape.

2. Calculates the number of hands in a shape (w,x,y,z). Let a be the number of floating suits.

#h= (4 choose 1)*((4-1) choose 1)*...*((a+1) choose 1)*(13 choose w)*

(13 choose x)(13 choose y)*(13 choose z)

3. Calculates the probability of a given hand shape:

P(w,x,y,z) = #h/(52 choose 13)

Results (Table)

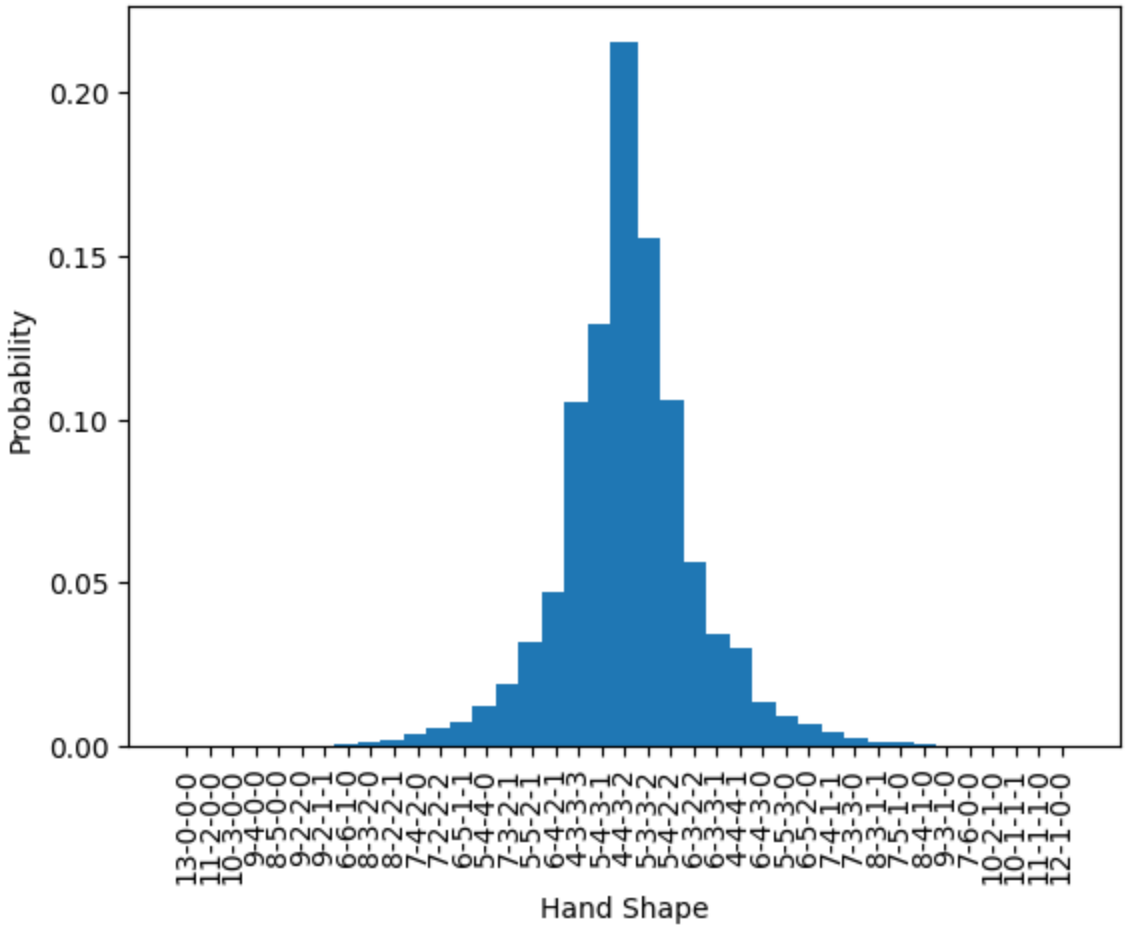
Shape	Expression =	# of hands	Probability
4-4-3-2	$\binom{4}{1}\binom{3}{1}\binom{13}{4}\binom{13}{4}\binom{13}{3}\binom{13}{2}$	136,852,887,600	0.215511756452
5-3-3-2	$\binom{4}{1}\binom{3}{1}\binom{13}{5}\binom{13}{3}\binom{13}{3}\binom{13}{2}$	98,534,079,072	0.155168464645
5-4-3-1	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{5}\binom{13}{4}\binom{13}{3}\binom{13}{1}$	82,111,732,560	0.129307053871
5-4-2-2	$\binom{4}{1}\binom{3}{1}\binom{13}{5}\binom{13}{4}\binom{13}{2}\binom{13}{2}$	67,182,326,640	0.105796680440
4-3-3-3	$\binom{4}{1}\binom{13}{4}\binom{13}{3}\binom{13}{3}\binom{13}{3}$	66,905,856,160	0.105361303154
6-3-2-2	$\binom{4}{1}\binom{3}{1}\binom{13}{6}\binom{13}{3}\binom{13}{2}\binom{13}{2}$	35,830,574,208	0.056424896235
6-4-2-1	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{6}\binom{13}{4}\binom{13}{2}\binom{13}{1}$	29,858,811,840	0.047020746862
6-3-3-1	$\binom{4}{1}\binom{3}{1}\binom{13}{6}\binom{13}{3}\binom{13}{3}\binom{13}{1}$	21,896,462,016	0.034481881032
5-5-2-1	$\binom{4}{1}\binom{3}{1}\binom{13}{5}\binom{13}{5}\binom{13}{2}\binom{13}{1}$	20,154,697,992	0.031739004132
4-4-4-1	$\binom{4}{1}\binom{13}{4}\binom{13}{4}\binom{13}{4}\binom{13}{1}$	19,007,345,500	0.029932188396
7-3-2-1	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{7}\binom{13}{3}\binom{13}{2}\binom{13}{1}$	11,943,524,736	0.018808298745

6-4-3-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{6}\binom{13}{4}\binom{13}{3}\binom{13}{0}$	8,421,716,160	0.013262261935
5-4-4-0	$\binom{4}{1}\binom{3}{1}\binom{13}{5}\binom{13}{4}\binom{13}{4}\binom{13}{0}$	7,895,358,900	0.012433370565
5-5-3-0	$\binom{4}{1}\binom{3}{1}\binom{13}{5}\binom{13}{5}\binom{13}{3}\binom{13}{0}$	5,684,658,408	0.008952026806
6-5-1-1	$\binom{4}{1}\binom{3}{1}\binom{13}{6}\binom{13}{5}\binom{13}{1}\binom{13}{1}$	4,478,821,776	0.007053112029
6-5-2-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{6}\binom{13}{5}\binom{13}{2}\binom{13}{0}$	4,134,297,024	0.006510564950
7-2-2-2	$\binom{4}{1}\binom{13}{7}\binom{13}{2}\binom{13}{2}\binom{13}{2}$	3,257,324,928	0.005129536021
7-4-1-1	$\binom{4}{1}\binom{3}{1}\binom{13}{7}\binom{13}{4}\binom{13}{1}\binom{13}{1}$	2,488,234,320	0.003918395572
7-4-2-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{7}\binom{13}{4}\binom{13}{2}\binom{13}{0}$	2,296,831,680	0.003616980528
7-3-3-0	$\binom{4}{1}\binom{3}{1}\binom{13}{7}\binom{13}{3}\binom{13}{3}\binom{13}{0}$	1,684,343,232	0.002652452387
8-2-2-1	$\binom{4}{1}\binom{3}{1}\binom{13}{8}\binom{13}{2}\binom{13}{2}\binom{13}{1}$	1,221,496,848	0.001923576008
8-3-1-1	$\binom{4}{1}\binom{3}{1}\binom{13}{8}\binom{13}{3}\binom{13}{1}\binom{13}{1}$	746,470,296	0.001175518672
8-3-2-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{8}\binom{13}{3}\binom{13}{2}\binom{13}{0}$	689,049,504	0.001085094158

7-5-1-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{7}\binom{13}{5}\binom{13}{1}\binom{13}{0}$	689,049,504	0.001085094158
6-6-1-0	$\binom{4}{1}\binom{3}{1}\binom{13}{6}\binom{13}{6}\binom{13}{1}\binom{13}{0}$	459,366,336	0.000723396106
8-4-1-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{8}\binom{13}{4}\binom{13}{1}\binom{13}{0}$	287,103,960	0.000452122566
9-2-1-1	$\binom{4}{1}\binom{3}{1}\binom{13}{9}\binom{13}{2}\binom{13}{1}\binom{13}{1}$	113,101,560	0.000178108890
9-3-1-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{9}\binom{13}{3}\binom{13}{1}\binom{13}{0}$	63,800,880	0.000100471681
9-2-2-0	$\binom{4}{1}\binom{3}{1}\binom{13}{9}\binom{13}{2}\binom{13}{2}\binom{13}{0}$	52,200,720	0.000082204103
7-6-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{7}\binom{13}{6}\binom{13}{0}\binom{13}{0}$	35,335,872	0.000055645854
8-5-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{8}\binom{13}{5}\binom{13}{0}\binom{13}{0}$	19,876,428	0.000031300793
10-2-1-0	$\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{13}{10}\binom{13}{2}\binom{13}{1}\binom{13}{0}$	6,960,096	0.000010960547
9-4-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{9}\binom{13}{4}\binom{13}{0}\binom{13}{0}$	6,134,700	0.000009660739
10-1-1-1	$\binom{4}{1}\binom{13}{10}\binom{13}{1}\binom{13}{1}\binom{13}{1}$	2,513,368	0.000003957975
10-3-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{10}\binom{13}{3}\binom{13}{0}\binom{13}{0}$	981,552	0.000001545718

11-1-1-0	$\binom{4}{1}\binom{3}{1}\binom{13}{11}\binom{13}{1}\binom{13}{1}\binom{13}{0}$	158,184	0.000000249103
11-2-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{11}\binom{13}{2}\binom{13}{0}\binom{13}{0}$	73,008	0.000000114971
12-1-0-0	$\binom{4}{1}\binom{3}{1}\binom{13}{12}\binom{13}{1}\binom{13}{0}\binom{13}{0}$	2,028	0.000000003194
13-0-0-0	$\binom{4}{1}\binom{13}{13}\binom{13}{0}\binom{13}{0}\binom{13}{0}$	4	0.000000000006

Results (Graph)



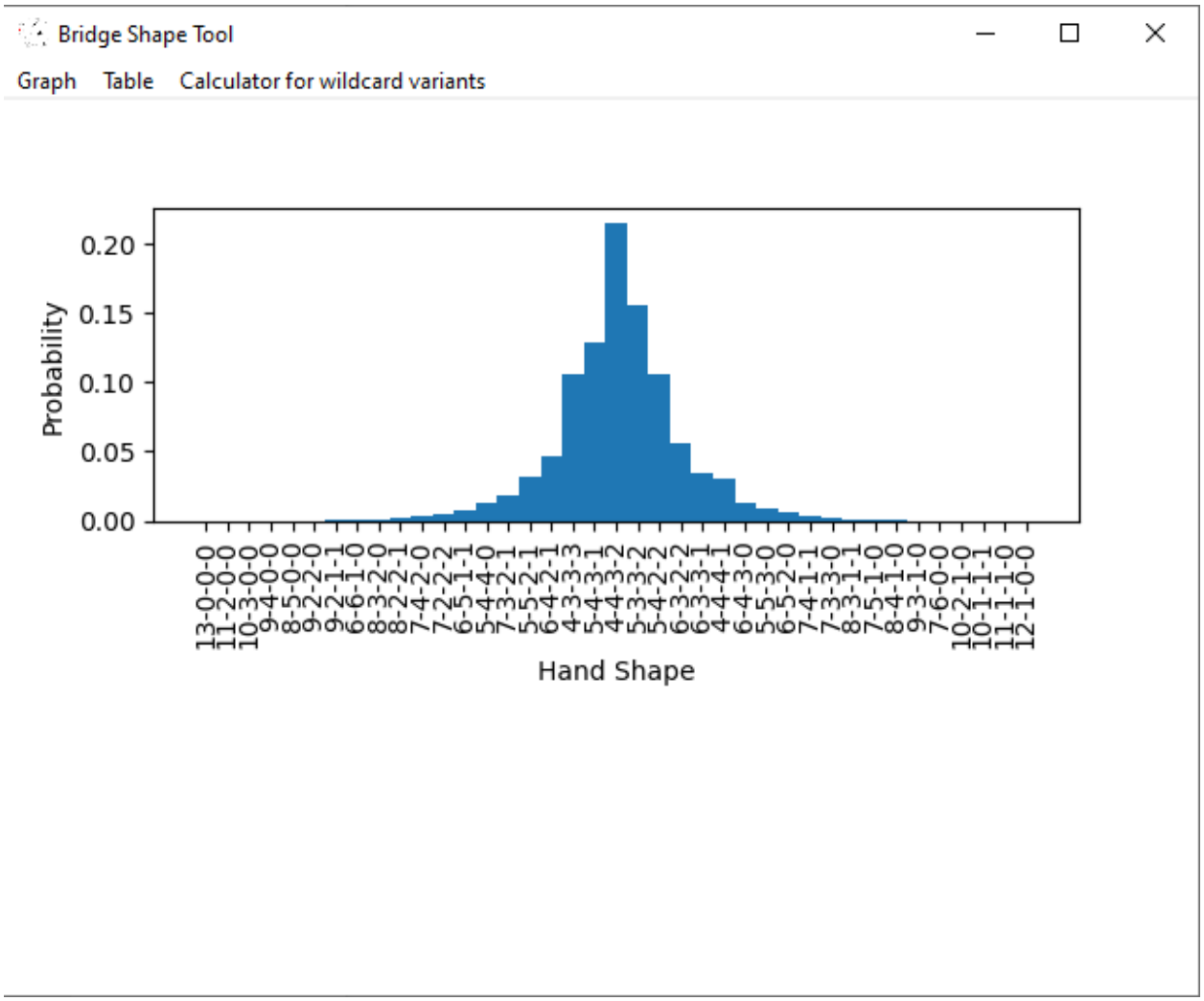
Description of wildcard feature

We also programmed a function that finds all the shapes associated with a given wildcard hand shape. If $x_1, \ldots, x_n$ ($n = 0,\ldots,4$) are the known numbers in the hand shape, then the code searches through all the hand shapes that have $x_1, \ldots, x_n$ in them, and outputs them. For example (6,4,X,X) outputs (6,4,2,1) and (6,4,3,0). (8,2,X,X) outputs (8,2,2,1) and (8,3,2,0).

User Guide

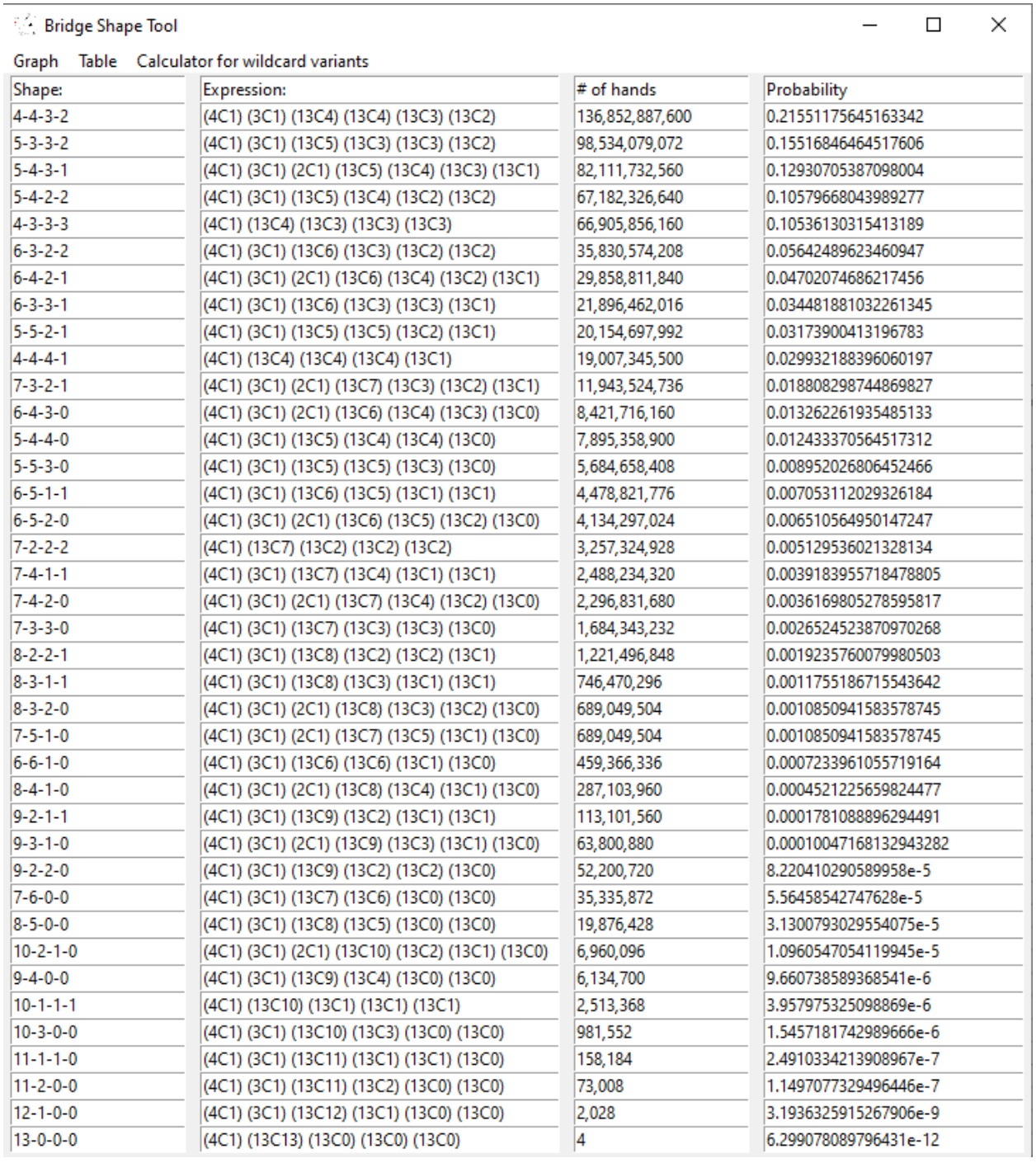
To run the program, simply open BridgeShapeTool.exe.

Once you run the program a window should pop up that looks like this.



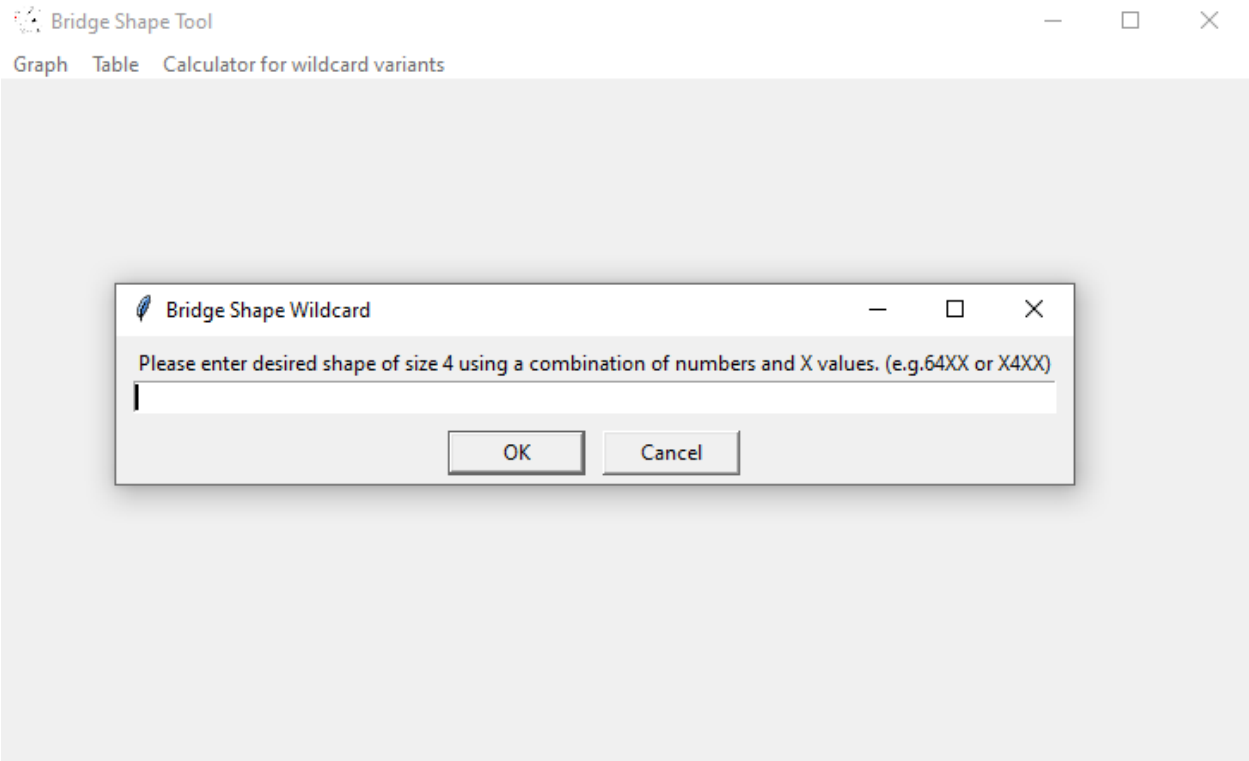
Click on “graph” to yield the plotted results

Click on “Table” to see the table of results

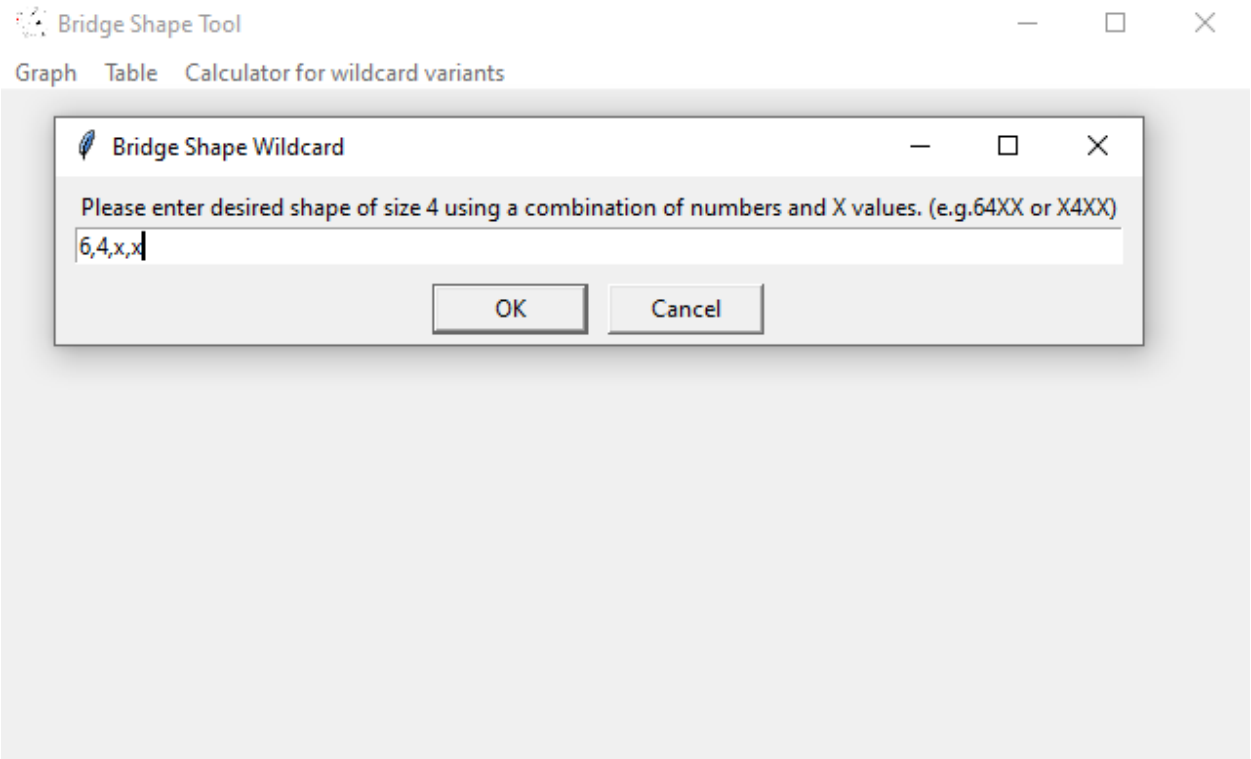


Shape:	Expression:	# of hands	Probability
4-4-3-2	(4C1) (3C1) (13C4) (13C4) (13C3) (13C2)	136,852,887,600	0.21551175645163342
5-3-3-2	(4C1) (3C1) (13C5) (13C3) (13C3) (13C2)	98,534,079,072	0.15516846464517606
5-4-3-1	(4C1) (3C1) (2C1) (13C5) (13C4) (13C3) (13C1)	82,111,732,560	0.12930705387098004
5-4-2-2	(4C1) (3C1) (13C5) (13C4) (13C2) (13C2)	67,182,326,640	0.10579668043989277
4-3-3-3	(4C1) (13C4) (13C3) (13C3) (13C3)	66,905,856,160	0.10536130315413189
6-3-2-2	(4C1) (3C1) (13C6) (13C3) (13C2) (13C2)	35,830,574,208	0.05642489623460947
6-4-2-1	(4C1) (3C1) (2C1) (13C6) (13C4) (13C2) (13C1)	29,858,811,840	0.04702074686217456
6-3-3-1	(4C1) (3C1) (13C6) (13C3) (13C3) (13C1)	21,896,462,016	0.034481881032261345
5-5-2-1	(4C1) (3C1) (13C5) (13C5) (13C2) (13C1)	20,154,697,992	0.03173900413196783
4-4-4-1	(4C1) (13C4) (13C4) (13C4) (13C1)	19,007,345,500	0.029932188396060197
7-3-2-1	(4C1) (3C1) (2C1) (13C7) (13C3) (13C2) (13C1)	11,943,524,736	0.018808298744869827
6-4-3-0	(4C1) (3C1) (2C1) (13C6) (13C4) (13C3) (13C0)	8,421,716,160	0.013262261935485133
5-4-4-0	(4C1) (3C1) (13C5) (13C4) (13C4) (13C0)	7,895,358,900	0.012433370564517312
5-5-3-0	(4C1) (3C1) (13C5) (13C5) (13C3) (13C0)	5,684,658,408	0.008952026806452466
6-5-1-1	(4C1) (3C1) (13C6) (13C5) (13C1) (13C1)	4,478,821,776	0.007053112029326184
6-5-2-0	(4C1) (3C1) (2C1) (13C6) (13C5) (13C2) (13C0)	4,134,297,024	0.006510564950147247
7-2-2-2	(4C1) (13C7) (13C2) (13C2) (13C2)	3,257,324,928	0.005129536021328134
7-4-1-1	(4C1) (3C1) (13C7) (13C4) (13C1) (13C1)	2,488,234,320	0.0039183955718478805
7-4-2-0	(4C1) (3C1) (2C1) (13C7) (13C4) (13C2) (13C0)	2,296,831,680	0.0036169805278595817
7-3-3-0	(4C1) (3C1) (13C7) (13C3) (13C3) (13C0)	1,684,343,232	0.0026524523870970268
8-2-2-1	(4C1) (3C1) (13C8) (13C2) (13C2) (13C1)	1,221,496,848	0.0019235760079980503
8-3-1-1	(4C1) (3C1) (13C8) (13C3) (13C1) (13C1)	746,470,296	0.0011755186715543642
8-3-2-0	(4C1) (3C1) (2C1) (13C8) (13C3) (13C2) (13C0)	689,049,504	0.0010850941583578745
7-5-1-0	(4C1) (3C1) (2C1) (13C7) (13C5) (13C1) (13C0)	689,049,504	0.0010850941583578745
6-6-1-0	(4C1) (3C1) (13C6) (13C6) (13C1) (13C0)	459,366,336	0.0007233961055719164
8-4-1-0	(4C1) (3C1) (2C1) (13C8) (13C4) (13C1) (13C0)	287,103,960	0.0004521225659824477
9-2-1-1	(4C1) (3C1) (13C9) (13C2) (13C1) (13C1)	113,101,560	0.0001781088896294491
9-3-1-0	(4C1) (3C1) (2C1) (13C9) (13C3) (13C1) (13C0)	63,800,880	0.00010047168132943282
9-2-2-0	(4C1) (3C1) (13C9) (13C2) (13C2) (13C0)	52,200,720	8.220410290589958e-5
7-6-0-0	(4C1) (3C1) (13C7) (13C6) (13C0) (13C0)	35,335,872	5.56458542747628e-5
8-5-0-0	(4C1) (3C1) (13C8) (13C5) (13C0) (13C0)	19,876,428	3.1300793029554075e-5
10-2-1-0	(4C1) (3C1) (2C1) (13C10) (13C2) (13C1) (13C0)	6,960,096	1.0960547054119945e-5
9-4-0-0	(4C1) (3C1) (13C9) (13C4) (13C0) (13C0)	6,134,700	9.660738589368541e-6
10-1-1-1	(4C1) (13C10) (13C1) (13C1) (13C1)	2,513,368	3.957975325098869e-6
10-3-0-0	(4C1) (3C1) (13C10) (13C3) (13C0) (13C0)	981,552	1.5457181742989666e-6
11-1-1-0	(4C1) (3C1) (13C11) (13C1) (13C1) (13C0)	158,184	2.4910334213908967e-7
11-2-0-0	(4C1) (3C1) (13C11) (13C2) (13C0) (13C0)	73,008	1.1497077329496446e-7
12-1-0-0	(4C1) (3C1) (13C12) (13C1) (13C0) (13C0)	2,028	3.1936325915267906e-9
13-0-0-0	(4C1) (13C13) (13C0) (13C0) (13C0)	4	6.299078089796431e-12

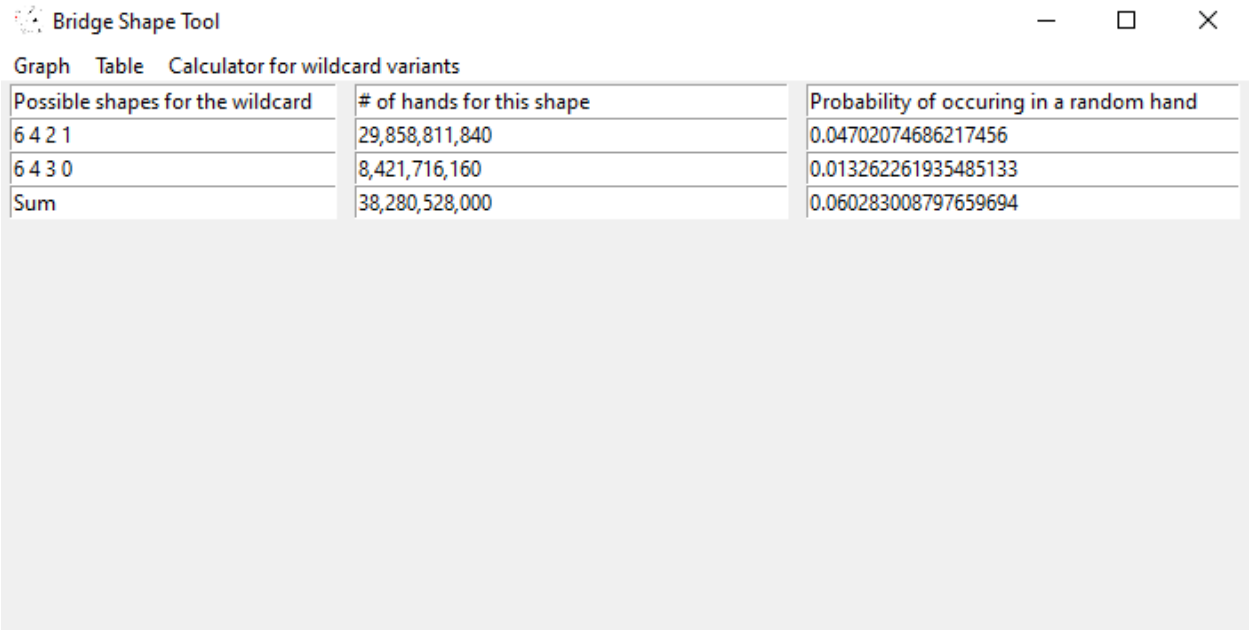
Click on “Calculator for wild variants” to get to the calculator that lets you input an incomplete hand shape and finds all the possible hand shapes that match and their probabilities



Input desired incomplete shape. Separate the numbers in the shape using commas to avoid confusing the program (will parse 112x as (11,2,x)).



Click “OK” or press Enter to calculate possible hand shapes. Clicking cancel results in an error message.



Possible shapes for the wildcard	# of hands for this shape	Probability of occuring in a random hand
6 4 2 1	29,858,811,840	0.04702074686217456
6 4 3 0	8,421,716,160	0.013262261935485133
Sum	38,280,528,000	0.060283008797659694

References / Resources Used

Resource	URL	Description
Python 3.8 or above	https://www.python.org/	A versatile and powerful programming language. We used this to create the Bridge Shape Tool.
matplotlib	https://matplotlib.org/	A library for creating good-looking graphs. Used to graph the probability distribution of a random bridge hand's shape.
Tk (as part of Python)	https://www.tcl.tk/	A graphical user interface toolkit. Provides components (buttons, textboxes, etc) that were used to create the interface of Bridge Shape Tool.
Ganesha Bridge	https://ganeshabridge.com/dist.html	Resource with the number of possible bridge hands that have any given shape. We verified the results of our calculations against this for correctness.
Course Notes	https://culearn.carleton.ca/moodle/pluginfile.php/4404944/mod_resource/content/1/Note05.pdf	A starting point for figuring out a way to count the number of hands that have a given shape.
Image used for executable file icon	https://www.hiclipart.com/free-transparent-background-png-clipart-icdim	The link supplied is where we found the image used for our executable file icon. We do not own the rights to the image, nor are we the original creators of the image.