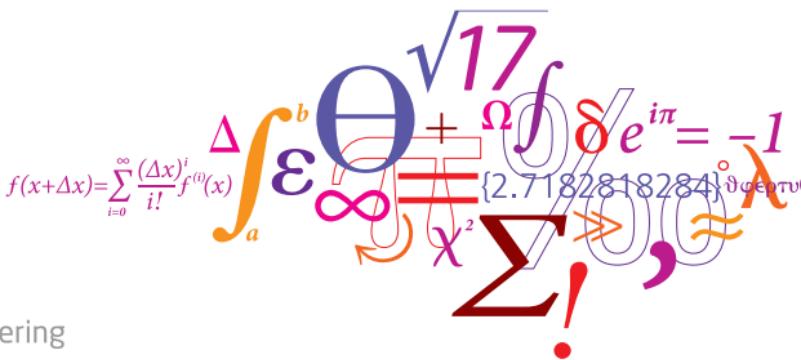


Topic Modeling - Latent Dirichlet Allocation

Francisco Pereira

Filipe Rodrigues

with contributions from: Haizheng Zhang, Moshe Ben-Akiva



- Introduction
- In a nutshell
 - Example with traffic incidents
- Topic Modeling
- Topic Modeling
- Concluding remarks

Introduction

- Sometimes some variables of our x are in form of human language
- Often called **unstructured** data
- How to bring it into statistical models?

Introduction

- Our goal is to have

$$\hat{y} = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

where \mathbf{x}_{text} are variables coming from text data, and \mathbf{x}_L are non-textual variables

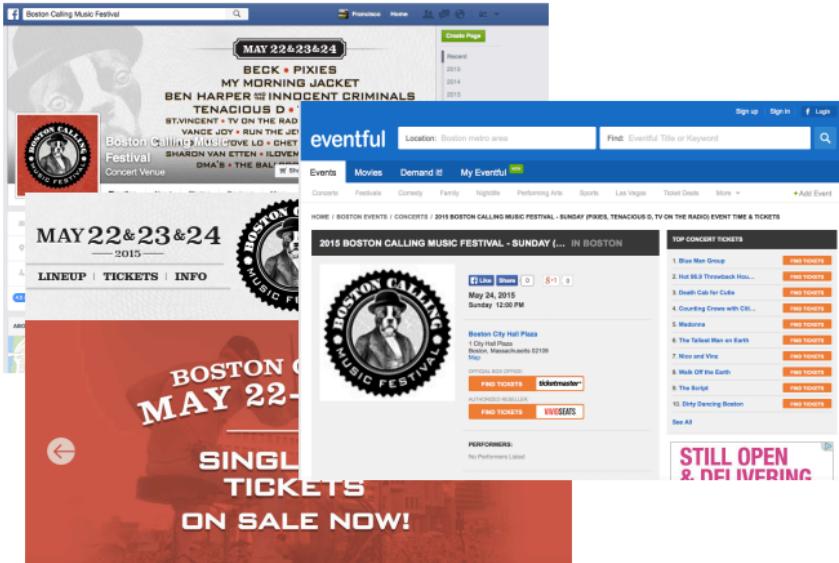
- we need to make \mathbf{x}_{text} usable in any multivariate model discussed in this course

Motivation: News



The screenshot shows the MIT Technology Review homepage. At the top, there's a navigation bar with links for NEWS & ANALYSIS, FEATURES, VIEWS, MULTIMEDIA, DISCUSSIONS, TOPICS, and POPULAR: ARTIFICIAL PHOTOSYNTHESIS. To the right of the navigation is a book cover for "TWELVE TOMORROWS" with a "BUY NOW" button. Below the navigation, there's a thumbnail image of three people in hard hats at a construction site. To the right of the thumbnail, the text reads: "PROGRESS IS EVERYONE'S BUSINESS See how Goldman Sachs is helping the Brooklyn Navy Yard grow. WATCH THE VIDEO". A "Goldman Sachs" logo is also present. Below this section, there's a link "Want to go ad free?". Underneath the main content area, there are social media sharing icons (Twitter, LinkedIn, Facebook, etc.) and a sidebar with the text "COMPUTING NEWS" and "10 COMMENTS". The main article title is "Software Predicts Tomorrow's News by Analyzing Today's and Yesterday's". Below the title, a subtitle reads: "Prototype software can give early warnings of disease or violence outbreaks by spotting clues in news reports."

Motivation: Special events



The image shows a screenshot of the Boston Calling Music Festival website and its integration with Eventful.com.

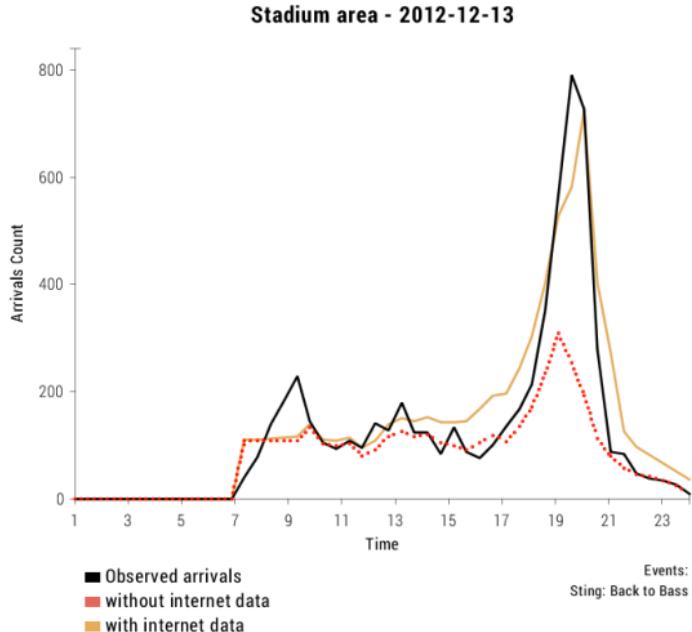
Boston Calling Music Festival Website:

- Header:** MAY 22 & 23 & 24 2015
- Performers:** BECK • PIXIES • MY MORNING JACKET • BEN HARPER & THE INNOCENT CRIMINALS • TENACIOUS D • STEVE VAI • VANCE JOY • RUN THE JEWELS • TOVE LO • CHET SHARON VAN ETEN • ILOVENOMA • THE BALLET
- Event Details:** Boston Calling Music Festival Concert Venue
- Tickets:** LINEUP | TICKETS | INFO
- Image:** A large red banner with a black and white photo of a person and the text "BOSTON CALLING MAY 22-24 SINGLE TICKETS ON SALE NOW!"

Eventful.com Integration:

- Header:** eventful | Location: Boston metro area | Find: Eventful Title or Keyword
- Navigation:** Events | Movies | Demand It! | My Eventful
- Search:** Location: Boston metro area | Find: Eventful Title or Keyword
- Category:** Concerts | Festivals | Comedy | Family | Nightlife | Performing Arts | Sports | Las Vegas | Ticket Deals | More | Add Event
- Content:** 2015 BOSTON CALLING MUSIC FESTIVAL - SUNDAY (PIXIES, TENACIOUS D, TV ON THE RADIO) EVENT TIME & TICKETS
- Details:** May 24, 2015, Sunday, 12:00 PM | City Hall Plaza, 1 City Hall Plaza, Boston, Massachusetts 02110 | Map
- Tickets:** FIND TICKETS | USTICKETMASTER | ADVANCED PURCHASE | FIND TICKETS | VIVISEATS
- Performers:** No Performers Listed
- Callout:** STILL OPEN & DEI INERING
- Top Concert Tickets:**
 1. Bruce Springsteen
 2. Hot 97's Throwback Show...
 3. Death Cab for Cutie
 4. Counting Crows with CB...
 5. Madonna
 6. The Tallest Man on Earth
 7. Nine and Nine
 8. Walk Off the Earth
 9. The Script
 10. Dirty Dancing Boston

Motivation: Special events



Motivation: traffic incidents



Motivation: traffic incidents

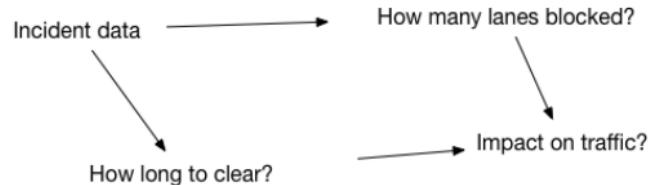
How many lanes blocked?

Impact on traffic?

How long to clear?



Motivation: traffic incidents



Motivation: traffic incidents



Motivation: traffic incidents

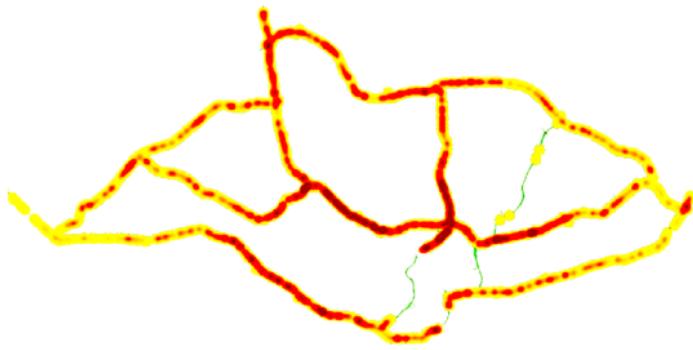


Incident report

Location type: 3
Lane blockage: Lane 1, Shoulder blocked
Down point: 20.32
Congestion status: 0
Queue length: 500m
Start time: 2010-08-20 22:50:01 **Communications sequence**
End time: 2010-08-20 23:31:45
Number of vehicles: 2
2250hrs - TP Joe X spots an accident. car and bike involved.
2255hrs - Passers-by shift the bike to the shoulder.
2300hrs - Ambulance arrives at location. LTM arrives at location.
2309hrs - Ambulance conveys rider to National University Hospital.
2310hrs - TP arrives at location.
2311hrs - Notify by LTM the rider is seriously injured. The accident involves a car and bike.
2331hrs - TP requests RC and LTM to resume patrolling. All other vehicles move off. Shoulder clear.

In a nutshell: an example

- **Context:** Singapore Expressways, 2010-2012



- **Objective:** Predict duration of each incident
- **Data:** Logs of all incidents, 2010-2012

In a nutshell: an example

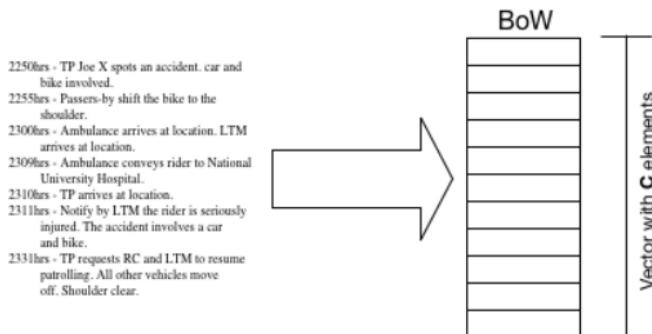
$$y = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

with

$$\mathbf{x}_L = \begin{bmatrix} \mathbf{x}_{expressway_ID} \\ \mathbf{x}_{num_block_lanes} \\ \mathbf{x}_{time_of_day} \\ \mathbf{x}_{day_of_week} \\ \mathbf{x}_{type_of_day} \\ \mathbf{x}_{congestion_status} \\ \mathbf{x}_{num_vehicles} \end{bmatrix}$$
$$\mathbf{x}_{text} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

In a nutshell: an example

- A document has a set of words, from a dictionary with size C
- How to represent it?
 - The bag-of-words (BoW) representation
 - Dimension C
 - Vector of word frequencies



BoW Representation

BoW

- 2250hrs - TP Joe X spots an accident. car and bike involved.
- 2255hrs - Passers-by shift the bike to the shoulder.
- 2300hrs - Ambulance arrives at location. LTM arrives at location.
- 2309hrs - Ambulance conveys rider to National University Hospital.
- 2310hrs - TP arrives at location.
- 2311hrs - Notify by LTM the rider is seriously injured. The accident involves a car and bike.
- 2331hrs - TP requests RC and LTM to resume patrolling. All other vehicles move off. Shoulder clear.

BoW Representation

Document

2250hrs - TP Joe X spots an accident. car and bike involved.
2255hrs - Passers-by shift the bike to the shoulder.
2300hrs - Ambulance arrives at location. LTM arrives at location.
2309hrs - Ambulance conveys rider to National University Hospital.
2310hrs - TP arrives at location.
2311hrs - Notify by LTM the rider is seriously injured. The accident involves a car and bike.
2331hrs - TP requests RC and LTM to resume patrolling. All other vehicles move off. Shoulder clear.

3	TP
1	Joe X
1	spots
2	accident
2	car
3	bike
1	passers-by
1	shoulder
2	ambulance
3	LTM
3	location
2	rider
1	seriously
1	injured
...	...
...	...
...	...
...	...

BoW representation - data preparation

- Replace words by their *stems* (e.g. injury, injured, injuries → injur)
- Remove words from the **stopwords** list (e.g. *and, or, that, it...*)
- The dictionary is a collection of C words (i.e. their stems)
- Calculate frequency of all words (ignore word sequence)

From text to regression

- We could use BoW frequencies directly in a model:

$$y = f(\mathbf{x}_{bow}, \mathbf{x}_L)$$

where \mathbf{x}_{bow} is the BoW vector

- Dimensionality= $C+L$, can be huge ($C \simeq 10^6$ for English)!
- Many words for same concept (e.g. synonyms, acronyms, abbreviations)
- Many words related (e.g. "ambulance", "injury", "hospital")
- Many words irrelevant (e.g. proper names)
- Manual work (e.g. replace synonyms with same word)?
 - VERY time consuming and prone to error!

Topic modeling

- Each document describes a set of K concepts (or **topics**), $K \ll C$
- Each topic is *latent*, i.e. we cannot observe it

In a nutshell: an example

Some examples of topics from our incident reports dataset

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*itm + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner +
0.047*nodamag
topic #11 - 0.101*assist + 0.056*taxi + 0.045*notifi + 0.032*requir + 0.029*came + 0.029*p3 + 0.027*passeng
topic #12 - 0.076*polic + 0.041*otm + 0.033*2nd + 0.033*last + 0.023*div + 0.022*tm1 + 0.016*1st
topic #13 - 0.077*damag + 0.071*call + 0.043*skid + 0.034*tp + 0.029*near + 0.028*self + 0.027*vig
topic #14 - 0.072*left + 0.057*activ + 0.050*tp + 0.045*2ln + 0.040*scdf + 0.035*open + 0.029*spill
topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail +
0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

Topic modeling

- Each document describes a set of K concepts (or **topics**), $K \ll C$
- Each topic is *latent*, i.e. we cannot observe it
- Examples of topic “labels”:
 - Medical assistance
 - Significant damage
 - Low damage
 - Oil spillage
 - ...

In a nutshell: an example

Medical assistance

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*itm + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner +
0.047*nodamag
topic #11 - 0.101*assist + 0.056*taxi + 0.045*notifi + 0.032*requir + 0.029*came + 0.029*p3 + 0.027*passeng
topic #12 - 0.076*polic + 0.041*otm + 0.033*2nd + 0.033*last + 0.023*div + 0.022*tm1 + 0.016*1st
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topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail +
0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

In a nutshell: an example

Significant damage

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*itm + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner +
0.047*nodamag
topic #11 - 0.101*assist + 0.056*taxi + 0.045*notifi + 0.032*requir + 0.029*came + 0.029*p3 + 0.027*passeng
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topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail +
0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

In a nutshell: an example

Low damage

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltn + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner +
0.047*nodamag
topic #11 - 0.101*assist + 0.056*taxi + 0.045*notifi + 0.032*requir + 0.029*came + 0.029*p3 + 0.027*passeng
topic #12 - 0.076*polic + 0.041*otm + 0.033*2nd + 0.033*last + 0.023*div + 0.022*tm1 + 0.016*1st
topic #13 - 0.077*damag + 0.071*call + 0.043*skid + 0.034*tp + 0.029*near + 0.028*self + 0.027*vig
topic #14 - 0.072*left + 0.057*activ + 0.050*tp + 0.045*2ln + 0.040*scdf + 0.035*open + 0.029*spill
topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail +
0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

In a nutshell: an example

Oil spillage

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltm + 0.000*1ln
topic #6 - 0.138*tow + 0.122*noinjur + 0.092*veh + 0.080*present + 0.066*clear + 0.061*owner +
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topic #12 - 0.076*polic + 0.041*otm + 0.033*2nd + 0.033*last + 0.023*div + 0.022*tm1 + 0.016*1st
topic #13 - 0.077*damag + 0.071*call + 0.043*skid + 0.034*tp + 0.029*near + 0.028*self + 0.027*vig
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topic #15 - 0.160*minor + 0.049*exchang + 0.047*particular + 0.032*advis + 0.029*refus + 0.025*detail +
0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

In a nutshell: an example

Some vehicle types (*taxis, truck/lorry*)

```
topic #1 - 0.085*tp + 0.071*convei + 0.070*ab + 0.064*tow + 0.062*hosp + 0.056*rider + 0.048*bike
topic #2 - 0.054*ir + 0.049*confirm + 0.045*congest + 0.034*traffic + 0.027*case + 0.023*soe + 0.018*heavi
topic #3 - 0.001*veh + 0.001*nch + 0.001*spot + 0.000*rc + 0.000*clear + 0.000*ltm + 0.000*1ln
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0.013*involv2
topic #18 - 0.112*report + 0.093*rc + 0.063*tm + 0.041*btw + 0.036*lorri + 0.034*bef + 0.032*actv
```

Topic modeling

- Can we quantify **how much** of each topic exists in a document?
 - E.g. 20% medical assistance, 50% significant damage, 30% oil spillage
 - A document would be a **linear combination** of K topics!
 - We go from C to $K \rightarrow$ **dimensionality reduction**

$$\mathbf{x}_{text} = \begin{bmatrix} \mathbf{x}_{topic_1} \\ \mathbf{x}_{topic_2} \\ \vdots \\ \mathbf{x}_{topic_K} \end{bmatrix}$$

Topic modeling

- Each topic:
 - Itself a vector with dimensionality C (i.e. one entry per word in dictionary)
 - Inferred from an unsupervised process (called *Latent Dirichlet Allocation*, LDA)
 - One can see a topic as a *prototypical document*

In a nutshell: an example

Topic assignment in our example text

2250hrs - TP Joe X spots an accident. car and bike involved.
2255hrs - Passers-by shift the bike to the shoulder.
2300hrs - Ambulance arrives at location. LTM arrives at location.
2309hrs - Ambulance conveys rider to National University Hospital.
2310hrs - TP arrives at location.
2311hrs - Notify by LTM the rider is seriously injured. The accident involves a car and bike.
2331hrs - TP requests RC and LTM to resume patrolling. All other vehicles move off. Shoulder clear.



0.43	topic #1
0.07	topic #3
...	...
0.10	topic #12
0.12	topic #18

Latent Dirichlet Allocation (LDA) algorithm

Generative model without probability distributions yet

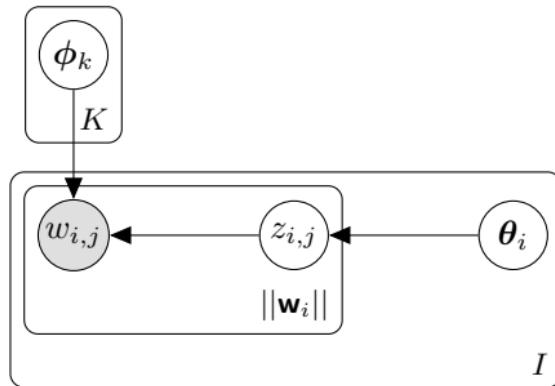
We want to generate a set of I documents ($i = 1, 2, \dots, I$), each one being a sequence of words, \mathbf{w}_i :

- ① For each topic k ($k=1\dots K$), there is a vector ϕ_k of C words (dictionary)
- ② For each document i :
 - a) We assign it a vector θ_i of K topic proportions
 - b) For each of the words, j ($j = 1, 2, \dots, ||\mathbf{w}_i||$), we have
 - i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, \dots, K\}$) according to θ_i
 - ii) A word, $w_{i,j}$, according to ϕ_k

Latent Dirichlet Allocation (LDA) algorithm

Generative model without probability distributions yet

- ① For each topic k ($k=1\dots K$), there is a vector ϕ_k of C words (dictionary)
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 - i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, \dots, K\}$) according to θ_i
 - ii) A word, $w_{i,j}$, according to ϕ_k



Some background

- Categorical distribution, $Cat(\theta)$

- With A categories

$$p(x = a | \theta) = \theta_a$$

where $\theta = [\theta_1, \theta_2, \dots, \theta_A]$

$$\sum_{a=1}^A \theta_a = 1$$

- When A=2, it's called *bernoulli* distribution
- Equivalent to *multinomial* with 1 trial

Some background

- Dirichlet distribution
 - Random vector, θ , of dimension A;

$$\theta \sim Dir(\alpha)$$

- $\alpha_1, \alpha_2, \dots, \alpha_A$, with $\alpha_i > 0$

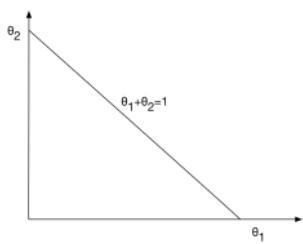
$$p(\theta|\alpha) = Dir(\alpha) = \frac{\Gamma\left(\sum_{i=1}^A \alpha_i\right)}{\prod_{i=1}^A \Gamma(\alpha_i)} \prod_{i=1}^A \theta_i^{\alpha_i - 1}$$

where $\sum_{i=1}^A \theta_i = 1$ and $\theta_i \in [0, 1]$.

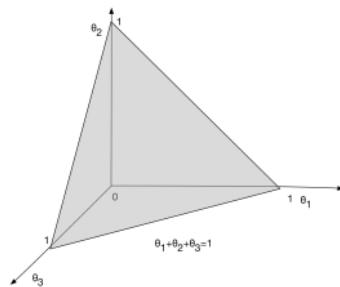
- $\Gamma(u)$ is the gamma function, a generalization of the factorial function, such that $\Gamma(u + 1) = u\Gamma(u)$

Some background - Dirichlet distribution

- It can be viewed as a probability distribution on an $A - 1$ dimensional simplex



1-D Simplex (segment)

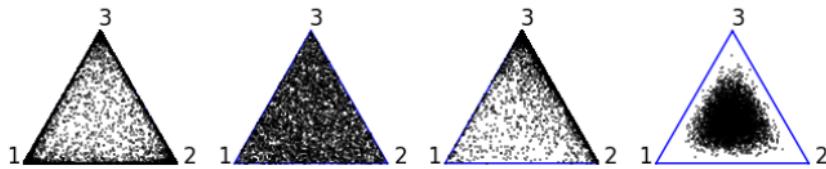


2-D Simplex (triangle)

Topic modeling

- A sample from the Dirichlet distribution with dimension A is a vector θ , such that $\sum_{i=1}^A \theta_i = 1$

$$\alpha = (0.200, 0.200, 0.200) \quad \alpha = (1.000, 1.000, 1.000) \quad \alpha = (0.100, 0.500, 0.990) \quad \alpha = (5.000, 5.000, 5.000)$$



- Perfect to generate arguments for the categorical distribution!

Playtime!



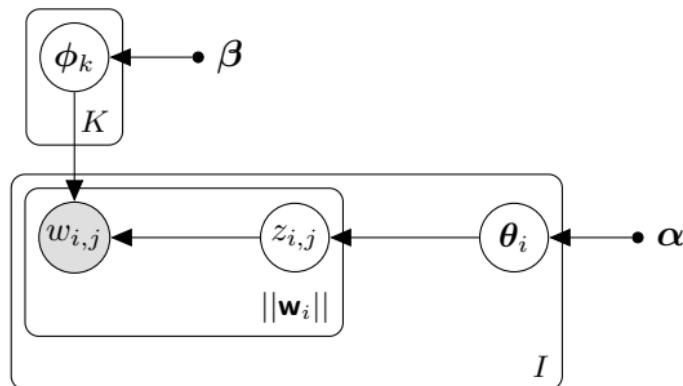
- Open "08. LDA.ipynb" notebook
- Do part 1
- Estimated time: 15 minutes

Latent Dirichlet Allocation (LDA) algorithm

- ① For each topic k , there is a vector of C words, ϕ_k , such that $p(\phi_k|\beta) = Dir(\beta)$
- ② For each document i :
 - a) There is a vector of K topics, θ_i , such that $p(\theta_i|\alpha) = Dir(\alpha)$
 - b) For each of the words, j , in each document i ($j = 1, 2, \dots, ||\mathbf{w}_i||$), we have
 - i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, \dots, K\}$) such that $p(z_{i,j}|\theta_i) = Cat(\theta_i)$
 - ii) A word, $w_{i,j}$, such that $p(w_{i,j}|\phi_{z_{i,j}}) = Cat(\phi_{z_{i,j}})$

Latent Dirichlet Allocation (LDA) algorithm

- ① For each topic k , there is a vector of C words, ϕ_k , such that $p(\phi_k|\beta) = Dir(\beta)$
- ② For each document i :
 - a) There is a vector of K topics, θ_i , such that $p(\theta_i|\alpha) = Dir(\alpha)$
 - b) For each of the words, j , in each document i ($j = 1, 2, \dots, ||\mathbf{w}_i||$), we have
 - i) A topic assignment, $z_{i,j}$ ($z_{i,j} \in \{1, 2, \dots, K\}$) such that $p(z_{i,j}|\theta_i) = Cat(\theta_i)$
 - ii) A word, $w_{i,j}$, such that $p(w_{i,j}|\phi_{z_{i,j}}) = Cat(\phi_{z_{i,j}})$



Playtime!



- Open "08. LDA.ipynb" notebook
- Do part 2
- Estimated time: 30 minutes

Topic modeling - LDA

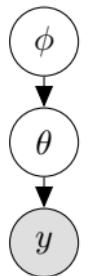
Joint distribution:

$$p(\mathbf{W}, \mathbf{Z}, \boldsymbol{\Theta}, \boldsymbol{\Phi} | \boldsymbol{\alpha}, \boldsymbol{\beta}) = \left(\prod_{i=1}^I p(\boldsymbol{\theta}_i | \boldsymbol{\alpha}) \prod_{j=1}^{||\mathbf{w}_i||} p(w_{i,j} | \boldsymbol{\phi}_{z_{i,j}}) p(z_{i,j} | \boldsymbol{\theta}_i) \right) \prod_{k=1}^K p(\boldsymbol{\phi}_k | \boldsymbol{\beta})$$

- No analytical solution
- Common implementations with Gibbs sampling or Variational Bayes

Discrete Latent Variables in STAN

- In its present version, one cannot explicitly use discrete latent variables in STAN
- Limitation with its MCMC inference process (explained in upcoming class)
- For example



$$\phi \sim \mathcal{N}(0, 1)$$

$$\theta \sim \text{Bernoulli}(\phi)$$

$$y \sim \mathcal{N}(k\theta, 10)$$

$$p(y, \theta, \phi) = p(y|\theta)p(\theta|\phi)p(\phi)$$

Discrete Latent Variables in STAN

- It would allow, however



$$p(y, \phi) = \sum_{\theta} p(y|\theta)p(\theta|\phi)p(\phi)$$

- Marginalization of the latent variable
- We "get rid" of the latent variable, but use its distribution correctly
- How do we do this in STAN?
- STAN takes care of the priors (because all their parameters are known!)
- So we need to focus on the likelihood

$$L(y) = p(y|\phi) = \sum_{\theta} p(y|\theta)p(\theta|\phi)$$

Discrete Latent Variables in STAN

- Notice that:

$$\theta \sim Bernoulli(\phi) \implies p(\theta|\phi) = \phi^\theta (1-\phi)^{(1-\theta)}$$

- So we have:

$$\begin{aligned} L(y) &= p(y|\phi) = \sum_{\theta} p(y|\theta)p(\theta|\phi) = \sum_{\theta} \left(\mathcal{N}(y|k\theta, 10) \phi^\theta (1-\phi)^{(1-\theta)} \right) = \\ &= \phi \mathcal{N}(y|k, 10) + (1-\phi) \mathcal{N}(y|0, 10) \end{aligned}$$

- STAN uses **log-probabilities**, so we should have:

$$LL(y) = \log \left(\phi \mathcal{N}(y|k, 10) + (1-\phi) \mathcal{N}(y|0, 10) \right)$$

Discrete Latent Variables in STAN

$$LL(y) = \log \left(\phi \mathcal{N}(y|k, 10) + (1 - \phi) \mathcal{N}(y|0, 10) \right)$$

- In fact, you'll need to use the function `log_sum_exp(vector v)`: *real*
 - Receives a vector **v** of real values
 - Transforms each element, \mathbf{v}_i , of the vector with $\exp(\mathbf{v}_i)$
 - Calculates the "log sum" to all elements \mathbf{v}_i
 - So, to use it, we need to apply $\log(\mathbf{v}_i)$ at each element
 - The model from above in STAN:

```
model {
    phi ~ normal(0,0.7); // prior
    real gamma[2];
    gamma[0] = log( phi*normal_lpdf(y| k, 10));
    gamma[1] = log( (1-phi)*normal_lpdf(y|0, 10));

    target += log_sum_exp(gamma); // likelihood;
}
```

Playtime!



- Open "08. LDA.ipynb" notebook
- Do part 3
- Estimated time: 1 hour

Back to the nutshell: our example

$$y = f(\mathbf{x}_{text}, \mathbf{x}_L)$$

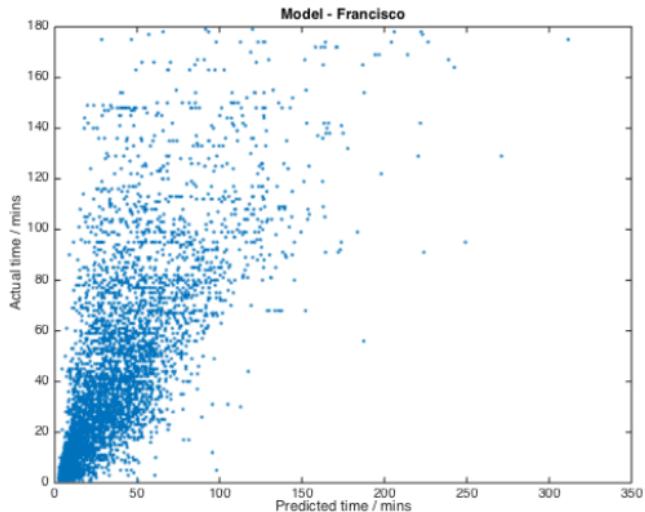
with

$$\mathbf{x}_L = \begin{bmatrix} \mathbf{x}_{expressway_ID} \\ \mathbf{x}_{num_block_lanes} \\ \mathbf{x}_{time_of_day} \\ \mathbf{x}_{day_of_week} \\ \mathbf{x}_{type_of_day} \\ \mathbf{x}_{congestion_status} \\ \mathbf{x}_{num_vehicles} \end{bmatrix}$$
$$\mathbf{x}_{text} = \begin{bmatrix} \mathbf{x}_{topic_1} \\ \mathbf{x}_{topic_2} \\ \dots \\ \mathbf{x}_{topic_K} \end{bmatrix}$$

Incident Duration Prediction

- Data:
 - 10000+ incidents
 - Each report modeled using 25 topics
 - Original information on time of day, location etc.
- Goal:
 - regression model for incident duration
- An accurate prediction for the duration is important for modelling the impact on traffic

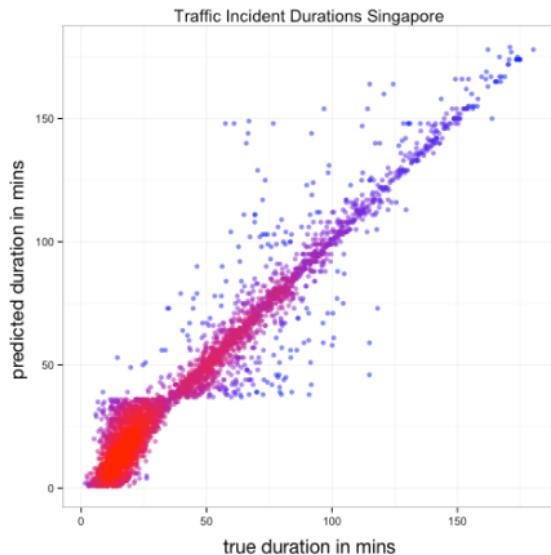
Non-parametric model ($R^2=0.461$)



- Predicted vs. Actual durations for test data.
- Without textual data, $R^2 = 0.230$

Incident Duration Prediction ($R^2=0.603$)

A slightly more complicated (non-parametric) model...



Concluding remarks



- In practice (e.g. SKLEARN), documents for LDA use BoW representation
 - Mechanics is exactly the same
 - We only mention so you know how to do it :-)
- LDA is super-optimized today - Variational approximation algorithm
- A bonus notebook (from 42184) on LDA with SKLean in Inside

Concluding remarks

- Topic modeling falls in the general family of *mixed membership* models
 - Each document is a mixture of components
- Similarities with Principal Components Analysis (PCA), Factor Analysis...
- What you saw was **unsupervised**...
 - What would be the supervised version?

Reference material

Introduction to Probabilistic Topic Models.

David Blei, 2011 (pp 1-8)

www.cs.princeton.edu/~blei/papers/Blei2011.pdf

More detail:

Probabilistic Topic Models, Mark Steyvers and Tom Griffiths

In T. Landauer, D McNamara, S. Dennis, and W. Kintsch (eds), Latent Semantic Analysis: A Road to Meaning. Laurence Erlbaum

psiexp.ss.uci.edu/research/papers/SteyversGriffithsLSABookFormatted.pdf

Appendix



- Another example: Demand prediction for special events

Demand prediction for special events

- **Context:**

- Singapore November 2011- February 2012
- 3 venues (Singapore Indoor Stadium, Kallang Theatre, Singapore Expo)

- **Objective:**

- Predict number of public transport arrivals (in 30 min blocks)
- Useful for operations planning (e.g. reallocate drivers)

- **Data:**

- EZLink (tap-in/tap-out) data, November 2011-February 2012
- 1500+ Event data from Eventful.org, Facebook, Google (600+ for selected venues)

Demand prediction for special events

- Model variables
 - Time of day (discrete and continuous), day of week, type of day
 - Time to next event/time since event started (in 30 min slots)
 - Number of Google hits, Facebook likes, Wikipedia page (dummy)
 - Topics from textual description ($K = 25$)
- Regression model(s)
 - Linear regression
 - Kernel method (Gaussian Processes)

Demand prediction for special events

