Exam 3

Dec 3, 2014

Instructor: Timothy Martin

Student Information

Name: ____________________

Student ID or email: ____________________

Circle your section:

013	Timothy Martin	11:00 am
014	Xinshuai Yan	11:00 am
015	Xinshuai Yan	8:00 am
016	Filmon Misgina	12:00 pm

Instructions

Answer the multiple choice questions on the test.

For the long form problems, make sure to show your work. A correct answer with no work will receive no points.

Electronic devices are forbidden during this test. (Including calculators and cell phones!)

Section	Score
M.C	
9	
10	
11	
Total	

Multiple choice [4 pts each]

1. If released to roll down the same slope, which of the following shapes will reach the bottom first? (Assume they roll around their axis of symmetry.)
   A. A sphere
   B. A hoop
   C. A disc
   D. Not enough information is given

> **Solution:** A. (The sphere has the lowest moment of inertia, and it doesn't matter what mass or radius they are.)

2. A block on a spring is oscillating back and forth. If it makes 4 oscillations in $2\,\mathrm{s}$, what is the angular frequency?
   A. $2\,\mathrm{rad/s}$
   B. $4\pi\,\mathrm{rad/s}$
   C. $4\,\mathrm{rad/s}$
   D. $8\pi\,\mathrm{rad/s}$

> **Solution:** B. 4 oscillations is $8\pi\,\mathrm{rad}$, so that's $4\pi\,\mathrm{rad/s}$

3. Two cats sit on either side of a massless seesaw. $4\,\mathrm{m}$ to the left of the fulcrum is a large, fat cat weighing $100\,\mathrm{N}$; $2\,\mathrm{m}$ to the right side is an *even fatter* cat cat weighing $150\,\mathrm{N}$. What's the net torque about the fulcrum when the seesaw is level with the ground?
   A. $50\,\mathrm{N\cdot m}$
   B. $100\,\mathrm{N\cdot m}$
   C. $250\,\mathrm{N\cdot m}$
   D. $700\,\mathrm{N\cdot m}$

> **Solution:** B. $400\,\mathrm{N/m}$ counterclockwise $300\,\mathrm{N\cdot m}$ clockwise, so a total of $100\,\mathrm{N\cdot m}$

4. Two planets of the same mass orbit the same star in perfectly circular orbits – the outer planet's orbital radius is exactly twice the inner planet's. How does the outer planet's orbital period compare to the inner planet's?
   A. It is exactly twice as long
   B. It is more than twice as long
   C. It is longer, but not twice as long
   D. It's the same

> **Solution:** A. From Kepler's 3rd Law, T^2 is proportional to R^3, so T is proportional to $R^{3/2}$. That means that when R doubles, T more than doubles.

5. A flywheel wheel is accelerated from rest at $4\,\mathrm{rad/s^2}$. How fast is it spinning after $3\,\mathrm{s}$?
   A. $3\,\mathrm{rad/s}$
   B. $4\,\mathrm{rad/s}$
   C. $12\,\mathrm{rad/s}$
   D. $18\,\mathrm{rad/s}$

> **Solution:** C. $\omega = \omega_0 + \alpha t = 4 * 3 = 12$

6. What is the gravitational field directly between a $3m$ mass and a $2m$ mass separated by a total distance d?
   A. $\frac{Gm}{d^2}$
   B. $\frac{2Gm}{d^2}$
   C. $\frac{4Gm}{d^2}$
   D. $\frac{5Gm}{d^2}$

> **Solution:** C. The point is a distance $r = d/2$ from each mass, and $g = G\frac{3m-2m}{r}$

7. Imagine a uniform, solid cube. It has the greatest moment of inertia around
   A. An axis passing through the center of mass and the middle of two faces
   B. An axis passing through the center of mass and two corners of the cube
   C. An axis touching, and parallel to, one of the edges
   D. An axis a great distance away from the cube

> **Solution:** D. The further the rotational axis is away from the center of mass, the large the moment of inertia.

8. Consider a lever, pivoted about one end. If I apply a force to the lever, the resultant angular acceleration about the pivot is greatest when the force is
   A. Farthest from the pivot point, parallel to the lever arm
   B. Right at the pivot point, parallel to the lever arm
   C. Farthest from the pivot point, perpendicular to the lever arm
   D. Right at the pivot point, perpendicular to the lever arm

> **Solution:** C. α is proportional to torque, which is proportional to the distance from the pivot and the sine of the angle.

9. A rotating rod of length L and mass M is spinning about its center of mass with speed ω_0. Gunter (mass $m = \frac{1}{6}M$) initially stands right above the center of the rod. Find the angular speed of the rod after Gunter has waddled to one end. (Assume that we can treat the penguin as a point mass, and that this is an isolated system.) [16 pts]

Solution: When Gunter is right at the center of mass, the initial moment of inertia is $I_0 = I_{rod} = \frac{1}{12}ML^2$. By the time he's at the end, the moment of inertia is $I_{rod} + \frac{M}{6}L^2 = \frac{3}{24}ML^2/$
Since angular momentum is conserved during this process,

$$I_0\omega_0 = I\omega$$

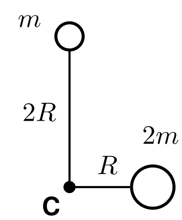
$$\frac{1}{12}ML^2\omega_0 = \frac{3}{24}ML^2\omega_f$$

$$\omega_f = \frac{2}{3}\omega_0$$

10. The illustrated configuration of masses pivots about the center point **C**.
(a) [12 pts] What is the moment of inertia of the system about the pivot? (Assume the two masses are point-like, connected to the pivot by rigid, massless rods, and that the pivot itself is also massless.)

Solution: They're both point-like masses, so $I = (2M)R^2 + M(2R)^2 = 6MR^2$

(b) [12 pts] If it is released from rest, what will the rotational speed ω of the system be when the larger mass passes directly under the pivot point?



Solution: As it falls, it loses potential energy and gains kinetic. The small mass falls a horizontal distance of $2R$, the large one a distance of R, so the amount of potential energy lost is $4MgR$.
The kinetic energy of a rotating object is $\frac{1}{2}I\omega^2$. So

$$\frac{1}{2}I\omega^2 = 3MR^2\omega^2 = 4MgR$$

$$\omega = \sqrt{\frac{4g}{3R}}$$

11. A satellite of mass m initially orbits the Earth (mass M_E) in a perfectly circular orbit of radius R. Astronuats manage to boost it into a higher orbit, (also circular) with radius $2R$.

(a) [12 pts] What is the change in gravitational potential energy?

Solution: Gravitational potential energy is $U = -\frac{Gm_1m_2}{r^2}$. So the change in potential energy is

$$\Delta U = \left(-\frac{GM_Em}{2R}\right) - \left(-\frac{GM_Em}{R}\right) = \frac{GM_Em}{2R}$$

(b) [12 pts] What is the change in kinetic energy?

Solution: Remember that the kinetic energy of a circular orbit is determined by its radius: $K = \frac{Gm_1m_2}{2r}$.
One way to get this is to remember that $E = K + U$, which means $K = E - U$. The total mechanical energy of an orbit is $-\frac{Gm_1m_2}{2a}$, and for a circular orbit $a = R$, so

$$K = \left(\frac{Gm_1m_2}{2r}\right) - \left(-\frac{Gm_1m_2}{r}\right) = \frac{Gm_1m_2}{2r}$$

Using that expression, the change in kinetic energy is:

$$\Delta K = \left(\frac{GM_Em}{4R}\right) - \left(\frac{GM_Em}{2R}\right) = -\frac{GM_Em}{4R}$$

(You could have also derived the equation for K by equating the centripetal force with the force of gravity: $m_2\frac{v^2}{r} = G\frac{m_1m_2}{r^2}$, and solving for $\frac{1}{2}mv^2$)

(c) [4 pts] Is the total work done by outside forces positive or negative?

Solution: Since the total energy increased, the net work by outside forces must be positive.

Formulas

Definition of v, a

$$\mathbf{v} = \frac{d\mathbf{x}}{dt}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

1D Kinematics

$$x(t) = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

$$v(t) = v_{0x} + a_x t$$

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

Newton's 2nd Law

$$\Sigma\mathbf{F} = m\mathbf{a}$$

Circular Motion

$$a_c = \frac{v_T^2}{r}$$

Circular kinematics

$$\theta = \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$$

Friction force

$$f_k = \mu_k n$$

$$f_s \le \mu_s n$$

Total kinetic energy

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Mechanical work done by a force

$$W_F = \int \mathbf{F}\cdot\mathbf{ds}$$

Spring potential energy

$$U_s = \tfrac{1}{2}kx^2$$

Gravitational potential energy (near surface)

$$U_g = mgy$$

Force and potential energy

$$F_x = -\frac{d}{dx}U$$

Hooke's Lqw

$$F_s = -kx$$

Power

$$P = \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v}$$

Momentum

$$\mathbf{p} = m\mathbf{v}$$

Impulse

$$\mathbf{I} = \Delta\mathbf{p} = \mathbf{F}_{avg}\Delta t$$

Two-body elastic head-on collision

$$v_1 - v_2 = -(v_1' - v_2')$$

Center of Mass

$$\mathbf{r}_{CM} = \frac{\Sigma m_i \mathbf{r}_i}{\Sigma m_i}$$

Moment of inertia (point)

$$I = mr^2$$

Rigid Object	Moment of Inertia
Solid disc	$\frac{1}{2}MR^2$
Hoop	MR^2
Solid sphere	$\frac{2}{5}MR^2$
Spherical shell	$\frac{2}{3}MR^2$
Rod about CM	$\frac{1}{12}ML^2$

Parallel axis theorem

$$I = I_{CM} + MD^2$$

Torque

$$\tau = rF\sin\phi_{rF}$$

Effect of net torque

$$\tau_{net} = \frac{dL}{dt} = I\alpha$$

Angular Momentum (point particle)

$$\mathbf{L} = \mathbf{r}\times\mathbf{p}$$

Angular Momentum (rigid, rotating object)

$$L = I\omega$$

Gravitational Force

$$F_g = G\frac{m_1 m_2}{r^2}$$

Gravitational Potential Energy

$$U_g = -G\frac{m_1 m_2}{r}$$

Kepler's Third Law

$$T^2 = ka^3$$

Total orbital energy about mass M

$$E_{mech} = -\frac{GMm}{2a}$$