Chapter 1

Introduction

A way to calculate the binding corrections to the spin g-factor of a charged particle of arbitrary spin is desired.

The g factor in this sense can be defined by the energy separation of two particles which differ only by spin orientation. (For particles in a bound state, a g-factor can be defined with respect to the angular momentum of the particle as well.) A free electron, with its spin oriented along the same axis as a weak magnetic field $B_z\hat{z}$, has energy

$$E = -\boldsymbol{\mu} \cdot \mathbf{B} = -g\frac{e}{2m}s_z B_z \tag{1.0.1}$$

The difference between such an electron and one with its spin flipped is then

$$\delta E = -g\frac{e}{m}s_z B_z = -g\mu_B B \tag{1.0.2}$$

The general definition of the g-factor then follows – the energy difference between two particles with spin projection parallel and anti-parallel to a small constant magnetic field will be proportional to $-\mu_B B$, and the coefficient of this difference defines the g-factor.

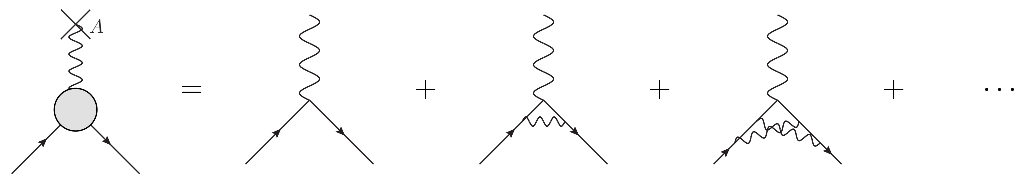
1.1 The free g-factor

The most well known g-factor is that of the free electron. It may be calculated from the Lagrangian of QED.

$$\mathcal{L} = \bar{\Psi}\left(i\partial\cdot\gamma - eA\cdot\gamma - m\right)\Psi \tag{1.1.1}$$

The Dirac equation itself predicts a magnitude of $g = 2$.

When calculated in the framework of QED, there will be corrections to this "natural" value of 2. The g-factor being determined by the behavior of the electron in a infinitesimal magnetic field, the relevant QED calculation is the process



When only the fundamental vertex is calculated, the value of 2 is obtained. The additional loop diagrams such as illustrated above will introduce corrections to this quantity. For an electron, these corrections will be quite small compared to the natural value of 2.

This correction to the g-factor is related to the anomalous magnetic moment $a_e = (g-2)/2$. Its calculation will be a series in the coupling constant α. (At very high orders additional interactions will enter into the calculation, but because of the light mass of the electron compared to other particles they are highly suppressed.) The well known first order result is

$$a_e = \frac{\alpha}{2\pi} + \mathcal{O}(\frac{\alpha^2}{4\pi^2}) \tag{1.1.2}$$

The full value has been calculated to extremely high accuracy, such that the uncertainty in g is

$$\frac{\Delta g}{g} \sim 10^{-13} \tag{1.1.3}$$

It has also been measured with a similar accuracy. A couple of the most recent measurements for the electron are

$$g = 2.0 - - - - - - - - \qquad \delta = - - \text{(Hanneke, Fogwell, Gavielese)} \tag{1.1.4}$$

Because both the experimental and theoretical values are known quite precisely, these measurements of the free electron's g-factor provide the best determination of the constant α.

For the electron, the leading order term came from the fundamental vertex, and the anomalous contribution coming from radiative corrections are relatively small. In the more general case this is not true. The proton has a g-factor of ~ 5.6, where because of its composite nature the anomalous part is quite large.

It is still necessarily the case that g is determined by the same process as the electron. And this process can be parametrized by two form factors, whose value wraps up the detailed information of the high energy physics or the small scale structure of the particle.

1.1.1 Bound g-factors

Even when the free g factor of a particle is known, there are corrections when the particle is placed into a bound state. Consider again the simplee case of the electron, but this time it sits in a hydrogenic bound state. It is immediately clear that the situation is much more complicated than the free case. There are several additional scales to the problem, and so the expression for g_b is no longer a series only in α.

- There are recoil corrections that occur when separating the internal degrees of freedom from the external motion of the whole bound system. The related parameter is the mass ratio m/M
- The relativistic motion of the particle contributes binding corrections. Because the velocity of a hydrogenic bound system is $v \sim Z\alpha$, corrections of this nature will be an expansion over $(Z\alpha)^2$.
- There can be effects due to the finite size of the nucleus, although these are not considered in this work.

All of these are in addition to the radiative corrections discussed earlier; the full series will contain mixtures of all types of corrections at higher orders.

The binding corrections for the electron with $g = 2$ were first calculated by Breit (1928). This can be done simply by taking the matrix element of the fundamental vertex (that responsible for the value $g = 2$) between the bound state wave functions. This calculation of $\langle e\gamma \cdot A \rangle$ gives

$$g_b = 2\left(1 - \frac{(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{12}\right) \tag{1.1.5}$$

1.1.2 Experimental relevance

There are experiments involving hydrogenic $^{12}C^{5+}$ and $^{16}O^{7+}$ that require a precise theoretical determination of the bound g-factor.

A hydrogenic system is placed in a weak magnetic field B. The spin flip frequency ω_L is measured, corresponding to transitions between Zeeman levels. It is through this quantity that dependence on g_b enters:

$$\omega_L = g_b \frac{e}{2m_e} B \tag{1.1.6}$$

The cyclotron frequency ω_C is

$$\omega_C = (Z-1)\frac{eB}{M} \tag{1.1.7}$$

So the ratio of these frequencies gives

$$\frac{\omega_L}{\omega_C} = \frac{f_L}{f_C} = g_b \frac{e}{2q}\frac{M}{m_e} \tag{1.1.8}$$

This no longer depends on B, but does depend upon g_b and the ratio of the electron mass ratio m_e/M. If g_b is known with sufficient precision, measuring this ratio is the best determination of m_e/M.

The best current measurements are, for $^{12}C^{5+}$ $(Z = 6)$

$$\frac{f_L}{f_C} = \tag{1.1.9}$$

And for $^{16}O^{7+}$ $(Z = 8)$

$$\frac{f_L}{f_C} = \tag{1.1.10}$$

With this generation of experiments the uncertainty is $\delta \sim 10^{-10}$, but in the future is expected to increase to $\delta \sim 10^{-12}$. To fully exploit such sensitivity requires g_b to be known with sufficient precision.

Here there is an issue connected with the overall spin magnitude of the nuclei. In both $^{16}O^{7+}$ and $^{12}C^{5+}$, the nucleon spins are arranged such that the overall spin is 0. It is entirely plausible that the binding corrections to the g-factor could depend on the spin of the particles. So some approach to this problem that works for nuclei of arbitrary spin would be useful.

There are already two extant approaches to this problem. An method based on the Bargmann-Michel-Telegdi (BMT) equation was utilised by Eides and Grotch. Leading relativistic and recoil corrections are calculated, with the result that they are in fact independent of spin. This follows from the form of the BMT equation, which itself carries no dependence on spin. The leading order correction of to g for a hydrogenic atom was found to be

$$\Delta g_{EG} = (1+Z)(Z\alpha)^2 \left(\frac{m_e}{M}\right)^2 \tag{1.1.11}$$

In (Faustov FIXME) a quasipotential framework was used to calculate g_b. Here the result was found to depend upon the spin of the particle. For a spin-half particle there is agreement between this and the above approach, but clearly in general there is disagreement. For a hydrogenic atom with a spin-0 nucleus, the result was

$$\Delta g_{FM} = \frac{Z}{3}(Z\alpha)^2 \left(\frac{m_e}{M}\right)^2 \tag{1.1.12}$$

To give a concrete idea of the discrepancy produced by the methods, here are the numerical results pertinent to the previously mentioned experiments:

	$^{12}C^{5+}$	$^{16}O^{7+}$
Δg_{EG}	0.28×10^{-10}	0.36×10^{-10}
Δg_{GM}	0.80×10^{-11}	0.11×10^{-10}
Discrepancy	0.2×10^{-10}	0.25×10^{-10}

Such discrepancies are relevant to the sorts of experiments previously discussed, so a resolution to the situation is important.

1.2 Attack

To resolve this disagreement, the following approach will be taken. The starting point will be a relativistic description of the electromagnetic current for particles of arbitrary spin. Just as for spin-half particles, all the details of the particles structure may be captured by form factors. This follows the path laid out by Khriplovich (XXXX)

Fully relativistic theories being cumbersome to apply to bound state problems, an effective field theory approach is adopted. A nonrelativistic Lagrangian for particles of arbitrary spin is constructed. Then by considering the same physical process in the relativistic and nonrelativistic theory, the particular parameters of the NRQED Lagrangian will be determined.

With the NRQED Lagrangian determined to the order necessary, it may then be used to attack the bound state nature of the problem. In a manner analogous to the calculation of the Breit potential (REF) an interaction Hamiltonian for the system can be developed. Finally, this may be used to calculate g_b at the necessary order.