From model-based learning to model-free behaviour with Meta-Interpretive Learning

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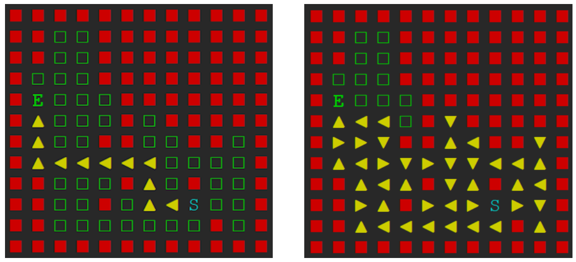
Abstract. A "model" is a theory that describes the state of an environment and the effects of an agent's decisions on the environment. A model-based agent can use its model to predict the effects of its future actions and so plan ahead, but must know the state of the environment. A model-free agent cannot plan, but can act without a model and without completely observing the environment. An autonomous agent capable of acting independently in novel environments must combine both sets of capabilities. We show how to create such an agent with Meta-Interpretive Learning used to learn a model-based Solver used to train a model-free Controller that can solve the same planning problems as the Solver. We demonstrate the equivalence in problem-solving ability of the two agents on grid navigation problems in two kinds of environment: randomly generated mazes, and lake maps with wide open areas. We find that all navigation problems solved by the Solver are also solved by the Controller, indicating the two are equivalent.

Keywords: ILP · Meta-Interpretive Learning · Planning.

1 Introduction

A *model* is a theory that describes the possible states of an environment and the way in which the state of the environment changes as a result of an agent's actions. An *environment* may be an abstract domain such as the set of all lists or the Cartesian plane, a virtual environment, such as a computer simulation or game, or a physical environment such as a real-world location. An *action* is a predicate that relates a description of the current state of an environment to a new state; in other words, a transition relation over states. An *agent* is a program that generates a sequence of actions connecting an initial state to a desired goal state; what we call a *plan*. To generate such a sequence of actions is to solve a *planning problem*. A *model-based* agent is an agent that solves planning problems given a model. A *model-free* agent is an agent that selects actions without a model and, therefore, without a plan. We call a model-based agent a "solver for planning problems" or, simply, *solver* and a model-free agent a *controller*.

Figure 1 illustrates the difference between planning by a model-based Solver and acting by a model-free Controller. In the Figure the environment is a map of a lake with islands (alternatively a cave system) where red tiles represent



(a) Model-based Solver (b) Model-free Controller

Fig. 1: A model-based Solver can predict the effect of its future actions and plan ahead, but requires knowledge of its environment. A model-free Controller cannot plan and must explore its environment but can act without knowledge of the environment. Green tiles: passable; red tiles: unpassable; S: start; E: goal.

unpassable locations, green tiles passable locations and the letters S and E a starting and goal location respectively. Yellow arrows show agent trajectories.

Planning with a model implies some knowledge of the structure of an environment. The Solver in Figure 1a is initially given a map of the environment, which it can fully observe, and uses this map to generate the set of all possible moves between adjacent, passable map tiles. The Solver then uses this navigation model to plan a direct path from S to E. The Controller in Figure 1b is not given the map of the environment, cannot directly observe S or E, and can only observe whether map tiles adjacent to its current location are passable or unpassable. The Controller explores its environment to find a path from S to E.

A model-based solver and a model-free controller have complementary advantages and disadvantages. An autonomous agent capable of acting independently in arbitrary environments must combine the capabilities of both. In this paper we show how the two sets of abilities can be combined by using Meta-Interpretive Learning (MIL) [9] to learn a solver, and use the solver to generate examples to learn a controller that solves the same problems as the solver.

Our ultimate goal is the development of an action-selection component of an Autonomous Agent Architecture that must guide a mobile robot to survery missions in environments with dynamic and unobserved features so that the autonomous agent guiding the robot must combine the capabilities of both solver and controller. Full details are reserved for future work.

Inductive Logic Programming as model-based learning *Plannning is the model-based approach to autonomous behaviour* [4] in the sense that a planning agent's behaviour is derived from a model and a planning problem by means of an inference procedure (typically a search algorithm such as A^*). Inductive Logic Programming (ILP)[6] can be seen as the model-based approach to *machine learning* where the "model" is a background theory and a new hypothesis is

$S(s_1, s_2) \leftarrow Step\_down(s_1, s_2).$
$S(s_1, s_2) \leftarrow Step\_left(s_1, s_2).$
$S(s_1, s_2) \leftarrow Step\_right(s_1, s_2).$
$S(s_1, s_2) \leftarrow Step\_up(s_1, s_2).$
$S(s_1, s_2) \leftarrow Step\_down(s_1, s_3), S(s_3, s_2).$
$S(s_1, s_2) \leftarrow Step\_left(s_1, s_3), S(s_3, s_2).$
$S(s_1, s_2) \leftarrow Step\_right(s_1, s_3), S(s_3, s_2).$
$S(s_1, s_2) \leftarrow Step\_up(s_1, s_3), S(s_3, s_2).$

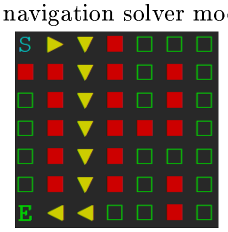
(a) Grid navigation solver

$Step\_down([id, x/y, t], [id, x/y^{-1}, t')$
$Step\_left([id, x/y, t], [id, x^{-1}/y, t')$
$Step\_right([id, x/y, t], [id, x^{+1}/y, t'])$
$Step\_up([id, x/y, t], [id, x/y^{+1}, t'])$

(b) Grid navigation solver model

$Step\_right([maze\_a, 0/6, s], [maze\_a, 1/6, f])$
$Step\_down([maze\_a, 2/6, f], [maze\_a, 2/5, f])$
$Step\_left([maze\_a, 2/0, f], [maze\_a, 1/0, f])$
$Step\_left([maze\_a, 1/0, f], [maze\_a, 0/0, e])$
% ... 56 more actions

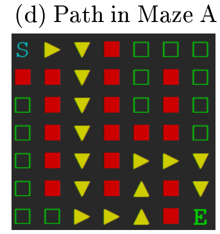
(c) Solver model instantiated to Maze A



(d) Path in Maze A

$Step\_right([maze\_b, 0/6, s], [maze\_b, 1/6, f])$
$Step\_down([maze\_b, 2/6, f], [maze\_b, 2/5, f])$
$Step\_up([maze\_b, 4/0, f], [maze\_b, 4/1, f])$
$Step\_down([maze\_b, 6/1, f], [maze\_b, 6/0, e])$
% ... 56 more actions

(e) Solver model instantiated to Maze B



(f) Path in Maze B

Fig. 2: A Solver for grid navigation problems learned with MIL.

derived by an inference procedure given a set of examples. To learn planning programs, i.e. solvers, with ILP, we can structure the background theory as a set of action predicates and give a set of planning problems as examples. For a solver to be general, it must be a recursive program, therefore we use MIL, a form of ILP capable of learning recursion.

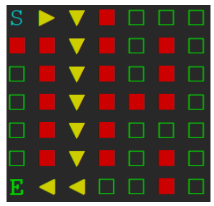
Figure 2 lists a grid navigation solver as a set of definite clauses. The solver's body consists of eight clauses of the predicate $S/2$, where each argument s_i is a Prolog list representing an initial or end state of the environment. Each clause of $S/2$ changes the environment state by moving an agent one step to each of the four directions up, right, down or left, recursively. The solver's model consists of the step actions $Step\_up, Step\_right, Step\_down, Step\_left$, implemented as dyadic predicates, listed in un-instantiated form in Figure 2b. The two arguments of each action, shared with $S/2$, represent the state of the environment before and after the action is taken as a list of first-order terms: id, a map identifier, x/y, the coordinates of an agent on the map, and, t, the type of map *tile* at x/y, one of the constants f, w, e, s for floor, wall, start and end tile, respectively; wall tiles are not passable and so they are not featured in actions' state arguments.

4 **Controller state labels:** $Q = \{q_0, q_1, q_2, q_3\}$
4 Action labels: $A = \{up, right, down, left\}$
15 Observation labels: $O = \{pppp, pppu, ppup, ppuu, pupp, pupu, puup, puuu, uppp, uppu, upup, upuu, uupp, uupu, uuup\}$
960 4-tuples: $|Q \times O \times A \times Q|$

(a) Finite State Controller labels

$(q_0, upuu, right, q_1)$
$(q_1, upup, right, q_1)$
$(q_1, uupp, down, q_2)$
$(q_2, pupu, down, q_2)$
$(q_2, ppup, left, q_3)$
$(q_3, upup, left, q_3)$

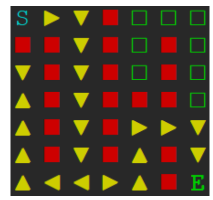
(b) FSC for Maze A



(c) Path in Maze A

$(q_1, upup, right, q_1)$.
$(q_1, pupp, down, q_2)$.
$(q_1, uupp, down, q_2)$.
$(q_2, ppup, right, q_1)$.
$(q_2, pupu, down, q_2)$.
$(q_0, uppu, right, q_1)$.
$(q_0, upuu, right, q_1)$.
$(q_0, pupu, up, q_0)$.
$(q_1, puup, up, q_0)$.

(d) FSC for Maze B



(e) Path in Maze B

Fig. 3: Finite State Controllers for mazes A, B trained with the Solver in Fig 2a.

The solver's model must be instantiated to the coordinates and tile types of a map before it can be used by the solver to solve that map. Two examples of instantiated models are listed in Figures 2c, 2e for two mazes, Maze A and B. The two mazes are identical save for the position of the end tile and so their instantiated models are almost identical. Figures 2d, 2f illustrate the solutions of the two mazes by the solver where each yellow arrow corresponds to a step action in the indicated direction.

The solver in Figure 2 was learned by the MIL system Louise [10] as described in Section 5.1.

Finite State Controllers for model-free behaviour When its model cannot be instantiated a model-based solver cannot plan. An alternative is found in Finite State Controllers (FSCs) [2] a formalisation of the concept of Finite State Machines from video games and robotics. A FSC is a set of 4-tuples (q, o, a, q') mapping pairs (q, o) of current *controller* state[1] and observation labels, to pairs (a, q') of action and next controller state labels. Figure 3 lists two maze-solving FSCs for mazes A and B in Figure 2, and their sets of labels. Each observation label is a string $\{U, R, D, L\} \in \{u, p\}$ denoting whether map tiles in the directions up, right, down or left, respectively, from the agent's current map location, are passable (p) or unpassable (u). For example, the string $upuu$ denotes that only the tile to the right of the agent is passable. Action labels denote that the agent must move up, right, down or left. The two FSCs in Figure 3 were learned by Louise from solutions of mazes A and B generated by the solver in Figure 2a.

FSCs are model-free in that they have no representation of the environment state, or the way actions modify that state; they are only a mapping between

[1] Controller states are not *environment* states.

pairs of labels. In particular, FSC observation labels represent an agent's *belief* about the state of the environment. Additionally, FSCs are *efficient* in that they do not need to search a large state-space like planners, or, indeed, solvers. Conversely, an FSC is equivalent to a Deterministic Finite State Transducer and so can only generate one sequence of actions, e.g. the maze FSC in Figure 3b can only solve mazes where the exit can be reached by moving right, down, or left, like Maze A in Figure 2d, whereas the FSC in figure 3d can only solve mazes where the exit can be reached by moving right, down or up, like Maze B in Figure 2f. Finally, the literature on FSCs does not describe a concrete method to *execute* FSCs, only their notation as sets of 4-tuples.

Contributions We identify FSCs as a promising framework for the development of model-free controllers solving the same problems as model-based solvers in partially observable environments. We make the following contributions.

- We formalise a notation for model-based learning of planning problem solvers as a higher-order background theory for MIL.
- We extend the framework of FSCs to Nondeterministic FSCs.
- We implement a set of FSC *executors*, stack-machines that take as input a set of FSC tuples and execute them in order.
- We implement a novel approach to Simultaneous Localisation and Mapping (SLAM) used by executors to avoid cycles.
- We show that a solver and a controller that solve the same problems can be learned by MIL using our model notation.
- We demonstrate empirically that our learned solver and controller can solve the same planning problems.
- We implement two new libraries in Prolog: *Controller Freak*, to learn controllers from solvers; and *Grid Master* to solve navigation problems on grids.

Implementation code and experiment data are available at the following URL: https://github.com/stassa/ijclr24.

2 Related Work

2.1 Meta-Interpretive Learning

Meta-Interpretive Learning (MIL)[9, 7] is a new form of Inductive Logic Programming (ILP)[6, 8] where the background theory is a higher-order program that includes both first- and second-order definite clauses. Hypotheses learned by MIL are first-order definite programs, i.e. logic programs in Prolog. Hypotheses are learned by SLD Refutation of training examples, which can be both positive or negative, given as Horn goals to a meta-interpreter capable of second-order SLD Resolution. During Resolution, substitutions of second-order variables in the higher-order background theory are derived which applied to their corresponding second-order clauses yield the first-order clauses of a hypothesis. MIL has been shown capable of learning recursion and of predicate invention without the limitations of earlier ILP systems[11].

2.2 Finite State Controllers

A Finite State Controller (FSC)[2] is a set of 4-tuples (q, o, a, q'), where $q, q' \in Q, o \in O, a \in A$ are constants serving as labels for controller states, observations and actions, respectively. A tuple (q, o, a, q') corresponds to a mapping between pairs (q, o) and (a, q') and the set of all tuples in an FSC to the *transition function* $Q \times O \rightarrow A \times Q$ that controls the actions of an agent, so that in controller state q, when the observation label o is input, the FSC outputs action label a and transitions to controller state q'. In [2] an approach is described to derive FSCs from contingent planning problems through a series of transformations to classical planning, via conformant planning, problems [4]. The approach is demonstrated in a number of domains including navigation on grids that we also demonstrate here, but transformation betwen planning problems is subject to high computational costs.

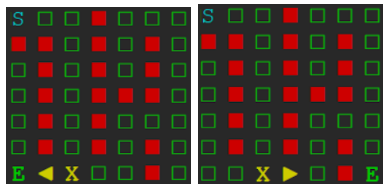
Compared to contingent plans and policies derived from Markov Decision Processes (MDPs) and Partially Observable MDPs (POMDPS)[2] claim that FSCs are: a) general, in that they can solve multiple problems in a domain; b) compact, in that they only need comprise a few controller states; and c) robust to uncertainty in initial and goal conditions and the effects of actions. Generality of FSCs is formalised in [1] and their compactness is self-evident. We could not find evidence of FSC robustness to uncertainty in the literature; we leave the matter to future work.

To be used as an action selection mechanism for an autonomous agent, an FSC must be executed in a loop by an external process. Such a process must connect the FSC to an environment by passing observation and action labels between the FSC and the environment. Such a mechanism has not, so far, been described in the literature. In Section 4 we implement a set of *executors* that perform this function in conjunction with a virtual environment for grid navigation problems.

FSCs are equivalent to Deterministic Finite State Transducers (FSTs), a.k.a. Mealy machines[2]. In an FSC's tuples, observation and action labels are the inputs and outputs, respectively, of a deterministic FST's transition function, and each pair $Q \times O$ is mapped to exactly one pair $A \times Q$, such that there exists a single tuple (q, o, a, q') for each pair (q, o) in an FSC. Being equivalent to *Deterministic* FSTs, in other words, Regular Automata, FSCs cannot deal with ambiguities in the sense that they can only successfully solve planning problems where the same action must always be taken in the same circumstances. While this may be sufficient, or even desired, that is not always the case.

As an example of the difficulty in dealing with ambiguities, consider the two mazes A and B in Figure 4, identical in structure, but with an end tile (E) placed in a different location. An FSC solving maze A must turn left at the cell marked with a yellow X in order to reach the exit, therefore the same FSC will not be able to solve maze B where it must turn *right* at the X to reach the

[2] This equivalence is not noted in earlier work.



(a) Maze A (b) Maze B

Fig. 4: An example of ambiguity in a maze environment. The two mazes, Maze A and Maze B have identical structure but the end tile E is placed in a different location. In Maze A, an agent must turn left at the yellow X to reach the exit at E, but in Maze B it must turn right instead, as indicated by the yellow arrows.

exit[3]. In Section 3 we extend FSCs to *Nondeterministic* FSCs and in Section 4 implement executors with a stack that permit an FSC to backtrack or reverse course to fully explore an environment and solve ambiguous environments.

3 Framework

3.1 MIL Planning Model

To learn a solver for planning problems with MIL we structure the higher-order background theory given to a MIL system as a model of the planning domain and give as an example a planning problem. We call such a background theory and example a *MIL Planning Model*.

Definition 1. *(MIL Planning Model) A MIL Planning Model $\mathcal{T}$ is a 5-tuple $\{\mathcal{F}, \mathcal{S}, \mathcal{A}, \mathcal{M}, e\}$ where:*

- $\mathcal{F}$ *is a set of first-order terms that we call* fluents*.*
- $\mathcal{S}$ *is a set of states $\{s_1, ..., s_m\}$, where each s_i is a set of* instances *of the fluents in $\mathcal{F}$, given as a Prolog list.*
- $\mathcal{A}$ *is a set of dyadic definite clauses with heads $A_1(s_{11}, s_{12}), ..., A_n(s_{n1}, s_{n2})$, where each $s_{i1}, s_{i2} \in \mathcal{S}$, and $\mathcal{A}$ defines a* transition relation *between states.*
- $\mathcal{M}$ *is the set containing the two second-order definite clauses: $P(x, y) \leftarrow Q(x, y)$ (called "Identity") and $P(x, y) \leftarrow Q(x, z), P(z, y)$ ("Tailrec") with P, Q existentially quantified over the set of predicate symbols in $\mathcal{A}$ and the symbol $S/2$, and x, y, z universally or existentially quantified over $\mathcal{S}$.*
- e *is a Horn goal $\leftarrow S(s_{in}, s_g)$ given as a training example, where $s_{in}, s_g \in \mathcal{S}$ are an initial state and a goal state of a planning problem, respectively.*

[3] The same limitation obtains when learning optimal policies from (PO)MDPs, as e.g. with Reinforcement Learning where this limitation was recently identified as *implicit partial observability*[5], the characteristic of a domain where an arbitrary instance of a problem does not include every observation possible in all problem instances.

We call s_{i1}, s_{i2} in an action A_i the input *and* output *states of A_i, respectively.*

Clauses of action predicates in $\mathcal{A}$ are selected for Resolution by unification so the fluents in actions' state-lists act as both preconditions and effects, unlike in STRIPs-like planning models where there is a cler distinction between the two.

Given a MIL Planning Model, a MIL system must return a *planning hypothesis* H such that $\mathcal{A} \cup H \models e$. We say that H solves the planning problem in e. We call H a *solver* for e. In particular, H is a *model-based* solver and its model is the set of ground instances of clauses in $\mathcal{A}$[4].

Definition 2. *(MIL Planning Hypothesis) A MIL Planning Hypothesis H is a set of first-order definite clause instances of* Identity *and* Tailrec *such that each P in a head literal $P(x, y)$ in* Identity *or* Tailrec *and the last body literal $P(z, y)$ in* Tailrec *is a literal of the predicate $S/2$ as in $e \in \mathcal{T}$, and each body literal $Q(x, y)$ or $Q(x, z)$ in* Identity *and* Tailrec *respectively is a literal of an action predicate $A_i(s_{i1}, s_{i2})$ in $\mathcal{A} \in \mathcal{T}$. If $\mathcal{A} \cup H \models e$, H is a* solver *for e.*

3.2 Nondeterministic Finite State Controllers

We extend the definition of Finite State Controllers, adapted, in Definition 3, from [2], to *Nondeterministic* Finite State Controllers.

Definition 3. *(Finite State Controller) Let Q be a set of controller state labels, O a set of observation labels, A a set of action labels, and let C be a function $Q \times O \to A \times Q$. C is a Finite State Controller, denoted as the set of 4-tuples (q, o, a, q') where $q, q' \in Q, o \in O, a \in A$ and $(q, o) \to (a, q') \in C$. Note that all FSC labels are constants, or strings of constants.*

Definition 4. *(Nondeterministic Finite State Controller) Let Q, O, A be as in Definition 3, let $R = Q \times O \times A \times Q$, and let $C \subseteq R$. We call C a Nondeterministic Finite State Controller for R. We denote C as the set of 4-tuples (q, o, a, q') that correspond to the subset of R in C, where $q, q' \in Q, o \in O, a \in A$.*

We will henceforth refer to Nondeterministic FSCs simply as FSCs, only clarifying the determinism property where it is not clear from the context. We call a sequence of FSC tuples the FSC's *behaviour*:

Definition 5. *(FSC Behaviour) Let C be a Nondeterministic FSC and $B = \langle T_1, ..., T_n \rangle$ a sequence of 4-tuples in C, such that for every $t_i, t_{i+1} \in B$ if $t_i = (q, o, a, q')$ then $t_{i+i} = (q', o', a', q'')$. We call B the* behaviour *of (an agent controlled by) C.*

4 Implementation

4.1 Controller Freak

We implement, in Prolog, a new library called *Controller Freak* that defines Prolog predicates to learn FSCs from solvers, by MIL. Predicates in Controller

[4] In this case the meaning of "model" converges to that in First Order Logic.

$Step\_down([id, x/y, t, [q|Qs], [o|Os], [a|As], [q'|Qs']], [id, x/y^{-1}, t', Qs, Os, As, Qs'])$
$Step\_left([id, x/y, t, [q|Qs], [o|Os], [a|As], [q'|Qs']], [id, x^{-1}/y, t', Qs, Os, As, Qs'])$
$Step\_right([id, x/y, t, [q|Qs], [o|Os], [a|As], [q'|Qs']], [id, x^{+1}/y, t', Qs, Os, As, Qs'])$
$Step\_up([id, x/y, t, [q|Qs], [o|Os], [a|As], [q'|Qs']], [id, x/y^{+1}, t', Qs, Os, As, Qs'])$

Table 1: Grid navigation solver model with controller labels used to generate FSC behaviours to learn a grid navigation FSC with MIL.

Freak take as input a set of planning problems E and a solver H where the set of fluents F in actions $\mathcal{A}$ in H's model includes the sets of labels Q, O, A that must be accepted by C, the FSC to be learned. A set of actions $\mathcal{A}$ extending the grid navigation solver model in Figure 2b with FSC labels is listed in Table 1.

Solving each $e \in E$ with H generates sequences of labels corresponding to example behaviours B of C used to compose the following MIL-learning problem:

- First-order Background theory: $Q \times O \times A \times Q$
- Second-Order Background theory: $\{Identity, Tailrec\}$
- Examples: Set of FSC behaviours B

Solving this MIL-learning problem results in a set of definite clauses, C_p, that represent C in the form of a recursive logic program. Further processing by Controller Freak predicates transforms C_p into the set of 4-tuples of C.

4.2 Grid Master

We implement, in Prolog, a library for grid navigation problems called *Grid Master*, defining predicates in three modules: a) A *Map* module is used to manage grid-style maps in text format, where each "cell" of a map is a string. A generator for maze maps called *MaGe* is included with Grid Master. b) An *Action Generator* module is used to generate actions for a MIL Planning Model given a map. c) A *Map Display* module is used to display maps and the progress of a grid navigation agent through a map, as in the map figures in this paper.

Grid Master allows the definition of game-like virtual environments with which an agent can interact to solve a map. An initial *Basic Environment* is included that manages navigation on mazes, and similar grids with a start and end location. The Basic Environment a) initialises an agent's starting location on a map, b) receives action labels from, and returns observation labels to, a controller (via an executor) as the agent navigates the map, and c) determines whether the agent has reached the end location on the map. Additionally, the Basic Environment can receive as input a sequence of action labels from a solver, and "play them back" to determine whether the solver has reached the end location.

4.3 Executors

An *executor* is an interpreter for FSCs implemented as a loop that passes a current pair (q, o) to an FSC, receives a pair (a, q') and passes the action label a of

```
executor_bck(Gs,_Q0,_O,As,Gs,As):- !.
executor_bck(Fs,Q0,O,[A|Acc],Gs,As):-
        controller_tuple(Q0,O,A,Q1)
        ,environment_action(Fs,A,O1,Fs1)
        ,executor_bck(Fs1,Q1,O1,Acc,Gs,As).
```

Fig. 5: Backtracking FSC Executor. Fs: initial fluents. $Q0$: current controller state label. O: current observation label. A: Next action label. Acc/As: Action label accumulator. Gs: Goal fluents. $Q1, O1$: next state and observation labels.

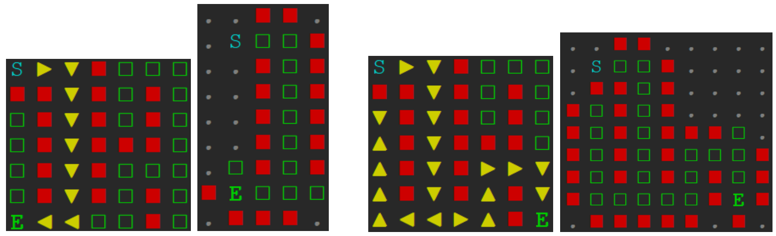
```
executor_rev(Gs,_T1,_Ss,As,Gs,As):- !.
executor_rev(_Fs,_T1,[],As,_Gs,As):- !.
% T1 is a partially ground 4-tuple (q,o,A,Q')
executor_rev(Fs,T1,Ss,[T0|Acc],Gs,As):-
        % All ground instances of T1 are pushed to the stack.
        push_tuples(T0,T1,Ss,Ss_1)
        % T2 is a ground instance of T1 (q,o,a,q')
        ,pop_tuple(Ss_1,T2,Ss_2)
        % T3 is a new, partially ground 4-tuple (q',o',A',Q'')
        ,environment_state_transition(Fs,T2,Fs1,T3)
        ,push_reverse(T2,Ss_2,Ss_r)
        ,executor_rev(Fs1,T3,Ss_r,[T2,T0|Acc],Gs,As).
```

Fig. 6: Reversing FSC Executor. Fs: initial fluents. $T1$: current FSC 4-tuple, partly ground. Ss: stack of FSC 4-tuples. $T0$: last FSC tuple, fully ground. Acc/As: FSC 4-tuple accumulator. Gs: goal fluents. *In body literals*: $T2$: next fully ground instance of $T1$. $T3$: new, partly ground FSC 4-tuple.

this pair to an environment to obtain a new observation label o, until a goal state is reached. Initial and goal states are represented as fluents not directly observed by the executor, or the FSC it is executing. Instead the environment, e.g. the Basic Environment, or the sensor and actor models on a robot, translate current environment states to observation labels and action labels to new environment states.

We define two kinds of executors: backtracking and reversing; and two variants, with and without SLAM.

Backtracking Executor We implement a Backtracking Executor that performs the loop described in Section 4.3 in the simplest possible way, as a recursive procedure in Prolog, that is allowed to backtrack and obtain further pairs (a, q') while the goal state has not been reached. Thus, the Backtracking Executor implicitly uses Depth First Search with backtracking (DFS) and a stack managed by the Prolog runtime, to "jump back" to locations on a map where alternative actions can be taken. The Backtracking Executor is listed, as Prolog code, in Figure 5.



(a) Maze A (b) Maze A SLAM (c) Maze B (d) Maze B SLAM

Fig. 7: Grid SLAM: mapping of an unseen environment during FSC execution with Reversing Executor (yellow arrows). Grey dots: unobservable tiles.

Reversing Executor Backtracking causes an environment to be "reset" to a previous state, e.g. with respect to an agent's coordinates in the environment. That may not be possible in all environments. In particular, simulator-like virtual environments such as the Basic Environment may allow backtracking but real-world environments such as the physical world, do not. Thus the Backtracking Executor cannot be used in the real world.

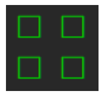
To allow FSCs to be used in real-world environments we implement a Reversing Executor that manages a stack explicitly, pushing on the stack the reverse of each FSC 4-tuple executed. When no more tuples can be pushed on the stack the first 4-tuple popped from the stack is the reverse of the last executed 4-tuple, causing the FSC to explicitly retrace its steps. When the last reverse-tuple is popped from the stack, the next 4-tuple is a not-yet tried alternative and the agent explores a new path, or the stack is empty and execution terminates.

The reverse of a tuple is determined by an FSC. Only the reverse of a pair (a, q') need be defined, rather than the reverse of every tuple[5]. Not all action-state pairs will have a reverse, thus not all decisions can be reversed.

The Reversing Executor is listed, as Prolog code, in Figure 6.

Grid SLAM When a map has wide areas of passable tiles an FSC can be stuck going around in loops. To avoid this we implement Simultaneous Localisation and Mapping (SLAM) as a Grid Master predicate used by an executor to update a SLAM-constructed map by placing obsevation labels on a grid that is continuously expanded as the FSC explores an environment. A SLAM executor marks the last location of an FSC on the SLAMming map as visited and an FSC is only allowed to re-visit a location when reversing course, with a Reversing (SLAM) executor, thus avoiding cycles. Details of the SLAM implementation are left for future work for lack of space. Figure 7 illustrates SLAMming maps constructed with our method.

[5] This does not violate the model-free nature of an FSC: the reverse-pair relation is specific to an FSC and does not depend on the environment. The same relation can be used to avoid oscillations, e.g. to avoid following a "down" with an "up" action.



(a) Zero map

Second-Order Definite Clauses M:
$(Identity)\ P(x, y) \leftarrow Q(x, y)$
$(Tailrec)\ P(x, y) \leftarrow Q(x, z), P(z, y)$
Actions $\mathcal{A}$:
$Step\_down([zero, 0/1, f], [zero, 0/0, f])$
$Step\_down([zero, 1/1, f], [zero, 1/0, f])$
$Step\_left([zero, 1/0, f], [zero, 0/0, f])$
$Step\_left([zero, 1/1, f], [zero, 0/1, f])$
$Step\_right([zero, 0/0, f], [zero, 1/0, f])$
$Step\_right([zero, 0/1, f], [zero, 1/1, f])$
$Step\_up([zero, 0/0, f], [zero, 0/1, f])$
$Step\_up([zero, 1/0, f], [zero, 1/1, f])$
Example e:
$s([zero, x_s/y_s, t_s], [zero, x_e/y_e, t_e).$

(b) Zero map MIL Planning Problem

Fig. 8: MIL Planning Problem for Map Zero.

5 Experiments

We show empirically that a grid navigation solver can be learned by MIL and used to learn an FSC capable of solving the same problems as the solver.

We carry out experiments in two types of environments with grid-based maps. The first type of environment are maps of mazes generated by the Grid Master *MaGe* module. The second type of environment, illustrated in Figure 1 are maps of open areas with obstacles, modelling lakes with islands, or caves, generated by the dungeon map generator in the R-language package *r.oguelike* [3]. We refer to the latter type of environment as "Lake maps".

Each MaGe map has a start and end tile placed automatically, at random. For Lake Maps we find it difficult to generate new maps programmatically, so we pre-generate 10 maps of dimensions 20×20 and create new planning problems by randomly placing a start and end tile in each map for each new problem.

Both types of environment are accessed via the Grid Master Basic Environment as described in Section 4.2. The salient difference between the two types of environment is that MaGe maps have no open areas of passable tiles where an agent can get stuck in a loop ("plazas"), whereas Lake Maps are predominantly made of such plazas. The MaGe maps thus effectively test the ability of an agent to find a path through a fully-connected graph without cycles, wheras the Lake maps test the ability of an agent to find a path through a graph with cycles.

5.1 Learning a Model-Based Solver with MIL

We find that we can learn a general grid navigation solver with MIL from a single example of a simplified grid-based map consisting of only four passable (floor) tiles. We name this map *Zero* and illustrate it in Figure 8a.

Experiment	Agent	Environment	Dimensions	Instances	Solved	Steps
Experiment 1	Solver	MaGe map	100×100	100	100%	816.04
		Lake map	20×20	10×50	100%	11.23
Experiment 2	FSC-BT	MaGe map	100×100	100	100%	816.04
	FSC-RE	MaGe map	100×100	100	100%	3942.78
	FSC-BT(S)	Lake map	20×20	10×50	92%	69.54
	FSC-RE(S)	Lake map	20×20	10×50	100%	132.98

Table 2: Experiment results. Instances: number of environment instances tested. Solved: percentage of instances solved. Steps: mean number of actions taken by an agent navigating between start and end tiles. FSC-BT: Backtracking executor. FSC-RE: Reversing executor. FSC-BT(S) Backtracking SLAM executor. FSC-RE(S) Reversing SLAM executor.

The initial, uninstantiated model for the target solver is listed in Figure 2b. To instantiate this model we use Grid Master's Action Generator module to generate the set of all 8 possible ground actions in the Zero map. We manually create a *generalised* training example where each fluent is left as a non-ground term, except for the map identifier. The resulting MIL Planning Model is listed in Figure 8b.

We train Louise on the MIL Planning Problem listed in Figure 8b and obtain the solver listed in Figure 2a. Inspecting the logic program in Figure 2a should leave no doubt that it is a general procedure for grid map navigation which systematically tries every available direction, recursively, until it reaches a goal state defined by the second argument in each head literal. Nevertheless we test this learned solver on two sets of environments: 100 maze maps with dimensions 100×100; and 10×50 Lake maps (i.e. we generate 50 instances for each of the 10 Lake maps)[6]. Results are listed in the *Experiment 1* rows of Table 2.

5.2 Learning a Model-Free Controller with MIL

The ambiguity illustrated in Figure 4 suggests a large number of diverse environments would be needed to use our solver to train a controller that can solve the same navigation problems as the solver. We observe that the effect of ambiguity is that not all observation labels will be included in FSC behaviours generated by the solver and so a complete mapping between observation and action labels will not be seen in training. Following from this observation we use the set of observation labels we want the target FSC to recognise, to generate 15 training maps of dimensions 3×3, one for each of the 15 observation labels in the target FSC, excluding the impossible *uuuu* (i.e. all-unpassable) label. We call these training maps, illustrated in Figure 9, *Observation Matrices*.

[6] We test fewer Lake map problems because the solver is executed in SWI-Prolog with tabling which avoids looping in plazas but requires a lot of RAM for experiments.

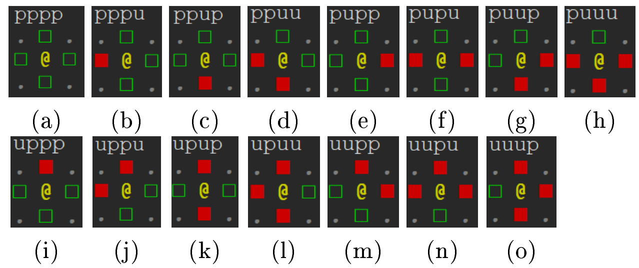


Fig. 9: Observation Matrices for Grid Navigation FSC. Yellow @ sign: agent.

Solving each Observation Matrix with our learned solver, given the model in Table 1, we generate one to four FSC behaviours, each mapping an observation label to the up to four action labels that correspond to it. We illustrate this mapping in supplemental material for lack of space.

We use this set of behaviours, and the controller labels listed in Figure 3a to compose a MIL-learning problem and learn an FSC with Controller Freak, as described in Section 4.1. The controller learned in this way is a non-deterministic FSC comprising 128 4-tuples in a subset of all 960 4-tuples in $Q \times O \times A \times Q$. We list this FSC in supplemental material for lack of space.

We test the learned FSC on the same planning problems as we tested the learned solver, using each of our four executors. Results are listed in the Experiment 2 rows of Table 9.

We make two observations. First, on the MaGe map the Backtracking executor (FSC-BT) returns paths with the same average number of steps as the solver. The solver is executed by DFS with backtracking like the Backtracking executor, so they find the same paths. Essentially the Backtracking executor plans, without a model. Second, the Backtracking-SLAM executor (FSC-BT(S)) takes a long time to solve some Lake maps because it keeps trying long, spiralling paths. We impose a time limit of 300 sec. to this executor only to measure the effect and find that it times out 8% of the time. With all other executors, the FSC learned from the solver, solves the same problems as the solver.

6 Conclusions and Future Work

We have shown how to train a model-based solver and use it to train a model-free controller with MIL. The two agents complement each other in that the solver can plan ahead but needs advance knowledge of an environment whereas the controller doesn't need advance knowledge but can't plan and must explore the environment instead. The solver is learned from a single, generalised example and the controller from 15 examples automatically generated from the controller's observation labels. The controller is a new kind of Nondeterministic Finite State Controller and can solve the same problems as the solver when executed by the right kind of executor. A SLAM-ming executor is needed to avoid loops in environment with open areas. We have focused on solvers and controllers for navigation on grids and have implemented a pair of new Prolog libraries used to learn solvers and controllers and execute them on grid maps.

Limitations and Future Work We have only studied the ability of MIL systems to learn solvers and equivalent FSCs for grid navigation problems. It remains to be seen whether the approach we have described can be further applied to environments that are not grids, and tasks other than navigation. Further, connecting our solver and FSC to a robot's sensors and actors and solving problems in physical, rather than virtual, environments remains to be done.

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A Supplemental I

$(q_0, upup, right, q_1)$.	$(q_1, pppp, right, q_1)$.	$(q_2, puuu, up, q_0)$.	$(q_3, ppuu, up, q_0)$.
$(q_0, upuu, right, q_1)$.	$(q_1, pppu, right, q_1)$.	$(q_2, pppp, right, q_1)$.	$(q_3, pupp, up, q_0)$.
$(q_0, ppup, up, q_0)$.	$(q_0, puuu, up, q_0)$.	$(q_2, pppu, right, q_1)$.	$(q_0, ppuu, right, q_1)$.
$(q_0, pppp, down, q_2)$.	$(q_1, ppup, right, q_1)$.	$(q_2, ppup, right, q_1)$.	$(q_3, pupu, up, q_0)$.
$(q_0, pppu, down, q_2)$.	$(q_1, ppuu, right, q_1)$.	$(q_2, ppuu, right, q_1)$.	$(q_3, puup, up, q_0)$.
$(q_0, pupp, down, q_2)$.	$(q_1, uppp, right, q_1)$.	$(q_2, uppp, right, q_1)$.	$(q_3, puuu, up, q_0)$.
$(q_0, pupu, down, q_2)$.	$(q_1, uppu, right, q_1)$.	$(q_2, uppu, right, q_1)$.	$(q_3, pppp, right, q_1)$.
$(q_0, ppuu, up, q_0)$.	$(q_1, upup, right, q_1)$.	$(q_2, upup, right, q_1)$.	$(q_3, pppu, right, q_1)$.
$(q_0, uppp, down, q_2)$.	$(q_1, upuu, right, q_1)$.	$(q_2, upuu, right, q_1)$.	$(q_3, ppup, right, q_1)$.
$(q_0, uppu, down, q_2)$.	$(q_1, pppp, down, q_2)$.	$(q_2, pppp, down, q_2)$.	$(q_3, ppuu, right, q_1)$.
$(q_0, uupp, down, q_2)$.	$(q_1, pppu, down, q_2)$.	$(q_2, pppu, down, q_2)$.	$(q_3, uppp, right, q_1)$.
$(q_0, uupu, down, q_2)$.	$(q_1, pupp, down, q_2)$.	$(q_2, pupp, down, q_2)$.	$(q_3, uppu, right, q_1)$.
$(q_0, pppp, left, q_3)$.	$(q_1, pupu, down, q_2)$.	$(q_0, pppp, right, q_1)$.	$(q_3, upup, right, q_1)$.
$(q_0, ppup, left, q_3)$.	$(q_1, uppp, down, q_2)$.	$(q_2, pupu, down, q_2)$.	$(q_3, upuu, right, q_1)$.
$(q_0, pupp, up, q_0)$.	$(q_1, uppu, down, q_2)$.	$(q_2, uppp, down, q_2)$.	$(q_3, pppp, down, q_2)$.
$(q_0, pupp, left, q_3)$.	$(q_1, uupp, down, q_2)$.	$(q_2, uppu, down, q_2)$.	$(q_3, pppu, down, q_2)$.
$(q_0, puup, left, q_3)$.	$(q_1, uupu, down, q_2)$.	$(q_2, uupp, down, q_2)$.	$(q_3, pupp, down, q_2)$.
$(q_0, uppp, left, q_3)$.	$(q_1, pppp, left, q_3)$.	$(q_2, uupu, down, q_2)$.	$(q_3, pupu, down, q_2)$.
$(q_0, upup, left, q_3)$.	$(q_1, ppup, left, q_3)$.	$(q_0, pppu, right, q_1)$.	$(q_3, uppp, down, q_2)$.
$(q_0, uupp, left, q_3)$.	$(q_1, pupp, left, q_3)$.	$(q_2, pppp, left, q_3)$.	$(q_3, uppu, down, q_2)$.
$(q_0, pupu, up, q_0)$.	$(q_1, puup, left, q_3)$.	$(q_2, ppup, left, q_3)$.	$(q_3, uupp, down, q_2)$.
$(q_0, uuup, left, q_3)$.	$(q_1, uppp, left, q_3)$.	$(q_2, pupp, left, q_3)$.	$(q_3, uupu, down, q_2)$.
$(q_1, pppp, up, q_0)$.	$(q_1, upup, left, q_3)$.	$(q_2, puup, left, q_3)$.	$(q_3, pppp, left, q_3)$.
$(q_1, pppu, up, q_0)$.	$(q_1, uupp, left, q_3)$.	$(q_2, uppp, left, q_3)$.	$(q_3, ppup, left, q_3)$.
$(q_1, ppup, up, q_0)$.	$(q_1, uuup, left, q_3)$.	$(q_0, pppu, up, q_0)$.	$(q_3, pupp, left, q_3)$.
$(q_1, ppuu, up, q_0)$.	$(q_2, pppp, up, q_0)$.	$(q_0, ppup, right, q_1)$.	$(q_3, puup, left, q_3)$.
$(q_1, pupp, up, q_0)$.	$(q_2, pppu, up, q_0)$.	$(q_2, upup, left, q_3)$.	$(q_3, uppp, left, q_3)$.
$(q_1, pupu, up, q_0)$.	$(q_2, ppup, up, q_0)$.	$(q_2, uupp, left, q_3)$.	$(q_0, uppp, right, q_1)$.
$(q_1, puup, up, q_0)$.	$(q_2, ppuu, up, q_0)$.	$(q_2, uuup, left, q_3)$.	$(q_3, upup, left, q_3)$.
$(q_0, puup, up, q_0)$.	$(q_2, pupp, up, q_0)$.	$(q_3, pppp, up, q_0)$.	$(q_3, uupp, left, q_3)$.
$(q_1, puuu, up, q_0)$.	$(q_2, pupu, up, q_0)$.	$(q_3, pppu, up, q_0)$.	$(q_3, uuup, left, q_3)$.
$(q_0, pppp, up, q_0)$.	$(q_2, puup, up, q_0)$.	$(q_3, ppup, up, q_0)$.	$(q_0, uppu, right, q_1)$.

Table 3: Four-tuples for the Finite State Controller for grid navigation learned in Section 5.2.

B Supplemental II

Observation matrix **FSC Behaviours**

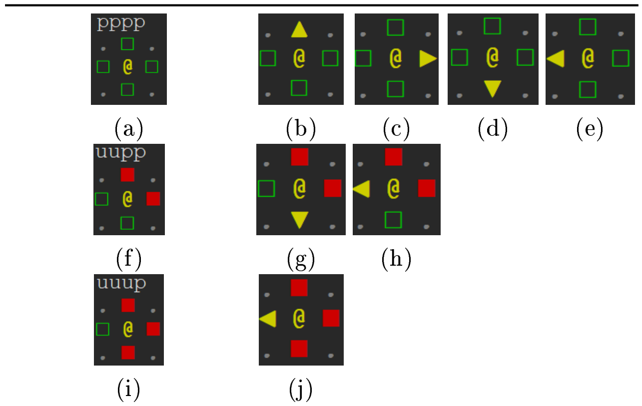

Table 4: Eexamples of FSC behaviours generated from Observation Matrices from Figure 9.

Table 4 illustrates the generation of FSC behaviours from Observation Matrices that are solved by our Grid Navigbation Solver and used to generate examples to train a Grid Navigation FSC, as described in Section 5.2. On the left-most column of the Table, Observation Matrices generated from the FSC observation labels *pppp*, *uupp* and *uuup* are shown. On the three right-hand side columns of the Table are illustrated all FSC behaviours possible to generate in each Observation Matrix. Single-action behaviours suffice to train an FSC for general grid navigation. Thus, in Table columns 4b to 4e, all the one-action FSC behaviours possible in the Observation Matrix 4a are listed. In that Observation Matrix the agent starts in the center cell and can take one step action to each of the four cardinal directions, so four FSC behaviours are generated. By contrast, in Table column 4i the agent can only take one step to the left and so a single FSC behaviour is generated.