# 1 Introduction

This is an introductory section.

## 1.1 A part of the intro

more stuff

# 2 Genral Spin Formalism

## 2.1 Formalism

First we need to work out a formalism that will apply to the general spin case. We want to represent the spin state of the particles by an object that looks like a generalization of the Dirac bispinor.

It is easiest to start with the Dirac basis, where the upper and lower components of the bispinor are objects of opposite helicity, each transforming as an object of spin 1/2.

To that end define an object

$$\Psi = \begin{pmatrix} \xi \\ \eta \end{pmatrix}$$

that we wish to have the appropriate properties. Each component should transform as a particle of spin s, but with opposite helicity. Under reflection the upper and lower components transform into each other.

The irreducible representations of the proper Lorentz group are spinors which are seperately symmetric in dotted and undotted indices. The spin of the particle will be half the total number of indices. So if $\xi$ is an object with $p$ undotted and $q$ dotted indices

$$\xi = \{\xi^{\alpha_1 \ldots \alpha_p}_{\dot{\beta}_1 \ldots \dot{\beta}_q}\}$$

Then this is a represenation of a particle of spin $s = (p+q)/2$.

We have some free choice in how to partition the dotted/undotted indices, and we cannot choose exactly the same scheme for all spin as long as both types of indices are present. However, we can make a consistent for integral and half-integral spin. For integral spin we can say $p = q = s$, while for the half-integral case we'll choose $p = s + \frac{1}{2}$, $q = s - \frac{1}{2}$.

We want the $\xi$ and $\eta$ to transform as objects of opposite helicity. Under reflection they will transform into each other. So

$$\eta = \{\eta^{\beta_1 \ldots \beta_q}_{\dot{\alpha}_1 \ldots \dot{\alpha}_p}\}$$

In the rest frame of the particle, they will have clearly defined and identical properties under rotation. The rest frame spinors are equivalent to rank $2s$ nonrelativistic spinors. So the bispinor in the rest frame looks like

$$\Psi = \begin{pmatrix} \xi_0 \\ \xi_0 \end{pmatrix}$$

where

$$\xi_0 = \{(\xi_0)_{\alpha_1 \ldots \alpha_p \beta_1 \ldots \beta_q}\}$$

and all indices are symmetric.

We can obtain the spinors in an arbitrary frame by boosting from the rest frame. The upper and lower components we have defined to have opposite helicity, and so will act in opposite ways under boost:

$$\xi = \exp\left(\frac{\mathbf{\Sigma} \cdot \phi}{2}\right)\xi_0, \qquad \eta = \exp\left(-\frac{\mathbf{\Sigma} \cdot \phi}{2}\right)\xi_0$$

What form should the operator $\mathbf{\Sigma}$ have? Under an infitesimal boost by a rapidity $\phi$, a spinor with a single undotted index is transformed as

$$\xi_\alpha \to \xi'_\alpha = \left(\delta_{\alpha\beta} + \frac{\boldsymbol{\phi} \cdot \boldsymbol{\sigma}_{\alpha\beta}}{2}\right)\xi_\beta$$

while one with a dotted index will transform as

$$\xi_{\dot{\alpha}} \to \xi'_{\dot{\alpha}} = \left(\delta_{\dot{\alpha}\dot{\beta}} - \frac{\boldsymbol{\phi} \cdot \boldsymbol{\sigma}_{\dot{\alpha}\dot{\beta}}}{2}\right)\xi_{\dot{\beta}}$$

The infiteismal transformation of a higher spin object would then be

$$\xi \to \xi' = \left(1 + \frac{\boldsymbol{\phi} \cdot \boldsymbol{\sigma}_a}{2}\right) \xi$$

$\boldsymbol{\Sigma}$ is an operator which acts seperately on each spinor index. How does each index of the spinor transform under a boost? It must

It can be written as

$$\boldsymbol{\Sigma} = \sum_{a=0}^{p} \boldsymbol{\sigma}_a - \sum_{a=p+1}^{p+q} \boldsymbol{\sigma}_a$$

We can think of it as acting seperately on two subsets on indices: the first $p$ and the last $q$.