Modeling the Depressurization Rate of a Space Station

Abstract

Our study considered the depressurization of a cylindrical space module in low Earth orbit. Utilizing Bernoulli's Equation as well as the Ideal Gas Law, we focus on the flow of pressurized air from the module into the vacuum of space. We construct a model that accounts for the variety in diameter and area of a puncture from objects of varying size in low Earth orbit. Within our first and second iteration of our model, we derive relationships between the changing density and pressure of air inside the module as the gas is evacuated into space. Additionally, we utilize the change in mass flow rate and the affect on the new pressure and density of the air. Our analysis concludes that for a spacecraft with a length of 50 meters, radius of 2 meters, with a puncture hole with a radius of 0.005 meters, it will take approximately 391 minutes or $23,460$ seconds for .7 of an atmosphere to be ejected from the spacecraft.

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1 Introduction

1.1 Background

Space stations, such as the International Space Station (ISS), play an important role in scientists' research of Earth and space. They allow experiments in low gravity while still being close enough to Earth to be easily accessible by rockets. Within the past few years, companies have also started offering space tourism to the public, and it's likely private vacation space stations will soon be operational.

Since space stations have the resources to host people for long periods of time, it's especially important to consider different risk factors and how they might impact life in space. Space stations like the ISS are pressurized with air in order to replicate the atmospheric conditions of Earth and to allow astronauts to breathe without having to wear an oxygen tank and helmet. Maintaining the air pressure inside the spacecraft is crucial, and an incident that compromises the barrier between the spacecraft's pressurized interior and the space's outside vacuum would be devastating. In an emergency such as this, it would be of the utmost importance to know how long until the station would be inhabitable and if there would be enough air to last the crew until they could be safely evacuated. Given the Earth's atmospheric composition (i.e. 75% Nitrogen and 25% Oxygen) and our final pressure .3 atm, we can calculate the partial pressure of oxygen to be .075 atm or 57 mm Hg. This is just below the threshold required to sustain human life (Hyun, 2024).

This paper analyzes the implications of different-sized holes in a space station's ends and determines how long it would take to deplete 70% of the air pressure. Our model could be used in an emergency situation to predict the depressurization time.

1.2 Problem Restatement

Consider a cylindrical-shaped space station with an interior length of 50 meters and a radius of 2 meters in orbit around the Earth. The air inside the space station has an initial temperature of 20° Celsius (293.15 Kelvin) and an initial pressure of 1 atmosphere ($101.3kPa$). A micrometeorite makes a hole in the center of one end of the space station, which has a radius of half a centimeter ($0.005m$). This causes the space station to begin depressurizing. We have created the following models to show how much time it would take for the air pressure inside the space station to reach 0.3 atmospheres, as well as how different-sized holes would change that time.

2 Model & Analysis

2.1 Notation

To begin our analysis, we cataloged our given values stated in the problem, as well as derived our own values from various equations. The first figure provides the variables we used throughout our model to represent our given values, as shown.

Pressure, P_i, (kPa)	**Temperature,** T, (K)	**Length,** L, (m)	**Radius,** R_{ss}, (m)	**Radius,** r_h, (m)
101.3	293.15	50	2	0.005

Figure 1: Given Values

Variable Calculated	Equation	Value
Area, A_h, (m^2)	$A_h = \pi r^2$	$2.5 \cdot 10^{-5}\pi$
Volume, V_{ss}, (m^3)	$V_{ss} = \pi R^2$	200π
Average Molecular Mass, μ, (kg/mol)	$\mu = .75M_{mol} + .25O_2$	.029
Mass of Air in Station, M_{air}, (kg)	$M_{air} = \rho_{ssi}V_{ss}$	757.31
Air Density, ρ_i, (kg/m^3)	$\rho_i = \frac{P\mu}{RT}$	1.205
Final Mass of Air, M_f, (kg)	$P \sim M_f$, $.3M \sim .3P$	227.19

Figure 2: Calculated Values

Assuming the air inside of the space station is similar to the Earth's atmospheric composition (i.e. 75% Nitrogen and 25% Oxygen), we used the molecular masses of each to calculate our average molecular mass (.75·molecular mass of nitrogen + .25·molecular mass of oxygen).

Since π is irrational, we chose to write our area and volume in terms of π, to ensure the minimum amount of rounding error for each calculation that was manually done.

2.2 Derivation of Equations

We begin with $PV = \frac{MRT}{\mu}$, which is a variation of the Ideal Gas Law with mass instead of mols (Australian Space Academy, 2011). We can use Bernoulli's Law to write down a conservation equation,

$$p_i + \rho v_i^2 = p_e + \frac{\rho v^2 e}{2}$$

The second term is zero due to lack of interior velocity, and the third term is 0 due to the vacuum assumption. This nets us:

$$p_i = \frac{\rho v^2 e}{2}$$

We model the air passing through the hole in one second as the area of the hole times the escape velocity times the density, $\frac{dm}{dt} = \rho A_h V_e$.(Benson, 2021) It's useful to do unit analysis to examine why this works, $\frac{M}{s} = \frac{kg}{m^3} m^2 \frac{m}{s}$. It can be seen this does net the correct units. We rearrange the simplified Bernoulli's Law for v_e, where:

$$v_e = \sqrt{\frac{2P}{\rho}}$$

We plug this into the other equation to net.

$$\frac{dm}{dt} = \rho A_h \sqrt{\frac{2P}{\rho}}$$

We further simplify this by bringing the Rho inside, netting our final equation for the Mass Flow Rate,

$$\frac{dm}{dt} = A_h \sqrt{2P\rho} \tag{1}$$

This nets our final useful equation for Mass Flow rate.
Using $PV = \frac{MRT}{\mu}$, we solve for pressure,

$$P = \frac{MRT}{\mu V}$$

We know that $\frac{M}{V} = \rho$, substituting this in, our final equation for pressure becomes

$$P = \frac{\rho RT}{\mu} \tag{2}$$

We calculate our density using the basic definition.

$$\rho = \frac{M}{V} \tag{3}$$

2.3 First Iteration

We started our first iteration of the model with the differential equation for mass flow rate that was previously derived:

$$\frac{dM}{dt} = A_h\sqrt{2P\rho}$$

The separation of variables technique was used to integrate the equation, and limits were put on the integrals in order to define the possible range we were working with.

$$\int_{M_i}^{M_f} dM = \int_0^t A_h\sqrt{2P\rho\, dt}$$

We added an absolute value around the difference of masses since our final mass is less than our initial mass, and that yields a negative result. Time cannot be a negative value, so the absolute value ensures that we get a value for time that is possible.

$$|M_f - M_i| = A_h\sqrt{2P\rho} \cdot t$$

The equation was then solved for t.

$$t = \frac{|M_f - M_i|}{A_h\sqrt{2P\rho}}$$

Finally, we plugged in the expected values for final pressure and density, according to equations 2 and 3 above, to find the amount of time it would take the space station to get down to 0.3 atm. Our result was 325 minutes.

2.3.1 Assumptions and Justifications

In our first model, we assumed laminar flow of air out of the hole. According to LibreTexts (2022), for incompressible fluids the flow rate can be modeled as constant. We did this because it would greatly simplify the model, while giving us a good ballpark estimate of the amount of time it would take for it to get down to 0.3 atm.

According to Biddle (2018), it is generally acceptable to model a fluid as incompressible if it is below Mach 0.3. When we calculated the initial escape velocity of the air through the hole, it was far above Mach 0.3, which told us that our model would be inaccurate. This prompted us to create a second version of the model which took this into account.

We decided that we did not need to factor in temperature as a variable that would affect the change in pressure because our analysis showed that T stayed constant over the desired interval. When $PV = MRT/\mu$ is solved for T, it becomes clear that the interior temperature of the space station depends on the ratio of its interior air pressure over the air density. Since this ratio changes at a constant rate, the effect of temperature change on the time it takes to depressurize is negligible.

2.4 Second Iteration

Given our initials, we utilize iteration to calculate. Figure 3 below will provide a basic example, including the numbering to indicate the order.

Mass Flow Rate (kg/s)	**Air Density** (kg/m^3)	**Pressure** (Pa)	**Mass** (kg)	**Mass Loss** ($kg/5s$)
1. $A_H\sqrt{2P_i\rho_i}$	ρ_i	P_i	m_i	2. $A_H\sqrt{2P_i\rho_i}\cdot dt$
6. $A_H\sqrt{2P_t\rho_t}$	4/5. $m_i/200\pi$	4/5. $P_i\cdot(M/M_{t-dt})$	3. M_{t-dt}	7. $A_H\sqrt{2P_t\rho_t}\cdot dt$

Figure 3: An example of the iterative calculations used with numbering to indicate order.

The order of steps calculated in Excel was as follows:

1. *Calculate mass flow rate using Eq. 1*
2. *Mass flow rate * dt = mass lost*
3. *Subtract mass lost from previous mass*
4. *Calculate new pressure and density from Eq 2 & 3*
5. *Repeat process from step 1*

Using this approach, Excel returned values every 5 seconds. The time it took for the space station to reach 0.3 atm was found to be $23,480$ seconds, or 391 minutes.

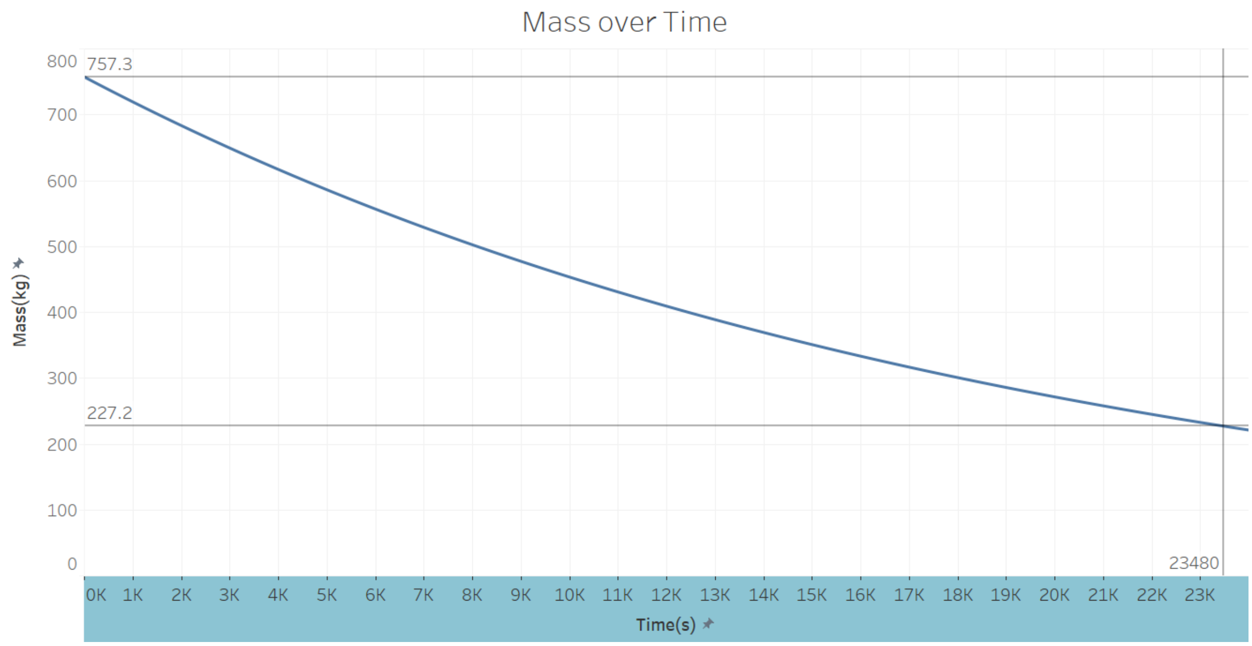

Figure 4: The mass at given time

Figure 4 shows the curve obtained, with lines highlighting the starting and final mass, and time.

2.4.1 Additional Assumptions

In addition to the assumption that T is constant from the first iteration of our model, this iteration assumes that the flow rate changes negligibly between each 5 second interval. This is a reasonable assumption, given that after the initial few minutes the flow rate will significantly slow down.

2.5 Exploration of Varying Hole Diameters

Intuitively, we should expect that the size of the hole will affect the rate of mass flow, and that's exactly whats seen in Eq 1. We expect that if the hole is doubled in size, the time needed to lose the mass will halve. The model was used again, this time with a varying area to calculate values for different area sizes. The area sizes must all be close in magnitude to the original size. If we expanded the hole too much, our model would fall apart likely because the hole would cause larger damage, along with the fact a large enough hole will take little time to depressurize.

Area(m^2)	Time(m)
1.00E-02	3
5.00E-03	6.0833
1.00E-03	30.6667
5.00E-04	61.41667
2.50E-04	122.9167
1.50E-04	204.91
1.00E-04	307.3333
8.00E-05	384.167
7.85E-05	391.3333
7.50E-05	405.23
7.00E-05	411.08
6.50E-05	417.8333
5.00E-05	446.21
3.00E-05	528.18

Figure 5: The affect of Area Size on time

Figure 5 below displays the results for given values. The inverse relationship can be seen.

3 Strengths & Weaknesses

3.1 Strengths

Our model is relatively simple to calculate while considering the most important factors affecting the depressurization time. It is also on the same order of magnitude as our predictions, so we can be certain that the values we obtain from our model are reasonably accurate.

Since our model is coded in Excel, calculations can be performed accurately and efficiently for holes of any size at any time interval. This would enable a space crew in an emergency situation to quickly calculate the approximate time they have left before the space station depressurizes. The potential for use in real-life scenarios is an important attribute of a model that is often overlooked.

3.2 Weaknesses

Our model does not consider certain factors, such as the velocity of the space station in its orbit around Earth. Depending on which end of the spacecraft the hole was in, this could cause some resistance and slow down the speed of the air leaving the space station.

Our model cannot accurately predict incredibly small holes or incredibly large ones. This can be shown using our area calculations, and we can see that our model predicts a linear inverse relationship. If we theoretically model this inverse linear relationship using the equation $t = \frac{k}{a}$. Using our known t and A values based on the solution to the main model, k was calculated and then utilized to plot a theoretical function predicting the values for a given Area. Figure 6 plots both the theoretical and model curves on a logarithmic scale, and it can be shown that while it lines up well for our reasonable values, when going extremely small it becomes less predictable.

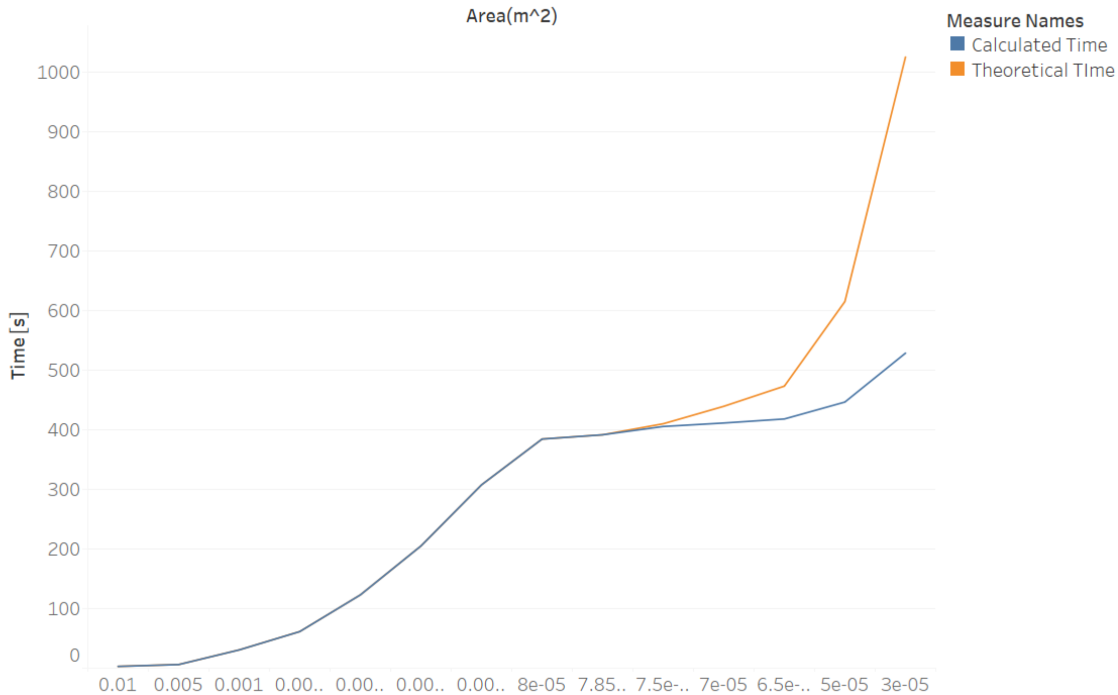

Figure 6: Theoretical and Computational Comparison

4 Conclusion

Using the last iteration of our model, we calculated the time for the space station to go from 1 atm to 0.3 atm with a hole diameter of 1 cm to be 391 minutes, or approximately 6.5 hours. This provides insight into analyzing the threat of large micrometeorites on human habitation modules. Based on the impact hole, the model can generate the amount of time that the module will remain over a pressure, likely the pressure required to survive, and thus analyze what sizes are truly dangerous. It is a severe limitation of our module that we cannot scale down. Since things smaller than a centimeter can puncture a hole into a spacecraft module, this could be explored as a module improvement.

References

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