Every vertex has three attributes:

1. Color
2. Distance from source
3. Parent

Colors: White, Gray, Black

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Lemma:1

For every (u,v) belongs to E, Sp = shortest distance

Sp(s,v) <= Sp(s,u) + 1

1. If u is reachable from s (u, v)
2. If u is not reachable -------------- Sp(s,v) <= infinity +1

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Lemma:2

Correctness of BFS i.e., to prove that for all v, that are reachable from s, upon termination following should be true: v.d = Sp(s,v). We first show that v.d bounds Sp(s,v) from above. Upon termination, for every vertex v belongs to V, v.d computes by BFS satisfies v.d >= Sp(s,v)

Using induction: inductive hypothesis will be v.d >= Sp(s,v)

Base case: s.d = 0 = Sp(s,s), v.d = infinity >= Sp(s,v) --- hypothesis holds //initialization of BFS

Inductive step: Line 15 of the algorithm

v.d = u.d + 1

>= Sp(s, u) + 1 -----assuming the hypothesis is true for u.d

>= Sp(s,v) ----- using Lemma:1

The hypothesis holds!!

Lemma:3 Upon termination, for every vertex v belongs to V, v.d computes by BFS satisfies v.d = Sp(s,v)