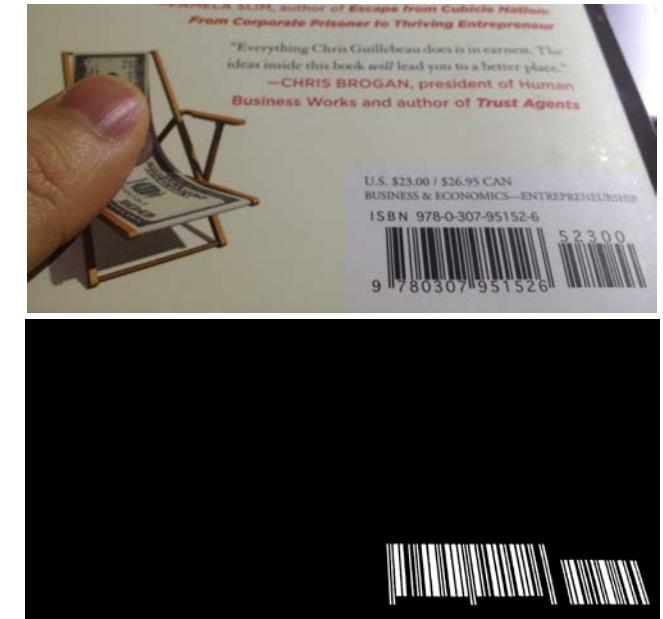
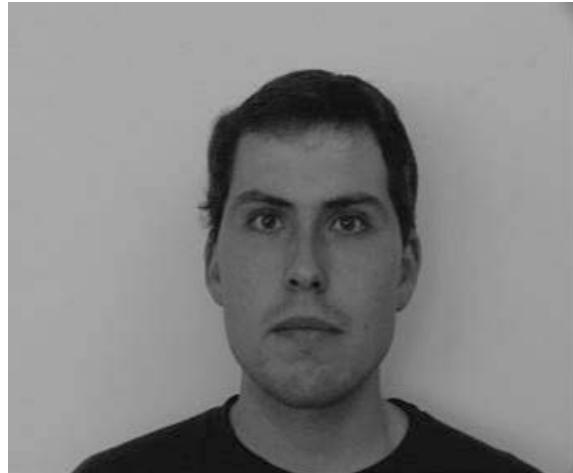


Image Segmentation

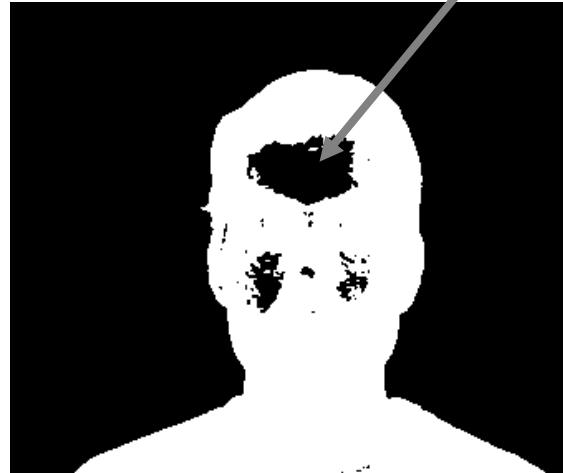
- Gray-level thresholding
- Supervised vs. unsupervised thresholding
- Binarization using Otsu's method
- Locally adaptive thresholding
- Maximally stable extremal regions
- Color-based segmentation
- Region labeling and counting
- Region moments



Gray-level thresholding



Original image
Peter $f[x, y]$



Thresholded
Peter $m[x, y]$

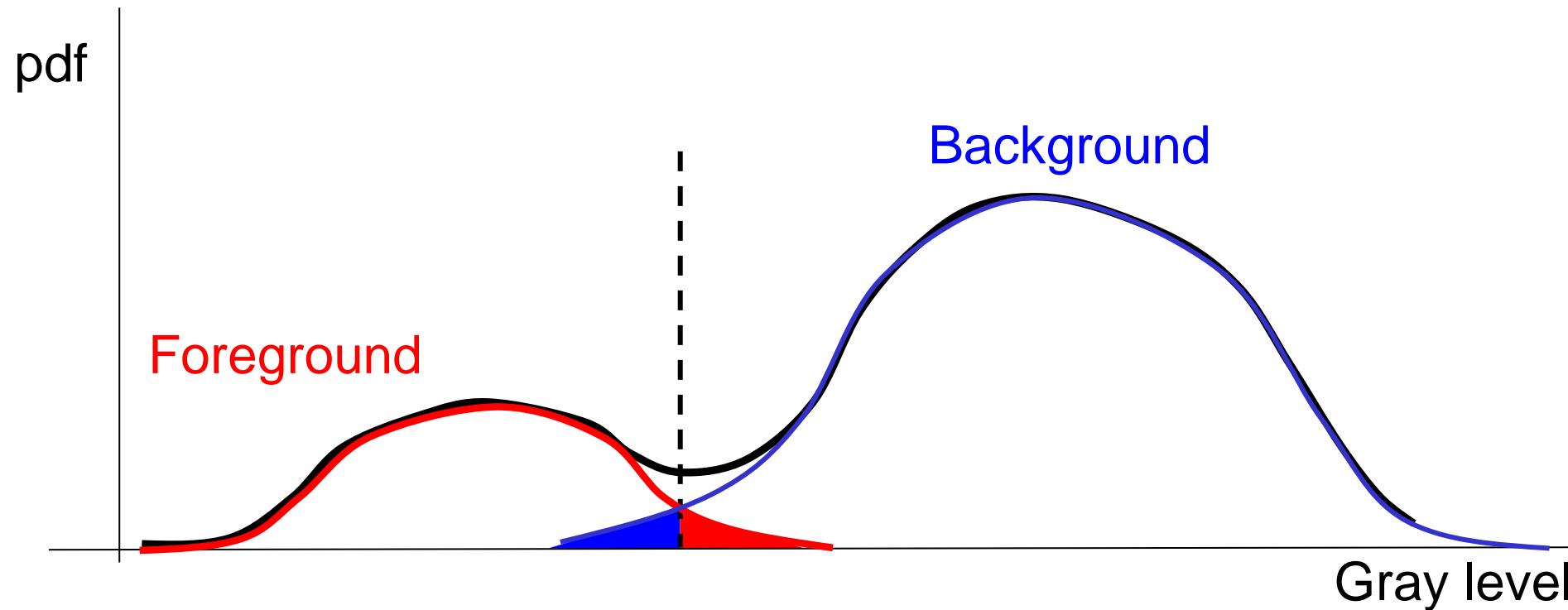


$$f[x, y] \cdot m[x, y]$$

How can holes be filled?



How to choose the threshold?



Unsupervised thresholding

- Idea: find threshold T that minimizes *within-class variance* of both foreground and background (same as k-means)

$$\sigma_{\text{within}}^2(T) = \frac{N_{\text{Fgrnd}}(T)}{N} \sigma_{\text{Fgrnd}}^2(T) + \frac{N_{\text{Bgrnd}}(T)}{N} \sigma_{\text{Bgrnd}}^2(T)$$

- Equivalently, maximize *between-class variance*

$$\begin{aligned}\sigma_{\text{between}}^2(T) &= \sigma^2 - \sigma_{\text{within}}^2(T) \\ &= \left(\frac{1}{N} \sum_{x,y} f^2[x,y] - \mu^2 \right) - \frac{N_{\text{Fgrd}}}{N} \left(\frac{1}{N_{\text{Fgrd}}} \sum_{x,y \in \text{Fgrnd}} f^2[x,y] - \mu_{\text{Fgrnd}}^2 \right) - \frac{N_{\text{Bgrnd}}}{N} \left(\frac{1}{N_{\text{Bgrnd}}} \sum_{x,y \in \text{Bgrnd}} f^2[x,y] - \mu_{\text{Bgrnd}}^2 \right) \\ &= -\mu^2 + \frac{N_{\text{Fgrnd}}}{N} \mu_{\text{Fgrnd}}^2 + \frac{N_{\text{Bgrnd}}}{N} \mu_{\text{Bgrnd}}^2 = \frac{N_{\text{Fgrnd}}}{N} (\mu_{\text{Fgrnd}} - \mu)^2 + \frac{N_{\text{Bgrnd}}}{N} (\mu_{\text{Bgrnd}} - \mu)^2 \\ &= \frac{N_{\text{Fgrnd}}(T) \cdot N_{\text{Bgrnd}}(T)}{N^2} (\mu_{\text{Fgrnd}}(T) - \mu_{\text{Bgrnd}}(T))^2\end{aligned}$$

[Otsu, 1979]

Unsupervised thresholding

- Idea: find threshold T that minimizes *within-class variance* of both foreground and background (same as k-means)

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- Equivalently, maximize *between-class variance*

$$\sigma_{\text{between}}^2(T) = \sigma^2 - \sigma_{\text{within}}^2(T) = \frac{N_{\text{Fgrnd}}(T) \cdot N_{\text{Bgrnd}}(T)}{N^2} \left(\mu_{\text{Fgrnd}}(T) - \mu_{\text{Bgrnd}}(T) \right)^2$$

[Otsu, 1979]

Unsupervised thresholding (cont.)

- Algorithm: Search for threshold T to maximize

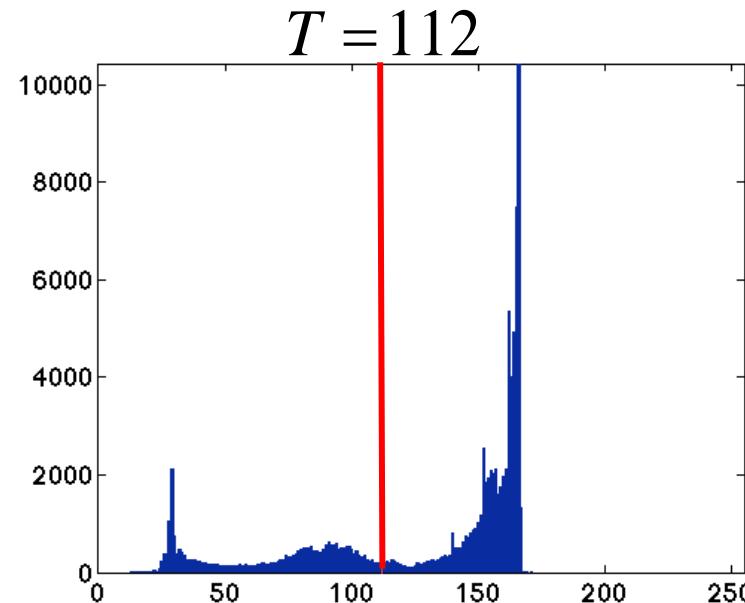
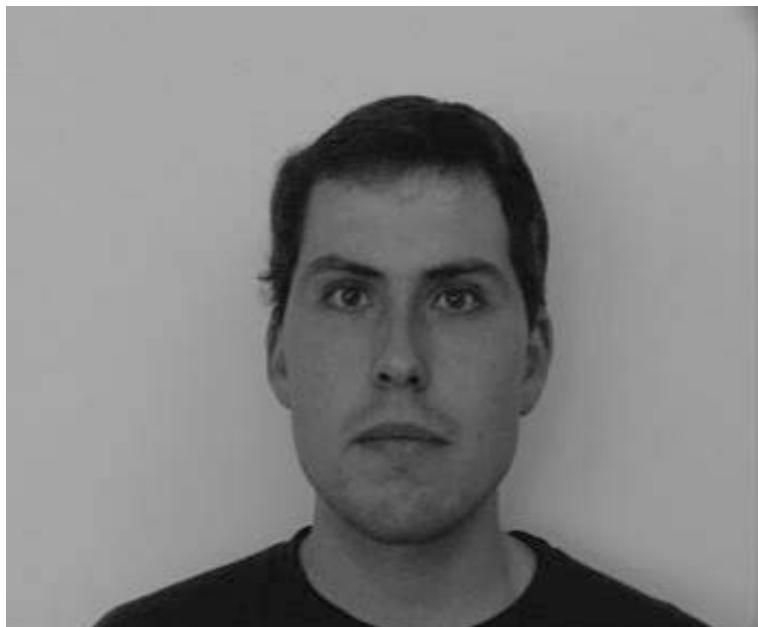
$$\sigma_{between}^2(T) = \frac{N_{Fgrnd}(T) \cdot N_{Bgrnd}(T)}{N^2} (\mu_{Fgrnd}(T) - \mu_{Bgrnd}(T))^2$$

- Useful recursion for sweeping T across histogram:

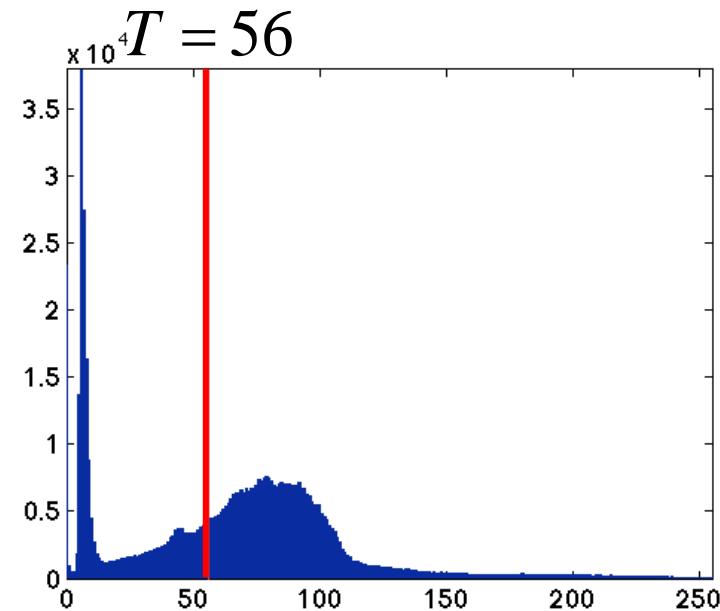
$$\begin{aligned} N_{Fgrnd}(T+1) &= N_{Fgrnd}(T) + n_T \\ N_{Bgrnd}(T+1) &= N_{Bgrnd}(T) - n_T \\ \mu_{Fgrnd}(T+1) &= \frac{\mu_{Fgrnd}(T)N_{Fgrnd}(T) + n_T T}{N_{Fgrnd}(T+1)} \\ \mu_{Bgrnd}(T+1) &= \frac{\mu_{Bgrnd}(T)N_{Bgrnd}(T) - n_T T}{N_{Fgrnd}(T+1)} \end{aligned}$$

[Otsu, 1979]

Unsupervised thresholding (cont.)



Unsupervised thresholding (cont.)



Unsupervised thresholding (cont.)

The Stanford Daily

Tuesday, September 18, 2012 ♦ 13



SIMON WARBY/The Stanford Daily

Stanford defensive lineman Josh Mauro put the pressure on USC's Matt Barkley. Mauro was relentless in the second half as Stanford's defense completely shut down Barkley and his touted wide receivers.

Handing out the USC game balls

By SAM FISHER
FOOTBALL EDITOR

about Andrew Luck.

Josh Mauro: The back-up defense end saw most of his action at nose tackle in the second half, where he completely took over the ball-game. As a back-up center, Cyrus Hobbs all had to provide the key pressure up the middle from the defensive line. The key! Matt Barkley missed. The rest of the defense did a great job in support, but Mauro went above and beyond the call of duty to help at the end. His 213 total yards of offense to go with a pair of TDs had fans on both sides forgetting

Please see AWARDS, page 15

FOOTBALL

The winding road ahead

By SAM FISHER
FOOTBALL EDITOR

Andrew Luck may be gone, but with Saturday night's win over USC, the Stanford Cardinal put itself in position to achieve beyond the path paved by number 12. You heard right; though they still have a lot left to do, the 2012 Stanford team showed that it is capable of playing at a national championship level.

Though Stanford survived one of its toughest tests in the games that is the BCS National Championship elimination, the road to Miami 2013 is no walk in the park. The toughest challenges will come on the road, with games at Notre Dame, Oregon and UCLA, all of whom are currently ranked in the top 20. The last two games at Wisconsin and then home against Arizona, are no pushovers either. And as Stanford has shown, top-ranked opponents can't beat any team on the Cardinal's schedule.

With the exception of Door Number Two for Stanford is a trip to a BCS bowl for the third straight year. Stanford has an all-time winning record against the BCS, thanks to a strong strength of schedule. However, the path to winning the national championship isn't completely out of the question, but it's still not the most likely ending to 2012.

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From Stanford's current vantage point, there are three paths the rest of the season could take:

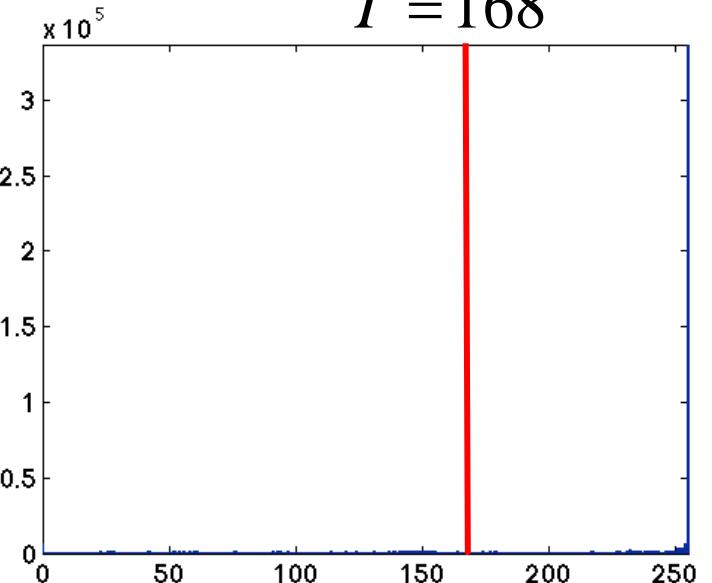
Please see AWARDS, page 15

For Stanford to even think about returning to the site of its 2011 Orange Bowl heat down of Virginia Tech, Josh Nunes will have to build off his recent career success and USC to play at a higher level consistently. From the 12-yard scramble on third-and-10 on, Nunes was good enough to earn a national championship. USC's defense is nowhere near a pushover, so Jordan Williamson's 0-3 kicking record also needs to improve.

With the way the Stanford defense played against USC over the last 41 minutes, it seems like a national championship isn't completely out of the question, but it's still not the most likely ending to 2012.

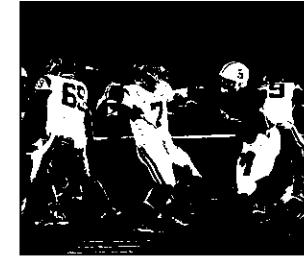
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$T = 168$



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Tuesday, September 18, 2012 ♦ 13



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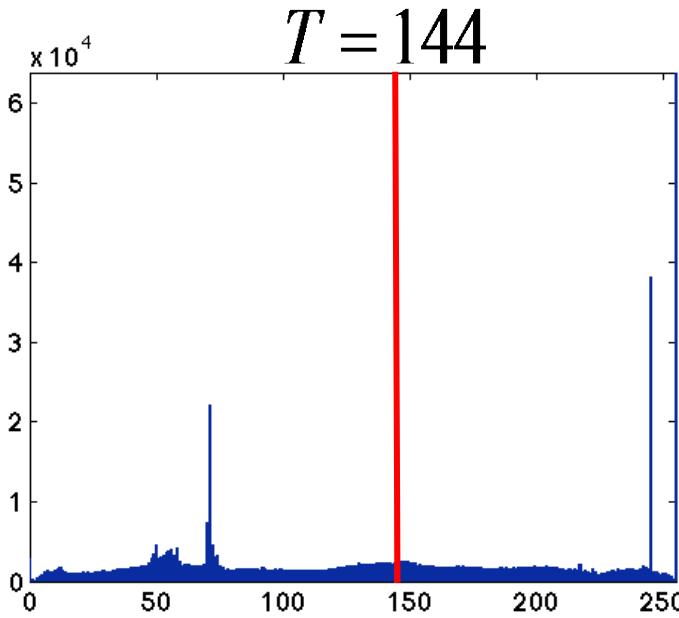
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Please see AWARDS, page 15

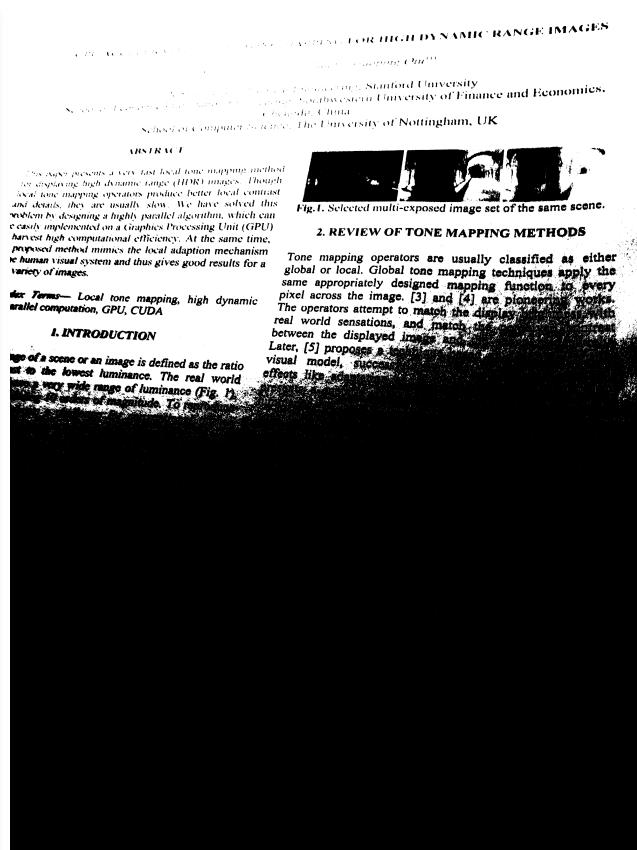


Unsupervised thresholding (cont.)



Sometimes, a global threshold does not work

Original image

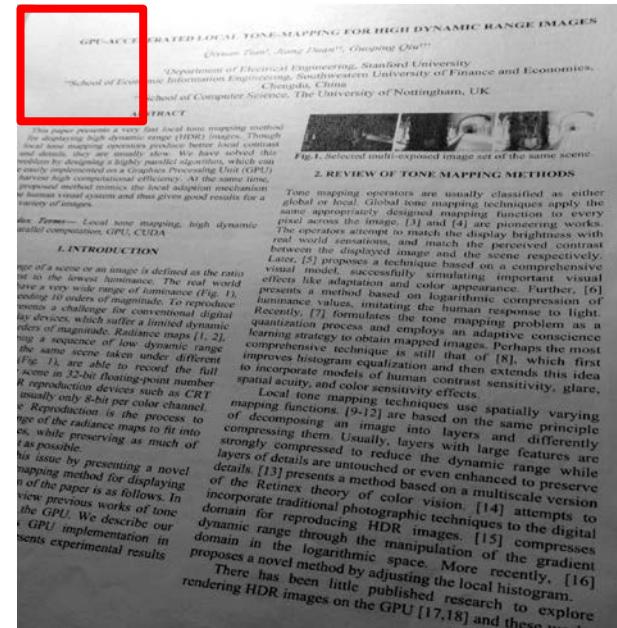


Thresholded with Otsu's Method



Locally adaptive thresholding

- Slide a window over the image
- For each window position, decide whether to perform thresholding
 - Thresholding should not be performed in uniform areas
 - Use variance or other suitable criterion
- Non-uniform areas: apply Otsu's method (based on local histogram)
- Uniform areas: classify the entire area as foreground or background based on mean value



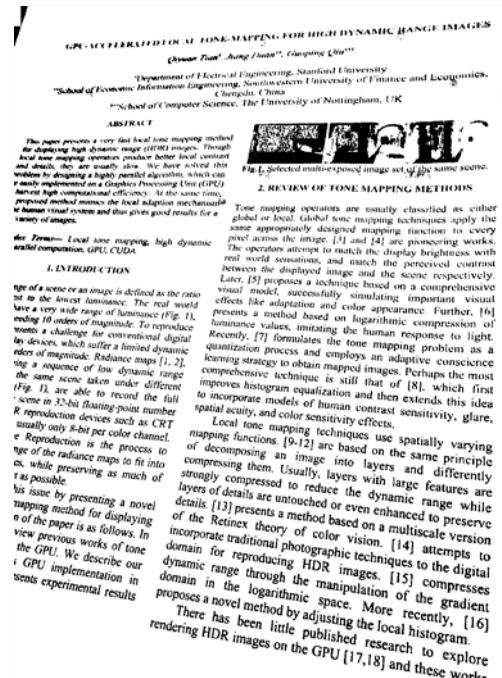
Locally adaptive thresholding (example)



Non-uniform areas



Local threshold values



Locally thresholded result



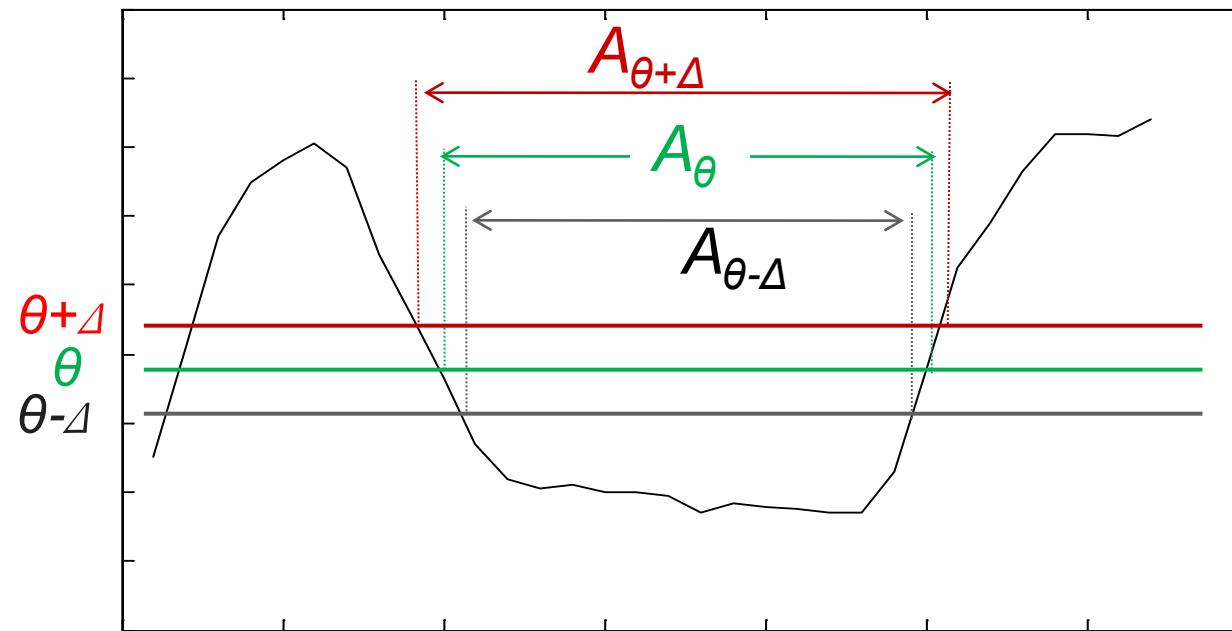
Maximally stable extremal regions

- Extremal region: any connected region in an image with all pixel values above (or below) a threshold
- Observations:
 - Nested extremal regions result when the threshold is successively raised (or lowered).
 - The nested extremal regions form a “component tree.”
- Key idea: choose thresholds θ such that the resulting bright (or dark) extremal regions are nearly constant when these thresholds are perturbed by $+/-\Delta$

→ “***maximally stable***” ***extremal regions (MSER)***

[Matas, Chum, Urba, Pajdla, 2002]

MSERs: illustration



$$\text{Local minimum of } \left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right| \rightarrow \text{MSER}$$

[Matas, Chum, Urba, Pajdla, 2002]

Level sets of an image

1	1	1	1	1	1	1	1	1	1	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	3	3	3	3	1	5	4	4	8
1	7	6	4	2	2	5	5	5	5	1	5	4	4	8
1	6	6	4	2	2	5	5	5	6	1	5	4	4	4
1	6	6	4	2	2	6	6	6	6	1	5	5	5	5
1	4	4	4	2	2	6	6	6	6	1	5	5	5	5
1	1	1	1	1	2	6	1	1	1	1	2	2	2	2
1	8	8	5	1	2	6	1	7	7	1	2	2	2	2
1	8	8	5	1	1	1	1	7	7	1	1	1	1	2
1	8	8	5	5	5	3	3	7	7	1	1	1	1	2
1	8	8	5	5	3	3	3	7	7	7	1	1	1	2
1	8	8	5	5	3	3	3	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

$f[x, y]$

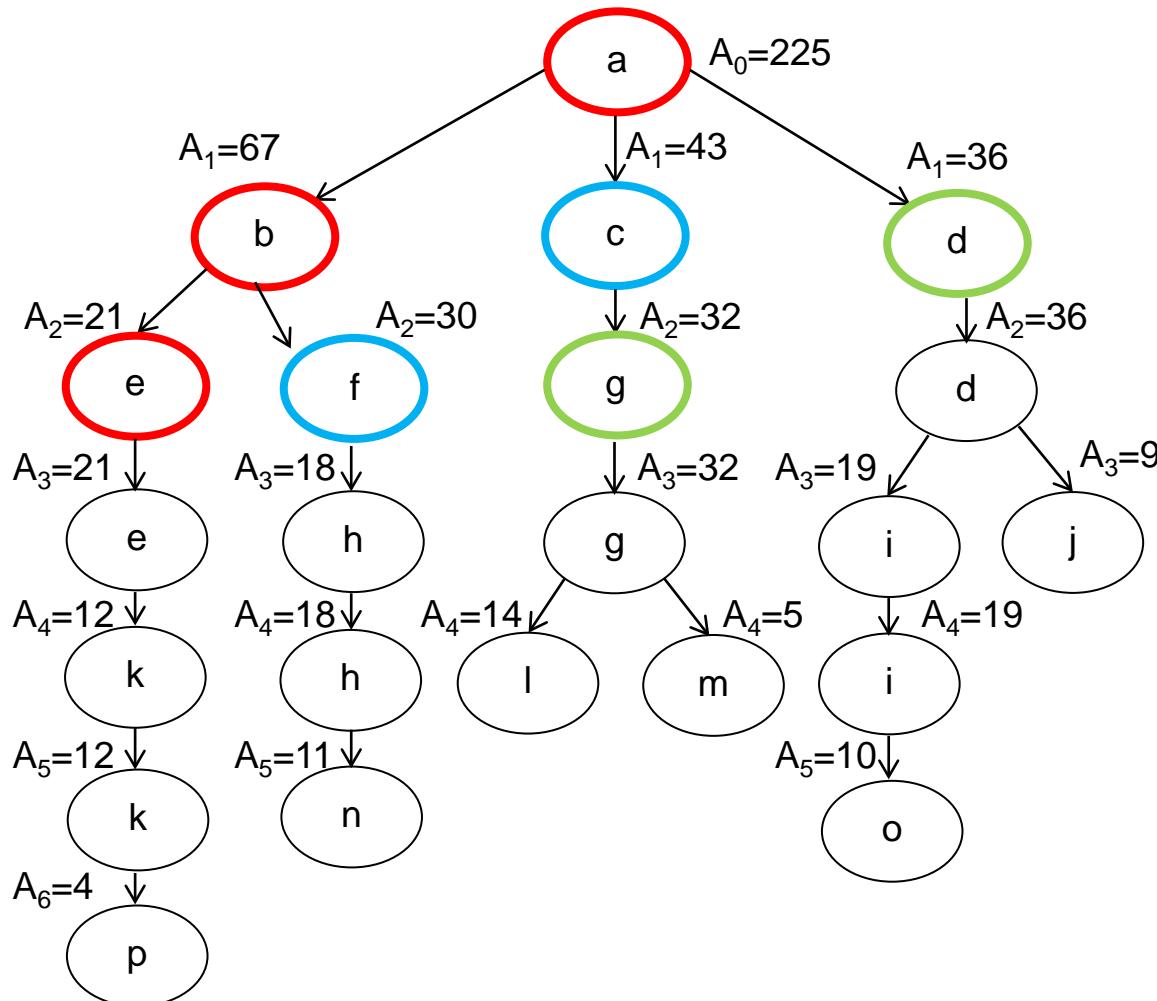
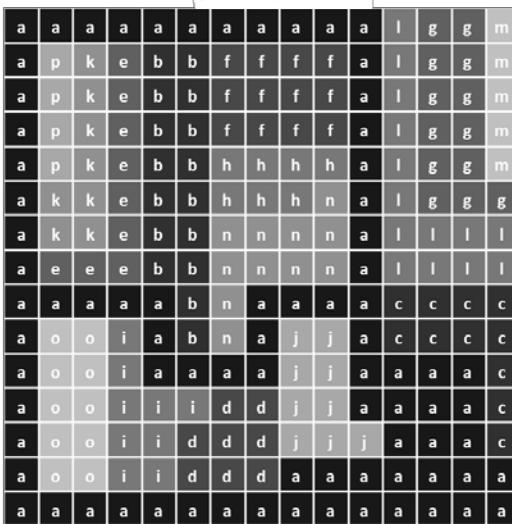
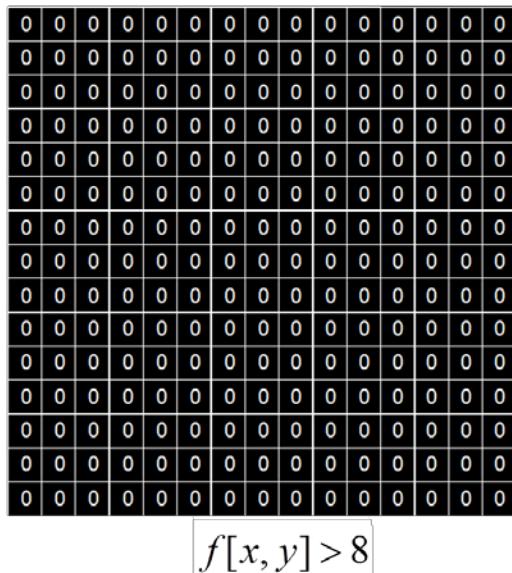
Image

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$f[x, y] > 8$

Level Set

Component tree of an image



Local minima of sequence

$$\left| \frac{A_{\theta-\Delta} - A_{\theta+\Delta}}{A_\theta} \right|$$

$\theta = \Delta, \Delta + 1, \dots \rightarrow \text{MSERs}$

MSER: examples



Dark MSERs, $\Delta=15$



Original image



Bright MSERs, $\Delta=15$

MSER: examples



Dark MSERs, $\Delta=15$

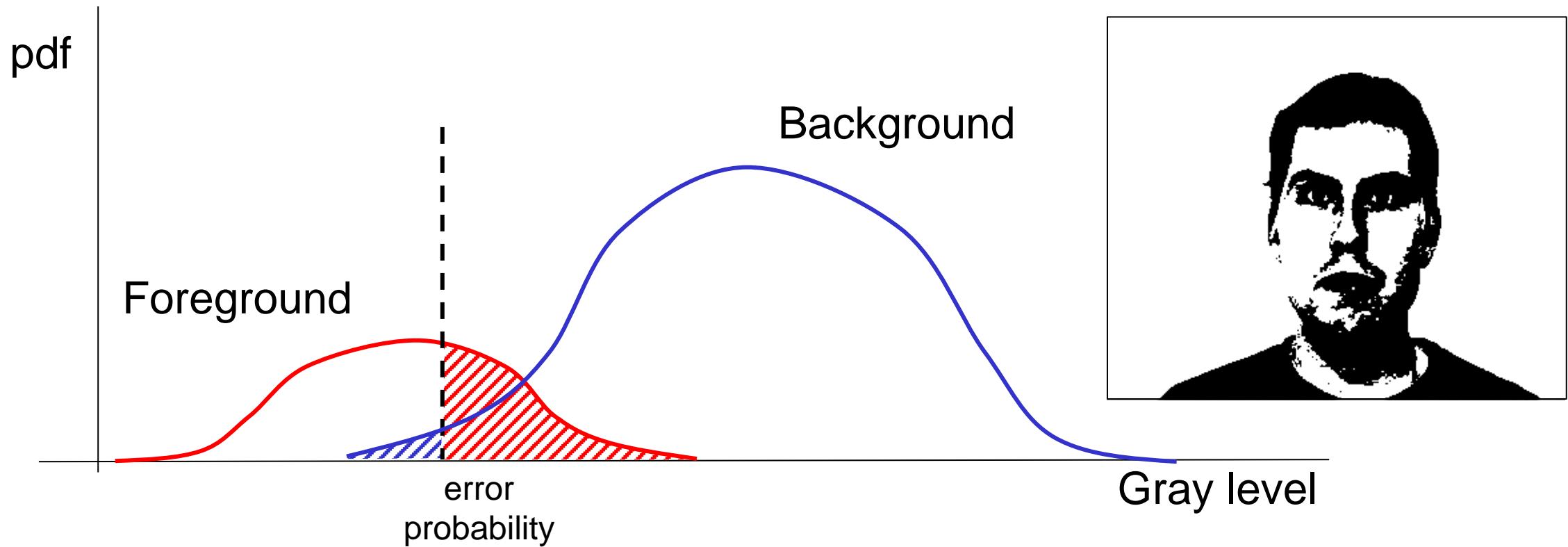


Original image

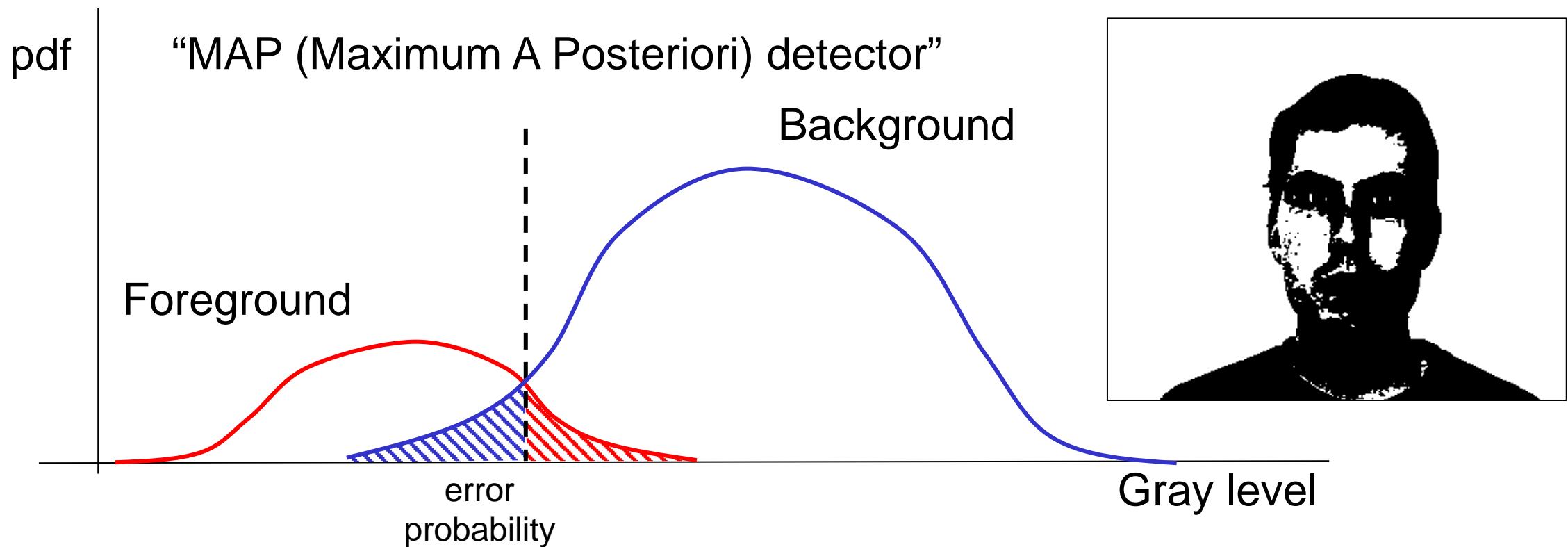


Bright MSERs, $\Delta=15$

Supervised thresholding



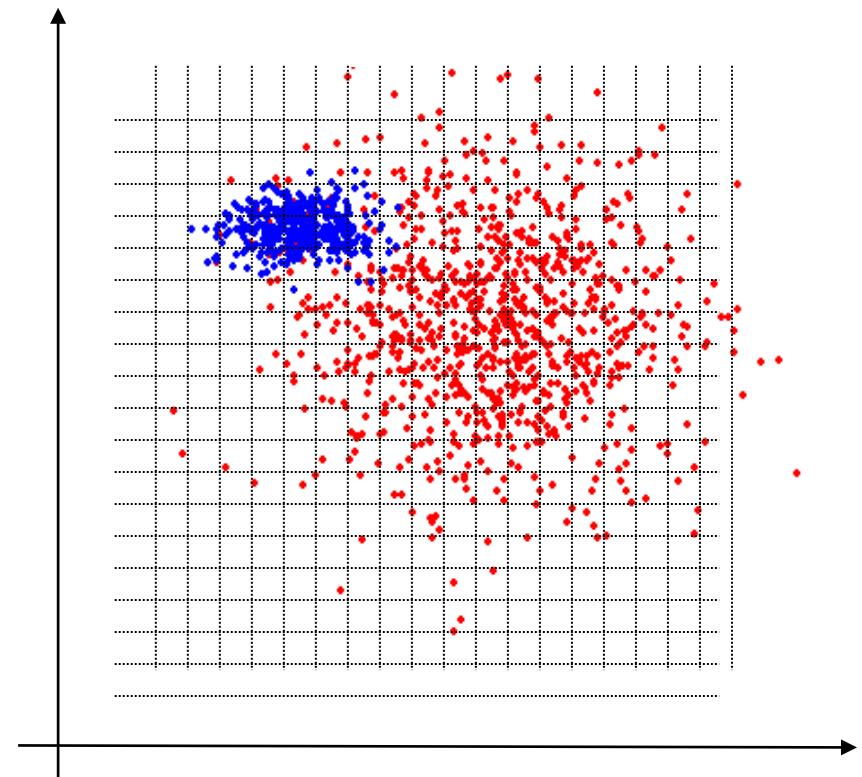
Supervised thresholding



If errors $\text{BG} \rightarrow \text{FG}$ and $\text{FG} \rightarrow \text{BG}$ are associated with different costs:
“Bayes minimum risk detector” is optimal.

Multidimensional MAP detector

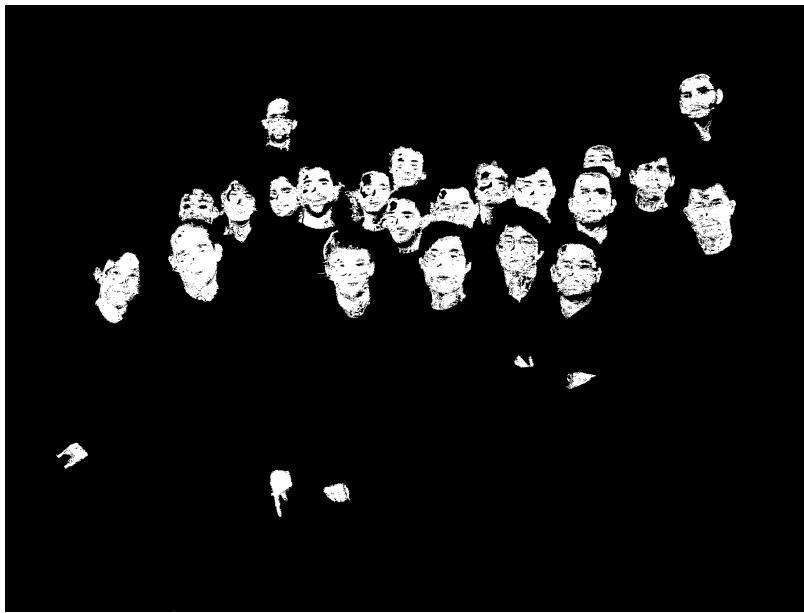
- Training
 - Provide labelled set of training data
 - Subdivide n-dimensional space into small bins
 - Count frequency of occurrence for each bin and class in training set, label bin with most probable class
 - (Propagate class labels to empty bins)
- For test data: identify bin, look up the most probable class



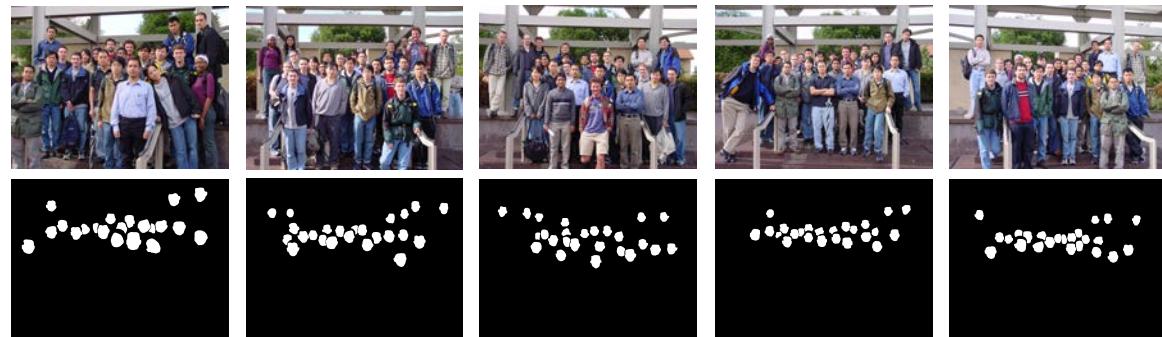
MAP detector in RGB-space



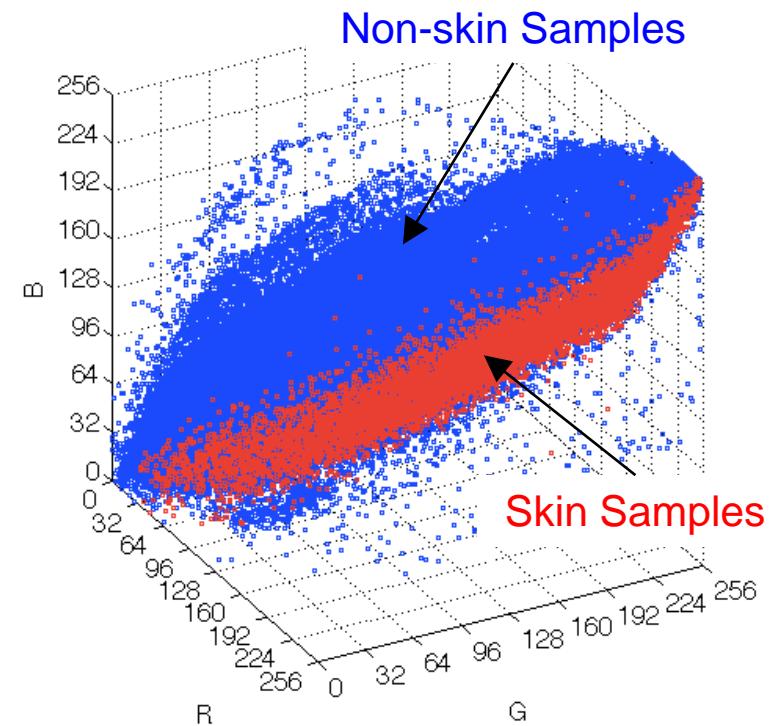
Original image

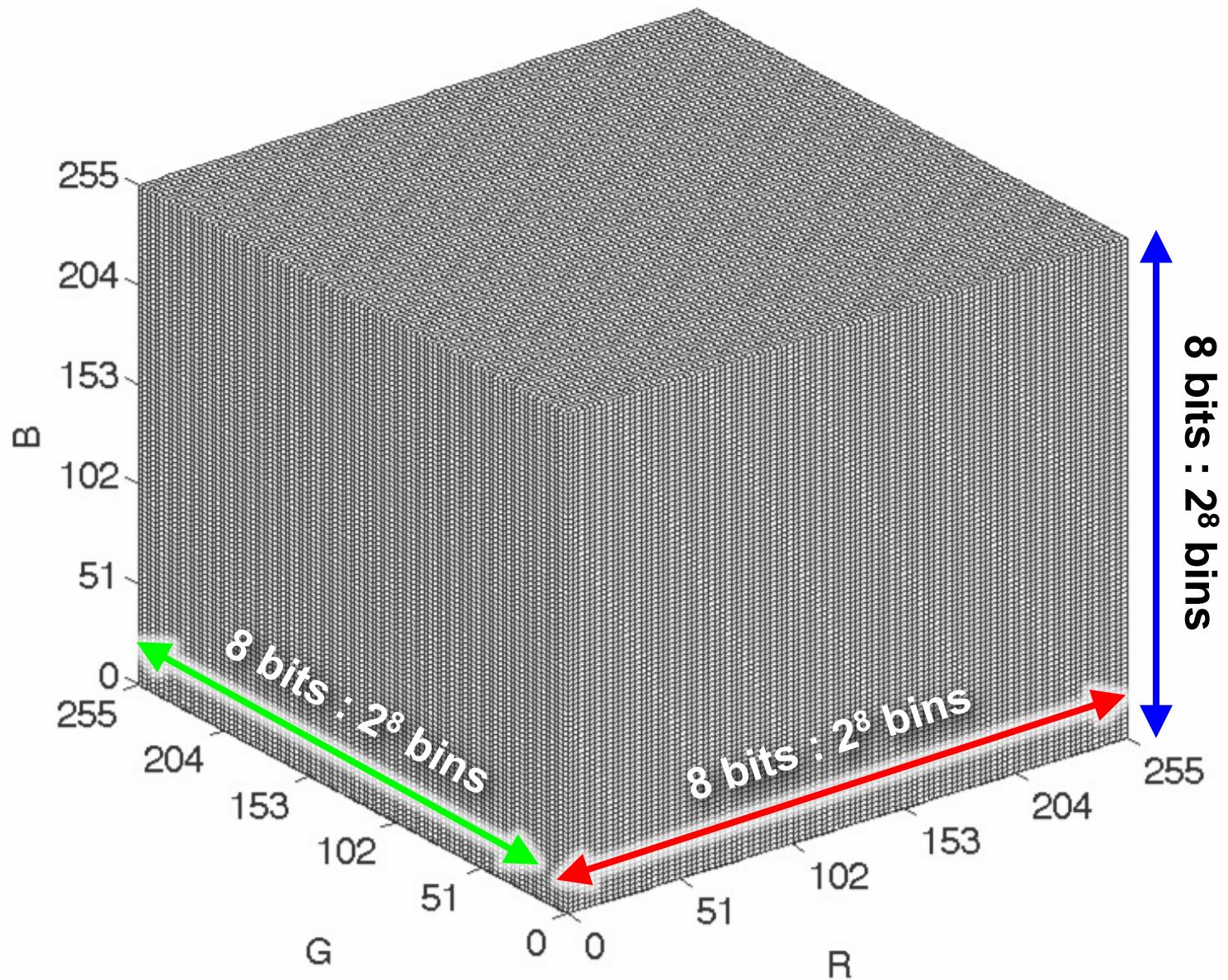


Skin color detector



Five training images



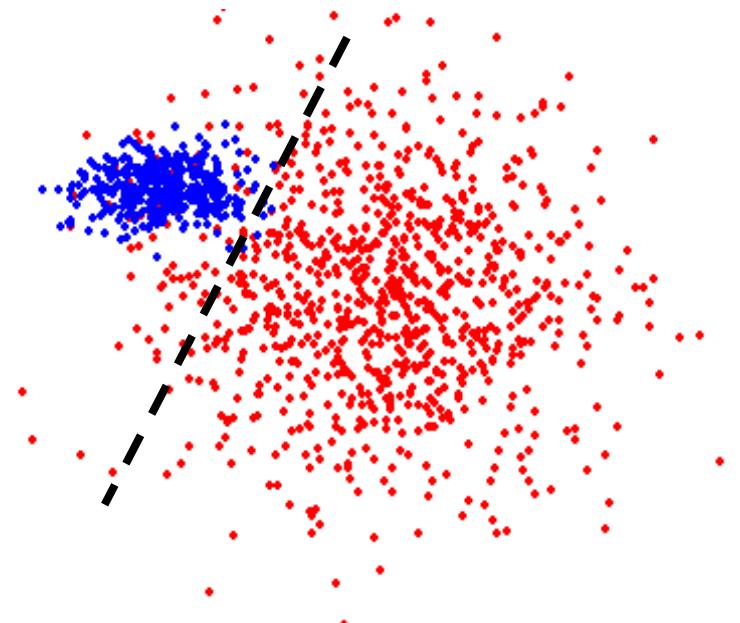


Linear discriminant function

- To segment image with n components $f_i, i=1,2,\dots,n$ into two classes, perform test

$$\sum_i w_i f_i + w_0 \geq 0 \quad ?$$

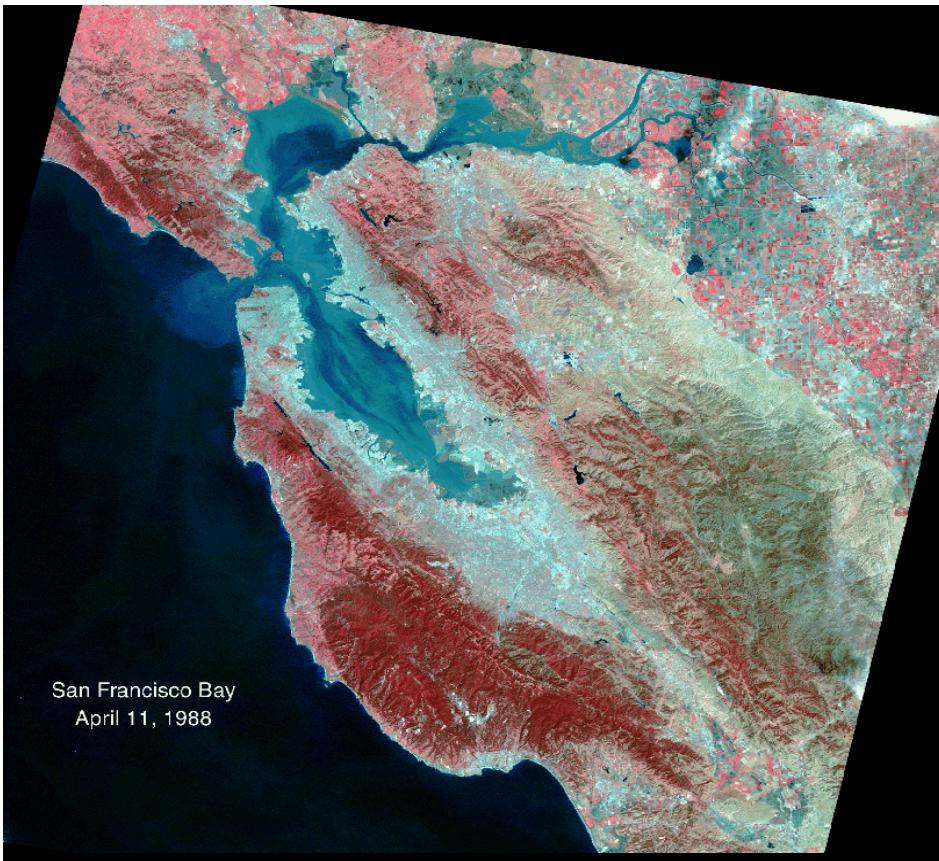
- Categories are separated by hyperplane in n -space
- Numerous techniques to determine weights
 $w_i, i=0,1,2,\dots,n$, see, e.g., [\[Duda, Hart, Stork, 2001\]](#)
- Can be extended to the intersection of several linear discriminant functions
- Can be extended to multiple classes



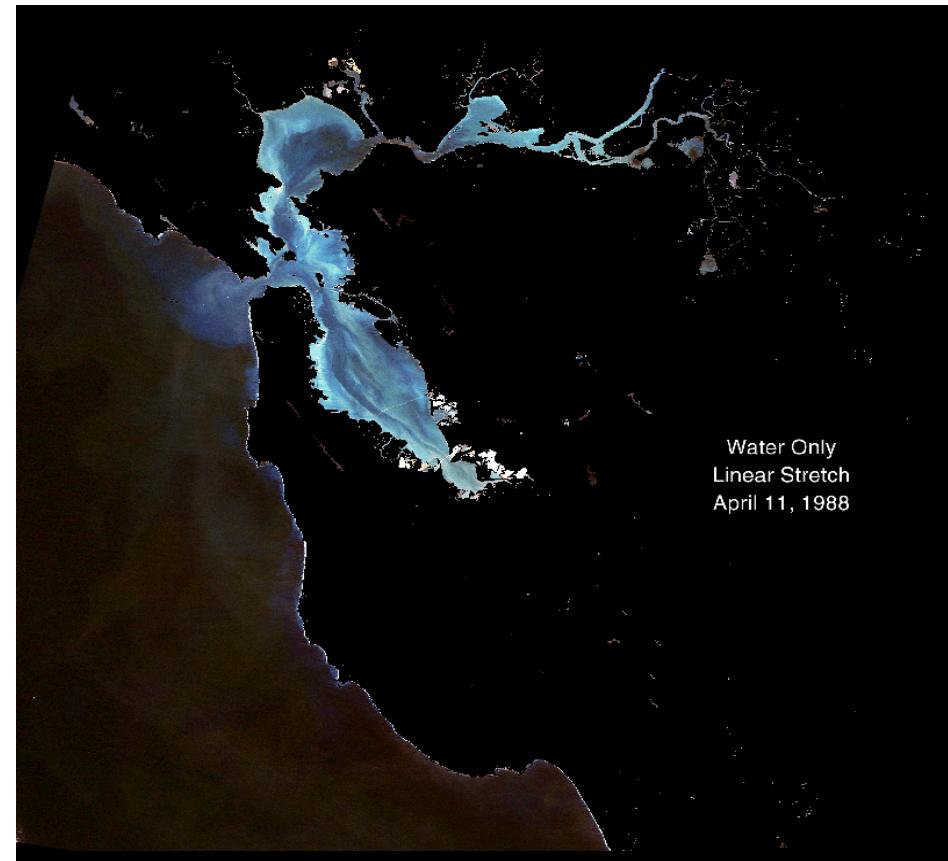
Chroma keying



Landsat image processing



Original Landsat image false color picture out
of bands 4,5,6

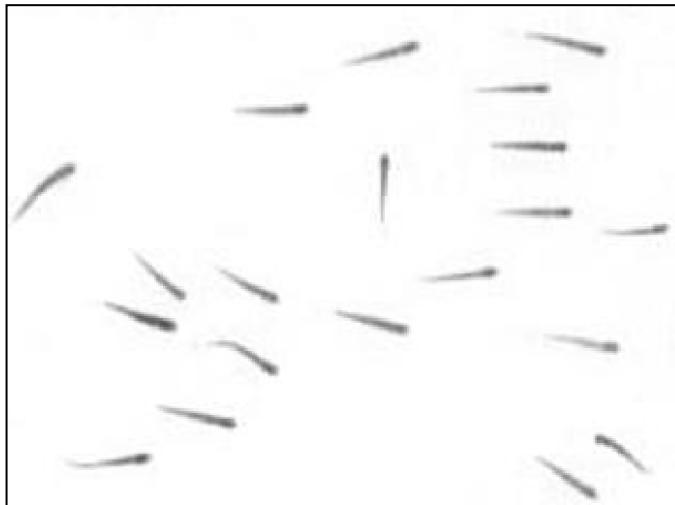


Water area segmented and enhanced to show
sediments

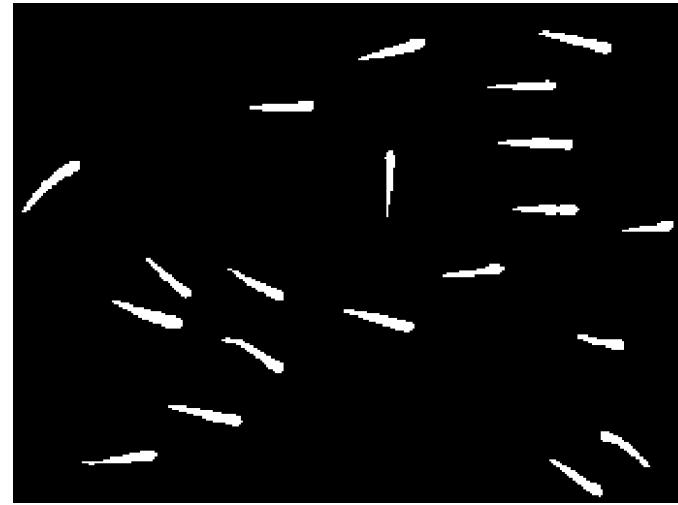
Source: US Geological Survey USGS, <http://sfbay.wr.usgs.gov/>

Region labeling and counting

- How many fish in this picture?



Original *Fish* image



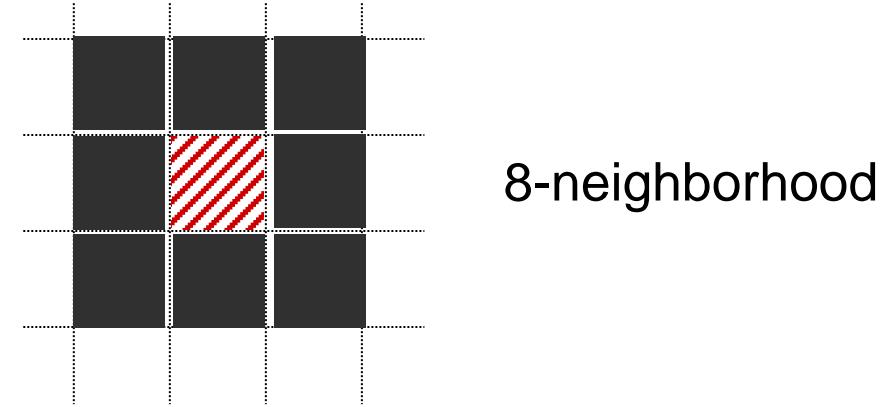
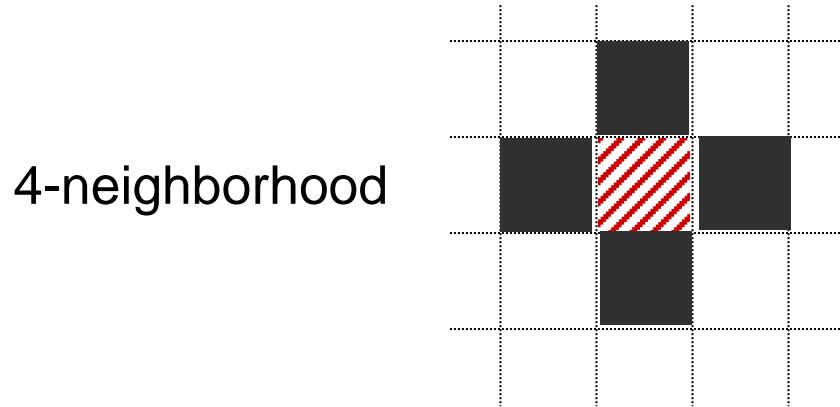
after thresholding

- Which pixels belong to the same object (region labeling)?
- How large is each object (region counting)?



4-connected and 8-connected neighborhoods

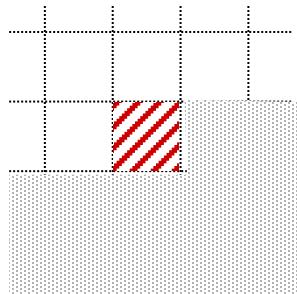
- Definition: a ***region*** is a set of pixels, where each pixel can be reached from any other pixel in the region by a finite number of steps, with each step starting at a pixel and ending in the neighborhood of the pixel.



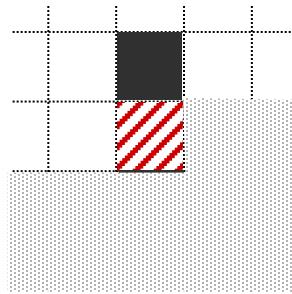
- Typically, either definition leads to the same regions, except when a region is only connected across diagonally adjacent pixels.

Region labeling algorithm (4-neighborhood)

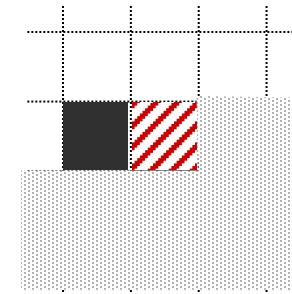
- Loop through all pixels $f[x,y]$, left to right, top to bottom
- If $f[x,y]=0$, do nothing.
- If $f[x,y]=1$, distinguish 4 cases



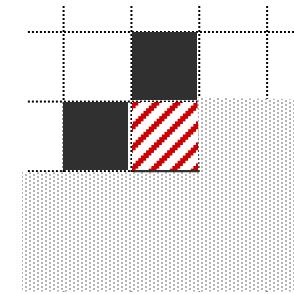
Generate new
region label



Copy label
from above



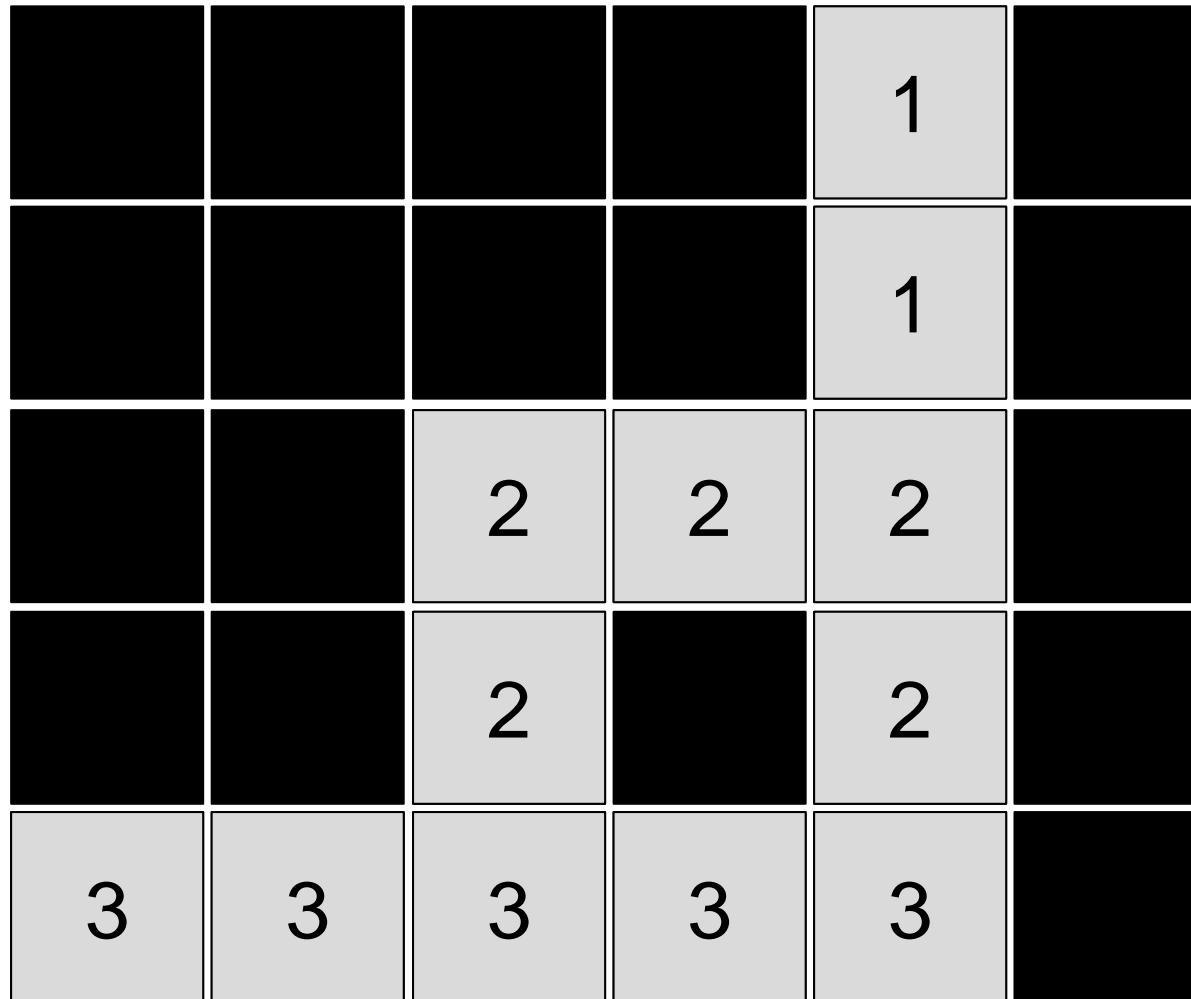
Copy label
from the left



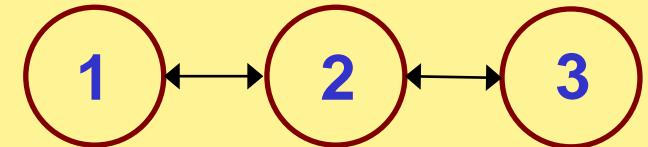
Copy label from the left. If
labels above and to the left are
different, store equivalence.

- Second pass through image to replace equivalent label by the same label.

Region labeling example (4-neighborhood)

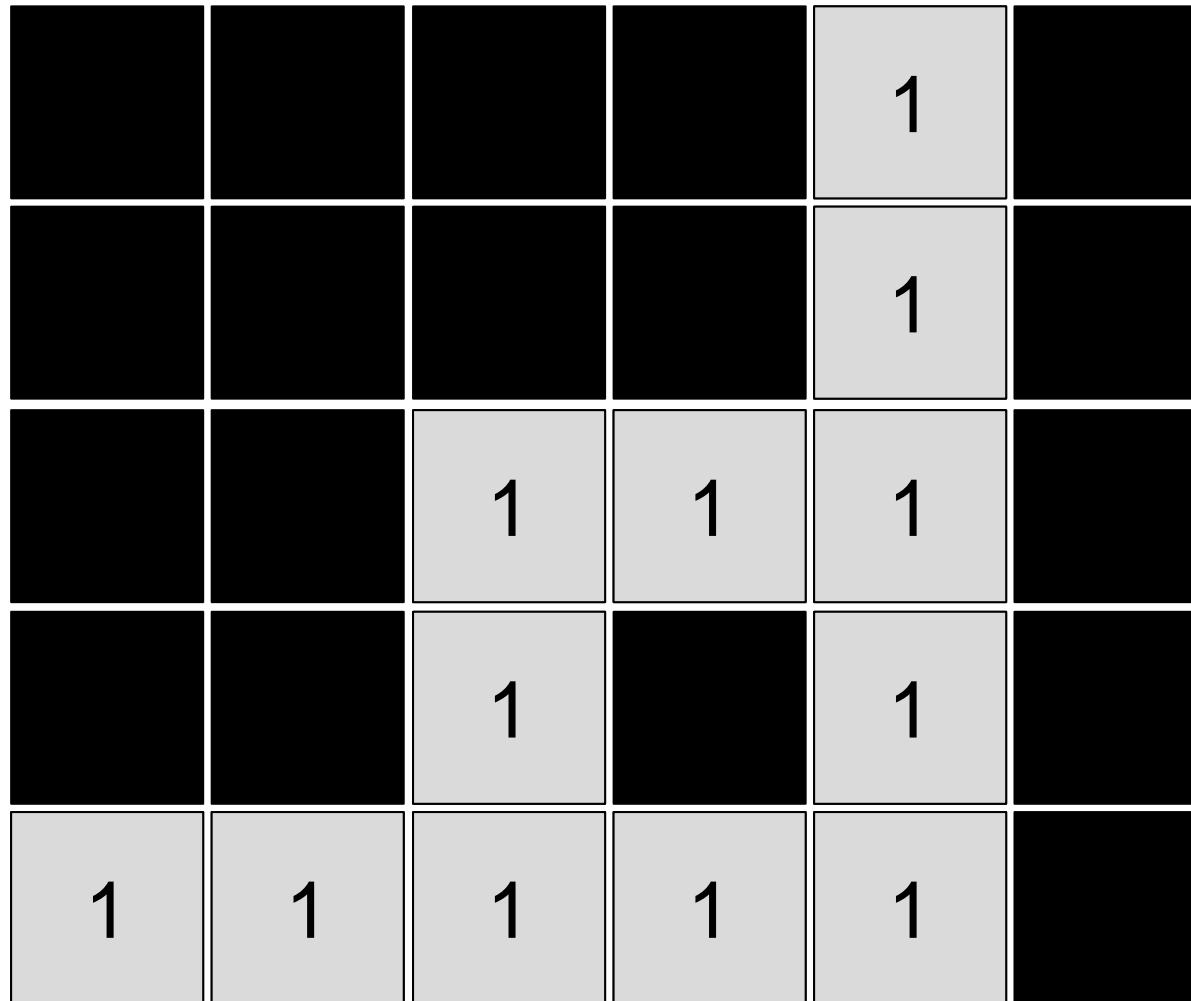


List of Region Labels



All three labels are equivalent, so merge into single label.

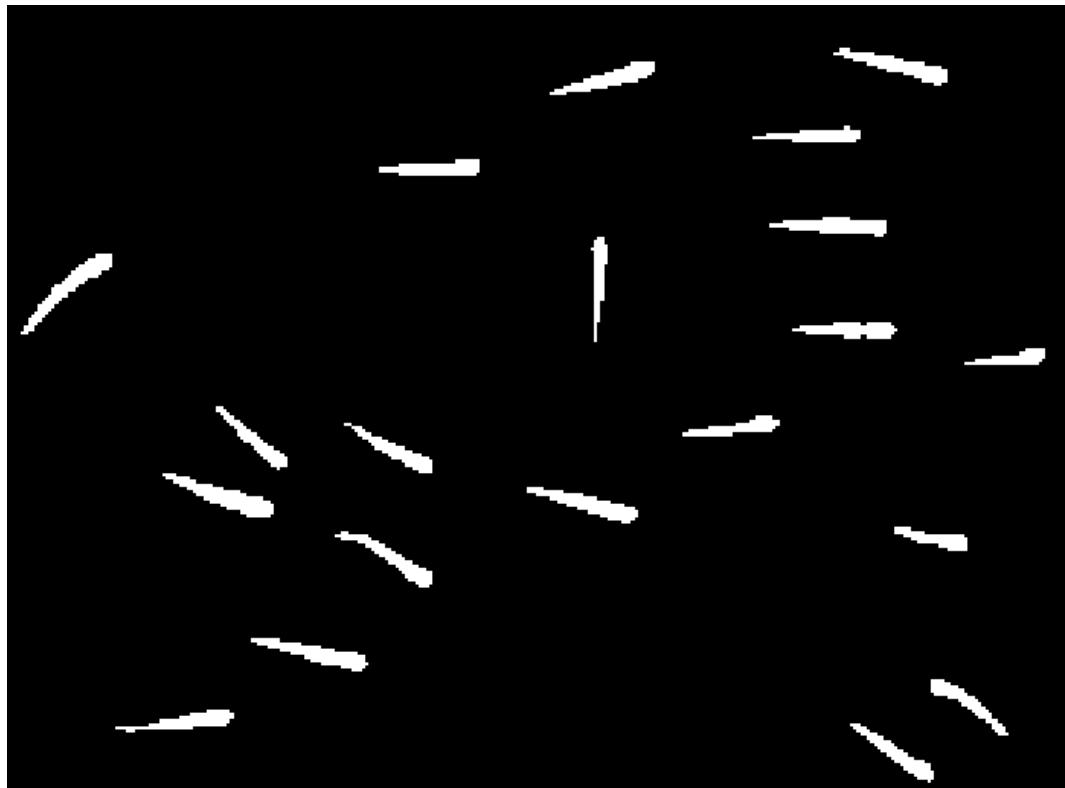
Region labeling example (4-neighborhood)



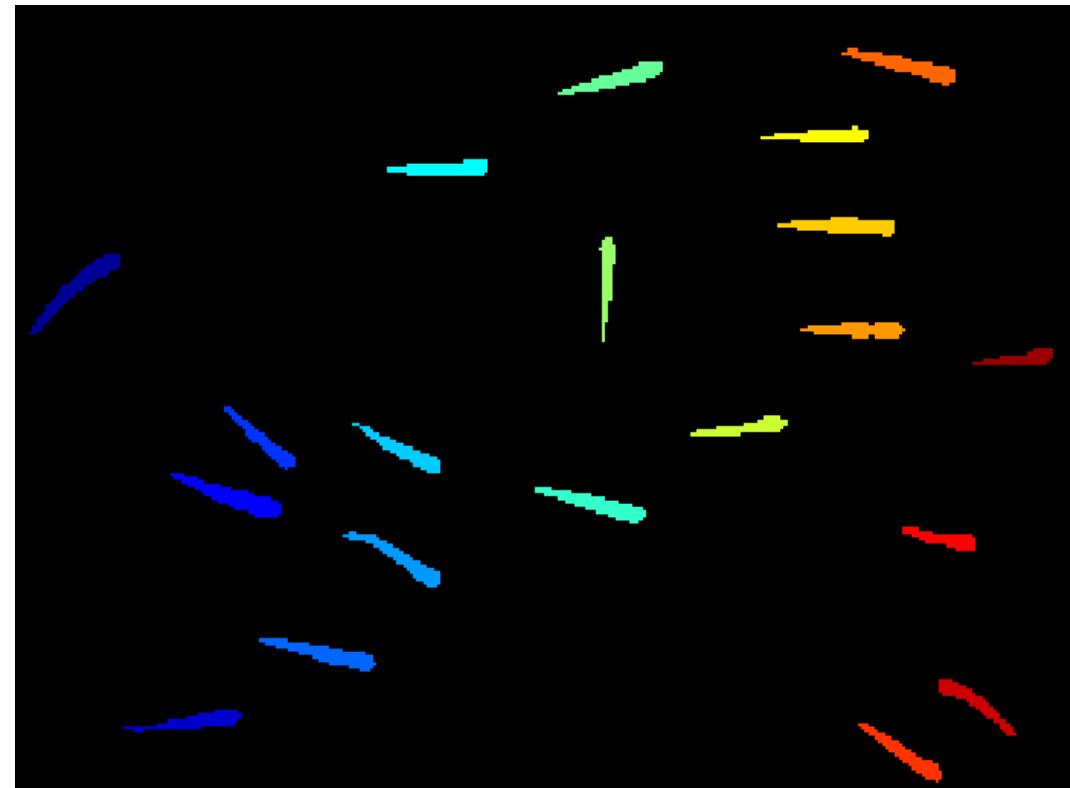
List of Region Labels

1

Example: region labeling



Thresholded image



20 labeled regions



Region counting algorithm

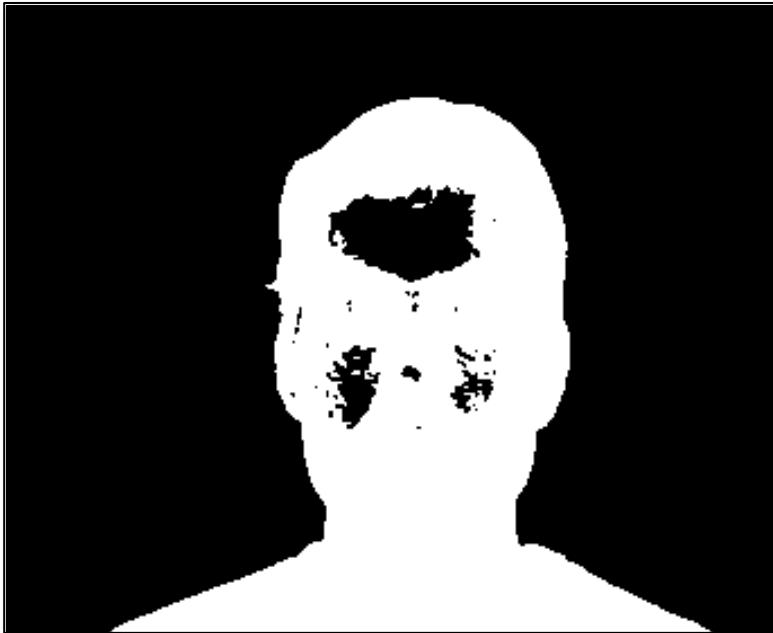
- Measures the size of each region
- Initialize $counter[label]=0$ for all $label$
- Loop through all pixels $f[x,y]$, left to right, top to bottom
 - If $f[x,y]=0$, do nothing.
 - If $f[x,y]=1$, increment $counter[label[x,y]]$

Small region removal

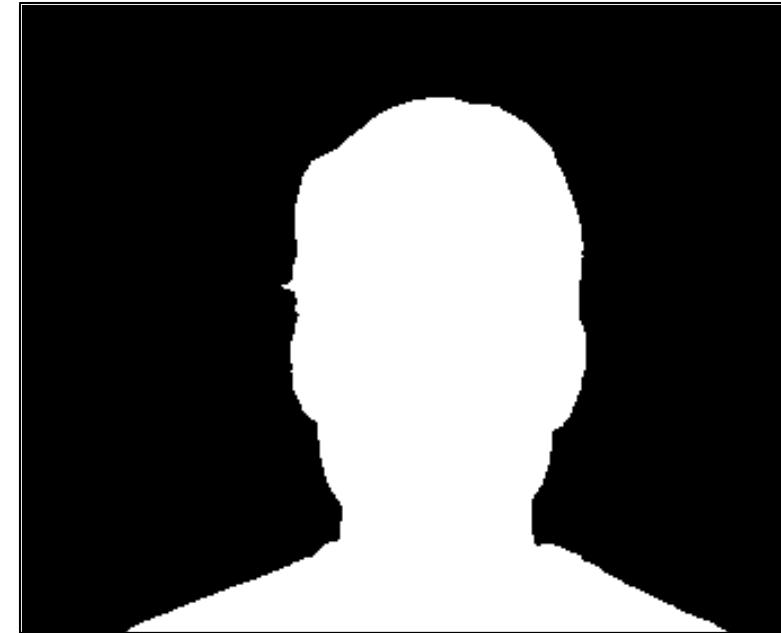
- Loop through all pixels $f[x,y]$, left to right, top to bottom
 - If $f[x,y]=0$, do nothing.
 - If $f[x,y]=1$ and $counter[label[x,y]] < S$, set $f[x,y]=0$
- Removes all regions smaller than S pixels

Hole filling as dual to small region removal

Mask with holes



After NOT operation, (background)
region labeling, small region removal,
and second NOT operation



Region moments

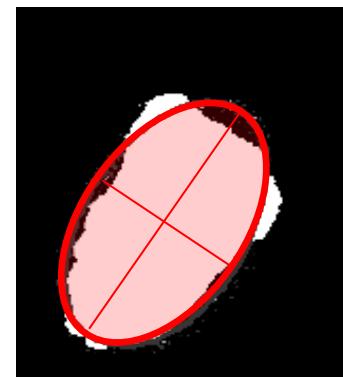
- Raw moments $M_{pq} = \sum_{x,y \in \text{Region}} x^p y^q$

- Central moments

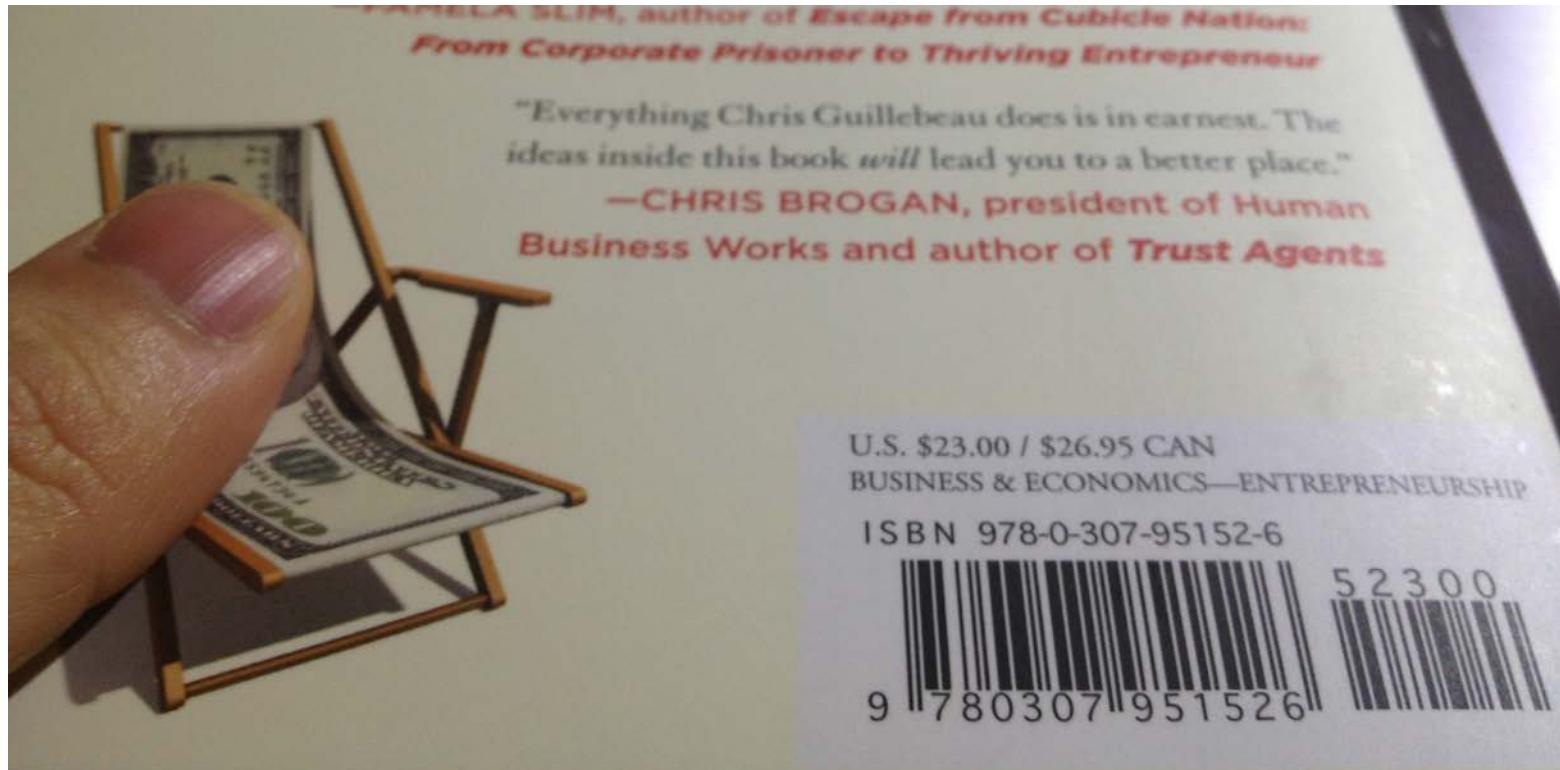
$$\mu_{pq} = \sum_{x,y \in \text{Region}} (x - \bar{x})^p (y - \bar{y})^q \quad \text{with } \bar{x} = \frac{M_{10}}{M_{00}} \text{ and } \bar{y} = \frac{M_{01}}{M_{00}}$$

- Region orientation and eccentricity:
calculate eigenvectors of covariance
matrix

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix}$$



Example: Detecting bar codes

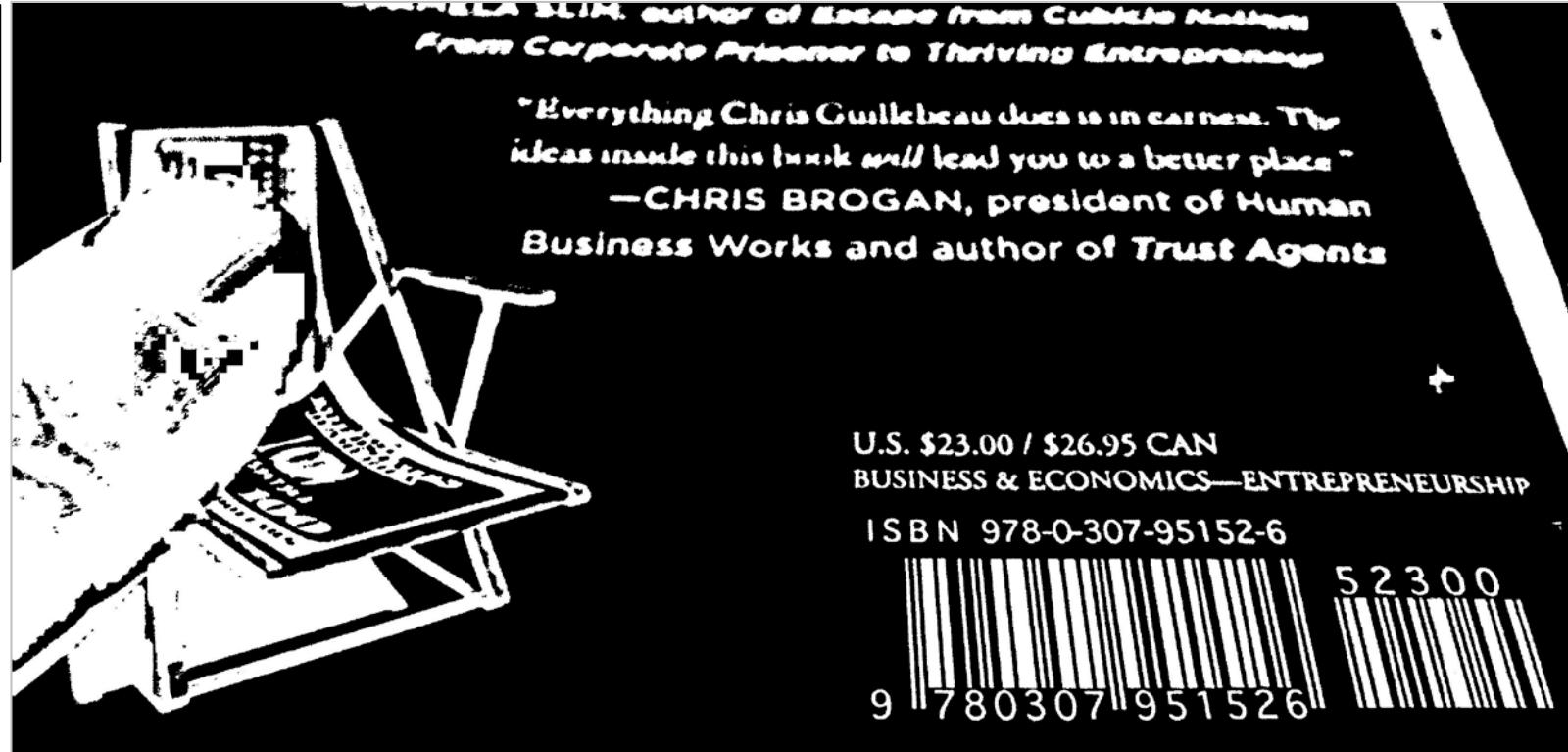


Original Image



Example: Detecting bar codes

Locally adaptive
thresholding



Example: Detecting bar codes

Locally adaptive
thresholding

Filtering by
eccentricity

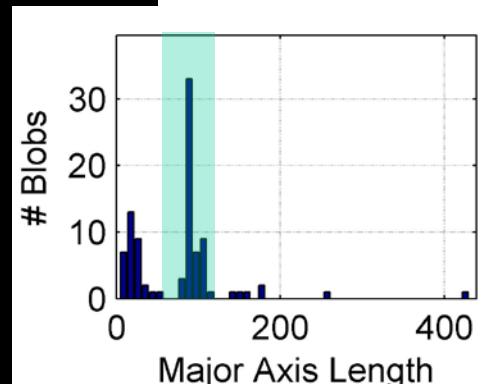


Example: Detecting bar codes

Locally adaptive
thresholding

Filtering by
eccentricity

Filtering by major
axis length



Example: Detecting bar codes

Locally adaptive
thresholding

Filtering by
eccentricity

Filtering by major
axis length

Filtering by
orientation

